COST & PROFIT ANALYSIS OF TWO-DIMENSIONAL STATE M/M/2 QUEUING MODEL WITH CORRELATED SERVERS, MULTIPLE VACATION, BALKING AND CATASTROPHES

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Abstract

The present study obtains the time-dependent solution of a two-dimensional state Markovian queuing model with infinite capacity, correlated servers, multiple vacation, balking and catastrophes. Inter arrival times follow an exponential distribution with parameters λ *and service times follow Bivariate exponential distribution BVE* (μ, μ, ν) *where* μ *is the service time parameter and* ν *is the correlation parameter. Both the servers go on vacation with probability one when there are no units in the system and the servers keeps on taking a sequence of vacations of random length each time the system becomes empty, till it finds at least one unit in the system to start each busy period referred as multiple vacation. The unit finds a long queue and decides not to join it; may be considered as balking. All the units are ejected from the system when catastrophes occur and the system becomes temporarily unavailable. The system reactivates when new units arrive. Occurrence of catastrophes follow Poisson distribution with rate* ξ. *Laplace transform approach has been used to find the time-dependent solution. By using differential-difference equations, the recursive expressions for probabilities of exactly* i *arrivals and* j *departures by time* t *are obtained. The probabilities of this model are consistent to the results of "Pegden & Rosenshine". The model estimates the total expected cost, total expected profit and obtained the optimal values by varying time* t *for cost and profit. These important key measures give a greater understanding of the model behaviour. Numerical analysis and graphical representations have been done by using Maple software.*

Keywords: Correlated servers, Multiple vacation, Balking, Catastrophes

1. Introduction

A two-dimensional state model has been used to deal with complicated transient analysis of some queuing problem. This model is used to examine the queuing system for exact number of arrivals and departures by given time t. In case of a one-dimensional state model, it is difficult to determine how many units have entered, left or waiting units in the system, while the two-dimensional state model exactly identifies the numbers of units that have entered, left, or waiting in the system. The idea of two-dimensional state model for the M/M/1 queue was first given by Pegden & Rosenshine [4]. After that, the two-dimensional state model has attracted the attention of a lot of researchers.

A system of queues in series or in parallel should ordinarily be studied taking into account the interdependence of servers, but this leads to very complicated mathematics even in very simple case of systems. So to reduce such complications of analysis the servers are considered to be independent. But this independence of servers cause impact in time bound operations such as vehicle inspection counters, toll booths, large bars and cafeterias *etc*. where for efficient system functioning the correlation between the servers contributes significantly. Nishida *et al.* [3] investigated a two-server Markovian queue assuming the correlation between the servers and obtained steady-state results for a limited waiting space capacity of two units. Sharma [6] investigated the transient solution to this problem again using only two units waiting spaces capacity. Sharma and Maheswar [8] developed a computable matrix approach to study a correlated two-server Markovian queue with finite waiting space. They also derived waiting time distribution for steady-state and obtained the transient probabilities through steady-state by using a matrix approach and Laplace transform approach.

Various studies have been conducted to evaluate different performance measures to verify the robustness of the system in which a server takes a break for a random period of time *i.e.* vacation. When the server returns from a vacation and finds the empty queue, it immediately goes on another vacation *i.e.* multiple vacation and if it finds at least one waiting unit, then it will commence service according to the prevailing service policy. Different queuing systems with multiple vacation have been extensively investigated and effectively used in several fields including industries, computer & communication systems, telecommunication systems *etc.* Different types of vacation policies are available in literature such as single vacation, multiple vacation and working vacations. Researches on multiple vacation systems have grown tremendously in the last several years. Cooper [2] was the first to study the vacation model and determined the mean waiting time for a unit arrive at a queue served in cyclic order. Doshi [5] and Wu & Zhang [15] have done outstanding researches on queuing system with vacations and released some excellent surveys. Xu and Zhang [12] considered the Markovian multi-server queue with a single vacation (e, d)-policy. They also formulated the system as a quasi-birth-and-death process and computed the various stationary performance measures. Altman and Yechiali [13] studied the customer's impatience in queues with server vacations. Kalidaas *et al.* [18] obtained the time-dependent solution of a single server queue with multiple vacation. Ammar [19] analysed M/M/1 queue with impatient units and multiple vacation. Sharma and Indra [24] investigated the dynamic aspects of a two-dimensional state single server Markovian queuing system with multiple vacation and reneging. Gahlawat *et al*. [25] studied the time-dependent first in first out queuing model with a single intermittently available server and variable-sized bulk arrivals and bulk departures by using the Laplace transform and inverse transform approaches.

Queues with balking have numerous applications in everyday life. Balking occurs if units avoid joining the queue, when they perceive the queue to be too long. Long queues at cash counters, ticket booths, banks, barber shops, grocery stores, toll plaza *etc*. Kumar *et al.* [7] obtained the time-dependent solution of an M/M/1 queue with balking. Chauhan and Sharma [10] derived an expression of the probability distribution for the number of customers in the service station for the M/M/r queuing model with balking and reneging. Zhang and Yue [11] analysed the M/M/1/N queuing system with balking, reneging and server vacation. Sharma and Kumar [17] studied a single-server Markovian feedback queuing system with balking. Bouchentouf and Medjahari [23] presented a single-server feedback queuing system under two differentiated multiple vacation with balking and obtained steady-state probabilities for the model. They also derived some important performance measures, including the average number of customers in the system, the average number of customers in the queue, the average balking rate *etc.*

Queuing systems with catastrophes are also getting a lot of attention nowadays and may be used to solve a wide range of real-world problems. Catastrophes may occur at any time, resulting in the loss of units and the deactivation of the service centre, because they are totally unpredictable in nature. Such type of queues with catastrophes plays an important role in computer programs, telecommunication and ticket counters *etc.* For example, virus or hacker attacking a computer system or program causing the system fail or become idle. Chao [9] obtained steady-state probability of the queue size and a product form solution of a queuing network system with catastrophes. Kumar *et al.* [14] obtained time-dependent solution for M/M/1 queuing system with catastrophes. Kalidass *et al.* [16] derived explicit closed form analytical expressions for the time-dependent probabilities of the system size. Dharamraja and Kumar [20] studied Markovian queuing system with heterogeneous servers and catastrophes. Chakravarthy [21] studied delayed catastrophic model in steady state using the matrix analytic method. Sampath and Liu [22] studied an M/M/1 queue with reneging, catastrophes, server failures and repairs using modified Bessel function, Laplace transform and probability generating function approach. Souza and Rodriguez [26] worked on fractional M/M/1 queue model with catastrophes.

