POWER KOMAL DISTRIBUTION WITH PROPERTIES AND APPLICATION IN RELIABILITY ENGINEERING

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Abstract

The statistical analysis and modeling of reliability data from engineering is really a challenge for statistician because the reliability data from engineering are stochastic in nature. Recently one parameter Komal distribution was introduced in statistics literature for the analysis and modeling of failure time data from engineering. Komal distribution, being one parameter distribution, does not provide good fit to some engineering data due to its theoretical or applied point of view. In this article we propose a two-parameter power Komal distribution, which includes Komal distribution as particular case, for the analysis and modeling of data from reliability engineering. Its statistical properties including behavior of probability density function and cumulative distribution function for varying values of parameters have been presented. The first four raw moments and the variance of the proposed distribution has been derived and given. The expressions for hazard rate function and mean residual life function have been obtained and their behaviors for varying values of parameters have been presented. The stochastic ordering which is very much useful comparing the stochastic nature has also been discussed. Method of maximum likelihood has been discussed for estimating the parameters. Application of the distribution has been investigated using a real lifetime dataset from engineering. The goodness of fit of power Komal distribution has been tested using Akaike Information criterion and Kolmogorov-Smirnov statistic. The goodness of fit of power Komal distribution shows that it gives much closure fit over two-parameter power Garima distribution, Power Shanker distribution and Weibull distribution and one parameter exponential distribution, Shanker distribution, Garima distribution and Komal distribution. As the power Komal distribution gives much better fit over Weibull distribution, which is very much useful for modeling and analysis of data from reliability engineering, the final recommendation is that the power Komal distribution should be preferred over the considered distributions including Weibull for modeling data from reliability engineering.

Keywords: *Komal distribution, Descriptive measures, Reliability properties, Maximum likelihood estimation, Application.*

1. Introduction

Shanker [1] introduced one parameter lifetime distribution named Komal distribution defined by probability density function (pdf) and cumulative distribution function (cdf) as

$$f(y;\theta)=\frac{\theta^2}{\theta^2+\theta+1}\left(1+\theta+y\right)e^{-\theta y};\ y>0,\theta>0 \tag{1}$$

$$F(y,\theta)=1-\left[1+\frac{\theta y}{\theta^2+\theta+1}\right]e^{-\theta y};\ y>0,\theta>0 \tag{2}$$

Komal distribution is a mixture of exponential (θ) and gamma $(2,\theta)$ distributions with mixing proportion $\frac{\theta(\theta+1)}{\theta^2+\theta+1}$. We have

$$f(y,\theta)=p\,g_1(y)+(1-p)\,g_2(y)$$

where $p=\frac{\theta+1}{\theta+2}$, $g_1(y)=\theta e^{-\theta y}$, and $g_2(y)=\theta^2 y e^{-\theta y}$.

Shanker [1] has discussed some of its statistical properties including shapes for varying values of parameter, moments, skewness, kurtosis, hazard rate function, mean residual life function and stochastic ordering. Estimation of parameter and applications of Komal distribution has also been discussed by Shanker [1].

The pdf and the cdf of Power Shanker distribution (PSD) obtained by Shanker and Shukla [2] are given by

$$f(x;\theta,\alpha)=\frac{\alpha\theta^2}{\theta^2+1}x^{\alpha-1}\left(\theta+x^{\alpha}\right)e^{-\theta x^{\alpha}}\ ;x>0,\theta>0,\alpha>0 \tag{3}$$

$$F(x;\theta,\alpha)=1-\left[1+\frac{\theta x^{\alpha}}{\theta^2+1}\right]e^{-\theta x^{\alpha}}\ ;x>0,\theta>0,\alpha>0 \tag{4}$$

It can be easily shown that at $\alpha=1$, PSD reduces to one parameter Shanker distribution, introduced by Shanker [3] having pdf and cdf

$$f(x;\theta)=\frac{\theta^2}{\theta+1}\left(1+x\right)e^{-\theta x};x>0,\theta>0 \tag{5}$$

$$F(x;\theta)=1-\left[1+\frac{\theta x}{\theta+1}\right]e^{-\theta x};x>0,\theta>0 \tag{6}$$

The pdf and the cdf of Power Garima distribution (PGD) obtained by Berhane et al [4] are given by

$$f(x;\theta,\alpha)=\frac{\alpha\theta}{\theta+2}\left(1+\theta+\theta x^{\alpha}\right)x^{\alpha-1}e^{-\theta x^{\alpha}},\ x>0,\theta>0,\ \alpha>0 \tag{7}$$

$$F(x;\theta,\alpha)=1-\frac{\left(\theta x^{\alpha}+\theta+2\right)}{\theta+2},\ \ x>0,\theta>0\,,\alpha>0 \tag{8}$$

It can be easily shown that at $\alpha=1$, PGD reduces to one parameter Garima distribution, introduced by Shanker [5] having pdf and cdf

$$f\left(x;\theta\right)=\frac{\theta}{\theta+2}\left(1+\theta+\theta x\right)e^{-\theta x};x>0,\theta>0 \tag{9}$$

$$F\left(x;\theta\right)=1-\left(1+\frac{\theta x}{\theta+2}\right)e^{-\theta x};x>0,\theta>0 \tag{10}$$

Komal distribution, being one parameter lifetime distribution, has less flexibility to model data of various natures. The main motivation of considering power Komal distribution lies in the fact that as the Komal distribution found to be better than exponential distribution, Shanker distribution of Shanker [3] and Garima distribution of Shanker [5], it is expected that power Komal distribution would provide better fit than power Shanker distribution (PSD) by Shanker and Shukla [2], Power Garima distribution (PGD) by Berhane et al [4] and Weibull distribution of Weibull [6]. In the present paper an attempt has been made to obtain two-parameter power Komal distribution using power transformation of Komal distribution. The statistical properties of the distribution including shapes of density for varying values of parameters, the moments, survival function, hazard rate function and mean residual life function have been discussed. The maximum likelihood estimation has been discussed. The goodness of fit of the proposed distribution has been discussed with a real lifetime dataset from engineering and fit shows quite satisfactory over other one parameter and two-parameter lifetime distributions.

