Redundancy Optimization for a System Comprising One Operative Unit and N Hot Standby Units

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Abstract

In many industries and applications, downtime or failure can have serious consequences, such as financial losses, safety hazards, or reputational damage. A hot standby unit can help minimize the impact of such events by providing a backup that can quickly and seamlessly take over in the event of a failure. Further, the question of as to how many hot standby units should be used also needs to be addressed. So, an N+1-Unit-system is investigated wherein N units are on hot standby, whereas one unit is operational and the system is such that the hot standby units can take over seamlessly if the single operative unit fails. The system breaks down completely when all the units fail. It is assumed that the failure rates of all the operational units and the redundant units will vary exponentially. To get different performability measurements, the regenerative point technique has been applied to optimize the value of N.

Keywords: Redundancy, Optimization, N Hot Standby Units, One Operative Unit, Profit Analysis, Regenerative Point Technique

1. Introduction

A crucial and difficult issue facing the manufacturing industry is reliability. Complexity makes it more challenging to successfully manage and run a system. Various system analyzers have had various issues as a result of expensive and unreliable components. Therefore, creating efficient modeling, monitoring, and control techniques is crucial for increasing the reliability of systems. To increase the dependability of a system, redundancy is necessary. In redundant systems, one or more than one unit runs while backups step in to take over as needed. Active redundancy and passive redundancy are the two types of redundancy. In the past, active redundancy has generally got greater attention. However, in actuality, a specific system design may include both active and cold-standby redundancies. Therefore, the challenge is to determine for each subsystem the appropriate redundancy approach, component, and level to maximize system dependability within system-level restrictions. Other considerations in redundancy optimization may include selecting the most appropriate type of standby units, such as identical or functionally equivalent units, and designing a failover mechanism that can quickly and seamlessly switch from one mode to the other. Reliability Models for different systems' dependability that take the idea into consideration of standby by taking some operative units have been developed by various researchers. For a 2-unit cold standby system, Ritu and Malhotra [13] discussed a stochastic analysis wherein both units might become operational depending on the demand. Wang et al. [16] considered the optimization of cold-standby systems that are subject to periodic inspection and maintenance, as well as the evaluation of system reliability and expected cost using an approximation method. Behboudi et al. [21] suggested a periodic switching approach for analyzing and evaluating the overall reliability of two-unit cold standby systems utilizing the idea of the virtual age. Shenyang et al. [23] presented an optimization model that employs a novel methodology using an imprecise and nonperiodic different types of switching strategy to improve the performance and efficiency of warm standby systems and various reliability functions are obtained using a recursive method. Zhang et al. [5] analyzed the performance of warm standby systems in which k units out of M+N must be active to provide complete system functioning with r repair facilities, and system availability and reliability are derived by considering a specific example. M. N. Gopalan [1] is focused on the availability and reliability analysis of a system with n units, single repair facility, and (n-1) warm standbys and discusses the cases n=2 and 3. Papageorgiou and Kokolakis [6], [10] studied n units parallel system in which 2 units are operative simultaneously and n-2 are in standby mode. When either operative unit fails, the other (n-2) warm/cold standbys immediately take over. By using recursive relations, the system's dependability is evaluated, unlike the majority of preceding findings, which offer boundaries for joint pdfs with limited information.

The reliability models considering hot standby systems have also been developed by various researchers. Goel et al. [1] analyzed three different sorts of failure mechanisms in a two-unit hot standby system with a single repair facility. A number of dependability characteristics that are important to system designers and operations managers have been assessed. Fujii and Sandoh [4] focused on the evaluation of the reliability of a 2-unit hot standby redundancy system using a Bayesian approach by taking both units identically. Rizwan, et al. [11] presented a theoretical analysis or mathematical model for the reliability enhancement of the hot standby industrial system and discussed various factors such as failure rates, repair times, availability, and downtime in the context of the system's reliability. Additionally, they provide simulation outcomes or case studies to demonstrate the effectiveness of the suggested reliability analysis method. Ebrahimipour et al. [9] addressed Optimizing Redundancy in Multi-State Series-Parallel k-out-of-n Systems with Hot Standby and discussed mathematical models, optimization algorithms, or simulation results to show that the suggested strategy is successful at maximizing redundancy and improving system reliability. Manocha et al. [14] focused on the profit analysis of a 2-unit hot standby database system to compare the costs associated with implementing and maintaining the redundant system with the potential benefits in terms of improved reliability, reduced downtime, and enhanced system availability. Venkatachalam and Parvatham [2] focused on the stochastic behavior analysis of a 1-server 2-unit hot standby system that deals with the probabilistic aspects of the system's performance and reliability. Shuhang and Zhang [8] developed a Markov model to analyze and evaluate the reliability of the hot standby repairable supply system and provide simulation results or case studies to validate the Markov model's effectiveness in assessing reliability.

Cold or warm standby redundancy can still be suitable for certain applications where downtime and response time are less critical or cost considerations are paramount, hot standby redundancy offers the highest level of reliability, availability, and quick recovery. It ensures continuous operation, minimizes disruptions, and provides a seamless user experience, making it essential for systems that demand high performance, real-time response, and minimal downtime.

Batra and Taneja [17] developed reliability models and optimized the number of hot standby units in a system having one or two operational units to ensure the desired level of system reliability while considering factors such as cost, availability, and other constraints by employing approaches for regenerative point techniques and the Markov process. However, more than two hot standby units may also be taken into consideration in a system after carrying out an optimum analysis as to how many hot standby units should be used in order to have a more reliable/available and economically viable system. So, this paper refers to a system design where there are N+1 units, with one unit being actively used (the single operative unit) and the remaining N units serving as hot standby backups. In an N+1 system, the extra unit provides an additional level of redundancy, which can increase system availability and reduce the likelihood of downtime. To get different performability measurements, the regenerating point technique has been applied to optimize the value of N which is laid out as follows. The system's assumptions and characterizations are covered in Section 2. The nomenclature used in this analysis is described in Section 3. The system's description is covered in Section 4. The model of the system, transition densities, and mean sojourn time are covered in Section 5. To increase the number of standby units that will be deployed, the Generalised Results for the different Measures of System Effectiveness and Profit Equation have been derived in Sections 6 and 7. For some specific circumstances, Section 8 gives graphical interpretation and numerical results. The study's conclusions are presented in Section 9.

2. Assumptions and System Characterizations

1. The system is designed so that only one operational unit is required at any given moment, therefore N additional units are kept on hot standby.
2. Both the active and standby unit's failure rates are modeled by an exponential distribution.
3. The item is as good as new after being repaired.
4. With the system, there is just one repairman.
5. Even with only one active unit, the system continues to function.
6. First come first serve service is followed.

3. Nomenclature

The terminology for various transition densities and probabilities is as follows:

λ_o/α_1	Rate of failure/ repair of operative unit
λ_1/α_2	Rate of failure/ repair of standby unit
Hs	Hot standby unit
Op	Operative unit.
NF_r	N Operative units are failed .
NF_{rh}	N standby units are failed .
HS_N	N units are on hot standby .
F_r	Operative unit under repair
Fr_h	Standby unit under repair
F_{wr}	A failed operational unit is awaiting repair.
F_{wr_h}	Failed hot standby unit waiting for the repair.

For the notation $\phi_i(t)$, $q_{ij}(t), Q_{ij}(t)$, $A_i(t)$, $Bi(t)$, $V_i(t)$, $M_i(t)$ one may refer [17].

4. System Description

Here we develop a reliability model consisting of one primary operating unit responsible for handling the system's operations. Hot standby units remain ready so that one of them takes over in case the primary unit fails or becomes unavailable. The hot standby units are fully functional and synchronized with the primary unit, ensuring a seamless transition in the event of a failure. The model examined here may be used in a variety of actual scenarios, including power distribution systems, network router systems, emergency power supply systems, and navigator components.

5. Transition Densities and Mean Sojourn Times

In this model, there are a total $N^2 + 3N + 1$ Number of states in the model out of which $N^2 + N + 1$ states are operative states and 2N number of states are failed states. Possible states of the system along with transitions are represented in Figure 1 and Figure 2

Repersentatinos of the States of the System;

$U_1(s,k)$: Operative state in which 1 represents the Main operative unit failed before any hot standby unit and then going for under repair and (s-2) the number of failed hot standby unit waiting for repair and k denotes the number of failed operative unit waiting for the repair and remaining [(N+1)-(s+k)] unit are on standby mode.

$U_2(s,k)$: Operative state in which 2 represent hot standby unit failed before Main operative unit and then going for under repair and (s-2) a number of failed hot standby unit waiting for repair and k denote the number of failed operative unit waiting for the repair and remaining [(N+1)-(s+k)] unit is on standby mode.

