STOCHASTIC ANALYSIS OF JUICE PLANT SUBJECT TO REPAIR FACILITY

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Abstract

The performance of a juice plant is analyzed by using the base state and the regenerative point graphical technique. The juice plant under consideration consists of three distinct units. It is considered that units A and B may be in a complete failed state through partial failure mode but unit C is in only partially failed state. If one of the units A or B or C partially fails then the system works in a reduced state. When any unit is completely failed then the system is in failed state and no unit can fail further when the system is in a failed state. A technician is always available to repair the failed unit. In this paper, the failure time and repair time follow general distributions. Tables are used to describe the reliability measures such as mean time to system failure, availability and profit values of juice plant.

Keywords: Reliability, juice plant, repair time, mean sojourn time and profit.

I. Introduction

Nowadays, manufacturers have to produce their products continuously to meet the increasing demands of their products which are possible by making their productions as efficient as possible. This paper discusses the MTSF, availability and profit of a juice plant with priority in repair using the regenerative point graphical technique under specified conditions. A large amount of research work has been done on repairable systems such that Bao and Mays [2] analyzed the hydraulic reliability of water distribution systems under demand, pipe roughness and pressure head. Gnedenko and Igor [7] explored reliability and probability studies for engineering purposes. Jack and Murthy [9] discovered the role of limited warranty and extended warranty for the product. Wang and Zhang [19] examined the repairable system of two non identical components under repair facility using geometric distributions. Diaz et al. [5] threw light on the warranty cost management system. Kumar and Goel [15] explored the idea of an imperfect switch on redundant systems in banking industry. Goyal [8] described the availability and behavior of single unit system under preventive maintenance and degradation after complete failure using RPGT. Kumar and Goel [14] analyzed the preventive maintenance in two unit cold standby system under general distributions. Malik and Rathee [17] threw light on the two parallel units system under preventive maintenance and maximum operation time. Kashid and Kumar [11] examined the availability of two unit system under degradation and subject to the repair facility. Kumar et al. [12] evaluate the effects of washing unit in the paper industry by using the regenerative point graphical technique. Levitin et al. [16] explored the results of optimal preventive replacement of failed units in a cold standby system by using the poisson process. Agarwal et al. [1] analyzed the performance and reliability of water treatment plant under repair facility. Barak et al. [3] threw light on the availability and profit values of milk plant under repair facility. Kumar et al. [13] described the cold standby redundant system under repair and refreshment facilities subject to inspection. Chaudhary and Sharma [4] explored the parallel non identical units system that gives priority to repair over preventive maintenance. Garg and Garg [6] analyzed the reliability and profit values of briquette machine under neglected faults like sound and overheating. Jia et al. [10] explored the two unit system under demand and energy storage techniques. Sengar and Mangey [18] examined the performance of complicated systems under inspection using copula methodology.

II. System Assumptions

There are following system assumptions:

- The juice plant consists of three distinct units.
- Unit A consists of a washing and storage tank.
- Unit B has grinding, blending, evaporation and pasteurization.
- Unit C has bottling, labeling and packing units.
- It is considered that units A and B may be in a complete failed state through partial failure mode but unit C is in only partially failed state.
- Failure and repair times follow general distributions.
- The failed unit works like a new unit after repair.

III. System Notations

There are following system notations:

$i \xrightarrow{Sr} j$	r^{th} directed simple path from state '*i*' to state '*j*' where '*r*' takes the positive integral values for different directions from state '*i*' to state '*j*'.
$\xi \xrightarrow{sff} i$	A directed simple failure free path from state ξ to state '*i*'.
$m - cycle$	A circuit (may be formed through regenerative or non regenerative / failed state) whose terminals are at the regenerative state '*m*'.
$m - \overline{cycle}$	A circuit (may be formed through the unfailed regenerative or non regenerative state) whose terminals are at the regenerative '*m*' state.
$U_{k,k}$	Probability factor of the state '*k*' reachable from the terminal state '*k*' of '*k*' cycle.
$U_{\overline{k,k}}$	The probability factor of state '*k*' reachable from the terminal state '*k*' of $k\ \overline{cycle}$.
μ_i	Mean sojourn time spent in the state '*i*' before visiting any other states.
μ_i'	Total unconditional time spent before transiting to any other regenerative state while the system entered regenerative state '*i*' at t=0.
η_i	Expected waiting time spent while doing a job given that the system entered to the regenerative state '*i*' at t=0.
$A / \overline{A} / a$	System's first unit is in the operative state/reduced state/failed state.
$B / \overline{B} / b$	System's second unit is in the operative state/reduced state/failed state.
$C / \overline{C} / c$	System's third unit is in the operative state/reduced state/failed state.
$\lambda_1, \lambda_2, \lambda_3$	The constant partial failure rate of the unit A/B/C respectively.
λ_4, λ_5	The constant complete failure rate of the unit A/B respectively.
w_1, w_2, w_3	Fixed repair rate of the unit A/B/C after partial failure respectively.
w_4, w_5	Fixed repair rate of unit A/B after the complete failure respectively.
○ ⬭ ▭	Upstate/ reduced state/ failed state

