Estimation of Stress-Strength Reliability Based on KME Model

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Abstract

In reliability theory the estimation of stress-strength reliability is an important problem. It has many applications in engineering and physics areas. In many practical situations, the assumption of identical strength distributions may not be quite realistic because components of a system are of different structure. Here we establish the estimation of stress-strength reliability of the KM-Exponential (KME) distribution. In this article, we consider the case that the stress-strength variables are independent. KME distribution is parsimonious in parameter and has decreasing failure rate. The stress-strength reliability based on KME model is estabished and using maximum likelihood estimation method, the estimation of the stress-strength reliability is derived and also the asymptotic distribution. Simulation method is used to show the performance of the parameters and the 95% confidence interval is also calculated. With the help of simulated data, we depicts the application of the stress-strength reliability of KME distribution.

Keywords: KM-Exponential Stress-Strength reliability Estimation Simulation study.

1. Introduction

The problem of estimation of stress-strength reliability has great attention in reliability theory. The term stress is defined as a failure inducing variable. That means the stress (load) which tends to produce a failure of a component or of a device of a material. For example, environment, pressure, load, velocity, resistance, temperature, humidity, vibrations, and voltage etc. The term strength is defined as it is failure resisting variable. The ability of component, device or a material to accomplish its required function (mission) satisfactorily without failure when subjected to the external loading and environment.

The stress-strength reliability model depicts the life of a component or item with a random strength X and is subjected to a random stress Y. If the stress on the component surpasses the strength, it fails instantaneously. Whenever $Y < X$ the item functions satisfactorily. The component reliability is defined as

$$R = P(Y < X) = \int_{-\infty}^{\infty} \int_{-\infty}^{x} f(x,\ y)\ dy\ dx,$$

where $f(x,\ y)$ is the joint pdf of X and Y. Suppose the random variable X and Y are independent, then R can be written as

$$R = \int_{-\infty}^{\infty} \int_{-\infty}^{x} f(x)\ g(y)\ dy\ dx,$$

where $f(x)$ and $g(y)$ are the marginal pdfs of X and Y. This is also can be written as

$$R = \int_{-\infty}^{\infty} f(x)\ G_y(x)\ dx.$$

where $G_y(x)$ is the cdf of $g(y)$.

The germ of this idea was proposed by Birnbaum [1] and was developed by Birnbaum and McCarty [2]. The formal term stress-strength firstly appears in the title of Church and Harris [3]. Based on certain parametric assumptions regarding X and Y, the first attempt to study R was undertaken by Owen et al. [4]. They also calculated the confidence interval for R when X and Y are independent or dependent normally distributed random variables. The estimation of R for major distributions like normal (Church and Harris [3], Downton [5];, Woodward and Kelley [6]), exponential (Kelly et al [7], Tong [8]), Pareto (Beg and Singh [9]), and exponential families (Tong [10]) was derived by the end of seventies. Enis and Geisser [11] contribute the Bayes estimation of R for exponentially or normally distributed X and Y. The other major works of the seventies include the introduction of a non-parametric empirical Bayes estimation of R by Ferguson [12] and Hollander and Korwar [13], and the study of system reliability (Bhattacharya and Johnson [14]).

Both stress and strength depend on some known covariates, Guttman et al. [15] and Weerahandi and Johnson [16] discussed the estimation and associated confidence interval of R. Using Bayesian approach Sun et al. [17] estimated the stress-strength reliability. Raqab and Kundu [18] carried out the estimation of stress-strength reliability, when Y and X two independent scaled Burr type X distribution. A comprehensive treatment of the different stress-strength models till 2001 can be found in the excellent monograph by Kotz et al. [19]. Some of the work on the estimation of stress-strength reliability can be obtained in Kundu and Gupta ([20], [21]), Kundu and Raqab [22], Krishnamoorthy et al. [23], Raqab et al. [24], Rezaei et al. [25], and Baklizi [26]. Baklizi and Eidous [27] introduced an estimator of stress-strength reliability based on kernel estimators. Estimation of stress-strength reliability using empirical likelihood method was studied by Jing et al. [28].

