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IDZ DISTRIBUTION: PROPERTIES AND APPLICATION

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Abstract

This paper introduces a novel two - parameter continuous distribution. This distribution is derived from the mixture of the Exponential, Weibull and Ailamujia distributions. The derived distribution is named as "Idz distribution". The probability density function of the Idz distribution is derived and some of its plots are presented. It can be observed that the Idz distribution can generate right tailed unimodal, non-monotonic decreasing and exponential shapes. Further, survival and hazard functions of the Idz distribution are derived. It reveals that the hazard function of the Idz distribution can accommodate three types of failure rate behaviors, namely, non-monotonic constant, right tailed unimodal and non-monotonic decreasing. Moreover, some properties of Idz distribution such as moments, mean, variance, moment generating function, order statistics and maximum likelihood estimates are derived. In addition, the proposed distribution is applied into a Breast Cancer data and compare with the Exponentiated Generalized Inverse Rayleigh distribution, the Ailamujia Inverted Weibull distribution and the New Extended Exponentiated Weibull distribution. Result shows that the Idz distribution gives better estimates as compared with the said distributions for a given dataset.

Keywords: Weibull distribution, Exponential distribution, Ailamujia distribution.

1. Introduction

Non-negative continuous probability distribution is important in modelling real lifetime data, specifically, in the field of reliability, engineering and biomedical science. There are popular classical limetime distributions such as the exponential, log-normal, log-logistic, Weibull, Rayleigh and the Frechet distributions. But due to the complexity of the lifetime data, the classical distributions need to generalize or extend in order to cater the complex behaviour of the data. One method for extending the classical distribution is by using the generated family of distributions like the Exponentiated - G family of distributions [7], Marshall-Orkin - G family of distributions [11], Beta - G family of distributions [4] and other existing families of distributions.

Another method of facing the complex behaviour of the lifetime data is by using mixture distribution of two or more probability distribution functions. A random variable X is assumed to have a mixture of two or more probability distribution functions $f_1(x)$, $f_2(x)$, $f_3(x)$,...,$f_n(x)$ if its probability density function $m(x) = \sum_{i=1}^{n} a_i f_i(x)$ with $a_i \in [0,1]$ and $\sum_{i=1}^{n} a_i = 1$. Years ago, several distributions have been derived from the mixing distributions, for example, the Aradhana distribution [13] which is a mixtures of the Gamma (2, θ), the Gamma (3, θ) and the Exponential (θ) distributions with corresponding mixing proportions $\frac{2\theta}{\theta^2+2\theta+2}$, $\frac{2}{\theta^2+2\theta+2}$ and $\frac{\theta^2}{\theta^2+2\theta+2}$. Other identified mixture distributions such as the Rama distribution [14], Darna distribution [15], Shanker distribution [12], Gharaibeh distribution [6], Alzoubi distribution [2] and Benrabia distribution [3].

In this paper, the concept of mixture distribution is used to propose a two - parameter distribution named as Idz distribution which is a mixture of three distributions, namely, the Weibull (λ, β) distribution [17], exponential (λ) distribution [8] and the Ailamujia (λ) distribution [10] with mixing proportions $\frac{\lambda\beta^2}{\lambda\beta^2+\lambda\beta+1}$, $\frac{\lambda\beta}{\lambda\beta^2+\lambda\beta+1}$ and $\frac{1}{\lambda\beta^2+\lambda\beta+1}$, respectively. Other goals of the paper are the following: (i) to derive some properties of Idz distribution such as its moments, moment generating function, mean, variance, order statistics and maximum likelihood estimates of the proposed distribution parameters; and (ii) to apply the proposed distribution into a real dataset and compare with the Exponentiated Generalized Inverse Rayleigh, the Ailamujia Inverted Weibull and the New Extended Exponentiated Weibull distributions.

This paper is arranged as follows: Idz distribution is introduced in section 2. In section 3, some properties of Idz distribution are derived. Order Statistics of the ID distribution is given in section 4 while the maximum likelihood estimates of the ID parameters is presented in section 5. In section 6, the application of Idz distribution is illustrated . Some concluding remarks is presented in section 7.

