A FUZZY INNOVATIVE ORDERING PLAN USING STOCK DEPENDENT HOLDING COST OF INSPECTION WITH SHORTAGES IN TIME RELIABILITY DEMAND USING TFN

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Abstract

The considerations in this paper are, the demand is consistent with time deterioration, the holding cost is dependent based on the quantity of stock available in the system, and the ordering cost is linear and time-dependent. This system should be considered in terms of fuzziness. It is assumed that the shortages are permitted partially, the order is inspected, defective items are identified, by using penalty cost, the defective items should be minimized. Under the classical model and fuzzy environment, the mathematical equation is arrived at to find the optimal solution of total relevant cost with optimal order quantity and time using triangular fuzzy numbers. Defuzzification has been accomplished through the use of the signed distance method of integration. The solutions have been arrived and the model numerical problem of three levels of values (lower, medium and upper) in parametric changes has been verified. Using Sensitivity analysis, the solution is used to validate the changes in different parameter values of the system. To demonstrate the convexity of the TRC function over time, it has used a three-dimensional mesh graph.

Keywords: Ordering plan, Triangular Fuzzy Numbers, Stock depending holding cost, Varying order Cost.

1. Introduction

In any type of business, maintenance of the stock plays crucial role. The stock should be with effective quality (with freshness); it means deterioration should be very less. In this paper, the demand is estimated so that the deterioration is reliable with time period. If the demand increases automatically the deterioration is to be minimized with time. The order placement depends on the demand of the system. Suppose the quantity supplied as part of order quantity is less than the demand, it will lead to shortage. The ordered stock should be in good condition if there is any defective product or service supplied, the firm will incur loss and also the goodwill of the customer. In order to meet the shortages, the lost sale cost is added, the items should be inspected. Sometimes when items are supplied by delay, then the penalty cost will also be added in the process. Here the ordering cost is not fixed as it is linearly time-dependent. Also, the holding cost is not fixed for the entire time period, whenever the stock reduces, the holding cost also reduces.

Abhishek et al., [1] developed a paper in a fuzzy economic production quantity model deteriorating production depends proportional to population, selling price and advertisement, in this paper he explain clearly how demand is work with population and selling price with advertisement. Dutta .D. et al., [2] presented a article in Optimal Inventory Shortages Fuzziness in Demand,. the model is developed in crisp environment. After that it is convert in to fuzzy environment. All the functions convert in to (TFN) and the fuzzy trapezoidal number. In order to Defuzzification, the SDM used. The EOQ, optimal total cost derived and in both environment. Magfura Pervin et al., [10] explained the combined vendor buyer of quadratic demand inspection ,preservation technology applied the vendor applied the (PT) to reduce deterioration cost using this technique to reduce the total cost and fount the optimal total cost. Pavan Kumar [3] deals with Optimal inventory model with shortages applied fuzzy environment. The shortages were allowed partially backlogged method were applied for manage the stock for genuine customer is was very useful for manufacturer. Sankar Kumar Roy et al., [4] established a model of Imperfection and inspection and varying demand in trade credit using inspection policy easily found the defect of the items and supply to the customer and get the goodwill of the customer. This model is very useful for improve the relationship of customer and supplier. Sivan.V et al., [5] formulated a model of retailer supplier of price dependent demand; To demand will improve automatically when we reduce the cost of the item. Srabani Shee et al.,[6] proposed a model in Fuzzy Supply Chain Varying Holding Cost of supplier and retailer , supplier will more benefit then the retailer since retailer will spend more amount for holding the items so the holding cost of retailer is more than the supplier all the calculation doing by fuzzy and crisp environment. Thirugnanasambandam. et al.,[7] developed model of estimation of EOQ Model negative exponential Demand of linear term. The drugs are maintained stock and problem formed using negative exponential demand so day by day the demand is diminishing. Two types of demand functions formulated and calculated the optimal total cost and more quantity. Tripathi [8] investigated the innovative stock sensitive demand of EOQ for deterioration by means of inconsistent here the newly found stock dependent holding cost using the model holding cost to minimized and linear and nonlinear holding cost considered and verified with parametric changes. Sudip Adak et al., [9] established inventory model reliability dependent partial backordering in fuzziness. Here the deterioration is minimized using the demand if demand is increases automatically the deterioration is reduced and partial shortages are balanced with backlogging here also the results were found and compared with crisp and fuzzy environment.

The present paper has eleven sections. Basic definitions and fuzzy preliminaries are followed by introduction provided. In Section 3, notations and assumptions are introduced. The problems are described and formulated in the fourth section. In Section 5, comes out with numerical solutions and sample problems. The Sensitivity Analysis, Graphical representation and the impact of parametric changes are portrayed in the sixth section. In the section seven represents detailed observation. In Section 8, the Inventory model in fuzzy environment is formulated. Some numerical problems are using Triangular Fuzzy Numbers with different data sets are solved. Illustrative examples are given in the Section nine. In Section 10, comparative studies of crisp and fuzzy optimal values are explored. Conclusions and further developments are distinguished in the final section.

2. Definitions and Fuzzy Preliminaries

Definition 1. Membership value : A fuzzy set $\tilde{\mho}$ is a universe of discourse. The following set of pairs is defined as X. $\tilde{\mho} = \{(x, \mu_{\tilde{\mho}}(x))/x \in R\}$, where $\mu_{\tilde{\mho}}(x) : X \longrightarrow [0, 1]$ is a mapping called **membership value** or degree of membership of $x \in R$ in the fuzzy set $\tilde{\mho}$.

Definition 2. Convex : A fuzzy set $\tilde{\mho}$ of the universe if and only if the discourse X is Convex,

$\forall x_1, x_2, \in R$ The following set of pairs is defined as X.

$$\mu_{\tilde{\mho}}(\rho x_1 + (1-\rho)x_2)) \geq \min\left[\mu_{\tilde{\mho}}(x_1), \mu_{\tilde{\mho}}(x_2)\right], \text{ when } 0 \leq \rho \leq 1.$$

Definition 3. Normal Fuzzy Set : A fuzzy set $\tilde{\mho}$ of the universe X is referred to as a Normal Fuzzy Set, meaning that at least one exists $x \in R$ such that $\mu_{\tilde{\mho}}(x) = 1$.

Definition 4. Triangular Fuzzy Number (TFN) : The Triangular Fuzzy Number $\tilde{\mho} = [a_{F1}, a_{F2}, a_{F3}]$ and is formed its continuous membership function $\mu_{\tilde{\mho}}(x) : X \longrightarrow [0,1]$ is,

$$\mu_{\tilde{\mho}}(x) = f(x) = \begin{cases} \dfrac{x - a_{F1}}{a_{F2} - a_{F1}}, & \text{for } a_{F1} \leq x \leq a_{F2}; \\ \dfrac{a_{F3} - x}{a_{F3} - a_{F2}}, & \text{for } a_{F2} \leq x \leq a_{F3}; \\ 0, & \text{Otherwise}; \end{cases}$$

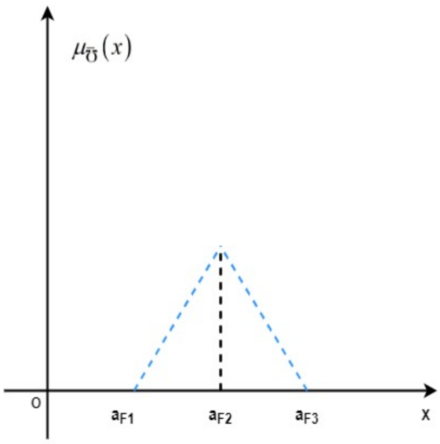

Figure 1: *Triangular Fuzzy Number*

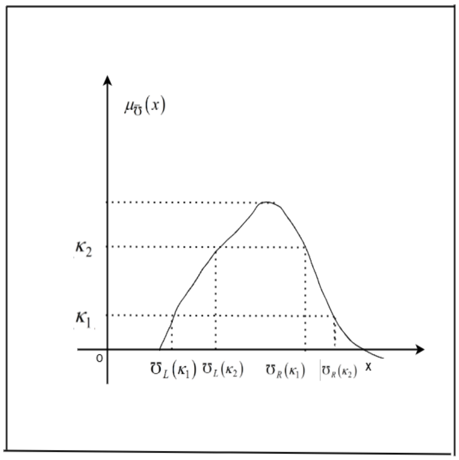

Figure 2: *Fuzzy number with cuts*

Definition 5. Signed Distance Method : Signed Distance Method: Defuzzification of $\tilde{\mho}$ can be discovered using the Signed Distance Method. If $\tilde{\mho}$ is a TFN then Sign distance from $\tilde{\mho}$ to 0 is described as:

$$d(\tilde{\mho}, 0) = \frac{1}{2}\int_0^1 [\tilde{\mho}_L(k), \tilde{\mho}_R(k), 0]\, dk.$$

