OPTIMIZATION OF PRIORITY SERVICE WITH EFFICIENT COORDINATION OF ADMISSION CONTROL, EMERGENCY VACATION OF AN UNRELIABLE SERVER

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Abstract

In this paper, we establish a single server retrial queueing system with two types of customers, admission control, balking, emergency vacation, differentiate breakdown, and restoration. There are two distinct factors which must be considered when classifying priority and ordinary customers. The non-preemptive priority discipline proposed by this concept. Ordinary and priority customers arrive in accordance with Poisson processes. For both priority and ordinary customers, the server continuously offers a single service that is distributed arbitrarily. In this study, we compute the Laplace transforms of the time-dependent probabilities of system states using a probability generating function and the supplementary variable technique. The sensitivity analysis of system descriptions is assisted by study of numerical findings.

Keywords: Batch Arrivals; Priority Queues; Admission Control; Emergency Vacation; Balking; Differentiate Breakdown; Restoration.
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1. Introduction

When a customer arrives and discovers the server is busy, they are asked to leave the service area and join a trial queue known as orbit, which separates queues with repeated efforts. Once a certain period has gone, the orbiting the customer may resubmit their service request. Any customer in the orbit can repeatedly request services without affecting the other customers in the orbit. In computer and communication systems, such queues have a unique function. Numerous researchers explored two various customer types in retrial queues Wang ([26]; Dimitriou [12]; Wu and Lian [27]; Rajadurai et al.[23]).

Several aspects of daily life involve priority queues, especially when specific groups of individuals are given special attention, e.g. telecommunications field. Priority systems are a inestimable scheduling tool that allows messages of several types to receive a wide range of service. Due to this, the priority queue has earned a lot of attention in the literature Ayyappan and Thilagavathy [5], Kim et al. [19], Bhagat and Madhu Jain [8], Rismawati et al. [24], Pandey and Pal [22].

The service channel will temporarily fail if the regularly busy server breaks down, which could happen at any time. In other words, the server is temporarily unavailable. Fiems et al. [13] focused at the single-server queueing system with two different kinds of server disruption. Jain and Agarwal [16] considered an unreliable server $M^{[X]}/M/1$ queueing system with multiple types of server breakdowns. Zhang and Zhu [29] investigated retrial queueing model with vacations and two types of breakdowns. Jayakumar and Senthilnathan [17] described the server breakdown without interruption in a batch arrival queueing system with multiple vacations and closedown. Yi-chih Hsieh and Andersland [28] explored steady-state queue length distribution and mean queue length of Markov queueing systems subject to random breakdowns. Ayyappan and Sankeetha [7] discussed single server that provides both regular and optional service with vacation, breakdown and repair.

In 1957, Haight conducted the first study on the phenomenon of balking, in which customers decide not to wait in queue if the server is not present. Customers may become frustrated in several types of scenarios, including call centres, computer systems, websites, and telephone switchboards. Artalejo and Lopez-Herrero [3] introduced an M/G/1 retrial queue with balking. A $M^{[X]}/G/1$ queue with variable vacations and balking was researched by Ke [18]. An $M/G/1$ retrial queue with non-persistent customers was mentioned by Gao and Wang [14] where the server was susceptible to failure because of the negative arrivals.

The following admission control policy was examined in this study. An arriving batch of low priority customers may be allowed to join the orbit with probability a or may not be allowed with probability $(1-a)$. For instance, the company may not be able to select one candidate from all the applicants during an interview. Some selection criteria could be used, such as a screening process, a group discussion, etc. A single server batch arrival queueing system with two service phases, an admission control system, and Bernoulli vacation was examined by Choudhury [9]. Artalejo et al. [4] generalised this queue to discrete-time. A $M/E_k/1$ queuing system's control approach was created by Madhu and Indhu [20]. A single server batch arrival retrial queueing system with two service phases and a Bernoulli admission algorithm was obtained by Choudhury and Deka [10].

Recent research regarding various vacation policies adopted by service providers has produced a considerable impact on queueing systems. This is a result of its widespread application in many kinds of real-life situations, particularly in flexible production systems, communication systems, and computer systems. Shekar et al. [25] introduced a single server queueing system's emergency vacation. This vacation policy states that the working server may take a vacation in an emergency without finishing the service to customers who are waiting in service interruption. A priority retrial queueing system that involves working vacations and vacation interruptions, emergency vacations, negative arrivals, and delayed repairs was studied by Ayyappan and Thamizhselvi [6]. With two different vacation possibilities, Anna Bagyam and Udhaya Chandrika [2] investigated a single server retrial queueing system. [1] has highlighted the transient behaviour of a multiple vacations queue with frustrated clients. [11] investigated a $M^{[X]}/G/1$ queueing system with a single vacation policy. Ayyappan and Meena developed the phase type queueing model with degrading service, breakdown and Vacation.

In this study, we investigate a single server retrial priority queueing system with admission control, balking, emergency vacation, differentiate breakdown and restoration. Incoming ordinary customers have the option of entering the orbit or exiting the system if the server is down. If the server is busy suddenly they go for vacation and the interrupted customer wait in the queue and get fresh service after return from vacation. The regular busy period server may breakdown at any instance. Hard and soft failure are the two kind of system failures. Hard failure is defined as an equipment failure that requires a repairman with specialized knowledge to be physically present, which is a time-consuming process. While soft failure is defined as failure brought on by circumstances rather than a physical problem and usually resolved by restarting the system. After breakdown that the system will take some time for its refunctioning. This recovering period is called the restoration.

The rest of the paper is organized as follows: Mathematical model is described in Section 2 and queue size distribution is analyzed in Section 3. An explicit expression for governing equation is enlisted in Section 4. Steady state analysis is discussed in Section 5. Stability condition discussed in Section 6. Particular cases are obtained in Section 7. The effect of system performance measures is illustrated in Section 8. Numerical and graphical results are derived and conclusion is obtained in Section 9 and 10.

