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DEVELOPING A NEW HUNTSBERGER TYPE SHRINKAGE ESTIMATOR FOR THE ENTROPY OF EXPONENTIAL DISTRIBUTION UNDER DIFFERENT LOSS FUNCTIONS

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Abstract

The aim of the paper is to develop a new Huntsberger type shrinkage estimator for the entropy function of the exponential distribution. The present paper proposes a Huntsberger type shrinkage estimator for the entropy function of the exponential distribution. This Huntsberger type shrinkage entropy estimator is based on test statistic, which eliminates arbitrariness of choice of shrinkage factor. For the developed estimator risk expressions under LINEX loss function and squared error loss function have been calculated. To assess the efficacy of the proposed estimator, numerical computations are performed, and graphical analysis is carried out for risk and relative risks for the proposed estimator. It is also compared with the existing best estimator for distinct degrees of asymmetry and different levels of significance. Based on the criteria of relative risk, it is found that the proposed Huntsberger type shrinkage estimator is better than the existing estimator for the entropy function of the exponential distribution for smaller values of level of significance and degrees of freedom.

Keywords: Exponential distribution, entropy function, shrinkage estimation, progressive censoring type sample, LINEX loss function, squared error loss function.

I. Introduction

The Exponential distribution is widely used models in reliability and life-testing research. It has been extensively examined by researchers in terms of inferential issues and its application in these fields. Many academics have studied how to estimate exponential distribution's parameters using both classical and Bayesian techniques. For example Bain [5], Chandrasekar et al. [8], Jaheen [12] and Ahmadi et al. [3], along with other references, have contributed to this body of knowledge.

If f and F be the probability density function and the distribution function of the random variable X respectively, then by Shannon [18], entropy function is given as

$$H(f) = E[-\log(f(X))] \tag{1}$$

For sharply peaked distribution entropy is very low and is much higher when the probability is spread out. Many authors worked on the estimation entropy for different life distributions. Noteworthy work in this direction may be refereed from Lazo and Rathee [15], Misra et al. [16], Jeevanand and Abdul- Sathar [13] and Kayal and Kumar [14] etc.

Suppose the random variable X has the probability distribution $f(x,\theta)$ where interest is to estimate entropy function as a function of θ. According to Thomson [20], shrinkage estimation can be accomplished by altering the usual estimator of the unknown parameter θ by bringing it closer to θ_0. Researchers have addressed several shrinkage estimators for various parameters or parametric functions under various sorts of distributions in statistical literature. Huntsberger [11] introduced weighted shrinkage estimator of the form

$$\tilde{\theta}_\phi = \phi(\hat{\theta})\hat{\theta} + (1-\phi(\hat{\theta}))\theta_0 ,$$

where $\phi(.)$, $0 \le \phi(.) \le 1$, represents a weighted function specifying the degree of belief in θ_0.

In this paper, we shall concentrate on obtaining a new Huntsberger type shrinkage estimation of entropy function under symmetric/asymmetric loss functions for progressive type-II censored sample, when the underlying distribution is assumed to follow an exponential distribution.
The form of density we consider is

$$f(x,\theta) = \frac{1}{\theta} e^{-\frac{x}{\theta}}, \qquad x \ge 0,\ \theta > 0 \tag{2}$$

Progressive censoring is indeed a useful scheme in the area of reliability and life time research. The number of authors including Cohen [9], Gibbons and Vance [10], Viveros and Balakrishan [22], Balakrishan and Aggarwala [6], Aggarwala [4], Adubisi and Adubisi [2], have contributed to the literature on inference problems related to progressive censoring for various probability distributions.

I. Shrinkage Estimators of H (f)

For exponential distribution with mean θ, the entropy function can be calculated as

$$H(f) = 1 + \ln(\theta) \tag{3}$$

Since $H(f)$ is linear function of $\ln(\theta)$, estimating $H(f)$ is correspondent to estimating $\ln(\theta)$. We shall write $I(\theta) = \ln(\theta)$ so that $H(f) = 1 + I(\theta)$. We will now talk about estimation of $I(\theta)$.

