SAMPLING PLANS BASED ON TRUNCATED LIFE TEST FOR LOGISTIC FAMILY OF DISTRIBUTIONS

Sriramachandran G V
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Department of Mathematics, Dr. N. G. P. Institute of Technology, Coimbatore, India
gvsriramachandran72@gmail.com

Abstract

In this article, we develops optimal sample size for acceptance number (zero and one) for single and double sampling plans by fixing consumer's risk and test completion time, with the assumption that, the life of the item follows logistic family of distributions (i.e. Logistic Rayleigh distribution/Logistic exponential distribution/Logistic Weibull distribution). The optimal size obtained for single and double sampling plans for logistic family of distributions are compared with baseline distributions and the results are discussed.

Keywords: Consumer risk, single sampling plans; double sampling plans; logistic Rayleigh distribution; logistic exponential distribution; logistic Weibull distribution.

I. Introduction

Based on the quality standards prescribed, quality assurance people uses acceptance sampling plan (ASP) to validate the lot. The main focus of ASP is to reduce consumer's risk (CR) as well as producer's risk (PR). By fixing the CR, the minimum sample sizes (MSS) to be used for ASP are obtained. If time parameter is included with ASP, to determine the MSS to guarantee certain median life of products (MLP), then ASP is called as ASP with truncated life test (TLT). Here it is assumed that MLP follows some probability distributions. ASP with TLT can be performed with a given acceptance number c (ACN) say zero and one to find the MSS that guarantees a given MLP at a given consumer confidence level 1- P* , where P* is called as consumer's risk (CR). Accordingly, the lot is rejected if; actual MLP is less than the specified MLP. If the decision to accept or reject the lot is based on the single sample taken from the lot, then it is called as single acceptance sampling plan (SASP) with TLT, otherwise a second sample is taken and decision is based on both first and second sample, then it is termed as double acceptance sampling plan (DASP) with TLT.

Many researchers have proposed these types of ASP with TLT. In the ASP literature, Epstein [6] was the first to consider TLT with an exponential distribution (ED), SASP and DASP for TLT based on transmuted Rayleigh distribution (RD) was studied by Mahendra Saha et al.[8], ASP based TLT for the Generalized Weibull Model by Shovan Chowdhury [13], ASP based on TLT for extended ED, by Amer I. Al-Omari et al. [1], TLT based ASP for generalized ED by Muhammad Aslam et al.[10], TLT in ASP based on exponentiated ED by Suresh et al.[11], ASP for Generalized RD based on TLT, by Tzong-ru tsai [14], ASP based on TLT for weighted ED [15] are important to mention.

An approach that helps in defining logistic compounded model was given by Yingjie Lan et.al [16] and they proposed logistic–exponential distribution (L-ED). In this line, by taking baseline distribution as RD, Logistic- Rayleigh distribution (L-RD) was defined by Arun Kumar Chaudhary et al. [2] and stated various properties and applications. By same process, Logistic-Weibull distribution (L-WD) was introduced by taking baseline distribution as WD by Arun Kumar Chaudhary et al. [3] and studied its properties and applications. In this article, we develop SASP and DASP with TLT, assuming that MLP follows L-RD or L-ED or L-WD. For CR fixed at 1% and 2% level, MSS to guarantee certain MLP are obtained for these plans.

The structure of the remaining article is: In Section 2, we described the L-RD/ L-ED /L-WD and studied some of its properties. Section 3 presents the design of SASP with TLT, if MLP follows L-RD, L-ED and L-WD. MSS required to maintain CR at 1% and 2% level are obtained for different test time and the results obtained are tabulated in this section. Section 4 presents the design of DASP with TLT if MLP follows L-RD, L-ED and L-WD and for this ASP, MSS required to maintain CR at 1% and 2% level are obtained for different test time. Section 5 compares the MSS obtained for SASP and DASP with some baseline distributions. Finally, conclusion is placed in Section 6.