IV. Circuits Descriptions

The individual circuit description is given by the table 1:

Table 1: *Circuit Descriptions*

Primary, Secondary and Tertiary Circuit at the vertex (*i*)			
i	(C1)	(C2)	(C3)
0	(0,1,0), (0,2,0), (0,3,0) (0,1,4,0), (0,2,5,0)	Nil	Nil
1	(1,0,1)	(0,2,0), (0,3,0)	Nil
2	(2,0,2)	(0,1,0), (0,3,0)	Nil
3	(3,0,3)	(0,1,0), (0,2,0)	Nil
4	(4,0,1,4)	(0,1,0), (0,2,0) (0,3,0), (1,0,1)	(2,0,2), (3,0,3)
5	(5,0,2,5)	(0,1,0), (0,2,0) (0,3,0), (2,0,2)	(1,0,1), (3,0,3)

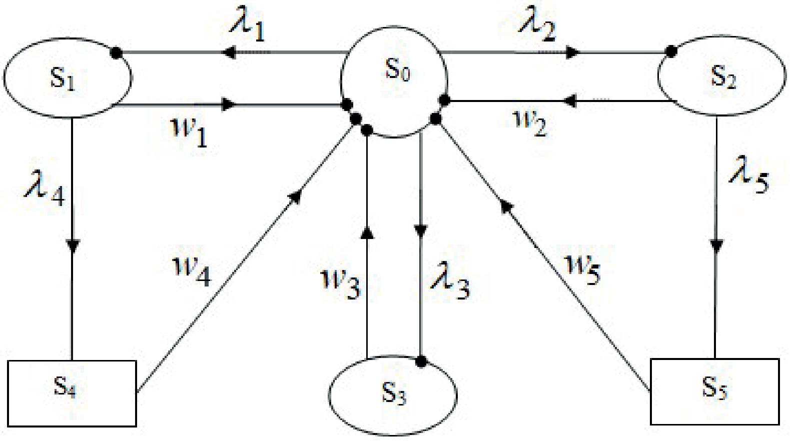

Figure 1: *State Transition Diagram*

where, $S_0 = ABC$, $S_1 = \bar{A}BC$, $S_2 = A\bar{B}C$, $S_3 = AB\bar{C}$, $S_4 = aBC$, $S_5 = AbC$

V. Transition Probabilities

The transition probabilities are following

$p_{0,1} = \lambda_1 /(\lambda_1 + \lambda_2 + \lambda_3)$, $p_{0,2} = \lambda_2 /(\lambda_1 + \lambda_2 + \lambda_3)$, $p_{0,3} = \lambda_3 /(\lambda_1 + \lambda_2 + \lambda_3)$

$p_{1,0} = w_1 /(w_1 + \lambda_4)$, $p_{1,4} = \lambda_4 /(w_1 + \lambda_4)$, $p_{2,0} = w_2 /(w_2 + \lambda_5)$

$$p_{2,5} = \lambda_5 /(w_2 + \lambda_5),\ p_{3,0} = p_{4,0} = p_{5,0} = 1 \quad (1)$$

It has been conclusively established that

$$p_{01} + p_{03} = 1,\ p_{10} + p_{12} + p_{14} = 1,\ p_{21} + p_{27} = 1,\ p_{31} + p_{38} = 1$$

$$p_{41} + p_{45} = 1,\ p_{56} = p_{76} = p_{86} = 1,\ p_{31} + p_{31.8(65)^n} = 1$$

$$p_{10} + p_{12} + p_{11.4} + p_{11.4(56)^n} = 1,\ p_{21} + p_{21.7(65)^n} = 1 \quad (2)$$