2. Power Komal Distribution

Assuming the power transformation $X=Y^{1/\alpha}$ in (1.1), the pdf of the random variable X can be obtained as

$$\begin{aligned} f\left(x;\theta,\alpha\right)&=\frac{\alpha\theta^2}{\theta^2+\theta+1}x^{\alpha-1}\left(1+\theta+x^{\alpha}\right)e^{-\theta x^{\alpha}},\ x>0,\theta>0,\ \alpha>0 \\ &=p\,g_1\left(x;\theta,\alpha\right)+\left(1-p\right)g_2\left(x;\theta,\alpha,2\right) \end{aligned} \tag{11}$$

where $p=\dfrac{\theta\left(\theta+1\right)}{\theta^2+\theta+1}$

$$g_1\left(x;\theta,\alpha\right)=\alpha\theta e^{-\theta x^{\alpha}}x^{\alpha-1};x>0,\theta>0,\alpha>0$$

$$g_2\left(x;\theta,\alpha,2\right)=\alpha\theta^2 e^{-\theta x^{\alpha}}x^{2\alpha-1};x>0,\theta>0,\alpha>0\,.$$

We would call the density in (2.1) as power Komal distribution (PKD) with parameters θ and α, and it is denoted by PKD$\left(\theta,\alpha\right)$. Like Komal distribution, PKD is also a convex combination of Weibull$\left(\theta,\alpha\right)$ distribution, a generalized gamma $\left(2,\alpha,\theta\right)$ distribution.

The corresponding cdf of PKD can be obtained as

$$F\left(x;\theta,\alpha\right)=1-\left[1+\frac{\theta x^{\alpha}}{\theta^2+\theta+1}\right]e^{-\theta x^{\alpha}},\ \ x>0,\theta>0\,,\alpha>0 \tag{12}$$

The nature of the pdf and the cdf of PKD has been studied with the help of the graphs for varying values of parameters and presented in figures 1 and 2 respectively.

On careful examination of the graphs of pdf of PKD it is obvious that the shapes of PKD are decreasing, symmetric, negatively skewed, positively skewed, platykurtic and leptokurtic for varying values of parameters.

For example, for fixed $\alpha = 1$ and $\theta \geq 1$, pdf of PKD is decreasing ,for fixed value of α i.e. $\alpha = 0.01$ and $\alpha = 0.05$ at $\theta \geq 2$ pdf of PKD becomes positively Skewed and symmetric. For $\alpha \geq 1$ and $\theta \geq 1$ the pdf graph becomes platykurtic and leptokurtic. This means that PKD is applicable for modeling lifetime data of various natures.