With above concepts in mind, we analyse a two-dimensional state M/M/2 queuing model with correlated servers, multiple vacation, balking and catastrophes.

Consider a situation in a company, where two colleagues work independently on the same project *i.e.* they are not able to share the information of project with each other and not helping each other. Then it will take a long time to complete the project, some information will be lost due to communication gap and the results obtained are not much reliable. But if both of them work together (interdependent servers) *i.e.* they will share all information of project and help each other, then there are more chances that it will take less time and results obtained will be more reliable. Hence interdependent servers are more reliable than the independent servers. When servers work interdependently then they termed as correlated servers. After project completion, the colleagues may take a break, when they find there is no further work, considered as vacation. During the project, if someone wants to work with these colleagues on different project but due to their busy schedule decides not to join them; it may be considered as balking. If due to disease or any other reason the colleagues are not working this may be considered as catastrophes.

The present paper has been structured as follows. In section 1 introduction and in section 2 the model assumptions, notations and description are given. In section 3 the differential-difference equations to find out the time-dependent solution are given and section 4 describes important performance measures. Section 5 investigates the total expected cost function and total expected profit function for the given queuing system. In section 6, we present the numerical results in the form of tables and section 7 contains the tables and graphs to illustrate the impact of various factors on performance measures. The last section contains discussion on the findings and suggestions for further work.

2. Model Assumptions, Notations and Description

- Arrivals follow Poisson distribution with parameter $\boldsymbol{\lambda}$.
- There are two servers and the service times follow Bivariate exponential distribution BVE* ($\boldsymbol{\mu}$, $\boldsymbol{\mu}$, $\boldsymbol{\nu}$) where $\boldsymbol{\mu}$ is the service time parameter and $\boldsymbol{\nu}$ is the correlation parameter.
- The vacation time of the server follows an exponential distribution with parameter $\boldsymbol{w}$.
- On arrival a unit either decides to join the queue with probability $\boldsymbol{\beta}$ or not to join the queue with probability $\boldsymbol{1-\beta}$.
- Occurrence of catastrophes follows Poisson distribution with parameter $\boldsymbol{\xi}$.
- Various stochastic processes involved in the system are statistically independent of each other.

  *introduced by Marshall and Olkin [1]

Initially, the system starts with zero units and the server is on vacation, *i.e.*

$$P_{0,0,V}(0)=1 \qquad ; \qquad P_{0,0,B}(0)=0 \tag{1}$$

$$\delta_{i,j}=\begin{cases}1 & ;for \quad i=j\\ 0 & ;for \quad i\neq j\end{cases} \qquad ; \qquad \sum_{i}^{j}=0 \quad for \quad j<i \tag{2}$$

The Two-Dimensional State Model

$P_{i,j,V}(t)=$ The probability that there are exactly ***i*** arrivals and ***j*** departures by time ***t*** and the server is on vacation.

$P_{i,j,B}(t)=$ The probability that there are exactly ***i*** arrivals and ***j*** departures by time ***t*** and the server is busy in relation to the queue.

$P_{i,j}(t)=$ The probability that there are exactly ***i*** arrivals and ***j*** departures by time ***t***.

3. The Differential-Difference Equations for the Queuing Model under Study

$$\frac{d}{dt}P_{i,i,V}(t)=-\lambda\beta P_{i,i,V}(t)+(\mu+\nu)P_{i,i\text{-}1,B}(t)(1-\delta_{i,0})+\nu P_{i,i\text{-}2,B}(t)(1-\delta_{i,0}-\delta_{i,1})+\xi(1-P_{i,i,V}(t)) \quad i\geq 0 \tag{3}$$

$$\frac{d}{dt}P_{i+1,i,B}(t)=-(\lambda\beta+\mu+\nu+\xi)P_{i+1,i,B}(t)+2\mu P_{i+1,i\text{-}1,B}(t)(1-\delta_{i,0})+\nu P_{i+1,i\text{-}2,B}(t)(1-\delta_{i,0}-\delta_{i,1})+wP_{i+1,i,V}(t) \quad i\geq 0 \tag{4}$$

$$\frac{d}{dt}P_{i,j,V}(t)=-(\lambda\beta+w+\xi)P_{i,j,V}(t)+\lambda\beta P_{i\text{-}1,j,V}(t) \quad i>j\geq 0 \tag{5}$$

$$\frac{d}{dt}P_{i,j,B}(t)=-(\lambda\beta+2\mu+\nu+\xi)P_{i,j,B}(t)+\lambda\beta P_{i\text{-}1,j,B}(t)(1-\delta_{i\text{-}1,j})+2\mu P_{i,j\text{-}1,B}(t)(1-\delta_{j,0}) +\nu P_{i,j\text{-}2,B}(t)+wP_{i,j,V}(t) \quad i>j+1 \tag{6}$$

The preceding equations (3) to (6) are solved by taking the Laplace transforms together with initial condition

$$\overline{P}_{0,0,V}(s)=\frac{(\xi+s)}{s(s+\lambda\beta+\xi)} \tag{7}$$

$$\overline{P}_{i,0,V}(s)=\frac{(\lambda\beta)^{i}(\xi+s)}{s(s+\lambda\beta+\xi)(s+\lambda\beta+w+\xi)^{i}} \quad i>0 \tag{8}$$

$$\overline{P}_{i,i,V}(s)=\left(\frac{\mu+\nu}{s+\lambda\beta+\xi}\right)P_{j,j\text{-}1,B}(s)+\left(\frac{\nu}{s+\lambda\beta+\xi}\right)P_{j,j\text{-}2,B}(s) \quad i>0 \tag{9}$$

$$\overline{P}_{i,0,B}(s)=\frac{w(\lambda\beta)^{i}(\xi+s)}{s(s+\lambda\beta+\xi)(s+\lambda\beta+w+\xi)(s+\lambda\beta+\mu+\nu+\xi)(s+\lambda\beta+2\mu+\nu+\xi)^{i-1}}+w(\lambda\beta)^{i}\sum_{m=1}^{i-1}\frac{(\xi+s)}{s(s+\lambda\beta+\xi)(s+\lambda\beta+w+\xi)^{m+1}(s+\lambda\beta+2\mu+\nu+\xi)^{i-m}} \quad i\geq 1 \quad (10)$$