From the exponential distribution given in (2), Let $X_{1:m:n}, X_{2:m:n}, \ldots\ldots, X_{m:m:n}$ be Type II progressive censored sample. The progressive censored sample's joint density is then calculated (see balakrishan and Aggarwala [6])

$$f(x_{1:m:n}, x_{2:m:n}, \ldots\ldots, x_{m:m:n}) = C\prod_{i=1}^{m} f(x_{i:m:n})(1-F(x_{i:m:n}))^{R_i},\ 0 \le x_{1:m:n} \le x_{2:m:n} \le \ldots.. \le x_{m:m:n}, \tag{4}$$

where

$$C = n(n-R_1-1)(n-R_1-R_2-2).....(n-R_1-R_2-\ldots-R_{m-1}-m+1)$$

Now substituting f and F in (4), we get

$$f(x_{1:m:n}, x_{2:m:n}, \ldots\ldots, x_{m:m:n}) = C(\frac{1}{\theta})^m \exp\left(-\frac{\sum_{i=1}^{m}(R_i+1)x_{i:m:n}}{\theta}\right),\ 0 \le x_{1:m:n} \le x_{2:m:n} \le \ldots.. \le x_{m:m:n} \tag{5}$$

Then MLE of θ can easily be obtained as

$$\hat{\theta} = \frac{\sum_{i=1}^{m}(R_i+1)x_{i:m:n}}{m} \tag{6}$$

Since $I(\theta)$ is continous function of θ, the MLE of $I(\theta)$ is obtained by replacing θ by its MLE $\hat{\theta}$ in $I(\theta)$. The MLE of entropy function for the exponential distribution is then

$$\hat{H}(f) = 1 + \ln(\hat{\theta}) \tag{7}$$

We can demonstrate that the distribution of $\hat{\theta}$ has

$$f(\hat{\theta};\theta) = \left(\frac{m}{\theta}\right)^m \frac{\hat{\theta}^{m-1}\exp(-\frac{m\hat{\theta}}{\theta})}{\Gamma(m)},\ \hat{\theta} > 0 \tag{8}$$

A symmetric loss function treats underestimation and overestimation equally, penalizing both types of errors in the same manner. However, in certain situations, the consequences of underestimation and overestimation may not be the same. To address this issue, many authors have used and promoted the usage of 'asymmetric' loss functions, particularly when discussing claim settlements and other related topics. Prominent researchers such as Varian [19], Zellner [23], Basu and Ebrahimi [7], Adegoke et al. [1] have highlighted the convenience and superiority of using asymmetric loss functions in various scenarios.
The loss function proposed by Varian [21] is defined as:

$$L(\Delta) = b(e^{a\Delta} - a\Delta - 1) \tag{9}$$

where $\Delta = \hat{\theta} - \theta$, **b** denotes scale parameter and **a** denotes shape parameter. When overestimation is more critical than underestimation then the positive value of **a** is used and for other cases, its negative value is used.

In section 2, the shrinkage estimator is defined. The expressions for the risk(s) are provided in section 3. In Section 4, the Relative Risk(s) are determined. Finally, in section 5, the proposed estimator is compared with the best estimator available.

II. Proposed Estimator

Srivastava and Shah [19] have proposed a shrinkage estimator of scale parameter in exponential distribution. The key contribution of their estimator is the removal of arbitrariness in the choice of shrinkage factor 'k' by making it dependent on the test statistics. Sahni and Kumar [17] proposed a Huntsberger type shrinkage estimator for the entropy of the exponential distribution by taking 'k' dependent on the test statistic. There could be several other choice of 'k'. Thus taking idea of the various choices of shrinkage factors and also with the help of sample and prior guess information a new Huntsberger type shrinkage entropy estimator for mean of exponential distribution can be proposed as follows:

$$\tilde{I}_1(\theta) = \begin{cases} \left(\frac{2m\hat{\theta}}{\theta_0\chi^2}\right)^2 \ln(\hat{\theta}) + \left(1 - \left(\frac{2m\hat{\theta}}{\theta_0\chi^2}\right)^2\right)\ln(\theta_0); & \text{if } {\chi_1}^2 \le \frac{2m\hat{\theta}}{\theta_0\chi^2} \le {\chi_2}^2 \\ \ln(\hat{\theta}) & ; \quad \text{Otherwise} \end{cases} \tag{10}$$

where k depends on the test statistic and is given as $k = \left(\frac{2m\hat{\theta}}{\theta_0\chi^2}\right)^2$ and $\chi^2 = ({\chi_2}^2 - {\chi_1}^2)$.

III. Derivation of Risk(s)

1. Risk of MLE, $\hat{I}(\theta)$

Risk of the estimator $\hat{I}(\theta)$ with respect to LLF is defined as follows:

$$R_{LLF}(\hat{I}(\theta)) = E(\hat{I}(\theta)/LLF)$$

$$= \int_0^\infty \left(\exp\left(a\left(\ln(\hat{\theta}) - \ln(\theta) \right) \right) - a\left(\ln(\hat{\theta}) - \ln(\theta) \right) - 1 \right) f(\hat{\theta};\, \theta) d(\hat{\theta})$$

$$= \int_0^\infty \left(\exp\left(a\left(\ln(\frac{\hat{\theta}}{\theta}) \right) \right) - a\left(\ln(\frac{\hat{\theta}}{\theta}) \right) - 1 \right) f(\hat{\theta};\, \theta) d(\hat{\theta})$$