2. Idz Distribution

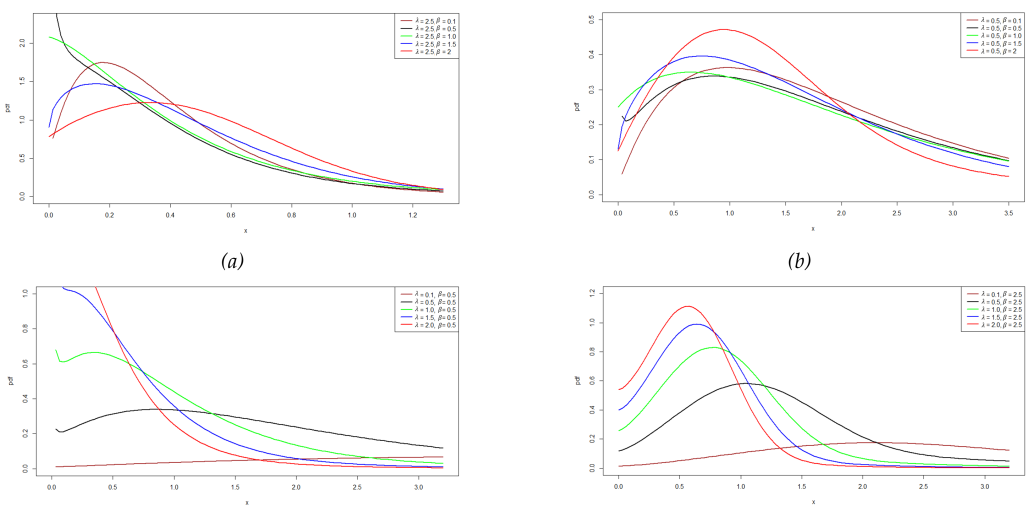
This section presents the definition of the Idz distribution and its special cases with the illustration of its pdf.

A random variable X is said to have an Idz distribution (ID) with parameters λ and β if the probability density function of X is given by

$$f(x,\lambda,\beta) = \frac{\lambda^2 e^{-\lambda x}\left[\beta + 4xe^{-\lambda x} + \beta^3 x^{\beta-1} e^{\lambda(x - x^{\beta})}\right]}{\lambda\beta^2 + \lambda\beta + 1}, \tag{1}$$

where $x \geq 0$, $\lambda > 0$ and $\beta > 0$. The corresponding cumulative distribution function of X is given by

$$F(x,\lambda,\beta) = 1 - \frac{\lambda\beta e^{-\lambda x} + (2\lambda x + 1)\, e^{-2\lambda x} + \lambda\beta^2 e^{-\lambda x^{\beta}}}{\lambda\beta^2 + \lambda\beta + 1}. \tag{2}$$



(a) (b) (c) (d)

Figure 1: *pdf plots of Idz distribution (ID) for different sets of values of the parameters: (a) $\lambda = 2.5$ and varying values of β; (b) $\lambda = 0.5$ and varying values of β; (c) $\beta = 0.5$ and varying values of λ ; and (d) $\beta = 2.5$ and varying values of λ.*

Figure 1 shows some possible density shapes of the ID distribution and it reveals that the pdf of the ID distribution can generate right tailed unimodal, non-monotonic decreasing and exponential shapes.

Special Cases of Idz distribution

1. If $\beta = 1$ then ID reduces to

$$f(x,\lambda) = \frac{2\lambda^2 e^{-\lambda x}(1 + 2xe^{-\lambda x})}{2\lambda + 1}. \tag{3}$$

2. If $\lambda = 1$ then ID reduces to

$$f(x,\beta) = \frac{e^{-x}[\beta + 4xe^{-x} + \beta^3 x^{\beta-1} e^{x-x^\beta}]}{\beta^2 + \beta + 1}. \tag{4}$$

3. If $\beta = 2$ then ID reduces to

$$f(x,\lambda) = \frac{2\lambda^2 e^{-\lambda x}[1 + 2xe^{-\lambda x} + 4xe^{\lambda(x-x^2)}]}{6\lambda + 1}. \tag{5}$$

We name the probability distribution functions (pdf) (3), (4) and (5) as the pdfs of the Edz distribution, Laks distribution and Alds distribution, respectively.