Basirat et al. [29] studied the estimation of stress-strength parameter using record values from proportional hazard model. Estimation of stress-strength reliability based on the generalized exponential distribution was developed by Asgharzadeh et al.[30]. Bai et al. [31] considered reliability inference of stress-strength model under progressively Type-II censored samples when stress and strength have truncated proportional hazard rate distributions. Bi and Gui [32] derived Bayesian estimation of R using inverse Weibull distribution. Ghitany et al. [33] discussed inference on stress-strength reliability based on power Lindley distribution. Sharma [34] proposed an upside-down bathtub shape distribution and estimate of stress-strength reliability of inverse Lindley distribution.

This paper is organized as follows. Preliminaries of the KME model are given in Section 2. In Section 3 the stress-strength reliability for the KME model is derived. The estimation of stress-strength reliability R is explained in Section 4. In Section 5 the asymptotic distribution and confidence interval are given. Simulation study and applications are discussed in Section 6 and Section 7 respectively. Finally we concluded the present work in Section 8.

2. Preliminaries of KME model

We obtain the KME model using the cumulative distribution function (cdf) of exponential distribution in KM transformation given in Kavya and Manoharan [36]. The probability distribution function (pdf) and cdf of the KME distribution are

$$\begin{aligned} f(x) =& \frac{\lambda e^{-\lambda x} e^{e^{-\lambda x}}}{e-1}, \quad x > 0, \ \ \lambda > 0, \\ F(x) =& \frac{e}{e-1}[1 - e^{-(1-e^{-\lambda x})}], \quad x > 0, \ \ \lambda > 0, \end{aligned}$$

3. Stress-strength reliability based on the model

The stress-strength reliability model depicts the life of a component or item with a random strength X and is subjected to a random stress Y. If the stress on the component surpasses the strength, it fails instantaneously. Whenever $Y < X$ the item functions satisfactorily. The component reliability is defined as $R = P(Y < X)$. It has applications in engineering fields such as failure of aircraft structures, deterioration of rocket motors, and the aging of concrete pressure vessels.

Suppose X and Y are two independent random variables. If $X \sim \text{KME}(\lambda_1)$ and $Y \sim \text{KME}(\lambda_2)$, then the stress-strength reliability is obtained as

$$\begin{aligned} R = P(Y < X) = &\int_0^\infty \frac{\lambda_1 e^{-\lambda_1 x} e^{e^{-\lambda_1 x}}}{e-1} \left[\frac{e}{e-1} \left(1 - e^{-(1-e^{-\lambda_2 x})} \right) \right] dx \\ = &\frac{\lambda_1 e}{(e-1)^2} \sum_{m=0}^{\infty} \frac{1}{m!} \left[I_1 + I_2 \right] \end{aligned} \tag{1}$$

where $I_1 = \int_0^\infty e^{-\lambda_1 x(m+1)} dx$ and $I_2 = \int_0^\infty e^{-\lambda_1 x(m+1)} e^{-(1-e^{-\lambda_2 x})} dx$. After integration, we get the values of $I_1 = \frac{1}{\lambda_1(m+1)}$ and $I_2 = \sum_{n=0}^\infty \sum_{i=0}^\infty \frac{(-1)^{n+i}}{n!} \binom{n}{i} \frac{1}{\lambda_1(m+1)+\lambda_2 i}$. Substituting these values in (1), the stress-strength reliability based on the KME model is obtained as

$$\begin{aligned} R &= P(Y < X) \\ &= \frac{\lambda_1 e}{(e-1)^2} \sum_{m=0}^{\infty} \frac{1}{m!} \left[\frac{1}{\lambda_1(m+1)} - \sum_{n=0}^{\infty} \sum_{i=0}^{\infty} \frac{(-1)^{n+i}}{n!} \binom{n}{i} \frac{1}{\lambda_1(m+1) + \lambda_2 i} \right]. \end{aligned} \tag{2}$$

