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A LITERATURE REVIEW ON DISCRETE-TIME QUEUEING MODELS

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Abstract

In this paper, a quantitative research survey is carried out on discrete-time queueing models. In real-life scenarios, the idea of discrete-time queues has taken on a new meaning. This survey mainly focuses on the unfolding of discrete-time queueing models in recent decades, challenges implied on them and their influence in various fields. The ultimate goal of this paper is to provide enough information to all the researchers and analysts who toil in this field and wish to know more about these models. A few open issues and intriguing future research paths has been discussed.

Keywords: Discrete-time queues, $Geo/G/1$ and $GI/Geo/1$ queues, $Geo/G/1$ retrial queues, Discrete-time retrial queues.

1. Introduction

Waiting in a queue has been a part of our day-to-day lives because, as a process, it has various significant purposes. Queueing theory analyses the entire structure of waiting in a queue. Erlang was a pioneer in the discipline of queueing theory in the early 20^{th} century, and now it has become a well-established research area in the past few decades. Initially, the research was done with the succor of continuous-time (CT) queueing models. Yet in the contemporary period, there has been a recognisable fluctuation from CT to discrete-time (DT) queueing systems (QS) and their counterparts as well. This change is primarily because of the noticeable relevance of DT queues in the fields of communication and computer systems, where the digital information is disseminated in the mould of fixed-length "packets," each of which requires a fixed-length transmission time known as slots. Events are constrained to take place during these slots. A DT queue, for example, might accept and serve at most one packet during a slot. Multiple events may occur during each slot on a network-wide basis. Herwig Bruneel [25] analysed a general single-server DT queueing model and its major principal performance measures were procured with the succor of a unifying analysis. Also, numerous fundamental relationships among the main quantitative measurements of a QS in general were derived, which was mainly helpful in studying the queueing phenomena, especially in the discipline of computerized messaging sectors. Many researchers then referred to books by Bruneel and Kim [24], Woodward [94], Takagi [72] and Miyazawa and Takagi [60], which helped them learn more about DT queueing models.

Due to network complexity and an expanding customer base, consumer behaviour and the retry phenomenon may have had a substantial impact on how well a system performs. Consumers who show up, discover the service completely occupied and hesitant to stay, choose to leave momentarily, but return at a subsequent period. Before retrying to occupy a server, such consumers are supposed to be in orbit, a virtual waiting room. Retrial queues (RQ) are frequently used to mimic these situations. A basic RQ model is presented in Fig. 1.

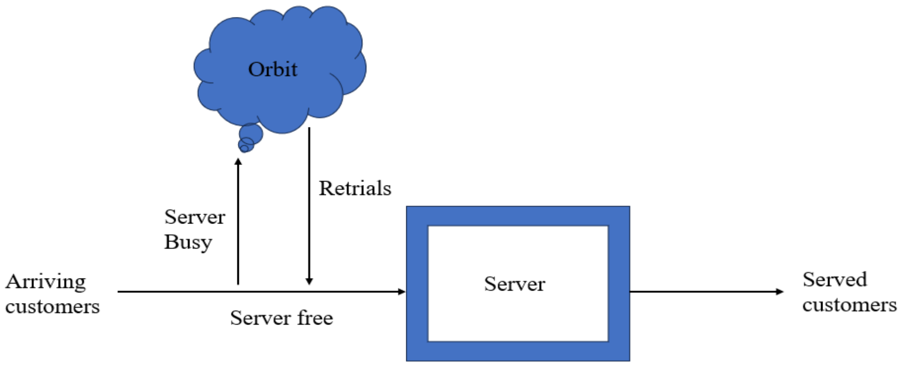

Figure 1: *A basic retrial queueing system*

To illustrate, if a consumer attempts to contact a call centre for assistance while the agents are busy, the consumers are likely to call at an inconvenient hour. This modelling is utilised in assorted fields, including call centres, cellular networks, and LAN. Meanwhile, in the recent two decades, researchers have begun to focus on DT retrial models. Rein Nobel [63] provided a high-level overview of several models for discrete-time late arrival retrial queues and demonstrated the importance of striking a balance between customers' active and passive roles in order to achieve optimal performance. Various DT retrial QS were investigated, and many results were established, which have been used in solving numerous issues in telecommunication networks, telephone switching sectors, computer systems, and networks in the past two decades.

A brief survey of various DT queueing models and DT retrial queueing models have been conducted. Readers who practice in these fields will find it quite useful. The remaining portion of this paper has been organised in the following manner: In Section 2, DT queueing models are reviewed. Section 3 presents the work on DT $Geo/G/1$ and $GI/Geo/1$ queues. DT $Geo/G/1$ retrial queues have been examined in Section 4. Section 5 reviews some of the other DT retrial queueing models. Section 6 details the difficulties in using the DT concepts, while Section 7 describes the methods employed in DT queueing models. Few distinct applications of DT queues are addressed in Section 8. Section 9 presents a few recent advancements in DT queues. Finally, the future scope and some open problems and the concluding remarks of the review are offered in Sections 10 and 11 respectively.

2. Discrete-time Queueing Models

In queueing theory, initially appearing only in scientific literature, DT queueing models in recent decades have been well-proportioned. It has largely been driven by potential applications that utilize slotted time. By analysing the proficiency of DT multi-server queues with priorities, K. Laevens and H. Bruneel [41] derived basic formulae to reckon some key metrics like variances, mean values, and tail probabilities. Dieter Fiems et al. [30] proved that DT single-server queue subjugated to server interruptions was able to reckon the proficiency of low-priority traffic in a two-priority Head-of-Line scheduling discipline. The issues of channel errors on wireless ATM multiplexers were investigated by Mehmet Ali et al. [59] by analysing the functioning of the DT queue with countless buffer. Dieter Fiems et al.[31] again studied a discrete $GI/G/1$ QS subjugated to server interruptions generated by a renewal process and further showed that this system can reckon the proficiency of a multi-class priority scheduling system. M. Mehmet Ali and X. Song [58] examined DT priority queues with correlated arrivals (CA) (i.e.with binary Markov sources) and derived a closed form of expression for the probability generating function (PGF) of queue length and finally extended the results to multiple priority queues. Alfa [4] presented an informative work about utilizing a matrix-analytic approach to study various queues in DT.

Jinting Wang et al. [84] analysed the DT correlated on-off source QS with negative customers (NC) and demonstrated that this system can simulate loss in the radio interface of HSDPA systems. Also, they came to the conclusion that the impression of NC should be acknowledged in pursuance of the wireless communication networks in an error-prone transmission environment. I. Atencia and A. V. Pechinkin [20] studied a DT QS with optional LCFS priority discipline and examined most of its principal characteristics. An $MMBP/Geo/1$ DT queue has been scrutinized by Jinting Wang et al. [85] with correlated positive and NC arrivals and its mean buffer content and the stationary probabilities have been determined. Ultimately, they concluded that when time slots with persistent size are defined, this queue will be supportive in analysing the radio link layer of HSDPA systems. I. Atencia [11] after analysing a DT QS with the customers arriving where the system follows LCFS discipline or to enroll in the queue where servers may fail and yet be repaired, proposed that the study of the server breakdowns (SB) can be brought into a realistic approach by reckoning with the modifications in the renewal times. With the succor of the Markovian approach, A. Senthil Vadivu et al. [80] scrutinized a DT infinite capacity QS with an on-off source and service time with two states of server and NC. There has been a rise in the popularity of non-deterministic service times in DT models because of the more intricate and erratic service mechanisms in today’s communications networks. As a result, Peixia Gao et al. [34] devised a generating-functions method for analyzing a DT multi-server QS with geometrically distributed service times. In the analysis of a finite buffer DT multiple working vacation queue with impatient clients and congestion-dependent service rates, Jyothsna et al. [37] determined the steady-state system length distributions via a matrix technique and give a recursive solution. From the recursive solution of the DT queue, the corresponding CT queue is further calculated.