$$\overline{P}_{i+1,i,B}(s)=\left(\frac{2\mu}{s+\lambda\beta+\mu+\nu+\xi}\right)P_{i+1,i-1,B}(s)+\left(\frac{\nu}{s+\lambda\beta+\mu+\nu+\xi}\right)P_{i+1,i-2,B}(s)+\left(\frac{(\mu+\nu)w\lambda\beta}{(s+\lambda\beta+\xi)(s+\lambda\beta+w+\xi)(s+\lambda\beta+\mu+\nu+\xi)}\right)P_{i-1,i-1,B}(s) \quad i>0 \quad (11)$$

$$\overline{P}_{i,j,V}(s)=\left(\frac{\mu+\nu}{s+\lambda\beta+\xi}\right)\left(\frac{\lambda\beta}{s+\lambda\beta+w+\xi}\right)^{i-j}P_{j,j-1,B}(s)+\left(\frac{\lambda\beta}{s+\lambda\beta+w+\xi}\right)^{i-j}\left(\frac{\nu}{s+\lambda\beta+\xi}\right)P_{j,j-2,B}(s) \quad i>j\geq 0 \quad (12)$$

$$\overline{P}_{i,j,B}(s)=\left(\frac{\lambda\beta}{s+\lambda\beta+2\mu+\nu+\xi}\right)P_{i-1,j,B}(s)+\left(\frac{2\mu}{s+\lambda\beta+2\mu+\nu+\xi}\right)P_{i,j-1,B}(s)+\left(\frac{\nu}{s+\lambda\beta+2\mu+\nu+\xi}\right)P_{i,j-2,B}(s)+\left(\frac{\mu+\nu}{s+\lambda\beta+\xi}\right)\left(\frac{w}{s+\lambda\beta+2\mu+\nu+\xi}\right)\left(\frac{\lambda\beta}{s+\lambda\beta+w+\xi}\right)^{i-j}\left[P_{j,j-1,B}(s)+P_{j,j-2,B}(s)\right] \quad i>j+1,\ j>0 \quad (13)$$

It is seen that

$$\sum_{i=0}^{\infty}\sum_{j=0}^{i}\left[\overline{P}_{i,j,V}(s)+\overline{P}_{i,j,B}(s)(1-\delta_{i,j})\right]=\frac{1}{s} \quad (14)$$

and hence

$$\sum_{i=0}^{\infty}\sum_{j=0}^{i}\left[P_{i,j,V}(t)+P_{i,j,B}(t)(1-\delta_{i,j})\right]=1 \quad (15)$$

a verification.

4. Performance Measures

(i) The Laplace transform of $P_{i.}(t)$ of the probability that exactly ***i*** units arrive by time ***t***; when initially there are no unit in the system is given by

$$\overline{P}_{i.}(s)=\sum_{j=0}^{i}\left[\overline{P}_{i,j,V}(s)+\overline{P}_{i,j,B}(s)(1-\delta_{i,j})\right]=\sum_{j=0}^{i}\overline{P}_{i,j}(s)=\frac{(\lambda\beta)^{i}}{(s+\lambda\beta)^{i+1}}$$

And its inverse Laplace transform is: $P_{i.}(t)=\frac{e^{-\lambda\beta t}(\lambda\beta t)^{i}}{i!}$ (16)

The arrivals follow a Poisson distribution as the probability of the total number of arrivals is not affected by vacation time of the server.

(ii) $P_{.j}(t)$ is the probability that exactly ***j*** units have been served by time ***t***. In terms of $P_{i,j}(t)$ we have

$$P_{.j}(t)=\sum_{i=j}^{\infty}P_{i,j}(t) \quad (17)$$

(iii) The Laplace transform of mean number of arrival is: $\sum_{i=0}^{\infty} i\overline{P}_{i.}(s) = \frac{\lambda\beta}{(s^2)}$ (18)

And its inverse Laplace transform is: $\sum_{i=0}^{\infty} iP_{i.}(t) = \lambda\beta t$ (19)

(iv)The mean number of units in the queue is calculated as follows

$$Q_L(t) = \sum_{N=0}^{\infty} NP_V(t) + \sum_{N=1}^{\infty} (N-1)P_B(t) \tag{20}$$

Where $N = i–j$.

5. Cost Function and Profit Function

For the given queuing system, the following notations have been used to represent various costs to find out the total expected cost and total expected profit per unit time:
Let
C_H: Cost per unit time for unit in the queue.
C_B: Cost per unit time for a busy server.
C_μ: Cost of service per unit time.
C_V: Cost per unit time when the server is on vacation.
If I is the total expected amount of income generated by delivering a service per unit time then

(i) Total expected cost per unit at time ***t*** is given by

$$TC(t) = C_H * Q_L(t) + C_B * P_B(t) + C_V * P_V(t) + \mu * C_\mu \tag{21}$$

(ii) Total expected income per unit at time ***t*** is given by

$$TE_I(t) = I * \mu * (1 - P_V(t)) = I * \mu * P_B(t) \tag{22}$$

(iii) Total expected profit per unit at time ***t*** is given by

$$TE_P(t) = TE_I(t) - TC(t) \tag{23}$$

6. Numerical Results

6.1. Numerical Validity Check

(i) For the state when the server is on vacation

$$P_V(t) = \sum_{j=0}^{i} P_{i,j,V}(t) \tag{24}$$

(ii) For the state when the server is busy in relation to the queue

$$P_B(t) = \sum_{j=0}^{i-1} P_{i,j,B}(t) \tag{25}$$

(iii) The probability $P_{i.}(t)$ that exactly i units arrive by time t is

$$\sum_{j=0}^{i} P_{i,j}(t) = \sum_{j=0}^{i} P_{i,j,V}(t) + \sum_{j=0}^{i-1} P_{i,j,B}(t) \tag{26}$$

(iv) A numerical validity check of inversion of $\overline{P}_{i,j}(s)$ is based on the relationship

$$\Pr\{\boldsymbol{i} \text{ arrivals in } (0, \boldsymbol{t})\} = \frac{e^{-\lambda\beta t}(\lambda\beta t)^{i}}{i!} = \sum_{j=0}^{\infty} P_{i,j}(t) = P_{i.}(t) \tag{27}$$