$F_1(s,k)$: Failed state in which one failed operating unit is under repair and (s-2) number of failed hot standby units awaiting repair and k+1 denotes the number of failed operative unit waiting for the repair and remaining [N-(s+k)] unit are on standby mode.

$F_2(s,k)$: Failed state in which one failed hot standby unit is under repair and (s-2) a number of failed hot standby units awaiting repair and k+1 denotes the number of failed operative unit waiting for the repair and remaining [N-(s+k)] unit are on standby mode.

Here, it has been assumed that if the value [(N+1)-(s+k)] and [N-(s+k)] is negative it means there is no hot standby unit available in the system.

The states where the main operative unit fails before any standby unit

State $U_1(2,0)$: (Op, F_r, HS_{N-1});
State $U_1(2,1)$: $(Op, F_r, F_{wr}, HS_{N-2})$;
State $U_1(2,2)$: $(Op, F_r, 2F_{wr}, HS_{N-3})$; . . .
State $U_1(2,N-3)$: $(Op, F_r, (N-3)F_{wr}, HS_2)$;
State $U_1(2,N-2)$: $(Op, F_r, (N-2)F_{wr}, WS_1)$;
State $U_1(2,N-1)$: $(Op, F_r, (N-1)F_{wr})$;
State $U_1(3,0)$: $(Op, F_r, F_{wr_h}, HS_{N-2})$;
State $U_1(3,1)$: $(Op, F_r, F_{wr}, F_{wr_h}, HS_{N-3})$;
State $U_1(3,2)$: $(Op, F_r, 2F_{wr}, F_{wr_h}, HS_{N-4})$; . . .
State $U_1(3,N-3)$: $(Op, F_r, (N-3)F_{wr}, F_{wr_h}, HS_1)$;
State $U_1(3,N-2)$: $(Op, F_r, (N-2)F_{wr}, F_{wr_h})$;
State $U_1(4,0)$: $(Op, F_r, 2F_{wr_h}, HS_{N-3})$;
State $U_1(4,1)$: $(Op, F_{wr}, F_r, 2F_{wr_h}, HS_{N-4})$;
State $U_1(4,2)$: $(Op, 2F_{wr}, F_r, 2F_{wr_h}, HS_{N-5})$;. . .
State $U_1(4,N-3)$: $(Op, F_r, (N-3)F_{wr}, 2F_{wr_h})$;
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State $U_2(N-1,0)$: $(Op, F_r, (N-3)F_{wr_h}, HS_2)$;
State $U_2(N-1,1)$: $(Op, F_r, F_{wr}, (N-3)F_{wr_h}, HS)$;
State $U_2(N-1,2)$: $(Op, F_r, 2F_{wr}, (N-3)F_{wr_h})$;
State $U_1(N,0)$: $(Op, F_r, (N-2)F_{wr_h}, HS)$;
State $U_1(N,1)$: $(Op, F_r, F_{wr}, (N-2)F_{wr_h})$;
State $U_1(N+1,0)$: $(Op, F_r, (N-1)F_{wr_h})$;
State $F_1(N+1,0)$: $(F_r, (N-1)F_{wr_h})$;
State $F_1(N,1)$: $(F_r, 2F_{wr}, (N-2)F_{wr_h})$;
State $F_1(N-1,2)$: $(F_r, 3F_{wr}, (N-3)F_{wr_h})$;. . .
State $F_1(4,N-3)$: $(F_r, (N-2)F_{wr}, 2F_{wr_h})$;
State $F_1(3,N-2)$: $(F_r, (N-1)F_{wr}, F_{wr_h})$;
State $F_1(2,N-1)$: (F_r, NF_{wr});

The states where the first hot standby unit fails before the main operative unit

State $U_2(2,0)$: (Op, F_{r_h}, HS_{N-1});
State $U_2(2,1)$: $(Op, F_{r_h}, F_{wr}, HS_{N-2})$;
State $U_2(2,2)$: $(Op, F_{r_h}, 2F_{wr}, HS_{N-3})$; . . .
State $U_2(2,N-3)$: $(Op, F_{r_h}, (N-3)F_{wr}, HS_2)$;
State $U_2(2,N-2)$: $(Op, F_{r_h}, (N-2)F_{wr}, HS_1)$;
State $U_2(2,N-1)$: $(Op, F_{r_h}, (N-1)F_{wr})$;
State $U_2(3,0)$: $(Op, F_{r_h}, F_{wr_h}, HS_{N-2})$;
State $U_2(3,1)$: $(Op, F_{r_h}, F_{wr}, F_{wr_h}, HS_{N-3})$;
State $U_2(3,2)$: $(Op, F_{r_h}, 2F_{wr}, F_{wr_h}, HS_{N-4})$; . . .
State $U_2(3,N-3)$: $(Op, F_{r_h}, (N-3)F_{wr}, F_{wr_h}, HS_1)$;
State $U_2(3,N-2)$: $(Op, F_{r_h}, (N-2)F_{wr}, F_{wr_h})$;

State $U_2(4,0)$: $(Op, F_{r_h}, 2F_{wr_h}, HS_{N-3})$;
State $U_2(4,1)$: $(Op, F_{wr}, F_{r_h}, 2F_{wr_h}, HS_{N-4})$;
State $U_2(4,2)$: $(Op, 2F_{wr}, F_{r_h}, 2F_{wr_h}, HS_{N-5})$;...
State $U_2(4,N-3)$: $(Op, F_{r_h}, (N-3)F_{wr}, 2F_{wr_h})$;

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State $U_2(N-1,0)$: $(Op, F_{r_h}, (N-3)F_{wr_h}, HS_2)$;
State $U_2(N-1,1)$: $(Op, F_{r_h}, F_{wr}, (N-3)F_{wr_h}, HS)$;
State $U_2(N-1,2)$: $(Op, F_{r_h}, 2F_{wr}, (N-3)F_{wr_h})$;
State $U_2(N,0)$: $(Op, F_{r_h}, (N-2)F_{wr_h}, HS)$;
State $U_2(N,1)$: $(Op, F_{r_h}, F_{wr}, (N-2)F_{wr_h})$;
State $U_1(N+1,0)$: $(Op, F_{r_h}, (N-1)F_{wr_h})$;
State $F_2(N+1,0)$: $(F_{r_h}, (N-1)F_{wr_h})$;
State $F_2(N,1)$: $(F_{r_h}, 2F_{wr}, (N-2)F_{wr_h})$;
State $F_2(N-1,2)$: $(F_{r_h}, 3F_{wr}, (N-3)F_{wr_h})$;. . .
State $F_2(4,N-3)$: $(F_{r_h}, (N-2)F_{wr}, 2F_{wr_h})$;
State $F_2(3,N-2)$: $(F_{r_h}, (N-1)F_{wr}, F_{wr_h})$;
State $F_2(2,N-1)$: (F_{r_h}, NF_{wr});