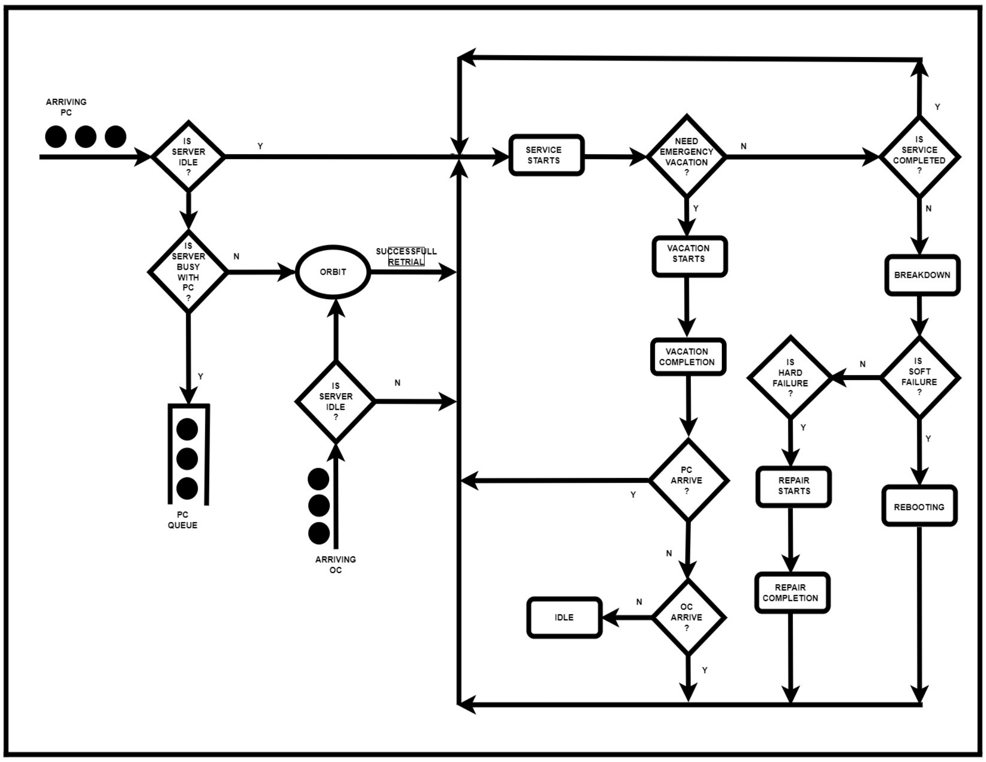

Figure 1: *Schematic representation*

2. Description of the Model

- **Arrival Process :** Two different types of units arrive in batches with independent Poisson compound process. Let $\lambda_p, \lambda_o > 0$ represent the corresponding arrival rates for priority and ordinary customers. Assume that the first order probabilities for priority and ordinary customers $\lambda_p c_i dt$ $(i = 1, 2, 3, ...)$ and $\lambda_o c_j dt$ $(j = 1, 2, 3, ...)$ respectively. The system has i and j batch size customers enters within a short period of time $(t, t + dt)$. Here, $0 \leq c_i \leq 1$, $\sum_{i=1}^{\infty} c_i = 1$, $0 \leq c_j \leq 1$, $\sum_{j=1}^{\infty} c_j = 1$.
- **Retrial Service Process :** Ordinary customers are known as retrial customers. These customers will go back to the orbit and will request repeatedly for their service after some time if the server is busy or unavailable. Retrial customers service time has rate $\beta(\nu)$ that follows the general distribution.
- **Regular Service Process :** Ordinary and priority customers ordinate in batches with distinct queues. Service rate follows general distribution and server renders single service for priority customers and ordinary customers with service rate $\mu_i(\nu)$, $i = 1, 2$ respectively. When the priority queue is empty, the service for ordinary customers begins.
- **Admission Control:** The server will follow the admission control policy for ordinary customers if it is overloaded in priority or ordinary customers. The server may grant ordinary customers admission with probability a or restricted entry with probability $(1 - a)$.
- **Differentiate Breakdown and restoration :** The rates of hard and soft failure are exponentially distributed with rate α_1 and α_2 respectively. For soft failure, the restoration time follows exponential distribution with rate η_1 and for hard failure restoration time follows

general distribution with rate $\eta_2(\nu)$.

- **Balking:** Incoming ordinary customers have the option of entering the orbit with probability b or exiting the system with probability $1-b$, if the server is down.
- **Emergency Vacation:** During the service term, the server has the opportunity to take an unexpected vacation at the exponentially distributed rate θ. The emergency vacation time for interruptions of priority and ordinary customers follows general distributions with rate $\gamma(\nu)$.

3. Analysis of queue size distribution

This section deals with the derivation of governing equations. On account of non-Markovian queueing system, probability generating function and supplementary variable have been used to solve this model.
Let
$N_1(t)$ = Number of priority customers in the queue at time t,
$N_2(t)$ = Number of ordinary customers in the orbit at time t,
$Y(t)$ = State of the server at time t.
Here $M^0(t)$, $B_i^0(t)$ for $i = 1, 2.$, $(R^{(2)})^0(t)$ and $E^0(t)$ indicates elapsed retrial time, elapsed service time for priority and ordinary customers, elapsed restoration period, elapsed emergency vacation at time t.

To obtain a bivariate Markov process $\{N_1(t), N_2(t), Y(t), t > 0\}$, $Y(t)$ denotes the server state. Here $Y(t)$ = (0,1,2,3,4,5), which mean as follows: 0, the server is idle; 1, server is in retrial state; 2, busy with priority customers; 3, busy with ordinary customers; 4, restoration; 5, on Emergency vacation.

Let us assume that, $M^0(0) = 0$, $M^0(\infty) = 1$, $B_i^0(0) = 0$, $B_i^0(\infty) = 1$, $(R^{(2)})^0(0) = 0$, $(R^{(2)})^0(\infty) = 1$ and $E^0(0) = 0$, $E^0(\infty) = 1$ be continuous at $\nu = 0$ for $i = 1, 2$.

If the elapsed time is ν, let function $\beta(\nu)$, $\mu_1(\nu)$, $\mu_2(\nu)$, $\eta_2(\nu)$ and $\gamma(\nu)$ represent the conditional probability of completion rates for the retrial period, high priority and low priority customer's service period, restoration period, and emergency vacation period.

$$\beta(\nu) = \frac{dM(\nu)}{1-M(\nu)}; \quad \mu_i(\nu) = \frac{dB_i(\nu)}{1-B_i(\nu)}, i = 1, 2 \quad \eta_2(\nu) = \frac{dR^{(2)}(\nu)}{1-R^{(2)}(\nu)}$$

$\gamma(\nu) = \dfrac{dE(\nu)}{1-E(\nu)}$; are the hazard rate functions of $M(.)$, $Bi(.)$, $R^{(2)}(.)$ and $E(.)$ respectively.

