Reliability Analysis of the Shaft Subjected to Twisting Moment and Bending Moment for Normally Distributed Strength and Stress

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Abstract

Shaft is the rotating component that transmits power from one place to another. The shafts are commonly subject to torsional and bending moments and combinations of these moments. In general, shafts are subjected to a combination of torsional and bending stresses. The design of a shaft is essential, subject to its strength and stress. This paper presents the ***reliability analysis of the shaft*** *subjected to (a) twisting moment, (b) bending moment and (c) combined twisting and bending moment for which* ***stress and strength follow the normal distribution****.*

Keywords: Reliability, normal distribution, twisting moment, bending moment, round solid shaft, maximum shear stress theory, maximum normal stress theory.

1. Introduction

Shaft is the most vital component used in almost every mechanical system and machine. Out of all power transmission components, the shaft is the main component that must be designed carefully for the efficient working of the machine. The shafts are designed based on their strength and rigidity considerations. Based strength includes shafts subjected to bending moments, twisting moments, combined bending and twisting moments, and fluctuating loads. While considering rigidity, torsional and lateral rigidity are considered for design.
Adekunle A. A. et al. [1] studied the development of CAD software for shafts under various loading conditions. Dr. Edward E. Osakue et al. [2] studied fatigue shaft design verification for bending and torsion. Dr. Edward E. Osakue et al. [3] studied the probabilistic fatigue design of shafts for bending and torsion. Frydrysek K. et al. [4] studied performance-based design applied to a shaft subjected to combined stress. Gowtham et al. [5] studied drive shaft design and analysis. Kamboh, M. S., et al. [7] discussed the design and analysis of the drive shaft with a critical review of advanced composite materials and the root causes of shaft failure. Kececioglu D. et al. [8] studied the reliability analysis of mechanical components and systems. Misra, A., et al. [9] studied the reliability analysis of drilled shaft behaviour using the finite difference method and Monte Carlo simulation. Munoz-Abella, B., et al. [10] discussed the determination of the critical speed of a cracked shaft from experimental data. Nayek et al. [11]

studied the reliability approximation for a solid shaft under a gamma setup. Patel, B., et al. [12] studied a critical review of the design of a shaft with multiple discontinuities and combined loadings. Villa-Covarrubias, B., et al. [14] discussed the probabilistic method to determine the shaft diameter and design reliability.

2. Statistical model

The normal distribution takes the well-known bell shape. This distribution is symmetrical about its mean value. **The probability density function** for a normally distributed stress χ and normally distributed strength ξ is given by [4]

$$f_\chi(\chi) = \frac{1}{\sigma_\chi\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{\chi-\mu_\chi}{\sigma_\chi}\right)^2\right] \text{ for } -\infty < \chi < \infty$$

$$f_\xi(\xi) = \frac{1}{\sigma_\xi\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{\xi-\mu_\xi}{\sigma_\xi}\right)^2\right] \text{ for } -\infty < \xi < \infty$$

where μ_χ = mean value of the stress

σ_χ = standard deviation of the stress

μ_ξ = mean value for the strength

σ_ξ = standard deviation of the strength

Let us define $y = \xi - \chi$. It is well known that the random variable y is normally distributed with mean of $\mu_y = \mu_\xi - \mu_\chi$ and standard deviation of $\sigma_y = \sqrt{\sigma_\xi^2 + \sigma_\chi^2}$.

The reliability R can be expressed in terms of y as

$$R = P(y > 0) = \frac{1}{\sigma_y\sqrt{2\pi}}\int_0^\infty \exp\left[-\frac{1}{2}\left(\frac{y-\mu_y}{\sigma_y}\right)^2\right] dy$$

If we let $z = (y-\mu_y)/\sigma_y$, then $\sigma_y\, dz = dy$.

When $y = 0$, the lower limit of z is given by [4]

$$z = \frac{0-\mu_y}{\sigma_y} = -\frac{(\mu_\xi - \mu_\chi)}{\sqrt{\sigma_\xi^2 + \sigma_\chi^2}}$$

and $y \to +\infty$, the upper limit of $z \to +\infty$.