3. Statistical Properties

In this section, we derive some properties of Idz distribution such as its moments, mean, variance, moment generating function, survival fuction and hazard function.

3.1. Moments

Theorem 1. Let X be a random variable that follows an Idz distribution then the rth moment of X denoted by μ'_r is given by

$$\mu'_r = \frac{\lambda^{1-r}}{\lambda\beta^2 + \lambda\beta + 1}\left[\beta\Gamma(r+1) + \frac{\Gamma(r+2)}{\lambda 2^r} + \frac{\beta^2\Gamma\left(\frac{r}{\beta}+1\right)}{\lambda^{\frac{r}{\beta}-r}}\right], \tag{6}$$

where $r = 1, 2, 3, ..., n$ and $\Gamma(\cdot)$ is a gamma function.

Proof. The rth moment of X is defined by

$$\begin{aligned}
\mu'_r &= \mathbb{E}[X^r] \\
&= \int_{-\infty}^{\infty} x^r f(x)dx \\
&= \int_0^{\infty} x^r \frac{\lambda^2 e^{-\lambda x}[\beta + 4xe^{-\lambda x} + \beta^3 x^{\beta-1} e^{\lambda(x-x^\beta)}]}{\lambda\beta^2 + \lambda\beta + 1} dx \\
&= \frac{\lambda^2}{\lambda\beta^2 + \lambda\beta + 1}\left(\beta \int_0^{\infty} x^r e^{-\lambda x} dx + 4\int_0^{\infty} x^{r+1} e^{-2\lambda x} dx + \beta^3 \int_0^{\infty} x^r x^{\beta-1} e^{-\lambda x^\beta} dx\right) \\
&= \frac{\lambda^2}{\lambda\beta^2 + \lambda\beta + 1}\left[\beta\left(\frac{1}{\lambda^{r+1}}\right)\Gamma(r+1) + 4\left(\frac{1}{2\lambda}\right)^{r+2}\Gamma(r+2) + \beta^3\left(\frac{1}{\beta\lambda^{\frac{r}{\beta}+1}}\right)\Gamma\left(\frac{r}{\beta}+1\right)\right] \\
&= \frac{\lambda^{1-r}}{\lambda\beta^2 + \lambda\beta + 1}\left[\beta\Gamma(r+1) + \frac{\Gamma(r+2)}{\lambda 2^r} + \frac{\beta^2\Gamma\left(\frac{r}{\beta}+1\right)}{\lambda^{\frac{r}{\beta}-r}}\right].
\end{aligned}$$

■

Corollary 1. Let X be a random variable with moment given in equation (6) then the mean μ and variance σ^2 of X are, respectively, given by

$$\mu = \frac{1}{c}\left[\beta\Gamma(2) + \frac{\Gamma(3)}{2\lambda} + \beta^2\lambda^{\frac{\beta-1}{\beta}}\Gamma\left(\frac{1}{\beta}+1\right)\right]$$

and

$$\sigma^2 = \frac{1}{c}\left\{\lambda^{-1}\left[\beta\Gamma(3) + \frac{\Gamma(4)}{4\lambda} + \frac{\beta^2\Gamma\left(\frac{2}{\beta}+1\right)}{\lambda^{\frac{2}{\beta}-2}}\right] - \frac{1}{c}\left[\beta\Gamma(2) + \frac{\Gamma(3)}{2\lambda} + \frac{\beta^2\Gamma\left(\frac{1}{\beta}+1\right)}{\lambda^{\frac{1-\beta}{\beta}}}\right]^2\right\},$$

where $c = \lambda\beta^2 + \lambda\beta + 1$.