The probability $I_{0,n}(\nu, t) = PrN_1(t) = 0, N_2(t) = 0, Y(t) = 0$ and probability densities are as follows:

$$\begin{aligned}
I_{0,n}(\nu, t)d\nu &= \Pr\{N_1(t) = 0, N_2(t) = n, Y(t) = 1; \nu \le I^0(t) \le \nu + d\nu\}, n \ge 1\\
P^{(1)}_{m,n}(\nu, t)d\nu &= \Pr\{N_1(t) = m, N_2(t) = n, Y(t) = 2; \nu \le B_1^0(t) \le \nu + d\nu\},\\
P^{(2)}_{m,n}(\nu, t)d\nu &= \Pr\{N_1(t) = m, N_2(t) = n, Y(t) = 3; \nu \le B_2^0(t) \le \nu + d\nu\},\\
R^{(2)}_{m,n}(\nu, t)d\nu &= \Pr\{N_1(t) = m, N_2(t) = n, Y(t) = 4; \nu \le R^0(t) \le \nu + d\nu\},\\
E_{m,n}(\nu, t)d\nu &= \Pr\{N_1(t) = m, N_2(t) = n, Y(t) = 5; \nu \le E^0(t) \le \nu + d\nu\}
\end{aligned}$$

for $\nu \ge 0$, $t \ge 0$, $m \ge 0$ and $n \ge 0$.

4. Equation Governing the System

$$\begin{aligned}\frac{\partial}{\partial t}P^{(1)}_{m,n}(\nu,t)+\frac{\partial}{\partial \nu}P^{(1)}_{m,n}(\nu,t)=&-(\lambda_p+a\lambda_o+\alpha_1+\alpha_2+\mu_1(\nu)+\theta)P^{(1)}_{m,n}(\nu,t)\\&+\lambda_p(1-\delta_{0m})\sum_{i=1}^{m}c_iP^{(1)}_{m-i,n}(\nu,t)\\&+a\lambda_o(1-\delta_{0n})\sum_{j=1}^{n}c_jP^{(1)}_{m,n-j}(\nu,t)\quad \text{for } m,n\geq 1.\end{aligned}\tag{1}$$

$$\begin{aligned}\frac{\partial}{\partial t}P^{(2)}_{m,n}(\nu,t)+\frac{\partial}{\partial \nu}P^{(2)}_{m,n}(\nu,t)=&-(\lambda_p+a\lambda_o+\alpha_1+\alpha_2+\mu_1(\nu)+\theta)P^{(2)}_{m,n}(\nu,t)\\&+\lambda_p(1-\delta_{0m})\sum_{i=1}^{m}c_iP^{(2)}_{m-i,n}(\nu,t)\\&+a\lambda_o(1-\delta_{0n})\sum_{j=1}^{n}c_jP^{(2)}_{m,n-j}(\nu,t)\quad \text{for } m,n\geq 1.\end{aligned}\tag{2}$$

$$\frac{\partial}{\partial t}I_{0,n}(\nu,t)+\frac{\partial}{\partial \nu}I_{0,n}(\nu,t)=-(\lambda_p+\lambda_o+\beta(\nu))I_{0,n}(\nu,t)\quad \text{for } n\geq 1.\tag{3}$$

$$\begin{aligned}\frac{\partial}{\partial t}E_{m,n}(\nu,t)+\frac{\partial}{\partial \nu}E_{m,n}(\nu,t)=&-(\lambda_p+b\lambda_o+\gamma(\nu))E_{m,n}(\nu,t)\\&+\lambda_p(1-\delta_{0m})\sum_{i=1}^{m}c_iE_{m-i,n}(\nu,t)\\&+b\lambda_o(1-\delta_{0n})\sum_{j=1}^{n}c_jE_{m,n-i}(\nu,t)\quad \text{for } m,n\geq 1.\end{aligned}\tag{4}$$

$$\begin{aligned}\frac{d}{dt}R^{(1)}_{m,n}(\nu,t)+\frac{d}{d\nu}R^{(1)}_{m,n}(\nu,t)=&-(\lambda_p+b\lambda_o+\eta_1)R^{(1)}_{m,n}(t)+\lambda_p(1-\delta_{0m})\sum_{i=1}^{m}c_iR^{(1)}_{m-i,n}(t)\\&+\alpha_1\int_0^\infty(P^{(1)}_{m,n}(\nu,t)+P^{(2)}_{m,n}(\nu,t))d\nu\\&+b\lambda_o(1-\delta_{0n})\sum_{j=1}^{n}c_jR^{(1)}_{m,n-i}(t)\quad \text{for } m,n\geq 1.\end{aligned}\tag{5}$$

$$\begin{aligned}\frac{\partial}{\partial t}R^{(2)}_{m,n}(\nu,t)+\frac{\partial}{\partial \nu}R^{(2)}_{m,n}(\nu,t)=&-(\lambda_p+b\lambda_o+\eta_2(\nu))R^{(2)}_{m,n}(\nu,t)\\&+\lambda_p(1-\delta_{0m})\sum_{i=1}^{m}c_iR^{(2)}_{m-i,n}(\nu,t)\\&+b\lambda_o(1-\delta_{0n})\sum_{j=1}^{n}c_jR^{(2)}_{m,n-i}(\nu,t)\quad \text{for } m,n\geq 1.\end{aligned}\tag{6}$$

$$\begin{aligned}\frac{d}{dt}I_{0,0}(t)=&-(\lambda_p+\lambda_o)I_{0,0}(t)+\int_0^\infty P^{(1)}_{0,0}(\nu,t)\mu_1(\nu)d\nu+R^{(1)}_{0,0}(t)\eta_1\\&+\int_0^\infty P^{(2)}_{0,0}(\nu,t)\mu_2(\nu)d\nu+\int_0^\infty R^{(2)}_{0,0}(\nu,t)\eta_2(\nu)d\nu\\&+\int_0^\infty E_{0,0}(\nu,t)\gamma(\nu)d\nu.\end{aligned}\tag{7}$$