Then the reliability is given by [4]

$$R = \frac{1}{2\pi}\int_{-\frac{(\mu_\xi-\mu_\chi)}{\sqrt{\sigma_\xi^2+\sigma_\chi^2}}}^{\infty} \exp\left(\frac{-z^2}{2}\right) dz \tag{1}$$

The random variable $z = \frac{(y-\mu_y)}{\sigma_y}$ is the standard normal variable.

a. Shaft subjected to twisting moment only

If the shaft is subjected to a twisting moment only, then torsion equation is given by [6]

$$\frac{T}{J} = \frac{\sigma_t}{r}$$

where T = Twisting moment acting upon the shaft

J = Polar moment of inertia of the shaft about the axis of rotation

σ_t = Torsional shear stress and

r = Distance from neutral axis to the outer-most fibre

$= \frac{d}{2}$ where d is the diameter of the shaft

For round solid shaft, polar moment of inertia is given by [6]

$$J = \frac{\pi}{32} \times d^4$$

From the torsion equation twisting moment is

$$T = \frac{\pi}{16} \times \sigma_t \times d^3 \text{ and}$$

The shear stress due to twisting moment is the normally distributed stress

$$\mu_\chi = \sigma_t = \frac{16 \times T}{\pi \times d^3}$$

Then the reliability of the shaft subjected to twisting moment for normally distributed strength and stress is

$$R_t = \frac{1}{2\pi} \int_{-\frac{\left(\mu_\xi - \frac{16\times T}{\pi\times d^3}\right)}{\sqrt{\sigma_\xi^2+\sigma_\chi^2}}}^{\infty} \exp\left(\frac{-z_t^2}{2}\right) dz \tag{2}$$

b. Shaft subjected to bending moment only

If the shaft is subjected to a bending moment only, then bending equation is given by [6]

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

where M = Bending moment

I = Moment of inertia of cross-sectional area of the shaft about the axis of rotation

σ_b = Bending stress and

y = Distance from neutral axis to the outer-most fibre

For solid shaft, moment of inertia is given by [6]

$$I = \frac{\pi}{64} \times d^4 \text{ and } y = \frac{d}{2}$$

From the bending equation bending moment is given by [8]

$$M = \frac{\pi}{32} \times \sigma_b \times d^3$$

The shear stress due to bending moment is the normally distributed stress

$$\mu_\chi = \sigma_b = \frac{32 \times M}{\pi \times d^3}$$

Then the reliability of the shaft subjected to bending moment for normally distributed strength and stress is

$$R_b = \frac{1}{2\pi} \int_{-\frac{\left(\mu_\xi - \frac{32\times M}{\pi\times d^3}\right)}{\sqrt{\sigma_\xi^2+\sigma_\chi^2}}}^{\infty} \exp\left(\frac{-z_b^2}{2}\right) dz \tag{3}$$

c. Shaft subjected to combined twisting moment and bending moment

When the shaft is subjected to combined twisting moment and bending moment, then the shaft must be designed on the basis of the two moments simultaneously.

According to **maximum shear stress theory**, the maximum shear stress in the shaft is

$$\sigma_{t_{max}} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4(\sigma_t)^2}$$

where σ_t = shear stress induced due to twisting moment
σ_b = bending stress induced due to bending moment
Therefore

$$\sigma_{t_{max}} = \frac{16}{\pi \times d^3}\sqrt{M^2 + T^2}$$

The expression $\sqrt{M^2 + T^2}$ is known as equivalent twisting moment and is denoted by T_e.
The equivalent twisting moment may be defined as that twisting moment, which when acting alone, produces same shear stress (σ_t) as the actual twisting moment.
By limiting the maximum shear stress ($\sigma_{t_{max}}$) equal to the allowable shear stress (σ_{ts}) for the material is the normally distributed stress

$$\mu_{\chi s} = \sigma_{ts} = \frac{16 \times T_e}{\pi \times d^3}$$

Then the reliability of the shaft subjected to combined twisting moment and bending moment for the normally distributed strength and stress is

$$R_{ts} = \frac{1}{2\pi} \int_{-\frac{\left(\mu_\xi - \frac{16 \times T_e}{\pi \times d^3}\right)}{\sqrt{\sigma_\xi^2 + \sigma_\chi^2}}}^{\infty} \exp\left(\frac{-z_{ts}^2}{2}\right) dz \tag{4}$$