Transition Diagram of the Model

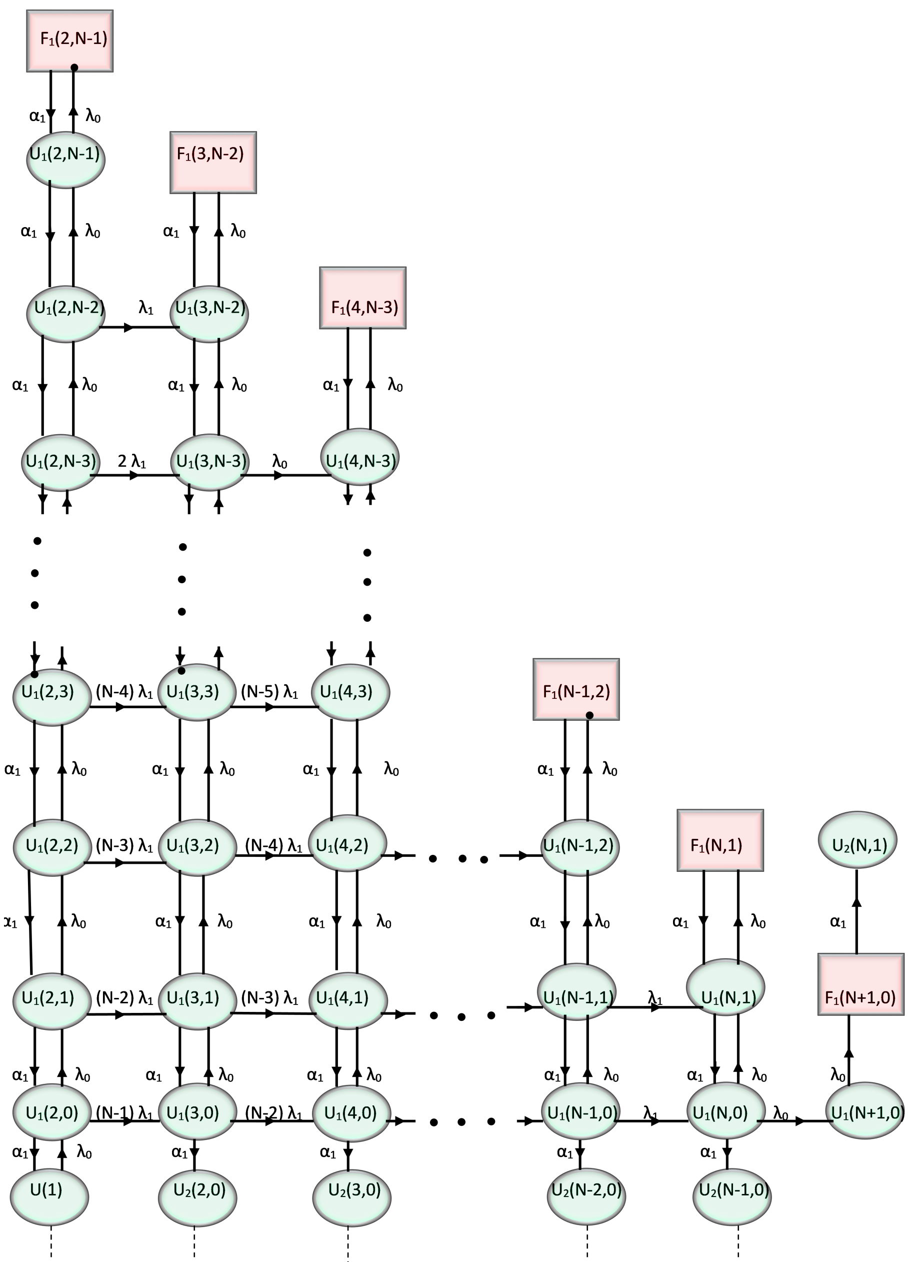

Figure 1: *State Transition Diagram (When main operative unit failed before any standby unit)*

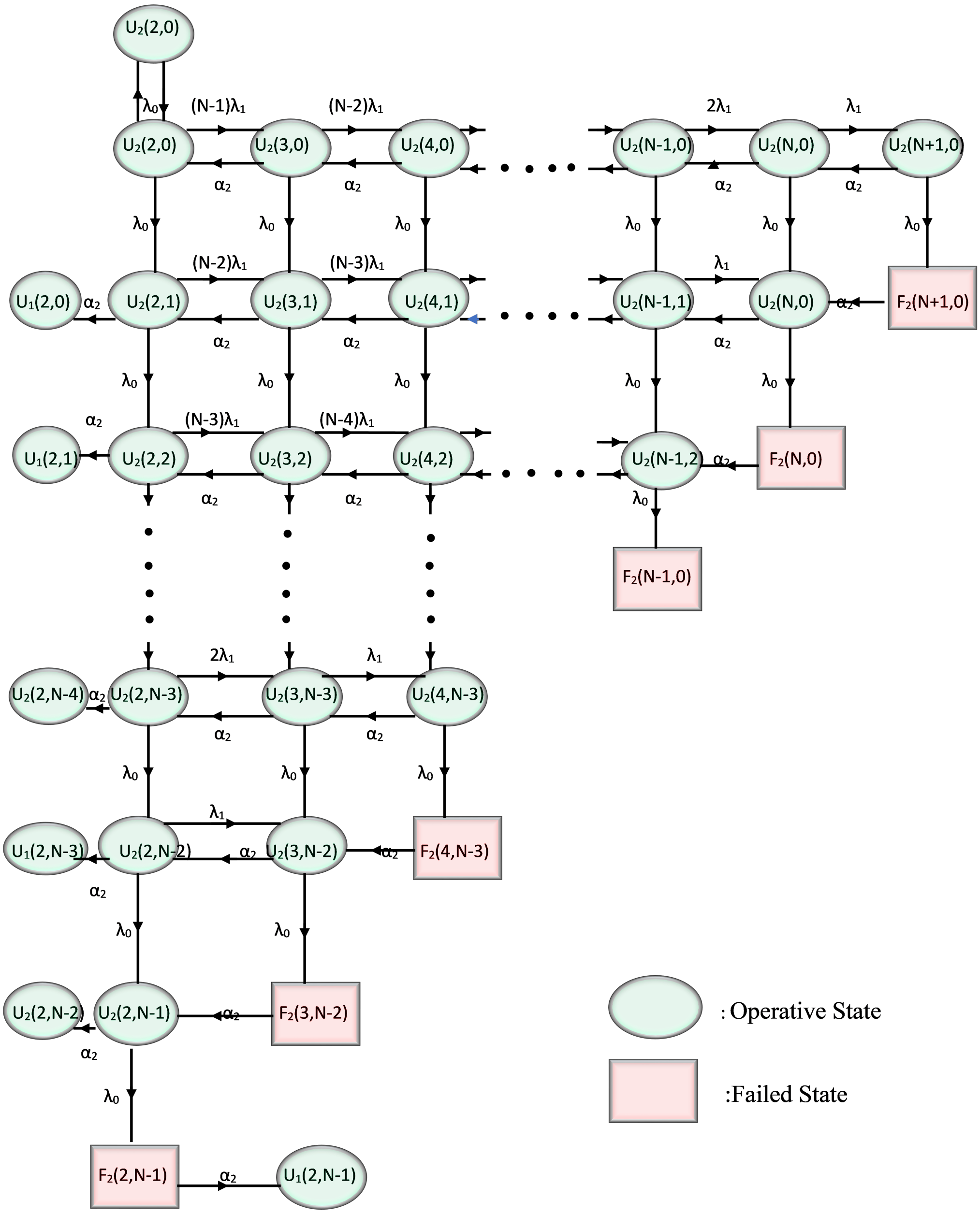

Figure 2: *State Transition Diagram (When first Standby unit failed before operational unit)*

The densities $q_{ij}(t)$ for transiting from state i to j are given by

$$q_{U(1),U_1(2,0)}(t) = \lambda_0 e^{-(\lambda_0+N\lambda_1)t}, \quad q_{U(1),U_2(2,0)}(t) = N\lambda_1 e^{-(\lambda_0+N\lambda_1)t}, \tag{1}$$

$$\begin{cases} q_{U_i(s,0),U_i(s,1)}(t) = \lambda_0 e^{-(\lambda_0+\alpha_i+[(M+1)-s]\lambda_1)t} \\ q_{U_i(s,0),U_i(s+1,0)}(t) = (M+1-s)\lambda_1 e^{-(\lambda+\alpha_i+[(M+1)-s]\lambda_1)t} \\ q_{U_i(s,0),U_2(s-1,0)}(t) = \alpha_i e^{-(\lambda+\alpha_i+[(M+1)-s]\lambda_1)t} \\ \textit{where } i = 1 \textit{ or } 2,\ 3 \le s \le N, 2 < M \le N \textit{ and } M \textit{ is an integer} \end{cases} \tag{2}$$

$$\begin{cases} q_{U_i(2,s),U_i(2,s+1)}(t) = \lambda_0 e^{-(\lambda_0+\alpha_i+[M-1-s]\lambda_1)t} \\ q_{U_i(2,s),U_i(3,s+1)}(t) = [M-1-s)]\lambda_1 e^{-(\lambda_0+\alpha_i+[M-1-s]\lambda_1)t} \\ q_{U_i(2,s),U_1(2,s-1)}(t) = \alpha_i e^{-(\lambda_0+\alpha_i+[M-1-s]\lambda_1)t} \\ \textit{where } i = 1 \textit{ or } 2,\ 0 \le s \le N-2, 1 < M \le N \textit{ and } M \textit{ is an integer} \end{cases} \tag{3}$$

$$\begin{cases} q_{U_i(s,k),U_i(s,k+1)}(t) = \lambda_0 e^{-(\lambda_0+\alpha_i+[(M+1)-(s+k)]\lambda_1)t} \\ q_{U_i(s,k),U_i(s+1,k)}(t) = [(M+1)-(s+k)]\lambda_1 e^{-(\lambda_0+\alpha_i+[(M+1)-(s+k)]\lambda_1)t} \\ q_{U_1(s,k),U_1(s,k-1)}(t) = \alpha_1 e^{-(\lambda_0+\alpha_1+[(M+1)-(s+k)]\lambda_1)t} \\ q_{U_2(s,k),U_2(s-1,k)}(t) = \alpha_2 e^{-(\lambda_0+\alpha_2+[(M+1)-(s+k)]\lambda_1)t} \\ where\ i = 1\ or\ 2,\ 3 \leq s \leq N-1,\ 1 \leq k \leq N-s, 3 < M \leq N\ and\ M\ is\ an\ integer \end{cases} \tag{4}$$