3. Notations and Assumption

3.1. Notations

1. $I_1(t)$ The inventory level in the time period $0 \leq t \leq t_1$
2. $I_2(t)$ The inventory level in the time period $t_1 \leq t \leq T$
3. t_1 The time stock reached to zero
4. T The total cycle time
5. c_{FO} Ordering cost depend of time dependent
6. $\widetilde{c_{FO}}$ Fuzzy Ordering cost depends of time dependent
7. $\theta(t) = \frac{t}{a^{\lambda}}$ Deterioration period $a, \lambda \geq 1$
8. c_{F2} : Deterioration cost per unit time

9. $\widetilde{c_{F2}}$: Fuzzy deterioration cost per unit time
10. c_{F3} : Holding cost per unit time
11. $\widetilde{c_{F3}}$: Fuzzy holding cost per unit time
12. c_{F4} : Shortage cost per unit time
13. $\widetilde{c_{F4}}$: Fuzzy shortage cost per unit time
14. c_{F5} : Inspection cost per unit time
15. $\widetilde{c_{F5}}$: Fuzzy inspection cost per unit time
16. c_{F6} : Penalty cost per unit time
17. $\widetilde{c_{F6}}$: Fuzzy penalty cost per unit time
18. Q_F : The maximum order level in the time period $(0 \leq t \leq t_1)$
19. $TRC(t_1, T)$: The total relevant cost per cycle
20. $\widetilde{TRC(t_1, T)}$: The Fuzzified total relevant cost per cycle
21. β_F : Unit of shortage cost $(0 \leq \beta_F \leq 1)$
22. μ_F : Unit of lost sale cost $(\mu_F \geq 0)$
23. δ_F : Unit of penalty cost $\delta_F \geq 0$
24. λ : Shape parameter $\lambda \geq 0$
25. D_I : The total items deteriorated

3.2. Assumption

1. The demand function is written $D(t) = p_1 t^a + p_2$ for $0 \leq t \leq t_1$, $t_1 \leq t \leq T$ for $p_1, p_2 \geq 0$
2. Deterioration per cycle $DC_F = \frac{t}{a^\lambda}$, $a \geq 1, \lambda \geq 1$
3. Ordering cost $c_{FO} = r_1 t_1 + r_2$, $r_1 \geq 0$, $r_2 > 0$
4. T is the complete cycle periods time horizon
5. t_1 is the period of time when inventory level reduces to finish
6. Lead time is negligible.
7. Shortages partially allowed is β_F, $0 \leq \beta_F < 1$ is backordered.
8. In some situation the demand may be considered high, in that period to maintain good relationship with customer and in to consider the inspection policy.
9. In the given cycle time, in the beginning of the process full inventory level is considered.
10. During the shortage period same demand is to be considered.

3.3. Decision Variables

- t_1 : The time Period first level $0 \leq t \leq t_1$
- T : The time Period second level $t_1 \leq t \leq T$

3.4. Objective Functions

1. $\text{TRC}(t_1, T)$: Total relevant cost per cycle
2. β_F : Cost of preservation technology investment per unit time
3. Q_F : The maximum order in the time period $0 \le t \le T$
4. t_1, T : Optimal time periods in $0 \le t \le t_1$, $t_1 \le t \le T$

4. Problem Description and Mathematical Equation

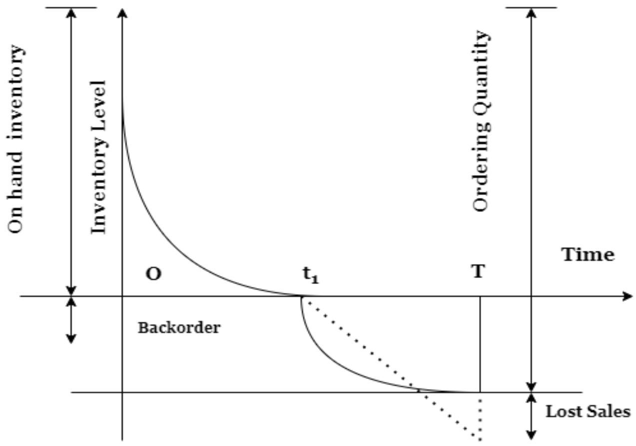

Figure 3: *Inventory level Vs Time.*

4.1. Problem description.

Initially the inventory level is Q_F, because of demand and deterioration the level of inventory is gradually reduced in time period $t = t_1$, so the shortages in this inventory is taken partially.

4.2. Mathematical Equation.

The following formula were used to create the Mathematical model for this paper using differential equations. In shortage period also, the same demand function is maintained.

$$\frac{dI_1(t)}{\text{dt}} + \frac{t}{a^{\lambda}} I_1(t) = -\ (p_1 t^a + p_2)\ ,\ for\ 0 \le t \le t_1, \tag{1}$$

$$\frac{dI_1(t)}{\text{dt}} = -\ \beta\,(p_1 t^a + p_2)\,,\ for\ t_1 \le t \le T, \tag{2}$$

Using the initial and the boundary conditions, let us find $I_1(t)$, and $I_2(t)$. In $I_1(t)$ and $I_2(t)$ put $t = t_1$, $t = T$ and $I_1(t_1) = 0$, $I_2(T) = 0$ and get the solution. In $I_1(t)$ put $t = 0$ get order quantity Q_F therefore $I_1(0) = Q_F$.

Solution of 1 and 2

$$\begin{aligned} I_1(t) &= \{p_2(t_1-t)\} + \frac{p_2}{6a^\lambda}(t_1^3-t^3) + \frac{p_1}{a+1}(t_1^{a+1}-t^{a+1}) + \frac{p_1}{2a^{\lambda+1}+6a^\lambda} \\ &\quad \times (t_1^{a+3}-t^{a+3}) + \frac{p_2}{2a^\lambda}(t^3-t_1t^2) + \frac{p_2}{12a^{2\lambda}}(t^5-t^2t_1^3) \\ &\quad + \frac{p_1}{(a+1)2a^\lambda}(t^{a+3}-t_1^{a+1}t^2) + \frac{p_1}{(2a^{\lambda+1}+6a^\lambda)(2a^\lambda)} \\ &\quad \times (t^{a+5}-t_1^{a+3}t^2) \end{aligned} \tag{3}$$

$$\begin{aligned} I_1(0) &= Q_F \\ &= \{p_2(t_1)\} + \frac{p_2}{6a^\lambda}(t_1^3) + \frac{p_1}{a+1}(t_1^{a+1}) + \frac{p_1}{2a^{\lambda+1}+6a^\lambda}(t_1^{a+3}) \end{aligned} \tag{4}$$

$$I_2(t) = \beta\left\{\frac{p_1}{a+1}[(T^{a+1}-t^{a+1})+p_2(T-t)]\right\} \tag{5}$$

The ordering cost is given by,

$$DC = c_{\mathrm{FO}} = r_1t_1 + r_2, \qquad r_1, r_2 > 0 \tag{6}$$

The total number of pieces that becomes deteriorated throughout that period of interval $0 \le t \le t_1$ is formed by,

$$\begin{aligned} D_I &= Q - \int_0^{t_1} D(t)\,dt \\ &= p_2(t_1) + \frac{p_2}{6a^\lambda}(t_1^3) + \frac{p_1}{a+1}(t_1^{a+1}) + \frac{p_1}{2a^{\lambda+1}+6a^\lambda}(t_1^{a+3}) - \int_0^{t_1}(p_1t^a+p_2)\,dt \\ &= \frac{p_2}{6a^\lambda}(t_1^3) + \frac{p_1}{2a^{\lambda+1}+6a^\lambda}t_1^{a+3} \end{aligned}$$

Therefore the deteriorating cost is formed by,

$$c_{F2}\left\{\frac{p_2}{6a^\lambda}(t_1^3) + \frac{p_1}{2a^{\lambda+1}+6a^\lambda}t_1^{a+3}\right\} \tag{7}$$