Recently, F. Verdonck et al. [82] focused on a DT multi-server QS with varying server availability by introducing server interruptions and a CA process. They also assumed the QS to alternate between two distinct dynamic characteristics having a random cluster of servers and determined the allocation of the system content at several observation points. The repercussions of assorted forms of variance on the setup behavior was also investigated and finally, they concluded that this paradigm is suitable for many real-life applications. Jens Baetens et al. [22] investigated a variable-capacity DT two-class batch-service queue in which all clients wait in a single queue and are served in the order in which they arrived and moreover clients of the same class are the only ones the server can service, hence the number of clients in a batch is equal to the number of successive clients of the same class. Using a supplementary variable technique (SVT), the repercussions of disasters on the processing of a DT single-server QS were studied by Mustafa Demircioglu et al. [29] and they concluded that the mean system content reduces with increasing irregularity for the QS with disasters. In order to evoke retransmission in communication packet networks, Kempa [38] has studied the time-dependent queue behavior (i.e., transient state) of a DT queue with a finite buffer and feedback. Larisa and Andrey [45] studied a discrete-time queueing system with a regenerative input flow and heterogeneous servers that may have independent interruptions, where the subsequent moments of breakdowns are specified by a renewal mechanism and are independent of the system state.

Alfa [5] introduced a new way to analyze the $GI/G/1/K$ system via the matrix-analytical method. The novelties stem from the Markov-based depiction of the arrival and service processes, at least one of which is based on the elapsed time approach, and the unusual organization of the state space. The packet loss distribution of a finite buffer single server queue subject to a DT batch Markovian arrival process (DBMAP) and a DT Markovian service process (DMSP)is investigated through a matrix-analytical method by Yung-Chung Wang et al. [90]. Using statistical measures, we will analyze the sporadic behavior of packet loss in terms of both loss period and loss distance. Using a DT $GI^{[X]}/G^{[Y]}/1$ queueing model, Alfa and He [6] investigated an algorithmic strategy based on the matrix-analytic approach. They also introduced and studied the Markov chain that underlies the ageing of service-receiving clients.

3. Discrete-time $Geo/G/1$ and $GI/Geo/1$ Queueing Models

Zhe George Zhang and Naishuo Tian [101] scrutinized the DT $Geo/G/1$ system with collective adjustable vacations and showed that the optimal control issue of vacation policies can be discussed with the succor of this paradigm. Again, a DT $GI/Geo/1$ queue with server vacations was analysed by Naishuo Tian and Zhe George Zhang [74]. The stationary distributions and the waiting times of the customers were procured by using the matrix-geometric method (MGM). Also, this model can be stretched to a vacation with a finite maximum value that is quite widely distributed. Ji-Hong Li and Nai-Shuo Tian [49] examined a DT $GI/Geo/1$ queue with working vacations (WV) and vacation interruption and showed that this QS can be helpful in modelling few realistic obstacles in communication networks and computers. Ji-Hong Li et al.[50] extended a $GI/M/1$ queue with WV to a DT $GI/Geo/1$ queue with umpteen WV, which was further discussed using MGM. For negative binomial distributions, the closed property of conditional probability was also determined and verified.

A DT $Geo^{[X]}/G/1$ queue with an uncertain server and collective adaptive delayed vacation policies was scrutinized by Yinghui Tang et al. [73] where the transient and steady-state distributions of the queue length were obtained and the stochastic decomposition (SD) property of the steady-state queue length was demonstrated. It is feasible to obtain a connection amidst the generating functions of steady-state queue length at departure epoch and an arbitrary epoch.The study of reliability problems became significant since the QS is unreliable. Furthermore, some key metrics were employed to assess the repercussions of diverse specifications on numerous functioning characteristics. A DT $Geo/G/1$ QS with a random threshold policy, namely a (p, N) policy, was analysed by Tsung-Yin Wang and Jau-Chuan Ke [91] and with reference to p and N, they conferred the convex combination of the QS characteristics. A randomised vacation policy was pioneered by Tsung-Yin Wang et al. [92] for a DT $Geo/G/1$ QS that takes at most J vacations. Using the generating function technique, few quantitative measurements of the QS were derived, and this model has great relevance in practical systems as well. The estimation of M/G/1 CT QS with D-policy from a DT $Geo/G/1$ system with D-policy was probed by Se Won Lee et al. [46]. Moreover, the repercussions of threshold D were also studied. Doo Ho Lee and Won Seok Yang [47] were the first ones to look into the conjuction of the disaster phenomenon and the N-policy under DT on $Geo/G/1$ QS and concluded that this could be useful to minimise power consumption in wireless sensor networks (WSNs). At the same time it also help network engineers to design and operate WSNs. Renbin Liu and Zhaohui Deng [53] examined a DT N-policy $Geo/G/1$ QS with repairable server and feedback, and the steady-state system size distribution (SSD) was derived by renewal process theory and probability analysis whose application is applied in a network access proxy system.

Jianjun Li and Liwei Liu [48] scrutinized a DT $Geo/G/1$ QS with vacations in a random environment and obtained the PGF using SVT and showed that it can be estimated to $M/G/1$ QS. A more realistic approach to real life problems was offered by the DT $Geo/G/1/$ QS with changes in vacation times, which was analysed by Ivan Atencia [12]. A state that occurs in a computer network centre with virus infection was investigated and compared with a DT $Geo/G/1$ QS with single vacation and disaster arrivals by S. Jeyakumar and P. Gunasekaran [35] and also the PGF of the queue length and the mean queue length of the QS were derived, which have proven to be valuable in a variety of real-world settings. Shaojun Lan and Yinghui Tang [42] were the initial ones to analyse the DT $Geo/G/1$ QS Bernoulli feedback, modified multiple vacations, and N-policy. The customers waiting time has been reduced, the system's queue length has been controlled, moreover the switching costs of the QS has been largely economised by this QS. Finally, to investigate the cost optimization problem, a cost structure was also established. F. P. Barbhuiya and U. C. Gupta [23] studied a DT batch arrival $GI/Geo/1$ QS with numerous WV. Using SVT and the shift operator method, the closed-form expressions of steady-state system content distribution have been derived, and the impression of assorted specification on the functionality of the QS have also been conferred. A repairable DT $Geo/G/1$ QS with tragedies and working breakdown was analysed by Shan Gao et al. [33] which contributes to a more practical service schedule for the growth of DT queues with tragedies and breakdowns. Futhermore, a manufacturing system can be feasibly modelled using this. Recently, Vaishnawi [81] looked at the accuracy of a DT $Geo^{[X]}/G/1$ recurrent model with Bernoulli feedback and two distinct forms of vacations.

3.1. Discrete Markovian Queueing models

Bara Kim and Jeongsim Kim [40] investigated a DT batch Markovian arrival process $(D-BMAP)/G/1$ RQ and found a light-tailed asymptotic distribution for the amount of consumers at embedded epochs. Jesus R. Artalejo and Quan-Lin Li [9] introduced an efficient generalisation of the DT Markovian arrival process (D-MAP) by analysing an RQ that follows a new discrete block state-dependent arrival (D-BSDA) distribution. Also, this model aids in creating more sophisticated models. Matrix analysis and the embedded Markov chain technique were used to determine the joint state probabilities at different epochs (arbitrary, pre-arrival, and departure) in Yu and Alfa [99] study of a DT single-server finite-buffer queue with a Markovian arrival process. Nandi and Samanta [86] provided a consolidated view of a system subject to shocks following the rules of a D-MAP, a stochastic model in which the arrival timings of events rely on one another. The results of this research can be used to real-world issues that manufacturing engineers encounter, improving a certain class of stochastic systems in which the shock magnitudes are isolated. By re-blocking the transition probability matrix in its desired $GI/G/1$ structure with rectangular boundary blocks, Das and Samanta [27] was able to determine the queue-length distribution at pre-arrival epochs for both the late arrival system with delayed access and the early arrival system of an $GI^{[X]}/DMSP^{(a,b)}/1$ queue using the RG-factorization method based on censoring technique. Similarly, Samanta and Das [68] examined $DBMAP/DBMSP/1$ queue by re-blocking the transition probability matrix to the desired $M/G/1$ structure via a censoring methodology-based UL-type RG-factorization approach in order to determine the stationary probability vectors at an external observer's epoch for a fixed-size batch service queue via a matrix-analytic approach. Samanta and Nandi [69] examines a single-server, infinite-buffer queueing system in which customers arrive in varying-sized groups in DT. The pre-arrival epoch probabilities are calculated using the UL-type RG-factorization for the Toeplitz type block-structured Markov chain, which is based on the censoring method. A renewal-input N-policy DT $GI/D-MSP/1/\infty$ QS was again studied by Samanta and Nandi [70]. The pre-arrival epoch duration distribution of the system is calculated using a matrix-geometric approach. We use the Markov renewal theory to calculate the random epoch distribution of system lengths. The optimal value of N is found by minimizing the expected cost function, which is assumed to be linear in terms of units of time.