Now, by using the transformation $x = \frac{m\hat{\theta}}{\theta}$ and substituting in the integral above, we get

$$R_{LLF}(\hat{I}(\theta)) = \frac{\Gamma(m+a)}{m^a \Gamma(m)} - a(\Psi(m) - \ln(m) - 1) \tag{11}$$

where

$$\Psi(n) = \frac{\frac{d}{dn}\Gamma(n)}{\Gamma(n)}$$

Also, under SELF, the risk of estimator $\hat{I}(\theta)$ is obtained as

$$R_{SELF}(\hat{I}(\theta)) = E\left(\ln(\hat{\theta}) - \ln(\theta) \right)^2$$

$$= \int_0^\infty \left(\ln(\hat{\theta}) - \ln(\theta) \right)^2 f(\hat{\theta};\, \theta) d(\hat{\theta}) = \int_0^\infty \left(\ln(\frac{\hat{\theta}}{\theta}) \right)^2 f(\hat{\theta};\, \theta) d(\hat{\theta})$$

$$= G(0, \infty, (\log(x))^2) - 2\ln(m)\psi(m) + (\ln(m))^2, \tag{12}$$

where

$$G(t_1, t_2, W) = \frac{\int_{t_1}^{t_2} W x^{n-1} e^{-x} dx}{\Gamma(n)}$$ and W is a function of x.

2. Risk of Shrinkage Estimator $\tilde{I}_1(\theta)$

Risk of the estimator $\tilde{I}_1(\theta)$ with respect to LLF is defined as follows:

$$R_{LLF}(\tilde{I}_1(\theta)) = E(\tilde{I}_1(\theta) / LLF)$$

$$= \int_{r_1}^{r_2} \begin{pmatrix} \exp(a((\frac{2m\hat{\theta}}{\theta_0 \chi^2})^2 \ln(\hat{\theta}) + (1 - (\frac{2m\hat{\theta}}{\theta_0 \chi^2})^2) \ln(\theta_0) - \ln(\theta))) \\ -a((\frac{2m\hat{\theta}}{\theta_0 \chi^2})^2 \ln(\hat{\theta}) + (1 - (\frac{2m\hat{\theta}}{\theta_0 \chi^2})^2) \ln(\theta_0) - \ln(\theta)) - 1 \end{pmatrix} f(\hat{\theta};\, \theta) d\hat{\theta}$$

$$+ \int_0^\infty \left(\exp(a(\ln(\hat{\theta}) - \ln(\theta))) - a(\ln(\hat{\theta}) - \ln(\theta)) - 1 \right) f(\hat{\theta};\, \theta) d\hat{\theta}$$

$$- \int_{r_2}^{r_1} \left(\exp(a(\ln(\hat{\theta}) - \ln(\theta))) - a(\ln(\hat{\theta}) - \ln(\theta)) - 1 \right) f(\hat{\theta};\, \theta) d\hat{\theta}$$

where r_1 and r_2 are the boundaries of the acceptance region of a test of the hypothesis $H_0 : \theta = \theta_0$ against the alternative $H_1 : \theta \neq \theta_0$. Define $r_1 = \frac{\theta_0 \chi_1{}^2}{2m}, r_2 = \frac{\theta_0 \chi_2{}^2}{2m}$, where $\chi_1{}^2$ and $\chi_2{}^2$ are respectively lower and upper α^{th} percentile values of the chi-square distribution with 2m degrees of freedom. Again, letting $x = \frac{m\hat{\theta}}{\theta}$ and solving the integrals in the expression for LLF we get

$$R_{LLF}(\hat{I}_1(\theta)) = I_1 + I_2 + I_3 + \frac{4am(m+1)\log(m\phi)}{(\phi\chi^2)^2}\left[I(r_2', m+2) - I(r_1', m+2)\right] - aG(0,\infty,\ln(x))$$
$$-\frac{\Gamma(a+m)}{\Gamma(m)\, m^a}\left[I(r_2', a+m) - I(r_1', a+m)\right] + \frac{\Gamma(a+m)}{\Gamma(m)\, m^a} - a\ln(m\phi)\left[I(r_2', m) - I(r_1', m)\right] + a\ln m - 1, \quad (13)$$

where

$$I_1 = \int_{r_1'}^{r_2'} \phi^a \left(\frac{t}{m\phi}\right)^{\frac{4at^2}{(\phi\chi^2)^2}} \frac{e^{-t}t^{m-1}}{\Gamma(m)}dt,\ I_2 = \frac{-4am(m+1)}{(\phi\chi^2)^2}\int_{r_1'}^{r_2'}(\log t)\frac{e^{-t}t^{m+1}}{\Gamma(m+2)}dt \text{ and } I_3 = \int_{r_1'}^{r_2'} a(\log t)\frac{e^{-t}t^{m-1}}{\Gamma(m)}dt$$

