

PROFIT ANALYSIS OF REPAIRABLE COLD STANDBY SYSTEM UNDER REFRESHMENTS

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Abstract

In the generation of science and technology, every company wants to increase the reliability of their products. So, they used the concept of cold standby redundancy, timely repair of the failed unit and providing limited refreshments to the available technician when required. This paper aims to explore the system of two identical units where the primary unit is operative and the secondary unit is in cold standby mode. When the primary unit fails due to any fault then secondary unit starts working immediately. Here, times of failure of unit and technician refreshment request follow the general distribution whereas times of repair of unit and refreshment follow the exponential distributions. Such types of systems are used in industries and education systems to prevent losses. The system's performance is calculated by using concepts of mean time to system failure, availability, busy period of the server, expected number of visits made by the server and profit function using the semi Markov process and regenerative point technique. Tables are used to explore the performance of the system.

Keywords: Cold standby, refreshment, regenerative point, semi Markov process

I. Introduction

Reliability and maintainability are the essential parameters of items and products that satisfy customers' requirements. In today's era, several approaches for performance improvement of maintainable systems have been adopted by scientists and engineers during designing them. A large amount of research work has been done on repairable systems such that Subramanian [17] explored the idea of preventive maintenance in two distinct units system under repair. Bao and Mays [3] analyzed the hydraulic reliability of water distribution systems under demand, pipe roughness and pressure head. Gnedenko and Igor [8] explored reliability and probability studies for engineering purposes. Diaz et al. [6] threw light on the warranty cost management system. Jack and Murthy [10] discovered the role of limited warranty and extended warranty for the product. Wang and Zhang [19] examined the repairable system of two non identical components under repair facility using geometric distributions. Mahmoud and Moshref [14] described the cold standby system under hardware failure and preventive maintenance using the semi Markov process. Deswal and Malik [5] evaluate the non identical units system under different working conditions by using the semi Markov process. Kumar et al. [11] examined the stochastic behavior of two unit system where one unit in cold standby mode and subject to maximum repair time using the regenerative point technique. Kumar and Goel [12] analyzed the preventive maintenance in two unit cold standby system under general distributions. Malik and Rathee [15] threw light on the two parallel units system under preventive maintenance and maximum operation time.

Temraz [18] analyzed the performance of two parallel components system under load sharing and degradation facility. Levitin et al. [13] explored the results of optimal preventive replacement of failed units in a cold standby system by using the poisson process. Barak et al. [2] threw light on the availability and profit values of milk plant under repair facility. Agarwal et al. [1] described the reliability and availability of water reservoir system under repair facility. Chaudhary and Sharma [4] explored the parallel non identical units system that gives priority to repair over preventive maintenance. Garg and Garg [7] analyzed the reliability and profit values of briquette machine under neglected faults like sound and overheating. Jia et al. [9] explored the two unit system under demand and energy storage techniques. Sengar and Mangey [16] examined the performance of complicated systems under inspection using copula methodology.

II. System Assumptions

There are following system assumptions:

- Initially, the system has two units such that one is an operative (primary) unit and the other is a cold standby (secondary) unit.
- When the operative unit fails then the cold standby unit starts working.
- An expert repairman is always available to repair the failed unit.
- The failed unit behaves like a new one after repair.
- Repair and refreshment times are exponentially distributed whereas times for failure of unit and server refreshment request are general.

III. System Notations

There are following system notations:

R	Collection of regenerative states S_i ($i = 0,1,2,3$)
O/Cs	Operative unit / cold standby unit of the system
a/b	The probability that the cold standby unit is working/ not working
λ / μ	Failure rate of the unit/ rate by which the server needs refreshment
$g(t)/G(t)$	PDF/ CDF of the repair time of the unit
$f(t)/F(t)$	PDF/ CDF of refreshments time that restores freshness to the server
$q_{r,s}(t)/Q_{r,s}(t)$	PDF/ CDF of first passage time from r^{th} to s^{th} regenerative state or s^{th} failed state without halting in any other $S_i \in R$ in $(0,1]$
$M_r(t)$	Represents the probability of the system that it initially works $S_r \in R$ at a time (t) without moving through another state $S_i \in R$
$W_r(t)$	Probability that up to time (t) the server is busy at the state S_r without transit to another state $S_i \in R$ or before return to the same state through one or more non regenerative states
\oplus/\otimes	Laplace convolution / Laplace Stieltjes Convolution
$*/**/'$	Symbol for Laplace Transform/ Laplace Stieltjes Transform/ Function's derivative
$\square / \bullet / \square$	Upstate/ regenerative state/ failed state