From operative states to failed states;

$$\begin{cases} q_{U_i(s,N+1-s),F_i(s,N+1-s)}(t) = \lambda_0 e^{-(\lambda_0+\alpha_i)t}, \quad where\ i = 1\ or\ 2,\ 2 \leq s \leq N+1 \\ q_{U_1(s,M+1-s),U_1(s,M-s)}(t) = \alpha_1 e^{-(\lambda_0+\alpha_1)t} \quad ;2 \leq s \leq N, 1 < M \leq N\ and\ M\ is\ an\ integer \\ q_{U_1(N+1,0),U_2(N,0)}(t) = \alpha_1 e^{-(\lambda_0+\alpha_1)t} \end{cases} \tag{5}$$

$$\begin{cases} q_{U_2(s,M+1-s),U_2(s-1,M+1-s)}(t) = \alpha_i e^{-(\lambda_0+\alpha_i)t} \\ q_{U_2(2,N-1),U_1(2,N-2)}(t) = \alpha_2 e^{-(\lambda_0+\alpha_2)t} \\ where\ 3 \leq s \leq N+1, 1 < M \leq N\ and\ M\ is\ an\ integer \end{cases} \tag{6}$$

From failed states to operative states;

$$\begin{cases} q_{F_1(s,M+1-s),U_1(s,M+1-s)}(t) = \alpha_1 e^{-\alpha_1 t}, \quad q_{F_1(N+1,0),U_2(N,1)}(t) = \alpha_1 e^{-\alpha_1 t} \\ where\ 2 \leq s \leq N, 1 < M \leq N\ and\ M\ is\ an\ integer \\ q_{F_2(s,M+1-s),U_2(s-1,M+2-s)}(t) = \alpha_2 e^{-\alpha_2 t}, \quad q_{F_2(2,N-1),U_1(2,N-2)}(t) = \alpha_2 e^{-\alpha_2 t} \\ where\ 3 \leq s \leq N+1, 1 < M \leq N\ and\ M\ is\ an\ integer \end{cases} \tag{7}$$

Thus, $p_{ij} = \lim_{s\to o} q^*_{ij}(s)$ ase given as:

$$p_{U(1),U_1(2,0)} = \frac{\lambda_0}{(\lambda_0 + N\lambda_1)}, \quad p_{U(1),U_2(2,0)} = \frac{N\lambda_1}{(\lambda_0 + N\lambda_1)}, \tag{8}$$

$$\begin{cases} p_{U_i(s,0),U_i(s,1)} = \frac{\lambda_0}{(\lambda_0+\alpha_i+[(M+1)-s]\lambda_1)} M, \quad p_{U_i(s,0),U_i(s+1,0)} = \frac{(M+1-s)\lambda_1}{(\lambda+\alpha_i+[(M+1)-s]\lambda_1)} \\ p_{U_i(s,0),U_2(s-1,0)} = \frac{\alpha_i}{(\lambda+\alpha_i+[(M+1)-s]\lambda_1)} \\ where\ i = 1\ or\ 2,\ 3 \leq s \leq N, 2 < M \leq N\ and\ M\ is\ an\ integer \end{cases} \tag{9}$$

$$\begin{cases} p_{U_i(s,k),U_i(s,k+1)} = \frac{\lambda_0}{(\lambda_0+\alpha_i+[(M+1)-(s+k)]\lambda_1)}, \quad p_{U_i(s,k),U_i(s+1,k)} = \frac{[(M+1)-(s+k)]\lambda_1}{(\lambda_0+\alpha_i+[(M+1)-(s+k)]\lambda_1)} \\ p_{U_1(s,k),U_1(s,k-1)} = \frac{\alpha_1}{(\lambda_0+\alpha_1+[(M+1)-(s+k)]\lambda_1)}, \quad p_{U_2(s,k),U_2(s-1,k)} = \frac{\alpha_2}{(\lambda_0+\alpha_2+[(M+1)-(s+k)]\lambda_1)t} \\ where\ i = 1\ or\ 2,\ 3 \leq s \leq N-1,\ 1 \leq k \leq N-s, 3 < M \leq N\ and\ M\ is\ an\ integer \end{cases} \tag{10}$$

$$\begin{cases} p_{U_i(2,s),U_i(2,s+1)} = \frac{\lambda_0}{(\lambda_0+\alpha_i+[M-1-s]\lambda_1)}, \quad p_{U_i(2,s),U_i(3,s+1)} = \frac{(M-1-s)\lambda_1}{(\lambda_0+\alpha_i+[M-1-s]\lambda_1)} \\ p_{U_i(2,s),U_1(2,s-1)} = \frac{\alpha_i}{(\lambda_0+\alpha_i+[M-1-s]\lambda_1)} \\ where\ i = 1\ or\ 2,\ 0 \leq s \leq N-2, 1 < M \leq N\ and\ M\ is\ an\ integer \end{cases} \tag{11}$$

From operative states to failed states;

$$\begin{cases} p_{U_i(s,N+1-s),F_i(s,N+1-s)} = \frac{\lambda_0}{(\lambda_0+\alpha_i)}, \quad where\ i = 1\ os\ 2,\ 2 \leq s \leq N+1 \\ p_{U_1(s,M+1-s),U_1(s,M-s)} = \frac{\alpha_1}{(\lambda_0+\alpha_1)} \quad ;2 \leq s \leq N, 1 < M \leq N\ and\ M\ is\ an\ integer \\ p_{U_1(N+1,0),U_2(N,0)}(t) = \frac{\alpha_1}{(\lambda_0+\alpha_1)} \end{cases} \tag{12}$$

$$\begin{cases} p_{U_2(s,M+1-s),U_2(s-1,M+1-s)} = \frac{\alpha_2}{(\lambda_0+\alpha_2)}, \quad p_{U_2(2,N-1),U_1(2,N-2)} = \frac{\alpha_2}{(\lambda_0+\alpha_2)} \\ where\ i = 1\ or\ 2,\ 3 \leq s \leq N+1, 1 < M \leq N\ and\ M\ is\ an\ integer \end{cases} \tag{13}$$

From failed states to operative states;

$$\begin{cases} p_{F_1(s,M+1-s),U_1(s,M+1-s)} = 1, \quad p_{F_1(N+1,0),U_2(N,1)} = 1 \\ \textit{where } 2 \leq s \leq N, 1 < M \leq N \textit{ and } M \textit{ is an integer} \\ p_{F_2(s,M+1-s),U_2(s-1,M+2-s)} = 1, \quad p_{F_2(2,N-1),U_1(2,N-2)} = 1 \\ \textit{where } 3 \leq s \leq N+1, 1 < M \leq N \textit{ and } M \textit{ is an integer} \end{cases} \tag{14}$$

We may, therefore, verify the following:

$$p_{U(1),U_1(2,0)} + p_{U(1),U_2(2,0)} = 1 \tag{15}$$

$$\left\{ p_{U_i(s,0),U_i(s,1)} + p_{U_i(s,0),U_i(s+1,0)} + p_{U_i(s,0),U_2(s-1,0)} = 1, \textit{where } i = 1 \textit{ or } 2,\ 3 \leq s \leq N \right. \tag{16}$$

$$\begin{cases} p_{U_1(s,k),U_i(s,k+1)} + p_{U_1(s,k),U_i(s+1,k)} + p_{U_1(s,k),U_1(s,k-1)} = 1 \\ p_{U_2(s,k),U_2(s,k+1)} + p_{U_2(s,k),U_i(s+1,k)} + p_{U_2(s,k),U_2(s-1,k)} = 1 \\ \textit{where } 3 \leq s \leq N-1,\ 1 \leq k \leq N-s, N > 3 \end{cases} \tag{17}$$

$$\begin{cases} p_{U_i(2,s),U_i(2,s+1)} + p_{U_i(2,s),U_i(3,s+1)} + p_{U_i(2,s),U_1(2,s-1)} = 1 \\ \textit{where } i = 1 \textit{ or } 2,\ 0 \leq s \leq N-2, N > 1 \end{cases} \tag{18}$$

$$\begin{cases} p_{U_1(s,M+1-s),F_1(s,M+1-s)} + p_{U_1(s,M+1-s),U_1(s,M-s)} = 1 \\ \textit{where } 2 \leq s \leq N, 1 < M \leq N \textit{ and } M \textit{ is an integer} \end{cases} \tag{19}$$

$$\begin{cases} p_{U_2(s,M+1-s),F_2(s,M+1-s)} + p_{U_2(s,M+1-s),U_2(s-1,M+1-s)} = 1 \\ \textit{where } 3 \leq s \leq N+1, 1 < M \leq N \textit{ and } M \textit{ is an integer} \end{cases} \tag{20}$$

$$\begin{cases} p_{U_1(N+1,0),F_1(N+1,0)} + p_{U_1(N+1,0),U_2(N,0)} = 1 \\ p_{U_2(2,N-1),F_2(2,N-1)} + p_{U_2(2,N-1),U_1(2,N-2)} = 1 \end{cases} \tag{21}$$

