ECONOMIC ORDER QUANTITY MODEL FOR IMPERFECT ITEMS WITH SHORTAGE BACKORDERING

Priyanka Singh[1], A. R. Nigwal[2], U. K. Khedlekar[3]

•

[1]Department of Mathematics, Government Degree College, Piprai, 473440, Ashoknagar, M.P., India
priyanka1singh2013@gmail.com
[2]Department of Mathematics, Ujjain Engineering College, Ujjain, 456010, Madhya Pradesh, India,
arnwnigwal@gmal.com
[3]Department of Mathematics and Statistics, Dr. Harisingh Gour Vishwavidyalaya,
Sagar, 470003, Madhya Pradesh, India, (A Central University)
uvkkcm@yahooo.co.in

Abstract

This study presents the development of an economic order quantity model (EOQ) specifically designed for imperfect quality items. The model takes into consideration three distinct scenarios: (a) Model I trigger a reorder when the inventory level reaches zero; (b) Model II initiates a reorder when the backordered quantity equals the imperfect quantity; (c) Model III initiates a reorder when the shortage persists. To distinguish between perfect and imperfect quality products, a screening process is implemented for each product lot. Upon product delivery from the supplier to the vendor, all received products undergo immediate inspection through the screening process. Following the EOQ ordering policy, the vendor sells imperfect products to customers at a reduced cost at the end of the cycle, rather than returning them to the supplier. To fulfil the remaining demand for high-quality products, the vendor procures such products from a local vendor at a higher price. This study optimizes the duration of positive inventory, selling price, and total profit per unit time. Model I, which exhibits the longest duration of positive inventory, demonstrates greater business stability compared to the other two models. The concavity property is analytically and numerically demonstrated, and a sensitivity analysis is provided to explore the impact of model parameters on outputs.

Keywords: EOQ inventory model, screening process, imperfect items, partial backordering

1. Introduction

In practice production does not lead to perfect items always, there may be a certain portion of imperfect items. Few fractions of imperfect items are usually present in the ordered lot size, shortage takes place due to the presence of imperfect items and also due to lead time. Initially, Rosenblatt and Lee [1] and Porteus[2] were thought to be the creation of things with imperfect quality. An interesting variant has been proposed by Roy et al. [3] who consider an economic order quantity model in which a percentage of imperfect quality lot size follows a uniform distribution function. The associated expected average profit function is generalized for the general distribution function. They discussed the model with partial backlogging and lost sale cases for imperfect quality products. An ideal inventory model for items of imperfect quality, inspection faults, shortfall backorders, and sales returns was proposed by Hsu and Hsu [4]. In their model, a closed-form solution is provided for the optimal order size, the optimal order point and the maximum number of backordering units. A method to estimate the vendor's best investment in lowering the defect rate in terms of lowering the joint expected yearly total cost was described in Dey and Giri [5].

The impact of the percentage of defective units on the ordering decisions is analysed in Paul et al. [6], who proposed a joint replenishment model with and without price discount for multiple items that indicates the relationship between family cycle length and the integer number of intervals that the replenishment quality of each item will last. The concavity of profit function is shown graphically in each case. According to the Jaber et al. [7], imperfect products make supply chains less sustainable and make the need for change to improve imperfect products worse. They presented two models for the fraction of imperfect quality items received by one shipment. The first model assumes that imperfect items are sent to a repair shop. While the second model assumes that imperfect items are replenished by perfect ones from a local supplier. An EOQ inventory model is developed Taleizadeh et al. ([8]) with partial backordering and studied four cases by taking into account the time when the lot of imperfect products comes back to the buyer's store after the reparation process. Studied a production lot size and backorders level under an EPQ inventory model for imperfect production and considered three cases depending on when the repaired items enter the inventory Taleizadeh et al. [9].

A maintenance policy was implemented by Liao et al. [10] to improve the dependability of a flawed production system. Two states of production operation are executed, namely: the type I state (out-of-control state) and the type II state (in-control state).An EOQ model was suggested by Eroglu and Ozdemir [11] for a production system with backordering for shortages and defective items. The basic assumption in their model is that 100$ screening of each order-lot contains good and defective items both. These defective items are a collection of imperfect quality and scrap items. They observed that the optimal total profit per unit time decreases with an increase in defective and scrap rates individually. An error in an EOQ model with uncertain supply, qualify by a random fraction of imperfect items, and 100$ screening process were fixed by Maddah and Jaber [12] revisiting Salameh and Jaber [14] paper. They proposed a new model that refine this flaw using renewal theory. The EPQ type inventory model with planned backorders was proposed by Cárdenas-Barrón [16] and determines the economic production quantity for a single item, which is manufactured in a single-stage manufacturing system that yield imperfect quality items as well. Also, all the defective items produced are reworked in the same cycle. Two EPQ models were presented by Hsu and Hsu [17], and the results demonstrate that the timing of when to sell the defective goods has a significant impact on the ideal production lot size and the backorder quantity.

Many researchers deliberated about lot-size inventory models with imperfect items. We refer the reader to Rosenblatt and Lee [1] for a detailed review of the defective items with rework and lot size. They considered that the defective quality items could be reworked instantaneously at a certain extra cost and established that the presence of defective items prompts smaller lot-size. According to Salameh and Jaber [14], it was proposed that selling imperfect quality items in a single batch at a reduced price prior to receiving the next shipment leads to an increase in the average percentage of flawed items, affecting the economic lot size. Besides imperfect production, other factors such as breakages and damages during the transportation and handling process may also result in defective items. This result was incorporated in Salameh and Jaber [14]. The problem of shortages of the item may occur when the imperfect quality items are withdrawn from stock. Due to high inventory holding cost, overproduction is not a solution of it assumed by El-Ansary and Robicheaux [18]. The impact of imperfect production processes on the economic lot-sizing policy was examined by Ben-daya and Hariga [19]. An economic production quantity model incorporating the rework of imperfect quality items was suggested by Chiu [20]. He considered that all imperfect items are not repairable, some parts of them are trash and are discarded. According to Chen [21]'s study, imperfect products frequently have an impact on the supply chain's efficiency, inventory holding costs, and production costs. He used a two-echelon supply chain consisting of a manufacturer and a retailer to deal with the imperfect manufacturing system problem. The correct equation and numerical outcomes for the error discovered in the paper by Salameh and Jaber [14] were presented by Cárdenas-Barrón [15]. This error only affects the optimum value of the order size. For a more recent study on inventory models with imperfect quality items, we mention the readers like Zhou et al. [22], Goyal et al. [23], Patro et al. [24], Keshavarzfard et al. [25], Liao et al. [10], Manna et al. [27], Öztürk [35].

Related recent literature considers that demand is met while screening is being executed, which, in practice, could result in shortages. In view of this, Maddah et al. [13] relaxed this consideration by suggesting an order "overlapping" schemes that provides satisfying demand during the screening process from the "previous order". An economic production quantity model with rework at a single-stage manufacturing system with planned backorders was studied by Chung [33]. He extended Cárdenas-Barrón [16] model and obtained two main results for the annual total relevant cost. Asadkhani et al. [29] developed four EOQ models with different types of imperfect items like salvage, repairable, scrap and reject items. The result show that learning in inspection errors has an important consequence on the profitability. An imperfect production inventory model that includes arbitrary carbon emissions under successive prepayments was developed by Manna et al. [26]. The average profit of the integrated model was maximised by optimising the production rate as well as the defective rate (Manna et al. [28]). This assumption was made that the production system may change from an in-control state to an out-of-control state after a time, which is a random variable. To determine the lower bound of the partial backordering ratio, Yu et al. [36] used an iterative method. By the optimality principle, they show that the shortage is allowed only if the ratio is greater than its lower bound. A periodic delivery policy was put forth for a production-inventory model with vendor-buyer integration by Wee et al. [31]. By combining the assumptions of permissible shortage backordering and the effects of varying backordering cost values, Wee et al. [30] extended Salameh and Jaber [14] model. In order to calculate the expected net profit per unit of time, Chang and Ho [32] revisited the work of Wee et al. [30] and also used the renewal-reward theorem. They used the algebraic methods to obtain the exact closed-form solutions for optimal lot size, backordering quantity and maximum expected profit. The existence of a particular optimal lot size that maximises the anticipated total profit is estimated by Moussawi-Haidar et al. [34]. Also, they studied the effect of deterioration and holding cost on the optimal lot size and backorder level.

It is significant for a manager of any organization to control and handle the inventories of perfect and imperfect quality items. However, one of the weaknesses of current inventory models is the unrealistic assumption that the demand rate is constant. In our study we assume a demand rate is price sensitive, demand varies with respect to selling price \$ p per unit. Each lot received comprises some percentage of imperfect quality, with a known probability density function $f(p)$. The imperfect items are sold in the nearby market by reducing the selling price per unit called salvage price. This type of perfect and imperfect quality item usually occurs in many industries. In this paper, we consider an EOQ inventory model with demand rate as a function of the selling price. At the start of the process, the items received from the supplier, which is far away from the buyer, are inspected. In the meantime, a 100\$ inspection process of the lot is carried out at a rate of α, items of imperfect quality are sorted, kept in one batch and sold at a salvage price of c_s per unit before receiving the next shipment. It is assumed that the imperfect quality items are collected as a single batch and replaced by the perfect items within the replenishment period T. Partial backordering is permissible. The optimum selling price and percentage of duration in which inventory level is positive are obtained by the optimization process to derive an optimal profit.