Define, the boundary conditions at $\nu = 0$

$$I_{0,n}(0,t) = \int_0^\infty E_{0,n}(\nu,t)\gamma(\nu)d\nu + \int_0^\infty P^{(1)}_{0,n}(\nu,t)\mu_1(\nu)d\nu + R^{(1)}_{0,n}(t)\eta_1 + \int_0^\infty P^{(2)}_{0,n}(\nu,t)\mu_2(\nu)d\nu + \int_0^\infty R^{(2)}_{0,n}(\nu,t)\eta_2(\nu)d\nu, \quad \text{for } n \geq 0. \tag{8}$$

$$P^{(1)}_{m,n}(0,t) = \int_0^\infty P^{(1)}_{m+1,n}(\nu,t)\mu_1(\nu)d\nu + R^{(1)}_{m+1,n}(t)\eta_1 + \int_0^\infty P^{(2)}_{m+1,n}(\nu,t)\mu_2(\nu)d\nu + \int_0^\infty E_{m+1,n}(\nu,t)\gamma(\nu)d\nu + \int_0^\infty R^{(2)}_{m+1,n}(\nu,t)\eta(\nu)d\nu + \lambda_p c_{m+1} I_{0,n}(t), \tag{9}$$

$$P^{(2)}_{0,0}(0,t) = \lambda_o c_1 I_{0,0}(t) + \int_0^\infty I_{0,1}(\nu,t)\beta(\nu)d\nu \tag{10}$$

$$P^{(2)}_{0,n}(0,t) = \lambda_o c_{n+1} I_{0,0}(t) + \int_0^\infty I_{0,n+1}(\nu,t)\beta(\nu)d\nu + \sum_{i=1}^{n} \lambda_o C_i(\nu,t) + \int_0^\infty I_{0,n+1-i}(\nu,t)d\nu \quad \text{for } n \geq 1. \tag{11}$$

$$P^{(2)}_{0,n}(0,t) = \lambda_o c_{n+1} I_{0,0}(t) + \int_0^\infty I_{0,n+1}(\nu,t)\beta(\nu)d\nu + \sum_{i=1}^{n} \lambda_o C_i(\nu,t) + \int_0^\infty I_{0,n+1-i}(\nu,t)d\nu \quad \text{for } n \geq 1. \tag{12}$$

$$R^{(2)}_{m,n}(0,t) = \alpha_2 \int_0^\infty P^{(1)}_{m,n}(\nu,t)\mu_1(\nu)d\nu + \alpha_2 \int_0^\infty P^{(2)}_{m,n}(\nu,t)d\nu \quad \text{for } m,n \geq 0. \tag{13}$$

$$P^{(1)}_{m,n}(0) = P^{(2)}_{m,n}(0) = E_{m,n}(0) = R^{(1)}_{m,n}(0) = R^{(2)}_{m,n}(0) = 0, \ \text{for } m,n \geq 0 \text{ and } I_{0,0} = 1, \ I_{0,n}(0) = 0, \quad \text{for } n \geq 1 \text{ are the initial conditions.} \tag{14}$$

Now, we define the Probability Generating Function (PGF),

$$I(\nu,t,z_o) = \sum_{n=1}^{\infty} z_o^n I_{0,n}(\nu,t); \quad A(\nu,t,z_p,z_o) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} z_o^m z_p^n A_{m,n}(\nu,t); \quad A(\nu,t,z_p) = \sum_{m=0}^{\infty} z_p^m A_m(\nu,t); \quad A(\nu,t,z_o) = \sum_{n=0}^{\infty} z_o^n A_n(\nu,t); \tag{15}$$

here $A = P^{(1)},\ P^{(2)},\ E,\ R^{(1)},\ R^{(2)}$.
By applying Laplace transforms to equations (1) to (13) and by using (14) and (15) , we obtain the following equations:

$$\overline{I}_0(\nu,s,z_o) = \overline{I}_0(0,s,z_o)e^{-(s+\lambda_p+\lambda_o)\nu - \int_0^\nu \beta(t)dt}, \tag{16}$$

$$\overline{P}^{(1)}(\nu,s,z_p,z_o) = \overline{P^{(1)}}(0,s,z_p,z_o)e^{-\phi_1(s,z)\nu - \int_0^\nu \mu_1(t)dt}, \tag{17}$$

$$\overline{P}^{(2)}(\nu,s,z_p,z_o) = \overline{P^{(2)}}(0,s,z_p,z_o)e^{-\phi_1(s,z)\nu - \int_0^\nu \mu_2(t)dt}, \tag{18}$$

$$\overline{E}(\nu,s,z_p,z_o) = \overline{E}(0,s,z_p,z_o)e^{-\phi_2(s,z)\nu - \int_0^\nu \gamma(t)dt}, \tag{19}$$

$$\overline{R}^{(2)}(\nu,s,z_p,z_o) = \overline{R}^{(2)}(0,s,z_p,z_o)e^{-\phi_2(s,z)\nu - \int_0^\nu \eta_2(t)dt}. \tag{20}$$

where,

$$\phi_1(s,z) = s + \lambda_p(1 - C(z_p)) + a\lambda_o(1 - C(z_o)) + \alpha_1 + \alpha_2 + \theta, \tag{21}$$

$$\phi_2(s,z) = s + \lambda_p(1 - C(z_p)) + b\lambda_o(1 - C(z_o)), \tag{22}$$

$$\phi_3(s,z) = s + \lambda_p(1 - C(z_p)) + b\lambda_o(1 - C(z_o)) + \eta_1, \tag{23}$$

$$\overline{P}^{(2)}(0,s,z_o) = \frac{\left\{\begin{array}{l}\lambda_o C(z_o)\overline{I}_{0,0}(s)[1-\lambda_p C(g(z_o))\\ \Big[\frac{1-\overline{M}(s+\lambda_p+\lambda_o)}{s+\lambda_p+\lambda_o}\Big]] - [\overline{I}_{0,0}(s)(s+\lambda_p+\lambda_o)-1]\\ [\overline{M}(s+\lambda_p+\lambda_o)+C(z_o)\lambda_o\Big[\frac{1-\overline{M}(s+\lambda_p+\lambda_o)}{s+\lambda_p+\lambda_o}\Big]]\end{array}\right\}}{\left\{\begin{array}{l}z_2[1-\lambda_p C(g(z_o))\Big[\frac{1-\overline{M}(s+\lambda_p+\lambda_o)}{s+\lambda_p+\lambda_o}\Big]]\\ -[\overline{M}(s+\lambda_p+\lambda_o)+C(z_o)\lambda_o\Big[\frac{1-\overline{M}(s+\lambda_p+\lambda_o)}{s+\lambda_p+\lambda_o}\Big]]\\ [\overline{B}_2(\sigma_1(z,s))+\theta z_o\overline{E}(\sigma_2(z,s))\Big[\frac{1-\overline{B}_2(\sigma_1(z,s))}{\sigma_1(z,s)}\Big]+\\ \Big[\frac{1-\overline{B}_2(\sigma_1(z,s))}{\sigma_1(z,s)}\Big]\Big[\frac{\alpha_1}{\sigma_3(z,s)}+\alpha_2\overline{R}^{(2)}(\sigma_2(z,s))\Big]]\end{array}\right\}}, \tag{24}$$