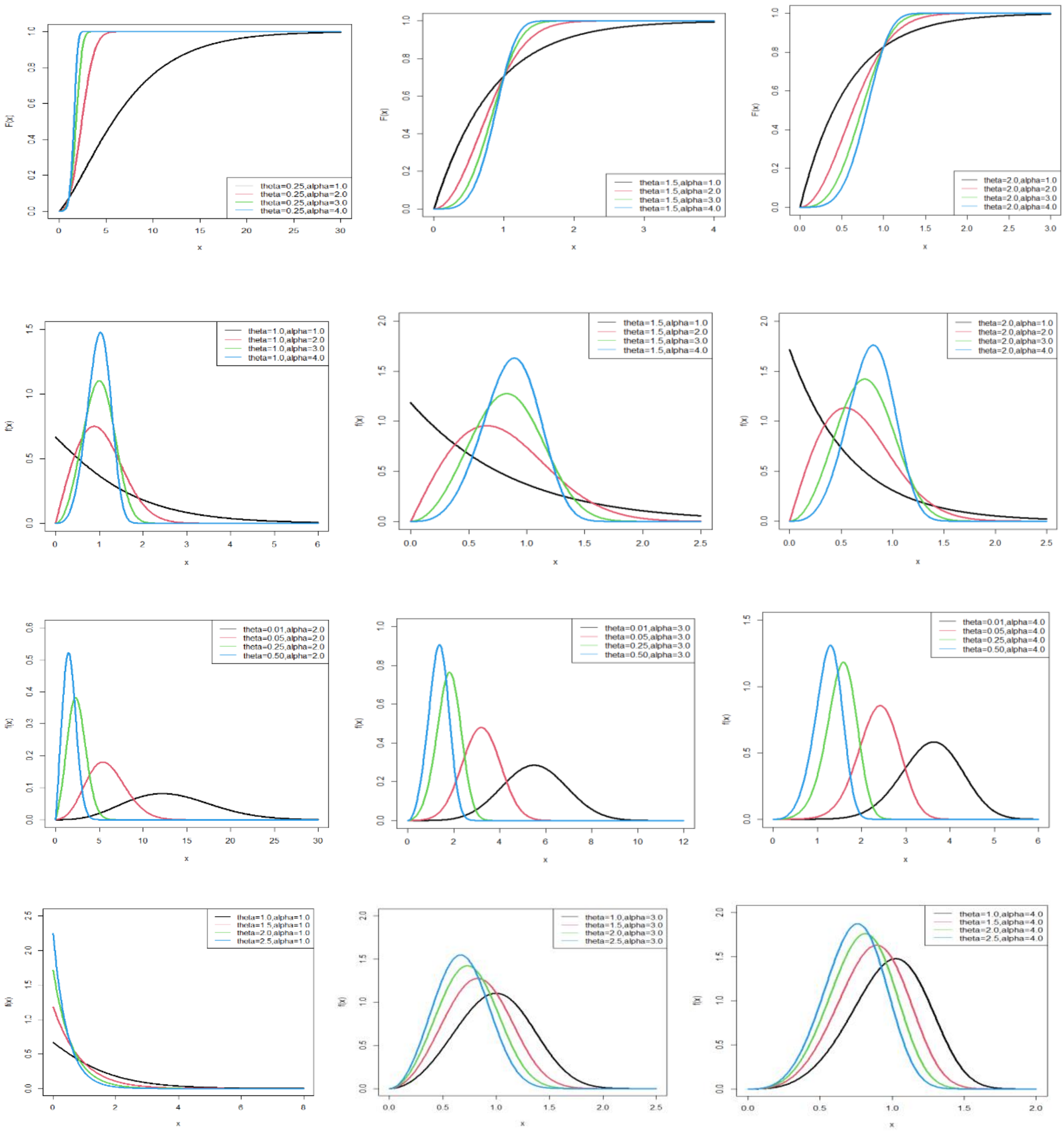

Figure 1: *Graphs of pdf of PKD for varying values of parameters* (θ, α)

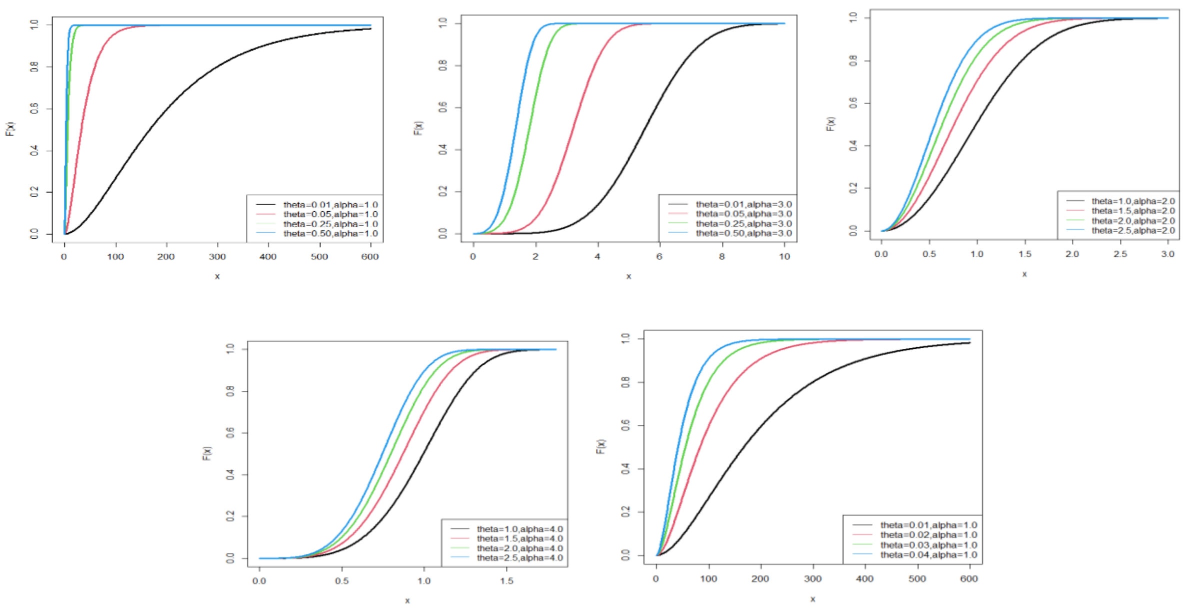

Figure 2: *Graphs of the cdf of PKD for varying values of the parameters* (θ,α)

3. Moments of Power Komal Distribution

Using the mixture representation in (2.2), the r th moment about the origin, μ_r' of PKD can be obtained as

$$\mu_r' = E\left(X^r\right) = p\int_0^\infty x^r g_1\left(x;\theta,\alpha\right)dx + \left(1-p\right)\int_0^\infty x^r g_2\left(x;\theta,\alpha,2\right)dx$$

$$= \frac{r\left(\alpha\theta^2+\alpha\theta+\alpha+r\right)\Gamma\left(\frac{r}{\alpha}\right)}{\alpha^2\left(\theta^2+\theta+1\right)\theta^{\frac{r}{\alpha}}};r=1,2,3,...$$

It should be noted that at $\alpha=1$, the above expression reduces to the r th moment about origin of Komal distribution given by

$$\mu_r' = \frac{r!\left(\theta^2+\theta+r+1\right)}{\theta^r\left(\theta^2+\theta+1\right)};r=1,2,3,...$$

Thus, the first four moments about origin of the PKD are thus obtained as

$$\mu_1' = \frac{\left(\alpha\theta^2+\alpha\theta+\alpha+1\right)\Gamma\left(\frac{1}{\alpha}\right)}{\alpha^2\left(\theta^2+\theta+1\right)\theta^{\frac{1}{\alpha}}}, \qquad \mu_2' = \frac{2\left(\alpha\theta^2+\alpha\theta+\alpha+2\right)\Gamma\left(\frac{2}{\alpha}\right)}{\alpha^2\left(\theta^2+\theta+1\right)\theta^{\frac{2}{\alpha}}}$$

$$\mu_3{}' = \frac{3\left(\alpha\theta^2+\alpha\theta+\alpha+3\right)\Gamma\left(\frac{3}{\alpha}\right)}{\alpha^2\left(\theta^2+\theta+1\right)\theta^{\frac{3}{\alpha}}}, \qquad \mu_4{}' = \frac{4\left(\alpha\theta^2+\alpha\theta+\alpha+4\right)\Gamma\left(\frac{4}{\alpha}\right)}{\alpha^2\left(\theta^2+\theta+1\right)\theta^{\frac{4}{\alpha}}}$$

