SYNTHETIC RELIABILITY MODELING AND PERFORMANCE ENHANCEMENT FOR MULTI-UNIT SERIAL SYSTEMS: UNVEILING INSIGHTS VIA GUMBEL-HOUGARD FAMILY COPULA APPROACH

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Abstract

This paper presents a comprehensive study of a series-parallel system comprising four interconnected subsystems: subsystem-1, subsystem-2, subsystem-3, and subsystem-4. Subsystem-1 stands as a single unit, subsystem-2 consists of three identical units in active parallel, subsystem-3 involves two identical units in series, while subsystem-4 incorporates two identical units in parallel. The system operates under good conditions, considering various failure rates and repair rates. The investigation employs Laplace transforms and Supplementary variable techniques to analyze the system's performance. Key reliability parameters, including Availability, Reliability, Mean Time to Failure (MTTF), Sensitivity, and Cost, are evaluated for specific values of failure and repair rates. The paper delves into the intricate analysis of a multi-unit series system, focusing on its reliability and performance evaluation. The study employs the Gumbel-Hougard Family Copula approach, a sophisticated and robust methodology to capture the interdependencies among system units. By utilizing this advanced technique, the paper provides a comprehensive understanding of the system's behavior under varying operating conditions. Various reliability and performance metrics, including Availability, Mean Time to Failure (MTTF), and Component Importance Measures, are thoroughly examined, offering valuable insights for optimizing the system's reliability and performance. The results are presented in a clear and visually appealing manner, utilizing tables and figures to aid in the comprehension of the findings.

Keywords: Availability; Reliability; Sensitivity; Mean time to system failure (MTTF); Cost Analysis;

I. Introduction

System reliability refers to the extent to which it can be relied upon to function correctly. Additionally, it encompasses critical aspects such as system usage, maintenance, and strategies for enhancing effectiveness by reducing failure occurrences and minimizing maintenance costs. By improving reliability, the risk of harm to maintenance personnel is reduced, as machine failures can result in significant injuries, revenue losses, reduced production output, and increased maintenance expenses. A key objective of system reliability analysis is to identify vulnerable components and assess the potential impacts of their failures.

The components of a serial or redundant system play a vital role in shaping its reliability and performance. The occurrence and nature of component failures within these systems significantly impact various key metrics, such as overall reliability, mean time to failure, dependability, availability, and revenue generation. Consequently, it becomes crucial to identify and assess the most critical components with the highest reliability in terms of the aforementioned factors. In today's manufacturing industries, traditional production systems are no longer enough to ensure the continued existence and success of an organization. The focus on steady-state operations is insufficient; constant improvement in the reliability and performance of systems is now essential to meet the demands of enhanced productivity, adaptability, and the ever-changing competitive landscape. Systems engineering, improvement, and setup are increasingly driven by the consideration of performance as a crucial factor. Merely assuming that systems function properly is inadequate; their effectiveness is equally important. Numerous instances in domains like telecommunication systems, industrial, and manufacturing systems have shown that enhancing system reliability and performance can lead to significant savings in disaster mitigation, time, costs, labor, risks, and even human lives. Therefore, reliability and performance analysis surveys are conducted to evaluate existing or planned systems, explore different configurations, and strive for an optimum design setup. The ongoing pursuit of improved reliability and performance is vital for organizations seeking to thrive in today's dynamic industrial landscape.

Numerous researchers have employed diverse methodologies to explore the performance and reliability of various systems, and their findings have demonstrated notable enhancements in operational efficiency. For instance, Teslyuk et al. (2021) proposed models for reliably assessing metrics related to testing the performance of local area networks. Additionally, Rotar et al. (2021) introduced a mathematical approach to determine the reliability of solar tracking systems by considering fault coverage aware metrics. In the realm of reliability analysis for various network systems, several researchers have made significant contributions using diverse methodologies. Bisht et al. (2021) devised an algorithm to compute reliability metrics, component measures, and critical measures for communication networks. Arora et al. (2020) developed models specifically for determining reliability metrics in parallel systems with fault coverage. On the other hand, Bisht and Singh (2019a) focused on analyzing reliability metrics of complex networks using universal generating functions. In the context of distributed networks, Huang et al. (2020) introduced their models for reliability analysis. Furthermore, Bisht and Singh (2019b) employed Markov processes to analyze the reliability measures for enhancing transmission network systems. In a separate study, Bisht and Singh (2020) delved into the analysis of profit and reliability in transmission networks using artificial neural networks and Markov processes.These diverse approaches and methodologies have contributed to advancing the understanding of reliability and performance evaluation in various network systems, paving the way for more robust and efficient designs.