3.2. Discrete-time $Geo/Geo/1$ Queueing Models

The enhancement of queueing theory on NC and disasters to the DT $Geo/Geo/1$ QS was probed by Ivan Atencia and P. Moreno [19] and some of the key metrics of the QS have been found along with its ergodicity condition. A restricted source DT $Geo/Geo/1$ QS with disasters was analysed by F. Jolai et al. [36] that has been used to design an email contact center plus with the succor of disasters, a clearing performance in a real-life source system can also be modelled. R. Sudhesh and R. Sebasthi Priya [71] investigated a DT $Geo/Geo/1$ QS with repair, feedback, and disaster. Busy-period distribution was determined in reference to Catalan numbers, and quantitative measurements like reliability and availability were determined, on top of some key metrics for steady and transient state system-size probabilities. Lee and Ke [52] constructed and studies the steady-state solutions of a $Geo/Geo/1$ queue under the QBD (quasi-birth-death) model, which accounts for server failures and reboots. In this case, the QBD model is level-independent up to a fixed threshold but level-dependent indefinitely above that. Cramer's rule, which goes along with QR factorization, can be used to solve the level-dependent structure of the suggested model. For this reason, they first calculated the states' probabilities using the finite level-independent technique, and then switch to the algorithm of infinite level-dependent QBDs. Artalejo et al. [8] investigated the performance metrics of a $Geo/Geo/c$ type DT queue with geometric repeated attempts and provided many algorithmic approaches for their efficient computation.

4. Discrete-time $Geo/G/1$ Retrial Queueing Models

Tao Yang and Hui Li [97] were the pioneer to peruse the steady-state distribution of the DT $Geo/G/1$ RQ and showed that the SD law holds for this QS. Also, they proved that this QS can be ultilized to imprecise the CT $M/G/1$ RQ. Not only that, but also, this model has great applications in communication and computer systems where consumer retrials are a conventional phenomenon. A DT $Geo_1, Geo_2/G/1$ RQ with two kind of calls was studied by B. D. Choi and J. W. Kim [26], which seems to have applicability in telephone switching systems and mobile cellular systems. The association between a CT $M/G/1$ RQ with a Bernoulli schedule and its DT counterpart was established by Hui Li and Tao Yang [51] by analysing a DT $Geo/G/1$ RQ with a Bernoulli schedule. I. Atencia and P.Moreno [18] studied a DT $Geo^{[X]}/G_H/1$ RQ with Bernoulli feedback and demonstrated that this QS is sometimes utilized to approximate $M^{[X]}/G_H/1$ RQ with Bernoulli feedback, plus they derived various quantitative measurements of the QS. An estimation of $M/G/1$ RQ with general retrial times was again given by I. Atencia and P.Moreno [15] by examining a DT $Geo/G/1$ RQ with general retrial times. With impatient consumers and a server prone to startup problems, Aboul-Hassan et al. [3] dealt with a $Geo/G/1$ RQ in DT. To simplify the computation of significant distributions, recursive formulas were constructed. To learn how impatience impacts system performance generally, a simulation study has also been performed constructed.

R. Artalejo et al. [7] scrutinized a DT $Geo^{[X]}/G/1$ RQ with batch arrivals where individual customers have admission control, plus the underlying Markov chain was also studied, and finally they concluded by approximating CT $M/G/1$ RQ with batch arrivals and admission control. I. Atencia and P.Moreno [16] investigated a DT $Geo/G/1$ RQ with starting failures once more. They devised two SD laws (SDL) and found the measure of proximity betwixt the SSDs. Also, they proved that this QS can be estimated to M/G/1 RQ with starting failures. I. Atencia and P. Moreno [17] studied a DT $Geo/G/1$ RQ with a breakdown and repairs again, which led to even more findings of DT RQ with active breakdowns. A stochastic distribution law was also derived and perhaps they even pioneered the idea of generalized service time. Jinting Wang and Qing Zhao [89] investigated a DT $Geo/G/1$ RQ contingent on starting fiaso and second optional service. Formulae for the stationary distribution along with two SDL were procured and few statistical paradigm were presented. The amount of consumers within the system's generating functions and in the orbit had been determined along with two SDLs and limits for the distance between the SSDs were given by Jinting Wang and Qing Zhao [88] by analysing a DT $Geo/G/1$ RQ with general retrial time and starting failures. A. Aboul-Hassan et al. [2] scrutinized a DT $Geo/G/1$ RQ with general retrial time and balking customers. They proved that as a fusion of two unrelated random variables, the SSD can be indicated. Also, a set of recursive formulae were obtained, and finally, they concluded that when the QS is considerably charged, the repercussions of balking is more apparent, whereas it has a slight impact on busy probability and the average system magnitude for light loads. Bara Kim and Jeongsim Kim [39] examined a DT $Geo/G/1$ RQ and they found the tail of the queue size distribution and proved that it was asymptotically geometric and also showed that it has been inconsistent with the CT M/G/1 retrial queue. A DT $Geo^{[X]}/G/1$ RQ with general retrial times had been scrutinized by Abdel-Karim Aboul-Hassan et al. [1] where the generating functions of the system state distribution plus the orbit magnitude and SSD were derived. Also, a study was conducted on the consequences of bulk arrivals and general retrial times on the stability region, and they concluded that the upper bound of the stability region rapidly reduces by boosting the average bulk size (i.e)., the whole mean batch size has a significant issue on the QS's functioning.

The repercussions of periodic customers was studied by I. Atencia et al. [13] by analysing a DT $Geo/G/1$ RQ with recurrent customers, that has various relevance on computers and communication networks. This QS is also affiliated to the vacation queue. Using Discrete Fourier transform inversion, the probability mass function of the QS was also obtained. I. Atencia et al. [14] scrutinized a DT $Geo/G/1$ RQ with Bernoulli feedback, starting failures, and general retrial times in order to obtain various analytical expressions and moreover, CT QS can be roughly estimated by this model. Also, this QS can be asigned to assorted versions of server interruptions. Jinbiao Wu et al. [95] studied a DT $Geo/G/1$ RQ with anticipatory restart and crash, and its ergodicity constraint was derived by analysing the Markov chain. Using the generating function technique, the system state distribution plus the orbit magnitude and SSD was obtained, and furthermore, the SD property was also investigated. Shan Gao et al. [32] examined a repairable $Geo/G/1$ RQ with Bernoulli feedback, recurrent cunsumers, and general retrial times. Using SVT, the Markov chain was studied, and some queueing measures were estimated by solving the Kolmogrov equations. Numerical illustrations were also done to prove the sensitiveness of the QS performance.

A DT $Geo_1, Geo_2^X/G_1, G_2/1$ RQ with two grouping of consumers and feedback was scrutinized by Zaiming Liu and Shan Gao [54] where they studied the Markov chain and procured some quantitative measurements of the QS in the steady-state. Also, the connection between DT system and its continuous counterpart was investigated. A DT $Geo/G/1$ RQ with favoured, intolerant consumers and general retrial times was analysed by Jinbiao Wu et al. [96]. The system state distribution plus the orbit size and SSD were determined. Apart from the SD property, its equivalent CT QS were also investigated. A DT $Geo/G/1$ RQ with general retrial time and Bernoulli vacation was probed by Jinting Wang [83] and he obtained its ergodicity condition along with its two SDL. This paradigm is mainly presented to enhance the view of DT scenarios with vacations from retrial queueing theory.