In traditional inventory models such as the economic order quantity (EOQ) the sole objective is to maximize the total profit, typically to minimize holding cost and ordering cost. These models do not consider the effect of price on the demand rate. Replenishment of imperfect items in an EOQ inventory model with partial backordering proposed model, which dealt with the optimum profit function but considered constant demand rate. However, due to the defective items and inspection process, demand may vary with respect to selling price p. In this direction, this paper develops an EOQ type inventory model with imperfect items and price-sensitive demand rate for determining the optimal unit price, percentage of duration in which inventory level is positive and total profit. The rest of the model explanation is as follows. In section 2, we presented notations and assumptions used throughout the proposed paper. In section 3, a mathematical model is developed for the three cases when partial backordering is allowed. Section 4 presents numerical examples to exemplify the important aspects of the model. Section 5 show the sensitivity analysis performed on the proposed models. Section 6 presents managerial implications of the models. Finally, a conclusion and future research of the present model are provided in section 7.

2. Notations and Assumptions

I. Notations

The following notations are incorporated to depict the proposed model:

- T the cycle length,
- t_1 the time duration in which inspection section occur,
- α the inspection rate,
- x the fraction of imperfect items,
- $g(x)$ the probability density function of imperfect items,
- y the fraction of backordered quantity,
- π the cost of lost sales,
- σ the backordered cost,
- h_e the holding cost of emergency purchased items,
- h the holding cost,
- c_o the buyer's ordering cost,
- c_p the purchasing cost per unit of an emergency order,
- c_u the initial purchasing cost of the unit item,
- c_s the salvage price of imperfect items,
- c_i the inspection cost,
- $D(p)$ the number of units demanded per year where $D(p) = a - bp > 0$, $a, b > 0$,
- p unit price,
- t the percentage of duration in which inventory level is positive,
- q the order quantity,
- TP the total profit.

II. Assumptions

The proposed model is constructed based on the subsequent assumptions:

1. The planning horizon is infinite.
2. Shortages are admissible. Partial backordering is allowed.
3. Replenishment is allowed and is equal to the imperfect quality items for one cycle period.

4. Demand is modelled as a price function. The demand rate at selling price p is formulated as $D(p) = a - bp$, where a is representing the maximum demand of the item. b is the coefficient which reflects the choice of price, $a, b > 0$, such that $a - bp > 0$.
5. The proportion of imperfect items x and its probability density function are driven at the end of the cycle length.
6. Purchasing cost c_p from a local supplier is higher than the initial purchasing cost c_u. Moreover, initial purchasing cost c_u is higher than salvage price c_s, $c_s < c_u < c_p$.
7. When the newly purchased items that replace the imperfect items are added to the stock, the inventory level is either zero or negative.
8. Inspection rate is constant and known.
9. The inventory level is I_{max}, at the beginning of the period.
10. In order to distinguish the imperfect items, all products are inspected carefully at the inspection rate α in time duration t_1, where $t_1 = I_{max}/\alpha$.
11. The inspection rate α per unit time is greater than $D(p)$.
12. The reordered items $\rho t T D(p)$ is used to meet the demand during the shortage period partially.

3. Mathematical Formulation

This section extends the work of Taleizadeh *et al.*, (2000) by assuming a price-sensitive demand rate. Three cases are explained with the defective items and partial backordering. Inventory management is a fundamental implementation of price and time control in any modern organization and retail industry. These days, this implementation is important to a supply manager of any supply chain to control and maintain the inventories of perfect and imperfect quality items due to advanced technologies, growing market competition and customer awareness. Consider a vendor who buys products at the beginning of the inventory cycle and satisfy the customer demands during the cycle.

I. *Model I: The reorder is made when the inventory level is zero*

Formulation and solution of the model:
In our model, the initial inventory level at the beginning of the cycle is I_{max} and is equal to $tTD(p)$. This inventory contains perfect and imperfect quality items. Inspection section separates the perfect and imperfect quality items with duration t_1 at an inspection rate α, i.e., $t_1 = \frac{tTD(p)}{\alpha}$. This section identifies $xtTD(p)$ of imperfect quality items and $(1-x)tTD(p)$ is of perfect quality items. The imperfect items are sold at a reduced price c_s so a part of demand of perfect items is still remain to fulfill. To fulfil the demand of buyers, the perfect quality items are purchased from a local supplier at a cost c_p, equal to the imperfect quality items. The reorder is made when the inventory level is zero. At the end of the cycle, the shortage is $(1-t)TD(p)$. y fraction of shortage is backordered at a charge of σ per unit and the rest is lost sales at a charge of π per unit. The total ordered quantity per cycle is $q = tTD(p) + y(1-t)TD(p)$. The logistic representation of the physical scenario is shown in Fig. 1. The inventory holding cost per cycle is calculated from Fig. 1 (summing up the areas) as

$$HC = h\left(\frac{(1-x)^2t^2T(a-bp)}{2} + \frac{xTt^2(a-bp)^2}{\alpha}\right) + \frac{h_e x^2 t^2 T(a-bp)}{2} \quad (1)$$

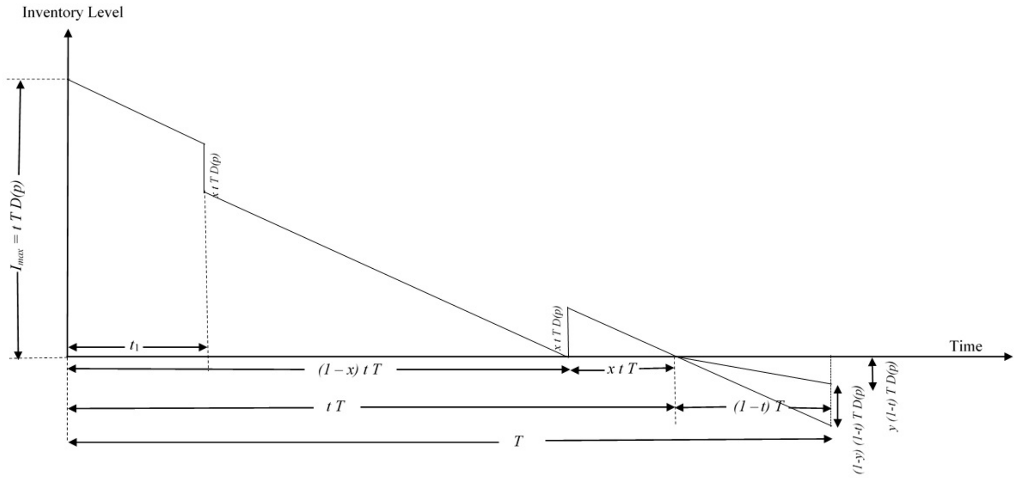

Figure 1: *Behaviour of the inventory level over time*

The shortage cost is given by

$$SC = \frac{(1-t)^2(a-bp)T\sigma y}{2} + \pi(1-y)(1-t)(a-bp) \tag{2}$$

The total revenue per cycle is $p\{tD(p) + y(1-t)D(p)\}$ where p is the unit price. The total profit is the difference of total revenue and the total cost per cycle, and is given as

$$\begin{aligned} TP(t,p) &= p\{t(a-bp) + y(1-t)(a-bp)\} + c_s xt(a-bp) - \frac{c_o}{T} - c_u(t + y(1-t))(a-bp) \\ &- c_p xt(a-bp) - \frac{(1-t)^2(a-bp)T\sigma y}{2} - h\left(\frac{(1-x)^2 t^2 T(a-bp)}{2} + \frac{xTt^2(a-bp)^2}{\alpha}\right) \\ &- \pi(1-y)(1-t)(a-bp) - c_i t(a-bp) - \frac{h_e x^2 t^2 T(a-bp)}{2} \end{aligned} \tag{3}$$

Our objective is to develop an optimal inventory model to determine the selling price and the percentage of duration in which the inventory level is positive and the total profit. The following cases are considered now

I.1 Case I:

Let the percentage of duration in which inventory level is positive is obtained in terms of p. Then, we have to maximize $TP(p)$, where $t(p)$ ia a function of p, satisfying the constraints $0 < y \leq 1$, $0 < x \leq 1$, $a - bp > 0$, $p > 0$.

In this case, for maximum of TP,

$$dTP(p)/dp = 0 \text{ and } d^2TP(p)/dp^2 < 0,$$

must be satisfied.

