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ON A CLASS OF LORENTZIAN PARA-KENMOTSU MANIFOLDS ADMITTING QUARTER-SYMMETRIC METRIC CONNECTION

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Abstract

In this present paper, a class of Lorentzian almost paracontact metric manifolds known as the LP-Kenmotsu (Lorentzian para-Kenmotsu) is considered that accepts a connection of quarter-symmetric. In this work, it was found that an LP-Kenmotsu manifold is either ϕ-symmetric or concircular ϕ-symmetric with respect to quarter-symmetric metric connection if and only if it is symmetric with respect to the Riemannian connection, provided the scalar curvature of Riemannian connection is constant.

Keywords: Lorentzian para-Kenmotsu manifold, quarter- symmetric metric connection, concircular curvature tensor.
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I. Introduction

In 1989, Matsumoto [4] introduced the notion of Lorentzian paracontact and in particular, Lorentzian para-Sasakian (*LP*-Sasakian) manifolds. Later, these manifolds have been widely studied by many geometers such as Matsumoto and Mihai [5], Mihai and Rosca [6], Mihai, Shaikh and De [7], Venkatesha and C. S. Baggewadi [13], Venkatesha, Pradeep Kumar and Bagewadi [14, 15] and obtained several results on these manifolds.

In 1995, Sinha and Sai Prasad [11] defined a class of almost paracontact metric manifolds namely para-Kenmotsu (briefly *P*-Kenmotsu) and special para-Kenmotsu (briefly *SP*- Kenmotsu) manifolds in similar to *P*-Sasakian and *SP*- Sasakian manifolds. In 2018, Abdul Haseeb and Rajendra Prasad [1] defined a class of Lorentzian almost paracontact metric manifolds namely Lorentzian para-Kenmotsu (briefly *LP*- Kenmotsu) manifolds. As an extension, many geometers have studied these Lorentzian para-Kenmotsu manifolds [8, 10, 12]. Sai Prasad, Sunitha Devi and Deekshitulu have considered *LP*-Kenmotsu manifolds admitting the Weyl-projective curvature tensor W_2 and shown that these manifolds admitting a Weyl-flat projective curvature tensor, an irrotational Weyl-projective curvature tensor and a conservative Weyl-projective curvature tensor are an Einstein manifolds of constant scalar curvature [9].

A linear connection $\widetilde{\nabla}$ in an n-dimensional differentiable manifold is said to be a quarter-symmetric connection [3] if its torsion tensor T is of the form

$$\begin{aligned} T(X,Y) &= \widetilde{\nabla}_X Y - \widetilde{\nabla}_Y X - [X,Y] \\ &= \eta(Y)\phi X - \eta(X)\phi Y, \end{aligned} \tag{1}$$

where η is a 1-form and ϕ is a tensor field of type (1,1). In particular, if we replace ϕX by X and ϕY by Y, then the quarter-symmetric connection reduces to the semi-symmetric connection [2]. Thus, the notion of quarter-symmetric connection generalizes the idea of semi-symmetric connection, and if quarter-symmetric linear connection $\widetilde{\nabla}$ satisfies the condition $\left(\widetilde{\nabla}_X g\right)(Y,Z) = 0$ for all $X, Y, Z \in \chi(M_n)$, where $\chi(M_n)$ is the Lie algebra of vector fields on the manifold M_n, then $\widetilde{\nabla}$ is said to be a quarter-symmetric metric connection.

Motivated by these studies, in the present paper, we study the geometry of Lorentzian para-Kenmotsu (LP-Kenmotsu) manifolds with respect to quarter-symmetric metric connection. The present paper is organized as follows. Section 2 is equipped with some prerequisites about Lorentzian para-Kenmotsu manifolds.

Further on, in relation to the quarter-symmetric metric connection in an Lorentzian para-Kenmotsu manifold, we derive the relations for the Ricci tensor $\tilde{S}$ and the Riemannian curvature tensor $\tilde{R}$ in section 3.