Proof. The mean of X is derived when $r = 1$ in (6). Hence,

$$\mu = \frac{1}{c}\left[\beta\Gamma(2) + \frac{\Gamma(3)}{2\lambda} + \beta^2\lambda^{\frac{\beta-1}{\beta}}\Gamma\left(\frac{1}{\beta}+1\right)\right],$$

where $c = \lambda\beta^2 + \lambda\beta + 1$. Next, the variance of X denoted by σ^2 can be computed as

$$\sigma^2 = \mu_2' - (\mu_1')^2.$$

Now, the 2nd raw moment μ_2' of X is obtained by setting $r = 2$ in equation (6). It follows that

$$\mu_2' = \frac{\lambda^{-1}}{c}\left[\beta\Gamma(3) + \frac{\Gamma(4)}{4\lambda} + \frac{\beta^2\Gamma\left(\frac{2}{\beta}+1\right)}{\lambda^{\frac{2}{\beta}-2}}\right].$$

Therefore, the variance σ^2 of X is

$$\begin{aligned}\sigma^2 &= \frac{\lambda^{-1}}{c}\left[\beta\Gamma(3) + \frac{\Gamma(4)}{4\lambda} + \frac{\beta^2\Gamma\left(\frac{2}{\beta}+1\right)}{\lambda^{\frac{2}{\beta}-2}}\right] - \left\{\frac{1}{c}\left[\beta\Gamma(2) + \frac{\Gamma(3)}{2\lambda} + \beta^2\lambda^{\frac{\beta-1}{\beta}}\Gamma\left(\frac{1}{\beta}+1\right)\right]\right\}^2 \\ &= \frac{1}{c}\left\{\lambda^{-1}\left[\beta\Gamma(3) + \frac{\Gamma(4)}{4\lambda} + \frac{\beta^2\Gamma\left(\frac{2}{\beta}+1\right)}{\lambda^{\frac{2}{\beta}-2}}\right] - \frac{1}{c}\left[\beta\Gamma(2) + \frac{\Gamma(3)}{2\lambda} + \frac{\beta^2\Gamma\left(\frac{1}{\beta}+1\right)}{\lambda^{\frac{1-\beta}{\beta}}}\right]^2\right\}.\end{aligned}$$

■

3.2. Moment Generating Function

Theorem 2. Let X be a random variable that follows an Idz distribution then the moment generating function of X is given by

$$M_x(t) = \sum_{r=0}^{\infty}\frac{t^r\lambda^{1-r}}{(\lambda\beta^2 + \lambda\beta + 1)\,r!}\left[\beta\Gamma(r+1) + \frac{\Gamma(r+2)}{\lambda 2^r} + \frac{\beta^2\Gamma\left(\frac{r}{\beta}+1\right)}{\lambda^{\frac{r}{\beta}-r}}\right],$$

where $t \in \mathbb{R}$.

Proof. The moment generating function of X is defined by

$$M_X(t) = \mathbb{E}(e^{tX}) = \int_{-\infty}^{\infty} e^{tx} f_X(x)dx.$$

Using equation (1), we have

$$M_X(t) = \int_0^{\infty} e^{tx} f(x, \lambda, \beta)dx.$$

Recall that $e^{tx} = \sum_{r=0}^{\infty} \frac{t^r}{r!} x^r$. Then,

$$M_X(t) = \int_0^{\infty} \sum_{r=0}^{\infty} \frac{t^r}{r!} x^r f(x,\lambda,\beta)dx = \sum_{r=0}^{\infty} \frac{t^r}{r!} \int_0^{\infty} x^r f(x,\lambda,\beta)dx = \sum_{r=0}^{\infty} \frac{t^r}{r!} \mu_r'.$$