According to **maximum normal stress theory**, the maximum normal stress in the shaft is

$$\sigma_{b_{max}} = \frac{1}{2}\sigma_b + \frac{1}{2}\sqrt{(\sigma_b)^2 + 4(\sigma_t)^2}$$

where σ_t = shear stress induced due to twisting moment
σ_b = bending stress induced due to bending moment
Therefore

$$\sigma_{b_{max}} = \frac{32}{\pi \times d^3}\left[\frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right)\right]$$

The expression $\frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right)$ is known as equivalent bending moment and is denoted by M_e.
The equivalent bending moment may be defined as that moment which when acting alone produces the same tensile or compressive stress (σ_b) as the actual bending moment.
By limiting the maximum normal stress ($\sigma_{b_{max}}$) equal to the allowable bending stress (σ_{bn}) for the material is the normally distributed stress

$$\mu_{\chi n} = \sigma_{bn} = \frac{32 \times M_e}{\pi \times d^3}$$

Then the reliability of the shaft subjected to combined twisting moment and bending moment for normally distributed strength and stress is

$$R_{bn} = \frac{1}{2\pi} \int_{-\frac{\left(\mu_\xi - \frac{32 \times M_e}{\pi \times d^3}\right)}{\sqrt{\sigma_\xi^2 + \sigma_\chi^2}}}^{\infty} \exp\left(\frac{-z_{bn}^2}{2}\right) dz \tag{5}$$

3. Numerical results and discussion

a. For twisting moment only

Table-1: $d = 110\ mm$, $\mu_\xi = 119.6584\ N/mm^2$.

T	z_t	R_t
100000	-1.993596824	0.976901934
2500000	-1.834327549	0.966697306
5000000	-1.659291647	0.951471481
7500000	-1.478628009	0.930380119
10000000	-1.296096181	0.902528825
13000000	-1.079424515	0.859800736
18000000	-0.736112792	0.769168971
20000000	-0.607809459	0.728343073
25000000	-0.313849897	0.623182477
30000000	-0.059258971	0.523627080

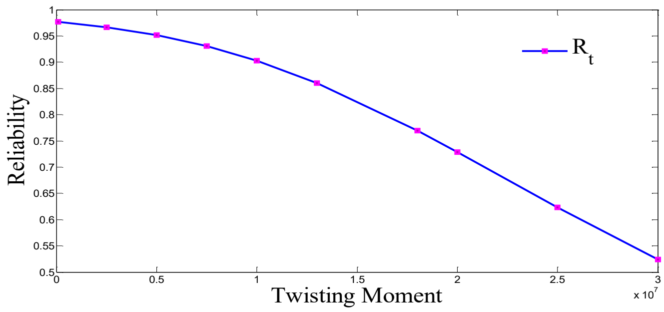

Figure 1: *Reliability and Twisting Moment*

Table-2: $T = 10^4\ N\text{-}mm$, $\mu_\xi = 119.6584\ N/mm^2$.

d	z_t	R_t
18	-0.437034097	0.668956690
20	-0.826725375	0.795803632
24	-1.323221625	0.907119157
30	-1.664313463	0.951975098
32	-1.725837528	0.957811677
40	-1.862933958	0.968764221
50	-1.930809365	0.973246684
58	-1.955923837	0.974762937
70	-1.975040282	0.975868212