The probabilities of this model shown in last column of table 1 given below are consistent to the last column of "Pegden & Rosenshine" [4] by keeping constant values of w=1, ξ=0, β=1 *and* ν=0.25 *shown in table*

Table-1: *Numerical validity check of inversion of* $\overline{P}_{i,j}(s)$

λ	μ	t	i	$\frac{e^{-\lambda t} * (\lambda t)^{i}}{i!}$	$\sum_{j=0}^{i} P_{i,j,V}(t)$	$\sum_{j=0}^{i-1} P_{i,j,B}(t)$	$\sum_{j=0}^{i} P_{i,j}(t)$
1	2	3	1	0.149361	0.129196	0.020165	0.149361
1	2	3	3	0.224041	0.158076	0.065965	0.224041
1	2	3	5	0.100818	0.057803	0.043016	0.100818
2	2	3	1	0.014873	0.012865	0.002008	0.014873
2	2	3	3	0.089235	0.062961	0.026274	0.089235
2	2	3	5	0.160623	0.092090	0.068533	0.160623
1	2	4	1	0.073263	0.065390	0.007873	0.073263
1	2	4	3	0.195367	0.148001	0.047366	0.195367
1	2	4	5	0.156294	0.100998	0.055296	0.156294
2	2	4	1	0.002683	0.002395	0.000288	0.002683
2	2	4	3	0.028626	0.021686	0.006940	0.028626
2	2	4	5	0.091604	0.059195	0.032409	0.091604
2	4	4	5	0.091604	0.073396	0.018208	0.091604
1	2	4	4	0.195367	0.136810	0.058557	0.195367
1	2	3	6	0.050409	0.025824	0.024585	0.050409

7. Sensitivity Analysis

This part focuses on the impact of the arrival rate (λ), service rate (μ), vacation rate (w), correlation parameter (ν), balking probability (1-β) and catastrophes rate (ξ) on the probability when the server is on vacation (**$P_V(t)$**), probability when the server is busy (**$P_B(t)$**), expected queue length (**$Q_L(t)$**), total expected cost (**$TC(t)$**), total expected income (**$TE_I(t)$**) and total expected profit (**$TE_P(t)$**) at time t. To determine the numerical results for the sensitivity of the queuing system one parameter varied while keeping all the other parameters fixed and taking cost per unit time for unit in the queue=10, cost per unit time for a busy server=8, cost per unit time when the server is on vacation=5, cost of service per unit time=4, total expected amount of income=100 and number of units in the system=8.

7.1. Impact of Arrival Rate (λ)

We examine the behaviour of the queuing system using measures of effectiveness along with cost and profit analysis by varying arrival parameter with time, while keeping all other parameters fixed; μ=5, w=3, v=0.25, ξ=0.0001 and β=1. In table 2, we observe that as the value of λ increases with time t, $P_B(t)$, $Q_L(t)$, $TC(t)$, $TE_I(t)$ and $TE_P(t)$ increases but $P_V(t)$ decreases.

Table-2: *Measures of Effectiveness versus λ*

t	λ	$P_V(t)$	$P_B(t)$	$Q_L(t)$	$TC(t)$	$TE_I(t)$	$TE_P(t)$
1	1.00	0.8550827	0.1449161	0.4618549	30.0532913	72.45805	42.4047587
2		0.8451239	0.1546386	0.4824289	30.2870173	77.31930	47.0322827
3		0.8427343	0.1534636	0.4779723	30.2211033	76.73180	46.5106967
4		0.8414541	0.1511897	0.4597185	30.0139731	75.59485	45.5808769
5		0.8403975	0.1485374	0.4210111	29.6003977	74.26870	44.6683023
1	1.10	0.8427120	0.1572855	0.5062882	30.5347260	78.64275	48.1080240
2		0.8321542	0.1673763	0.5277639	30.7774204	83.68815	52.9107296
3		0.8279867	0.1661032	0.5192708	30.6614671	83.05160	52.3901329
4		0.8278007	0.1634085	0.4891622	30.3378935	81.70425	51.3663565
5		0.8264318	0.1599687	0.4312130	29.7240386	79.98435	50.2603114
1	1.20	0.8306335	0.1693615	0.5505522	31.0135815	84.68075	53.6671685
2		0.8194239	0.1797142	0.5725572	31.2604051	89.85710	58.5966949
3		0.8165607	0.1777708	0.5579035	31.0840049	88.88540	57.8013951
4		0.8147421	0.1744598	0.5117878	30.5872669	87.22990	56.6426331
5		0.8130834	0.1702170	0.4412499	29.8396520	85.10850	55.2688480

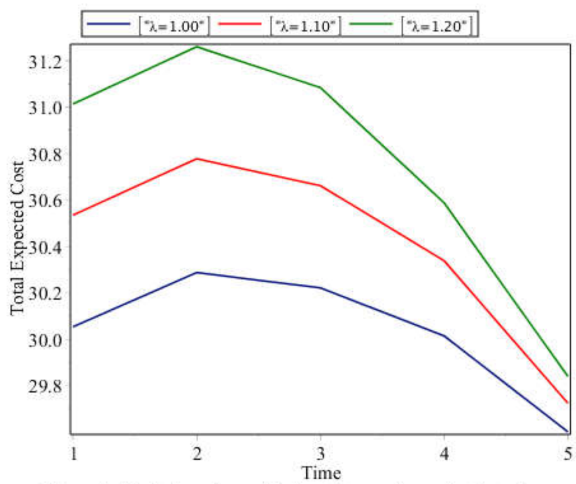

Figure 1: *Variation of cost with time t by varying arrival rate λ*

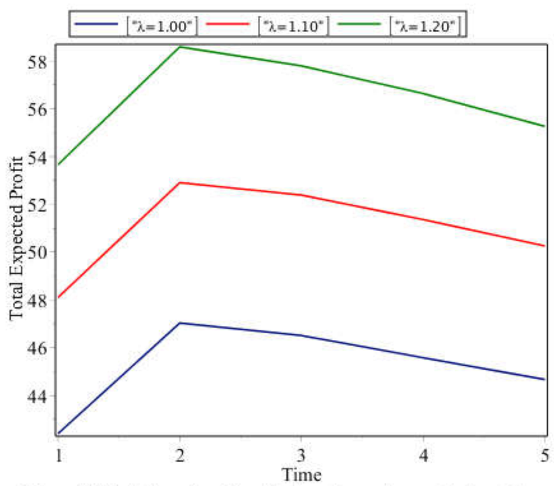

Figure 2: *Variation of profit with time t by varying arrival rate λ*

Figure 1 shows the variation of cost with time by varying arrival rate while keeping the other parameters fixed.