The research landscape on reliability assessment for various systems has witnessed several significant studies employing different methodologies. Ye et al. (2020) conducted an investigation on the reliability of a repairable machine, exploring its behavior under shocks and degradation caused by low-quality feedstocks. Sharifi et al. (2019) tackled a redundancy allocation problem, aiming to optimize the reliability and cost of weighted-k-of-n parallel systems. They employed a combination of a universal generating function, a non-dominated sorting genetic algorithm, and a non-dominated ranked genetic algorithm to determine the reliability and cost of each subsystem. In the field of power systems, Jia et al. (2020) introduced a multi-state decision diagram method for evaluating system reliability. Their approach incorporated a multi-state performance sharing mechanism and warm standby units. Lin et al. (2021) contributed to the reliability modeling domain by establishing a copula-based Bayesian model. This model effectively captures the interdependence between components in parallel systems and allows for the estimation of system failure rates. Additionally, Jia et al. (2021) presented a model for power systems that integrate warm standby and energy storage components. Their reliability assessment was calculated using the multi-valued decision diagram technique. These diverse research endeavors have enriched the understanding of reliability assessment and paved the way for more robust and efficient designs across different systems and industries.

The realm of reliability analysis has seen diverse studies that explore different methodologies and applications. Pundir et al. (2021) conducted an analysis of reliability metrics for a system comprising two non-identical cold standby units, considering different types of priors for unknown parameters. Kumar et al. (2019a) introduced a novel approach inspired by the hierarchical and fishing behavior of gray wolves (Canis lupus) to enhance the technical specifications of a Nuclear Power Plant's safety system's residual heat removal system (RHRS). In another study, Kumar et al. (2019b) utilized a multi-objective gray wolf optimizer algorithm to optimize the reliability-cost trade-off for a space capsule's life support system. Fuzzy reliability evaluation was the focus of Kumar et al.'s (2020) investigation, specifically for series, parallel, and linear consecutive k-out-of-n: F systems. Hesitant fuzzy sets, triangular fuzzy numbers, and the Weibull distribution were employed to obtain fuzzy reliability measures for different types of systems. Mellal and Zio (2020) proposed a new cuckoo optimization algorithm to address reliability redundancy allocation problems, specifically with a cold-standby strategy. This innovative approach holds promise for optimizing the reliability and performance of redundant systems.

Numerous studies in the field of reliability engineering have shown that effective performance analysis can help to enhanced the reliability, avoid disasters and save time, money, or both. Xie et al. (2021) investigated and examined the performance of a safety system that is vulnerable to cascading failures that cause the appearance of further failures. In the paper, a unique technique for mitigating and preventing cascading failure is provided. Xie et al. (2019) suggested performance and an approximation approach for medium-frequency hazardous failures in safety instrumental systems prone to cascade failures. Yemane and Colledani (2019) offer a method for evaluating the performance of unstable manufacturing systems that takes into account unknown machine reliability predictions. Zhao et al. (2021) investigate and optimize the economic performance of a cold standby system susceptible to -shocks and imperfect repairs, proposing geometric process models to quantify the lifetime and repair time.

Numerous researchers have previously presented copula methods in the field of reliability and performance analysis of systems by examining system performance under various conditions. To name a few, Rawal et al. (2022) have concentrated on the reliability assessment of multi-computer systems consisting of n clients and k-out-of-n: G operational scheme with copula repair policy in the ongoing reliability investigations. Sha (2021) conducted research on a copula approach to reliability analysis for hybrid systems. Through the copula repair approach, Sanusi et al. (2022) estimate the dependability metrics of automated teller machine using Gumbel Hougaaard family copula.Yusuf et al. (2022) focus on reliability assessment and estimation of multi-unit of serial system. Maihulla et al. (2021) used the Gumbel-Hougaard family Copula to model and assess the dependability and performance of solar photovoltaic systems. Yusuf and Sanusi (2023) present copula technique in assessing and estimating the reliability characteristics of automated teller machine system. Maihulla and Yusuf (2022) analyzed the reliability of solar PV system through copula techniques. Yusuf et al. (2021) carried out a study on reliability analysis of distributed systems utilizing copula technique. Singh et al. (2021) examine the performance of a multi-unit k-out-of-n: G system through copula linguistic scheme. Singh et al. (2022) suggest a copula linguistic technique for analyzing the performance and effectiveness of a redundant k-out-of-n: G system with multiple successive state degradation. Abubakar and Singh (2019) analyses the Performance assessment of an industrial system (African Textile Manufactures Ltd.) through copula linguistic approach. Maihulla et al. (2021) used the Gumbel-Hougaard family Copula to model and assess the dependability and performance of solar photovoltaic systems.