82	-1.984508267	0.976400396

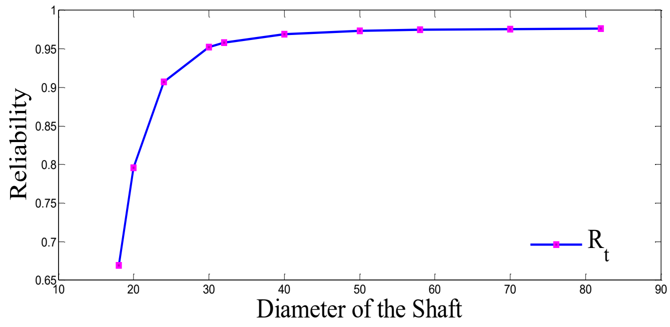

Figure 2: *Reliability and Diameter of the Shaft*

b. For bending moment only

Table-3: $d = 110\ mm$, $\mu_\xi = 119.6584\ N/mm^2$.

M	z_b	R_b
100000	-1.98717341	0.976548408
1200000	-1.841158255	0.967200815
2000000	-1.730194966	0.958202276
2400000	-1.673552215	0.952890682
3800000	-1.471338225	0.929400165
4800000	-1.325278842	0.907460658
6500000	-1.079424515	0.859800736
7800000	-0.897319098	0.815225666
8600000	-0.789017221	0.784949029
12000000	-0.369480494	0.644115195

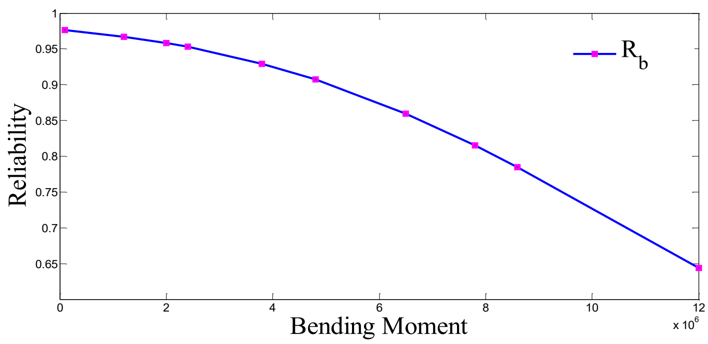

Figure 3: *Reliability and Bending Moment*

Table-4: $M = 10^4$ *N-mm*, $\mu_\xi = 119.6584$*N/mm²*.

d	z_b	R_b
22	-0.313849897	0.623182477
26	-0.928635716	0.823461047
32	-1.433116930	0.924087788
36	-1.608706553	0.946159739
40	-1.718965160	0.957189642
52	-1.875536297	0.969640510
62	-1.927365578	0.973032957
82	-1.968900245	0.975517726
100	-1.982910074	0.976311262
110	-1.987173410	0.976548408

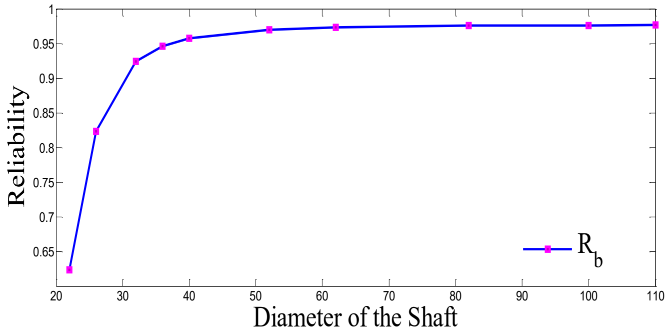

Figure 4: *Reliability and Diameter of the Shaft*

c. For combined twisting moment and bending moment

Table-5: $M = 10^4$ *N-mm*, $d = 110$ *mm*, $\mu_\xi = 119.6584$ *N/mm²*.