$$\overline{I}(0,s,z_o) = \frac{\left\{\begin{array}{l}\lambda_o C(z_o)\overline{I}_{0,0}(s)[\overline{B}_2(\sigma_1(z,s)+\theta z_o\overline{E}(\sigma_2(z,s))\\ \Big[\frac{1-\overline{B}_2(\sigma_1(z,s))}{\sigma_1(z,s)}\Big]+\Big[\frac{1-\overline{B}_2(\sigma_1(z,s))}{\sigma_1(z,s)}\Big]\\ \Big[\frac{\alpha_1}{\sigma_3(z,s)}+\alpha_2\overline{R}^{(2)}(\sigma_2(z,s))\Big]]\\ -[\overline{I}_{0,0}(s)(s+\lambda_p+\lambda_o)-1]z_o\end{array}\right\}}{\left\{\begin{array}{l}z_2[1-\lambda_p C(g(z_o))\Big[\frac{1-\overline{M}(s+\lambda_p+\lambda_o)}{s+\lambda_p+\lambda_o}\Big]]\\ -[\overline{M}(s+\lambda_p+\lambda_o)+C(z_o)\lambda_o\Big[\frac{1-\overline{M}(s+\lambda_p+\lambda_o)}{s+\lambda_p+\lambda_o}\Big]]\\ [\overline{B}_2(\sigma_1(z,s))+\theta z_o\overline{E}(\sigma_2(z,s))\Big[\frac{1-\overline{B}_2(\sigma_1(z,s))}{\sigma_1(z,s)}\Big]+\\ \Big[\frac{1-\overline{B}_2(\sigma_1(z,s))}{\sigma_1(z,s)}\Big]\Big[\frac{\alpha_1}{\sigma_3(z,s)}+\alpha_2\overline{R}^{(2)}(\sigma_2(z,s))\Big]]\end{array}\right\}}, \tag{25}$$

$$\overline{P}^{(1)}(0,s,z_p,z_o) = \frac{\left\{\begin{array}{l}\lambda_p[C(z_p)-C(g(z_o))]\Big[\frac{1-\overline{M}(s+\lambda_p+\lambda_o)}{s+\lambda_p+\lambda_o}\Big]\overline{I}_0(0,s,z_o)\\ +[\overline{B_2}(\phi_1(z,s))-\overline{B_2}(\sigma_1(z,s))+\theta z_o\overline{E}(\phi_2(z,s))\\ \Big[\frac{1-\overline{B}_2(\phi_1(z,s))}{\phi_1(z,s)}\Big]-\theta z_o\overline{E}(\sigma_2(z,s))\Big[\frac{1-\overline{B}_2(\sigma_1(z,s))}{\sigma_1(z,s)}\Big]\\ +\Big[\frac{\alpha_1}{\phi_3(z,s)}+\alpha_2\overline{R}^{(2)}(\phi_2(z,s))\Big]\Big[\frac{1-\overline{B_2}(\phi_1(z,s))}{\phi_1(z,s)}\Big]\\ -\Big[\frac{1-\overline{B}_2(\sigma_1(z,s))}{\sigma_1(z,s)}\Big]\Big[\frac{\alpha_1}{\sigma_3(z,s)}+\alpha_2\overline{R}^{(2)}(\sigma_2(z,s))\Big]]\end{array}\right\}}{\left\{\begin{array}{l}z_p-[\overline{B_1}(\phi_1(z,s))+\theta z_o\overline{E}(\phi_2(z,s))\Big[\frac{1-\overline{B}_1(\phi_1(z,s))}{\phi_1(z,s)}\Big]\\ +\Big[\frac{\alpha_1}{\phi_3(z,s)}+\alpha_2\overline{R}^{(2)}(\phi_2(z,s))\Big]\Big[\frac{1-\overline{B_1}(\phi_1(z,s))}{\phi_1(z,s)}\Big]]\end{array}\right\}}, \tag{26}$$

$$\begin{aligned}\overline{E}(0,s,z_p,z_o) = \theta z_p\overline{P}^{(1)}(0,s,z_p,z_o)\Big[\frac{1-\overline{B_1}(\phi_1(z,s))}{\phi_1(z,s)}\Big]\\ +\theta z_o\overline{P}^{(2)}(0,s,z_o)\Big[\frac{1-\overline{B_2}(\phi_1(z,s))}{\phi_1(z,s)}\Big],\end{aligned} \tag{27}$$

$$\begin{aligned}\overline{R}^{(2)}(0,s,z_p,z_o) = \alpha_2\overline{P}^{(1)}(0,s,z_1,z_2)\Big[\frac{1-\overline{B_1}(\phi_1(z,s))}{\phi_1(z,s)}\Big] \\ + \alpha_2\overline{P}^{(2)}(0,s,z_2)\Big[\frac{1-\overline{B_2}(\phi_1(z,s))}{\phi_1(z,s)}\Big],\end{aligned} \tag{28}$$

$$\begin{aligned}\sigma_1(s,z) &= s+\lambda_p(1-C(g(z_o)))+a\lambda_o(1-C(z_o))+\alpha_1+\alpha_2+\theta,\\ \sigma_2(s,z) &= s+\lambda_p(1-C(g(z_o)))+b\lambda_o(1-C(z_o)),\\ \sigma_3(s,z) &= s+\lambda_p(1-C(g(z_o)))+b\lambda_o(1-C(z_o))+\eta_1.\end{aligned}$$

Theorem.1 When the system is in regular service, breakdown, emergency vacation and repair by using the Laplace transforms the probability generating function of the number of customers in the respective queue is given by.

$$\overline{I}_0(s,z_o) = \overline{I}_0(0,s,z_o)\Big[\frac{1-\overline{M}(s+\lambda_p+\lambda_o)}{s+\lambda_p+\lambda_o}\Big], \tag{29}$$

$$\overline{P}^{(1)}(s,z_p,z_o) = \overline{P}^{(1)}(0,s,z_p,z_o)\Big[\frac{1-\overline{B}_1(\phi_1(s,z))}{\phi_1(s,z)}\Big], \tag{30}$$

$$\overline{P}^{(2)}(s,z_p,z_o) = \overline{P}^{(2)}(0,s,z_o)\Big[\frac{1-\overline{B}_2(\phi_1(s,z))}{\phi_1(s,z)}\Big], \tag{31}$$

$$\overline{E}(s,z_p,z_o) = \overline{E}(0,s,z_p,z_o)\Big[\frac{1-\overline{E}(\phi_2(s,z))}{\phi_2(s,z)}\Big], \tag{32}$$

$$\overline{R}^{(2)}(s,z_p,z_o) = \overline{R}^{(2)}(0,s,z_p,z_o)\Big[\frac{1-\overline{R}^{(2)}(\phi_2(s,z))}{\phi_2(s,z)}\Big]. \tag{33}$$

Proof: Integrating the preceding equations (29) to (33) with respect to ν and applying the solution of renewal theory we obtain the following

$$\int_0^\infty \big[1-H(\nu)\big]e^{-s\nu}d\nu = \frac{1-\overline{h(s)}}{s}. \tag{34}$$

Here, the LST of the distribution function of a random variable H(ν) is denoted as $\overline{h(s)}$. The overall results of the probability generating functions for the following states, $\overline{I}_0(s,z_o)$, $\overline{P}^{(1)}(s,z_p,z_o)$, $\overline{P}^{(2)}(s,z_p,z_o)$, $\overline{E}(s,z_p,z_o)$, and $\overline{R}^{(2)}(s,z_p,z_o)$ are obtained by using equation (29) to (33).