Yue Dequan and Zhang Feng [100] studied a DT $Geo/G/1$ RQ with general retrial times and a J-vacation policy. The generating abilities of the amount of consumers in the orbit and in the QS, along with the SD outcome for the system size, were procured, and finally, the end result of some frameworks of the QS were also investigated. A DT $Geo/G/1$ RQ with J vacations where the server subjugated to two contrasting kinds of breakdowns was examined by Feng Zhang and Zhifeng Zhu [102] and a few quantitative measurements of the QS were derived. Further numerical analysis was carried out to look into the implications of vacations and collapse of the system. Feng Zhang and Zhifeng Zhu [103] studied DT $Geo/G/1$ RQ with two contrasting kinds of vacations, the non-exhaustive random vacation policy and the exhaustive single vacation policy, and illustrated that the system magnitude has a solely unique SD property. Finally, the repercussions of assorted parameters on few of the main key metrics of the system were presented.

Sheweta Upadhyaya [76] shown how to obtain expressions for system size, orbit size, and other critical metrics using the generating function methodology and other key metrics by analysing a DT $Geo^{[X]}/G/1$ RQ using Bernoulli feedback. The congestion issues endured frequently in various digital systems can be feasibly cleared up with the succor of this paradigm, and it was also proved that this model assit network developers in detaching its system size into two queueing models, one with lacking retrials and the other being the conventional DT RQ. Lately, Shweta Upadhyaya [77] employed the generating function approach and the SVT to analyze a DT $Geo^{[X]}/G/1$ RQ with Bernoulli feedback and a starting failure, and then she deployed the particle swarm optimisation technique to achieve the optimal values for a few critical system parameters, for instance the total system cost, by minimizing it. The initial work on a DT $Geo/G/1$ RQ with non-pre-emptive priority, general retrial times, working vacations, and vacation disruption was probed by Shaojun Lan and Yinghui Tang [43]. The closed-form expressions for the PGF of the stationary distribution of diverse server states, the amount of consumers in the priority queue, in the orbit, and in the QS were found using SVT and the generating function method, and this model can be executed in many real-life congestion scenarios. Again, the pioneer work on a DT $Geo/G/1$ RQ with probabilistic pre-emptive priority, balking consumers, initial defects, and substitutions of repair times was reported by Shaojun Lan and Yinghui Tang [44] where they introduced the concept of replacements. Some quantitative measurements were obtained, and a cost structure for determining the optimal replacement probability while minimising the system cost was also established.

Pavai Madheswari et al. [55] investigated a general service RQ with recurring clients that operated on a single server in DT and as a special case of the suggested model, several of the current results are generated. Malik and Upadhyaya [57] have recently discussed a DT $Geo/G/1$ RQ, with retrials, preferred and impatient units, single vacations, and state-dependent policies and again they [56] focused on studying a DT $Geo/G/1$ RQ in which, in order to ensure client satisfaction and enhance service quality, the server offers both essential and discretionary services. Shweta Upadhyaya [78] discussed about the periodic customers, high-priority customers, and impatient customers make up the arrival stream in an unreliable DT $Geo/G/1$ RQ with Bernoulli feedback. Recent research conducted by Rajasudha et al. [67] compares the performance of a batch arrival single server DT RQ under three distinct vacation regimes.

4.1. Discrete-time $Geo/Geo/1$ Retrial Queueing Models

An investigation on DT $Geo/Geo/1$ RQ with a server subjugated to failures and a general server lifetime was probed by P. Moreno [61] and diverse quantitative measurements were also estimated. Finally, some numerical analyses were presented. Jin-Ting Wang and Peng Zhang [87] extended the evaluation of retrial queueing theory on NC to DT retrial G QS by analysing a DT $Geo/Geo/1$ retrial G-queue with server breakdowns and repairs. Using G-queues, the reliability modelling was widen to the DT scenario for the maiden time using this model. The marginal functions of the orbit size, besides the ergodicity condition had also been determined. Sheweta Upadhyaya's [75] study on a DT $Geo^{[X]}/Geo/1$ RQ with a functioning vacation plan, aided to design numerous realistic as well as economic gridlock circumstances. Using the MGM, diverse quantitative measurements were obtained. Futhermore, with the succor of a straightforward delve technique gleaned from a heuristic method, joint optimal values were also determined. Also, this model is feasibly regarded as an economical tool. An elementary and basic study on DT $Geo/Geo/1$ retrial G-queues with server breakdown and repair was probed by A.Azhagappan et al. [21] where they acquired the model's ergodicity condition by analysing the Markov chain underlying the QS.

5. **Other Discrete-time Retrial Queueing Models**

R. Artalejo and M. J. Lopez-Herrero [10] conducted a contemplation on DT multi-server RQ with a defined population. In the study conducted by Rein Nobel and Pilar Moreno [64] on a DT single-server QS with retrials, they calculated ergodicity conditions, the vestigial service epoch, and average orbit size, besides some quantitative measurements. A distinct aspect to handle the combinations of various conditions like positive arrival, negative arrival, server breakdowns, and customer retrial was given by Jinting Wang and Peng Zhang [93] by analysing a DT single-server RQ with NC and an unreliable server. A DT inventory system with retrial demands that tends to access in aggrement with a Bernoulli process where the inventory has been refilled in accordance with (s, S) policy was probed by C. Periyasamy [65]. A DT $GI/G/1$ RQ with Bernoulli retrials and time restraint vacation was examined by Jin-ting Wang et al. [86] which further, with the aid of the Markov-based methodology, feasibly be analysed by a level-dependent quasi-birth-and-death (LDQBD) method that enhances the system's computability with a technical perspective. Recently, Upadhyaya et al. [79] analysed a DT bulk-entry recurrent- RQ to examine problems with traffic management and control in ATM networks. First, they used the generating function approach to obtain the required performance indices, and then they utilized an adaptive neuro-fuzzy interface system (ANFIS) to get a rough approximation of all the estimated findings. Finally, they used computational analysis of the model, involving particle swarm optimisation and genetic algorithm techniques, to make the system more cost-effective. Rajasudha and Arumuganathan [66] adopted a hybrid method to simulate and analyze the behavior of a system infected with malware. Malware is filtered out and system performance is enhanced due to a likelihood estimation. To account for the peculiarities of the damaged system, a DT single-server RQ with two kinds of arrivals is addressed. Xiaoyun Yu et al. [98] introduced an innovative k-out-of-n : G repairable QS that incorporates Bernoulli shocks and retrial into it under the DT assumption as a mechanism of redundancy to considerably increase the system's reliability. Additionally,the DT Markov processes theory is used to assess the system's performance.

6. Challenges on using Discrete Time Queueing models

While many of the current contributions focus on single arrival DT systems, there are various practical scenarios where batch arrival DT systems can exist but have received little attention. An additional difficulty in DT queues is posed by working vacation polices. Moreover, multi-stage DT systems have not been explored due to the focus on single-stage DT systems until this point. Traditional queuing systems are well-known and widely utilized, but their assumptions are often too far-fetched for comfort, therefore fuzzy queues are employed to reflect real scenarios instead. Fuzzy queues are more accurate and realistic than the usual queue method. Fuzzy parameter analysis of DT queues is a growing problem.

7. Techniques employed in Discrete-time models

Many authors have presented new methods for analyzing queueing models that take DT into consideration. They employ strategies such as supplementary variable technique, generating function method, matrix-geometric method (matrix analytic method), quasi-birth-and-death process, quasi Newton method, Markov based approach, UL-type RG-factorization, (s,S) policy, Newton–Raphson's method, method of exclusion, discrete Fourier transform method, partial repeat-after-interruption service strategy, truncation method, transform technique, routine method, direct search method based on heuristic approach to obtain the optimal cost, numerical inversion and maximum entropy techniques, theory of difference equation, renewal process theory, probabilistic analysis method, embedded Markov chain technique, etc. In the study of DT queueing systems, most of the parameters we consider display a stochastic process, therefore the variables we denote arrival rate, retrial rate, service rate, balking rate, reneging rate, breakdown rate, repair rate, vacation rate, feedback rate as random variables.