T	z_{ts}	z_{bn}	R_{ts}	R_{bn}
100000	-1.990938603	-1.984506864	0.976756181	0.976400318
2500000	-1.834190835	-1.827344797	0.966687164	0.966176028
5000000	-1.659220259	-1.652076253	0.951464292	0.950740496
7500000	-1.478579423	-1.471289619	0.930373622	0.929393595
10000000	-1.296059727	-1.288770095	0.902522546	0.901260987
13000000	-1.079397107	-1.072274996	0.859794630	0.858201733
18000000	-0.736094575	-0.729543208	0.769163428	0.767165276
20000000	-0.607793789	-0.601533305	0.728337875	0.726257582
25000000	-0.31383893	-0.308363433	0.623178312	0.621097098
30000000	-0.059251123	-0.054549707	0.523623955	0.521751396

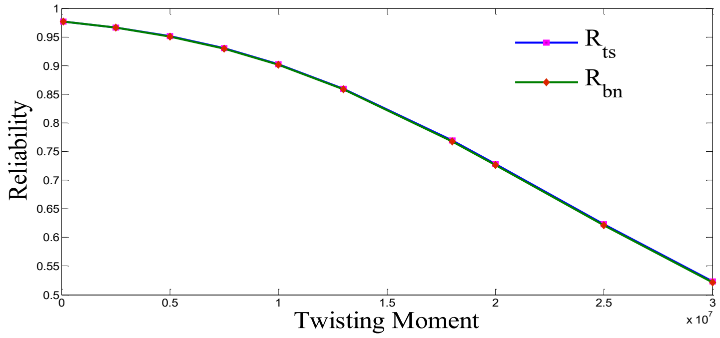

Figure 5: *Reliability and Twisting Moment*

Table-6: $T = 10^4$ *N-mm*, $d = 110$ *mm*, $\mu_\xi = 119.6584$ *N/mm²*.

M	z_{ts}	z_{bn}	R_{ts}	R_{bn}
100000	-1.990938603	-1.984506864	0.976756181	0.976400318
1200000	-1.921595252	-1.840874437	0.972671647	0.967180019
2000000	-1.868157213	-1.730019145	0.969129920	0.958186573
2400000	-1.841016165	-1.673403924	0.967190405	0.952876097
3800000	-1.744153719	-1.471242305	0.959433855	0.929387200
4800000	-1.673478046	-1.325202811	0.952883388	0.907448053
6500000	-1.551268837	-1.079369702	0.939581364	0.859788524
7800000	-1.456703987	-0.897275084	0.927400946	0.815213926
8600000	-1.398287099	-0.788978462	0.918986565	0.784937703
12000000	-1.151035866	-0.369456983	0.875141260	0.644106434

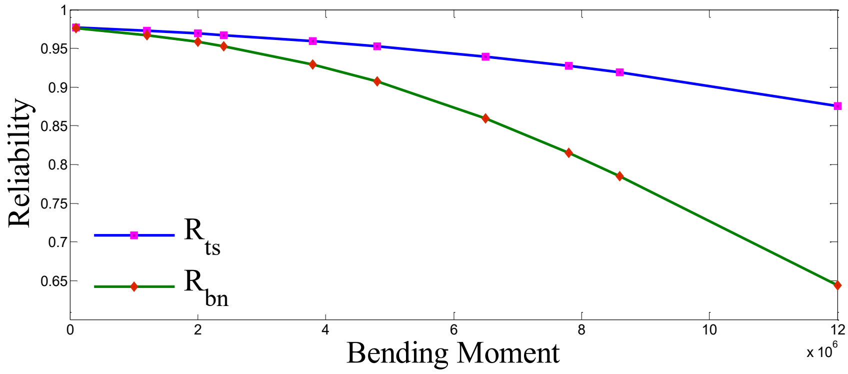

Figure 6: *Reliability and Bending Moment*

Table-7: $T = 10^4\ N - mm,\ M = 15 \times 10^4$ *N-mm*, $\mu_\xi = 119.6584$ *N/mm²*.

d	z_{ts}	z_{bn}	R_{ts}	R_{bn}
24	-0.778536422	-0.208090258	0.781873578	0.582420753
28	-1.228383905	-0.802787694	0.890348557	0.788951272
32	-1.491547172	-1.195487826	0.932091052	0.884051755
36	-1.649076771	-1.442464991	0.950434046	0.925414380
40	-1.747806723	-1.599826782	0.959751250	0.945181492
52	-1.888095775	-1.823839541	0.970493453	0.965911833
62	-1.934633552	-1.897558264	0.973482360	0.971122852
82	-1.971987598	-1.956288280	0.975694489	0.974784398
100	-1.984601679	-1.976010500	0.976405597	0.975923206
110	-1.988441857	-1.982002340	0.976618578	0.976260511