II. Logistic Family of Distributions

Yingjie Lan et.al [16] have presented an approach to define the logistic compounded model and introduced the logistic exponential survival distribution. Based on Yingjie Lan et.al [16], the L-RD was defined by Arun Kumar Chaudhary et al. [2] as follows: Let τ be a non-negative random variable with a positive shape parameter φ and positive scale parameter ω then CDF of L-RD can be defined as

$$F(\tau;\varphi,\omega)=1-\frac{1}{1+\left(e^{\omega\tau^2/2}-1\right)^{\varphi}}\quad,\quad \tau\geq 0 \text{ and } \varphi,\omega>0 . \tag{1}$$

The corresponding PDF of L-RD is given by

$$f(\tau;\varphi,\omega)=\frac{\omega\,\varphi\,\tau\,e^{\omega\tau^2/2}\left(e^{\omega\tau^2/2}-1\right)^{\varphi-1}}{\left[1+\left(e^{\omega\tau^2/2}-1\right)^{\varphi}\right]^2},\quad \tau\geq 0 \text{ and } \varphi,\omega>0 , \tag{2}$$

The median of the L-RD is given by $Md=\left[\frac{2}{\omega}\log 2\right]^{1/2}$.

The plots of CDF and PDF of L-RD are given in Fig 1 and Fig 2 respectively for shape parameters φ =0.25, 1, 2 and the scale parameter $\omega=1$.

Let τ be a non-negative random variable with a positive shape parameter φ and positive scale parameter ω then CDF of L-ED as defined by Sajid Ali et al. [12] as in equation 3.

$$F(\tau;\varphi,\omega)=1-\frac{1}{1+\left(e^{\omega\tau}-1\right)^{\varphi}},\ \tau\geq 0 \text{ and } \varphi,\omega>0 \tag{3}$$

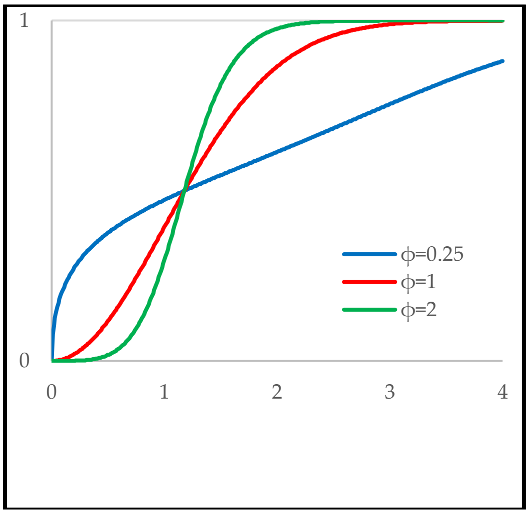

Figure 1 : *cdf plot of L-RD*

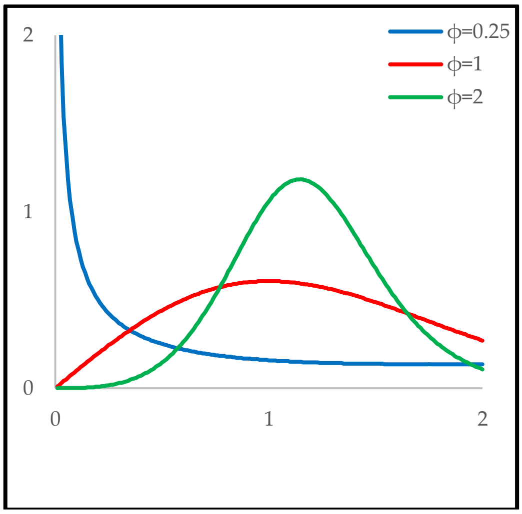

Figure 2 : *pdf plot of L-RD*

Note that the exponential distribution can be obtained as a special case of L-ED when $\phi=1$. The corresponding PDF of L-ED is given in equation 4.

$$f(\tau;\varphi,\omega)=\frac{\omega\,\varphi\,e^{\omega\tau}\left(e^{\omega\tau}-1\right)^{\varphi-1}}{\left[1+\left(e^{\omega\tau}-1\right)^{\varphi}\right]^{2}},\ \tau\geq 0 \text{ and } \varphi,\omega>0 \tag{4}$$

The median of the L-ED is given by $Md=\left[\frac{2}{\omega}\log 2\right]$.

The plots of CDF and PDF of L-ED are given in Fig 3 and Fig 4 respectively for shape parameters φ =0.25, 1, 2 and the scale parameter $\omega=1$.