Thus Mean Sojourn Time (μ_i) are:

$\mu_{U(1)} = \frac{1}{(\lambda_0 + N\lambda_1)}$

$\mu_{U_i(s,0)} = \frac{1}{(\lambda_0+\alpha_i+[(N+1)-s]\lambda_1)}$ *where* $i = 1$ *or* $2,\ 3 \leq s \leq N, N > 2$

$\mu_{U_i(s,k)} = \frac{1}{(\lambda_0+\alpha_i+[(N+1)-(s+k)]\lambda_1)}$ *where* $i = 1$ *or* $2,\ 3 \leq s \leq N-1,\ 1 \leq k \leq N-s, N > 3$

$\mu_{U_i(2,s)} = \frac{1}{(\lambda_0+\alpha_i+[N-1-s]\lambda_1)}$ *where* $i = 1$ *or* $2,\ 0 \leq s \leq N-2, N > 1$

$\mu_{U_i(s,N+1-s)} = \frac{1}{(\lambda_0+\alpha_i)}$ *where* $i = 1$ *or* $2,\ 2 \leq s \leq N+1$

$\mu_{F_i(s,N+1-s)} = \frac{1}{\alpha_i}$ *where* $i = 1$ *or* $2,\ 2 \leq s \leq N+1$

Using state i as the starting point, the unconditional mean times (m_{ij}) are computed as

$$m_{U(1),U_1(2,0)} = \frac{\lambda_0}{(\lambda_0 + N\lambda_1)^2}, \quad m_{U(1),U_2(2,0)} = \frac{N\lambda_1}{(\lambda_0 + N\lambda_1)^2} \tag{22}$$

$$\begin{cases} m_{U_i(s,0),U_i(s,1)} = \frac{\lambda_0}{(\lambda_0+\alpha_i+[(M+1)-s]\lambda_1)^2}, \quad m_{U_i(s,0),U_i(s+1,0)} = \frac{(M+1-s)\lambda_1}{(\lambda+\alpha_i+[(M+1)-s]\lambda_1)^2} \\ m_{U_i(s,0),U_2(s-1,0)} = \frac{\alpha_i}{(\lambda+\alpha_i+[(M+1)-s]\lambda_1)^2} \\ \textit{where } i = 1 \textit{ or } 2,\ 3 \leq s \leq N, 2 < M \leq N \textit{ and } M \textit{ is an integer} \end{cases} \tag{23}$$

$$\begin{cases} m_{U_i(s,k),U_i(s,k+1)} = \frac{\lambda_0}{(\lambda_0+\alpha_i+[(M+1)-(s+k)]\lambda_1)^2}, \ m_{U_i(s,k),U_i(s+1,k)} = \frac{[(M+1)-(s+k)]\lambda_1}{(\lambda_0+\alpha_i+[(M+1)-(s+k)]\lambda_1)^2} \\ m_{U_1(s,k),U_1(s,k-1)} = \frac{\alpha_1}{(\lambda_0+\alpha_1+[(M+1)-(s+k)]\lambda_1)^2}, \ m_{U_2(s,k),U_2(s-1,k)} = \frac{\alpha_2}{(\lambda_0+\alpha_2+[(M+1)-(s+k)]\lambda_1)^2} \\ \textit{where } i = 1 \textit{ or } 2,\ 3 \leq s \leq N-1,\ 1 \leq k \leq N-s, 3 < M \leq N \textit{ and } M \textit{ is an integer} \end{cases} \tag{24}$$

$$\begin{cases} m_{U_i(2,s),U_i(2,s+1)} = \frac{\lambda_0}{(\lambda_0+\alpha_i+[M-1-s]\lambda_1)^2}, \quad m_{U_i(2,s),U_i(3,s+1)}(t) = \frac{[M-1-s)]\lambda_1}{(\lambda_0+\alpha_i+[M-1-s]\lambda_1)^2} \\ m_{U_i(2,s),U_1(2,s-1)} = \frac{\alpha_i}{(\lambda_0+\alpha_i+[M-1-s]\lambda_1)^2} \\ \textit{where } i = 1 \textit{ or } 2,\ 0 \leq s \leq N-2, 1 < M \leq N \textit{ and } M \textit{ is an integer} \end{cases} \tag{25}$$

From operative states to failed states

$$\begin{cases} m_{U_i(s,N+1-s),F_i(s,N+1-s)} = \frac{\lambda_0}{(\lambda_0+\alpha_i)^2}, \quad where\ i = 1\ or\ 2,\ 2 \le s \le N+1 \\ m_{U_1(s,M+1-s),U_1(s,M-s)} = \frac{\alpha_1}{(\lambda_0+\alpha_1)^2} \quad ; 2 \le s \le N, 1 < M \le N\ and\ M\ is\ an\ integer \\ m_{U_1(N+1,0),U_2(N,0)}(t) = \frac{\alpha_1}{(\lambda_0+\alpha_1)^2} \end{cases} \tag{26}$$

$$\begin{cases} m_{U_2(s,M+1-s),U_2(s-1,M+1-s)} = \frac{\alpha_2}{(\lambda_0+\alpha_2)^2}, \quad m_{U_2(2,N-1),U_1(2,N-2)} = \frac{\alpha_2}{(\lambda_0+\alpha_2)^2} \\ where\ 3 \le s \le N+1, 1 < M \le N\ and\ M\ is\ an\ integer \end{cases} \tag{27}$$

From failed states to operative state;

$$\begin{cases} m_{F_1(s,M+1-s),U_1(s,M+1-s)} = \frac{1}{\alpha_1}, \quad m_{F_1(N+1,0),U_2(N,1)} = \frac{1}{\alpha_1} \\ where\ i = 1\ or\ 2,\ 2 \le s \le N, 1 < M \le N\ and\ M\ is\ an\ integer \\ m_{F_2(s,M+1-s),U_2(s-1,M+2-s)} = \frac{1}{\alpha_2}, \quad m_{F_2(2,N-1),U_1(2,N-2)} = \frac{1}{\alpha_2} \\ where\ 3 \le s \le N+1, 2 < M \le N\ and\ M\ is\ an\ integer \end{cases} \tag{28}$$

It may be verified that

$$m_{U(1),U_1(2,0)} + m_{U(1),U_2(2,0)} = \mu_{U(1)} \tag{29}$$

$$\begin{cases} m_{U_i(s,0),U_i(s,1)} + m_{U_i(s,0),U_i(s+1,0)} + m_{U_i(s,0),U_2(s-1,0)} = \mu_{U_i(s,0)} \\ where\ i = 1\ or\ 2,\ 3 \le s \le N, N > 2 \end{cases} \tag{30}$$

$$\begin{cases} m_{U_1(s,k),U_i(s,k+1)} + m_{U_1(s,k),U_i(s+1,k)} + m_{U_1(s,k),U_1(s,k-1)} = \mu_{U_1(s,k)} \\ m_{U_2(s,k),U_2(s,k+1)} + m_{U_2(s,k),U_i(s+1,k)} + m_{U_2(s,k),U_2(s-1,k)} = \mu_{U_2(s,k)} \\ where\ 3 \le s \le N-1,\ 1 \le k \le N-s, N > 3 \end{cases} \tag{31}$$

$$\begin{cases} m_{U_i(2,s),U_i(2,s+1)} + m_{U_i(2,s),U_i(3,s+1)} + m_{U_i(2,s),U_1(2,s-1)} = \mu_{U_i(2,s)} \\ where\ i = 1\ or\ 2,\ 0 \le s \le N-2, N > 1 \end{cases} \tag{32}$$

$$\begin{cases} m_{U_1(s,M+1-s),F_1(s,M+1-s)} + m_{U_1(s,M+1-s),U_1(s,N-s)} = \mu_{U_1(s,M+1-s)} \\ where\ 2 \le s \le N, 1 < M \le N\ and\ M\ is\ an\ integer \end{cases} \tag{33}$$

$$\begin{cases} m_{U_2(s,M+1-s),F_2(s,M+1-s)} + m_{U_2(s,M+1-s),U_2(s-1,M+1-s)} = \mu_{U_2(s,M+1-s)} \\ where\ 3 \le s \le N+1, 1 < M \le N\ and\ M\ is\ an\ integer \end{cases} \tag{34}$$

$$\begin{cases} m_{U_1(N+1,0),F_1(N+1,0)} + m_{U_1(N+1,0),U_2(N,0)} = \mu_{U_1(N+1,0)} \\ m_{U_2(2,N-1),F_2(2,N-1)} + m_{U_2(2,N-1),U_1(2,N-2)} = \mu_{U_2(2,N-1)} \end{cases} \tag{35}$$