5. Steady State Analysis

According to Tauberian property,

$$\lim_{s\to 0} s\overline{f}(s) = \lim_{t\to\infty} f(t).$$

Despite of the state of the system, the probability generating function of the queue size is as follows:

$$W_q(z_1,z_2) = \frac{Nr(z_1,z_2)}{Dr(z_1,z_2)}, \tag{35}$$

where

$$Nr(z_p, z_o) = N_3(z)D_1(z)\phi_1(z)\phi_2(z)\phi_3(z)\Big[\frac{1-\overline{M}(s+\lambda_p+\lambda_o)}{s+\lambda_p+\lambda_o}\Big] + N_1(z)D_2(z)(1-\overline{B_1}(\phi_1(z,s)))F_1(z) + N_2(z)D_1(z)(1-\overline{B_2}(\phi_1(z,s)))F_2(z)$$

$$Dr(z_p, z_o) = D_1(z)D_2(z)\phi_1(z)\phi_2(z)\phi_3(z),$$

$$N_1(z) = \lambda_p[C(z_p) - C(g(z_o))]\Big[\frac{1-\overline{M}(\lambda_p+\lambda_o)}{\lambda_p+\lambda_o}\Big]\overline{I}_0(0,z_o) + [\overline{B_2}(\phi_1(z)) - \overline{B_2}(\sigma_1(z)) + \theta z_o\overline{E}(\phi_2(z)) + \Big[\frac{\alpha_1}{\phi_3(z)}\Big[\frac{1-\overline{B}_2(\phi_1(z))}{\phi_1(z)}\Big] - \theta z_o\overline{E}(\sigma_2(z))\Big[\frac{1-\overline{B}_2(\sigma_1(z))}{\sigma_1(z)}\Big] + \alpha_2\overline{R}^{(2)}(\phi_2(z))\Big]\Big[\frac{1-\overline{B_2}(\phi_1(z))}{\phi_1(z)}\Big] - \Big[\frac{1-\overline{B}_2(\sigma_1(z))}{\sigma_1(z)}\Big]\Big[\frac{\alpha_1\eta_1}{\sigma_3(z)} + \alpha_2\overline{R}^{(2)}(\sigma_2(z))\Big]],$$

$$N_2(z) = -[\lambda_p(1-C(g(z_o))) + b\lambda_o(1-C(z_o))][1-\lambda_pC(g(z_o))\Big[\frac{1-\overline{M}(\lambda_p+\lambda_o)}{\lambda_p+\lambda_o}\Big]] [\overline{M}(\lambda_p+\lambda_o) + C(z_o)\lambda_o\Big[\frac{1-\overline{M}(\lambda_p+\lambda_o)}{s+\lambda_p+\lambda_o}\Big],$$

$$N_3(z) = -[\lambda_p(1-C(g(z_o))) + b\lambda_o(1-C(z_o))][\overline{B}_2(\sigma_1(z)) + \theta z_o\overline{E}(\sigma_2(z)) \Big[\frac{1-\overline{B}_2(\sigma_1(z))}{\sigma_1(z)}\Big] + \Big[\frac{1-\overline{B}_2(\sigma_1(z))}{\sigma_1(z)}\Big]\Big[\frac{\alpha_1\eta_1}{\sigma_3(z)} + \alpha_2\overline{R}^{(2)}(\sigma_2(z))\Big]],$$

$$D_1(z) = (z_p - [\overline{B_1}(\phi_1(z)) + \theta z_o\overline{E}(\phi_2(z))\Big[\frac{1-\overline{B}_1(\phi_1(z))}{\phi_1(z)}\Big] + \Big[\frac{\alpha_1\eta_1}{\phi_3(z)} + \alpha_2\overline{R}^{(2)}(\phi_2(z))\Big]\Big[\frac{1-\overline{B_1}(\phi_1(z))}{\phi_1(z)}\Big]],$$

$$D_2(z) = z_o[1-\lambda_pC(g(z_o))\Big[\frac{1-\overline{M}(\lambda_p+\lambda_o)}{s+\lambda_p+\lambda_o}\Big]] - [\overline{M}(\lambda_p+\lambda_o) + C(z_o)\lambda_o\Big[\frac{1-\overline{M}(\lambda_p+\lambda_o)}{\lambda_p+\lambda_o}\Big]][\overline{B}_2(\sigma_1(z)) + \theta z_o\overline{E}(\sigma_2(z)) \Big[\frac{1-\overline{B}_2(\sigma_1(z))}{\sigma_1(z)}\Big] + \Big[\frac{1-\overline{B}_2(\sigma_1(z))}{\sigma_1(z)}\Big]\Big[\frac{\alpha_1\eta_1}{\sigma_3(z)} + \alpha_2\overline{R}^{(2)}(\sigma_2(z))\Big]].$$

6. Stability Condition

We apply the normalising condition to determine $I_{0,0}$.

$$I_{0,0} + I_0(1) + P^{(1)}(1,1) + P^{(2)}(1,1) + E(1,1) + R^{(1)}(1,1) + R^{(2)}(1,1) = 1 \tag{36}$$

$$I_{0,0} = \frac{D_2'\phi_1\phi_2'\phi_3 - N_2'F'(1-\overline{B}_2(\phi_1(z)))}{D_2'\phi_1\phi_2'\phi_3} \tag{37}$$

and the utilization factor is given by

$$\rho = \frac{N_2'F'(1-\overline{B}_2(\phi_1(z)))}{D_2'\phi_1\phi_2'\phi_3} \tag{38}$$

The stability condition for the model under which steady state exists is $\rho < 1$