where $r_1' = \frac{2\chi_1^{\,2}}{\lambda}$, $r_2' = \frac{2\chi_2^{\,2}}{\lambda}$, $\phi = \frac{\theta_0}{\theta}$ and $I(x, n)$ is the cumulative distribution function of gamma distribution given as

$$I(x,n) = \frac{\int_0^\infty t^{n-1}e^{-t}dt}{\Gamma(n)}$$

Under SELF, risk of the estimator $\tilde{I}_1(\theta)$ is defined as follows:

$$R_{SELF}(\tilde{I}_1(\theta)) = E\left(\tilde{I}_1(\theta) - \ln(\theta)\right)^2$$
$$= \int_{r_1}^{r_2}\left((\frac{2m\hat{\theta}}{\theta_0\chi^2})^2 \ln(\hat{\theta}) + (1 - (\frac{2m\hat{\theta}}{\theta_0\chi^2})^2)\ln(\theta_0) - \ln(\theta)\right)^2 f(\hat{\theta};\theta)d\hat{\theta}$$
$$+\int_0^\infty \left(\ln(\hat{\theta}) - \ln(\theta)\right)^2 f(\hat{\theta};\theta)d\hat{\theta} - \int_{r_1}^{r_2}\left(\ln(\hat{\theta}) - \ln(\theta)\right)^2 f(\hat{\theta};\theta)d\hat{\theta}$$

Again by using the transformation $x = \frac{m\hat{\theta}}{\theta}$ and substituting in the integrals above, we get

$$R_{SELF}(\tilde{I}_1(\theta)) = I_4 + I_5 + I_6 + \left((\ln\phi)^2 - (\ln m)^2\right)\left[I(r_2', m) - I(r_1', m)\right] + 2(\ln m)G(r_1', r_2, (\ln x))$$
$$\left(-2\ln m\right)G(0,\infty,(\ln x)) - G\left(r_1', r_2', \left(\ln x\right)^2\right) - \frac{8m(m+1)(\ln m\phi)(\ln\phi)}{\phi^2\chi^4}\left[I(r_2', (m+2)) - I(r_1', (m+2))\right]$$
$$+\frac{16m(m+1)(m+2)(m+3)(\ln m\phi)^2}{\phi^4\chi^8}\left[I(r_2', (m+4)) - I(r_1', (m+4))\right] + G(0,\infty,(\ln x)^2) + (\ln m)^2, \quad (14)$$

where

$$I_4 = \frac{16\Gamma(m+4)}{\phi^4\chi^8\Gamma(m)}\int_{r_1'}^{r_2'}(\ln t)^2\frac{e^{-t}t^{m+3}}{\Gamma(m+4)}dt,\ I_5 = -\frac{32\ln(m\phi)\Gamma(m+4)}{\phi^4\chi^8\Gamma(m)}\int_{r_1'}^{r_2'}(\ln t)\frac{e^{-t}t^{m+3}}{\Gamma(m+4)}dt \quad \text{and}$$

$$I_6 = \frac{8\ln(\phi)\Gamma(m+2)}{\phi^2\chi^4\Gamma(m)}\int_{r_1'}^{r_2'}(\ln t)\frac{e^{-t}t^{m+1}}{\Gamma(m+2)}dt$$

IV. Relative Risk(s)

To investigate the properties of the proposed estimator under LLF and SELF, we can compare the relative risks of the estimator with the MLE $\hat{I}(\theta)$.

The relative risk of $\tilde{I}_1(\theta)$ under LLF compared to $\hat{I}(\theta)$ is

$$RR_{LLF}(\tilde{I}_1(\theta)) = \frac{R_{LLF}(\hat{I}(\theta))}{R_{LLF}(\tilde{I}_1(\theta))}$$

Additionally, under SELF, the relative risk of $\tilde{I}_1(\theta)$ w.r.t. $\hat{I}(\theta)$

$$RR_{SELF}(\tilde{I}_1(\theta)) = \frac{R_{SELF}(\hat{I}(\theta))}{R_{SELF}(\tilde{I}_1(\theta))}$$

V. Numerical Computations And Graphical Analysis

We observe that the expressions $RR_{LLF}(\tilde{I}_1(\theta))$, $RR_{SELF}(\tilde{I}_1(\theta))$ depend on m, a, ϕ and α. To show the performance of this considered estimator under LLF and SELF, we have taken some values of these constants as given in Sahni and Kumar [17], i.e. a= -1,-2,-3, 1, 2, 3, m= 5, 8, α = 0.01, 0.05, ϕ = 0.2(0.2)1.6.