Using equation (6) and hence,

$$M_x(t) = \sum_{r=0}^{\infty} \frac{t^r \lambda^{1-r}}{(\lambda\beta^2 + \lambda\beta + 1)\, r!} \left[\beta\Gamma(r+1) + \frac{\Gamma(r+2)}{\lambda 2^r} + \frac{\beta^2 \Gamma\left(\frac{r}{\beta}+1\right)}{\lambda^{\frac{r}{\beta}-r}} \right],$$

where $t \in \mathbb{R}$. ■

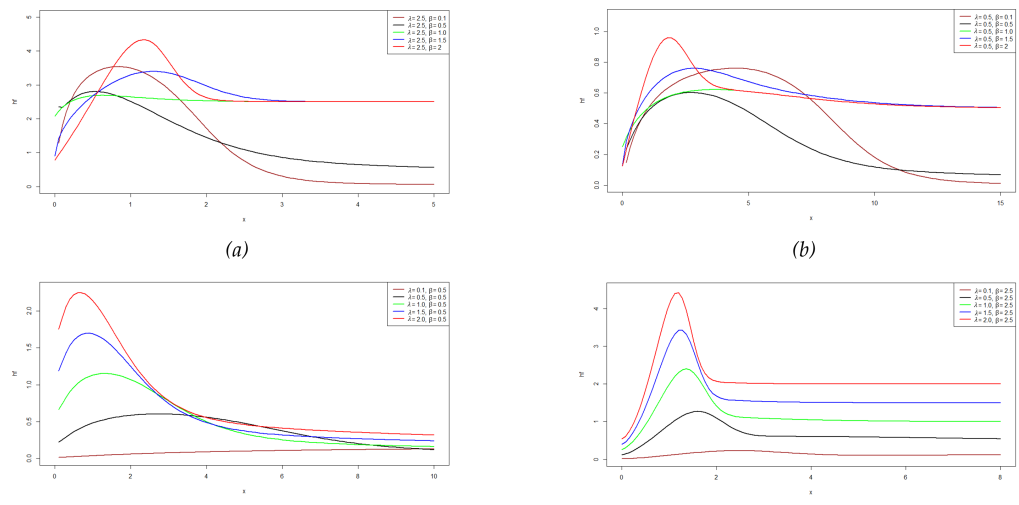
3.3. Reliability Analysis

Let X be a random variable with cdf (2) and pdf (1) then the survival $S(x,\lambda,\beta)$ and hazard $h(x,\lambda,\beta)$ functions of X are respectively, given by

$$S(x,\lambda,\beta) = \frac{\lambda\beta e^{-\lambda x} + (2\lambda x+1)\, e^{-2\lambda x} + \lambda\beta^2 e^{-\lambda x^{\beta}}}{\lambda\beta^2 + \lambda\beta + 1}, x \geq 0, \lambda > 0, \beta > 0$$

and

$$h(x,\lambda,\beta) = \frac{\lambda^2[\beta + 4xe^{-\lambda x} + \beta^3 x^{\beta-1} e^{\lambda(x-x^{\beta})}]}{\lambda\beta + (2\lambda x+1)\, e^{-\lambda x} + \lambda\beta^2 e^{-\lambda\left(x-x^{\beta}\right)}}.$$



(a) (b) (c) (d)

Figure 2: *hf plots of Idz distribution (ID) for different sets of values of the parameters: (a) $\lambda = 2.5$ and varying values of β; (b) $\lambda = 0.5$ and varying values of β; (c) $\beta = 0.5$ and varying values of λ ; and (d) $\beta = 2.5$ and varying values of λ.*

Figure 2 presents some possible shapes of the hazard function of the ID distribution and it reveals that the hazard function of the ID distribution can accommodate non-monotonic constant, right tailed unimodal and non-monotonic decreasing behaviors.