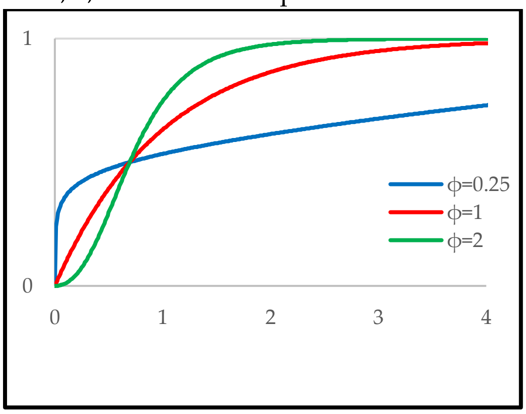

Figure 3 : *cdf plot of L-ED*

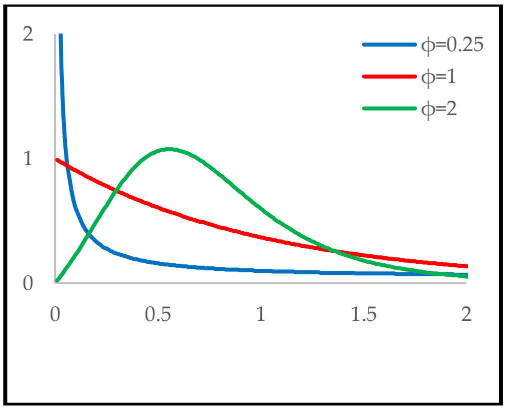

Figure 4 : *pdf plot of L-ED*

Let τ be a non-negative random variable with a positive shape parameter φ and positive scale parameter ω, β then CDF of L-WD as defined by Arun Kumar Chaudhary [3] is given in equation 5.

$$F(\tau;\varphi,\beta,\omega) = 1 - \frac{1}{1+\left(e^{\omega\tau^{\beta}}-1\right)^{\varphi}}, \ \tau \ge 0 \text{ and } \beta,\varphi,\omega > 0 \tag{5}$$

The corresponding PDF of L-WD is given by

$$f(\tau;\varphi,\beta,\omega) = \frac{\omega\,\varphi\,\beta\,\tau^{\beta-1}\,e^{\omega\tau^{\beta}}\left(e^{\omega\tau^{\beta}}-1\right)^{\varphi-1}}{\left[1+\left(e^{\omega\tau^{\beta}}-1\right)^{\varphi}\right]^{2}}, \quad \tau \ge 0 \text{ and } \beta,\varphi,\omega > 0, \tag{6}$$

The median of the L-WD is given by $Md = \left[\frac{2}{\omega}\log 2\right]^{1/\beta}$. The plots of CDF and PDF of L-WD are given in Fig 5 and Fig 6 respectively for shape parameters φ =0.25, 1, 2 and the scale parameter $\omega = 1$ and $\beta = 0.25$.

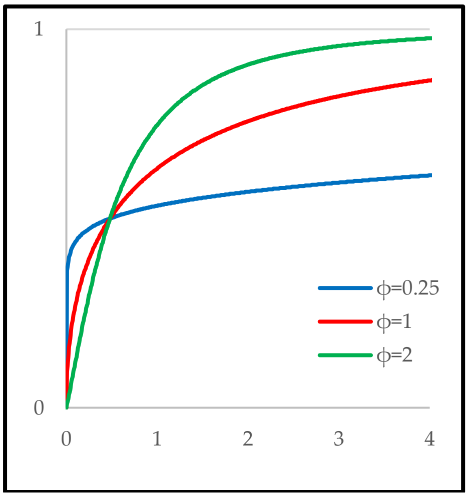

Figure 5 : *cdf plot of L-WD*

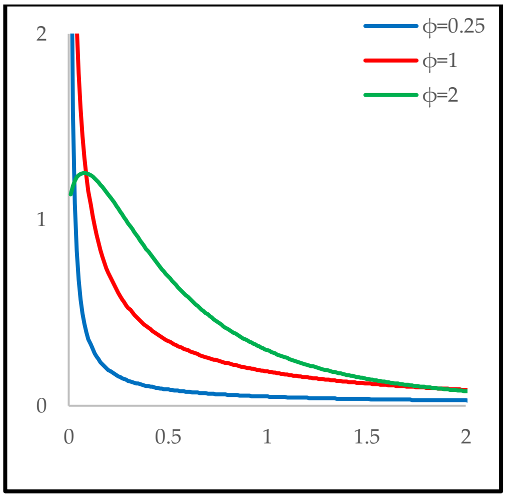

Figure 6 : *pdf plot of L-WD*

III. Design of SASP with TLT Based on Median

The assumption made in designing the SASP with TLT is that the products under study have an MLP of υ. To test the above assumption, we follow the hypothesis test, with null hypothesis H0: $\upsilon \ge \upsilon_0$, the alternative hypothesis H1: $\upsilon < \upsilon_0$ then it is accepted, where υ_0 is a precise MLP of the product. With P^* as CR, the value $1-P^*$ is called the level of significance for the test. Here it may be noted that binomial distribution is applied as the size of the sample is considerably excessive.