IV. State Descriptions

The individual state description is given by the table 1:

Table 1: State Descriptions

States	Descriptions
S_0	It is a regenerative upstate with two units such that one is operative (O) and other is cold standby (Cs).
S_1	This regenerative upstate has two units such that one is failed under repair (Fur) and the other is in operative mode (O).
S_2	It is a regenerative upstate under refreshment facility (sut) where one unit is failed & waiting for repair (Fwr) and the other is in operative mode (O).
S_3	It is a regenerative down state and the system has two units such that one is failed under repair (Fur) and the other is failed and waiting for repair (Fwr).
S_4	It is a down state where one unit fails under repair (FUR) continuously from the prior state and the other unit is failed & waiting for repair (Fwr).
S_5	It is a down state that has two units under refreshment facility (sut) such that one is failed and waiting for repair (FWR) continuously from the previous state and the other is failed and waiting for repair (Fwr).
S_6	At this down state, the system has two units such that one is failed under repair (FUR) continuously from the previous state and the other unit is failed and waiting for repair (FWR) continually from the prior state.
S_7	This down state has two units under continuous refreshment facility (SUT) such that one is failed and waiting for repair (Fwr) and the other is failed and waiting for repair (FWR) continuously from the previous state.
S_8	This down state has two units under refreshment facility (sut) such that one is failed and waiting for repair (Fwr) and the other is failed and waiting for repair (FWR) continuously from the previous state.

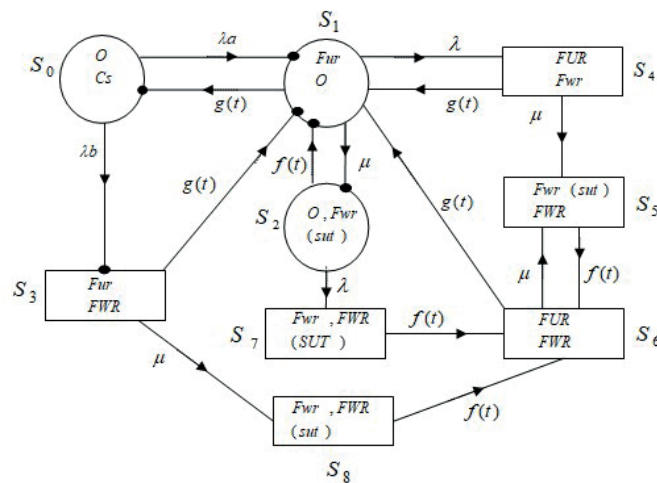


Figure 1: State Transition Diagram

V. Transition Probabilities

The transition probabilities are calculated using $f(t) = \theta e^{-\theta t}$, $g(t) = \phi e^{-\phi t}$ and get

$$\begin{aligned}
 p_{01} &= a, \quad p_{03} = b, \quad p_{10} = \frac{\phi}{\phi + \mu + \lambda}, \quad p_{12} = \frac{\mu}{\phi + \mu + \lambda}, \quad p_{14} = \frac{\lambda}{\phi + \mu + \lambda} \\
 p_{21} &= \frac{\theta}{\theta + \lambda}, \quad p_{27} = \frac{\lambda}{\theta + \lambda}, \quad p_{31} = \frac{\phi}{\phi + \mu}, \quad p_{38} = \frac{\mu}{\phi + \mu}, \quad p_{41} = \frac{\phi}{\phi + \mu}, \quad p_{45} = \frac{\mu}{\phi + \mu} \\
 p_{61} &= \frac{\phi}{\phi + \mu}, \quad p_{65} = \frac{\mu}{\phi + \mu}, \quad p_{56} = p_{76} = p_{86} = 1
 \end{aligned} \tag{1}$$