The researchers mentioned above have made remarkable contributions in enhancing the reliability and performance of complex repairable systems using various techniques. However, there is a need for a new model that offers substantiated and comprehensive evaluations. With this in mind, this present paper focuses on the reliability and performance analysis of a serial system comprising five components. The paper introduces a novel technique called the copula repair technique to analyze the optimization of reliability and performance in this serial system. The main objective is to predict the system's performance optimization by employing two repair strategies. When the system experiences partial failure, the general repair technique is applied to fix it. On the other hand, if the system encounters complete failure, the copula repair technique is employed to fully recover from the failure.In pursuit of these objectives, the paper develops expressions for availability, reliability, mean time to failure (MTTF), sensitivity, and cost function. Through numerical analysis, the behavior of availability, reliability, and cost function over time is determined. This comprehensive approach aims to shed light on the dynamics of the serial system's performance and reliability, providing valuable insights for its optimization.

II. State Description, Notation, and assumptions

State descriptions

S_0 At the outset, the system is in an optimal operational state, where units B2, B3, and D2 are in hot standby mode, while units A1, B1, C1, C2, and D1 are actively functioning.

S_1 At this point, the system experiences a complete failure as subsystem 1 malfunctions.

S_2 At this moment, the system encounters a partial failure with units B3 and D2 in hot standby mode, units A1, B2, C1, C2, and D1 in working mode, and unit B1 undergoing repair.

S_3 At this point, the system experiences a complete failure as unit C1 malfunctions.

S_4 At this moment, the system faces a complete failure as unit C2 malfunctions.

S_5 At this point, the system encounters a partial failure with units B2 and B3 in hot standby mode, units A1, B1, C1, C2, and D2 in working mode, and unit D1 undergoing repair.

S_6 At this moment, the system experiences a partial failure with unit D2 in hot standby mode, units A1, B3, C1, C2, and D1 in working mode, and units B1 and B2 undergoing repair.

S_7 At this point, the system encounters a partial failure with unit B3 in hot standby mode, units A1, B2, C1, C2, and D2 in working mode, and units B1 and D1 undergoing repair..

S_8 At this moment, the system experiences a complete failure as all the units in subsystem 2 malfunction.

S_9 At this point, the system encounters a complete failure as all the units in subsystem 4 malfunction.

3.2 Notations

- t Stands for Time variable on a time scale.
- s Stands for Laplace transform variable for all expressions.
- λ_1 Stands for Failure rate of the unit in the subsystem 1.
- λ_2 Stands for Failure rate of any unit in subsystem 2.
- λ_3 Stands for Failure rate of the unit C_1 in subsystem 3.

- λ_4 Stands for Failure rate of the unit C_2 in subsystem 3.
- λ_5 Stands for Failure rate of any unit in subsystem 4.
- $\phi(x)$ Stands for Repair rates of the unit of subsystem 1.
- $\phi(y)$ Stands for Repair rate of units in the subsystem 2.
- $\phi(z)$ Stands for Repair rate of the unit C_1 in the subsystem 3.
- $\phi(m)$ Stands for Repair rate of the unit C_2 in the subsystem 3.
- $\phi(n)$ Stands for Repair rate of units in the subsystem 4.
- $P_i(t)$ Stands for the probability that the system is in S_i state at instants for i = 0 to 9.
- $\bar{P}(s)$ Stands for Laplace transformation of the state transition probability P(t).
- $P_1(x,t)$ Stands for the probability that a system is in state S_1 the system is running under repair and elapse repair time is (x, t) with repair variable x and time variable t.
- $E_p(t)$ Stands for Expected profit during the time interval [0, t).
- K_1, K_2 Stands for Revenue and service cost per unit time respectively.
- $P_i(y,t)$ Stands for Probability that the system is in state S_i for i=2, 6, 8, and the system is running under repair and elapse repair time is (y, t), with repair variable y and time variable t.
- $P_3(z,t)$ Stands for the probability that a system is in state S_3 the system is running under repair and elapse repair time is (z, t) with repair variable z and time variable t.
- $P_4(m,t)$ Stands for the probability that a system is in state S_4 the system is running under repair and elapse repair time is (m, t) with repair variable m and time variable t.
- $P_i(n,t)$ Stands for Probability that the system is in state S_i for i=5, 7, 9, and the system is running under repair and elapse repair time is (n, t), with repair variable n and time variable t.

1.3 Assumptions

- At the beginning, all the subsystems are in an ideal working mode
- One unit in subsystem 1, 2, 4 and all units in subsystem 3 are necessary for the system to be in operative mode
- All failure rates are unvarying and considered to undergo exponential distribution
- The repairs undergo a general distribution.
- It is considered that a repaired system performs like a new system and no damage seen during repair.
- Immediately the failed unit gets repaired, it is ready to undergo the task.