Now, differentiate Eq. (3) with respect to t and equating to zero, we obtain

$$t(p) = \frac{\alpha(-c_i + \pi + p - c_u - \pi y - py + \sigma Ty + c_u y - c_p x + c_s x)}{T\left[h\{\alpha(x-1)^2 + 2D(p)x\} + \alpha(\sigma y + h_e x^2)\right]} \tag{4}$$

The expected value of variable x is substituted in Eq. (4)

$$t(p) = \frac{\alpha(-c_i + \pi + p - c_u - \pi y - py + \sigma Ty + c_u y - c_p E(x) + c_s E(x))}{T\left[h\{\alpha(E(x)-1)^2 + 2D(p)E(x)\} + \alpha(\sigma y + h_e E(x)^2)\right]}$$

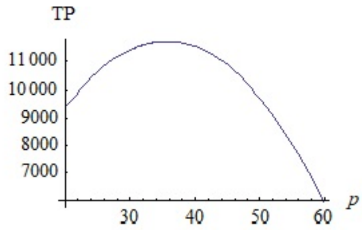

Figure 2: *Graphical representation of the total profit $TP(p)$ with respect to p*

Now Substituting this value in Eq. (3) and differentiating with respect to p leads to

$$\begin{aligned}
\frac{dTP(p)}{dp} &= \frac{-2bhD(p)h_e\alpha^2x^3(M2)^2}{T(M1)^3} - \frac{2bh^2D(p)\alpha x(M3)(M2)^2}{T(M1)^3} - \frac{D(p)h_e\alpha^2(y-1)x^2(M2)}{T(M1)^2} \\
&- \frac{hD(p)\alpha(y-1)(M3)(M2)}{T(M1)^2} + \frac{bhD(p)\alpha x(M2)^2}{T(M1)^2} + \frac{bh_e\alpha^2x^2(M2)^2}{2T(M1)^2} + \frac{bh\alpha(M3)(M2)^2}{2T(M1)^2} \\
&+ \frac{2bc_ihD(p)\alpha x(M2)}{T(M1)^2} + \frac{2bc_phD(p)\alpha x^2(M2)}{T(M1)^2} - \frac{2bhD(p)c_s\alpha x^2(M2)}{T(M1)^2} + \frac{c_iD(p)\alpha(y-1)}{T(M1)} \\
&+ \frac{D(p)c_s\alpha(1-y)x}{T(M1)} + \frac{c_pD(p)\alpha(y-1)x}{T(M1)} + \frac{bc_i\alpha x(M4)}{T(M1)} - \frac{bc_s\alpha x(M4)}{T(M1)} \\
&+ b\pi(y-1)\left[-1-\frac{x(M2)}{T(M1)}\right] + \frac{1}{2}b\sigma Ty\left[-1-\frac{x(M2)}{T(M1)}\right]^2 \\
&+ (D(p)-bp+bc_u)\left[y+\frac{\alpha y(M2)}{T(M1)}-\frac{x(M2)}{T(M1)}\right] - \frac{D(p)\sigma\alpha y(M5)(M4)}{T(M1)^3} \\
&+ \frac{(\pi+p-c_u)D(p)\alpha(y-1)(M4)}{T(M1)^2}
\end{aligned} \tag{5}$$

where

$$M1 = h(M3) + \alpha(\sigma y + h_ex^2),\; M2 = c_i - p + c_u + \pi(y-1) + py - \sigma Ty - c_uy + c_px - c_sx,$$
$$M3 = \alpha(x-1)^2 + 2D(p)x,\; M4 = \alpha(y-1)(\sigma y + h_ex^2) + h\left[\alpha(y-1)(x-1)^2 + 2x\{a(y-1)\right.$$
$$\left. +b\,(c_i + c_u \; +\pi(y-1) - \sigma Ty - c_uy + c_px - c_sx)\}\right],\; M5 = c_i\alpha - p\alpha + hT\alpha + c_u\alpha + \pi\alpha(y-1)$$
$$+ p\alpha y - c_u\alpha y + 2D(p)hTx + c_p\alpha x - c_s\alpha x - 2hT\alpha x + (h+h_e)T\alpha x^2$$

whose solution is given by setting the Eq. (5) to zero and solving for p. Clearly, $\frac{d^2TP(p)}{dp^2}$ is negative (see Fig. 2), which implies that $TP(p)$ is concave.

I.2 Case II.

Let p and the percentage of duration in which inventory levels are positive t be decision variables.
Here, we have to maximize $TP(t,p)$
such that $0 < y \leq 1$, $0 < x \leq 1$, $a - bp > 0$, $p > 0$.

The expression for $\frac{\partial TP(t,p)}{\partial t} = 0$ and $\frac{\partial TP(t,p)}{\partial p} = 0$, are complicated and it is possible to obtain the optimal values of p and t analytically by using Mathematica Software and satisfying the conditions $\frac{\partial TP^2(t,p)}{\partial t^2} < 0$, $\frac{\partial TP^2(t,p)}{\partial p^2} < 0$ and $\frac{\partial TP^2(t,p)}{\partial t^2}.\frac{\partial TP^2(t,p)}{\partial p^2} - \left(\frac{\partial TP^2(t,p)}{\partial p \partial t}\right)^2 > 0$.
Therefore, we draw a graph of Eq. (3) for the parameter values interpreted in the numerical example 1. The Fig. 7 demonstrate that the function $TP(p,t)$ is concave having a clear maximum.

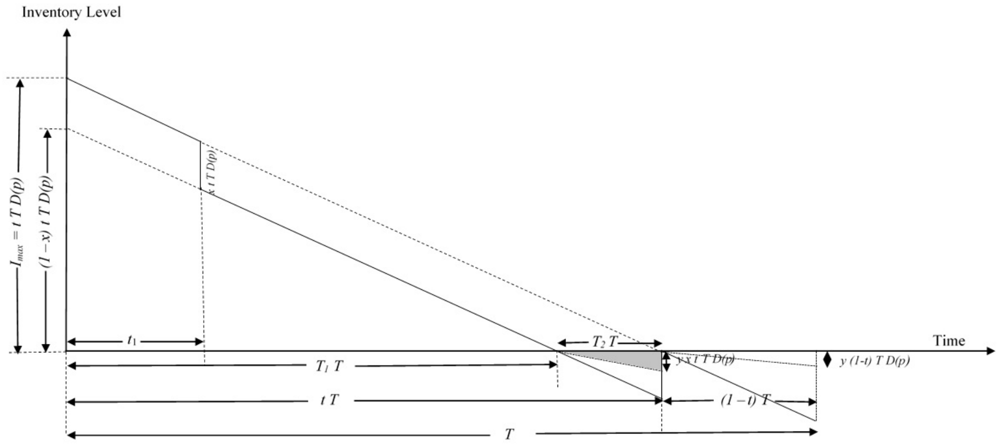

Figure 3: *Behaviour of the inventory level over time*

For this we have,

$$\frac{\partial TP^2(t,p)}{\partial t^2}.\frac{\partial TP^2(t,p)}{\partial p^2} - \left(\frac{\partial TP^2(t,p)}{\partial p \partial t}\right)^2$$
$$= \frac{1}{\alpha^2}\Big[2bD(p)T\{\alpha y + t\alpha(1-y) + bt^2hTx\}(M1) - \{a(\alpha - \alpha y + 4bthTx)$$
$$+b\,(c\alpha - 2p\alpha + thT\alpha + c_u\alpha + \pi\alpha(y-1) + 2p\alpha y - \sigma T\alpha y + t\sigma T\alpha y - c_u\alpha y$$
$$-4bthpTx + c_p\alpha x - c_s\alpha x - 2thT\alpha x + thT\alpha x^2 + tTh_e\alpha x^2\Big)\}^2\Big] \tag{6}$$

For a particular case it can be proved numerically by considering the data given in section 4, Example 1, $\frac{\partial^2 TP}{\partial p^2} = -19.52 < 0$, $\frac{\partial^2 TP}{\partial t^2} = -150.48 < 0$, $\frac{\partial^2 TP}{\partial p^2}.\frac{\partial^2 TP}{\partial t^2} - \left(\frac{\partial^2 TP}{\partial p \partial t}\right)^2 = 2893.28 > 0$. This implies that $TP(p,t)$ is concave in nature with respect to p and t.

II. *Model II: The reorder is made when the backordered quantity is equal to the imperfect quality items*

Note that t is the percentage of duration in which the inventory level is positive. In the beginning of cycle, the inventory level begins with the order quantity $I_{max} = tTD(p)$. These products are inspected to distinguish perfect and imperfect items separately. At the end of the inspection process at a rate of α units per unit time during the period t_1, a part of the perfect quality items is used to serve the customer demand at a price of p per unit. At the end of the cycle, the imperfect quality items $xtTD(p)$ are withdrawn and sold at a salvage price c_s. The reorder items are received when the shortage is equal to $xtTD(p)$. After adding these products to the negative inventory lot, the inventory level reaches zero. The shortage is $(1-t)TD(p)$. The backordered percentage of shortage is y and the rest is lost sale. Let x be the fraction of imperfect items. Specifically, we have $t = T_1 + T_2$ where $T_1 = (1-x)t$, $T_2 = xt$. The total ordered quantity per cycle be $q = T_1TD(p) + yT_2TD(p) + y(1-t)TD(p)$. The behaviour of the inventory level over time is illustrated in Fig. 3.