8. Applications of Discrete-time Queueing models

Several DT queueing models can be extrapolated from practical situations. The creators of these DT queueing models can use the resulting overview to direct their decision-making. This section provides a brief summary of some application-focused research articles that have addressed DT queueing models.

Wireless sensor networks: While digital communication protocols are managed by a " slot,"which is equivalent to a time interval, DT queues are better suited for this analysis. Since data packets in wireless sensor networks are vulnerable to being lost due to external attacks or shocks, DT queues are applied to a power-saving technique. Understanding the appropriate operation of the N-policy for power saving in a wireless sensor networks is realistically vital when dealing with sensor nodes under unstable network connections. Since the N-policy lessens the amount of initial power used to toggle a radio server in sensor nodes between its busy and idle states, it is an effective power-saving method. Thus, Doo and Won [47] analysed a DT queue with N-policy and applied in wireless sensor networks.

Wireless ATM multiplexer: Mehmet Ali et al. [59] looked into how wireless ATM is affected by channel faults. Their efforts will be useful for a cell in a wireless ATM network that includes a base station and a number of users on mobile devices. Since the wireless access point acts as an interface to the wired ATM network, the latter can now serve mobile users. While all the mobile users are assumed to experience the same channel condition at any one time, the uplink

wireless channel can be modeled as a DT queueing system fed by On-Off sources and served by a two-state Markovian server, and then analyzed accordingly.
Wireless local communication: In order to put a number on how much of an impact multimedia services have over a wireless local communications channel with Rayleigh fading, a DBMAP queuing model [90] is used. In this case, a wireless link with sporadic defects and a high data rate is used, with the time axis slotted. Error processes of varying granularity can be modeled using this framework. It can monitor problems at multiple levels, including the bit and packet levels, among others.
Email contact center: The DT model may find use in the increasingly common incoming email contact center. Users from various offices, faculties, and research centers send and receive emails over the network using the office automation system to communicate with potential customers.Everything from requesting a channel to sending and receiving data happens at regular intervals. Thus, with these systems, sending emails, preprocessing, and processing requests are all done in a defined amount of time. While this may be fine for email, it is blatantly unacceptable for incoming phone calls. This is why a DT model of the system is more appropriate here [[36], [92]].
Computer and communication systems: Like bits and bytes, the inter-arrival times of packets and their forward transmission periods are the elementary units of time in computer and communication systems (such as time division multiple access (TDMA)).DT queueing activities can only happen at regularly spaced epochs, which has provided a strong impetus for studying this phenomenon. Furthermore, DT queues can be used to approximate the continuous systems, but not the other way around, making them a better fit for defining the behaviors of data communication and computer networks [42], [74].

9. Advancements in Discrete Time Queueing models

Once a model has been built and is ready to go into operation, the first thing a manufacturer or analyst will want to know is whether or not it is cost effective. This highlights the importance of 'cost analysis' in determining a model's sufficiency. As a result, system designers are able to make more informed choices and reduce potential dangers. In view of this, many authors [[79],[77], [81], [66]] employing methods such as particle swarm optimization, artificial bee colony, genetic algorithm, etc., have attained the cost optimization for their queueing models.
Combining the strengths of neural networks and fuzzy logic systems, an adaptive neuro fuzzy inference system (ANFIS) is a form of artificial intelligence. Like a neural network, ANFIS can learn and make judgments based on data, but it can also deal with fuzzy or incomplete data in the same way as a fuzzy logic system can. Because of this, ANFIS is particularly well-suited for uses where data is volatile or unreliable. Recently, many studies [[79],[81]] have been conducted recently comparing the numerical findings of retry QS to those obtained using neuro-fuzzy methods.

10. Future Scope

The focus of research has shifted in recent years from continuous-time queues to their DT analogues. Applications of DT queues in environments where time is slotted are mostly responsible for this change. However, there have been many studies and publications in the field of DT models, there are still many questions that need to be answered. This section primarily suggests some potential avenues for future study.
Many potential extensions on the DT models with heterogeneous service have not been investigated because previous research mostly focused on single service phase DT models. These unanswered questions may, in part, be attributable to the " curse of dimensionality, "which results from the complexity of the underlying systems. One promising area of study is the formation of approximations to these intricate DT models. Moreover, few works have been done on quorum queues, so investigating DT quorum queues is a promising area for future study.

Additionally, the transient behaviour of DT queues will be an intriguing aspect to investigate as well. Further, complex vacation policies, such as hybrid vacation, coxian vacation, and Bernoulli working vacation, are yet another path to explore while analyzing DT queues.

11. Conclusion

A comprehensive investigation on DT queueing models and DT retrial queueing models has been conducted in this paper. The primary purpose of this review is to access the current level of understanding in the discipline of DT queueing models, identify key authors, research papers, theories and conclusions, and identify the knowledge gaps in this field. The applications of these QS are widely applied in our day-to-day life, precisely in the field of communication and computer systems, where time is slotted. The ideas affiliated to the DT Geo/G/1 QS and retrial QS scrutinized in several research papers have been synthesized. Distinct techniques and various challenges involved in DT queues are discussed. Further, recent advancements in DT queues are also presented. Thus, a diverse collection of literature has been reviewed, with appropriate references mentioned.

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References

[1] Aboul-Hassan, A. K., Rabia, S. I. and Taboly, F. A. (2009). Performance evaluation of a discrete-time $Geo^{[X]}/G/1$ retrial queue with general retrial times. *Computers and Mathematics with Applications*, 58(3):548-557. https://doi.org/10.1016/j.camwa.2009.03.101.

[2] Aboul-Hassan, A. K., Rabia, S. I. and Taboly, F. A. (2008) Discrete-time $Geo/G/1$ retrial queue with general retrial times and balking customers. *Journal of the Korean Statistical Society*, 37(4):335-348 .

[3] Aboul-Hassan, A. K., Rabia, S. I. and Al-Mujahid, A. A. (2010). A discrete-time $Geo/G/1$ retrial queue with starting failures and impatient customers. *Transactions on computational science VII*, 22-50. https://doi.org/10.1007/978-3-642-11389-5_2

[4] Alfa, A. S. (2002). Discrete time queues and matrix-analytic methods. *Top*, 10:147-185. https://doi.org/10.1007/BF02579008

[5] Alfa, A. S. (2003). An alternative approach for analyzing finite buffer queues in discrete time. *Performance Evaluation*, 53(2):75-92. https://doi-org.egateway.vit.ac.in/10.1016/S0166-5316(02)00225-0

[6] Alfa, A. S. and He, Q. M. (2008). Algorithmic analysis of the discrete time $GI^{[X]}/G^{[Y]}/1$ queueing system. *Performance Evaluation*, 65(9):623-640. https://doi-org.egateway.vit.ac.in/10.1016/j.peva.2008.02.001

[7] Artalejo, J. R., Atencia, I. and Moreno, P. (2005). A discrete-time $Geo^{[X]}/G/1$ retrial queue with control of admission. *Applied Mathematical Modelling*, 29(11):1100-1120. https://doi.org/10.1016/j.apm.2005.02.005.

[8] Artalejo, J. R., Economou, A. and G?mez-Corral, A. (2008). Algorithmic analysis of the $Geo/Geo/c$ retrial queue. *European Journal of Operational Research*, 189(3):1042-1056. https://doi-org.egateway.vit.ac.in/10.1016/j.ejor.2007.01.060

[9] Artalejo, J. R. and Li, Q. L. (2009). Performance Analysis of a Block-Structured Discrete-Time Retrial Queue with State-Dependent Arrivals. *Discrete Event Dynamic Systems*, 20:325-347. https://doi.org/10. 1007/s10626-009-0075-6

[10] Artalejo, J. R. and Lopez-Herrero, M. J. (2007). A simulation study of a discrete time multi-server retrial queue with finite population. *Journal of Statistical Planning and Inference*, 137(8):2536-2542. https://doi.org/10.1016/j.jspi.2006.04.018.