7. Performance Measures

The expected queue size for priority customer is as follows:

$$L_{q_1} = \frac{d}{dz_p} W_q(z_p, 1)|_{z_p=1} \tag{39}$$

The expected orbit size for ordinary customer is as follows:

$$L_{q_2} = \frac{d}{dz_o} W_q(1, z_o)|_{z_o=1} \tag{40}$$

then

$$L_{q_1} = \frac{Dr''(1)Nr^{(''')}(1) - Dr^{''')}(1)Nr''(1)}{3(Dr''(1))^2}, \tag{41}$$

$$L_{q_2} = \frac{Dr'''(1)Nr^{(iv)}(1) - Dr^{(iv)}(1)Nr'''(1)}{4(Dr'''(1))^2}. \tag{42}$$

The expected waiting time for priority queue is as follows:

$$W_{q_1} = \frac{L_{q_1}}{\lambda_p} \tag{43}$$

The expected waiting time for orbit is as follows:

$$W_{q_2} = \frac{L_{q_2}}{\lambda_o}. \tag{44}$$

8. Particular Cases

Case 1:
Without a priority queue, all arriving customers are accepted into the system; customers do not balk to orbit, take vacations and no failures, the above model becomes

$$I_0(z) = \frac{I_{0,0}[C(z_o)\overline{B}(\phi(z)) - z_o][1 - \overline{M}(\lambda_o)]}{\overline{B}(\phi(z))[C(z_o) + \overline{M}(\lambda_o)(1 - C(z_o))] - z_o},$$

$$P^{(2)}(z) = \frac{I_{0,0}(1 - \overline{B}(\phi(z)))\overline{M}(\lambda_o)}{\overline{B}(\phi(z))[(1 - C(z_o))\overline{M}(\lambda_o) - z_o]}.$$

This result is associated with Gomez-Corral [15].

Case 2:
In the absence of priority queue, when there is no breakdown, no retrial, no balking, no repair and no vacation then the above model becomes

$$P^{(2)}(z) = \frac{I_{0,0}(1 - \overline{B}_2(\phi(z)))}{\overline{B}_2(\phi(z)) - z_o}$$

This result is associated with Medhi [21].

9. Numerical Results

The numerical and graphical analyses of this model are covered in this section. We assumed that the distribution of service time, breakdown, repair, and vacation time are all exponential.

Table 1: *Effect of priority arrival rate (λ_p)*

λ_p	$I_{0,0}$	ρ	L_{q_1}	W_{q_1}	L_{q_2}	W_{q_2}
0.5	0.9971	0.0029	1.3928	2.7856	0.2013	0.1006
0.6	0.9963	0.0037	1.8271	3.0452	0.2497	0.1248
0.7	0.9955	0.0045	2.4404	3.4863	0.3016	0.1508
0.8	0.9945	0.0055	3.3321	4.1651	0.3574	0.1787
0.9	0.9934	0.0066	4.6809	5.2010	0.4174	0.2087
1.0	0.9922	0.0078	6.8391	6.8391	0.4819	0.2410

Table 1 demonstrates that the probability of the server being idle reduces as the arrival rate (λ_p) of priority customers for the priority queue rises. However, average queue lengths, busy period and customers average waiting times all rises: we assume the values as $\lambda_o = 2$, $\alpha_1 = 0.3$, $\alpha_2 = 0.5$ $\mu = 5$, $\eta_1 = 3$,$\eta_2 = 5$,$\theta = 0.5$, $\beta = 15$,$\gamma = 10$, $a = 0.5$,$b = 0.6$ and $\lambda_p = 0.5$ to 1.0.

Table 2: *Effect of service rate (μ)*

μ	$I_{0,0}$	ρ	L_{q_1}	W_{q_1}	L_{q_2}	W_{q_2}
2	0.6611	0.3389	3.7025	1.9418	7.0960	4.3513
3	0.9627	0.0373	2.2242	1.8339	6.3020	3.1121
4	0.9906	0.0094	1.7731	1.5863	5.7105	2.3861
5	0.9966	0.0034	0.3434	1.3856	5.2686	1.1717
6	0.9985	0.0015	0.1792	1.2542	4.9155	0.0896

Table 2 indicates that when the service rate (μ) increases, the probability of the server being busy reduces. However, average queue lengths, idle time, and customers' average waiting times all reduces: we assume the values as $\lambda_1 = 0.5$, $\lambda_2 = 2$, $\alpha_1 = 0.3$, $\alpha_2 = 0.5$, $\eta_1 = 1$, $\eta_2 = 6$ $\beta = 10$, $\theta = 0.5$,$\gamma = 10$, $\beta = 15$,$a = 0.5$,$b = 0.6$and $\mu = 2$ to 6.

Table 3: *Effect of retrial rate (β)*

β	$I_{0,0}$	ρ	L_{q_1}	W_{q_1}	L_{q_2}	W_{q_2}
10	0.9725	0.0275	2.6038	5.2076	0.7172	0.3586
11	0.9724	0.0276	2.3409	4.6817	0.6998	0.3499
12	0.9722	0.0278	2.1251	4.2503	0.6856	0.3428
13	0.9721	0.0279	1.9449	3.8899	0.6737	0.3369
14	0.9720	0.0280	1.7922	3.5843	0.6636	0.3318
15	0.9719	0.0281	1.6610	3.3220	0.6549	0.3275

Table 3 indicates the probability of the server being busy period rises as the retrial rate rises. However, average queue lengths, idle time, and customers' average waiting times all reduces: we assume the values as $\lambda_p = 0.5$,$\lambda_o = 2$, $\alpha_1 = 0.3$,$\alpha_2 = 0.6$, $\eta_1 = 3$,$\mu = 3$,$\theta = 0.5$,$\eta_2 = 5$,$a = 0.5$,$b = 0.6$ and $\beta = 10 to 15$.

We assumed to be follow the Erlang-2 distribution for service time, breakdown, repair, and vacation time in graphical representations. The two-dimensional graphs are shown in Figure 2 - 4. Figure 2 exhibits that the expected length of the queue (L_{q_1}, L_{q_2}) rises, the expected length of the queue extends together with the priority arrival rate (λ_p). The behaviour of the queue sizes (L_{q_1}, L_{q_2}), which depends on the service rate (μ), is shown in Figure 3, the length of the queue as the service rate decreases. Figure 4 shows the expected queue length (L_{q_1}, L_{q_2}), which depends on the retrial rate (β), the length of the queue as the service rate decreases.