As proposed by the ASP, to locate the mss, we have to iterate the inequality

$$\sum_{i=0}^{c}\binom{n}{i}p^{i}(1-p)^{n-i} \le 1-P^{*} \tag{7}$$

where $F(\tau;\varphi,\omega)$ as in (1) for L-RD , (3) for L-ED and (5) for L-WD. The MSS required for inspection of the lot for L-RD, L-ED and L-WD for different values of $\frac{\tau}{\upsilon_0}$ with CR = 2% and 1% level with for shape parameters φ =0.25, 1, 2 and the scale parameter $\omega = 1$ are calculated and tabulated in the Tables 1 respectively.

Table 1: *MSS for SASP*

CR	Distribution	φ	C	$\frac{\tau}{\upsilon_0}$					
				0.5	0.6	0.7	0.8	0.9	1
1%	L-RD	0.25	0	10	9	8	8	7	7
			1	14	13	12	12	11	11
		1	0	27	19	14	11	9	7
			1	39	28	21	16	13	11
		2	0	131	60	31	17	11	7
			1	-	87	45	25	16	11
	L-ED	0.25	0	8	8	8	8	7	7
			1	12	12	11	11	11	11
		1	0	14	12	10	9	8	7
			1	20	17	15	13	12	11
		2	0	30	20	14	11	9	7
			1	43	29	21	16	13	11
	L-WD	0.25	0	8	8	7	7	7	7
			1	11	11	11	11	11	11
		1	0	10	9	8	8	8	7
			1	15	13	12	12	11	11
		2	0	14	12	10	9	8	7
			1	21	17	15	13	12	11
2%	L-RD	0.25	0	8	8	7	7	6	6
			1	13	12	11	10	10	9
		1	0	23	16	12	9	7	6
			1	35	24	18	14	11	9
		2	0	112	51	26	15	9	6
			1	167	77	40	23	14	9
	L-ED	0.25	0	7	7	7	6	6	6
			1	11	11	10	10	10	9
		1	0	12	10	9	8	7	6
			1	18	15	13	12	10	9
		2	0	25	17	12	9	7	6
			1	38	26	19	14	11	9
	L-WD	0.25	0	7	7	6	6	6	6
			1	10	10	10	10	10	9
		1	0	8	8	7	7	6	6
			1	13	12	11	10	10	9
		2	0	12	10	9	8	7	6
			1	18	15	13	12	10	9

IV. Design of DASP with TLT Based on Median

To give more protection to both consumer as well as producers a two stage ASP called the DASP is preferred. As the name says it provides double protection to producers because we are testing the second sample before taking the final decision on the lot. Hence it provides a total protection to producers and hence minimizes the PR. The parameters of the DASP with TLT are n1 first sample size, it's AN c1, n2 second sample size, it's AN c2 and the testing time t. To accept the lot, it is necessary that the sample supports the hypothesis that sample median to be greater than the median specified. Otherwise, the lot will be rejected. Now, we fix the CR not more than (1-P*).

Then PA of the lot is

$$PA = \sum_{i=0}^{c_1} \binom{n1}{i} p^i (1-p)^{n1-i} + \sum_{x=c_1+i}^{c_2} \binom{n1}{x} p^x (1-p)^{n1-x} \sum_{j=0}^{c_2} \binom{n2}{j} p^j (1-p)^{n2-j} \tag{8}$$

where p is defined in equation (1) for L-RD/ equation (3) for L-ED and equation (5) for L-WD and depends on ratio $\frac{\tau}{\upsilon_0}$. As we are considering only zero-one failure form i.e., c1 =0 and c2 =1,

$$PA = (1-p)^{n1}\left[1 + n1\, p (1-p)^{n_2-1}\right] \tag{9}$$

where p is defined in equation (1) for L-RD/ equation (3) for L-ED and equation (5) for L-WD.

Our aim is to find the MSS for DASP, for this we have to minimize equation 9.