Further in sections 4 and 5, we study the ϕ-symmetry and concircular ϕ-symmetry of Lorentzian para-Kenmotsu manifolds with respect to quarter-symmetric metric connection respectively.

II. Preliminaries

An n-dimensional differentiable manifold M_n admitting a (1,1) tensor field ϕ, contravariant vector field ξ, a 1-form η and the Lorentzian metric $g(X,Y)$ satisfying

$$\eta(\xi) = -1, \tag{2}$$

$$\phi^2 X = X + \eta(X)\xi, \tag{3}$$

$$g(\phi X, \phi Y) = g(X,Y) + \eta(X)\eta(Y), \tag{4}$$

$$g(X,\xi) = \eta(X), \tag{5}$$

$$\phi\xi = 0, \quad \eta(\phi X) = 0, \quad rank\phi = n-1. \tag{6}$$

is called Lorentzian almost paracontact manifold [4].

In a Lorentzian almost paracontact manifold, we have

$$\Phi(X,Y) = \Phi(Y,X) \quad where \quad \Phi(X,Y) = g(\phi X, Y). \tag{7}$$

A Lorentzian almost paracontact manifold M_n is called Lorentzian para-Kenmotsu manifold if [1]

$$(\nabla_X \phi)\, Y = -g\,(\phi X,\ Y)\,\xi - \eta\,(Y)\,\phi X, \tag{8}$$

for any vector fields X and Y on M_n, and ∇ is the operator of covariant differentiation with respect to the Lorentzian metric g.

It can be easily seen that in a LP-Kenmotsu manifold M_n, the following relations hold [1]:

$$\nabla_X \xi = -\phi^2 X = -X - \eta\,(X)\,\xi, \tag{9}$$

$$(\nabla_X \eta)\, Y = -g\,(X,\ Y)\,\xi - \eta\,(X)\ \eta\,(Y)\,, \tag{10}$$

for any vector fields X and Y on M_n.

Also, in an LP-Kenmotsu manifold, the following relations hold [1]:

$$g(R(X,Y)Z,\xi) = \eta(R(X,Y)Z) = g(Y,Z)\eta(X) - g(X,Z)\eta(Y) \tag{11}$$

$$R(\xi, X)Y = g(X, Y)\xi - \eta(Y)X, \tag{12}$$

$$R(X, Y)\xi = \eta(Y)X - \eta(X)Y, \tag{13}$$

$$S(X, \xi) = (n-1)\eta(X), \tag{14}$$

$$S(\phi X, \phi Y) = S(X, Y) + (n-1)\eta(X)\eta(Y) \tag{15}$$

for any vector fields X, Y and Z, where R is the Riemannian curvature tensor and S is the Ricci tensor of M_n.

Definition 1. An *LP*- Kenmotsu manifold M_n is said to be symmetric if

$$(\nabla_W R)(X,\ Y) Z = 0, \tag{16}$$

for all vector fields X, Y, Z and W.

Definition 2. An *LP*-Kenmotsu manifold M_n is said to be ϕ-symmetric if

$$\phi^2 (\nabla_W R)(X,\ Y) Z = 0, \tag{17}$$

for all vector fields X, Y, Z and W.

Definition 3. An *LP*-Kenmotsu manifold M_n is said to be concircular symmetric if

$$\left(\nabla_W \tilde{C}\right)(X,\ Y) Z = 0, \tag{18}$$

for all vector fields X, Y and Z. Here $\tilde{C}$ is the concircular curvature tensor and is given by [16]

$$\tilde{C}(X,\ Y) Z = R(X, Y) Z - \frac{r}{n(n-1)} \left[g(Y, Z) X - g(X, Z) Y\right], \tag{19}$$

for all vector fields X, Y and Z, where R and r are the Riemannian curvature tensor and scalar curvature respectively.

Definition 4. An *LP*-Kenmotsu manifold M_n is said to be concircular ϕ-symmetric if

$$\phi^2 \left(\nabla_W \tilde{C}\right)(X, Y) Z = 0, \tag{20}$$

for all vector fields X, Y, Z and W.