Tables I and Table II and Figures 1 to 9 present the behaviour of relative risks of the estimators w.r.t α for varing values of m and a.

i. For $m = 5$, $\alpha = 1\%$ and for all values of 'a' (+ve as well as -ve), $\tilde{I}_1(\theta)$ yield better results than the conventional estimator for the whole scale of ϕ.

ii. Further if we switch α to 5%, the same type of behaviour comes under notice for RR. However, the magnitude of relative risk values was smaller as compared to $\alpha = 1\%$ values.

iii. We have also taken $\alpha = 10\%$ in order to explore the pattern at a higher level of significance and it is found that $\tilde{I}_1(\theta)$ still gives the better results as compared to the conventional estimator but the magnitude of RR values become lower but even then it remains mostly above unity.

iv. After comparing these relative risk values, a lower value i.e. $\alpha = 1\%$ is preferred. Similarly, when varing the value of 'm', higher relative risk values were obtained for $m = 5$ compared to other values of m as 8, 10 and 12. Thus, a smaller 'm' is advised. Higher RR shows better control over risk. Therefore, we can conclude that selecting appropriate values of 'a' and 'α' will result in a higher gain in terms of performance of $\tilde{I}_1(\theta)$.

Table 1: *Relative risk of estimator $\tilde{I}_1(\theta)$ under LLF*

$\alpha = 0.01$		ϕ							
m	a	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6
5	-1	0.8691	0.8752	1.4756	3.3650	5.0900	3.1115	1.7132	1.0904
	-2	0.8377	0.9944	1.8405	3.6257	4.0222	2.6767	1.7404	1.2428
	-3	0.8736	1.2787	2.1754	2.7952	2.4724	1.9211	1.5151	1.2499
	1	0.9372	0.8379	1.1346	2.2949	4.3252	2.9016	1.3766	0.7639
	2	0.9577	0.8565	1.0623	1.9232	3.5375	2.6712	1.2620	0.6679
	3	0.9719	0.8839	1.0193	1.6401	2.8481	2.4762	1.2146	0.6180
8	-1	1.0069	0.8980	1.0572	2.1134	3.9030	2.4284	1.2104	0.7311
	-2	0.9973	0.9040	1.2027	2.5358	4.0023	2.4178	1.3193	0.8545
	-3	0.9767	0.9473	1.4398	2.9114	3.5163	2.2076	1.3708	0.9713
	1	1.0087	0.9249	0.9117	1.5052	2.9003	2.1365	0.9868	0.5372
	2	1.0071	0.9420	0.8762	1.2987	2.3910	1.9649	0.9106	0.4742
	3	1.0054	0.9573	0.8546	1.1334	1.9524	1.8058	0.8639	0.4319
11	-1	1.0187	1.0611	0.9111	1.4573	2.8868	2.0297	0.9805	0.5833
	-2	1.0267	1.0523	0.9918	1.7352	3.2296	2.0971	1.0709	0.6729
	-3	1.0372	1.0475	1.1122	2.0790	3.3339	2.0673	1.1487	0.7707
	1	1.0093	1.0687	0.8137	1.0657	2.0586	1.7619	0.8195	0.4447
	2	1.0066	1.0655	0.7829	0.9228	1.6948	1.6147	0.7613	0.3962
	3	1.0048	1.0588	0.76	0.8022	1.3789	1.4688	0.7199	0.36

$\alpha = 0.05$									
5	-1	1.0199	1.1053	1.4350	2.0083	2.1672	1.6757	1.1896	0.8884
	-2	1.0424	1.2268	1.6473	2.0502	1.9381	1.5303	1.1891	0.9676
	-3	1.1248	1.4121	1.6966	1.7260	1.5322	1.3070	1.1320	1.0110
	1	1.0110	1.0230	1.1380	1.5109	1.8990	1.6317	1.0834	0.7190
	2	1.0084	1.0128	1.0600	1.2972	1.6186	1.5025	1.0252	0.6601
	3	1.0060	1.0084	1.0108	1.1343	1.3559	1.3457	0.9790	0.6262
8	-1	1.0342	1.1634	1.1458	1.4046	1.7189	1.4285	0.9832	0.7179
	-2	1.0519	1.1999	1.2771	1.6149	1.7994	1.4232	1.0246	0.7930
	-3	1.0808	1.2642	1.4479	1.7602	1.7307	1.3597	1.0488	0.8670
	1	1.0158	1.1184	0.9860	1.0348	1.3173	1.2794	0.8806	0.5906
	2	1.0111	1.0987	0.9389	0.8979	1.0998	1.1502	0.8300	0.5442
	3	1.0078	1.0794	0.9064	0.7884	0.9040	1.0001	0.7804	0.5101
11	-1	1.0134	1.2545	1.0347	1.0059	1.3095	1.2578	0.8958	0.6649
	-2	1.0219	1.2928	1.1210	1.1756	1.4639	1.2951	0.9356	0.7257
	-3	1.0371	1.3347	1.2319	1.36	1.5541	1.2935	0.9693	0.7898
	1	1.0056	1.1827	0.9107	0.7387	0.9443	1.0796	0.8089	0.5627
	2	1.0039	1.1499	0.8650	0.6358	0.7728	0.9503	0.7631	0.5232
	3	1.0028	1.12	0.8277	0.5483	0.6191	0.8031	0.7127	0.4911