Now, for the given consumer's confidence level P*, the MSS for both the samples n1 and n2, which ensure $\upsilon \geq \upsilon_0$, can be found by the solution of the following optimization problem, given as:

$$\begin{gathered} Min\ \text{ASN} = \text{n}_1 + \text{n}_1\text{n}_2\, p(1-p)^{n_1-1} \\ \text{Subject to: } (1-p)^{n_1}\left[1 + \text{n}_1 p (1-p)^{n_2-1}\right] \leq 1 - P^* \\ 1 \leq n_2 \leq n_1 \\ n_1 \text{ and } n_2 \text{ are integers} \end{gathered} \tag{10}$$

While solving the above optimization problem, it provides many solutions for both n1 and n2. We take the solution which minimizes our objective function i.e. our ASN as our best solution. MSS obtained for P* = 0.98 and 0.99 and for different $\frac{\tau}{\upsilon_0}$ are presented in Table 2.

V. Comparative Study

As we are discussing the ASP with TLT based on L-RD, L-ED and L-WD, the CDF and PDF plots of these three distributions are compared at the parameter values at φ =0.25, 1, 2 in Fig 1 and Fig 2. The MSS obtained for L-RD, L-ED and L-WD plans studied here are compared with transmuted RD by Mahendra Saha et al [8], generalized RD by Tzong-ru tsai et al.[14], compound RD by Bhupendra

Singh [4], Wenhao Gui [15], generalized ED by Muhammad Aslam [10], Extended WD by M. S. Hamed et al [7], Generalized WD by Shovan Chowdhury[13], and the results obtained are tabulated in Table 3.