The holding cost per cycle is given as

$$HC = h\left(\frac{(1-x)^2t^2T(a-bp)}{2} + \frac{xTt^2(a-bp)^2}{\alpha}\right) \tag{7}$$

and the shortage cost per cycle is

$$SC = \sigma\frac{yx^2t^2TD(p)}{2} + \sigma\frac{y(1-t)^2TD(p)}{2} + \pi(1-y)T_2D(p) + \pi(1-y)(1-t)D(p)$$

Then the shortage cost can be rewritten as

$$SC = \sigma\frac{yx^2t^2TD(p)}{2} + \sigma\frac{y(1-t)^2TD(p)}{2} + \pi(1-y)(1-T_1)(a-bp) \tag{8}$$

Therefore, the total profit per cycle of length T becomes

$$\begin{aligned} TP(t,p) = {} & p\{(1-x)tD(p) + y(1-(1-x)t)D(p)\} + c_sxtD(p) - \frac{c_o}{T} - c_u(t+y(1-t))D(p) \\ & - c_itD(p) - c_pxtD(p) - h\left(\frac{(1-x)^2t^2TD(p)}{2} + \frac{xt^2TD(p)^2}{\alpha}\right) - \frac{\sigma yx^2t^2TD(p)}{2} \\ & - \frac{\sigma y(1-t)^2TD(p)}{2} - \pi(1-y)(1-(1-x)t)D(p) \end{aligned} \tag{9}$$

The following cases are considered

II.1 Case I:

Let the percentage of duration in which inventory level is positive is obtained in terms of p. Then, we have to maximize $TP(p)$, where $t(p)$ ia a function of p, satisfying the constraints $0 < y \leq 1$, $0 < x \leq 1$, $a - bp > 0$, $p > 0$. In this case, for maximum of TP, $dTP(p)/dp = 0$ and $d^2TP(p)/dp^2 < 0$ must be satisfied. Now, differentiating Eq. (9) with respect to t and equating to zero, we obtain $t(p) = -\frac{\alpha(A6)}{T(A2)}$. Substituting this in Eq. (9) and differentiating with respect to p gives

$$\begin{aligned} \frac{\partial TP}{\partial p} = {} & \frac{-2bhD(p)\sigma\alpha^2yx^3(A1)^2}{T(A2)^3} - \frac{-2bh^2D(p)\alpha x(A5)(A1)^2}{T(A2)^3} + \frac{2bhD(p)c_s\alpha x^2(A1)}{T(A2)^2} + \frac{bh\alpha(A1)^2(A5)}{2T(A2)^2} \\ & - \frac{D(p)\alpha^2\sigma(y-1)(x-1)yx^2(A1)}{T(A2)^2} - \frac{hD(p)\alpha(y-1)(x-1)(A1)(A5)}{T(A2)^2} + \frac{bhD(p)\alpha x(A1)^2}{T(A2)^2} \\ & + \frac{2bc_ihD(p)\alpha x(A6)}{T(A2)^2} - \frac{bc_i\alpha(A6)}{T(A2)} + \frac{2bc_phD(p)\alpha x^2(A6)}{T(A2)^2} + \frac{D(p)c_s\alpha(y-1)(x-1)x}{T(A2)} \\ & + \frac{c_pD(p)\alpha x(A4)}{T(A2)} - \frac{bc_s\alpha x(A1)}{T(A2)} - \frac{bc_p\alpha x(A6)}{T(A2)} + \frac{b\sigma Ty}{2}\left[\frac{\alpha(A1)}{T(A2)} - 1\right]^2 \\ & - b\pi(y-1)\left[1 + \frac{\alpha(x-1)(A1)}{T(A2)}\right] - bp\left[y + \frac{\alpha(y-1)(x-1)(A1)}{T(A2)}\right] + \frac{D(p)\sigma\alpha y}{T(A2)^3}\Big[c_i\alpha \\ & - p\alpha + hT\alpha + c_u\alpha + p\alpha y - c_u\alpha y + 2ahTx - 2bhpTx + c_p\alpha x + p\alpha x - c_s\alpha x - 2hT\alpha x \\ & - p\alpha yx + hT\alpha x^2 + \sigma T\alpha yx^2 + \pi\alpha(A4)\Big]\Big[\sigma\alpha(y-1)y(A3) + h\{\alpha(y-1)(x-1)^3 \\ & + 2x\left(a(y-1)(x-1)b(A7)\right)\}\Big] + \frac{D(p)c_u\alpha(y-1)}{T(A2)^2}\Big[\sigma\alpha(y-1)y(A3) + h\{\alpha(y-1)(x-1)^3 \\ & + 2x\left(a(y-1)(x-1) + b(A7)\right)\}\Big] + \\ & \frac{\pi D(p)\alpha(y-1)(x-1)}{T(A2)^2}\Big[\sigma\alpha(y-1)y(A3) + h\{\alpha(y-1)(x-1)^3 + 2x\left(a(y-1)(x-1) + b(A7)\right)\}\Big] \\ & + \frac{pD(p)\alpha(y-1)(x-1)}{T(A2)^2}\Big[\sigma\alpha(y-1)y(A3) + h\{\alpha(y-1)(x-1)^3 + 2x\left(a(y-1)(x-1) + b(A7)\right)\}\Big] \\ & + \frac{pD(p)\alpha(y-1)(x-1)}{T(A2)^2}\Big[\sigma\alpha(y-1)y(A3) + h\{\alpha(y-1)(x-1)^3 + 2x\left(a(y-1)(x-1) + b(A7)\right)\}\Big] \\ & + \frac{c_iD(p)\alpha(A4)}{T(A2)} + bc_u\left[y + \frac{\alpha(A1)}{T(A2)} - \frac{\alpha y(A1)}{T(A2)}\right] + D(p)\left[y + \frac{\alpha(y-1)(x-1)(A1)}{T(A2)}\right] \end{aligned}$$

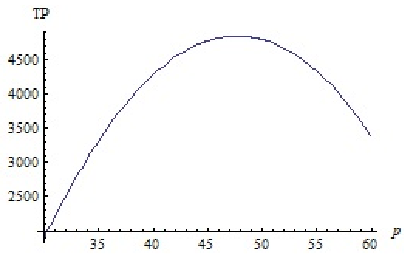

Figure 4: *Graphical representation of the total profit $TP(p)$ with respect to p*

where

$A1 = (A7) - p(A4),\ A2 = h\{\alpha(x-1)^2 + 2D(p)x\} + \sigma\alpha y(1+x^2),\ A3 = -(1-x)(1+x^2),$
$A4 = -(1-x)(1-y),\ A5 = \alpha(x-1)^2 + 2D(p)x,\ A6 = p(A4) + (A7) - Ty(\alpha+\sigma),$
$A7 = -c_i - c_u + \sigma Ty + c_u y - \pi(A4) - c_P x + c_s x,$

$$A8 = \{c_i\alpha - p\alpha + hT\alpha + c_u\alpha + p\alpha y - c_u\alpha y + 2ahTx - 2bhpTx + c_p\alpha x + p\alpha x - c_s\alpha x - 2hT\alpha x - p\alpha yx + hT\alpha x^2 + \sigma T\alpha y x^2 + \pi\alpha(A4)\}\left[\sigma\alpha(y-1)y(A3) + h\{\alpha(y-1)(x-1)^3 + 2x\left(-a(A4) + b(A7)\right)\}\right]$$

Note that as long as $a - bp > 0$, it can be shown by differentiating $TP(p)$ with respect to p twice that $TP(p)$ is concave in p (see Fig. 4).

II.2 Case II.

Let p and the percentage of duration in which inventory levels are positive t be decision variables. Here, we have to maximize $TP(t,p)$ such that $0 < y \leq 1$, $0 < x \leq 1$, $a - bp > 0$, $p > 0$. The expression for $\frac{\partial TP(t,p)}{\partial t} = 0$ and $\frac{\partial TP(t,p)}{\partial p} = 0$, are complicated and it is possible to obtain the optimal values of p and t analytically by using Mathematica Software and satisfying the conditions $\frac{\partial TP^2(t,p)}{\partial t^2} < 0$, $\frac{\partial TP^2(t,p)}{\partial p^2} < 0$ and $\frac{\partial TP^2(t,p)}{\partial t^2}.\frac{\partial TP^2(t,p)}{\partial p^2} - \left(\frac{\partial TP^2(t,p)}{\partial p\partial t}\right)^2 > 0$. Therefore, we draw a graph of Eq. (9) for the parameter values interpreted in the numerical example 1. The Fig. 8 demonstrate that the function $TP(p,t)$ is concave having a clear maximum. For this, we have

$$\frac{\partial TP^2(t,p)}{\partial t^2}.\frac{\partial TP^2(t,p)}{\partial p^2} - \left(\frac{\partial TP^2(t,p)}{\partial p\partial t}\right)^2 = \frac{1}{\alpha^2}\Big[2b(a-bp)T\{bht^2Tx + (t(1-x)(1-y)+y)\alpha\}$$
$$\{2hx(a-bp) + \alpha\left(h(x-1)^2 + (1+x^2)y\sigma\right)\} - \{a\left(4bhtTx + (1-x)(1-y)\alpha\right)$$
$$+b\left(-4bhptTx + \alpha\left(c_i + c_u + c_p x - c_s x + (1-x)^2 htT - c_u y - (2p+\pi)(1-x)(1-y) - Ty\sigma\right.\right.$$
$$\left.\left.+(1+x^2)tTy\sigma\right)\right)\}^2\Big] \tag{10}$$