Figure 2 shows the variation of profit with time by varying arrival rate while keeping the other parameters fixed.

7.2. Impact of Service Rate (μ)

The behaviour of the queuing system measures of effectiveness along with cost and profit analysis by varying μ with time t, while keeping all other parameters fixed; λ=1, w=3, v=0.25, ξ=0.0001 and β=1. In table 3, we observe that as the value of μ increases with time t, $P_B(t)$, $Q_L(t)$, $TC(t)$, $TE_I(t)$ and $TE_P(t)$ increases but $P_V(t)$ decreases.

Table-3: *Measures of Effectiveness versus μ*

t	μ	$P_V(t)$	$P_B(t)$	$Q_L(t)$	$TC(t)$	$TE_I(t)$	$TE_P(t)$
1	2.00	0.7369325	0.2630663	0.6177736	19.9669289	52.613260	32.6463311
2		0.6762899	0.3234726	0.7090209	21.0594393	64.694520	43.6350807
3		0.6699756	0.3262223	0.7084844	21.0445004	65.244460	44.1999596
4		0.6667285	0.3239153	0.6797388	20.7273529	64.783060	44.0557071
5		0.6649823	0.3189526	0.6203355	20.0798873	63.790520	43.7106327
1	2.50	0.7655438	0.2344550	0.5762758	21.4661170	58.613750	37.1476330
2		0.7243328	0.2754297	0.6356142	22.1812436	68.857425	46.6761814
3		0.7211500	0.2750479	0.6303259	22.1093922	68.761975	46.6525828
4		0.7177601	0.2728837	0.6050702	21.8275721	68.220925	46.3933529
5		0.7134529	0.2684820	0.5531662	21.2567825	67.120500	45.8637175
1	3.00	0.7897314	0.2102673	0.5431578	23.0623734	63.080190	40.0178166
2		0.7609462	0.2388163	0.5847948	23.5632094	71.644890	48.0816806
3		0.7587053	0.2374926	0.5788371	23.4818383	71.247780	47.7659417
4		0.7561697	0.2354741	0.5561210	23.2258513	70.642230	47.4163787
5		0.7501976	0.2317373	0.5088841	22.6937274	69.521190	46.8274626

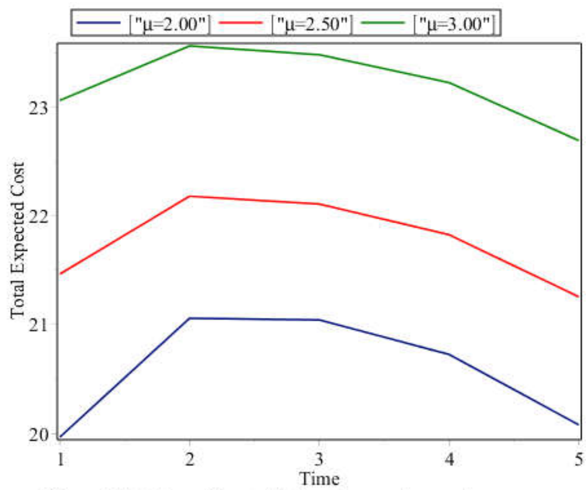

Figure 3: *Variation of cost with time t by varying service rate μ*

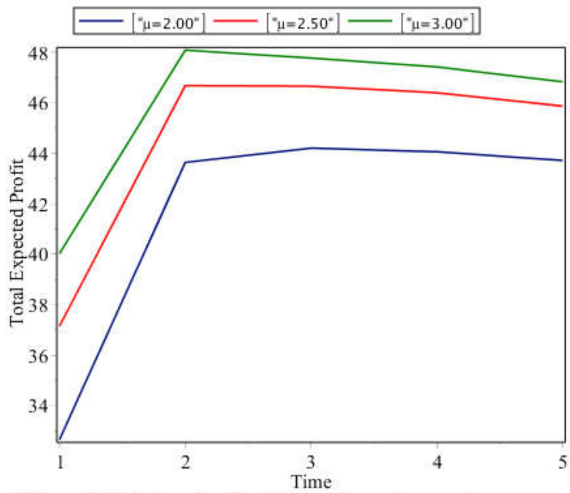

Figure 4: *Variation of profit with time t by varying service rate μ*

Figure 3 shows the variation of cost with time by varying service rate while keeping the other parameters fixed**.**

Figure 4 shows the variation of profit with time by varying service rate while keeping the other parameters fixed.

7.3. Impact of Vacation Rate (w)

We observe that the behaviour of the queuing system measures of effectiveness along with cost and profit by varying w with time t, while keeping all other parameters fixed; λ=1, μ=5, v=0.25, ξ=0.0001 and β=1. In table 4, we observe that as the value of w increases with time t, $P_B(t)$, $Q_L(t)$, $TC(t)$, $TE_I(t)$ and $TE_P(t)$ increases but $P_V(t)$ decreases.