Table 2: *Relative risk of estimator* $\tilde{I}_1(\theta)$ *under SELF*

α=0.01				ϕ				
m	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6
5	0.9076	0.8377	1.2584	2.7861	5.0209	3.1094	1.5457	0.9090
8	1.0094	0.9085	0.9679	1.7697	3.4465	2.3068	1.0904	0.6229
11	1.0131	1.0673	0.8548	1.2395	2.4663	1.9064	0.8938	0.5068
α=0.05								
5	1.0140	1.0478	1.2579	1.7682	2.1238	1.7027	1.1452	0.7987
8	1.0231	1.1390	1.0523	1.2039	1.5375	1.3767	0.9329	0.6491
11	1.0085	1.2178	0.9664	0.8608	1.1278	1.1841	0,8531	0.6102
α=0.1								
5	1.0384	1.1420	1.2480	1.4216	1.4917	1.2946	1.0070	0.7862
8	1.0206	1.2009	1.0874	1.0031	1.0871	1.0675	0.8857	0.7147
11	1.0059	1.2144	1.0072	0.7265	0.7954	0.9163	0.8559	0.7344

5.1. ***Graphs of Relative Risk for*** $\tilde{I}_1(\theta)$ ***under LLF***

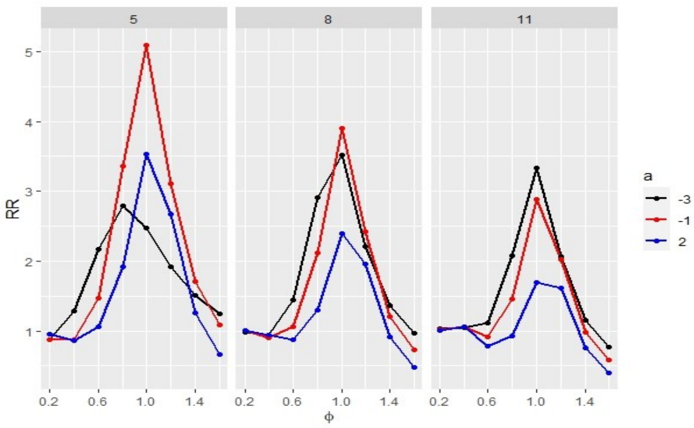

Figure 1: *For α=0.01*

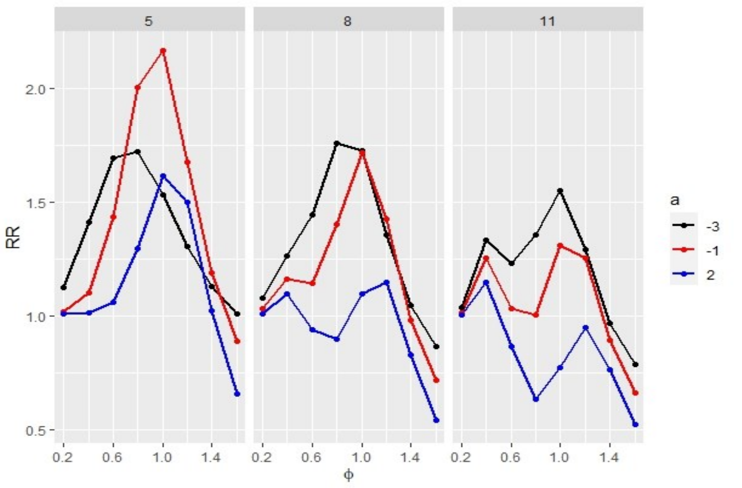

Figure 2: *For α=0.05*

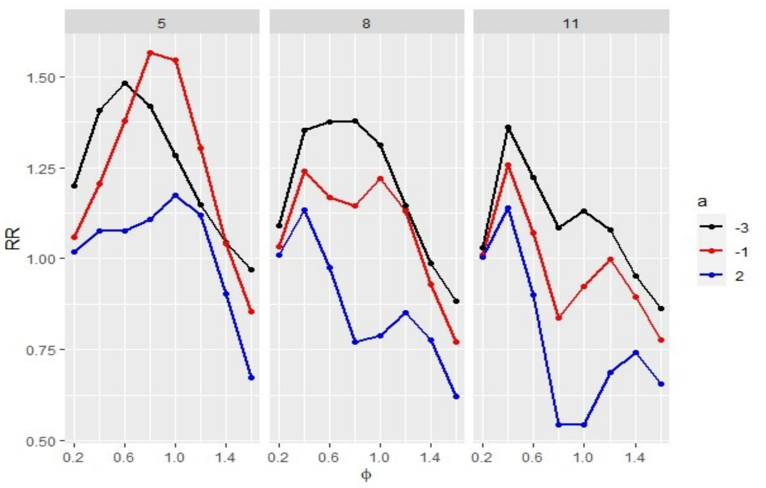

Figure 3: *For α=0.1*

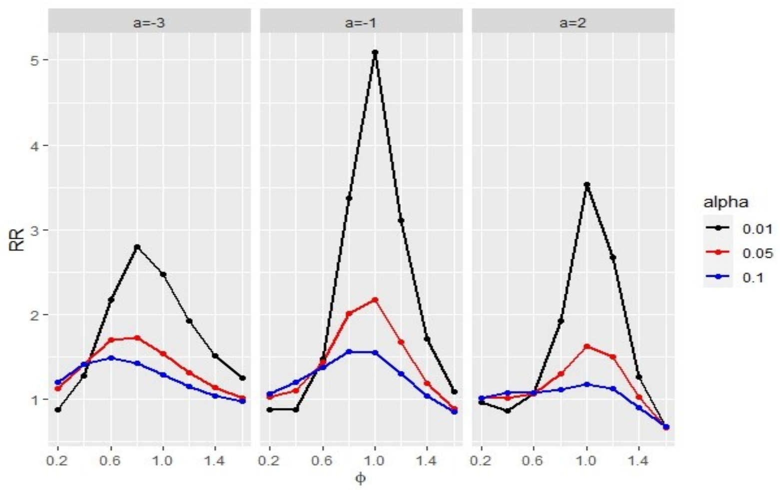

Figure 4: *For m=5*

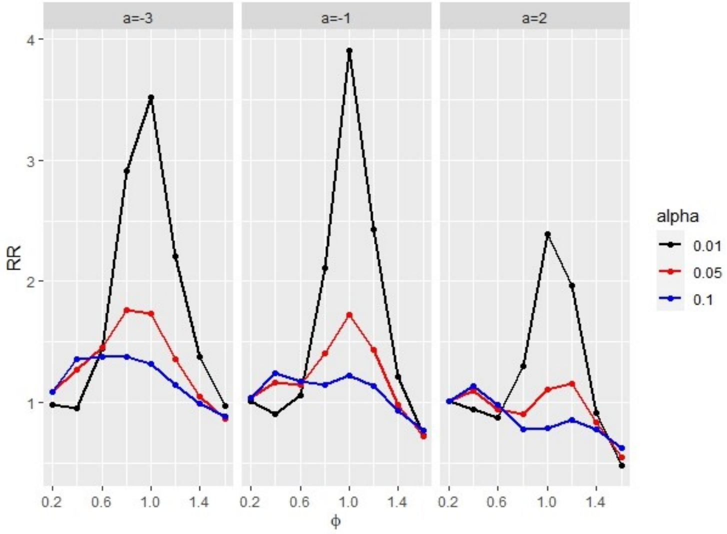

Figure 5: *For m=8*

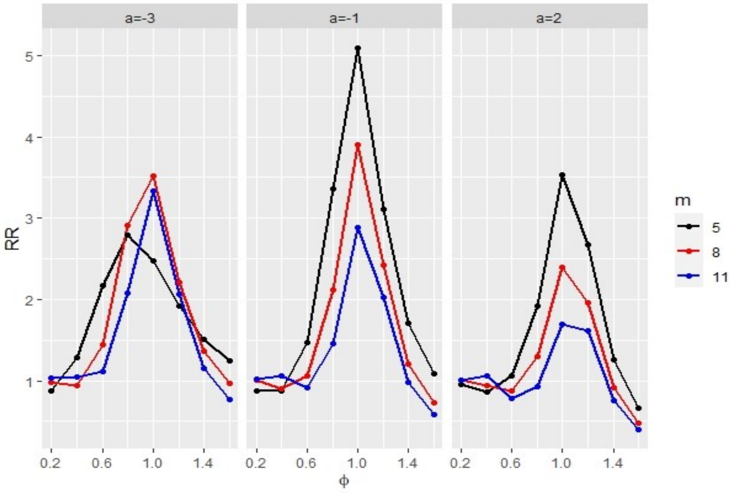

Figure 6: *For α=0.01*

5.2. *Graphs of Relative Risk for* $\tilde{I}_1(\theta)$ *under SELF*

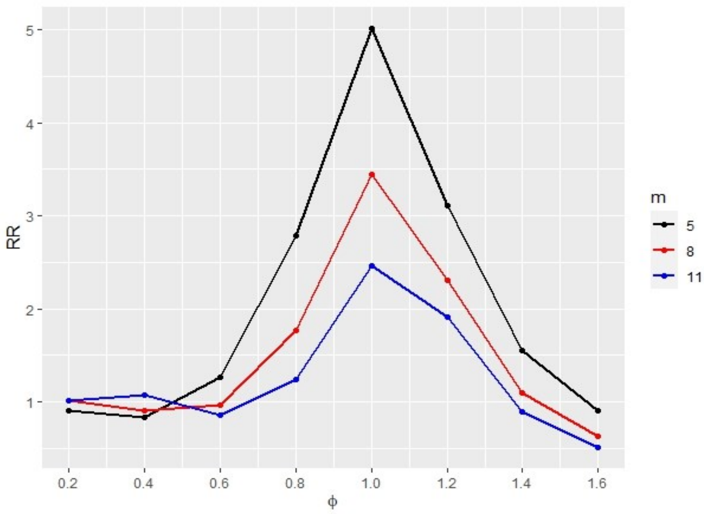

Figure 7: *For α=0.01*

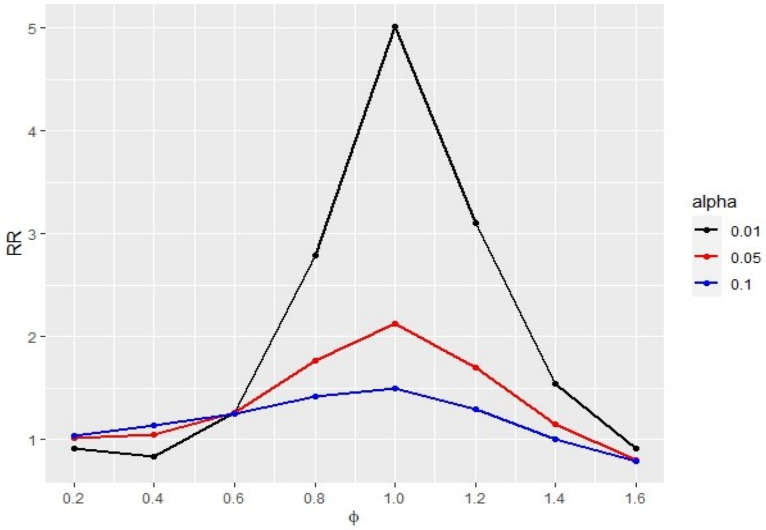