III. Expression of $\tilde{R}(X, Y) Z$ in terms of $R(X, Y) Z$

In this section we express $\tilde{R}(X, Y) Z$, the curvature tensor with respect to quarter-symmetric metric connection, in terms of $R(X, Y) Z$ which is the curvature tensor with respect to Riemannian connection.

Let $\widetilde{\nabla}$ be a linear connection and ∇ be a Riemannian connection of an almost contact metric manifold M_n such that

$$\widetilde{\nabla}_X Y = \nabla_X Y + U(X, Y), \tag{21}$$

where U is a tensor of type (1, 1). For $\widetilde{\nabla}$ to be a quarter-symmetric metric connection in M_n, we have

$$U(X, Y) = \frac{1}{2}\left[T(X, Y) + T'(X, Y) + T'(Y, X)\right] \tag{22}$$

and

$$g(T'(X, Y), Z) = g(T(Z, X), Y). \tag{23}$$

From (1) and (23), we get

$$T'(X,Y) = \eta(Y)\phi X - g(\phi X, Y)\xi. \tag{24}$$

Using (1) and (24) in (22), we obtain

$$U(X,Y) = \eta(Y)\phi X - g(\phi X, Y)\xi.$$

Thus the quarter-symmetric metric connection $\widetilde{\nabla}$ in an *LP*-Kenmotsu manifold is given by

$$\widetilde{\nabla}_X Y = \nabla_X Y + \eta(Y)\phi X - g(\phi X, Y)\xi, \tag{25}$$

which is the relation between Riemannian connection and the quarter-symmetric metric connection on Lorentzian para-Kenmotsu manifolds.

Similarly, on simplication, we obtain the relation between the curvature tensor $\tilde{R}(X,Y)Z$ of M_n with respect to the quarter-symmetric metric connection $\widetilde{\nabla}$ and the curvature tensor $R(X,Y)Z$ of Riemannian connection ∇ as follows:

$$\begin{aligned} \tilde{R}(X,Y)Z = R(X,Y)Z &+ [g(\phi Y, Z) + g(Y,Z)\xi]\phi X] \\ &- [g(\phi X, Z) + g(X,Z)\xi]\phi Y \\ &+ Xg(\phi Y, Z) - Yg(\phi X, Z). \end{aligned} \tag{26}$$

Then from (26), it follows that

$$\tilde{S}(Y,Z) = S(Y,Z), \tag{27}$$

where $\tilde{S}$ and S are the Ricci tensors of the connections $\widetilde{\nabla}$ and ∇ respectively.

Further contracting (27), we get

$$\tilde{r} = r, \tag{28}$$

where $\tilde{r}$ and r are the scalar curvatures of the connections $\widetilde{\nabla}$ and ∇ respectively.

IV. Symmetry of *LP*-Kenmotsu manifold with respect to quarter-symmetric metric connection

By the definition of symmetric *LP*-Kenmotsu manifold with respect to Riemannian connection, we define a symmetric *LP*-Kenmotsu manifold with respect to quarter-symmetric metric connection by

$$\left(\widetilde{\nabla}_W \tilde{R}\right)(X,\ Y)Z = 0, \tag{29}$$

where

$$\begin{aligned} \left(\widetilde{\nabla}_W \tilde{R}\right)(X,\ Y)Z = \widetilde{\nabla}_W\left(\tilde{R}(X,\ Y)Z\right) &- \tilde{R}\left(\left(\widetilde{\nabla}_W X, Y\right)Z\right) \\ &- \tilde{R}\left(\left(X,\ \widetilde{\nabla}_W Y\right)Z\right) - \tilde{R}\left((X,Y)\,\widetilde{\nabla}_W Z\right), \end{aligned} \tag{30}$$

for all vector fields X, Y, Z and W.