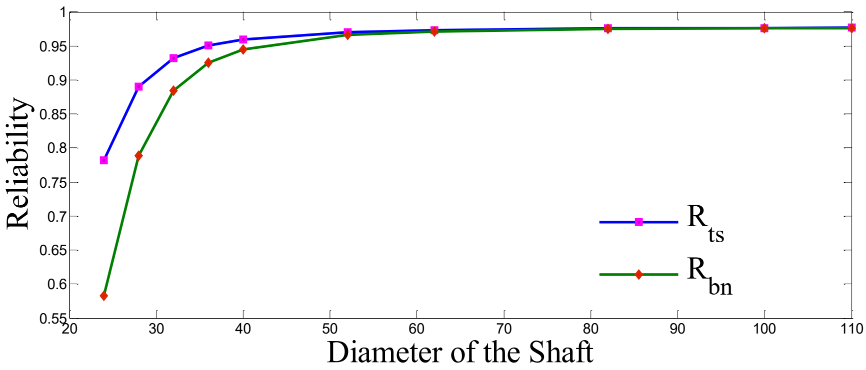

Figure 7: *Reliability and Diameter of the Shaft*

Table-8: $T = 4 \times 10^4\ \ N - mm, M = 2 \times 10^4$ *N-mm*, $d = 110$ *mm*.

μ_ξ	z_{ts}	z_{bn}	R_{ts}	R_{bn}
2.5	-0.521239161	-0.013920898	0.698899912	0.505553455
3.5	-0.918703097	-0.477954094	0.820874555	0.683658561
6.5	-1.425158981	-1.157220313	0.922944375	0.87640882
10.5	-1.652400548	-1.487687204	0.950773538	0.931583298
20.5	-1.826770123	-1.745803964	0.966132830	0.959577489
40.5	-1.913822863	-1.874254579	0.972178603	0.969552328
60.5	-1.942677032	-1.916561189	0.973972403	0.972353149
120.5	-1.971410740	-1.958499434	0.975661543	0.974914281
210.5	-1.983682381	-1.976343042	0.976354371	0.975942031
350.5	-1.990215750	-1.985824871	0.976716413	0.976473613

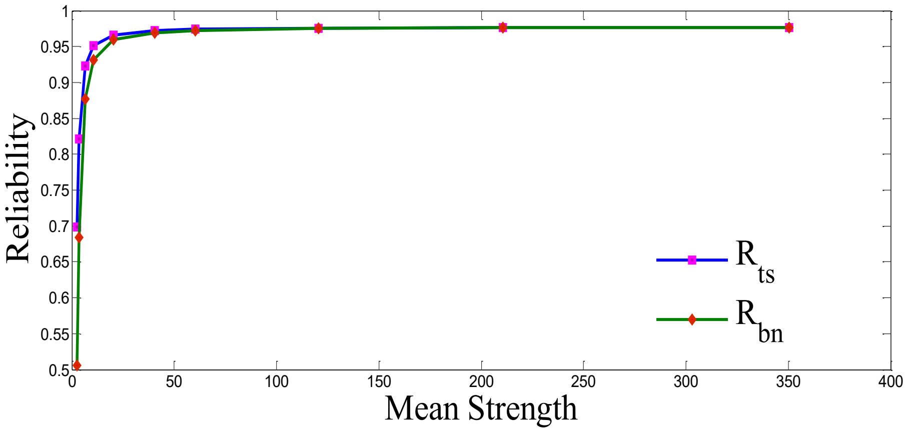

Figure 8: *Reliability and Diameter of the Shaft*

4. Conclusion

The reliability of the shaft is derived from the twisting and bending moments and the combined twisting and bending moments. According to the computations, the reliability of the shaft decreases when the twisting and bending moments increase. Reliability increases when the diameter and strength of the shaft increase.

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