Therefore, the variance of PKD can be obtained as

$$\mu_2 = \mu_2{}' - \left(\mu_1{}'\right)^2 = \frac{2\left(\alpha\theta^2+\alpha\theta+\alpha+2\right)\left(\theta^2+\theta+1\right)\Gamma\left(\frac{2}{\alpha}\right) - \left(\alpha\theta^2+\alpha\theta+\alpha+1\right)^2\left(\Gamma\left(\frac{1}{\alpha}\right)\right)^2}{\alpha^2\left(\theta^2+\theta+1\right)^2\theta^{\frac{2}{\alpha}}}$$

The higher order central moments are not being given here because their expressions are big. However, higher order central moments, if required, can be easily obtained using relationship between moments about mean and moments about origin. Finally, coefficient of variation, skewness, kurtosis and index of dispersion, if needed, can be obtained using their formulae in terms of central moments.

4. Reliability Properties of Power Komal Distribution

The survival function of PKD can be obtained as

$$S\left(x;\theta,\alpha\right) = 1 - F\left(x;\theta,\alpha\right) = \left(1+\frac{\theta x^{\alpha}}{\theta^2+\theta+1}\right)e^{-\theta x^{\alpha}}\ ;\ x>0, \theta>0, \alpha>0$$

The hazard rate function $h\left(x;\theta,\alpha\right)$ and the mean residual function $m\left(x;\theta,\alpha\right)$ of PKD are given respectively as:

$$h\left(x;\theta,\alpha\right) = \frac{f\left(x;\theta,\alpha\right)}{S\left(x;\theta,\alpha\right)} = \frac{\alpha\theta^2\left(1+\theta+x^{\alpha}\right)x^{\alpha-1}}{\theta^2+\theta+1+\theta x^{\alpha}}.$$

$$m\left(x;\theta,\alpha\right) = E\left(X-x \mid X \ge x\right) = \frac{1}{S\left(x;\theta,\alpha\right)}\int_x^{\infty} t\, f\left(t;\theta,\alpha\right)dt - x$$

$$= \frac{\left(\theta^2+\theta\right)\Gamma\left(\frac{1}{\alpha}+1, \theta x^{\alpha}\right) + \Gamma\left(\frac{1}{\alpha}+2, \theta x^{\alpha}\right)}{\theta^{\frac{1}{\alpha}}\left(\theta^2+\theta+1+\theta x^{\alpha}\right)e^{-\theta x^{\alpha}}} - x$$

It can be easily verified that at $x=0$ it reduces to the expression for the mean of PKD. The nature of hazard rate function of PKD for varying values of parameters has been shown graphically in figure 3. Depending upon the values of the parameter the shapes of hazard rate function of PKD is increasing very quickly and slowly in nature, for example, for $\alpha \ge 1$ and $\theta < 1$, the hazard rate is increasing very quickly and for $\alpha \ge 2$ and $\theta > 1$ it is increasing slowly. Again, the nature of mean residual life function of PKD for varying values of parameters has been shown in figure 4. The graphs of mean residual life function of PKD are monotonically decreasing for varying values of parameters.