6. Measures of System Effectiveness

6.1. Mean Time to System Failure (MTSF)

The following may result from viewing the failed state as an absorbing state:

$$\phi_{U(0)}(t) = Q_{U(1),U_1(2,0)}(t) \circledS \phi_{U_1(2,0)}(t) + Q_{U(1),U_2(2,0)}(t) \circledS \phi_{U_2(2,0)}(t) \tag{36}$$

$$\begin{aligned} \phi_{U_i(s,0)}(t) = {} & Q_{U_i(s,0),U_i(s,1)}(t) \circledS \phi_{U_i(s,1)}(t) + Q_{U_i(s,0),U_i(s+1,0)}(t) \circledS \phi_{U_i(s+1,0)}(t) \\ & + Q_{U_i(s,0),U_2(s-1,0)}(t) \circledS \phi_{U_2(s-1,0)}(t) where\ i = 1\ or\ 2,\ 3 \le s \le N, N > 2 \end{aligned} \tag{37}$$

$$\begin{aligned} \phi_{U_1(s,k)}(t) = {} & Q_{U_1(s,k),U_1(s,k+1)}(t) \circledS \phi_{U_1(s,k+1)}(t) + Q_{U_1(s,k),U_1(s+1,k)}(t) \circledS \phi_{U_1(s+1,k)}(t) \\ & + Q_{U_1(s,k),U_1(s,k-1)}(t) \circledS \phi_{U_1(s,k-1)}(t) \\ & where\ 3 \le s \le N-1,\ 1 \le k \le N-s, N > 3 \end{aligned} \tag{38}$$

$$\phi_{U_2(s,k)}(t) = Q_{U_2(s,k),U_2(s,k+1)}(t)ⓢ\phi_{U_2(s,k+1)}(t) + Q_{U_2(s,k),U_2(s+1,k)}(t)ⓢ\phi_{U_2(s+1,k)}(t) + Q_{U_2(s,k),U_2(s-1,k)}(t)ⓢ\phi_{U_2(s-1,k)}(t) \quad \textit{where } 3 \le s \le N-1,\ 1 \le k \le N-s, N > 3 \tag{39}$$

$$\phi_{U_i(2,s)}(t) = Q_{U_i(2,s),U_i(2,s+1)}(t)ⓢ\phi_{U_i(2,s+1)}(t) + Q_{U_i(2,s),U_i(3,s+1)}(t)ⓢ\phi_{U_i(3,s+1)}(t) \quad \textit{where } i = 1 \textit{ or } 2,\ 0 \le s \le N-2, N > 1 \tag{40}$$

$$\phi_{U_1(N+1,0)}(t) = Q_{U_i(N+1,0),F_1(N+1,0)}(t) + Q_{U_1(N+1,0),U_2(N,0)}(t)ⓢ\phi_{U_2(N,0)}(t) \tag{41}$$

$$\phi_{U_1(s,N+1-s)}(t) = Q_{U_1(s,N+1-s),F_1(s,N+1-s)}(t) + Q_{U_1(s,N+1-s),U_1(s,N-s)}(t)ⓢ\phi_{U_1(s,N-s)}(t) \textit{where } 2 \le s \le N, N > 1 \tag{42}$$

$$\phi_{U_2(s,N+1-s)}(t) = Q_{U_2(s,N+1-s),U_2(s-1,N+1-s)}(t)ⓢ\phi_{U_2(s-1,N+1-s)}(t) + Q_{U_2(s,N+1-s),F_2(s,N+1-s)}(t) \textit{where } 3 \le s \le N+1, N > 1 \tag{43}$$

$$\phi_{U_2(2,N-1)}(t) = Q_{U_2(2,N-1),F_2(2,N-1)}(t) + Q_{U_2(2,N-1),U_1(2,N-2)}(t)ⓢ\phi_{U_1(2,N-2)}(t) \tag{44}$$

Laplace transform can solve these equations, and the function $\phi_0^{(N)}(t)$ can be obtained after inversion. So the MTSF is determined by

$$\phi_0^{(N)} = \lim_{s\to 0} \frac{K^{(N)}(s) - L^{(N)}(s)}{sK^{(N)}(s)} = \frac{L^{(N)}}{K^{(N)}} \tag{45}$$

where the value of $L^{(N)}$ and $K^{(N)}$ in determinant form can be evaluated using MATLAB or MATHEMATICA software.

6.2. Availability of the System

By definition of $AT_m(t)$ and the transitions that occur, we have:

$$A_{U(0)}(t) = M_{U(0)}(t) + q_{U(1),U_1(2,0)}(t)ⓒA_{U_1(2,0)}(t) + q_{U(1),U_2(2,0)}(t)ⓒA_{U_2(2,0)}(t) \tag{46}$$

$$A_{U_i(s,0)}(t) = M_{U_i(s,0)}(t) + q_{U_i(s,0),U_i(s,1)}(t)ⓒA_{U_i(s,1)}(t) + q_{U_i(s,0),U_i(s+1,0)}(t)ⓒA_{U_i(s+1,0)}(t) + q_{U_i(s,0),U_2(s-1,0)}(t)ⓒA_{U_2(s-1,0)}(t) \quad \textit{where } i = 1 \textit{ or } 2,\ 3 \le s \le N, N > 2 \tag{47}$$

$$A_{U_1(s,k)}(t) = M_{U_1(s,k)}(t) + q_{U_1(s,k),U_1(s,k+1)}(t)ⓒA_{U_1(s,k+1)}(t) + q_{U_1(s,k),U_1(s+1,k)}(t)ⓒA_{U_1(s+1,k)}(t) + q_{U_1(s,k),U_1(s,k-1)}(t)ⓒA_{U_1(s,k-1)}(t) \quad \textit{where } 3 \le s \le N-1,\ 1 \le k \le N-s, N > 3 \tag{48}$$

$$A_{U_2(s,k)}(t) = M_{U_2(s,k)}(t) + q_{U_2(s,k),U_2(s,k+1)}(t)ⓒA_{U_2(s,k+1)}(t) + q_{U_2(s,k),U_2(s+1,k)}(t)ⓒA_{U_2(s+1,k)}(t) + q_{U_2(s,k),U_2(s-1,k)}(t)ⓒA_{U_2(s-1,k)}(t) \quad \textit{where } 3 \le s \le N-1,\ 1 \le k \le N-s, N > 3 \tag{49}$$

$$A_{U_i(2,s)}(t) = M_{U_i(2,s)}(t) + q_{U_i(2,s),U_i(2,s+1)}(t)ⓒA_{U_i(2,s+1)}(t) + q_{U_i(2,s),U_i(3,s+1)}(t)ⓒA_{U_i(3,s+1)}(t) \textit{ where } i = 1 \textit{ or } 2,\ 0 \le s \le N-2, N > 1 \tag{50}$$

$$A_{U_1(s,N+1-s)}(t) = M_{U_1(s,N+1-s)}(t) + q_{U_1(s,N+1-s),F_1(s,N+1-s)}(t)ⓒA_{F_1(s,N+1-s)}(t) + q_{U_1(s,N+1-s),U_1(s,N-s)}(t)ⓒA_{U_1(s,N-s)}(t) \textit{ where } 2 \le s \le N, N > 1 \tag{51}$$

$$A_{U_1(N+1,0)}(t) = M_{U_1(N+1,0)}(t) + q_{U_i(N+1,0),F_1(N+1,0)}(t)ⓒA_{F_1(N+1,0)}(t) + q_{U_1(N+1,0),U_2(N,0)}(t)ⓒA_{U_2(N,0)}(t) \tag{52}$$

$$\begin{aligned} A_{U_2(2,N-1)}(t) &= M_{U_2(2,N-1)}(t) + q_{U_2(2,N-1),F_2(2,N-1)}(t)©A_{F_2(2,N-1)}(t) \\ &+ q_{U_2(2,N-1),U_1(2,N-2)}(t)©A_{U_1(2,N-2)}(t) \end{aligned} \tag{53}$$

$$\begin{aligned} A_{U_2(s,N+1-s)}(t) &= M_{U_2(s,N+1-s)}(t) + q_{U_2(s,N+1-s),F_2(s,N+1-s)}(t)©A_{F_2(s,N+1-s)}(t) \\ &+ q_{U_2(s,N+1-s),U_2(s-1,N+1-s)}(t)©A_{U_2(s-1,N+1-s)}(t) \\ &\textit{where } 3 \le s \le N+1, N > 1 \end{aligned} \tag{54}$$

$$A_{F_1(s,N+1-s)}(t) = q_{F_1(s,N+1-s),U_1(s,N+1-s)}(t)©A_{U_1(s,N+1-s)}(t) \textit{ where } 2 \le s \le N, N > 1 \tag{55}$$