$$\begin{aligned} \widetilde{\nabla}_W\left(\tilde{R}(X,\ Y)Z\right) = \nabla_W\left(\tilde{R}(X,\ Y)Z\right) &+ \eta\left(\tilde{R}(X,\ Y)Z\right)\phi W \\ &- g\left(\phi W,\ \left(\tilde{R}(X,\ Y)Z\right)\right)\xi, \end{aligned} \tag{31}$$

$$\begin{aligned} \tilde{R}\left(\left(\widetilde{\nabla}_W X, Y\right)Z\right) = \tilde{R}(\nabla_W X, Y)Z &+ \eta(X)\tilde{R}(\phi W, Y)Z \\ &- g(\phi W,\ X)\tilde{R}(\xi,\ Y)Z, \end{aligned} \tag{32}$$

$$\begin{aligned} \tilde{R}\left(\left(X,\ \widetilde{\nabla}_W Y\right)Z\right) = \tilde{R}(X,\ \nabla_W Y)Z &+ \eta(Y)\tilde{R}(X,\ \phi W)Z \\ &- g(\phi W,\ Y)\tilde{R}(X,\xi)Z, \end{aligned} \tag{33}$$

$$\tilde{R}\left((X,Y)\,\widetilde{\nabla}_W Z\right) = \tilde{R}\,(X,Y)\,\nabla_W Z + \eta\,(Z)\,\tilde{R}\,(X,Y)\,\phi W \\ g\,(\phi W,\ Z)\,\tilde{R}\,(X,\ Y)\,\xi, \tag{34}$$

$$\tilde{R}(\xi,Y)Z = g(Y,Z)\xi - \eta(Z)Y - \eta(Z)\phi Y\xi + g(\phi Y,Z)\xi, \tag{35}$$

$$\tilde{R}(X,\xi)Z = \eta(Z)X - g(X,Z)\xi - \eta(Z)\phi X\xi + g(\phi X,Z)\xi, \tag{36}$$

$$\tilde{R}(X,Y)\xi = \eta(Y)X - \eta(X)Y + \eta(Y)\phi X\xi - \eta(X)\phi Y\xi. \tag{37}$$

By using (25), (31) to (37) in (30), we get

$$\begin{aligned}\left(\widetilde{\nabla}_W\tilde{R}\right)(X,\ Y)\,Z = {} & \left(\nabla_W\tilde{R}\right)(X,\ Y)\,Z + \eta\left(\tilde{R}\,(X,Y)\,Z\right)\phi W \\ & - g\left(\phi W,\ \tilde{R}\,(X,Y)\,Z\right)\xi - \eta\,(X)\,\tilde{R}\,(\phi W,Y)\,Z \\ & - \eta\,(Y)\,\tilde{R}\,(X,\phi W)\,Z + g\,(W,\ \phi X)\,\tilde{R}\,(\xi,Y)\,Z \\ & + g\,(W,\ \phi Y)\,\tilde{R}\,(X,\xi)\,Z + g\,(W,\ \phi Z)\,\tilde{R}\,(X,Y)\,\xi \\ & - \eta\,(Z)\,\tilde{R}\,(X,Y)\,\phi W.\end{aligned} \tag{38}$$

Then by differentiating (26) with respect to W and on using (6), (7) and (10), we get

$$\begin{aligned}(\nabla_W\tilde{R})(X,Y)Z = {} & (\nabla_W R)(X,Y)Z - [g(\phi W,Y)\eta(Z) + g(\phi W,Z)\eta(Y) \\ & + Wg(Y,Z) + \eta(W)g(Y,Z)\xi]\phi X \\ & + [g(\phi W,X)\eta(Z) + g(\phi W,Z)\eta(X) \\ & + Wg(X,Z) + \eta(W)g(X,Z)\xi]\phi Y \\ & + g(\phi X,Z)[g(\phi W,Y)\xi + \eta(Y)\phi W] \\ & - g(\phi Y,Z)[g(\phi W,X)\xi + \eta(X)\phi W] \\ & + [\eta(Y)g(X,Z)\xi - \eta(X)g(Y,Z)\xi]\phi W \\ & + [Yg(\phi W,X) - Xg(\phi W,Y)]\eta(Z) \\ & + g(\phi W,Z)[\eta(X)Y - \eta(Y)X].\end{aligned} \tag{39}$$