Table-4: *Measures of Effectiveness versus w*

t	w	$P_V(t)$	$P_B(t)$	$Q_L(t)$	$TC(t)$	$TE_I(t)$	$TE_P(t)$
1	3.00	0.8550827	0.1449161	0.4618549	30.0532913	72.45805	42.4047587
2		0.8451239	0.1546386	0.4824289	30.2870173	77.31930	47.0322827
3		0.8427343	0.1534636	0.4779723	30.2211033	76.73180	46.5106967
4		0.8414541	0.1511897	0.4597185	30.0139731	75.59485	45.5808769
5		0.8403975	0.1495374	0.4210111	29.6083977	74.76870	45.1603023
1	4.00	0.8462857	0.1537131	0.4015245	29.4763783	76.85655	47.3801717
2		0.8411568	0.1586057	0.4083021	29.5576506	79.30285	49.7451994
3		0.8389988	0.1571991	0.4040524	29.4931108	78.59955	49.1064392
4		0.8378927	0.1547511	0.3886032	29.3135043	77.37555	48.0620457
5		0.8321931	0.1517418	0.3558749	28.9336489	75.87090	46.9372511
1	5.00	0.8415912	0.1584076	0.3613974	29.0891908	79.20380	50.1146092
2		0.8386494	0.1611131	0.3640154	29.1223058	80.55655	51.4342442
3		0.8365297	0.1596682	0.3602046	29.0620401	79.83410	50.7720599
4		0.8345421	0.1575017	0.3464072	28.8967961	78.75085	49.8540539
5		0.8300883	0.1548467	0.3171915	28.5611301	77.42335	48.8622199

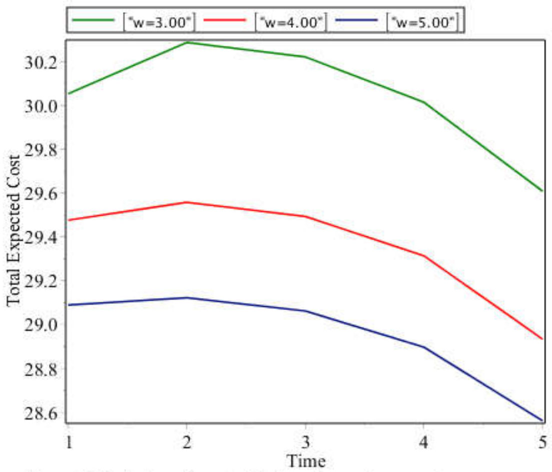

Figure 5: *Variation of cost with time t by varying vacation rate w*

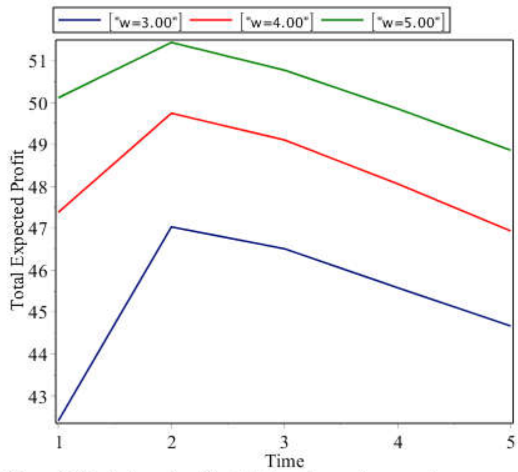

Figure 6: *Variation of profit with time t by varying vacation rate w*

Figure 5 shows the variation of cost with time by varying vacation rate while keeping the other parameters fixed.

Figure 6 shows the variation of profit with time by varying vacation rate while keeping the other parameters fixed.

7.4. Impact of Correlation Parameter (*v*)

We see that the behaviour of the queuing system using measures of effectiveness along with cost and profit analysis by varying ***v*** with time ***t***, while keeping all other parameters fixed; λ=1, μ=5, w=3, ξ=0.0001 and β=1. In table 5, we observe that as the value of ***v*** increases with time ***t***, $P_B(t)$, $Q_L(t)$, $TC(t)$, $TE_I(t)$ and $TE_P(t)$ increases but $P_V(t)$ decreases.

Table-5: *Measures of Effectiveness versus v*

t	v	$P_V(t)$	$P_B(t)$	$Q_L(t)$	$TC(t)$	$TE_I(t)$	$TE_P(t)$
1	0.25	0.8550827	0.1449161	0.4618549	30.0532913	72.45805	42.4047587
2		0.8451239	0.1546386	0.4824289	30.2870173	77.31930	47.0322827
3		0.8427343	0.1534636	0.4779723	30.2211033	76.73180	46.5106967
4		0.8414541	0.1511897	0.4597185	30.0139731	75.59485	45.5808769
5		0.8403975	0.1475374	0.4210111	29.5923977	73.76870	44.1763023
1	0.50	0.8612902	0.1387085	0.4555353	29.9714720	69.35425	39.3827780
2		0.8522410	0.1475215	0.4756205	30.1975820	73.76075	43.5631680
3		0.8497855	0.1464124	0.4712878	30.1331047	73.20620	43.0730953
4		0.8472639	0.1433799	0.4532707	29.9160657	71.68995	41.7738843
5		0.8456738	0.1392611	0.4150683	29.4931408	69.63055	40.1374092
1	0.75	0.8670184	0.1329803	0.4497140	29.8960744	66.49015	36.5940756
2		0.8587324	0.1410301	0.4694068	30.1159708	70.51505	40.3990792
3		0.8562189	0.1399790	0.4651801	30.0527275	69.98950	39.9367725
4		0.8544773	0.1361665	0.4473789	29.8355075	68.08325	38.2477425
5		0.8524003	0.1315346	0.4096382	29.4106603	65.76730	36.3566397

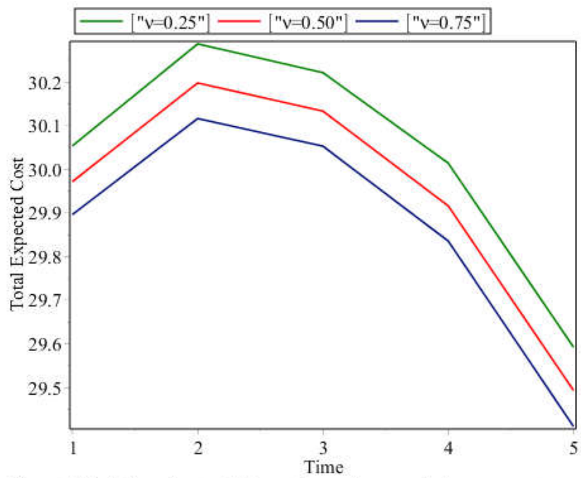

Figure 7: *Variation of cost with time t by varying correlation parameter v*

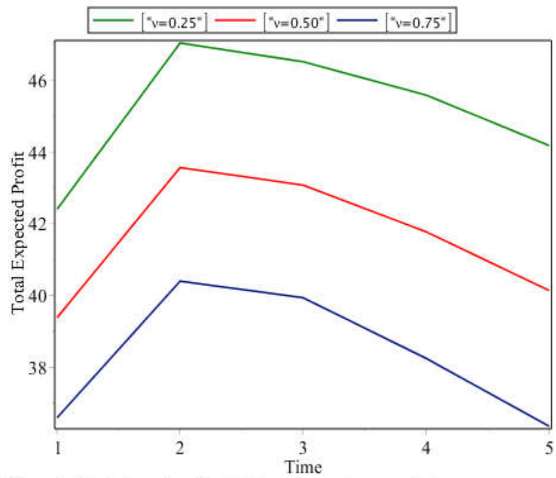

Figure 8: *Variation of profit with time t by varying correlation parameter v*

Figure 7 shows the variation of cost with time by varying correlation parameter while keeping the other parameters fixed**.**
Figure 8 shows the variation of profit with time by varying correlation parameter while keeping the other parameters fixed.