4. Estimation of R

Suppose we drawn a random sample $x_1, x_2, ..., x_n$ of size p from KME(λ_1) and $y_1, y_2, ..., y_n$ of size q from KME(λ_2). The likelihood function is obtained as

$$L = \left(\frac{1}{e-1} \right)^p \lambda_1^p e^{-\lambda_1 \sum_{i=1}^p x_i} e^{\sum_{i=1}^p e^{-\lambda_1 x_i}} \left(\frac{1}{e-1} \right)^q \lambda_2^q e^{-\lambda_2 \sum_{j=1}^q y_j} e^{\sum_{j=1}^q e^{-\lambda_2 y_j}} \tag{3}$$

The log likelihood function is

$$\begin{aligned} \log\ L = &p \log(\frac{1}{e-1}) + p \log(\lambda_1) - \lambda_1 \sum_{i=1}^p x_i + \sum_{i=1}^p e^{-\lambda_1 x_i} \\ &q \log(\frac{1}{e-1}) + q \log(\lambda_2) - \lambda_2 \sum_{j=1}^q y_j + \sum_{j=1}^q e^{-\lambda_2 y_j} \end{aligned} \tag{4}$$

The partial derivatives of the log likelihood function with respect to λ_1 and λ_2 are

$$\frac{\partial \log\ L}{\partial \lambda_1} = \frac{p}{\lambda_1} - \sum_{i=1}^p x_i \left(1 + e^{-\lambda_1 x_i} \right)$$

and

$$\frac{\partial \log\ L}{\partial \lambda_2} = \frac{q}{\lambda_2} - \sum_{j=1}^q y_j \left(1 + e^{-\lambda_2 y_j} \right)$$

The maximum likelihood estimates of the parameters are obtained as the solution of the above non-linear equations.

The second partial derivatives of the log likelihood function with respect to λ_1 and λ_2 are

$$\frac{\partial^2 \log\ L}{\partial \lambda_1^2} = \frac{-p}{\lambda_1^2} + \sum_{i=1}^{p} x_i^2 e^{-\lambda_1 x_i}$$

and

$$\frac{\partial^2 \log\ L}{\partial \lambda_2^2} = \frac{-q}{\lambda_2^2} + \sum_{j=1}^{q} y_j^2 e^{-\lambda_2 y_j}$$

The maximum likelihood estimate of Stress-Strength reliability R is

$$\widehat{R}_{ML} = \frac{\widehat{\lambda_1} e}{(e-1)^2} \sum_{m=0}^{\infty} \frac{1}{m!} \left[\frac{1}{\widehat{\lambda_1}(m+1)} - \sum_{n=0}^{\infty} \sum_{i=0}^{\infty} \frac{(-1)^{n+i}}{n!} \binom{n}{i} \frac{1}{\widehat{\lambda_1}(m+1) + \widehat{\lambda_2} i} \right] \tag{5}$$

We obtain the expression of the maximum likelihood estimate of Stress-Strength reliability R by substituting the estimated parameters in the Equation (2).

5. Asymptotic distribution and confidence interval

In this section we focused on the asymptotic distribution and confidence interval of the maximum likelihood estimate of R. To obtain the asymptotic variance of the maximum likelihood estimate of R, we consider the Fisher information matrix of λ and is denoted as I.

$$I = -\begin{pmatrix} E(\frac{\partial^2 \log\ L}{\partial \lambda_1^2}) & E(\frac{\partial^2 \log\ L}{\partial \lambda_1 \partial \lambda_2}) \\ E(\frac{\partial^2 \log\ L}{\partial \lambda_2 \partial \lambda_1}) & E(\frac{\partial^2 \log\ L}{\partial \lambda_2^2}) \end{pmatrix}$$

Using the standard method of asymptotic properties of maximum likelihood estimate, we derive the asymptotic normality of R as

$$d(\lambda) = \left(\frac{\partial R}{\partial \lambda_1}, \frac{\partial R}{\partial \lambda_2} \right)' = (d_1, d_2)'$$