The Holding cost (HC) during the interval $[0, t_1]$ is formed by,

$$\begin{aligned} HC &= c_3\int_0^{t_1} e + f[I_1(t)]dt \\ &= c_{F3}\left(et_1 + f\left[\left(\frac{p_2}{2}\right)t_1^2 + \left(\frac{p_2}{8a^\lambda}\right)\frac{t_1^4}{2} + \frac{p_1}{a+2}t_1^{a+2} + \left(\frac{p_1}{2a^{\lambda+1}+6a^\lambda}\right)\right.\right. \\ &\quad \times \left(\frac{a+3}{a+4}\right)t_1^{a+4} - \left(\frac{p_2}{72a^{2\lambda}}\right)t_1^6 - \left(\frac{p_1}{3(2a^{\lambda+1}+2a^\lambda}\right)\left(\frac{a+1}{a+4}\right)t_1^{a+4} \\ &\quad \left.\left. - \left(\frac{p_1}{6a^\lambda(2a^{\lambda+1}+6a^\lambda}\right)\left(\frac{a+3}{a+6}\right)t_1^{a+6}\right]\right) \end{aligned} \tag{8}$$

Shortage Cost is formed by,

$$\begin{aligned} Sh.C &= c_{F4}\beta_F\int_{t_1}^{T}(p_1t^a+p_2)\,\mathrm{dt} \\ &= c_{F4}\beta_F\left\{\frac{p_1}{(a+1)}[T^{a+1}-{t_1}^{a+1}] + p_2(T-t_1)\right\} \end{aligned} \tag{9}$$

The lost sale cost is formed by,

$$\begin{aligned} Lo.SC &= \mu_F(1-\beta_F)\int_{t_1}^{T}(p_1t^a+p_2)\,\mathrm{dt} \\ &= \mu_F(1-\beta_F)\left\{\frac{p_1}{(a+1)}[T^{a+1}-{t_1}^{a+1}]+p_2(T-t_1)\right\} \end{aligned} \tag{10}$$

The Inspection cost in the interval $[0, t_1]$ is given by,

$$\begin{aligned} In.C &= c_{F5}Q_F \\ &= c_{F5}\left\{p_2t_1+\frac{p_2}{6a^{\lambda}}t_1^3+\frac{p_1}{(a+1)}t_1^{a+1}+\frac{p_1}{2a^{\lambda+1}+6a^{\lambda}}t_1^{a+3}\right\} \end{aligned} \tag{11}$$

The penalty cost (PC) during the interval $[0, t_1]$ is formed by,

$$\begin{aligned} Pn.C &= c_{F6}\delta_F\int_0^{t_1}(p_1t^a+p_2)\,\mathrm{dt} \\ &= c_{F6}\delta_F\left(\frac{p_1}{(a+1)}t_1^{a+1}+p_2t_1\right) \end{aligned} \tag{12}$$

The Total Average Relevant Cost,

$$\begin{aligned} \mathrm{TRC}(t_1,T) &= \frac{1}{T}\Big\{\text{Ordering Cost + Deteriorating Cost + Holding Cost +} \\ &\qquad \text{Shortage Cost + Lost Cost + Inspection cost + Penalty cost}\Big\} \\ &= \frac{1}{T}\Bigg[r_1t_1+r_2+c_{F2}\left\{\frac{p_2}{6a^{\lambda}}(t_1^3)+\frac{p_1}{2a^{\lambda+1}+6a^{\lambda}}t_1^{a+3}\right\} \\ &\quad +c_{F3}\left(et_1+f\left[\left(\frac{p_2}{2}\right)t_1^2+\left(\frac{p_2}{8a^{\lambda}}\right)\frac{t_1^4}{2}+\frac{p_1}{a+2}t_1^{a+2}\right]\right) \\ &\quad +\left(\frac{p_1}{2a^{\lambda+1}+6a^{\lambda}}\right)\left(\frac{a+3}{a+4}\right)t_1^{a+4}-\left(\frac{p_2}{72a^{2\lambda}}\right)t_1^6 \\ &\quad -\left(\frac{p_1}{3(2a^{\lambda+1}+2a^{\lambda})}\right)\left(\frac{a+1}{a+4}\right)t_1^{a+4} \\ &\quad -\left(\frac{p_1}{6a^{\lambda}(2a^{\lambda+1}+6a^{\lambda})}\right)\left(\frac{a+3}{a+6}\right)t_1^{a+6} \\ &\quad +c_{F4}\beta_F\left\{\frac{p_1}{(a+1)}[T^{a+1}-{t_1}^{a+1}]+p_2(T-t_1)\right\} \\ &\quad +\mu_F(1-\beta_F)\left\{\frac{p_1}{(a+1)}[T^{a+1}-{t_1}^{a+1}]+p_2(T-t_1)\right\} \\ &\quad +c_{F5}\left\{p_2t_1+\frac{p_2}{6a^{\lambda}}t_1^3+\frac{p_1}{(a+1)}t_1^{a+1}+\frac{p_1}{2a^{\lambda+1}+6a^{\lambda}}t_1^{a+3}\right\} \\ &\quad +c_{F6}\delta_F\left(\frac{p_1}{(a+1)}t_1^{a+1}+p_2t_1\right)\Bigg] \end{aligned} \tag{13}$$

For the convenience let us do this substitution,

$$\varphi_1 = \frac{p_2}{6a^{\lambda}} \qquad \varphi_2 = \frac{p_1}{2a^{\lambda+1}+6a^{\lambda}} \qquad \varphi_3 = \frac{p_2}{2}$$

$$\varphi_4 = \frac{p_2}{8a^{\lambda}} \qquad \varphi_5 = \frac{p_1}{a+2} \qquad \varphi_6 = \frac{p_1}{2a^{\lambda+1}+6a^{\lambda}}\left(\frac{a+3}{a+4}\right)$$

$$\varphi_7 = \frac{p_2}{24a^{\lambda}} \qquad \varphi_8 = \frac{p_2}{72a^{2\lambda}} \qquad \varphi_9 = \frac{p_1}{3\left(2a^{\lambda+1}+6a^{\lambda}\right)}\left(\frac{a+1}{a+4}\right)$$

$$\varphi_{10} = \frac{p_1}{6a^{\lambda}\left(2a^{\lambda+1}+6a^{\lambda}\right)}\left(\frac{a+3}{a+6}\right) \qquad \varphi_{11} = \frac{p_1}{(a+1)}$$

Substitute the above φ_1, φ_2,φ_4,φ_5,φ_6,φ_7,φ_8,φ_9,φ_{10},φ_{11} in equation (13)
The Total Relevant Cost will become

$$\begin{aligned}
\text{TRC}(t_1,T) \quad = \quad & \frac{1}{T}\Big[r_1t_1 + r_2 + c_{F2}(\varphi_1 t_1^3 + \varphi_2 t_1^{a+3}) \\
& + c_{F3}\left(et_1 + f\left[\varphi_3 t_1^2 + \varphi_1\frac{t_1^4}{2} + \varphi_5 t_1^{a+2} + \varphi_6 t_1^{a+4} - \varphi_8 t_1^6 - \varphi_9 t_1^{a+4}\right.\right. \\
& \left.\left. - \varphi_{10} t_1^{a+6}\right]\right) + c_{F4}\beta_F\left\{\varphi_{11}[T^{a+1} - {t_1}^{a+1}] + p_2(T-t_1)\right\} \\
& + \mu_F(1-\beta_F)[\varphi_{11}[T^{a+1} - {t_1}^{a+1}] + p_2(T-t_1) \\
& + c_{F5}\left\{p_2t_1 + \varphi_1 t_1^3 + \varphi_{11}t_1^{a+1} + \varphi_2 t_1^{a+3}\right\} \\
& + c_{F6}\delta_F\left(\varphi_{11}t_1^{a+1} + p_2t_1\right)\Big]
\end{aligned} \tag{14}$$

$$\begin{aligned}
\text{TRC}(t_1,T) \quad = \quad & \frac{1}{T}\Big[r_1t_1 + r_2 + c_{F2}(\varphi_1 t_1^3 + \varphi_2 t_1^{a+3}) \\
& + c_{F3}\left(et_1 + f\left[\varphi_3 t_1^2 + \varphi_1\frac{t_1^4}{2} + \varphi_5 t_1^{a+2} + \varphi_6 t_1^{a+4} - \varphi_8 t_1^6 - \varphi_9 t_1^{a+4}\right.\right. \\
& \left.\left. - \varphi_{10} t_1^{a+6}\right]\right) + [c_{F4}\beta_F + \mu(1-\beta)]\left\{\varphi_{11}[T^{a+1} - {t_1}^{a+1}]\right. \\
& \qquad \left. + p_2(T-t_1)\right\} + c_{F5}\left\{p_2t_1 + \varphi_1 t_1^3 + \varphi_{11}t_1^{a+1} + \varphi_2 t_1^{a+3}\right\} \\
& + c_{F6}\delta_F\left(\varphi_{11}t_1^{a+1} + p_2t_1\right)\Big]
\end{aligned} \tag{15}$$

5. Numerical Solutions and Sample Problems

5.1. Numerical Solutions of Fuzzy Innovative Ordering Plan

For the solution purpose, MATLAB R2018b and Excel solver are used to find all the optimal solutions, all the graphs and convex mesh using MATLAB R2018b software.