7.5. Impact of Joining Probability (β)

We see that the behaviour of the queuing system measures of effectiveness along with cost and profit analysis by varying β with time t. While keeping all other parameters fixed; λ=1, μ=5, w=3, v=0.25 and ξ=0.0001. In table 6, we observe that as the value of β increases with time t, $P_B(t)$, $Q_L(t)$, $TC(t)$, $TE_I(t)$ and $TE_P(t)$ increases but $P_V(t)$ decreases.

Table-6: *Measures of Effectiveness versus β*

t	β	$P_V(t)$	$P_B(t)$	$Q_L(t)$	$TC(t)$	$TE_I(t)$	$TE_P(t)$
1	0.65	0.8876633	0.0990264	0.2868039	28.0985667	49.51320	21.4146333
2		0.8840590	0.1068084	0.3068008	28.3427702	53.40420	25.0614298
3		0.8880948	0.1069225	0.3132331	28.4281850	53.46125	25.0330650
4		0.8895493	0.1065419	0.3154762	28.4548437	53.27095	24.8161063
5		0.8871414	0.1053500	0.3133391	28.4118980	52.67500	24.2631020
1	0.75	0.8767085	0.1125818	0.3349279	28.6334759	56.29090	27.6574241
2		0.8720983	0.1210185	0.3563601	28.8922405	60.50925	31.6170095
3		0.8751685	0.1210047	0.3615710	28.9595901	60.50235	31.5427599
4		0.8746624	0.1200356	0.3610913	28.9445098	60.01780	31.0732902
5		0.8732121	0.1183004	0.3533446	28.8459097	59.15020	30.3042903
1	0.85	0.8671069	0.1257705	0.3844829	29.1865275	62.88525	33.6987225
2		0.8608738	0.1347799	0.4064078	29.4466862	67.38995	37.9432638
3		0.8623335	0.1344946	0.4090881	29.4785053	67.24730	37.7687947
4		0.8615314	0.1334039	0.4037567	29.4124552	66.70195	37.2894948
5		0.8598611	0.1310590	0.3868133	29.2159105	65.52950	36.3135895

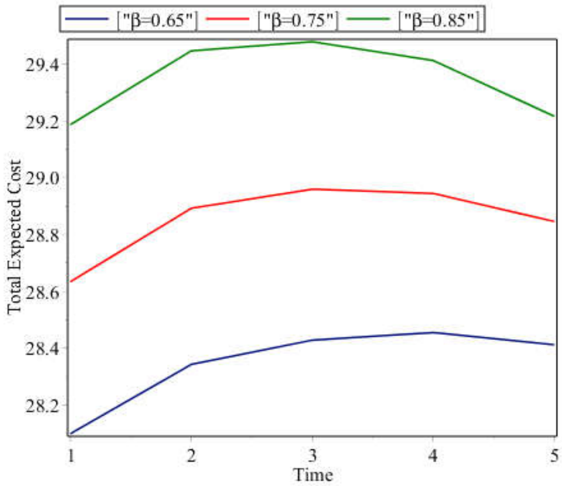

Figure 9: *Variation of cost with time t by varying joining probability β*

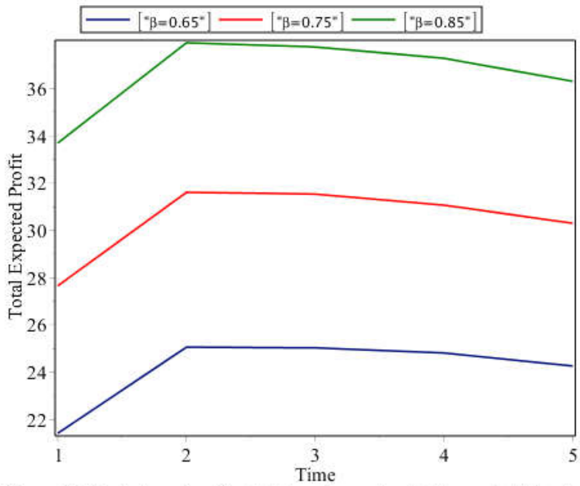

Figure 10: *Variation of profit with time t by varying joining probability β*

Figure 9 shows the variation of cost with time by varying joining probability while keeping the other parameters fixed.
Figure 10 shows the variation of profit with time by varying joining probability while keeping the other parameters fixed.

7.6. Impact of Catastrophes Rate (ξ)

We see that the behaviour of the queuing system measures of effectiveness along with cost and profit analysis by varying ξ with time t, while keeping all other parameters fixed; λ=1, μ=5, w=3, v=0.25 and β=1. In table 7, we observe that as the value of ξ increases with time t, $P_B(t)$, $Q_L(t)$, $TC(t)$, $TE_I(t)$ and $TE_P(t)$ increases but $P_V(t)$ decreases.