It has been conclusively established that

$$\begin{aligned}
 p_{01} + p_{03} &= 1, \quad p_{10} + p_{12} + p_{14} = 1, \quad p_{21} + p_{27} = 1, \quad p_{31} + p_{38} = 1 \\
 p_{41} + p_{45} &= 1, \quad p_{56} = p_{76} = p_{86} = 1, \quad p_{31} + p_{31.8(65)^n} = 1 \\
 p_{10} + p_{12} + p_{11.4} + p_{11.4(56)^n} &= 1, \quad p_{21} + p_{21.7(65)^n} = 1
 \end{aligned} \tag{2}$$

VI. Mean Sojourn Time

In the cold standby redundant system, μ_i represents the mean sojourn time. Mathematically, time

consumed by a system in a particular state is, $\mu_i = \sum_j m_{i,j} = \int_0^{\infty} P(T > t) dt$. Then

$$\begin{aligned}
 \mu_0 &= m_{01} + m_{03} = \frac{1}{\lambda}, \quad \mu_1 = m_{10} + m_{12} + m_{14} = \frac{1}{\phi + \mu + \lambda}, \quad \mu_2 = m_{21} + m_{27} = \frac{1}{\theta + \lambda} \\
 \mu_3 &= m_{31} + m_{38} = \frac{1}{\phi + \mu}, \quad \mu_4 = m_{41} + m_{45} = \frac{1}{\phi + \mu}, \quad \mu_6 = m_{61} + m_{65} = \frac{1}{\phi + \mu} \\
 \mu_5 &= \mu_7 = \mu_8 = \frac{1}{\theta}, \quad \mu'_1 = m_{10} + m_{12} + m_{11.4} + m_{11.4(56)^n} = \frac{\theta\phi + \lambda(\theta + \mu)}{\theta\phi(\phi + \mu + \lambda)} \\
 \mu'_2 &= m_{21} + m_{21.7(65)^n} = \frac{\theta\phi + \lambda(\theta + \phi + \mu)}{\theta\phi(\theta + \lambda)}, \quad \mu'_3 = m_{31} + m_{31.8(65)^n} = \frac{(\theta + \mu)}{\theta\phi}
 \end{aligned} \tag{3}$$

VII. Reliability Measures Evaluations

I. Mean Time to System Failure (MTSF)

Let the cumulative distribution function of the first elapsed time be $\varphi_i(t)$ from the regenerative state S_i to the failed state of the system. Treating the failed states as an absorbing state then the repetitive interface for $\varphi_i(t)$ being

$$\begin{aligned}
 \varphi_0(t) &= Q_{0,3}(t) + Q_{0,1}(t) \otimes \varphi_1(t) \\
 \varphi_1(t) &= Q_{1,4}(t) + Q_{1,2}(t) \otimes \varphi_2(t) + Q_{1,0}(t) \otimes \varphi_0(t) \\
 \varphi_2(t) &= Q_{2,7}(t) + Q_{2,1}(t) \otimes \varphi_1(t)
 \end{aligned} \tag{4}$$

Taking LST of the relation (4) and solving for $\varphi_0^{**}(s)$ then

$$MTSF = \lim_{s \rightarrow 0} \frac{1 - \varphi_0^{**}(s)}{s} \tag{5}$$

System reliability can be obtained by using the inverse LT of equation (5). We have

$$MTSF = \frac{[1 - p_{12}p_{21}]\mu_0 + p_{01}\mu_1 + p_{01}p_{12}\mu_2}{1 - p_{01}p_{10} - p_{12}p_{21}} \quad (6)$$

$$MTSF = \frac{\theta\phi + \lambda\theta + \phi\lambda + \mu\lambda + \lambda^2 + a\lambda\theta + a\lambda^2 + a\mu\lambda}{\theta\phi + \lambda\theta + \phi\lambda + \mu\lambda + \lambda^2 - a\theta\phi - a\lambda\phi} \quad (7)$$