Here

$$\frac{\partial R}{\partial \lambda_1} = -\frac{e}{e-1} \sum_{m=0}^{\infty} \frac{1}{m!} \sum_{n=0}^{\infty} \sum_{i=0}^{\infty} \frac{(-1)^{n+i}}{n!} \binom{n}{i} \frac{\widehat{\lambda_2} i}{(\widehat{\lambda_1}(m+1) + \widehat{\lambda_2} i)^2}$$

and

$$\frac{\partial R}{\partial \lambda_2} = -\frac{\widehat{\lambda_1} e}{(e-1)^2} \sum_{m=0}^{\infty} \frac{1}{m!} \sum_{n=0}^{\infty} \sum_{i=0}^{\infty} \frac{(-1)^{n+i}}{n!} \binom{n}{i} \frac{i}{(\widehat{\lambda_1}(m+1) + \widehat{\lambda_2} i)^2}$$

Now we obtain the asymptotic distribution of $\widehat{R}_{ML}$ as

$$\sqrt{p+q}(\widehat{R}_{ML} - R) \to^d N(0, d'(\lambda) I^{-1} d(\lambda)).$$

The asymptotic variance of the $\widehat{R}_{ML}$ is

$$\begin{aligned} AV(\widehat{R}_{ML}) =& \frac{1}{p+q} 0, d'(\lambda) I^{-1} d(\lambda) \\ =& V(\widehat{\lambda_1}) d_1^2 + V(\widehat{\lambda_2}) d_2^2 + 2 d_1 d_2 Cov(\widehat{\lambda_1}, \widehat{\lambda_2}). \end{aligned}$$

Hence an asymptotic $100(1-\xi)\%$ confidence interval for R can be obtained as

$$\widehat{R}_{ML} \pm Z_{\frac{\xi}{2}} \sqrt{AV(\widehat{R}_{ML})},$$

where $Z_{\frac{\xi}{2}}$ is the upper $\frac{\xi}{2}$ quantile function of the standard normal distribution.

6. Simulation study

In this section we check the performance of estimators in R using simulation technique. For this purpose we generate 1000 pseudo random samples using Newton-Raphson method. The random samples are generated for different population parameters of (λ_1, λ_2) as (0.5,1), (0.9,0.5), and (1.0,0.9) and sample sizes (p, q) as (10,10),(15,25),(20,20),(30,30),(40,40), and (50,50). The maximum likelihood estimates, their mean square error (MSE) and 95% confidence interval (CI) are calculated and the results are given in the following tables.

Table 1: *The ML estimates, MSEs and confidence interval of different estimators of R when* $\lambda_1 = 0.5$ *and* $\lambda_2 = 1.0$

(p, q)	ML estimates	MSEs	CI
(10,10)	λ_1 =0.57077	0.24634	(0.08795, 1.05359)
	λ_2 =1.14347	0.06295	(0.80201, 1.26686)
(15,25)	λ_1 =0.53939	0.20729	(0.13312, 0.94566)
	λ_2 =1.03657	0.12229	(0.80124, 1.06082)
(20,20)	λ_1 =0.53241	0.32406	(-0.10276, 1.16757)
	λ_2 =1.05715	0.04907	(0.96097, 1.15333)
(30,30)	λ_1 =0.52109	0.13712	(0.25233, 0.78985)
	λ_2 =1.04110	0.01237	(0.80140, 1.28079)
(40,40)	λ_1 =0.51019	0.22229	(0.07450, 0.94588)
	λ_2 =1.02810	0.02570	(0.97773, 1.07847)
(50,50)	λ_1 =0.51125	0.19753	(0.12409, 0.89841)
	λ_2 =1.02751	0.02640	(0.97577, 1.07925)

Table 2: *The ML estimates, MSEs and confidence interval of different estimators of R when* $\lambda_1 = 0.9$ *and* $\lambda_2 = 0.5$