$$\begin{aligned} A_{F_2(s,N+1-s)}(t) &= q_{F_2(s,N+1-s),U_2(s-1,N+2-s)}(t)©A_{U_2(s-1,N+2-s)}(t) \\ &\textit{where } 3 \le s \le N+1, N > 1 \end{aligned} \tag{56}$$

$$A_{F_1(N+1,0)}(t) = q_{F_1(N+1,0),U_2(N,1)}(t)©A_{U_2(N,1)}(t); 2 \le s \le N \tag{57}$$

$$A_{F_2(2,N-1)}(t) = q_{F_2(2,N-1),U_1(2,N-2)}(t)©A_{U_1(2,N-2)}(t) \tag{58}$$

where

$$\begin{cases} M_{U(0)}(t) = e^{-(\lambda_0+N\lambda_1)t} \\ M_{U_i(s,0)}(t) = e^{-(\lambda_0+\alpha_i+[(N+1)-s]\lambda_1)t} \textit{ where } i = 1 \textit{ or } 21', \ 3 \le s \le N \\ M_{U_i(s,k)}(t) = e^{-(\lambda_0+\alpha_i+[(N+1)-(s+k)]\lambda_1)t} \textit{ where } i = 1 \textit{ or } 2, 3 \le s \le N-1, \ 1 \le k \le N-s \\ M_{U_i(2,s)}(t) = e^{-(\lambda_0+\alpha_i+[N-1-s]\lambda_1)t} \textit{ where } i = 1 \textit{ or } 2, \ 0 \le s \le N-2, N > 1 \\ M_{U_1(s,N+1-s)}(t) = e^{-(\lambda_0+\alpha_i)t}; \ 2 \le s \le N, N > 1 \\ M_{U_2(s,N+1-s)}(t) = e^{-(\lambda_0+\alpha_i)t}; \ 3 \le s \le N+1, N > 1 \\ M_{U_1(N+1,0)}(t) = e^{-(\lambda_0+\alpha_1)t} \\ M_{U_2(2,N-1)}(t) = e^{-(\lambda_0+\alpha_2)t} \end{cases} \tag{59}$$

Laplace transform can solve these equations, and the function $A_0^{(N)}(t)$ can be obtained after inversion. As a result, the availability of the system is determined by

$$A_0^{(N)} = \lim_{s\to 0} sAT_0^*(s) = \lim_{s\to 0} \frac{sN_1^{(N)}(s)}{D_1^{(N)}(s)} = \frac{N_1^{(N)}}{D_1^{(N)}} \tag{60}$$

where the value of $N_1^{(N)}$ and $D_1^{(N)}$ in determinant form can be evaluated using MATLAB or MATHEMATICA software.

6.3. Other Measures

Using the definitions of B_0 and V_0 (specified in section 3) and the same procedures described in the section preceding it, The Expected Busy Period, the expected number of Visits for Repair, is given as:

6.3.1 Expected Busy Period

$$B_0^{(N)} = \lim_{s\to 0} \frac{sN_2^{(N)}(s)}{D_1^{(N)}(s)} = \frac{N_2^{(N)}}{D_1^{(N)}} \tag{61}$$

6.3.2 Expected Number Visits for Repair

$$V_0^{(N)} = \lim_{s\to 0} sV_0^{**}(s) = \lim_{s\to 0} \frac{sN_3^{N}(s)}{D_1^{(N)}(s)} = \frac{N_3^{(N)}}{D_1^{(N)}} \tag{62}$$

where the value of $N_2^{(N)}$, $N_3^{(N)}$ and $D_1^{(N)}$ in determinant form can be evaluated using MATLAB or MATHEMATICA software.

7. Profit Analysis

The profit equation, therefore, is

$$Profit(P_N) = C_0 A_0^{(N)} - C_1 B_0^{(N)} - C_2 V_0^{(N)} \tag{63}$$

C_0, C_1, C_2 are revenue, repair cost, and repairman visit cost respectively and all the costs are per unit time.

8. Graphical Interpretation and Numerical Results

Here, reliability metrics are determined for a system of N+1 units using arbitrary parameter values. For parameters with fixed values, the reliability measure's trend has been graphically displayed.

(i) For $\lambda_1 = 0.02, \alpha_1 = 0.35, \alpha_2 = 0.45$ the numerical values and of MTSF for various values of λ are provided in Table (1)

Table 1: *MTSF w.r.t. Failure Rate (λ) for N=1,2 and 3*

	MTSF		
λ	$N = 1$	$N = 2$	$N = 3$
0.1	15.9994	38.298	81.9643
0.2	9.118	18.4515	32.6082
0.3	6.2559	11.6205	18.735
0.4	4.7245	8.3449	12.7916
0.5	3.7813	6.4651	9.609
0.6	3.1456	5.2588	7.6582
0.7	2.6897	4.4239	6.3505
0.8	2.3475	3.8139	5.4171
0.9	2.132	3.534	5.381

From Table (1) It can be noted that decreasing the MTSF by increasing the failure rate of operational units is in contrast to an almost linear trend observed when standby units are increasing.

(ii)For $\lambda = 0.1, \alpha_1 = 0.35, \alpha_2 = 0.45$ the numerical values of MTSF for various values of λ_1 are provided in Table (2)

Table 2: *MTSF w.r.t. Failure Rate of standby units(λ_1) for N=1,2 and 3*

	MTSF		
λ_1	$N = 1$	$N = 2$	$N = 3$
0.1	33.5714	67.5989	98.7371
0.2	25.9677	40.602	48.1599
0.3	22.0732	30.3347	33.501
0.4	19.7059	25.0817	26.8425
0.5	18.1148	21.932	23.0889
0.6	16.9718	19.8471	20.6911
0.7	16.1111	18.3709	19.0302
0.8	15.4396	17.2734	17.813
0.9	14.901	16.4268	16.88

From Table (2) It can be seen that as the failure rate of hot standby units increases, the MTSF reduces as the trend for the number of standby units increases almost linearly.

(iii) For $\lambda_1 = 0.001, \alpha_1 = 0.2, \alpha_2 = 0.3$ Availability when N=1,2 and 3 w.r.t λ

Table 3: *Availability of the system (N=1,2,3) w.r.t. (λ)*

λ	Availability $N=1$	$N=2$	$N=3$
0.1	0.8268	0.8814	0.9151
0.2	0.6262	0.6826	0.7143
0.3	0.4871	0.5248	0.5505
0.4	0.3932	0.4173	0.4543
0.5	0.3276	0.3434	0.3606
0.6	0.2799	0.2906	0.3155
0.7	0.2438	0.2513	0.272
0.8	0.2157	0.2212	0.2361
0.9	0.1933	0.1974	0.2154

Table (3) shows that the availability is decreasing while the failure rate is increasing. However, it increases as the number of standby units increases.

(iv) For $\lambda_1 = 0.001, \alpha_1 = 0.2, \alpha_2 = 0.3, C_0 = 3000, C_1 = 1000, C_2 = 500$ the graph below depicts the profit when N=1,2, and 3 w.r.t *lambda*. , as seen in Figure (3)

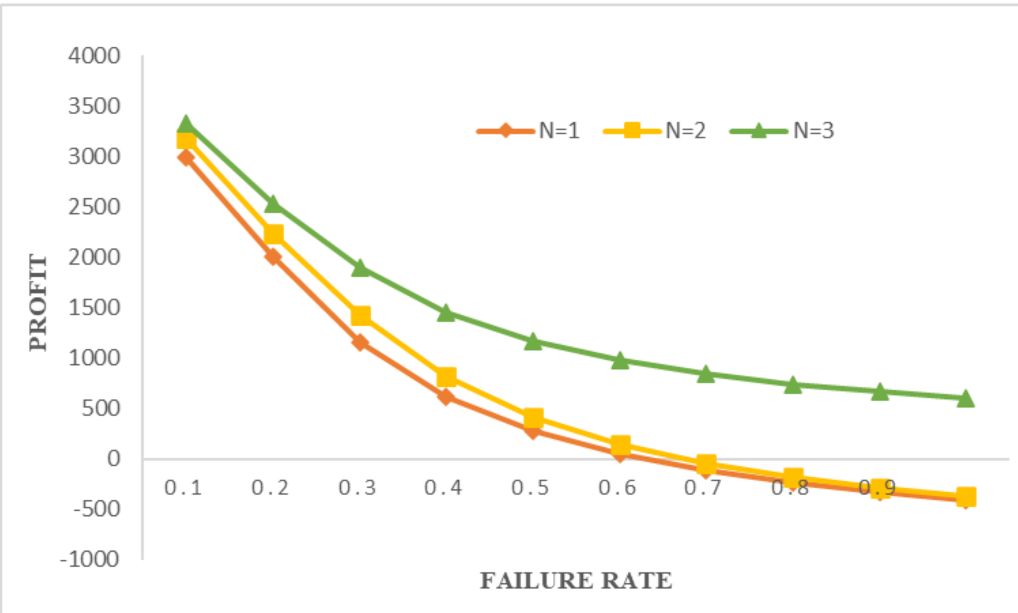

Figure 3: *Impact of Numbers of Standby Units on Profit w. r. t. λ*

From Figure (3), it is found that the Profit decreases with an increase in failure rate and the increase in the number of standby units.