[11] Atencia, I. (2015). A discrete-time queueing system with server breakdowns and changes in the repair times. *Annals of Operations Research*, 235:37-49. https: //doi.org/10.1007/s10479-015-1940-3.

[12] Atencia, I. (2016). A Discrete-time Queueing Systems with changes in the Vacation Times. *International Journal of Applied Mathematics and Computer Science*, 26(2):379-390. https://doi.org/10.1515/amcs-2016-0027.

[13] Atencia, I., Fortes, I., Nishimura, S. and Sanchez, S. (2010). A discrete-time retrial queueing system with recurrent customers. *Computers and Operations Research*, 37(7):1167-1173. https://doi.org/10.1016/j.cor.2009.03.029.

[14] Atencia, I., Fortes, I. and Sanchez, S. (2009). A discrete-time retrial queueing system with starting failures, Bernoulli feedback and general retrial times. *Computers and Industrial Engineering*, 57(4):1291-1299. https://doi.org/10. 1016/j.cie.2009.06.011.

[15] Atencia, I. and Moreno, P. (2004). A Discrete-Time $Geo/G/1$ Retrial Queue with General Retrial Times. *Queueing Systems*,48:5-21. https://doi.org/10.1023/B:QUES.0000039885.12490.02.

[16] Atencia, I. and Moreno, P. (2006). A Discrete-Time $Geo/G/1$ retrial queue with the server subject to starting failures. *Annals of Operations Research*,141:85-107. https://doi.org/10.1007/s10479-006-5295-7.

[17] Atencia, I. and Moreno, P. (2006). A Discrete-time $Geo/G/1$ Retrial Queue with Server Breakdowns. *Asia-Pacific Journal of Operational Research*, 23(02):247-271. https://doi.org/10.1142/S0217595906000929.

[18] Atencia, I. and Moreno, P. (2004). Discrete-Time $Geo^{[X]}/G_H/1$ Retrial Queue with Bernoulli Feedback. *Computers and Mathematics with Applications*,47(8-9):1273-1294. https://doi.org/10.1016/S0898-1221(04)90122-8.

[19] Atencia, I. and Moreno, P. (2004). The discrete-time $Geo/Geo/1$ queue with negative customers and disasters. *Computers and Operations Research*, 31(9):1537-1548.https://doi.org/10.1016/S0305-0548(03)00107-2.

[20] Atencia, I. and Pechinkin, A.V. (2013). A discrete-time queueing system with optional LCFS discipline. *Annals of Operations Research*, 202:3-17. https: //doi.org/10.1007/s10479-012-1097-2.

[21] Azhagappan, A., Sridevi, A. and Deepa, T. (2018). Discrete Time $Geo/Geo/1$ Retrial G – Queue with Server Break Down and Repair. *International Journal of Engineering Research and Technology*.

[22] Baetens, J., Steyaert, B., Claeys, D. and Bruneel, H. (2016). System occupancy of a two-class batch-service queue with class-dependent variable server capacity. *Proceedings In Analytical and Stochastic Modelling Techniques and Applications: 23rd International Conference, ASMTA 2016, Cardiff, UK, Springer International Publishing*, 32-44. https://doi.org/10.1007/978-3-319-43904-4_3

[23] Barbhuiya, F. P. and Gupta, U. C. (2020). A Discrete-Time $GI^{[X]}/Geo/1$ Queue with Multiple Working Vacations Under Late and Early Arrival System. *Methodology and Computing in Applied Probability*, 22:599-624. https://doi. org/10.1007/s11009-019-09724-6.

[24] Bruneel, H. and Kim, B.G. (2012). Discrete-time models for communication systems including ATM. *Springer Science & Business Media*,205.

[25] Bruneel, H. (1993). Performance of Discrete-Time Queueing Systems. *Computers Operation Research*, 20(3):303-320. https://doi.org/10.1016/ 0305-0548(93)90006-5.

[26] Choi, B.D. and Kim, J.W. (1997). Discrete-Time $Geo_I, Geo_2/G/1$ Retrial Queueing Systems with Two Types of Calls. *Computers and Mathematics with Applications*, 33(10):79-88.

[27] Das, K. and Samanta, S. K. (2023). Computational analysis of GI [X]/D-MSP (a, b)/1 queueing system via RG-factorization. *Mathematical Methods of Operations Research*, 1-39.https://doi.org/10.1007/s00186-023-00816-1

[28] Das, K. and Samanta, S. K. (2021). Modelling and analysis of D-BMAP/D-MSP/1 queue using RG-factorization. *Quality Technology & Quantitative Management*, 18(3):355-381. https://doi.org/10.1080/16843703.2020.1830477

[29] Demircioglu, M., Bruneel, H. and Wittevrongel, S. (2021) Analysis of a Discrete-Time Queueing Model with Disasters. *Mathematics*, 9(24):3283. https://doi.org/10.3390/math9243283.

[30] Fiems, D., Steyaert, B. and Bruneel, H. (2001). Discrete-time queues with general service times and general server interruptions. *In Internet Quality and Performance and Control of Network Systems*, 4211: 93-104. SPIE. https: //doi.org/10.1117/12.417475.

[31] Fiems, D., Steyaert, B. and Bruneel, H. (2004). Discrete-time queues with generally distributed service times and renewal-type server interruption. *Performance Evaluation*, 55(3-4):277-298. https: //doi.org/10.1016/j.peva.2003.08.004.

[32] Gao, S., Liu, Z. and Dong, H. (2012). A repairable discrete-time retrial queue with recurrent customers, Bernoulli feedback and general retrial times. *Operational Research*, 12(3):367-383.https://doi. org/10.1007/s12351-010-0098-7.

[33] Gao, S., Wang, J. and Van Do, T. (2019). Analysis of a discrete-time repairable queue with disasters and working breakdowns. *RAIRO-Operations Research*, 53(4):1197-1216. https://doi.org/10.1051/ro/2018057.

[34] Gao, P., Wittevrongel, S. and Bruneel, H. (2004). Discrete-time multiserver queues with geometric service times. *Computers & Operations Research*, 31(1):81-99. https://doi-org.egateway.vit.ac.in/10.1016/S0305-0548(02)00155-7

[35] Jeyakumar, S. and Gunasekaran, P. (2017). An analysis of Discrete Queue with Disaster and Single Vacation. *International Journal of Pure and Applied Mathematics*, 113:82–90.

[36] Jolai, F., Asadzadeh, S. M. and Taghizadeh, M. R. (2008). Performance estimation of an email contact center by a finite source discrete time Geo/Geo/1 queue with disasters. *Computers & Industrial Engineering*, 55(3):543-556. https: //doi.org/10.1016/j.cie.2008.01.009.

[37] Jyothsna, K., Vijaya Kumar, P. and Gopala Rao, Ch. (2022). Discrete tiem working vacations queue with impatient clients and congestion dependent service rates. *Reliability: Theory & Applications*, 17(3). URL

[38] Kempa, W. M. (2019). Transient study of a discrete-time queueing model with feedback mechanism. *In AIP Conference Proceedings,AIP Publishing*, 2172(1). https://doi.org/10.1063/1.5133595

[39] Kim, B. and Kim, J. (2009). Tail asymptotics for the queue size distribution in a discrete-time $Geo/G/1$ retrial queue. *Queueing Systems*, 61(2-3):243-254. https://doi.org/10.1007/s11134-009-9106-0.

[40] Kim, B. and Kim, J. (2010). Queue size distribution in a discrete-time $D - BMAP/G/1$ retrial queue. *Computers and Operations Research*, 37(7):1220-1227. https://doi.org/10.1016/j.cor.2009.04.016.

[41] Laevens, K. and Bruneel, H. (1998). Discrete-time mult-iserver queues with priorities. *Performance Evaluation*, 33(4):249-275.https:// doi.org/10.1016/S0166-5316(98)00024-8.

[42] Lan, S. and Tang, Y. (2019). An N-policy discrete-time $Geo/G/1$ queue with modified multiple server vacations and Bernoulli feedback. *RAIRO-Operations Research*, 53(2):367-387. https://doi.org/10.1051/ro/2017027.