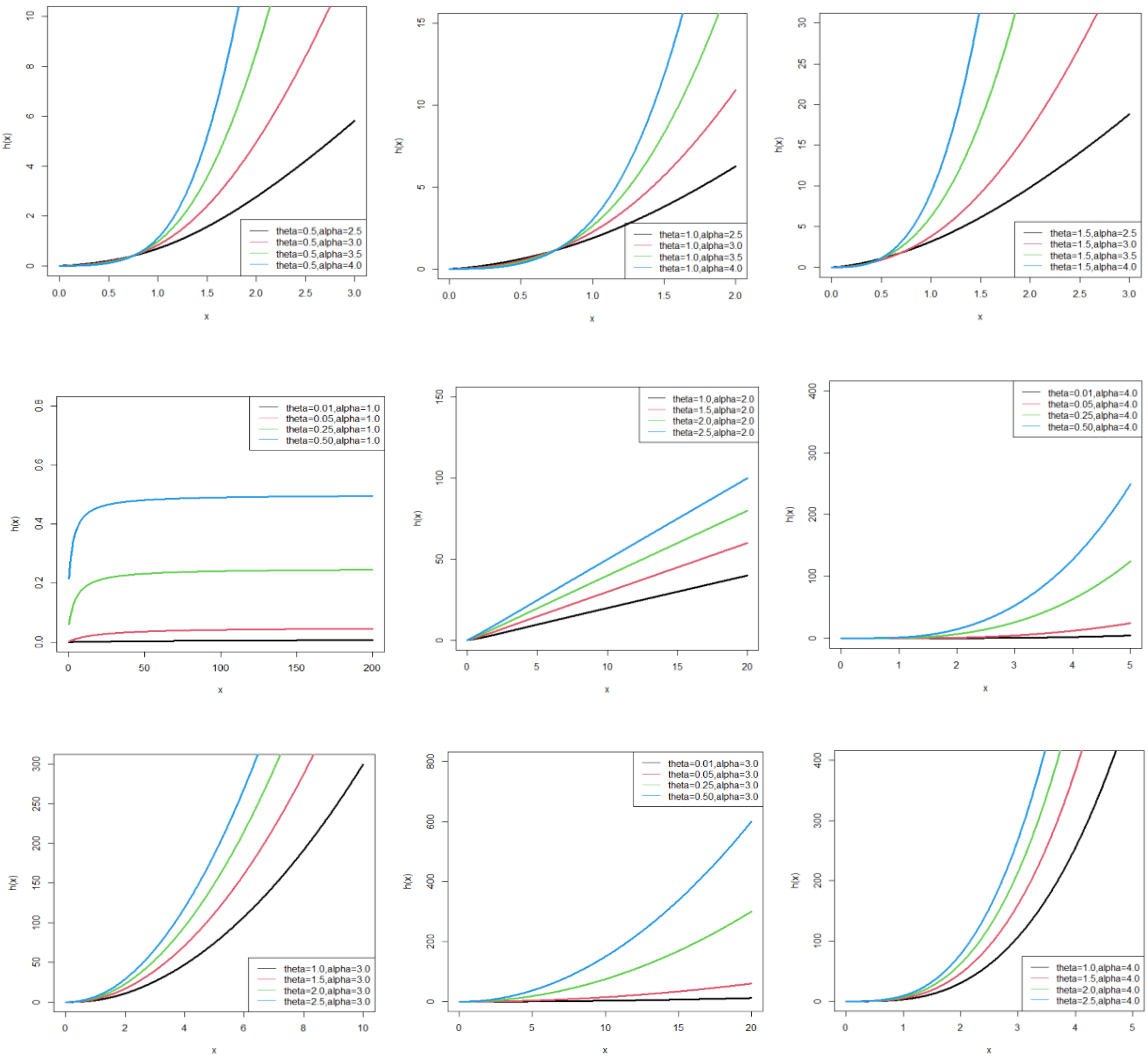

Figure 3: *Hazard rate function of PKD for varying values of the parameters* (α, θ)

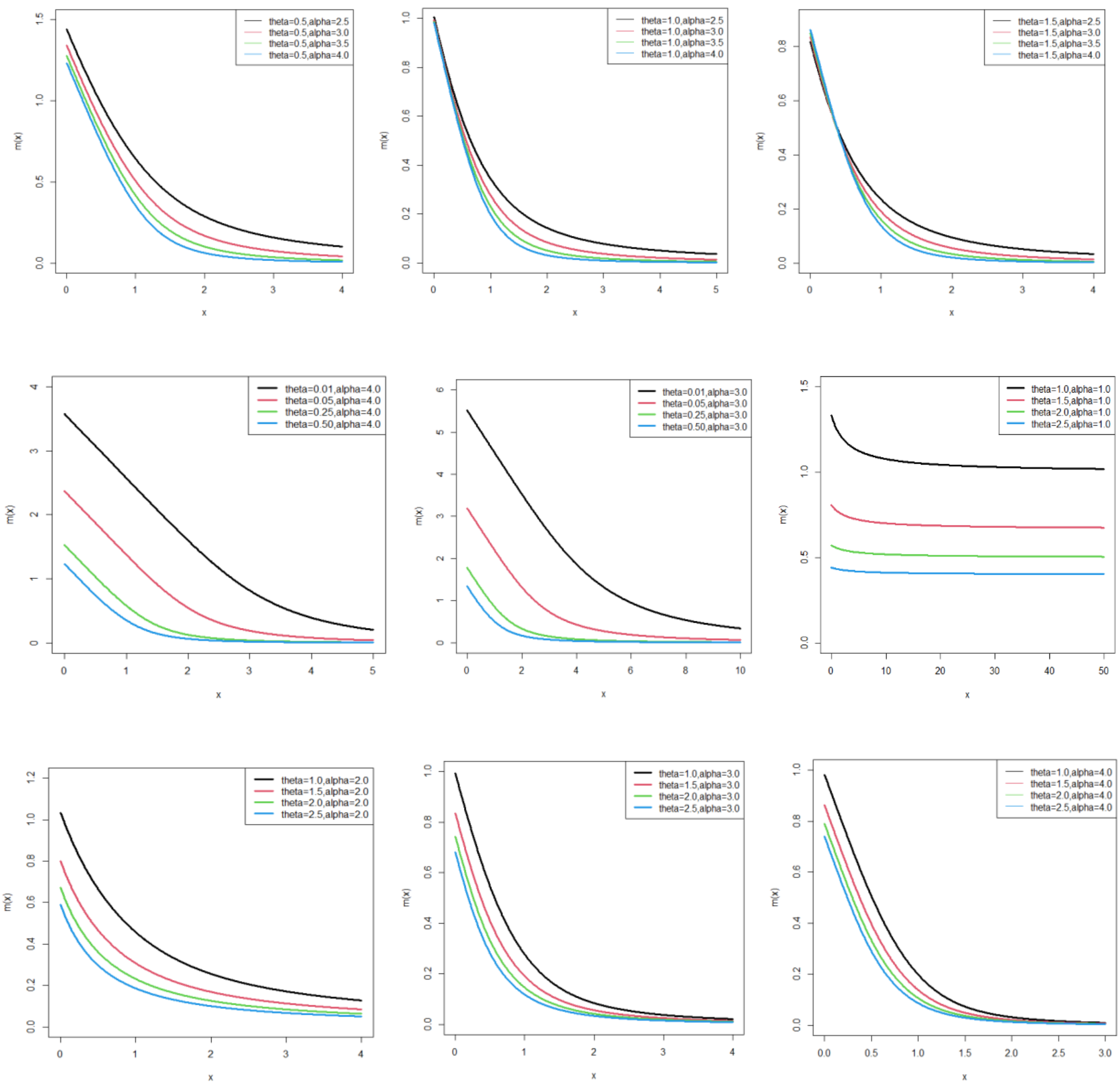

Figure 4: *Mean residual life function of PKD for varying values of the parameters* (α, θ)