II. Availability of the system

From the transition diagram, the system is available at the regenerative up states S_0, S_1 and S_2 . Let $A_i(t)$ is the probability that the system is in upstate at time (t) specified that the system arrives at the regenerative state S_i at $t = 0$. Then the repetitive interface for $A_i(t)$ is

$$\begin{aligned} A_0(t) &= M_0(t) + q_{01}(t) \oplus A_1(t) + q_{03}(t) \oplus A_3(t) \\ A_1(t) &= M_1(t) + q_{10}(t) \oplus A_0(t) + q_{12}(t) \oplus A_2(t) \\ &\quad + q_{11.4}(t) \oplus A_1(t) + q_{11.4(56)}^n(t) \oplus A_1(t) \\ A_2(t) &= M_2(t) + q_{21}(t) \oplus A_1(t) + q_{21.7(65)}^n(t) \oplus A_1(t) \\ A_3(t) &= q_{31}(t) \oplus A_1(t) + q_{31.8(65)}^n(t) \oplus A_1(t) \end{aligned} \quad (8)$$

$$\text{where, } M_0(t) = e^{-\lambda t}, M_1(t) = e^{-(\mu+\lambda_2)t} \overline{G(t)}, M_2(t) = e^{-\lambda t} \overline{G(t)} \quad (9)$$

Using LT of the above relation (8), there exist

$$\therefore A_0 = \lim_{s \rightarrow 0} \frac{N_A}{D_1} = \frac{[\mu_0 p_{1,0} + \mu_1 + \mu_2 p_{1,2}]}{[\mu_0 p_{1,0} + \mu_1' + \mu_2' p_{1,2} + \mu_3' p_{0,3} p_{1,0}]} \quad (10)$$

$$\therefore A_0 = \frac{\theta\phi[\phi(\theta + \lambda) + (a - b)(\theta + \lambda)\lambda + \lambda\mu]}{\left[\begin{aligned} &\theta\phi^2(\theta + \lambda) + (\theta\phi + \lambda(\theta + \mu))(a - b)\lambda(\theta + \lambda) \\ &+ (\theta\phi + \lambda(\theta + \phi + \mu))\mu(a - b)\lambda - (\theta + \mu)b\lambda\phi(\theta + \lambda) \end{aligned} \right]} \quad (11)$$

III. Busy Period of the Server

From the transition diagram, it is clear that the technician is busy at states S_1, S_2 and S_3 . Let $B_i(t)$ is the probability that the repairman is busy due to the repair of the failed unit at time 't' specified that the system arrives at the regenerative state S_i at $t = 0$. Then the repetitive interface for $B_i(t)$ is

$$\begin{aligned} B_0(t) &= q_{01}(t) \oplus B_1(t) + q_{03}(t) \oplus B_3(t) \\ B_1(t) &= W_1(t) + q_{10}(t) \oplus B_0(t) + q_{12}(t) \oplus B_2(t) \\ &\quad + q_{11.4}(t) \oplus B_1(t) + q_{11.4(56)}^n(t) \oplus B_1(t) \\ B_2(t) &= W_2 + q_{21}(t) \oplus B_1(t) + q_{21.7(65)}^n(t) \oplus B_1(t) \\ B_3(t) &= W_3(t) + q_{31}(t) \oplus B_1(t) + q_{31.8(65)}^n(t) \oplus B_1(t) \end{aligned} \quad (12)$$

where, $W_1(t) = \overline{G_1(t)} e^{-(\lambda+\mu)t} + \overline{G_1(t)} \lambda e^{-(\lambda+\mu)t} \oplus \overline{G_1(t)} e^{-\mu t} + \dots$

$$\begin{aligned} W_2(t) &= \overline{G_1(t)} \oplus \lambda e^{-\lambda t} \overline{F_1(t)} \oplus f_1(t) \\ W_3(t) &= \overline{G_1(t)} e^{-\mu t} + \overline{G_1(t)} \mu e^{-\mu t} \oplus f_1(t) \oplus \overline{G_1(t)} e^{-\mu t} + \dots \end{aligned} \quad (13)$$