VI. Mean Sojourn Time

Let μ_i represents the mean sojourn time. Mathematically, the time taken by a system in a particular state becomes

$$\mu_i = \sum_j m_{i,j} = \int_0^\infty P(T > t)\,dt .$$

and $\mu_0 = 1/(\lambda_1 + \lambda_2 + \lambda_3), \mu_1 = 1/(w_1 + \lambda_4)$, $\mu_2 = 1/(w_2 + \lambda_5)$

$$\mu_3(t) = 1/(w_3),\ \mu_4 = 1/(w_4),\ \mu_5 = 1/(w_5) \quad (3)$$

VII. Evaluation of Parameters

All reliability parameters (such as mean time to system failure, availability, busy period of the server and expected number of visits) are determined by using the regenerative point graphical technique. The probability factors of all the reachable states from the base state '0' are given below

$$U_{0,0} = (0,1,0) + (0,2,0) + (0,3,0) = 1,\ U_{0,1} = \frac{\lambda_1}{\lambda_1 + \lambda_2 + \lambda_3},\ U_{0,2} = \frac{\lambda_2}{\lambda_1 + \lambda_2 + +\lambda_3}$$

$$U_{0,3} = \frac{\lambda_3}{(\lambda_1 + \lambda_2 + \lambda_3)},\ U_{0,4} = \frac{\lambda_1\lambda_4}{(\lambda_1 + \lambda_2 + \lambda_3)(w_1 + \lambda_4)},\ U_{0,5} = \frac{\lambda_2\lambda_5}{(\lambda_1 + \lambda_2 + \lambda_3)(w_2 + \lambda_5)}$$

I. Mean Time to System Failure (MTSF)

The regenerative un-failed states (i=0, 1, 2, 3) to which the system can transit (with initial state 0) before entering to any failed state (using base state ξ=0) then MTSF becomes

$$T_0 = \left[\sum_{i=0}^{3} Sr \left\{ \frac{\left\{ pr\left(0 \xrightarrow{Sr(sff)} i\right) \right\} . \mu_i}{\Pi_{k_1 \neq 0} \left\{ 1 - V_{\overline{k_1 k_1}} \right\}} \right\} \right] \div \left[1 - \sum Sr \left\{ \frac{\left\{ pr\left(0 \xrightarrow{Sr(sff)} 0\right) \right\}}{\Pi_{k_2 \neq 0} \left\{ 1 - V_{\overline{k_2 k_2}} \right\}} \right\} \right]$$

$$T_0 = \frac{(w_1 + \lambda_4)(w_2 + \lambda_5)(w_3 + \lambda_3) + w_3[\lambda_1(w_2 + \lambda_5) + \lambda_2(w_1 + \lambda_4)]}{w_3[(\lambda_1 + \lambda_2 + \lambda_3)(w_1 + \lambda_4)(w_2 + \lambda_5) - \lambda_1 w_1(w_2 + \lambda_5) - \lambda_2 w_2(w_1 + \lambda_4)]} \quad (4)$$

II. Availability of the system

The system is available for use at regenerative states j=0, 1, 2, 3 with ξ=0 then the availability of system is defined as

$$A_0 = \left[\sum_{j=0}^{3} Sr \left\{ \frac{\left\{ pr\left(0 \xrightarrow{Sr} j\right) \right\} . f_j . \mu_j}{\prod_{k_1 \neq 0} \left\{ 1 - V_{\overline{k_1 k_1}} \right\}} \right\} \right] \div \left[\sum_{i=0}^{5} Sr \left\{ \frac{\left\{ pr\left(0 \xrightarrow{Sr} i\right) \right\} . \mu_i'}{\prod_{k_2 \neq 0} \left\{ 1 - V_{\overline{k_2 k_2}} \right\}} \right\} \right]$$

$$A_0 = \frac{w_4 w_5 [(w_1 + \lambda_4)(w_2 + \lambda_5)(w_3 + \lambda_3) + w_3 \{\lambda_1 (w_2 + \lambda_5) + \lambda_2 (w_1 + \lambda_4)\}]}{\left[\begin{array}{l} w_4 w_5 (w_1 + \lambda_4)(w_2 + \lambda_5)(w_3 + \lambda_3) + \lambda_1 w_3 w_5 (w_2 + \lambda_5)(w_4 + \lambda_4) \\ \qquad + \lambda_2 w_3 w_4 (w_1 + \lambda_4)(w_5 + \lambda_5) \end{array} \right]} \quad (5)$$