To find the solution of the equation (15) using the below necessary and sufficient condition. The necessary condition for the least value of TRC (t_1, T) are,

$$\frac{\partial\left(\text{TRC}\left(t_1,T\right)\right)}{\partial\, t_1} = 0$$

and

$$\frac{\partial\left(\mathrm{TRC}\left(t_1, T\right)\right)}{\partial\ T} = 0$$

The Sufficient condition for optimal TRC (t_1, T), $\ t_1 > 0,\ T > 0$.

$$\frac{\partial^2(\mathrm{TRC})}{\partial t_1^2} > 0$$

and

$$\frac{\partial^2(\mathrm{TRC})}{\partial T^2} > 0$$

$$\begin{vmatrix} \frac{\partial^2(\mathrm{TRC})}{\partial t_1^2} & \frac{\partial^2(\mathrm{TRC})}{\partial t_1 \partial T} \\ \frac{\partial^2(\mathrm{TRC})}{\partial T \partial t_1} & \frac{\partial^2(\mathrm{TRC})}{\partial T^2} \end{vmatrix} > 0$$

Therefore the optimal solutions of t_1^*, T^*, Q_F^* and TRC^* are found and given in the table.

Table 1: *The optimal solution using crisp*

t_1^*	T^*	Q_F^*	$\mathrm{TRC}^*(t_1, T)$(Rs.)
0.83244	0.86903	10.1671	148.603

5.2. Sample Problems

Example 1. Let's take the input: p_1= 4.2, p_2 = 9.5 , a =1.2, e = 1.8,f= 0.2, β_F= 0.004, δ_F= 3.85, μ_F=11, λ = 3.99, c_{F2} =Rs.15, c_{F3} = Rs.10, c_{F4} =Rs.5 , c_{F5} =Rs.0.5, c_{F6} =Rs.1.03, r_{F1} = Rs.9.8, r_{F2} = Rs.40.

The Optimal Solutions are Q_F^* = 9.58728, $t_1^* = 0.82247$,T^* =0.90941 & TRC^* = Rs.145.406.

Example 2. Let's take the input: p_1= 4.2, p_2 = 9.99, a =1.2 , e = 1.8, f = 0.2, β_F= 0.004, δ_F= 3.85, μ_F=11,λ = 3.99, c_{F2} =Rs.15, c_{F3} = Rs.10, c_{F4} =Rs.5, c_{F5} =Rs.0.5, c_{F6} =Rs.1.03, r_{F1} =Rs. 9.8, r_{F2} = Rs.40.

The Optimal Solutions are Q_F^* = 10.1671, $t_1^* = 0.83244$, $T^* = 0.86903$ & TRC^* = Rs.148.603

Example 3. Let's take the input: p_1 = 4.2, p_2 = 10.25, a = 1.2, e = 1.8, f = 0.2, β_F = 0.004, δ_F = 3.85, μ_F = 11, λ = 3.99, c_{F2} =Rs.15, c_{F3} = Rs.10, c_{F4} =Rs.5, c_{F5} =Rs. 0.5, c_{F6} =Rs.1.03, r_{F1} = Rs.9.8, r_{F2} = Rs.40.

The Optimal Solutions are Q_F^* = 10.4737, $t_1^* = 0.83734$, $T^* = 0.84657$ & TRC^* = Rs.150.252.

5.3. Convexity of the optimal function

Convexity of Optimal total cost TRC(t_1, T) versus t_1 and T using Matlab R2018b are shown graphically Figure (4) and Figure (6).

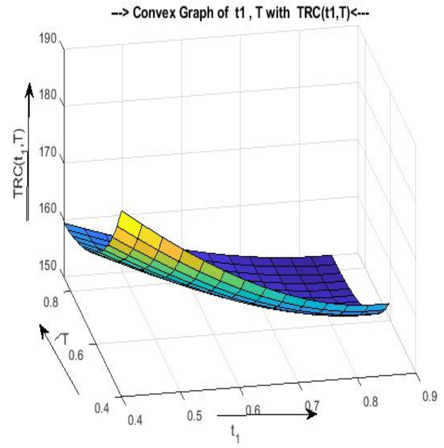

Figure 4: *Convexity Graph of t_1, T with $TRC(t_1, T)$*

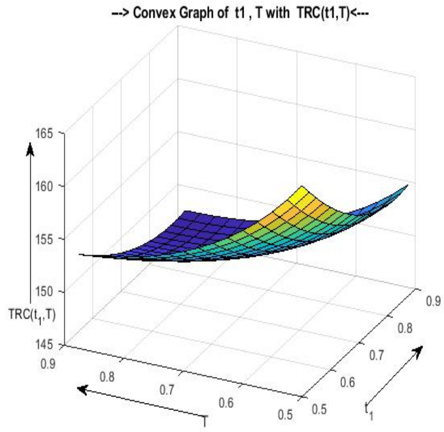

Figure 5: *Convexity Graph of t_1, T with $TRC(t_1, T)$*

6. Sensitivity Analysis and Graphical representation

6.1. Sensitivity analysis of Fuzzy Innovative Economic Order Quantity

Table 2: *Sensitivity analysis of time reliability demand (Parameters P_1, P_2, a, e and f)*

Parameter	Changed values	t_1^*	T^*	Q_F^*	TRC^* (Rs.)
P_1	4.1580	0.8318	0.8698	10.1431	148.2532
	4.2000	0.8324	0.8647	10.1671	148.3721
	4.2420	0.8331	0.8598	10.1912	148.4893
	4.2840	0.8337	0.8548	10.2152	148.6049
P_2	10.0879	0.8343	0.8563	10.2826	148.9947
	10.1898	0.8362	0.8474	10.4028	149.6376
	10.2917	0.8381	0.8384	10.5228	150.2749
	10.4975	0.8418	0.8199	10.765	151.5447
a	1.2120	0.8459	0.8566	10.3447	147.8646
	1.2240	0.8595	0.8485	10.5236	147.3534
	1.2362	0.8733	0.8403	10.7076	146.8281
	1.2485	0.8872	0.8321	10.893	146.2988
e	1.6940	0.8422	0.8454	10.32	147.3337
	1.7464	0.8324	0.8647	10.1671	148.3721
	1.7820	0.8324	0.8647	10.1671	147.8557
	1.8000	0.8341	0.8615	10.1932	148.1983
f	0.18822	0.846	0.8532	10.3799	147.7543
	0.19404	0.8393	0.859	10.2739	148.0635
	0.19800	0.8347	0.8628	10.2028	148.2694
	0.20000	0.8324	0.8647	10.1671	148.3721

Table 3: *Sensitivity analysis of time reliability demand (Parameters β_F, δ_F, μ_F, λ and c_{F2})*