5. Stochastic Ordering

The stochastic ordering of positive continuous random variables is an important tool for judging their comparative behavior. A random variable X is said to be smaller than a random variable Y in the

(i) stochastic order $\left(X \le_{st} Y\right)$ if $F_X(x) \ge F_Y(x)$ for all x

(ii) hazard rate order $\left(X \le_{hr} Y\right)$ if $h_X(x) \ge h_Y(x)$ for all x

(iii) mean residual life order $\left(X \le_{mrl} Y\right)$ if $m_X(x) \le m_Y(x)$ for all x

(iv) likelihood ratio order $\left(X \le_{lr} Y\right)$ if $\dfrac{f_X(x)}{f_Y(x)}$ decreases in x.

The following interrelationships due to Shaked and Shanthikumar [7] are well known for establishing stochastic ordering of distributions

$$\begin{array}{c} X \le_{lr} Y \Rightarrow X \le_{hr} Y \Rightarrow X \le_{mrl} Y \\ \Downarrow \\ X \le_{st} Y \end{array}$$

It can be easily shown that PKD is ordered with respect to the strongest 'likelihood ratio' ordering. The stochastic ordering of PKD has been explained in the following theorem:

Theorem: Suppose $X \sim \text{PKD}\left(\theta_1, \alpha_1\right)$ and $Y \sim \text{PKD}\left(\theta_2, \alpha_2\right)$. If $\alpha_1 \le \alpha_2$ and $\theta_1 > \theta_2$ (or $\alpha_1 < \alpha_2$ and $\theta_1 \ge \theta_2$), then $X \le_{lr} Y$ and hence $X \le_{hr} Y$, $X \le_{mrl} Y$ and $X \le_{st} Y$.

Proof: We have

$$\frac{f_X\left(x;\theta_1,\alpha_1\right)}{f_Y\left(x;\theta_2,\alpha_2\right)} = \left[\frac{\alpha_1\theta_1^{\,2}\left(\theta_2^{\,2}+\theta_2+1\right)}{\alpha_2\theta_2^{\,2}\left(\theta_1^{\,2}+\theta_1+1\right)}\right]\left(\frac{1+\theta_1+x^{\alpha_1}}{1+\theta_2+x^{\alpha_2}}\right)x^{\alpha_1-\alpha_2}\,e^{-\left(\theta_1 x^{x^{\alpha_1}}-\theta_2 x^{\alpha_2}\right)};x>0$$

Now, taking logarithm both sides, we get

$$\log\frac{f_X\left(x;\theta_1,\alpha_1\right)}{f_Y\left(x;\theta_2,\alpha_2\right)} = \log\left[\frac{\alpha_1\theta_1^{\,2}\left(\theta_2^{\,2}+\theta_2+1\right)}{\alpha_2\theta_2^{\,2}\left(\theta_1^{\,2}+\theta_1+1\right)}\right] + \log\left(\frac{1+\theta_1+x^{\alpha_1}}{1+\theta_2+x^{\alpha_2}}\right) + \left(\alpha_1-\alpha_2\right)\log x - \left(\theta_1 x^{x^{\alpha_1}} - \theta_2 x^{\alpha_2}\right)$$

This gives

$$\frac{d}{dx}\ln\frac{f_X\left(x;\theta_1,\alpha_1\right)}{f_Y\left(x;\theta_2,\alpha_2\right)} = \frac{\alpha_1-\alpha_2}{x} + \frac{\left(\alpha_1 x^{\alpha_1-1}-\alpha_2 x^{\alpha_2-1}\right)+\left(\alpha_1\theta_2 x^{\alpha_1-1}-\alpha_2\theta_1 x^{\alpha_2-1}\right)+\left(\alpha_1-\alpha_2\right)x^{\alpha_1+\alpha_2-1}}{\left(1+\theta_1+x^{\alpha_1}\right)\left(1+\theta_2+x^{\alpha_2}\right)}$$
$$-\left(\theta_1\alpha_1 x^{\alpha_1-1}-\theta_2\alpha_2 x^{\alpha_2-1}\right)$$

.

Thus for $\alpha_1 \le \alpha_2$ and $\theta_1 > \theta_2$ (or $\alpha_1 < \alpha_2$ and $\theta_1 \ge \theta_2$), $\dfrac{d}{dx}\ln\dfrac{f_X\left(x;\theta_1,\alpha_1\right)}{f_Y\left(x;\theta_2,\alpha_2\right)} < 0$. This means that $X \le_{lr} Y$ and hence $X \le_{hr} Y$, $X \le_{mrl} Y$ and $X \le_{st} Y$.