Using LT on relations (12) then we get

$$B_0 = \lim_{s \rightarrow 0} \frac{N_B}{D_1} = \frac{W_1^*(0) + W_2^*(0)p_{1,2} + W_3^*(0)p_{0,3}p_{1,0}}{[\mu_0 p_{1,0} + \mu_1' + \mu_2' p_{1,2} + \mu_3' p_{0,3} p_{1,0}]} \quad (14)$$

$$B_0 = \frac{\lambda\mu[\theta\phi + \lambda(\phi + \theta)]}{\left[\begin{array}{l} \theta\phi^2(\theta + \lambda) + (\theta\phi + \lambda(\theta + \mu))(a - b)\lambda(\theta + \lambda) \\ + (\theta\phi + \lambda(\theta + \phi + \mu))\mu(a - b)\lambda - (\theta + \mu)b\lambda\phi(\theta + \lambda) \end{array} \right]} \quad (15)$$

IV. Estimated number of visits made by the server

The transition diagram explores that the technician visits at states S_1 and S_2 . Let $N_i(t)$ is the estimated number of visits made by the repairman for repair in $(0, t]$ specified that the system arrives at the regenerative state S_i at $t = 0$. Then the repetitive interface for $N_i(t)$ is

$$\begin{aligned} N_0(t) &= Q_{01}(t) \otimes [1 + N_1(t)] + Q_{03}(t) \otimes [1 + N_3(t)] \\ N_1(t) &= Q_{10}(t) \otimes N_0(t) + Q_{12}(t) \otimes N_2(t) \\ &\quad + Q_{11.4}(t) \otimes N_1(t) + Q_{11.4(56)}^n(t) \otimes N_1(t) \\ N_2(t) &= Q_{21}(t) \otimes N_1(t) + Q_{21.7(65)}^n(t) \otimes N_1(t) \\ N_3(t) &= Q_{31}(t) \otimes N_1(t) + Q_{31.8(65)}^n(t) \otimes N_1(t) \end{aligned} \quad (16)$$

Using LST of the above relations (16) then we get

$$N_0 = \lim_{s \rightarrow 0} \frac{N_v}{D_1'} = \frac{P_{1,0}}{[\mu_0 P_{1,0} + \mu_1' + \mu_2' P_{1,2} + \mu_3' P_{0,3} P_{10}]} \quad (17)$$

$$N_0 = \frac{\lambda\theta\phi^2(\theta + \lambda)}{\left[\begin{array}{l} \theta\phi^2(\theta + \lambda) + (\theta\phi + \lambda(\theta + \mu))(a - b)\lambda(\theta + \lambda) \\ + (\theta\phi + \lambda(\theta + \phi + \mu))\mu(a - b)\lambda - (\theta + \mu)b\lambda\phi(\theta + \lambda) \end{array} \right]} \quad (18)$$

V. Profit Analysis

It is an integral part of reliability measures that tell customers and system developers whether the system is beneficial or not. The profit values depend upon the MTSF, availability of the system, busy period of server and extended number of visits. Then the profit function of the system is defined by

$$P = T_0 A_0 - T_1 B_0 - T_2 N_0 \quad (19)$$

where, $T_0 = 1500$ (Revenue per unit up-time)

$T_1 = 500$ (Charge per unit for server busy period)

$T_2 = 200$ (Charge per visit made by the server)

4. Discussion

The transition diagram is used to calculate the system reliability measures like MTSF, availability of the system, busy period of the server, expected number of visits made by the server and profit values. It can be seen from the table 2 that the tendency of MTSF increases smoothly with respect to increments in refreshment rate (θ); however, other parameters such as failure rate of unit ($\lambda=0.55$), server refreshment request rate ($\mu=0.4$), repair rate of unit ($\phi=0.5$), cold stand by unit working probability ($a=0.8$) and not working probability ($b=0.2$) have fixed values. It is clear that when failure rate (λ) increases then MTSF declines. When technician refreshment rate (μ) enhances then MTSF also declines but when repair rate (ϕ) increases then MTSF enhances. Thus, the concept of refreshment is beneficial for the owner and technician. When MTSF enhances then system reliability also enhances.