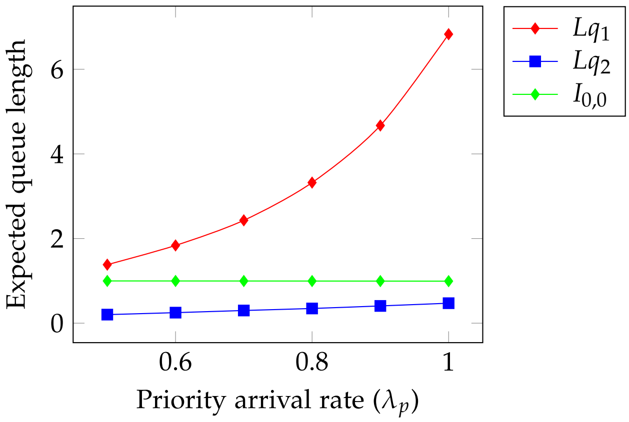

Figure 2: *Expected queue length vs priority arrival rate* λ_p

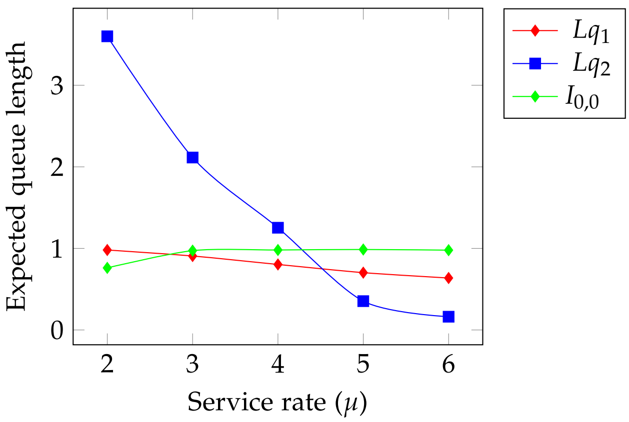

Figure 3: *Expected queue length vs service rate* μ

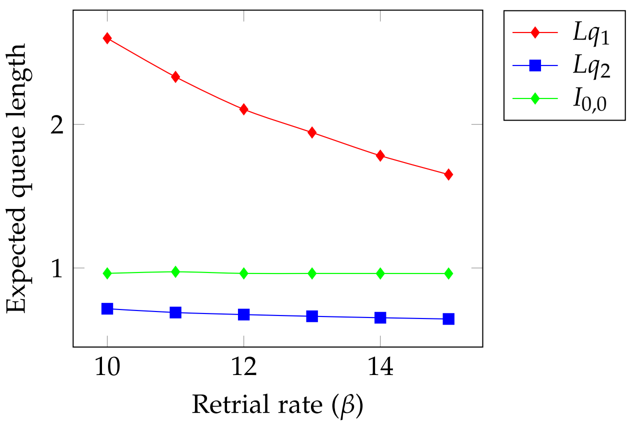

Figure 4: *Expected queue length vs Retrial rate* β

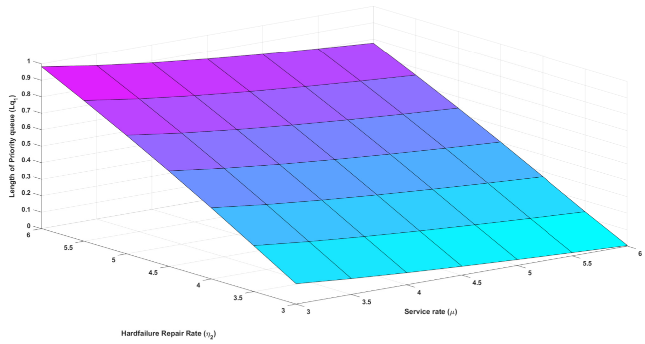

Figure 5: *Lq_1 Vs μ and η_2*

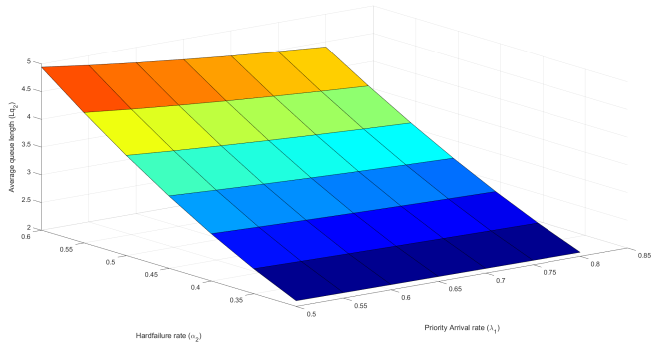

Figure 6: *Lq_2 Vs λ_p and α_2*

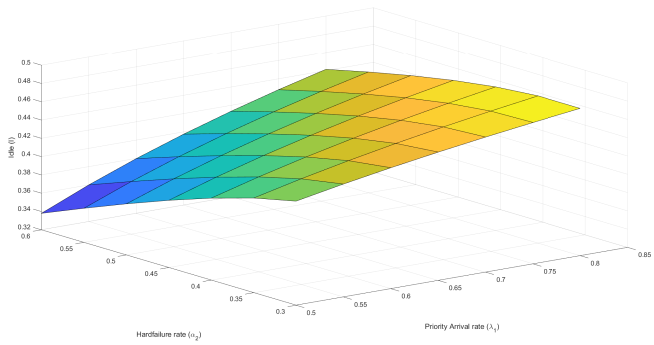

Figure 7: *Idle Vs λ_p and α_2*

Graphs in three dimensions can be found in Figures 5 - 7. Figure 5 in the reference indicate that the service rate (μ) and hard failure repair rate (α_2) increase, the expected queue size (Lq_2) decease. Figure 6 in the reference indicate that as the priority arrival rate (λ_1) and breakdown(hard failure) rate (α_2) increase, the expected queue size (Lq_2) rises. Figure 6 in the reference indicate that as the priority arrival rate (λ_p) and breakdown (hard failure) rate (α_2) increase, the idle is $(I_0, 0)$ rises.

10. Conclusion

In this inquiry, we investigated a single server retrial queueing system with admission control, balking, non-preemptive priority service, and emergency vacation where the server is susceptible to various breakdown and restoration periods. The analytical findings that are supported by numerical examples can be applied to design outputs in a variety of real-world scenarios. The supplementary variable technique is used to determine the PGFs for the number of customers in the system when it is free, busy and restoration period.The average queue length of the orbit and system contains explicit expressions. The mean busy period and other significant system performance measures are obtained. Finally, it is demonstrated that this retrial queueing method works well with the conditional decomposition law. Our technique is more adaptable in dealing with real-time systems of numerous sectors in many real-life queueing scenarios. This work can be expanded in various directions by considering the concept of:

- Multi server batch arrival priority queueing model with production inventory system.
- Batch arrival bulk service double orbit retrial queueing system with priority service.

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