Parameter	Changed values	t_1^*	T^*	Q_F^*	TRC^* (Rs.)
β_F	0.00202	0.8339	0.8619	10.1902	148.3776
	0.00288	0.8333	0.8631	10.1801	148.3753
	0.00360	0.8327	0.8642	10.1718	148.3732
	0.00400	0.8324	0.8647	10.1671	148.3721
β_F	3.7357	0.8469	0.8397	10.3946	147.0331
	3.7734	0.8422	0.8481	10.3197	147.4823
	3.8115	0.8373	0.8565	10.2438	147.9286
	3.8500	0.8324	0.8647	10.1671	148.3721
μ_F	11.1100	0.8459	0.8377	10.3787	148.3928
	11.3322	0.8592	0.8098	10.5892	148.3414
	11.3333	0.8724	0.7807	10.7987	148.2116
	11.4433	0.8853	0.7504	11.0072	147.9958
λ	4.02990	0.8349	0.8633	10.2017	148.2932
	4.07020	0.8374	0.8618	10.2368	148.2132
	4.11090	0.84	0.8603	10.2723	148.1320
	4.15201	0.8426	0.8587	10.3082	148.0497
c_{F2}	14.5545	0.8419	0.8591	10.3146	148.0689
	14.7015	0.8387	0.861	10.2652	148.1704
	14.8500	0.8356	0.8629	10.2161	148.2714
	15.0000	0.8324	0.8647	10.1671	148.3721

Table 4: *Sensitivity analysis of time reliability demand (Parameters c_{F3}, c_{F4} c_{F5}, c_{F6}, r_{F1} and r_{F2})*

Parameter	Changed values	t_1^*	T^*	Q_F^*	TRC^* (Rs.)
c_{F3}	9.7030	0.8442	0.8492	10.3512	147.5376
	9.8010	0.8403	0.8544	10.2902	147.8166
	9.9000	0.8364	0.8596	10.2289	148.0947
	10.0000	0.8324	0.8647	10.1671	148.3721
c_{F4}	5.0500	0.8325	0.8647	10.1675	148.3722
	5.1005	0.8325	0.8646	10.1679	148.3723
	5.1515	0.8325	0.8646	10.1683	148.3724
	5.2030	0.8325	0.8645	10.1687	148.3725
c_{F5}	0.4851	0.8346	0.8615	10.2007	148.1968
	0.4901	0.8339	0.8626	10.1896	148.2548
	0.4950	0.8332	0.8636	10.1784	148.3132
	0.5000	0.8324	0.8647	10.1671	148.3721
c_{F6}	0.9994	0.8498	0.8352	10.4391	146.7880
	1.0095	0.845	0.8436	10.3642	147.2414
	1.0197	0.8373	0.8565	10.2438	147.9286
	1.0300	0.8353	0.8604	10.2117	148.1393
r_{F1}	9.5089	0.8351	0.8595	10.2093	148.0905
	9.6050	0.8342	0.8612	10.1954	148.1837
	9.7020	0.8333	0.863	10.1813	148.2776
	9.8000	0.8324	0.8647	10.1671	148.3721
r_{F2}	39.4020	0.8324	0.8561	10.1671	147.9095
	39.8000	0.8324	0.8647	10.1671	148.3721
	40.1980	0.8324	0.8733	10.1671	148.8301
	40.6000	0.8324	0.8818	10.1671	149.2881