Therefore, by using (2), (8) and (39) in (38), we obtain

$$\left(\widetilde{\nabla}_W\tilde{R}\right)(X,\ Y)\,Z = (\nabla_W R)\,(X,\ Y)\,Z. \tag{40}$$

Thus we can state the following:

Theorem 1. An *LP*-Kenmotsu manifold is symmetric with quarter-symmetric metric connection $\widetilde{\nabla}$ if and only if it is symmetric with respect to Riemannian connection ∇.

Corollary 1. An *LP*-Kenmotsu manifold is ϕ-symmetric with respect to quarter-symmetric metric connection $\widetilde{\nabla}$ if and only if it is symmetric with respect to Riemannian connection ∇.

V. Concircular symmetry of *LP*-Kenmotsu manifold with respect to quarter-symmetric metric connection

An *LP*-Kenmotsu manifold M_n is said to be a concircular symmetric with respect to quarter-symmetric metric connection if

$$(\widetilde{\nabla}_W\widetilde{\widetilde{C}})(X,Y)Z = 0, \tag{41}$$

for all vector fields X, Y, Z and W. Here the concircular curvature tensor $\widetilde{\widetilde{C}}$ with respect to quarter-symmetric metric connection is given by

$$\widetilde{\widetilde{C}}(X,Y)Z = \widetilde{R}(X,Y)Z - \frac{\widetilde{r}}{n(n-1)}[g(Y,Z)X - g(X,Z)Y], \tag{42}$$

where $\widetilde{R}$ is the Riemannian curvature tensor and $\widetilde{r}$ is the scalar curvature of the quarter-symmetric metric connection $\widetilde{\nabla}$.

Using (25), we can write

$$\begin{aligned}(\widetilde{\nabla}_W\widetilde{\widetilde{C}})(X,Y)Z = &(\nabla_W\widetilde{\widetilde{C}})(X,Y)Z + \eta(\widetilde{\widetilde{C}}(X,Y)Z)\phi W \\ &- g(\phi(\widetilde{\widetilde{C}}(X,Y)Z,W))\xi - \eta(X)\widetilde{\widetilde{C}}(\phi W,Y)Z \\ &- \eta(Y)\widetilde{\widetilde{C}}(X,\phi W)Z - \eta(Z)\widetilde{\widetilde{C}}(X,Y)\phi W \\ &+ g(\phi W,X)\widetilde{\widetilde{C}}(\xi,Y)Z + g(\phi W,Y)\widetilde{\widetilde{C}}(X,\xi)Z \\ &+ g(\phi W,Z)\widetilde{\widetilde{C}}(X,Y)\xi.\end{aligned} \tag{43}$$

Now on differentiating (42) with respect to W, we obtain

$$(\nabla_W\widetilde{\widetilde{C}})(X,Y)Z = (\nabla_W\widetilde{R})(X,Y)Z - \frac{\nabla_W\widetilde{r}}{n(n-1)}[g(Y,Z)X - g(X,Z)Y]. \tag{44}$$

Therefore, by using of (28) and (39), we get from (44) that

$$\begin{aligned}(\nabla_W\widetilde{\widetilde{C}})(X,Y)Z = &(\nabla_W R)(X,Y)Z - [g(\phi W,Y)\eta(Z) + g(\phi W,Z)\eta(Y) \\ &+ Wg(Y,Z) + \eta(W)g(Y,Z)\xi]\phi X + [g(\phi W,X)\eta(Z) \\ &+ g(\phi W,Z)\eta(X) + Wg(X,Z) + \eta(W)g(X,Z)\xi]\phi Y \\ &+ g(\phi X,Z)[g(\phi W,Y)\xi + \eta(Y)\phi W] \\ &- g(\phi Y,Z)[g(\phi W,X)\xi + \eta(X)\phi W] \\ &+ [\eta(Y)g(X,Z)\xi - \eta(X)g(Y,Z)\xi]\phi W \\ &+ [Yg(\phi W,X) - Xg(\phi W,Y)]\eta(Z) \\ &+ g(\phi W,Z)[\eta(X)Y - \eta(Y)X] \\ &- \frac{\nabla_W r}{n(n-1)}[g(Y,Z)X - g(X,Z)Y].\end{aligned} \tag{45}$$