Table-7: *Measures of Effectiveness versus ξ*

t	ξ	$P_V(t)$	$P_B(t)$	$Q_L(t)$	$TC(t)$	$TE_I(t)$	$TE_P(t)$
1	0.0001	0.8550827	0.1449161	0.4618549	30.0532913	72.45805	42.4047587
2		0.8451239	0.1546386	0.4824289	30.2870173	77.31930	47.0322827
3		0.8427343	0.1534636	0.4779723	30.2211033	76.73180	46.5106967
4		0.8414541	0.1491897	0.4597185	29.9979731	74.59485	44.5968769
5		0.8403975	0.1445374	0.4210111	29.5683977	72.26870	42.7003023
1	0.0002	0.8550885	0.1449103	0.4618404	30.0531289	72.45515	42.4020211
2		0.8451309	0.1546316	0.4824119	30.2868263	77.31580	47.0289737
3		0.8427420	0.1534569	0.4779565	30.2209302	76.72845	46.5075198
4		0.8414661	0.1491850	0.4597091	29.9979015	74.59250	44.5945985
5		0.8404256	0.1445380	0.4210193	29.5686250	72.26900	42.7003750
1	0.0003	0.8550943	0.1449045	0.4618259	30.0529665	72.45225	42.3992835
2		0.8451380	0.1546246	0.4823948	30.2866348	77.31230	47.0256652
3		0.8427496	0.1534503	0.4779406	30.2207564	76.72515	46.5043936
4		0.8414782	0.1491802	0.4596998	29.9978306	74.59010	44.5922694
5		0.8404537	0.1445385	0.4210276	29.5688525	72.26925	42.7003975

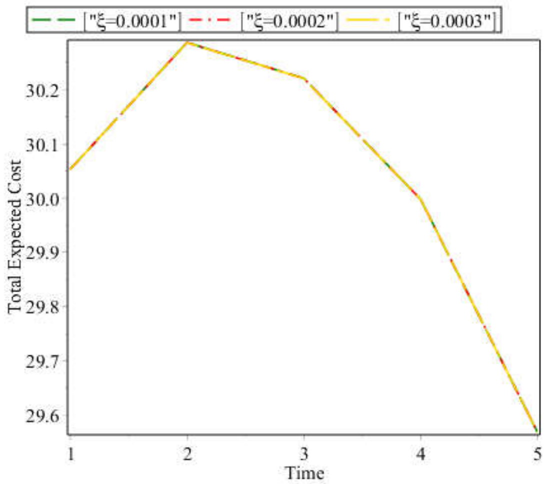

Figure 11: *Variation of cost with time t by varying catastrophes rate ξ*

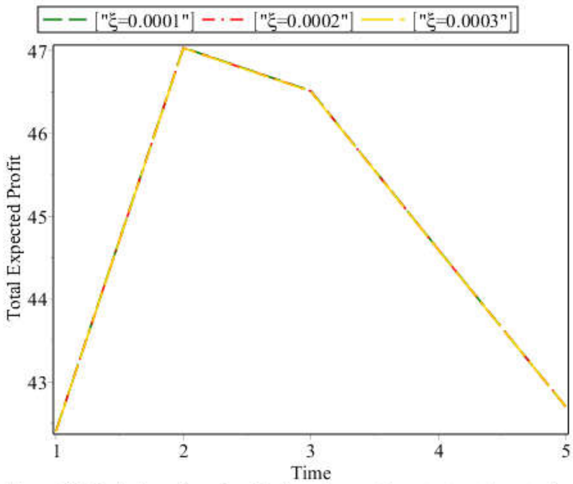

Figure 12: *Variation of profit with time t by varying catastrophes rate ξ*

Figure 11 shows the variation of cost with time by varying catastrophes rate while keeping the other parameters fixed.
Figure 12 shows the variation of profit with time by varying catastrophes rate while keeping the other parameters fixed.

8. Discussion

Figure 1 & figure 2 show the variation of cost & profit respectively with time t by varying λ(=1.00, 1.10, 1.20). The value of both cost & profit increases with increase in t upto t(=2.00) then decreases slightly. Hence we get the optimal value at t=5 when λ=1.00 and t=2 when λ=1.20 for minimum cost and maximum profit respectively.

Figure 3 show the variation of cost with time t by varying μ(=2.00, 2.50, 3.00). The value of cost increases with increase in t upto t(=2.00) then decreases slightly. The variation in profit with time t represented in figure 4 by varying μ(=2.00, 2.50, 3.00). The profit increases with increase in time up to (i) t=3 when μ=2.00 (ii) t=2 when μ=2.50, 3.00 respectively then decreases slightly. Hence we get the optimal value at t=1 when μ=2.00 and t=2 when μ=3.00 for minimum cost and maximum profit respectively.

Figure 5 & figure 6 show the variation of cost & profit respectively with time t by varying w(=3.00, 4.00, 5.00). The value of both cost & profit increases with increase in t upto t(=2.00) then decreases slightly. Hence we get the optimal value at t=5 when w=5.00 and t=2 when w=5.00 for minimum cost and maximum profit respectively.

Figure 7 & figure 8 show the variation of cost & profit respectively with time t by varying v(=0.25, 0.50, 0.75). The value of both cost & profit increases with increase in t upto t(=2.00) then decreases slightly. Hence we get the optimal value at t=5 when v=0.75 and t=2 when v=0.25 for minimum cost and maximum profit respectively.

Figure 9 show the variation of cost with time t by varying β(=0.65, 0.75, 0.85). The value of cost increases with increase in t upto (i) t=4 when β=0.65 (ii) t=3.00 when β=0.75, 0.85 respectively then decreases slightly. The variation in profit with time t represented in figure 10 by varying β(=0.65, 0.75, 0.85). The profit increases with increase in time up to t(=2.00) then decreases slightly. Hence we get the optimal value at t=1 when β=0.65 and t=2 when β=0.85 for minimum cost and maximum profit respectively.

Figure 11 & figure 12 show the variation of cost & profit respectively with time t by varying ξ(=0.0001, 0.0002, 0.0003). The value of both cost & profit increases with increase in t upto t(=2.00) then decreases slightly. Hence we get the optimal value at t=5 when ξ=0.0001 and t=2 when ξ=0.0001 for minimum cost and maximum profit respectively. Finally, the variation in rate of catastrophes shows the minor effect on cost and profit.

9. Conclusions and Future Work

The time-dependent solution, for the two-dimensional state M/M/2 queuing model with correlated servers, multiple vacation, balking and catastrophes, has been obtained. The model estimates the total expected cost and total expected profit, the best optimal value is at t=1 when service rate(=2.00) and t=2 when arrival rate(=1.20) for minimum cost and maximum profit respectively. These key measures give a greater understanding of model behaviour. Finally, the numerical analysis clearly demonstrates the meaningful impact of the correlated servers and multiple vacation on the system performances. This model finds its applications in communication networks, computer networks, supermarkets, hospital administrations, financial sector and many others.

As part of future study, this model may be examined further for Non-Markovian queues, bulk queues, tandem queues *etc*.

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