For a particular case it can be proved numerically by considering the data given in section 4, Example 2, $\frac{\partial^2 TP}{\partial p^2} = -19.48$, $\frac{\partial^2 TP}{\partial t^2} = -150.62$, $\frac{\partial^2 TP}{\partial p^2}.\frac{\partial^2 TP}{\partial t^2} - \left(\frac{\partial^2 TP}{\partial p\partial t}\right)^2 = 2892.35$. Note that we have $\partial^2 TP(p,t)/\partial p^2 < 0$, $\partial^2 TP(p,t)/\partial t^2 < 0$, and $\partial^2 TP(p,t)/\partial p\partial t > 0$, which implies that there exist unique values of p and t that maximize Eq. (9). The optimal solution can be obtained by setting Eqs. (**??**) and (**??**) to zero in Mathematica Software, which leads to $TP(p,t)$ is a concave function.

III. *Model III: The reorder is made when the shortage is yet continued*

We considered the problem of a lot size at the beginning of the cycle $tTD(p)$, which is inspected at an inspection rate α in a time duration t_1 where $t_1 = tTD(p)/\alpha$. We assumed

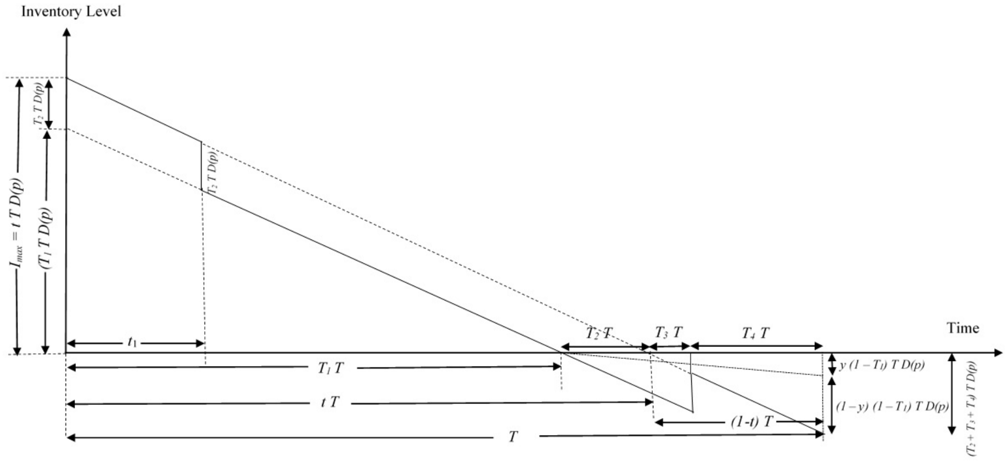

Figure 5: *Behaviour of the inventory level over time*

that the duration length is divided into four parts. The parts are symbolize by T_i, $i = 1, 2, 3, 4$. Let t be the percentage of duration in which inventory level is positive and $t = T_1 + T_2$. Also $1 - t = t_3 + t_4$. Let x be the fraction of imperfect items found and are sold at the end of the cycle at a salvage price c_s. Then $T_1 = (1 - x)t$, $T_2 = xt$. The imperfect items are replaced by perfect items from the local supplier at the cost of c_p. The reorder items are obtained when the shortage level is $(T_2 + T_3)TD(p)$. After adding these items to the inventory system, the inventory level is negative means shortage is still continued. Let y be the fraction of backordered quantity and rest is considered as lost sale. The backordered quantity has backordered cost σ per unit and the lost sale cost per unit is π. At the end of the cycle, the shortage level is $(T_2 + T_3 + T_4)TD(p)$. The ordered quantity per cycle is $q = TD(p) + y(1 - t)TD(p)$. The behaviour of the inventory system is illustrated in Fig. 5. The holding cost per cycle is determined from Fig. 5

$$HC = h\left(\frac{(1-x)^2t^2T(a-bp)}{2} + \frac{xTt^2(a-bp)^2}{\alpha}\right) \tag{11}$$

The shortage cost per cycle is given by

$$SC = \sigma\frac{(T_2+T_3+T_4)(T_3+T_4)yTD(p)}{2} + \pi(1-y)(T_3+T_4)(a-bp) \tag{12}$$

The total profit per cycle of length T becomes

$$\begin{aligned} TP(p,t) = {} & pD(p)\{t+y(1-t)\} + c_sxtD(p) - \frac{c_o}{T} - c_uD(p)\{t+y(1-t)\} - c_itD(p) \\ & - h\left[\frac{D(p)t^2T(1-x)^2}{2} + \frac{xTt^2D(p)^2}{x}\right] - \frac{\sigma y(1-(1-x)t)(1-t)D(p)T}{2} \\ & - \pi(1-y)(1-t)D(p) - c_pxtD(p) \end{aligned} \tag{13}$$

III.1 Case I:

Let the percentage of duration in which inventory level is positive is obtained in terms of p. Then, we have to maximize $TP(p)$, where $t(p)$ ia a function of p, satisfying the constraints $0 < y \leq 1$, $0 < x \leq 1$, $a - bp > 0$, $p > 0$. In this case, for maximum of TP, $dTP(p)/dp = 0$ and $d^2TP(p)/dp^2 < 0$ must be satisfied.
Differentiating Eq. (13) with respect to t and equating to zero, we obtain $t(p) = -\frac{\alpha(B2)}{2T(B1)}$.

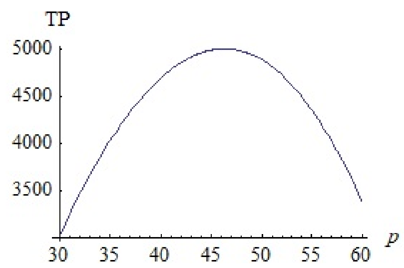

Figure 6: *Graphical representation of the total profit $TP(p)$ with respect to p*

Now, substituting this value in Eq. (13) and differentiating with respect to p, we have

$$\begin{aligned}\frac{\partial TP}{\partial p} &= \frac{2bc_i hD(p)\alpha x(B2)}{T(B1)^2} + \frac{2bc_p hD(p)\alpha x^2(B2)}{T(B1)^2} - \frac{2bhD(p)c_s \alpha x^2(B2)}{T(B1)^2} \\ &+ \frac{ac_i D(p)\alpha(y-1)}{T(B1)} + \frac{2(c_p - c_s)D(p)\alpha(y-1)x}{T(B1)} - \frac{bc_i\alpha(B2)}{T(B1)} - \frac{b\alpha x(c_p - c_s)(B2)}{T(B1)} \\ &+ \frac{b(p-c_u)(B5)}{T(B1)} - \frac{D(p)(B5)}{T(B1)} + 2b\pi(1-y)\left(1 + \frac{\alpha(B2)}{2T(B1)}\right) + b\sigma Ty\left(1 + \frac{\alpha(B2)}{2T(B1)}\right) \\ &\left(1 - \frac{\alpha(x-1)(B2)}{2T(B1)}\right) - \frac{h\alpha(B2)}{4T(B1)^3}\{4bhD(p)\alpha(x-1)^2 x(B2) + 8bhD(p)^2 x^2(B2) \\ &+ 4D(p)\alpha(y-1)(x-1)^2(B1) + 8D(p)^2(y-1)x(B1) - b\alpha(x-1)^2(B1)(B2) \\ &- 4bD(p)x(B1)(B2)\} + \frac{2D(p)\alpha(\pi + p - c_u)(y-1)(B6)}{T(B1)^2} + \frac{\sigma\alpha y D(p)(x-1)(B6)(B7)}{2T(B1)^3} \\ &- \frac{\sigma\alpha y D(p)(B6)}{(B1)^2}\left(1 - \frac{\alpha(x-1)(B2)}{2T(B1)}\right)\end{aligned} \tag{14}$$