III. Busy Period of the Server

The server is busy due to repair of the failed unit at regenerative states j= 1, 2, 3, 4, 5 with ξ = 0 then the fraction of time for which the server remains busy is defined as

$$B_0 = \left[\sum_{j=1}^{5} Sr \left\{ \frac{\left\{ pr\left(0 \xrightarrow{Sr} j\right) \right\} . \eta_j}{\prod_{k_1 \neq 0} \left\{ 1 - V_{\overline{k_1 k_1}} \right\}} \right\} \right] \div \left[\sum_{i=0}^{5} Sr \left\{ \frac{\left\{ pr\left(0 \xrightarrow{Sr} i\right) \right\} . \mu_i'}{\prod_{k_2 \neq 0} \left\{ 1 - V_{\overline{k_2 k_2}} \right\}} \right\} \right]$$

$$B_0 = \frac{\left[\begin{array}{l} w_4 w_5 \lambda_3 (w_1 + \lambda_4)(w_2 + \lambda_5) + \lambda_1 w_3 w_5 (w_2 + \lambda_5)(w_4 + \lambda_4) \\ \qquad + \lambda_2 w_3 w_4 (w_1 + \lambda_4)(w_5 + \lambda_5) \end{array} \right]}{\left[\begin{array}{l} w_4 w_5 (w_1 + \lambda_4)(w_2 + \lambda_5)(w_3 + \lambda_3) + \lambda_1 w_3 w_5 (w_2 + \lambda_5)(w_4 + \lambda_4) \\ \qquad + \lambda_2 w_3 w_4 (w_1 + \lambda_4)(w_5 + \lambda_5) \end{array} \right]} \quad (6)$$

IV. Estimated number of visits made by the server

The repairman visits at regenerative states j= 1, 2, 3 with ξ=0 then the number of visits by the repairman is defined as

$$V_0 = \left[\sum_{j=1}^{3} Sr \left\{ \frac{\left\{ pr\left(0 \xrightarrow{Sr} j\right) \right\}}{\prod_{k_1 \neq 0} \left\{ 1 - V_{\overline{k_1 k_1}} \right\}} \right\} \right] \div \left[\sum_{i=0}^{5} Sr \left\{ \frac{\left\{ pr\left(0 \xrightarrow{Sr} i\right) \right\} . \mu_i'}{\prod_{k_2 \neq 0} \left\{ 1 - V_{\overline{k_2 k_2}} \right\}} \right\} \right]$$

$$V_0 = \frac{\left[\begin{array}{l} w_4 w_5 \lambda_3 (w_1 + \lambda_4)(w_2 + \lambda_5)) + \lambda_1 w_3 w_4 w_5 (w_2 + \lambda_5) \\ \qquad + \lambda_2 w_3 w_4 w_5 (w_1 + \lambda_4) \end{array} \right]}{\left[\begin{array}{l} w_4 w_5 (w_1 + \lambda_4)(w_2 + \lambda_5)(w_3 + \lambda_3) + \lambda_1 w_3 w_5 (w_2 + \lambda_5)(w_4 + \lambda_4) \\ \qquad + \lambda_2 w_3 w_4 (w_1 + \lambda_4)(w_5 + \lambda_5) \end{array} \right]} \quad (7)$$

V. Profit Analysis

The profit function may be used to do a profit analysis of the system and it is given by

$$P = E_0 A_0 - E_1 B_0 - E_2 V_0 \quad (8)$$

where, E_0 =5000 (Pay per unit uptime of the system)

$E_1 = 1000$ (Charge per unit time for which server is busy due to repair)

$E_2 = 500$ (Charge per visit of the server)

VIII. Discussion

Table 2 describes the nature of mean time to system failure of the juice plant having an increasing trend corresponding to increment in repair rate (w_2). In this table, the values of parameters λ_1=0.2, λ_2=0.3, λ_3=0.15, λ_4=0.25, λ_5=0.4, w_1=0.35, w_3=0.4, w_4=0.45, w_5=0.5 respectively taking as constant for the simplicity. When λ_1=0.2 changing into λ_1=0.25; λ_2=0.3 changing into λ_2=0.35 and λ_3=0.15 changing into λ_3=0.2 then MTSF values have decreasing trends.