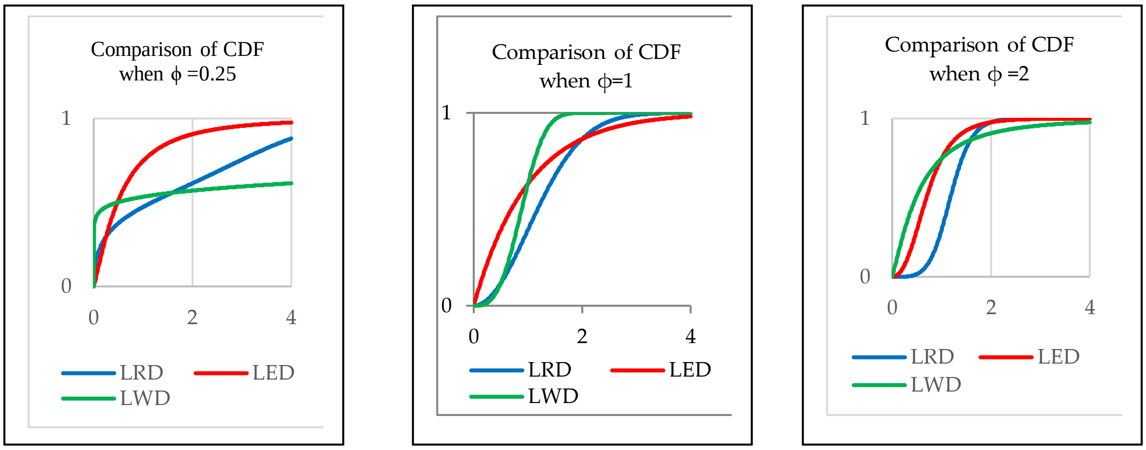

Figure 7: *Comparison of CDF at φ = 0.25, 1 and 2*

Table 2: *Sample size n1, n2 and ASN for DASP*

CR	Distribution	φ	$\frac{\tau}{\upsilon_0}$							
			0.6		0.8		1		1.2	
			(n1, n2)	ASN	(n1, n2)	ASN	(n1, n2)	ASN	(n1, n2)	ASN
1%	L-RD	0.25	(9,6)	9.28	(8,5)	8.24	(7,5)	7.27	(7,3)	7.12
		1	(19,15)	19.71	(11,7)	11.33	(7,5)	7.27	(5,3)	5.18
		2	(61,49)	63.16	(18,11)	18.47	(7,5)	7.27	(4,2)	4.1
	L-ED	0.25	(8,5)	8.25	(8,4)	8.16	(7,5)	7.27	(7,4)	7.19
		1	(12,7)	12.29	(9,5)	9.23	(7,5)	7.27	(6,4)	6.21
		2	(20,17)	20.81	(11,8)	11.39	(7,5)	7.27	(5,4)	5.24
	L-WD	0.25	(8,4)	8.16	(7,6)	7.36	(7,5)	7.27	(7,4)	7.2
		1	(9,7)	9.36	(8,5)	8.24	(7,5)	7.27	(7,3)	7.12
		2	(12,7)	12.31	(9,5)	9.23	(7,5)	7.27	(6,4)	6.21
2%	L-RD	0.25	(8,5)	8.36	(7,4)	7.31	(6,5)	6.47	(6,3)	6.21
		1	(17,11)	17.76	(10,5)	10.33	(6,5)	6.47	(5,2)	5.12
		2	(52,47)	55.53	(15,13)	16.04	(6,5)	6.47	(4,1)	4.05
	L-ED	0.25	(7,5)	7.4	(7,3)	7.2	(6,5)	6.47	(6,4)	6.33
		1	(10,8)	10.64	(8,4)	8.28	(6,5)	6.47	(5,4)	5.41
		2	(18,11)	18.75	(10,6)	10.41	(6,5)	6.47	(5,2)	5.12
	L-WD	0.25	(7,3)	7.2	(7,3)	7.18	(6,5)	6.47	(6,4)	6.35
		1	(8,5)	8.39	(7,4)	7.31	(6,5)	6.47	(6,3)	6.22
		2	(10,8)	10.68	(8,4)	8.28	(6,5)	6.47	(5,4)	5.41

From Table 3, it is found that the value of MSS for L-WD distribution is low when compared to these distributions. Also the plans studied in this article are compared with respect to MSS at CR=1% and 2%, from table 1, among the three, the L-WD ASP takes MSS for inspection.

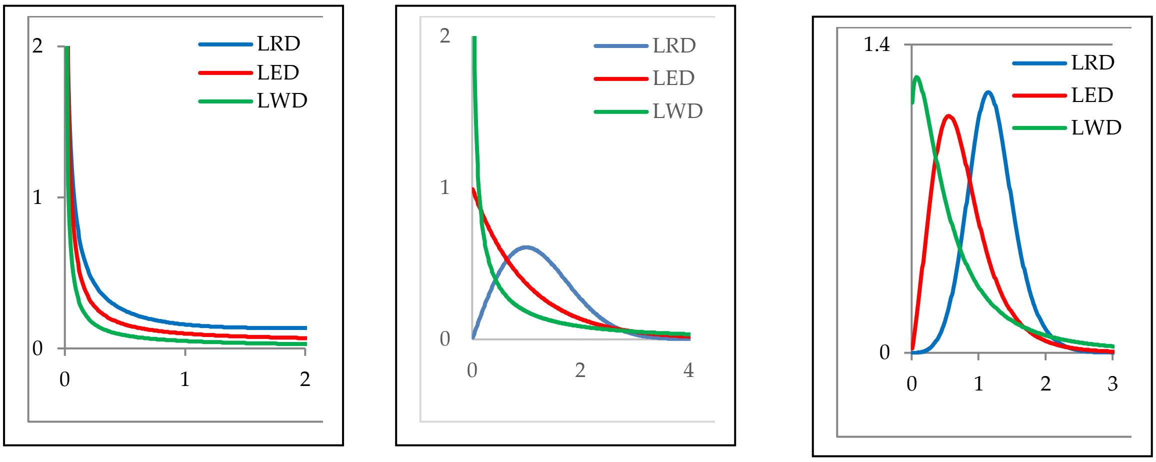

Figure 8 : *Comparison of PDF at φ = 0.25, 1 and 2*

Table 3: *Comparison of MSS for SASP with baseline distributions for SASP*

Distribution	c=0	c=1
Transmuted RD	16	-
Generalized RD	17	24
Compounded RD	17	24
L-RD at $\varphi = 1$	27	39
Generalized ED	14	21
Weighted ED	19	28
L-ED at $\varphi = 1$	30	43
Extended WD	6	9
Generalized WD	6	10
L-WD at $\varphi = 1$	10	15

VI. Conclusion

In this paper we develop single and double sampling plans by fixing consumer's risk and test completion time, with the assumption that, the life of the item follows logistic family of distributions (i.e. L-RD/L-ED/L-WD). MSS for acceptance number (zero and one) for both SASP and DASP are obtained. The MSS obtained for single and double sampling plans for logistic family of distributions are compared with baseline distributions and the results are discussed.

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