4. Order Statistics

Let $X_{(1)}$, $X_{(2)}$,..., $X_{(n)}$ be the order statistics of a random sample X_1, X_2,..., X_n drawn from the continuous population with probability density function (pdf) $f_X(x)$ and cumulative distribution function $F_X(x)$, then the pdf of rth order statistics $X_{(r)}$ is given by

$$f_{X_{(r)}}(x) = \frac{n!}{(r-1)!(n-r)!} f_X(x)[F_X(x)]^{r-1}[1-F_X(x)]^{n-r}. \tag{7}$$

The pdf of rth order statistics $X_{(r)}$ of the ID distribution is derived by inserting (2) and (1) into (7) and is

$$\begin{aligned} f_{X_{(r)}}(x,\lambda,\beta) =& \frac{n!\lambda^2 e^{-\lambda x}[\beta + 4xe^{-\lambda x} + \beta^3 x^{\beta-1} e^{\lambda(x-x^\beta)}]}{(r-1)!(n-r)!\,(\lambda\beta^2+\lambda\beta+1)} \\ & \left[1 - \frac{\lambda\beta e^{-\lambda x} + (2\lambda x+1)\, e^{-2\lambda x} + \lambda\beta^2 e^{-\lambda x^\beta}}{\lambda\beta^2+\lambda\beta+1}\right]^{r-1} \\ & \left[\frac{\lambda\beta e^{-\lambda x} + (2\lambda x+1)\, e^{-2\lambda x} + \lambda\beta^2 e^{-\lambda x^\beta}}{\lambda\beta^2+\lambda\beta+1}\right]^{n-r}. \end{aligned} \tag{8}$$

The pdf of the smallest or $1st$ order statistics of the ID distribution is obtained by setting $r = 1$ in equation (8) and is

$$\begin{aligned} f_{X_{(1)}}(x,\lambda,\beta) =& \frac{n!\lambda^2 e^{-\lambda x}[\beta + 4xe^{-\lambda x} + \beta^3 x^{\beta-1} e^{\lambda(x-x^\beta)}]}{(n-1)!\,(\lambda\beta^2+\lambda\beta+1)^n} \\ & \left[\lambda\beta e^{-\lambda x} + (2\lambda x+1)\, e^{-2\lambda x} + \lambda\beta^2 e^{-\lambda x^\beta}\right]^{n-1}. \end{aligned}$$

If $r = n$ then the pdf of the nth or largest order statistics of ID distribution is given by

$$\begin{aligned} f_{X_{(n)}}(x,\lambda,\beta) =& \frac{n!\lambda^2 e^{-\lambda x}[\beta + 4xe^{-\lambda x} + \beta^3 x^{\beta-1} e^{\lambda(x-x^\beta)}]}{(n-1)!\,(\lambda\beta^2+\lambda\beta+1)} \\ & \left[1 - \frac{\lambda\beta e^{-\lambda x} + (2\lambda x+1)\, e^{-2\lambda x} + \lambda\beta^2 e^{-\lambda x^\beta}}{\lambda\beta^2+\lambda\beta+1}\right]^{n-1}. \end{aligned}$$

5. Maximum Likelihood Estimation

Let X_1, X_2,..., X_n be a random sample of size n from Idz distribution (ID). Then the likelihood function of ID is given by

$$L = \prod_{i=1}^{n} \frac{\lambda^2 e^{-\lambda x_i}[\beta + 4x_i e^{-\lambda x_i} + \beta^3 x_i^{\beta-1} e^{\lambda(x_i - x_i^\beta)}]}{\lambda\beta^2+\lambda\beta+1}. \tag{9}$$

Then, the log-likelihood function of ID is

$$l = 2n\log(\lambda) - \lambda\sum_{i=1}^{n} x_i + \sum_{i=1}^{n}\log[\beta - 4x_i e^{-\lambda x_i} + \beta^3 x_i^{\beta-1} e^{\lambda(x_i - x_i^\beta)}] - n\log(\lambda\beta^2+\lambda\beta+1). \tag{10}$$