6.2. The graphical representation using Matlab 2018b

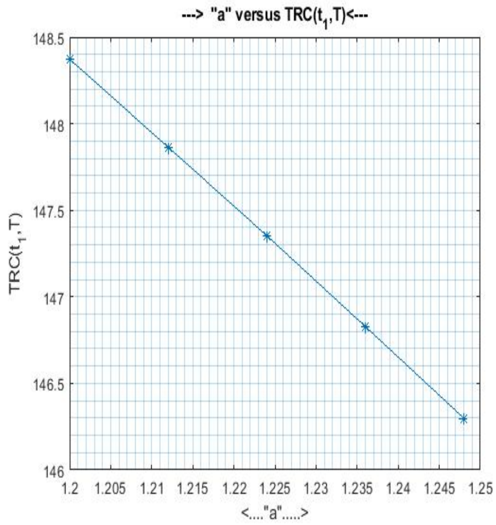

Figure 6: *The impact of a is compared with $TRC(t_1, T)$*

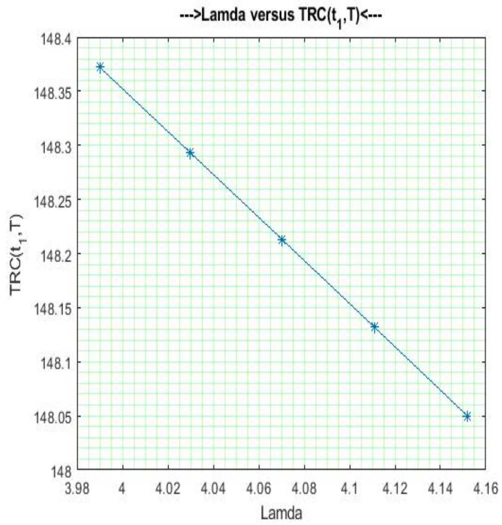

Figure 7: *The impact of λ is compared with $TRC(t_1, T)$*

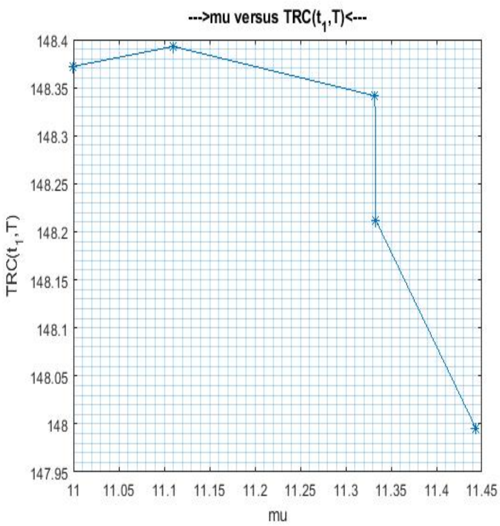

Figure 8: *The impact of μ_F (LSC) is compared with $TRC(t_1, T)$*

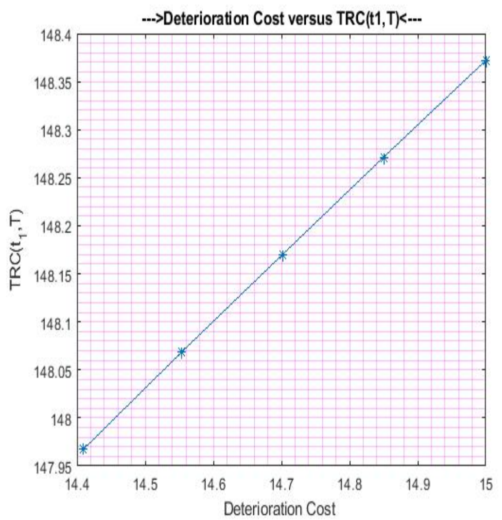

Figure 9: *The impact of Deteriorating cost is compared with $TRC(t_1, T)$*

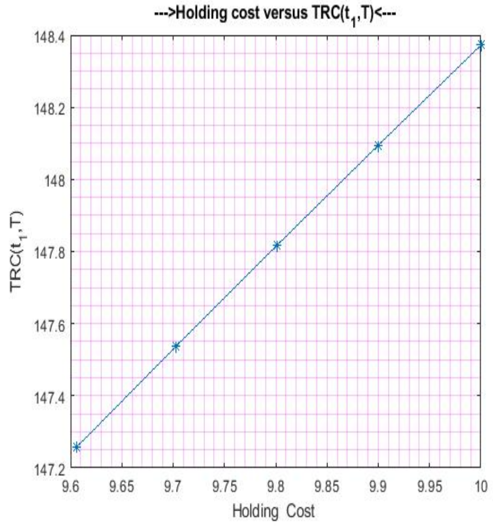

Figure 10: *The impact of Holding cost is compared with $TRC(t_1, T)$*

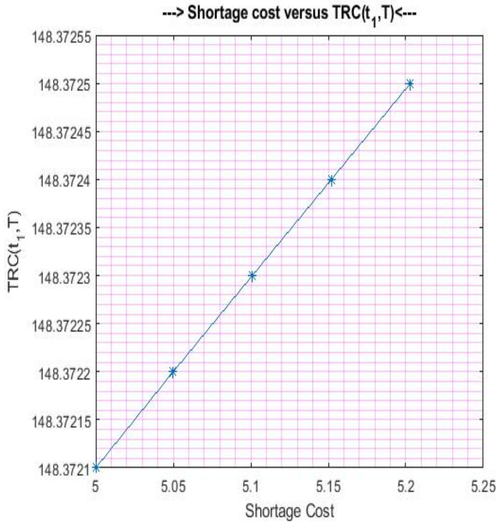

Figure 11: *The impact of Shortage cost is compared with $TRC(t_1, T)$*

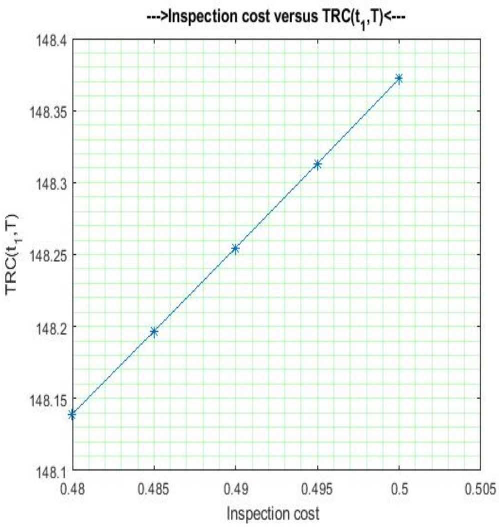

Figure 12: *The impact of Inspection cost is compared with $TRC(t_1, T)$*

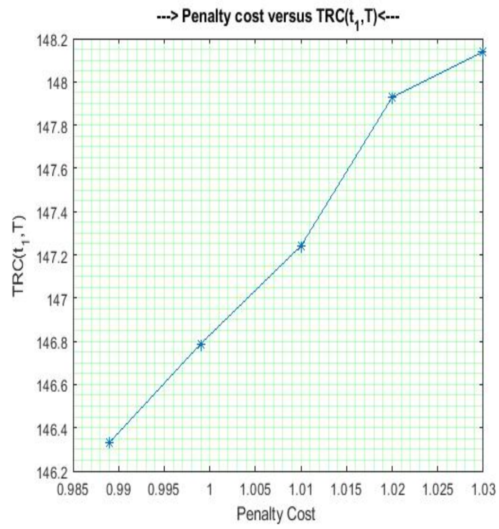

Figure 13: *The impact of Panelty cost is compared with $TRC(t_1, T)$*

7. Observations using table values

Here the investigations are done by using tabular values, let us observe the following progress.

1. While p_2 is raising, the following values t_1, T, Q_F and TRC are oscillating.
2. While the values of a, μ_F & λ are raising, the value of t_1 is raising, T is reducing, Q_F is mounting and TRC is gradually turning down.
3. During the augmentation of the following values, e, f, δ_F, c_{F2}, c_{F3}, c_{F5}, c_{F6} and r_{F1}, t_1 is diminishing, T is growing, Q_F is turning down and TRC is gradually raising.
4. During the mounting of the c_{F4}, the following value of $t_1$is raising , T is growing, Q_F turns up and TRC is gradually leading.
5. While the value of $p_1$is raising, the following values t_1 is raising, T is reducing, Q_F is mounting and TRC is gradually raising
6. While the value of β_F is raising, t_1 is reducing, T is raising, Q_F and TRC are turning down
7. While the value of $p_1$is raising, t_1 is raising, T is reducing, Q_F is mounting and TRC is gradually raising
8. While the value of r_{F2} is raising, the same values of t_1 are repeated, T is raising, the same values of Q_F are repeated and TRC is gradually raising.

8. The Proposed Inventory Model Produced in a Fuzzy Environment

Due to the decision making problem, sometimes the output will be uncertaint and vague, and so some new ideas can be applied to meet the difficulties in characterizing the vagueness and uncertainty. Let us apply the fuzzy environment using Triangular Fuzzy Numbers,

$$\begin{aligned}
TRC(t_1,T) &= \frac{1}{T}\Bigg[r_1 t_1 + r_2 + c_{F2}(\varphi_1\, t_1^3 + \varphi_2\, t_1^{a+3}) \\
&+ c_{F3}\Bigg\{e\, t_1 + f\Bigg[\varphi_3\, t_1^2 + \varphi_1\,\frac{t_1^4}{2} + \varphi_5\, t_1^{a+2} + \varphi_6\, t_1^{a+4} - \varphi_8\, t_1^6 \\
&- \varphi_9\, t_1^{a+4} - \varphi_{10}\, t_1^{a+6}\Bigg]\Bigg\} + c_{F4}\beta_F\left\{\varphi_{11}\,[T^{a+1} - {t_1}^{a+1}] + p_2\,(T-t_1)\right\} \\
&+ \mu_F(1-\beta_F)\,\varphi_{11}\,[T^{a+1} - {t_1}^{a+1}] + p_2\,(T-t_1) + c_{F5}\Bigg\{p_2\, t_1 + \varphi_1 t_1^3 \\
&+ \varphi_{11}\, t_1^{a+1} + \varphi_2\, t_1^{a+3}\Bigg\} + c_{F6}\delta_F(\varphi_{11}\, t_1^{a+1} + p_2 t_1\Bigg] \qquad (16)
\end{aligned}$$

For the convenience, let us do the following suitable substitution.

$$\begin{aligned}
A_1 &= \varphi_1\, t_1^3 + \varphi_2\, t_1^{a+3} \\
A_2 &= e\, t_1 + f\left[\varphi_3\, t_1^2 + \varphi_1\,\frac{t_1^4}{2} + \varphi_5\, t_1^{a+2} + \varphi_6\, t_1^{a+4} - \varphi_8\, t_1^6 - \varphi_9\, t_1^{a+4} - \varphi_{10}\, t_1^{a+6}\right] \\
A_3 &= \beta_F\left\{\varphi_{11}\,[T^{a+1} - {t_1}^{a+1}] + p_2\,(T-t_1)\right\} \\
A_4 &= \mu(1-\beta_F)\,[\varphi_{11}\,[T^{a+1} - {t_1}^{a+1}] + p_2\,(T-t_1)] \\
A_5 &= p_2\, t_1 + \varphi_1 t_1^3 + \varphi_{11}\, t_1^{a+1} + \varphi_2\, t_1^{a+3} \\
A_6 &= \delta_F(\varphi_{11}\, t_1^{a+1} + p_2 t_1)
\end{aligned}$$

In equation (16), substitute the above A_1, A_2, A_3, A_4, A_5, A_6

$$TRC\ (t_1, T) = \frac{1}{T}\Big[r_1 t_1 + r_2 + c_{F2} A_1 + c_{F3} A_2 + c_{F4} A_3 + A_4 + c_{F5} A_5 + c_{F6} A_6\Big] \tag{17}$$

The parameters and costs should be fuzzified using Triangular Fuzzy Number (TFN).

$$\widetilde{r_{F1}} = (r_{11},\ r_{12}, r_{13})\,, \widetilde{r_{F2}} = (r_{21},\ r_{22}, r_{23}),$$

$$\widetilde{c_{F2}} = (c_{F21}, c_{F22}, c_{F23},)\ \widetilde{c_{F3}} = (c_{F31}, c_{F32}, c_{F33}), \widetilde{c_{F4}} = (c_{F41}, c_{F42}, c_{F43}),$$

$$\widetilde{c_{F5}} = (c_{F51}, c_{F52}, c_{F53}), \widetilde{c_{F6}} = (c_{F61}, c_{F62}, c_{F63})$$

$$\begin{aligned} \widetilde{TRC(t_1, T)} &= \frac{1}{T}\Big[(\tilde{r}_{F1}\, t_1 + \tilde{r}_{F2}) + \Big\{\tilde{c}_{F2}\,(\varphi_1\, t_1^3 + \varphi_2\, t_1^{a+3})\Big\} \\ &+ \tilde{c}_{F3}\,(e\, t_1 + f\Big[\varphi_3\, t_1^2 + \varphi_1\, \frac{t_1^4}{2} + \varphi_5\, t_1^{a+2} + \varphi_6\, t_1^{a+4} - \varphi_8\, t_1^6 \\ &- \varphi_9\, t_1^{a+4} - \varphi_{10}\, t_1^{a+6}\Big]\Big\} + \tilde{c}_{F4}\,\beta_F\,\Big\{\varphi_{11}\,[T^{a+1} - {t_1}^{a+1}] + p_2\,(T - t_1)\Big\} \\ &+ \mu_F(1-\beta_F)\,\varphi_{11}\,[T^{a+1} - {t_1}^{a+1}] + p_2\,(T - t_1) + \tilde{c}_{F5}\Big\{p_2\, t_1 + \varphi_1 t_1^3 \\ &+ \varphi_{11}\, t_1^{a+1} + \varphi_2\, t_1^{a+3}\Big\} + \tilde{c}_{F6}\delta_F(\varphi_{11}\, t_1^{a+1} + p_2 t_1\Big] \end{aligned} \tag{18}$$