Figure 8: *For m=5*

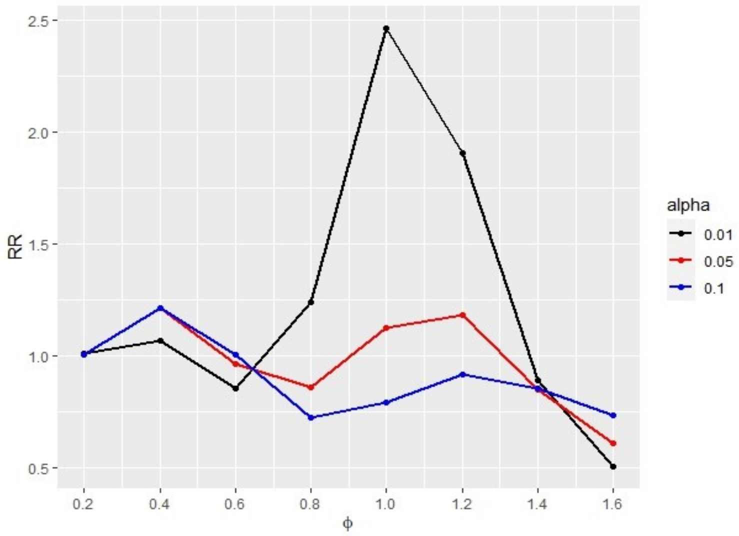

Figure 9: *For m=11*

VI. Conclusion

In this paper, a new Huntsberger type shrinkage entropy estimator for Exponential distribution have been proposed and its properties have been examined under different loss functions. On the basis of relative risk, it is found that the proposed estimator gives better results for smaller values of degrees of freedom and level of significance. And it is also concluded that the proposed estimator gives better results than the estimator proposed by Sahni and Kumar [17], when the estimated value is close to the actual value.

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