The partial derivatives of (10) with respect to parameters β and λ are presented as follow:

$$\frac{\partial l}{\partial\beta} = \sum_{i=1}^{n} \frac{1 + \beta^3 x_i^{\beta-1}\left[\frac{3}{\beta} + \left(1 - \lambda x_i^\beta\right)\log(x_i)\right] e^{\lambda\left(x_i - x_i^\beta\right)}}{\beta - 4x_i e^{-\lambda x_i} + \beta^3 x_i^{\beta-1} e^{\lambda\left(x_i - x_i^\beta\right)}} - \frac{n\lambda(2\beta+1)}{\lambda\beta^2+\lambda\beta+1}; \tag{11}$$

and

$$\frac{\partial l}{\partial\lambda} = \frac{2n}{\lambda} - \sum_{i=1}^{n} x_i + \sum_{i=1}^{n} \frac{\beta^3 x_i^{\beta-1}\left(x_i - x_i^\beta\right) e^{\lambda\left(x_i - x_i^\beta\right)} - x_i e^{-\lambda x_i}}{\beta - 4x_i e^{-\lambda x_i} + \beta^3 x_i^{\beta-1} e^{\lambda\left(x_i - x_i^\beta\right)}} - \frac{n\beta(\beta+1)}{\lambda\beta^2+\lambda\beta+1}. \tag{12}$$

The maximum likelihood estimates of the parameters β and λ of Idz distribution can be computed by setting equations (11) and (12) equal to zero. This can be done by using any numerical method like the Newton-Raphson iterative method.

6. Application

This section presents the application of Idz distribution to a medical dataset. In this application, we use breast cancer data from Lee [9]. This dataset is taken from a large hospital in a period from 1929 to 1938 and it represents the survival times of 121 patients with breast cancer. The observations are given as follow: 0.3, 0.3, 4.0, 5.0, 5.6, 6.2, 6.3, 6.6, 6.8, 7.4, 7.5, 8.4, 8.4, 10.3, 11.0, 11.8, 12.2, 12.3, 13.5, 14.4, 14.4, 14.8, 15.5, 15.7, 16.2, 16.3, 16.5, 16.8, 17.2, 17.3, 17.5, 17.9, 19.8, 20.4, 20.9, 21.0, 21.0, 21.1, 23.0, 23.4, 23.6, 24.0, 24.0, 27.9, 28.2, 29.1, 30.0, 31.0, 31.0, 32.0, 35.0, 35.0, 37.0, 37.0, 37.0, 38.0, 38.0, 38.0, 39.0, 39.0, 40.0, 40.0, 40.0, 41.0, 41.0, 41.0, 42.0, 43.0, 43.0, 43.0, 44.0, 45.0, 45.0, 46.0, 46.0, 47.0, 48.0, 49.0, 51.0, 51.0, 51.0, 52.0, 54.0, 55.0, 56.0, 57.0, 58.0, 59.0, 60.0, 60.0, 60.0, 61.0, 62.0, 65.0, 65.0, 67.0, 67.0, 68.0, 69.0, 78.0, 80.0, 83.0, 88.0, 89.0, 90.0, 93.0, 96.0, 103.0, 105.0, 109.0, 109.0, 111.0, 115.0, 117.0, 125.0, 126.0, 127.0, 129.0, 129.0, 139.0 and 154.0.

Fatima [5] used the above dataset for their proposed model named as the Exponentiated Generalized Inverse Rayleigh distribution (EGIR) and compared with the Exponentiated Inverse Rayleigh (EIR), the Generalized Inverse Rayleigh (GIR) and the Inverse Rayleigh (IR) distributions. They found that the EGIR had the best fit for the Breast Cancer dataset.

Here, we compare the proposed distribution with the EGIR, the Ailamujia Inverted Weibull distribution (AIW) [16] and the New Extended Exponentiated Weibull distribution (NEEW) [1]. The probability density functions of EGIR, AIW and NEEW are given as follow:

$$f_{EGIR}(x) = 2\frac{\alpha\gamma}{\lambda^2 x^3} e^{-(\lambda x)^{-2}} \left(1 - e^{-(\lambda x)^{-2}}\right)^{\alpha-1} \left[1 - \left(1 - e^{-(\lambda x)^{-2}}\right)^{\alpha}\right]^{\gamma-1},\ x > 0, \alpha, \lambda, \gamma > 0;$$

$$f_{AIW}(x) = 4\alpha\theta^2 x^{-2\alpha-1} e^{-2\theta x^{-\alpha}},\ x > 0, \theta, \alpha > 0;$$

and

$$f_{NEEW}(x) = \frac{\alpha\lambda x^{\lambda-1} e^{-\alpha x^{\lambda}} (1 - e^{-\alpha x^{\lambda}}) \left[e^{\theta(1-e^{\alpha x^{\lambda}})}(2 + \theta - \theta e^{-\alpha x^{\lambda}}) + 2\right]}{e^{\theta} + 1},\ x \geq 0, \alpha, \lambda, \theta > 0.$$