6. Estimation of Parameters of Power Komal Distribution

Suppose $(x_1, x_2, ..., x_n)$ be a random sample of size n from PKD(θ,α) The log-likelihood function of PKD can be expressed as

$$\log L = \sum_{i=1}^{n} \log f(x;\theta,\alpha)$$

$$= 2n\log\theta - n\log\left(\theta^2+\theta+1\right) + (\alpha-1)\sum_{i=1}^{n}\log x_i + \sum_{i=1}^{n}\log\left(1+\theta+x_i^{\alpha}\right) - \theta\sum_{i=1}^{n}x_i^{\alpha}$$

The maximum likelihood estimates $(\hat{\theta},\hat{\alpha})$ of parameters (θ,α) of PKD are the solution of the following log-likelihood equations

$$\frac{\partial \log L}{\partial\theta} = \frac{2n}{\theta} - \frac{n(2\theta+1)}{\theta^2+\theta+1} + \sum_{i=1}^{n}\frac{1}{1+\theta+x_i^{\alpha}} - \sum_{i=1}^{n}x_i^{\alpha} = 0$$

$$\frac{\partial \log L}{\partial\alpha} = \sum_{i=1}^{n}\log x_i + \sum_{i=1}^{n}\frac{x_i^{\alpha}\log x_i}{1+\theta+x_i^{\alpha}} - \theta\sum_{i=1}^{n}x_i^{\alpha}\log(x_i)$$

These two natural log-likelihood equations do not seem to be solved directly, because they cannot be expressed in closed forms. The (MLE's) $(\hat{\theta},\hat{\alpha})$ of (θ,α) can be computed directly by solving the natural log-likelihood equation using Newton-Raphson iteration available in R-software till sufficiently close values of $\hat{\theta}$ and $\hat{\alpha}$ are obtained.

For Fisher's scoring method, we have

$$\frac{\partial^2 \log L}{\partial\theta^2} = \frac{-2n}{\theta^2} - \frac{n\left(2\theta^2+2\theta-1\right)}{\left(\theta^2+\theta+1\right)^2} - \sum_{i=1}^{n}\frac{1}{\left(1+\theta+x_i^{\alpha}\right)^2}$$

$$\frac{\partial^2 \log L}{\partial\theta\partial\alpha} = -\sum_{i=1}^{n}\frac{x_i^{\alpha}\log x_i}{\left(1+\theta+x_i^{\alpha}\right)^2} - \sum_{i=1}^{n}x_i^{\alpha}\log x_i = \frac{\partial^2 \log L}{\partial\alpha\partial\theta}$$

$$\frac{\partial^2 \log L}{\partial\alpha^2} = \sum_{i=1}^{n}\frac{\left(\log x_i\right)^2\left\{\left(1+\theta+x_i^{\alpha}\right)x_i^{\alpha} - x_i^{2\alpha}\right\}}{\left(1+\theta+x_i^{\alpha}\right)^2} - \theta\sum_{i=1}^{n}\left(\log x_i\right)^2 x_i^{\alpha}$$

For finding the MLEs $(\hat{\theta},\hat{\alpha})$ of parameters (θ,α) of PKD, following equations can be solved

$$\begin{bmatrix} \frac{\partial^2 \ln L}{\partial\theta^2} & \frac{\partial^2 \ln L}{\partial\theta\partial\alpha} \\ \frac{\partial^2 \ln L}{\partial\theta\partial\alpha} & \frac{\partial^2 \ln L}{\partial\alpha^2} \end{bmatrix}_{\substack{\hat{\theta}=\theta_0 \\ \hat{\alpha}=\alpha_0}} \begin{bmatrix} \hat{\theta}-\theta_0 \\ \hat{\alpha}-\alpha_0 \end{bmatrix} = \begin{bmatrix} \frac{\partial \ln L}{\partial\theta} \\ \frac{\partial \ln L}{\partial\alpha} \end{bmatrix}_{\substack{\hat{\theta}=\theta_0 \\ \hat{\alpha}=\alpha_0}}$$

where θ_0 and α_0 are the initial values of θ and α, as given by the method of moments. These equations are solved iteratively till close estimates of parameters are obtained.

7. Goodness of Fit

In this section, the goodness of fit of PKD using maximum likelihood estimates of parameters has been discussed with one real dataset. The goodness of fit has been compared with other one parameter and two-parameter lifetime distributions. Following dataset from engineering has been considered.

Dataset 1: The following symmetric dataset, discussed by Murthy et al [8], is the failure times of windshields and the values are:
0.04, 0.3, 0.31, 0.557, 0.943, 1.07, 1.124, 1.248, 1.281, 1.281, 1.303, 1.432, 1.48, 1.51, 1.51, 1.568, 1.615, 1.619, 1.652, 1.652, 1.757, 1.795, 1.866, 1.876, 1.899, 1.911, 1.912, 1.9141, 0.981, 2.010, 2.038, 2.085, 2.089, 2.097, 2.135, 2.154, 2.190, 2.194, 2.223, 2.224, 2.23, 2.3, 2.324, 2.349, 2.385, 2.481, 2.610, 2.625, 2.632, 2.646, 2.661, 2.688, 2.823, 2.89, 2.9, 2.934, 2.962, 2.964, 3, 3.1, 3.114, 3.117, 3.166, 3.344, 3.376, 3.385, 3.443, 3.467, 3.478, 3.578, 3.595, 3.699, 3.779, 3.924, 4.035, 4.121, 4.167, 4.240, 4.255, 4.278, 4.305, 4.376, 4.449, 4.485, 4.570, 4.602, 4.663, 4.694

In order to compare the considered distributions, values of $\mathrm{MLE}\left(\hat{\theta},\hat{\alpha}\right)$ along with their standard errors, $-2\log L$, AIC (Akaike Information Criterion), K-S (Kolmogorov-Smirnov Statistics) and p-values for the real lifetime dataset have been computed and presented in table1. The AIC and K-S Statistics are computed using the following formulae: $AIC=-2\log L+2k$ and $\text{K-S}=\sup_{x}\left|F_n\left(x\right)-F_0\left(x\right)\right|$, where $k=$ the number of parameters, $n=$ the sample size , $F_n\left(x\right)$ is the empirical (sample) cumulative distribution function, and $F_0\left(x\right)$ is the theoretical cumulative distribution function. The best distribution is the distribution corresponding to lower values of $-2\log L$, AIC, and K-S statistic.