(p, q)	ML estimates	MSEs	CI
(10,10)	λ_1 =1.00947	0.29601	(0.42928, 1.58966)
	λ_2 =0.55836	0.10123	(0.35994, 0.75677)
(15,25)	λ_1 =0.96646	0.12012	(0.73102, 1.2019)
	λ_2 =0.52620	0.01846	(0.35258, 0.52656)
(20,20)	λ_1 =0.95902	0.08818	(0.78620, 1.13185)
	λ_2 =0.53293	0.01209	(0.43045, 0.53541)
(30,30)	λ_1 =0.94372	0.06563	(0.81508, 1.07237)
	λ_2 =0.51937	0.00126	(0.49567, 0.54307)
(40,40)	λ_1 =0.92773	0.00185	(0.82411, 0.93135)
	λ_2 =0.51179	0.00033	(0.41115, 0.51243)
(50,50)	λ_1 =0.91653	0.00167	(0.89133, 0.91980)
	λ_2 =0.51249	0.00017	(0.45121, 0.51283)

Results from the simulation study reveals that sample sizes p and q increase, the estimated parameter values tends to population parameter values. Also the MSEs are decreasing with increase in sample sizes (p, q).

7. Application

In this section we have generated two data sets ($p = q = 20$) using KME model with parameter values $\lambda_1 = 1$ and $\lambda_2 = 0.5$. Therefore the value of R is obtained as 0.17633. The data points are adjusted in two decimal points and the data sets are presented in the following tables.

Table 3: *The ML estimates, MSEs and confidence interval of different estimators of R when* $\lambda_1 = 1.5$ *and* $\lambda_2 = 0.9$

(p, q)	ML estimates	MSEs	CI
(10,10)	λ_1 =1.70398	0.11763	(1.06944, 1.73853)
	λ_2 =1.01417	0.04031	(0.13567, 1.01478)
(15,25)	λ_1 =1.62178	0.09751	(1.10266, 1.64089)
	λ_2 =0.93769	0.03302	(0.27297, 1.00240)
(20,20)	λ_1 =1.59399	0.08241	(1.23654, 1.65144)
	λ_2 =0.95547	0.00594	(0.43819, 0.96712)
(30,30)	λ_1 =1.56481	0.08071	(1.39415, 1.54773)
	λ_2 =0.93981	0.00556	(0.63479, 0.98262)
(40,40)	λ_1 =1.55355	0.05150	(1.39382, 1.71329)
	λ_2 =0.92423	0.00625	(0.71198, 0.93647)
(50,50)	λ_1 =1.53238	0.00228	(1.32790, 1.53686)
	λ_2 =0.92832	0.00279	(0.81279, 0.94385)

Table 4: *Data set I*

4.02	0.44	1.43	0.09	0.49	0.27	0.54	0.02	0.48	1.77
3.04	4.30	0.94	3.08	1.42	0.09	3.05	2.17	0.21	0.64

Table 5: *Data set II*

0.09	0.26	0.20	0.11	2.08	1.48	0.85	2.57	1.04	0.26
0.01	0.40	1.37	0.71	0.29	1.10	0.81	0.13	1.73	2.25

In this case the maximum likelihood estimates of λ_1 and λ_2 are obtained respectively as 0.854 and 0.542. Here the estimated value of R, $\widehat{R}_{ML}$ is obtained as 0.21336. The corresponding 95% confidence interval based on asymptotic distribution is (0.19153, 0.23519).

8. Conclusion

In this paper we consider the estimation of the stress-strength reliability for the KME model for independent stress and strength random variables when the parameters are unknown. The maximum likelihood estimators of the unknown parameters are calculated. Then provide the asymptotic distributions of the maximum likelihood estimators, which have been used to construct the asymptotic confidence intervals. Simulation study is carried out to examine the performance of the estimators. The study reveals that MSEs are decreasing with increase in sample sizes. Using a simulated data set, we find the estimates of the parameters, $\widehat{R}_{ML}$ value and 95% confidence interval.

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