[43] Lan, S. and Tang, Y. (2019). Performance analysis of a discrete-time Geo/G/1 retrial queue with non-preemptive priority, working vacations and vacation interruption. *Journal of Industrial & Management Optimization*, 15(3). https://doi.org/10.3934/jimo.2018102

[44] Lan, S. and Tang, Y. (2020). An unreliable discrete-time retrial queue with probabilistic pre-emptive priority, balking customers and replacements of repair times. *AIMS Mathematics*, 5(5):4322-4344. https://doi.org/10. 3934/math.2020276.

[45] Larisa G. Afanasyeva. and Andrey Tkachenko. (2018). Stability of Discrete Multi-Server Queueing Systems with Heterogeneous Servers, Interruptions and Regenerative Input Flow. *Reliability: Theory & Applications*, 13(1).

[46] Lee, S. W., Lee, H. W. and Baek, J. W. (2011). Analysis of discrete-time $Geo/G/1$ queue under the D-policy. *In Proceedings of the 6th international conference on queueing theory and network applications*, 107-115. https://doi.org/10.1145/2021216.2021232.

[47] Lee, D. H. and Yang, W. S. (2013). The N-policy of a discrete time $Geo/G/1$ queue with disasters and its application to wireless sensor networks. *Applied Mathematical Modelling*, 37(23):9722-9731. https://doi.org/10.1016/ j.apm.2013.05.012.

[48] Li, J. and Liu, L. (2016). On the Discrete-Time $Geo/G/1$ Queue with Vacations in Random Environment. *Discrete Dynamics in Nature & Society*. http://dx.doi.org/10.1155/2016/4029415.

[49] Li, J. H. and Tian, N. S. (2007). The discrete-time $GI/Geo/1$ queue with working vacations and vacation interruption. *Applied Mathematics and Computation*, 185(1):1-10. https://doi.org/10.1016/j.amc.2006.07.008.

[50] Li, J. H., Tian, N. S. and Liu, W. Y. (2007). Discrete-time $GI/Geo/1$ queue with multiple working vacations. *Queueing Systems*, 56:53-63. https: //doi.org/10.1007/s11134-007-9030-0.

[51] Li, H. and Yang, T. (1998). $Geo/G/1$ discrete time retrial queue with Bernoulli schedule. *European Journal of Operational Research*, 111(3): 629-649. https://doi. org/10.1016/S0377-2217(97)90357-X.

[52] Lin, C. H. and Ke, J. C. (2011). On the discrete-time system with server breakdowns: Computational algorithm and optimization algorithm. *Applied Mathematics and Computation*, 218(7):3624-3634. https://doi-org.egateway.vit.ac.in/10.1016/j.amc.2011.09.003

[53] Liu, R. and Deng, Z. (2014). On the steady-state system size distribution for a discrete-time $Geo/G/1$ repairable queue. *Discrete Dynamics in Nature and Society*. http://dx.doi.org/10.1155/2014/ 924712

[54] Liu, Z. and Gao, S. (2011). Discrete-time $Geo_1, Geo_2^X/G_1, G_2/1$ retrial queue with two classes of customers and feedback. *Mathematical and Computer Modelling*, 53(5-6):1208-1220. https://doi.org/10.1016/j.mcm.2010.11.090

[55] Madheswari, S. P., Suganthi, P. and Vasanthamani, K. (2021). A Study on $Geo/G/1$ Retrial Queueing System with Starting Failure and Customer Feedback. *Mathematical Modeling and Computation of Real-Time Problems*, 79-98. https://doi.org/10.3934/jimo.2018102

[56] Malik, G. and Upadhyaya, S. (2021). Performance Modelling of a Discrete-Time Retrial Queue with Preferred and Impatient Customers, Bernoulli Vacation and Second Optional Service. I*n Recent Trends in Mathematical Modeling and High Performance Computing: M3HPCST-2020, Ghaziabad, India, Springer International Publishing*, 331-345.https://doi.org/10.1007/978-3-030-68281-1_25

[57] Malik, G. and Upadhyaya, S. (2020). Single vacation policy for discrete-time retrial queue with two types of customers. *Strategic System Assurance and Business Analytics*, 335-349. https://doi.org/10.1007/978-981-15-3647-2_25

[58] Mehmet Ali, M. and Song, X. (2004). A performance analysis of a discrete-time priority queueing system with correlated arrivals. *Performance Evaluation*, 57(3):307-339. https://doi.org/10.1016/j.peva. 2004.01.001.

[59] Mehmet Ali, M., Zhang, X. and Hayes, J. F. (2003). A performance analysis of a discrete-time queueing system with server interruption for modelling wireless ATM multiplexer. *Performance Evaluation*, 51(1):1-31. https://doi.org/10.1016/S0166-5316(02)00087-1.

[60] Miyazawa, T. and Takagi, H. (1994) Advances in discrete-time queues. *Queueing Systems*.

[61] Moreno, P. (2006). A Discrete-time retrial queue with Unreliable Server and General Server Lifetime. *Journal of Mathematical Sciences*, 132:643-655. https: //doi.org/10.1007/s10958-006-0009-x.

[62] Nandi, R. and Samanta, S. K. (2018). Performance Analysis of Shock Models Governed by a Discrete-Time Markovian Arrival Process. *Journal of the Indian Society for Probability and Statistics*, 19:455-468. https://doi.org/10.1007/s41096-018-0053-0

[63] Nobel, R. (2016). Retrial queueing models in discrete time: a short survey of some late arrival models. *Annals of Operations Research*, 247:37-63. https://doi.org/10.1007/s10479-015-1904-7

[64] Nobel, R. and Moreno, P. (2008). A discrete-time retrial queueing model with one server. *European Journal of Operational Research*, 189(3):1088-1103. https://doi.org/10.1016/j.ejor.2007.03.051.

[65] Periyasamy, C. (2013). A Discrete Time Inventory System with Retrial Demands. *International Journal of Engineering Research and Technology*, 2(7):165-170 .

[66] Rajasudha, R. and Arumuganathan, R. (2022). Reliability and Optimum Cost Analysis of Malware Infected System by Discrete-Time Negative Arrival Retrial Queue. *IETE Journal of Research*, 1-20. https://doi.org/10.1080/03772063.2022.2125091

[67] Rajasudha, R., Arumuganathan, R. and Dharmaraja, S. (2022). Performance analysis of discrete-time GeoX/G/1 retrial queue with various vacation policies and impatient customers. *RAIRO-Operations Research*, 56(3):1089-1117. https://doi.org/10.1051/ro/2022042

[68] Samanta, S. K. and Das, K. (2023). Detailed Analytical and Computational Studies of $D-BMAP/D-BMSP/1$ Queueing System. *Methodology and Computing in Applied Probability*, 25(1):12. https://doi.org/10.1007/s11009-023-10012-7

[69] Samanta, S. K. and Nandi, R. (2021). Analysis of $GI^{[X]}/D\text{-}MSP/1/\infty$ queue using *RG*-factorization. *Journal of Industrial and Management Optimization*, 17(2):549-573. https://doi.org/10.3934/jimo.2019123

[70] Samanta, S. K. and Nandi, R. (2020). Performance analysis of the GI/D-MSP/1 queue with N-policy and its optimal control. *Quality Technology & Quantitative Management*, 17(4):399-422. https://doi.org/10.1080/16843703.2019.1662217

[71] Sudhesh, R. and Priya, R. S. (2019). An analysis of discrete-time Geo/Geo/1 queue with feedback, repair and disaster. *International Journal of Operational Research*, 35(1):37-53. https://doi.org/10.1504/IJOR.2019.

[72] Takagi, H. (1993). Queueing Analysis: A Foundation of Performance Evaluation in Discrete-time Systems, *Vol.3, North-Holland, Amsterdam*, (1993).