(v) For $\lambda = 0.1, \lambda_1 = 0.1, \alpha_1 = 0.2, \alpha_2 = 0.3, C_0 = 4000, C_1 = 1000$, As indicated in Figure (4), the following graph has been generated.

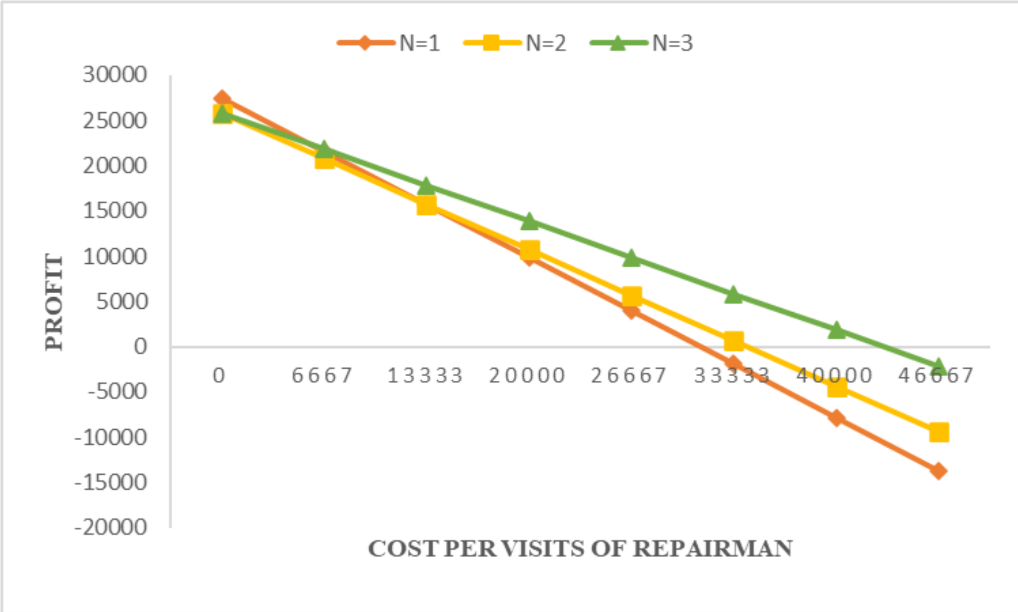

Figure 4: *Profit versus the repairman's visit costs* (C_2) *for N=1,2,3*

One might suggest that based on the above graph

- It is suggested to bring one unit along for standby if $C_2 < 6623$.
- Three or fewer units should be kept on standby if $C_2 > 6623$ for this value of C_2 since using more than three standby units causes revenue to decrease.
- Further C_2 should not exceed 32245.76 for $N = 1$, 35351.23 for $N = 2$, 44267.5 for $N = 3$ the system to be profitable. However, the above two conditions may also be kept in view while deciding about the number of standby units that will be needed.

(vi) Figure (5) and Figure (6) shows the change in profit for varied Revenue and N=1,2,3 and with fixed values for $\lambda = 0.02, \lambda_1 = 0.01, \alpha_1 = 0.1, \alpha_2 = 0.1, C_1 = 2000, C_2 = 1000$

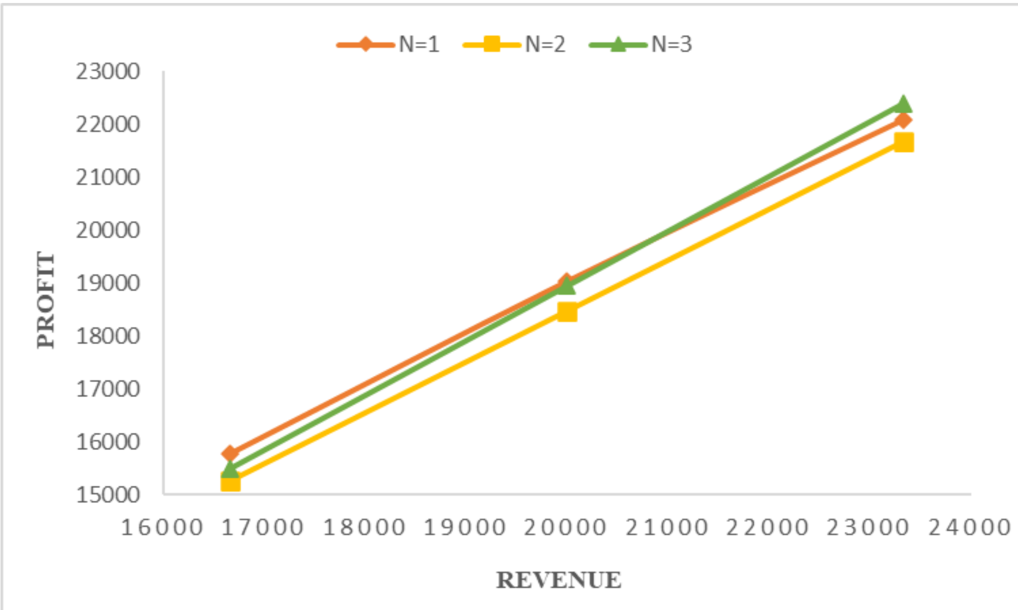

Figure 5: *Profit versus Revenue* (C_0) *for N=1,2,3*

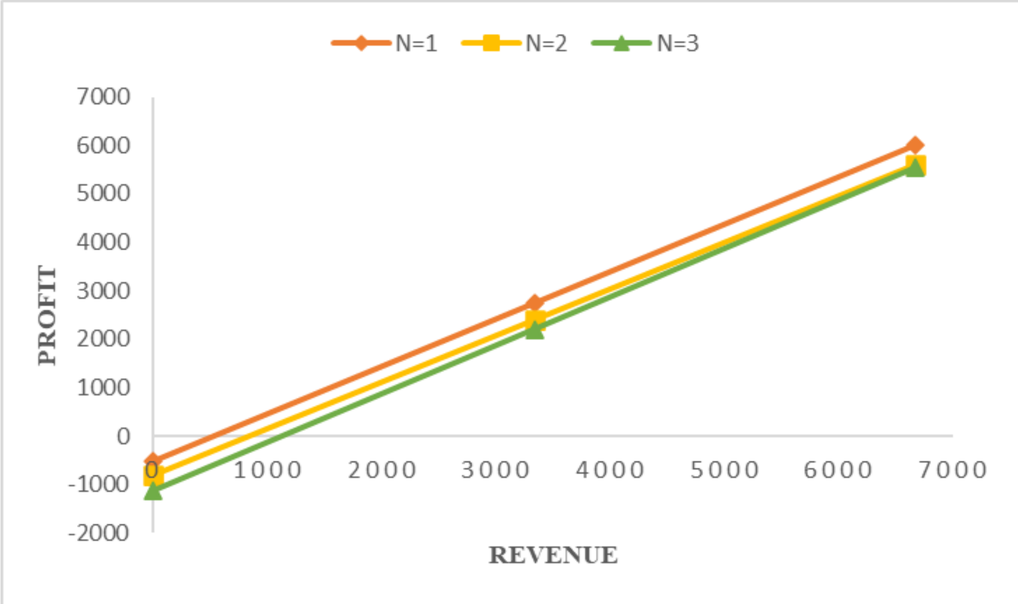

Figure 6: *Profit versus Revenue* (C_0) *for N=1,2,3*

From the graphs, the following can be determined

- From Figure (5) We notice that when C_0 is less than 20842.53, It would be profitable to keep one standby unit, and if C_0 is greater than 20842.53, keeping three standby units would be profitable.
- From Figure (6) we come to the decision that the cost of the products that the system will produce should be fixed in such a way that the value of C_0 is not less than 674.22 for N=1, 896.87 for N=2, 1144.12 for N=3, so as to make the system always profitable.

9. Conclusion

This study evaluates a crucial repairable system with a operational unit and N hot standby units. Arbitrary distributions have been used to get general findings for reliability metrics. However, the model has been validated taking specific parametric values for N=1,2,3. It has been observed taking a number of standby units is situational and may use 1 or 2 or 3 standby depending upon the cut-off points obtained in different situations.

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