where

$$\begin{aligned}B1 &= -\sigma\alpha y(x-1) + h\{\alpha(x-1)^2 + 2D(p)x\} \\ B2 &= 2c_i - 2p + 2c_u + 2\pi(y-1) + 2py - 2\sigma Ty - 2c_u y + 2c_p x - 2c_s x + \sigma Tyx \\ B3 &= -2p\alpha + 2c_u\alpha - 2c_i\alpha(y-1) - 2\pi\alpha(y-1)^2 + 4p\alpha y - 2hT\alpha y - 2\sigma T\alpha y - 4c_u\alpha y \\ &\quad - 2p\alpha y^2 + 2c_u\alpha y^2 + 2c_p\alpha x - 2c_s\alpha x - 4ahTyx + 4bhpTyx - 2c_p\alpha yx + 2c_s\alpha yx \\ &\quad + 4hT\alpha yx + \sigma T\alpha yx + \sigma T\alpha y^2 x - 2hT\alpha yx^2 \\ B4 &= 2c_i + 2c_u + 2\pi(y-1) - 2\sigma Ty - 2c_u y + 2c_p x - 2c_s x + \sigma Tyx \\ B5 &= -2p\alpha + 2c_u\alpha - 2c_i\alpha(y-1) - 2\pi\alpha(y-1)^2 + 4p\alpha y - 2hT\alpha y - 2\sigma T\alpha y - 4c_u\alpha y \\ &\quad - 2p\alpha y^2 + 2c_u\alpha y^2 + 2c_p\alpha x - 2c_s\alpha x - 4ahTyx + 4chpTyx - 2c_p\alpha yx + 2c_s\alpha yx \\ &\quad + 4hT\alpha yx + \sigma T\alpha yx + \sigma T\alpha y^2 x - 2hT\alpha yx^2 \\ B6 &= \sigma\alpha y(A4) + h\left[\alpha(y-1)(x-1)^2 + x\{2a(y-1) + b(B4)\}\right] \\ B7 &= 2c_i\alpha - 2p\alpha + 2hT\alpha + 2c_u\alpha + 2\pi\alpha(y-1) + 2p\alpha y - 2c_u\alpha y + 4ahTx - 4bhpTx \\ &\quad + 2c_p\alpha x - 2c_s\alpha y - 4hT\alpha x - \sigma T\alpha yx + 2hT\alpha x^2\end{aligned}$$

Now, $\frac{dTP(p)}{dp} = 0$ implies a unique value of p using Mathematica software, which is complex in nature. Clearly, $\frac{d^2TP(p)}{dp^2}$ is negative (see Fig. 6), which implies that $TP(p)$ is concave.

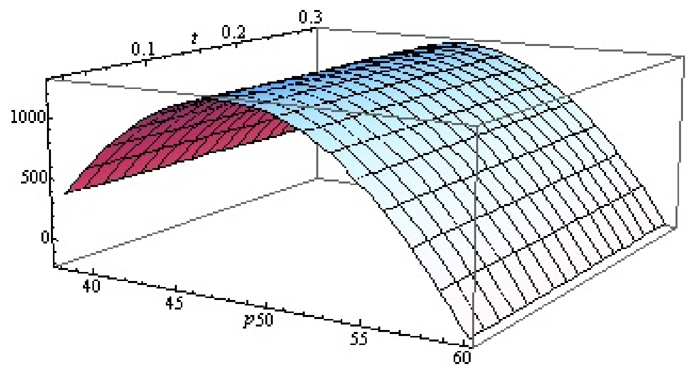

Figure 7: *Graphical representation of the total profit $TP(p,t)$ with respect to p and t*

Table 1: *Optimal value for the decision variables*

	p^*	t^*	$TP^*(p^*,t^*)$
Model I	47.71	21.00%	1278.10
Model II	47.69	14.20%	1276.41
Model III	47.00	16.70%	1272.97

III.2 Case II.

Let p and the percentage of duration in which inventory levels are positive t be decision variables. Here we have maximize $TP(t,p)$ such that $0 < y \leq 1$, $0 < x \leq 1$, $a - bp > 0$, $p > 0$. The expression for $\frac{\partial TP(t,p)}{\partial t} = 0$ and $\frac{\partial TP(t,p)}{\partial p} = 0$, are complicated and it is possible to obtain the optimal values of p and t analytically by using Mathematica Software and satisfying the conditions $\frac{\partial TP^2(t,p)}{\partial t^2} < 0$, $\frac{\partial TP^2(t,p)}{\partial p^2} < 0$ and $\frac{\partial TP^2(t,p)}{\partial t^2}.\frac{\partial TP^2(t,p)}{\partial p^2} - \left(\frac{\partial TP^2(t,p)}{\partial p \partial t}\right)^2 > 0$, where

$$\left(\frac{\partial^2 TP}{\partial p^2}\right)\left(\frac{\partial^2 TP}{\partial t^2}\right) - \left(\frac{\partial^2 TP}{\partial p \partial t}\right)^2 = -\frac{1}{4\alpha^2}\Big[-8b(a-bp)T\{bht^2Tx + (t+y-ty)\alpha\}\{2hx(a-bp) +(x-1)\alpha(h(x-1) - y\sigma)\} + \{2a(4bhtTx + \alpha - y\alpha) + b(-8bhptTx + \alpha\{2c_i + 2c_u + 2c_p x - 2c_s x +2htT(1-x)^2 + 4p(y-1) - 2c_u y - 2\pi + 2y\pi - 2Ty\sigma + 2tTy\sigma + Txy\sigma - 2tTxy\sigma\})\}^2\Big] \tag{15}$$

For a particular case it can be proved numerically by considering the data given in section 4, Example 3, $\frac{\partial^2 TP}{\partial p^2} = -19.50 < 0$, $\frac{\partial^2 TP}{\partial t^2} = -151.49 < 0$, $\frac{\partial^2 TP}{\partial p^2}.\frac{\partial^2 TP}{\partial t^2} - \left(\frac{\partial^2 TP}{\partial p \partial t}\right)^2 = 2906.43 > 0$. Our objective is to determine the optimum values of p and t, which maximise the total profit per cycle. We set $\frac{\partial TP}{\partial p} = 0 = \frac{\partial TP}{\partial t}$ in Mathematica Software, to get the optimum values of p and t. Therefore, we draw a graph of Eq. (13) for the parameter values interpreted in the numerical example 3. The Fig. 9 demonstrate that the function $TP(p,t)$ is concave having a clear maximum.

4. Numerical Examples

Example 1**.** *Model I:* We considered the values of the parameters in suitable units such that cycle length $T = 0.028$ years, market potential $a = 700$, price sensitivity of demand $b = 10$, salvage

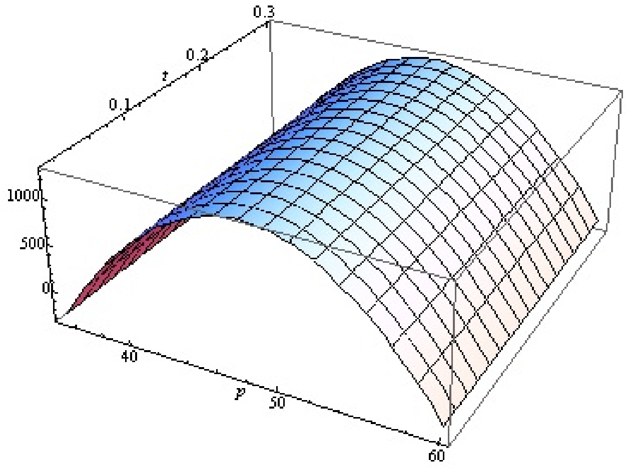

Figure 8: *Graphical representation of the total profit $TP(p,t)$ with respect to p and t*

price $c_s = \$20$ per unit, initial purchasing cost $c_u = \$25$ per unit, inspection cost $c_i = \$0.5$ per unit, purchasing cost of emergency order $c_p = \$40$ per unit, holding cost of emergency order $h_e = \$8$ per unit per year, fraction of backordered quantity $y = 97\%$, fraction of imperfect items $x = 0.03$, ordering cost $c_o = \$100$ per order, inventory holding cost $h = \$5$ per unit per year, inspection rate $\alpha = 175200$ units per year, backordered cost $\sigma = \$20$ per unit per unit time, lost sales cost $\pi = \$0.5$ per unit, $D(p) = a - bp = 222.89$ units per year, which gives optimal unit price $p^* = \$47.71$ per unit, optimal percentage of duration $t^* = 21\%$, optimal total profit $TP^*(p^*, t^*) = 1278.10$ (see Fig. 7).

We assume that the fraction of imperfect quality items follow a probability density function with $x = U\ (0, 0.04)$, $g(x) = 1/(0.04 - 0)$. **Example** 2**.** *Model II:* We considered a situation with the following parameters: $T = 0.028$, $a = 700$, $b = 10$, $c_s = 20$, $c_u = 25$, $c_i = 0.5$, $c_p = 40$, $h_e = 8$, $y = 97\%$, $x = 0.03$, $c_o = 100$, $h = 5$, $\alpha = 175200$, $\sigma = 20$, $\pi = 0.5$. Then, the optimal unit price and optimal percentage of duration in which inventory level is positive are found by maximizing the total profit function. We obtain the following optimal solution $p^* = \$47.7$, $t^* = 14.2\%$ and $TP^*(p^*, t^*) = \$1276.41$ (see Fig. 8). **Example** 3**.** *Model III:* We assumed a situation with the following parameters: $T = 0.028$, $a = 700$, $b = 10$, $c_s = 20$, $c_u = 25$, $c_i = 0.5$, $c_p = 40$, $h_e = 8$, $\sigma = 20$, $y = 97\%$, $x = 0.03$, $c_o = 100$, $h = 5$, $\alpha = 175200$, $\pi = 0.5$, which gives $p^* = \$47$, $t^* = 16.7\%$ and $TP^*(p^*, t^*) = \$1272.97$ (see Fig. 9).