$$\begin{aligned} \widetilde{TRC(t_1, T)} &= \frac{1}{T}\Big[((r_{11},\ r_{12}, r_{13})t_1 + (r_{21},\ r_{22}, r_{23})) + \Big\{(c_{F21}, c_{F22}, c_{F23}) \\ &\times(\varphi_1\, t_1^3 + \varphi_2\, t_1^{a+3})\Big\} + (c_{F31}, c_{F32}, c_{F33})(e\, t_1 + f\Big[\varphi_3\, t_1^2 + \varphi_1\, \frac{t_1^4}{2} \\ &+\varphi_5\, t_1^{a+2} + \varphi_6\, t_1^{a+4} - \varphi_8\, t_1^6 - \varphi_9\, t_1^{a+4} - \varphi_{10}\, t_1^{a+6}\Big]\Big\} \\ &+(c_{F41}, c_{F42}, c_{F43})\beta_F\,\Big\{\varphi_{11}\,[T^{a+1} - {t_1}^{a+1}] + p_2\,(T - t_1)\Big\} \\ &+\mu(1-\beta)\,\varphi_{11}\,[T^{a+1} - {t_1}^{a+1}] + p_2\,(T - t_1) + (c_{F51}, c_{F52}, c_{F53}) \\ &\times\Big\{p_2\, t_1 + \varphi_1 t_1^3 + \varphi_{11}\, t_1^{a+1} + \varphi_2\, t_1^{a+3}\Big\} + (c_{F61}, c_{F62}, c_{F63})\,\delta_F \\ &\times(\varphi_{11}\, t_1^{a+1} + p_2 t_1\Big] \\ &= (U_F, V_F, W_F) \end{aligned} \tag{19}$$

where,

$$\begin{aligned} U_F &= \frac{1}{T}\Big[(r_{11})t_1 + r_{21} + c_{F21}\Big\{\varphi_1\, t_1^3 + \varphi_2\, t_1^{a+3}\Big\} + c_{F31}\, e\, t_1 + f\Big[\varphi_3\, t_1^2 + \varphi_1\, \frac{t_1^4}{2} \\ &+\varphi_5\, t_1^{a+2} + \varphi_6\, t_1^{a+4} - \varphi_8\, t_1^6 - \varphi_9\, t_1^{a+4} - \varphi_{10}\, t_1^{a+6}\Big] + \Big[c_{F41}\,\beta_F + \mu_F(1-\beta_F)\Big] \\ &\times\Big\{\varphi_{11}\,[T^{a+1} - {t_1}^{a+1}] + p_2\,(T - t_1)\Big\} + c_{F51}\,\Big\{p_2\, t_1 + \varphi_1 t_1^3 + \varphi_{11}\, t_1^{a+1} \end{aligned}$$

$$+\varphi_2\, t_1^{a+3}\Big\} + c_{F61}\,\delta_F(\varphi_{11}\, t_1^{a+1} + p_2 t_1)\Big] \tag{20}$$

$$\begin{aligned} V_F &= \frac{1}{T}\Bigg[(r_{12})t_1 + r_{22} + c_{F22}\Big\{\varphi_1\, t_1^3 + \varphi_2\, t_1^{a+3}\Big\} + c_{F32}\, e\, t_1 + f\Big[\varphi_3\, t_1^2 + \varphi_1\, \frac{t_1^4}{2} \\ &+\varphi_5 t_1^{a+2} + \varphi_6\, t_1^{a+4} - \varphi_8\, t_1^6 - \varphi_9\, t_1^{a+4} - \varphi_{10}\, t_1^{a+6}\Big] + \Big[c_{F42}\,\beta_F + \mu_F(1-\beta_F)\Big] \\ &\times\Big\{\varphi_{11}\,[T^{a+1} - {t_1}^{a+1}] + p_2\,(T - t_1)\Big\} + c_{F52}\Big\{p_2\, t_1 + \varphi_1 t_1^3 + \varphi_{11}\, t_1^{a+1} \\ &+\varphi_2\, t_1^{a+3}\Big\} + c_{F62}\delta_F(\varphi_{11}\, t_1^{a+1} + p_2 t_1)\Bigg] \end{aligned} \tag{21}$$

$$\begin{aligned} W_F &= \frac{1}{T}\Bigg[(r_{13})t_1 + r_{23} + \Big\{c_{F23}(\varphi_1\, t_1^3 + \varphi_2\, t_1^{a+3})\Big\} + c_{F33}\, e\, t_1 + f\Big[\varphi_3\, t_1^2 + \varphi_1\, \frac{t_1^4}{2} \\ &+\varphi_5\, t_1^{a+2} + \varphi_6\, t_1^{a+4} - \varphi_8\, t_1^6 - \varphi_9\, t_1^{a+4} - \varphi_{10}\, t_1^{a+6}\Big] + \Big[c_{F43}\,\beta_F + \mu_F(1-\beta_F)\Big] \\ &\times\Big\{\varphi_{11}\,[T^{a+1} - {t_1}^{a+1}] + p_2\,(T - t_1)\Big\} + c_{F53}\,\Big\{p_2\, t_1 + \varphi_1 t_1^3 + \varphi_{11}\, t_1^{a+1} \\ &+\varphi_2\, t_1^{a+3}\Big\} + c_{F63}\,\delta_F(\varphi_{11}\, t_1^{a+1} + p_2 t_1)\Bigg] \end{aligned} \tag{22}$$

The κ-cuts $\mho_L(\kappa)\,\&\,\mho_R(\kappa)$ of Triangular Fuzzy Numbers. $TRC\widetilde{(t_1, T)}$ are given by

$$\begin{aligned} \mho_L(\kappa) &= U_F + (V_F - U_F)\kappa \\ &= \frac{1}{T}\Bigg[(r_{11})t_1 + r_{21} + c_{F21}\, A_1 + c_{F31}\, A_2 + c_{F41}\, A_3 + A_4 \\ &+c_{F51}\, A_5 + c_{F61}\, A_6 + \Big\{(r_{21} - r_{11})t_1 + (r_{22} - r_{21}) + (c_{F22} - c_{F21})A_1 \\ &+(c_{F32} - c_{F31})A_2 + (c_{F42} - c_{F41})A_3 + A_4 + (c_{F52} - c_{F51})A_5 \\ &+(c_{F62} - c_{F61})A_6\Big\}\kappa\Bigg] \end{aligned} \tag{23}$$

$$\begin{aligned} \mho_R(\kappa) &= W_F - (W_F - V_F)\kappa \\ &= \frac{1}{T}\Bigg[(r_{13})t_1 + r_{23} + c_{F23}\, A_1 + c_{F33}\, A_2 + c_{F43}\, A_3 + A_4 \\ &+c_{F53}\, A_5 + c_{F63}\, A_6 + \Big\{(r_{13} - r_{12})t_1 + (r_{23} - r_{23}) + (c_{F23} - c_{F23})A_1 \\ &+(c_{F33} - c_{F33})A_2 + (c_{F43} - c_{F43})A_3 + A_4 + (c_{F53} - c_{F53})A_5 \\ &+(c_{F63} - c_{F62})A_6\Big\}\kappa\Bigg] \end{aligned} \tag{24}$$

By apply the Signed Distance Method, the defuzzified value of average TRC, using the fuzzy number

$$
\begin{aligned}
TRC(t_1,T) &= \frac{1}{2}\left[\int_0^1 \{\mho_L\kappa + \mho_R\kappa\}\, d\kappa\right] \\
&= \frac{1}{4T}\Big[(r_{11}+2r_{12}+r_{13})t_1 + (r_{21}+2r_{22}+r_{23}) + (c_{F21}+2c_{F22} \\
&\quad +c_{F23})A_1 + (c_{F31}+2c_{F32}+c_{F33})A_2 + [c_{F41}+2c_{F42}+c_{F43}]\,A_3 \\
&\quad +4A_4 + (c_{F51}+2c_{F52}+c_{F53})A_5 + (c_{F61}+2c_{F62} \\
&\quad +c_{F63})A_6\Big] \qquad (25)
\end{aligned}
$$

9. Solutions and numerical problems using triangular fuzzy numbers of different data

9.1. Solutions using triangular fuzzy numbers

For the solution purpose of equation (19), MATLAB R2018b and Excel 2010 solver are used to find all the optimal solutions.
For optimization let us do the following:

The necessary condition for the least value of $TRC\,(t_1,T)$ are,

$$\frac{\partial(\widetilde{TRC}\,(t_1,T)}{\partial t_1} = 0 \text{ and } \frac{\partial(\widetilde{TRC}\,(t_1,T)}{\partial T} = 0$$

The sufficient condition for optimal $TRC\,(t_1,T)$, $t_1 > 0, T > 0$.

$$\begin{vmatrix} \dfrac{\partial^2(\widetilde{TRC})}{\partial t_1^2} & \dfrac{\partial^2(\widetilde{TRC})}{\partial t_1\,\partial T} \\ \dfrac{\partial^2(\widetilde{TRC})}{\partial T\,\partial t_1} & \dfrac{\partial^2(\widetilde{TRC})}{\partial T^2} \end{vmatrix} > 0$$

Therefore the optimal fuzzy solutions of t_1^*, T^*, Q_F^* and TRC^* are found and given in the table.