Table 2: MTSF vs. Refreshment Rate

θ ↓	$\lambda=0.55, \mu=0.4$ $\phi=0.5, a=0.8$ $b=0.2$	$\lambda=0.65,$	$\mu=0.6$	$\phi=0.7$
0.1	3.8613371	3.218144	3.775294	4.0647311
0.2	3.8894957	3.235483	3.806505	4.1010786
0.3	3.9129156	3.250148	3.833119	4.1312067
0.4	3.9327001	3.262713	3.856081	4.1565858
0.5	3.9496349	3.273599	3.876094	4.1782569
0.6	3.9642941	3.283122	3.893693	4.1969772
0.7	3.9771076	3.291522	3.90929	4.2133109
0.8	3.9884034	3.298987	3.923207	4.2276872
0.9	3.998436	3.305664	3.935703	4.240438
1	4.007406	3.311673	3.946984	4.2518242

The availability of the redundant system is affected by the refreshment rate (θ), repair rate (ϕ), unit failure rate (λ) and server refreshment request rate (μ). Table 3 explores the availability of the system and its value increase corresponding to increments in refreshment rate (θ) when the system's other parameters $\lambda=0.55, \mu=0.4, \phi=0.5, a=0.8, b=0.2$ possess constant values. When the failure rate of unit changes ($\lambda=0.55$ to 0.65) then the availability of system declines. Also, when the technician request rate changes ($\mu=0.4$ to 0.6) then the system's availability declines but when the repair rate of unit changes ($\phi=0.5$ to 0.7) then the availability of the system enhances.

Table 3: Availability vs. Refreshment Rate

θ ↓	$\lambda=0.55, \mu=0.4$ $\phi=0.5, a=0.8$ $b=0.2$	$\lambda=0.65,$	$\mu=0.6$	$\phi=0.7$
0.1	0.214076	0.185918	0.190417	0.25028
0.2	0.319498	0.281347	0.293845	0.364639
0.3	0.381465	0.338725	0.358019	0.427923
0.4	0.421888	0.376681	0.401314	0.467056
0.5	0.450148	0.403461	0.432266	0.493101
0.6	0.470912	0.423258	0.455361	0.511375
0.7	0.48675	0.438421	0.473168	0.524719
0.8	0.499191	0.450366	0.487264	0.534774
0.9	0.509198	0.45999	0.498663	0.542545
0.1	0.517404	0.46789	0.508046	0.548679

It is an important part of the system that tells the customers about the performance of the product that it is beneficial or not. So, the cold standby redundant system is used to enhance the system's profit. It is evident from table 4 that the system uses constant parameters such that $\lambda=0.55, \mu=0.4, \phi=0.5, a=0.8, b=0.2$ and the trend of profit values enhanced with respect to increments in refreshment rate (θ). When the failure rate of unit (λ) changes from 0.55 to 0.65 then the profit of system declines. Also, when the technician request rate (μ) changes from 0.4 to 0.6 then profit values decline but when the repair rate of unit (ϕ) changes from 0.5 to 0.7 then the profit of the system enhances.

Table 4: Profit vs. Refreshment Rate

θ ↓	$\lambda=0.55, \mu=0.4$ $\phi=0.5, a=0.8$ $b=0.2$	$\lambda=0.65,$	$\mu=0.6$	$\phi=0.7$
0.1	2046.417	1588.805	1690.491	2692.9
0.2	3640.771	3027.839	3256.171	4428.411
0.3	4576.626	3891.418	4226.372	5389.1
0.4	5186.276	4461.578	4880.064	5983.344
0.5	5611.935	4863.085	5346.797	6378.977
0.6	5924.286	5159.353	5694.6	6656.653
0.7	6162.259	5385.877	5962.46	6859.482
0.8	6348.978	5563.998	6174.235	7012.365
0.9	6498.993	5707.276	6345.293	7130.568
1	6621.895	5824.711	6485.948	7223.9

7. Conclusion

The results of the study show that providing refreshments to the server during the job generally enhances his efficiency which is crucial for any repairable system. From the above discussion, the MTSF, availability and profit values of the system increase with respect to increments in refreshment rate as well as repair rate but the reliability values decline when server refreshment request rate and failure rate of unit are enhanced. It is clear from tables that the server has to override his emotions and try to satisfy the customers. The idea of refreshment is used by corporate sectors, industries, cybercafés, education, university systems, etc.

8. Future Scope

Refreshment to the server plays an essential role in the water-boosting station system where one unit is operative and another is kept on cold standby.

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