5. Comments on Numerical Examples

In this section, we prepared numerical analysis for the models. First, we find the optimal profit for a given set of parameters. Table 1, indicates that Model I is the best choice as it gives maximum profit. But if we think in reference of duration, then Model II is the good choice as it gives more profit in less time. Comparative studies in the example show that the total profit per year using the Model I is usually highest than the Model II and Model III. Also, the total profit of the Model II is higher than the Model III.

We then execute sensitivity analysis by varying some model parameters and calculating the optimal price p, optimal percentage of duration in which inventory level is positive t and optimal total profit $TP(p,t)$ for each parameter set. We performed sensitivity analysis of the optimal solutions with respect to the length of cycle, T, and price sensitivity of demand, b, the other parameters are kept fixed at their pre-assumed values. The results are presented in Table 2 and Table 3. In the first column of Table 2, we vary the cycle length, which assumes the values -20%, -10%, $+50\%$, $+60\%$, $+70\%$, $+80\%$. The results indicate that as the cycle length increases, the

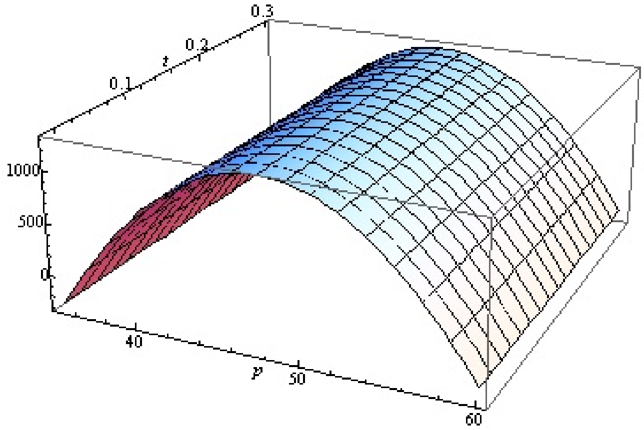

Figure 9: *Graphical representation of the total profit $TP(p,t)$ with respect to p and t*

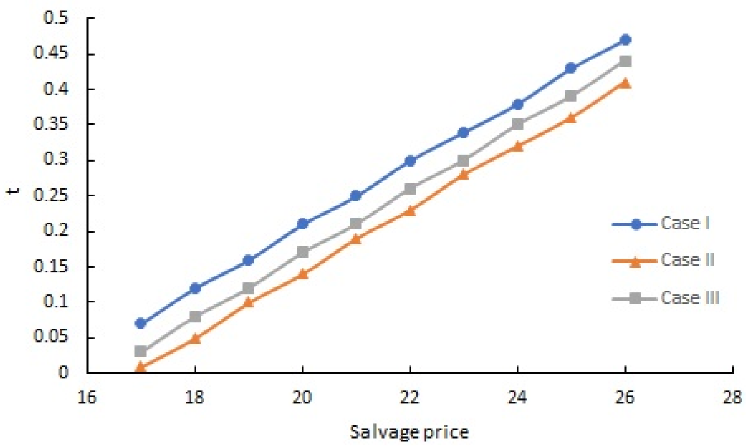

Figure 10: *Behaviour of Percentage of duration t for the three cases against Salvage price c_s*

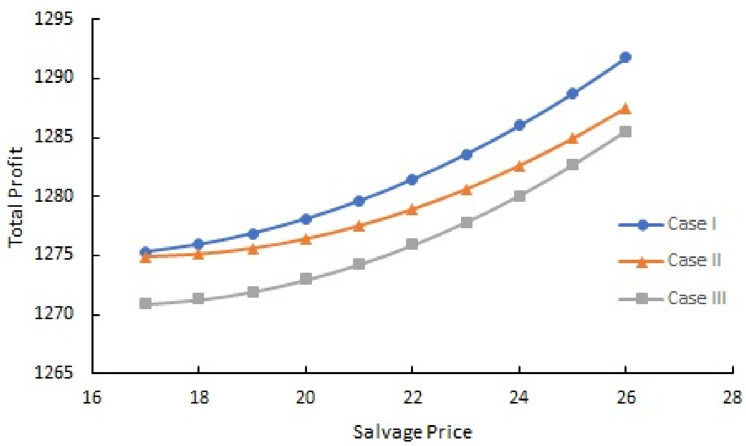

Figure 11: *Behaviour of Total Profit TP for the three models against salvage price c_s*

Table 2: *Model I: Optimal solutions for different cycle time, data used here are* $a = 700$, $b = 10$, $c_s = 20$, $c_u = 25$, $c_i = 0.5$, $c_p = 40$, $h_e = 8$, $\sigma = 20$, $y = 97\%$, $x = 0.03$, $c_o = 100$, $h = 5$, $\alpha = 175200$, $\pi = 0.5$.

T (years)		p^*	t^*	$TP(p^*, t^*)$
− 20%	0.022	47.63	4%	314.00
− 10%	0.025	47.68	13%	854.03
+ 50%	0.042	47.81	41%	2453.80
+ 60%	0.045	47.83	44%	2610.10
+ 70%	0.048	47.84	46%	2746.70
+ 80%	0.050	47.85	47%	2828.58

Table 3: *Optimal solutions for different values of parameter b, data used here are* $T = 0.028$, $a = 700$, $c_s = 20$, $c_u = 25$, $c_i = 0.5$, $c_p = 40$, $h_e = 8$, $\sigma = 20$, $y = 97\%$, $x = 0.03$, $c_o = 100$, $h = 5$, $\alpha = 175200$, $\pi = 0.5$.

b	Model I			Model II			Model III		
	p^*	t^*	$TP(p^*, t^*)$	p^*	t^*	$TP(p^*, t^*)$	p^*	t^*	$TP(p^*, t^*)$
7	63.02	89%	5969.72	62.98	80.0%	5957.21	63.02	89.7%	5969.54
8	56.62	60%	3965.11	56.59	52.5%	3957.94	56.62	60.5%	3964.64
9	51.67	38%	2451.49	51.64	31.2%	2447.66	51.67	38.0%	2451.04
10	47.71	21%	1278.10	47.69	14.2%	1276.41	47.00	16.7%	1272.97
11	44.48	6%	350.14	44.47	0.3%	349.86	44.48	5.2%	350.05

optimal solutions p, t and $TP(p.t)$ increase, for fixed a, b, c_s, c_u, c_i, c_p, h_e, σ, y, x, c_o, h, α and π. In Table 3, the b takes the values 7, 8, 9, 10 and 11. The results indicate that as the value of b increases, the optimal solutions decrease, for fixed a, T, c_s, c_u, c_i, c_p, h_e, σ, y, x, c_o, h, α and π.

Figure 10, represents the variation of variable t with respect to salvage price c_s for the three cases for fixed p^*, a, T, c_u, c_i, c_p, h_e, σ, y, x, c_o, h, α and π.
For the fixed p^* and the above-mentioned values of parameters, Figure 11, represents the variation of total profit function TP for the three models with respect to salvage price c_s.
The main results from the sensitivity analysis are as follows:

(1) As depicted in Fig. 10, when the salvage value of the imperfect quality items increases, the percentage of the duration in which inventory level is positive, t(Model I), t(Model II) and t(Model III) also increase. The value of t(Model III) is higher than that of t(Model II). But t(Model I) is highest always. This reveals the percentage of the duration of positive inventory is high which shows more stability of the business. Also, t is high that is the shortages time is less, which also shows a better strategy. This shows that Model I with partial backordering performs better than the other two models.

(2) As depicted in Fig. 11, when the salvage price of the imperfect quality items increases, the optimal total profit per year TP for Models I, II and III also increase. The value of TP for Model I is always higher than that of Model II and Model III. So, Model I is the more perfect choice for the proposed objective.
It is noted that TP(Model I) $>$ TP(Model II) $>$ TP(Model III).

6. Managerial Implications

Replenishment of perfect items equal to imperfect quality items is well established and studies in the literature. In this paper, we integrate the EOQ model with replenishment of perfect quality items equal to imperfect quality items after going through the screening process. We find the optimal unit price, optimal percentage of duration in which inventory level is positive and optimal profit for the models with imperfect quality items, and compare the results of the models.

We examine the effect of changing various model parameters, such as cycle time, salvage price and holding cost of emergency purchased items. Also, we incorporated partial backordering into our model. We notice that as the salvage price increases, it is optimal to have more percentage of duration (t) and total profit (TP). In all three cases, the optimal unit price increases as the cycle time (T) increases. The effect of price-sensitive parameter (b) significantly impacts the unit price (p), percentage of duration (t) and total profit (TP). Several studies discuss the techniques to reduce the effect of imperfect quality items at the vendor level. The cost of reducing the effect of imperfect quality items and partial backordering by employing appropriate techniques would help to increase the total profit.