Table 2: *MTSF vs. Repair Rate*

w_2 ↓	λ_1=0.2, λ_2=0.3 λ_3=0.15, λ_4=0.25 λ_5=0.4, w_1=0.35 w_3=0.4, w_4=0.45 w_5=0.5	λ_1=0.25	λ_2=0.35	λ_3=0.2
0.1	4.0647311	3.8613371	3.218144	3.775294
0.2	4.1010786	3.8894957	3.235483	3.806505
0.3	4.1312067	3.9129156	3.250148	3.833119
0.4	4.1565858	3.9327001	3.262713	3.856081
0.5	4.1782569	3.9496349	3.273599	3.876094
0.6	4.1969772	3.9642941	3.283122	3.893693
0.7	4.2133109	3.9771076	3.291522	3.90929
0.8	4.2276872	3.9884034	3.298987	3.923207
0.9	4.240438	3.998436	3.305664	3.935703
1	4.2518242	4.007406	3.311673	3.946984

Table 3 explores the increasing trends of availability corresponding to increments in repair rate (w_2) where the system's other parameters possess constant values. When the failure rate of unit changes (λ_1=0.2 to 0.25), (λ_2=0.3 to 0.35) and (λ_3=0.15 to 0.2) then the availability of system declines.

Table 3: *Availability vs. Repair Rate*

w_2 ↓	λ_1=0.2, λ_2=0.3 λ_3=0.15, λ_4=0.25 λ_5=0.4, w_1=0.35 w_3=0.4, w_4=0.45 w_5=0.5	λ_1=0.25	λ_2=0.35	λ_3=0.2
0.1	0.25028	0.214076	0.185918	0.190417
0.2	0.364639	0.319498	0.281347	0.293845
0.3	0.427923	0.381465	0.338725	0.358019
0.4	0.467056	0.421888	0.376681	0.401314
0.5	0.493101	0.450148	0.403461	0.432266
0.6	0.511375	0.470912	0.423258	0.455361
0.7	0.524719	0.48675	0.438421	0.473168
0.8	0.534774	0.499191	0.450366	0.487264
0.9	0.542545	0.509198	0.45999	0.498663
1	0.548679	0.517404	0.46789	0.508046

Table 4 explores the trend of profit values with respect to repair rate (w_2) and its value increase corresponding to increments in repair rate (w_2) where the system's other parameters possess constant values. When the failure rate of unit changes (λ_1=0.2 to 0.25), (λ_2=0.3 to 0.35) and (λ_3=0.15 to 0.2) then the profit of system declines.

Table 4: *Profit vs. Repair Rate*

w_2 ↓	λ_1=0.2, λ_2=0.3 λ_3=0.15, λ_4=0.25 λ_5=0.4, w_1=0.35 w_3=0.4, w_4=0.45 w_5=0.5	λ_1=0.25	λ_2=0.35	λ_3=0.2
0.1	2692.9	2046.417	1588.805	1690.491
0.2	4428.411	3640.771	3027.839	3256.171
0.3	5389.1	4576.626	3891.418	4226.372
0.4	5983.344	5186.276	4461.578	4880.064
0.5	6378.977	5611.935	4863.085	5346.797
0.6	6656.653	5924.286	5159.353	5694.6
0.7	6859.482	6162.259	5385.877	5962.46
0.8	7012.365	6348.978	5563.998	6174.235
0.9	7130.568	6498.993	5707.276	6345.293
1	7223.9	6621.895	5824.711	6485.948

IX. Conclusion

The performance of the juice plant is discussed using the regenerative point graphical technique (RPGT). The above tables explore that when the repair rate increases then the MTSF, system's availability and profit values also increase but when the failure rate increases then the MTSF, availability and profit values decrease. It is clear that RPGT is helpful for industries to analyze the behaviour of the products and components of system.

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