Then by making use of (19), we rewrite the above equation (45) as

$$\begin{aligned}(\nabla_W\widetilde{\widetilde{C}})(X,Y)Z = &(\nabla_W\widetilde{C})(X,Y)Z - [g(\phi W,Y)\eta(Z) + g(\phi W,Z)\eta(Y) \\ &+ Wg(Y,Z) + \eta(W)g(Y,Z)\xi]\phi X + [g(\phi W,X)\eta(Z) \\ &+ g(\phi W,Z)\eta(X) + Wg(X,Z) + \eta(W)g(X,Z)\xi]\phi Y \\ &+ g(\phi X,Z)[g(\phi W,Y)\xi + \eta(Y)\phi W] \\ &- g(\phi Y,Z)[g(\phi W,X)\xi + \eta(X)\phi W] \\ &+ [\eta(Y)g(X,Z)\xi - \eta(X)g(Y,Z)\xi]\phi W \\ &+ [Yg(\phi W,X) - Xg(\phi W,Y)]\eta(Z) \\ &+ g(\phi W,Z)[\eta(X)Y - \eta(Y)X].\end{aligned} \tag{46}$$

Using (2), (6) and (46) in (43), we get

$$(\widetilde{\nabla}_W\widetilde{\widetilde{C}})(X,Y)Z = (\nabla_W\widetilde{C})(X,Y)Z. \tag{47}$$

Hence we can state the following:

Theorem 2. An *LP*-Kenmotsu manifold is concircular symmetric with respect to $\widetilde{\nabla}$ if and only if it is so with respect to Riemannian connection ∇.

Corollary 2. An *LP*-Kenmotsu manifold is concircular ϕ-symmetric with respect to $\widetilde{\nabla}$ if and only if it is so with respect to Riemannian connection ∇.

Now taking (2), (6) and (45) in (43), we get

$$(\widetilde{\nabla}_W \widetilde{\widetilde{C}})(X,Y)Z = (\nabla_W R)(X,Y)Z - \frac{\nabla_W r}{n(n-1)}[g(Y,Z)X - g(X,Z)Y]. \tag{48}$$

If scalar curvature r is constant, the above equation (48) reduces to

$$(\widetilde{\nabla}_W \widetilde{\widetilde{C}})(X,Y)Z = (\nabla_W R)(X,Y)Z. \tag{49}$$

Thus we have the following assertion.

Theorem 3. An *LP*-Kenmotsu manifold is concircular symmetric with respect to quarter-symmetric metric connection $\widetilde{\nabla}$ if and only if it is symmetric with respect to Riemannian connection ∇, provided the scalar curvature r is constant.

Corollary 3. An *LP*-Kenmotsu manifold is concircular ϕ-symmetric with respect to quarter-symmetric metric connection $\widetilde{\nabla}$ if and only if it is symmetric with respect to Riemannian connection ∇, provided the scalar curvature r is constant.

VI. Conclusion

We explore a class of Lorentzian almost paracontact metric manifolds known as the Lorentzian para-Kenmotsu that accepts a quarter-symmetric connection. In relation to the quarter-symmetric metric connection, the relations for the Ricci tensor and the Riemannian curvature tensor in a Lorentzian para-Kenmotsu manifold were derived. Further, it was found that an *LP*-Kenmotsu manifold is either ϕ-symmetric or concircular ϕ-symmetric with respect to quarter-symmetric metric connection if and only if it is symmetric with respect to the Riemannian connection, provided the scalar curvature of Riemannian connection is constant. The paper ends with a handful of bibliography.

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