[73] Tang, Y., Yun, X. and Huang, S. (2008). Discrete-time $Geo^{[X]}/G/1$ queue with unreliable server and multiple adaptive delayed vacations. *Journal of Computational and Applied Mathematics*, 220(1-2):439-455. https: //doi.org/10.1016/j.cam.2007.08.019

[74] Tian, N. and Zhang, Z. G. (2002). The discrete-time GI/Geo/1 queue with multiple vacations. *Queueing systems*, 40:283-294. https://doi. org/10.1023/A:1014711529740

[75] Upadhyaya, S. (2015). Working vacation policy for a discrete-time $Geo^{[X]}/Geo/1$ retrial queue. *Opsearch*, 52(4):650-669. https://doi.org/10. 1007/s12597-015-0200-2.

[76] Upadhyaya, S. (2016). Performance prediction of a discrete-time batch arrival retrial queue with Bernoulli feedback. *Applied Mathematics and Computation*, 283:108-119. https://doi.org/10.1016/j.amc.2016.02.026.

[77] Upadhyaya, S. (2020). Cost optimisation of a discrete-time retrial queue with Bernoulli feedback and starting failure. *International Journal of Industrial and Systems Engineering*, 36(2):165-196. https://doi.org/10.1504/IJISE.2020.110245

[78] Upadhyaya, S. (2021). Performance Analysis of a Discrete-Time Retrial Queue with Bernoulli Feedback, Starting Failure and Single Vacation Policy. *In Recent Trends in Mathematical Modeling and High Performance Computing: M3HPCST-2020, Ghaziabad, India, Springer International Publishing*, 365-379. https://doi.org/10.1007/978-3-030-68281-1_27

[79] Upadhyaya, S., Malik, G. and Sharma, R. (2022). Neuro-fuzzy computing and optimisation results for batch discrete time retrial queue. *International Journal of Mathematics in Operational Research*, 23(1):119-146. https://doi.org/10.1504/IJMOR.2022.126049

[80] Vadivu, A. S., Arumuganathan, R. and Kumar, M. S. (2016). Analysis of discrete-time queues with correlated arrivals, negative customers and server interruption. *RAIRO-Operations Research*, 50(1):67-81. https://doi.org/ 10.1051/ro/2015012.

[81] Vaishnawi, M., Upadhyaya, S. and Kulshrestha, R. (2022). Optimal Cost Analysis for Discrete-Time Recurrent Queue with Bernoulli Feedback and Emergency Vacation. *International Journal of Applied and Computational Mathematics*, 8(5):254. https://doi.org/10.1007/s40819-022-01445-8

[82] Verdonck, F., Bruneel, H. and Wittevrongel, S. (2019). Analysis of a 2-state discrete-time queue with stochastic state-period lengths and state-dependent server availability and arrivals. *Performance Evaluation*, 135:102026. https://doi.org/10.1016/j.peva.2019.102026.

[83] Wang, J. (2012). Discrete-time $Geo/G/1$ retrial queues with general retrial time and Bernoulli vacation. *Journal of Systems Science and Complexity*, 25:504-513. https://doi.org/10.1007/s11424-012-0254-7.

[84] Wang, J., Huang, Y. and Dai, Z. (2011). A discrete-time on–off source queueing system with negative customers. *Computers & Industrial Engineering*, 61(4):1226-1232. https://doi.org/10.1016/j.cie.2011.07.013.

[85] Wang, J., Huang, Y. and Van Do, T. (2013). A single-server discrete-time queue with correlated positive and negative customer arrivals. *Applied Mathematical Modelling*, 37(9):6212-6224. https://doi.org/10.1016/j.apm. 2012.12.021.

[86] Wang, J. T., Wang, N. and Alfa, A. S. (2013). Discrete-time $GI/G/1$ retrial queues with time-controlled vacation policies. *Acta Mathematicae Applicatae Sinica, English Series*, 29(4):689-704. https://doi.org/ 10.1007/s10255-013-0244-0.

[87] Wang, J. T. and Zhang, P. (2009). A single-server discrete-time retrial G-queue with server breakdowns and repairs. *Acta Mathematicae Applicatae Sinica, English Series*, 25(4):675-684. https://doi.org/10.1007/ s10255-008-8823-1.

[88] Wang, J. and Zhao, Q. (2007). Discrete-time $Geo/G/1$ retrial queue with general retrial times and starting failures. *Mathematical and Computer Modelling*, 45(7-8):853-863. https://doi.org/10.1016/j.mcm.2006.08.005.

[89] Wang, J. and Zhao, Q. (2007). A discrete-time $Geo/G/1$ retrial queue with starting failures and second optional service. *Computers & Mathematics with Applications*, 53(1):115-127. https://doi.org/10.1016/j.camwa.2006.10.024.

[90] Wang, Y. C., Chou, J. H. and Wang, S. Y. (2011). Loss pattern of $DBMAP/DMSP/1/K$ queue and its application in wireless local communications. *Applied Mathematical Modelling*, 35(4):1782-1797.https://doi-org.egateway.vit.ac.in/10.1016/j.apm.2010.10.009

[91] Wang, T. Y. and Ke, J. C. (2009). The randomized threshold for the discrete-time $Geo/G/1$ queue. A*pplied Mathematical Modelling*, 33(7):3178-3185. https://doi.org/10.1016/j.apm.2008.10.010.

[92] Wang, T. Y., Ke, J. C. and Chang, F. M. (2011). On the discrete-time $Geo/G/1$ queue with randomized vacations and at most J vacations. *Applied Mathematical Modelling*, 35(5):2297-2308. https://doi.org/10.1016/j.apm. 2010.11.021.

[93] Wang, J. and Zhang, P. (2009). A discrete-time retrial queue with negative customers and unreliable server. *Computers & Industrial Engineering*, 56(4):1216-1222. https://doi.org/10.1016/j.cie.2008.07.010.

[94] Woodward, M.E. (1994). Communication and Computer Networks: Modelling with Discrete-Time Queues, *IEEE Computer Society, London*.

[95] Wu, J., Liu, Z. and Peng, Y. (2011). A discrete-time Geo/G/1 retrial queue with pre-emptive resume and collisions. *Applied Mathematical Modelling*, 35(2):837-847. https://doi.org/10.1016/j.apm.2010.07.039.

[96] Wu, J., Wang, J. and Liu, Z. (2013). A discrete-time $Geo/G/1$ retrial queue with preferred and impatient customers. *Applied Mathematical Modelling*, 37(4):2552-2561. https://doi.org/10.1016/j.apm.2012.06.011.

[97] Yang, T. and Li, H. (1995). On the steady-state queue size distribution of the discrete-time $Geo/G/1$ queue with repeated customers. *Queueing systems*, 21:199-215. https://doi.org/10.1007/BF01158581.

[98] Yu, X., Hu, L. and Ma, M. (2023). Reliability measures of discrete time k-out-of-n: G retrial systems based on Bernoulli shocks. *Reliability Engineering & System Safety*, 109491.https://doi.org/10.1016/j.ress.2023.109491

[99] Yu, M. and Alfa, A. S. (2015). Algorithm for computing the queue length distribution at various time epochs in $DMAP/G^{(1,a,b)}/1/N$ queue with batch-size-dependent service time.

European Journal of Operational Research, 244(1):227-239. https://doi.org/10.1016/j.ejor.2015.01.056

[100] Yue, D. and Zhang, F. (2013). A discrete-time $Geo/G/1$ retrial queue with J-vacation policy and general retrial times. *Journal of Systems Science and Complexity*, 26:556-571. https://doi.org/10.1007/s11424-013-1121-x.

[101] Zhang, Z. G. and Tian, N. (2001). Discrete time $Geo/G/1$ queue with multiple adaptive vacations. *Queueing Systems*, 38: 419-429. https://doi. org/10.1023/A:1010947911863.

[102] Zhang, F. and Zhu, Z. (2013) A Discrete-Time $Geo/G/1$ Retrial Queue with J-Vacations and Two Types of Breakdowns. *Journal of Applied Mathematics*. http://dx.doi.org/10.1155/2013/834731.

[103] Zhang, F. and Zhu, Z. (2015). A Discrete-Time $Geo/G/1$ Retrial Queue with Two Different Types of Vacations. *Mathematical Problems in Engineering*, http://dx.doi.org/10.1155/2015/835158.