In this application, we use the following diagnostics statistics: (i) Akaike Information Criterion (AIC); (ii) Bayesian Information Criterion (BIC); (iii) Kolmogorov - Smirnov (K-S); (iv) Cramer - von Mises (W^*); and (v) Anderson - Darling (A). Furthermore, a package "fitdistrplus" in R software is also used. In addition, the results are shown in the following tables. Table 1 presents the maximum likelihood estimates of the fitted models for Breast Cancer dataset while Table 2 indicates that Idz distribution gives better estimate for the given dataset since it has a smallest values of some diagnostics statistics as compared with the EGIR, AIW and NEEW distributions. Also, same result is noticed from Figure 3.

Table 1: *ML estimates of the fitted models using different distributions*

Distribution	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\theta}$	$\hat{\lambda}$	$\hat{\gamma}$
ID		1.68335961		0.02058629	
EGIR	0.3331558			5986.6441628	2239.7157204
AIW	0.5159137		4.8684926		
NEEW	0.1099727		1.7984615	0.7582980	

Table 2: *Some diagnostic statistics of the fitted models using different distributions*

Distribution	AIC	BIC	$K-S$	A	W^*
ID	1164.159	1169.751	0.05341806	0.51366194	0.06178559
EGIR	1279.365	1287.753	0.2180867	9.7828973	1.5945699
AIW	1250.729	1256.321	0.1723414	7.0402261	1.1024897
NEEW	1167.178	1175.565	0.07776119	0.45898966	0.06532455

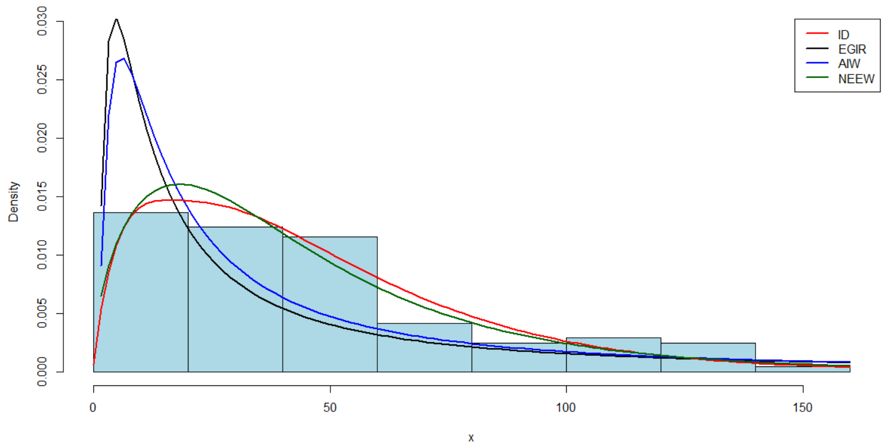


Figure 3: *Estimated pdf of the fitted models for the Breast Cancer dataset.*

7. Concluding Remarks

This paper derives a novel two - parameter continuous distribution called as Idz distribution. Some properties of Idz distribution such as moments, mean, variance, moment generating function, survival function, hazard function and order statistics were derived. Maximum likelihood method was used to estimate the parameters of Idz distribution. The applicability of the proposed distribution was examined by applying into a breast cancer data and compared with the Exponentiated Generalized Inverse Rayleigh (EGIR), the Ailamujia Inverted Weibull (AIW) and the New Extended Exponentiated Weibull (NEEW) distributions. It was found that the Idz distribution provides better fit for the given dataset as compared with the said distributions.

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