7. Conclusion and Future Research

In this paper, we have developed three different models for the same problem and then selected the most suitable model that is the optimal inventory model with perfect and imperfect quantity items, the inspection process, partial backorders and reorders, which gives better results than the other two. We developed three models: The first model assumes that reordered items are received when the inventory level is zero. Our analytical and numerical results show that there exists a unique optimal sales price, lot size and percentage of duration in which inventory level is positive that maximizes the total profit. The second model assumes that reordered items are received when the backordered quantity is equal to the imperfect quantity items. The third model assumes that reordered items are received when backordered is yet continued. We provided numerical examples and sensitivity analysis to illustrate the outcomes of the above models. Sensitivity analysis performs to study the effect of different parameters of the system like salvage price, cycle time, holding cost of emergency purchased items and price-sensitive parameter on the optimal solutions. Numerical computations show that when the price-sensitive parameter (b) increases, the optimal unit price (p), the optimal percentage of the duration in which inventory level is positive (t) and the optimal total profit (TP) decrease for all three models. The percentage of the duration of positive inventory t is high which shows more stability of the business. Also, t is high that is the shortages time is less, which also show a better strategy. Model I is the more perfect choice for the proposed objective as it gives the maximum value of total profit.

Several extensions of the current model are possible. Future studies can be carried out by considering complete backordering. Another interesting extension is to offering price discounts on the stock-out items to the EOQ models with imperfect quality items and the screening process. One way to extend the model is to consider stochastic demand.

References

[1] Rosenblatt, M. J. and Lee, H. L. (1986). Economic production cycles with imperfect production processes. *IIE Transactions*, 18(1):48–55.

[2] Porteus, E. L. (1986). Optimal lot sizing, process quality improvement and setup cost reduction. *Operations Research*, 34:137–144.

[3] Roy, M. D., Sana S. S. and Chaudhuri, K. (2011). An economic order quantity model of imperfect quality items with partial backlogging. *International Journal of Systems Science*, 42(8):1409–1419.

[4] Hsu, J. T. and Hsu, L. F. (2013b). An EOQ model with imperfect quality items, inspection errors, shortage backordering, and sales returns. *International Journal of Production Economics*, 143(1):162–170.

[5] Dey, O. and Giri, B. C. (2014). Optimal vendor investment for reducing defect rate in a vendorâĂŞbuyer integrated system with imperfect production process. *International Journal of Production Economics*, 155:222–228.

[6] Paul, S., Wahab, M. I. M. and Ongkunaruk, P. (2014). Joint replenishment with imperfect items and price discount. *Computers and Industrial Engineering*, 74:179–185.

[7] Jaber, M. Y., Zanoni, S. and Zavanella, L. E. (2014). Economic order quantity models for imperfect items with buy and repair options. *International Journal of Production Economics*, 155:126–131.

[8] Taleizadeh, A. A., Khanbaglo, M. P. S. and Cárdenas-Barrón, L. E. (2016). An EOQ inventory model with partial backordering and reparation of imperfect products. *International Journal of Production Economics*, 182(2016):418–434.

[9] Taleizadeh, A. A., Khanbaglo, M. P. S. and Cárdenas-Barrón, L. E. (2017). Outsourcing rework of imperfect items in the economic production quantity (EPQ) inventory model with backordered demand. *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 49(12):2688–2699.

[10] Liao, J. J., Huang, K. N., Chung, K. J., Lin, S. D., Chuang, S. T. and Srivastava, H. M. (2020). Optimal ordering policy in an economic order quantity (EOQ) model for non-instantaneous deteriorating items with defective quality and permissible delay in payments. *Revista de la Real Academia de Ciencias Exactas, Físicas y Naturales. Serie A. Matemáticas*, 114(1):41, 1–26.

[11] Eroglu, A. and Ozdemir, G. (2007). An economic order quantity model with defective items and shortages. *International Journal of Production Economics*, 106(2):544–549.

[12] Maddah, B. and Jaber, M. Y. (2008). Economic order quantity for items with imperfect quality: Revisited. *International Journal of Production Economics*, 112(2):808–815.

[13] Maddah, B., Salameh, M. K. and Moussawi-Haidar, L. (2010). Order overlapping: A practical approach for preventing shortages during screening. *Computers and Industrial Engineering*, 58(4):691–695.

[14] Salameh, M. K. and Jaber M. Y. (2000). Economic production quantity model for items with imperfect quality. *International Journal of Production Economics*, 64(1–3):59–64.

[15] Cárdenas-Barrón, L. E. (2000). Observation on: "Economic production quantity model for items with imperfect quality". *International Journal of Production Economics*, 67(2):201.

[16] Cárdenas-Barrón, L. E. (2009). Economic production quantity with rework process at a single-stage manufacturing system with planned backorders. *Computers and Industrial Engineering*, 57(3):1105–1113.

[17] Hsu, J. T. and Hsu, L. F. (2013a). Two EPQ models with imperfect production processes, inspection errors, planned backorders, and sales returns. *Computers and Industrial Engineering*, 64(1):389–402.

[18] El-Ansary, A. I. and Robicheaux, R. A. (1974). A theory of channel control: revisited. *Journal of Marketing*, 30:2–7.

[19] Ben-Daya, M. and Hariga, M. (2000). Economic lot scheduling problem with imperfect production processes. *Journal of the Operational Research Society*, 51:875–881.

[20] Chiu, Y. P. (2003). Determining the optimal lot size for the finite production model with random defective rate, the rework process, and backlogging. *Engineering Optimization*, 35(4):427–437.

[21] Chen, T. H. (2017). Optimizing pricing, replenishment and rework decision for imperfect and deteriorating items in a manufacturer-retailer channel. *International Journal of Production Economics*, 183:539–550.

[22] Zhou, Y. W., Chen, J., Wu, Y. and Zhou, W. (2015). EPQ models for items with imperfect quality and one-timeonly discount. *Applied Mathematical Modelling*, 39(3–4):1000–1018.

[23] Goyal, S. K., Singh, S. R. and Yadav, D. (2017). Economic order quantity model for imperfect lot with partial backordering under the efect of learning and advertisement dependent imprecise demand. *International Journal of Operational Research*, 29(2):197–218.

[24] Patro, R., Acharya, M., Nayak, M. M. and Patnaik, S. (2018). A fuzzy EOQ model for deteriorating items with imperfect quality using proportionate discount under learning effects. *International Journal of Management and Decision Making*, 17(2):171–198.

[25] Keshavarzfard, R., Makui, A., Tavakkoli-Moghaddam, R. and Taleizadeh, A. A. (2019). Optimization of imperfect economic manufacturing models with a power demand rate dependent production rate. *SÄĄdhanÄĄ*, 44(9):206.

[26] Manna, A. K., Das, B. and Tiwari, S. (2020(a)). Impact of carbon emission on imperfect production inventory system with advance payment base free transportation. *RAIRO - Operations Research*, 54(4):1103–1117.

[27] Manna, A. K., Dey, J. K. and Mondal, S. K. (2020(b)). Effect of inspection errors on imperfect production inventory model with warranty and price discount dependent demand rate. *RAIRO Operations Research*, 54(4):1189–1213.

[28] Manna, A. K., Dey, J. K. and Mondal, S. K. (2017). Two layers supply chain in an imperfect production inventory model with two storage facilities under reliability consideration. *Journal of Industrial and Production Engineering*, 35(2):57–73.

[29] Asadkhani, J., Mokhtari, H. and Tahmasebpoor, S. (2021). Optimal lot-sizing under learning effect in inspection errors with different types of imperfect quality items. *Operational Research* 22:2631–2665.

[30] Wee, H., Yu, J. and Chen, M. C. (2007). Optimal inventory model for items with imperfect quality and shortage backordering. *Omega*, 35(1):7-11.

[31] Wee, H. M., Yu, J. C. P. and Wang, K. J. (2006). An integrated production-inventory model for deteriorating items with imperfect quality and shortage backordering considerations. *Lecture Notes in Computer Science*, 885–897.

[32] Chang, H. C. and Ho, C. H. (2010). Exact closed-form solutions for "optimal inventory model for items with imperfect quality and shortage backordering". *Omega*, 38(3–4):233–237.

[33] Chung, K. J. (2011). The economic production quantity with rework process in supply chain management. *Computers and Mathematics with Applications*, 62(6):2547–2550.

[34] Moussawi-Haidar, L., Salameh, M. and Nasr, W. (2014). Effect of deterioration on the instantaneous replenishment model with imperfect quality items. *Applied Mathematical Modelling*, 38(24):5956–5966.

[35] Öztürk, H. (2020). Economic order quantity models for the shipment containing defective items with inspection errors and a sub-lot inspection policy. *European Journal of Industrial Engineering*, 14(1):85–126.

[36] Yu, J. C. P., Wee, H. M. and Chen, J. M. (2005). Optimal ordering policy for a deteriorating item with imperfect quality and partial backordering. *Journal of the Chinese Institute of Industrial Engineers*, 22(6):509–520.