Table 1: MLE's, $-2\log L$, $S.E\left(\hat{\theta},\hat{\alpha}\right)$, AIC, K-S and P-value of the fitted distributions of dataset 1.

Distributions	ML estimates and Standard Error		$-2\log L$	AIC	K-S	P-Value
	$\hat{\theta}$ $S.E\left(\hat{\theta}\right)$	$\hat{\alpha}$ $S.E\left(\hat{\alpha}\right)$				
PKD	0.2572 (0.0401)	1.8786 (0.1329)	**271.8212**	**275.8212**	**0.0540**	**0.9826**
PGD	0.1563 (0.0373)	2.1489 (0.1823)	272.7053	276.7053	0.1009	0.4340
PSD	0.3422 (0.0483)	1.7015 (0.1203)	272.7581	276.7581	0.0912	0.5729
WD	0.0829 (0.0223)	2.3563 (0.2031)	274.1806	278.1806	0.2990	0.0000
KD	0.5826 (0.0445)	…	325.0655	327.0655	0.2189	0.0015
GD	0.5572 (0.0506)	…	331.4615	333.4615	0.5131	0.0000
SD	0.6430 (0.0451)	…	314.4891	316.4891	0.2075	0.0031
ED	0.3893 (0.0415)	…	342.0450	344.045	0.5256	0.0000

In order to see the closeness of the fit given by one parameter exponential, Shanker and Garima distributions and two-parameter Weibull distribution, PGD and PSD, the fitted plot of pdfs of these distributions for the given dataset have been shown in figure 5. It is also obvious from the goodness of fit given in the table 1 and the fitted plots of the distributions in figure 5 that the PKD gives much closer fit as compared to other considered distributions.

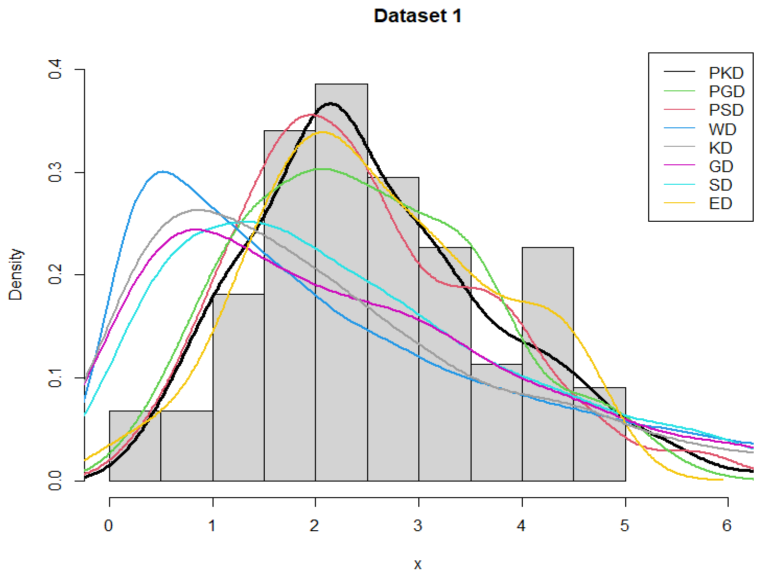

Figure 5: *Fitted pdf plots of distributions for dataset 1*

8. Conclusions

A two-parameter Power Komal distribution (PKD) has been introduced which includes Komal distribution, introduced by Shanker (2023), as a particular case. The statistical and reliability properties including shapes of density for varying values of parameters, the moments about the origin, the variance, survival function, hazard rate function, mean residual function of PKD have been discussed. The method of maximum likelihood for estimating the parameters has been discussed. Finally, the goodness of fit of PKD has been discussed with a real lifetime dataset and the fit has been found quite satisfactory as compared to one parameter exponential Shanker and Garima distributions and two-parameter Power Shanker distribution (PSD), power Garima distribution (PGD) exponential, and Weibull distribution. Therefore, PKD can be considered as an important lifetime distribution for modeling lifetime data from engineering.

Conflict of Interest

The Authors declare that there is no conflict of interest.

References

[1] Shanker, R. (2023). Komal Distribution with properties and Application in Survival Analysis, *Biometrics & Biostatistics International Journal*, 12(2): 40 – 44.

[2] Shanker, R. and Shukla, K.K. (2017). Power Shanker Distribution and Application, *Turliye Kliniklery Journal of Biostatistics* ,9(3):175-87.

[3] Shanker, R. (2015). Shanker distribution and its Applications, *International Journal of Statistics and Applications,* 5(6): 338 – 348.

[4] Berhane, A., Tesfay, M., Tesfalem, E., Shanker, R. (2019). A Two Parameter Power Garima Distribution with Properties and Applications, Annals of Biostatistics and Biometric applications ,1(3):1-7.

[5] Shanker, R., (2016). Garima distribution and Its Application to model behavioral science data, Biometrics and Biostatistics International Journal, 4(7): 1-9.

[6] Weibull, W. (1951). A statistical distribution of wide applicability, Journal of Applied Mathematics, 18: 293 – 297.

[7] Shaked, M. and Shanthikumar, J.G. (1994): Stochastic Orders and Their Applications, Academic Press, New York.

[8] Murthy, D.N.P., Xie, M., Jiang, R. (2004). Weibull Models, John Wiley & Sons Inc., Hoboken.