Table 5: *Optimal solution using fuzzy Numbers*

$t_1{}^*$	T^*	Q_F^*	$TRC(t_1,T)^*(Rs.)$
0.54526521	0.6455852	20.4035902	135.998429

9.2. Sample problems using triangular fuzzy numbers

Example 4. Let's take the input: p_1 = 8.25, p_2 = 31.75, a = 1.25, e = 1.825, f = 0.385, β_F = 0.0044, δ_F = 4.2, μ_F = 13.75, λ = 4.25, $\widetilde{c_{F2}} = (8, 11.5, 15)$, $\widetilde{c_{F3}} = (2, 2.5, 3)$, $\widetilde{c_{F4}} = (1.25, 2.125, 3)$, $\widetilde{c_{F5}} = (0.2, 0.3, 0.4)$, $\widetilde{c_{F6}} = (0.22, 0.33, 0.44)$, $\widetilde{r_{F1}}$ = (8.55,10.525,12.5), $\widetilde{r_{F2}} = (20.5, 21.5, 22.5)$.
The optimal solutions are Q_F^* = 19.6661, t_1^* =0.57313, T^* =0.70023, & TRC^* = Rs.127.092.

Example 5. Let's take the input: p_1 = 8.25, p_2 = 34.75, a = 1.2, e = 1.825, f = 0.385, β_F = 0.0044, δ_F = 4.2, μ_F = 13.75, λ = 4.25, $\widetilde{c_{F2}} = (8, 11.5, 15)$, $\widetilde{c_{F3}} = (2, 2.5, 3)$, $\widetilde{c_{F4}} = (1.25, 2.125, 3)$,

$\widetilde{c_{F5}} = (0.2, 0.3, 0.4)$, $\widetilde{c_{F6}} = (0.22, 0.33, 0.44)$, $\widetilde{r_{F1}}$ =(8.55,10.525,12.5),$\widetilde{r_{F2}} = (20.5, 21.5, 22.5\ )$.
The optimal solutions are $Q_F^* = 20.4035902$, t_1^* =0.54526521, $T^* = 0.6455852$, & TRC^* = Rs.135.998429.

Example 6. Let's take the input: p_1 = 8.25, p_2 = 36, a = 1.2, e = 1.825 , f = 0.385, β_F = 0.0044, δ_F = 4.2, μ_F = 13.75, λ = 4.25, $\widetilde{c_{F2}} = (8, 11.5, 15)$, $\widetilde{c_{F3}} = (2, 2.5, 3)$, $\widetilde{c_{F4}} = (1.25, 2.125, 3)$, $\widetilde{c_{F5}} = (0.2, 0.3, 0.4)$, $\widetilde{c_{F6}} = (0.22, 0.33, 0.44)$, $\widetilde{r_{F1}}$ = (8.55,10.525,12.5), $\widetilde{r_{F2}} = (20.5, 21.5, 22.5)$.
The optimal solutions are Q_F^* = 21.2316, t_1^* =0.54833, T^* =0.60289, & TRC^*= Rs.138.968.

10. Comparison of Crisp and Fuzzy Optimal Solutions

Table 6: *Comparison of crisp and fuzzy solutions*

	$t_1{}^*$	T^*	Q_F^*	$TRC(t_1, T)^*(Rs.)$
Crisp	0.83244	0.86903	10.1671	148.603
Fuzzy	0.54526521	0.6455852	20.4035902	135.998429

11. Conclusion & Extending investigation scope

In this study, an attempt is made to formulate an inventory model of innovative economic order with the quantity of items. The considerations in this paper are (i) the demand is consistent with time deterioration,(ii) the holding cost has been used as dependent on the amount of stock available in the system, and (iii) the ordering cost is linear and time-dependent. This system should be considered in terms of crisp and fuzziness. It is assumed that the shortages are permitted partially and the quantity ordered is inspected to reduce defective items. To use the penalty cost delay of supplying items should be minimized. Under the classical model and fuzzy environment, a mathematical equation is arrived. The optimal solution of total relevant cost with optimal order quantity and time using triangular fuzzy numbers has been found. Defuzzification has been accomplished through the use of the signed distance method of integration. The solutions have been arrived at and verified by using model with a few numerical problems of three levels of values (lower, medium, and upper) in parametric changes. Sensitivity analysis is used to validate the changes in different values of the system's parameters. To demonstrate the convexity of the total relevant cost function over time, a three-dimensional mesh graph has been used. This model can be modified and developed further by changing the demand into probabilistic, price, advertisement dependent etc.

References

[1] Abhishek Kanti Biswas and Sahidul Islam, 2019, "A fuzzy epq model for non-instantaneous deteriorating items where production depends on demand which is proportional to population, selling price as well as advertisement". Independent journal of management and production, Volume (10), issue (5), pp. 1679-1703.

[2] Dutta D and Kumar P, 2013, "Optimal Policy for an Inventory Model without Shortages considering Fuzziness in Demand Holding Cost and Ordering Cost". International Journal of Advanced and Innovative Research, Volume (2), issue (3), pp. 320-325.

[3] Pavan Kumar, 2021, "Optimal policies for inventory model with shortages time-varying holding and ordering costs in trapezoidal fuzzy environment". Independent journal of management and production, Volume (12), issue (2), pp. 557-574.

[4] Sankar Kumar Roy and Magfura Pervin, 2019, " Imperfection with inspection policy and variable demand under trade-credit a deteriorating inventory model". numerical Algebra Control and optimization, Volume (10), issue (05), pp. 45-74.

[5] Sivan V and Thirugnanasambandam K, 2020, "A Joint activity of single retailer and single supplier with price dependent demand in changeable decision of lead time with shortages". Advances in Mathematics Scientific Journal, Volume (9), issue (7), pp. 4641-4658.

[6] Srabani Shee and Tripti Chakrabarti, 2020, "A Fuzzy Two-Echelon Supply Chain Model for Deteriorating Items with Time Varying Holding Cost Involving Lead Time as a Decision Variable". Optimization and Inventory Management, Asset Analytics, Springer book series (ASAN), chap (21) pp. 391-406.

[7] Thirugnanasambandam K and Sivan V, 2020, "Estimation of EOQ Model for Drug Inventory in Wellness Industries Using Time Dependent Negative Exponential Demand and Holding Cost of Linear Equation Without Constant Term". Journal of advanced research and dynamical control system, Volume (12), issue (5), pp. 91-106.

[8] Tripathi R.P., 2020, "Innovative investigation of stock sensitive demand induced economic order quantity (EOQ) model for deterioration by means of inconsistent holding cost functions". Springer Nature Switzerland AGSN Applied Sciences, Volume (2), issue (4), pp. 1-8.

[9] Sudip Adak and Mahapatra G.S., 2018, "An inventory model of time and reliability Dependent demand with deterioration and partial backordering under fuzziness". Recent Advances in Intelligent Information Systems and Applied Mathematics, Book series (SCI, volume 863), pp. 610-627.

[10] Magfura Pervin, Sankar Kumar Roy and Gerhard Wilhelm Weber, 2019, "An integrated vendor-buyer model with quadratic demand under inspection policy and preservation technology". Hacettepe Journal of Mathematics and Statistics Hacet, Volume (20), issue(10) pp. 1-22.