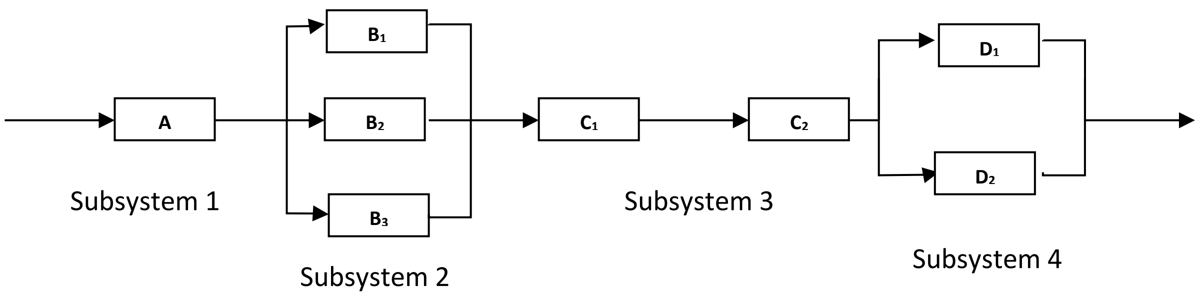

Figure 1 *Reliability Block Diagram of the System*

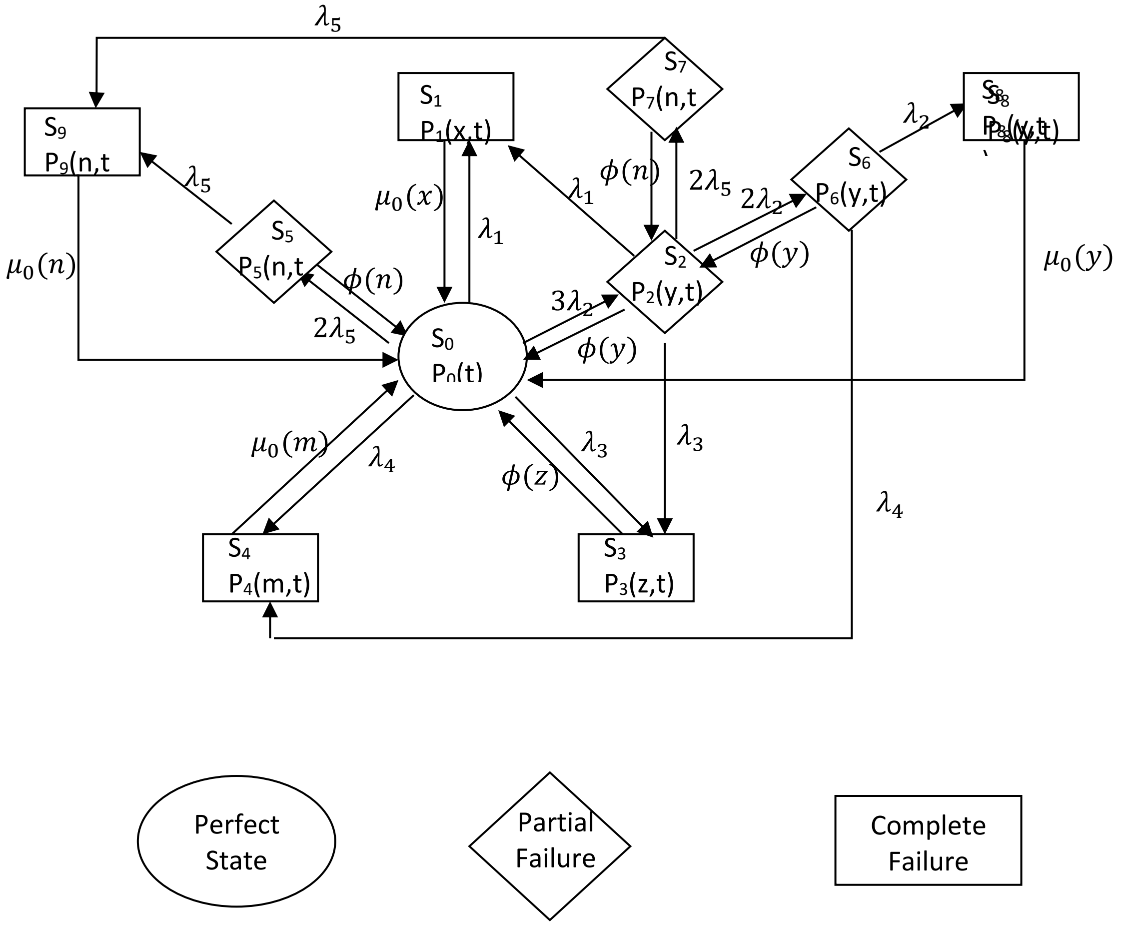

Figure 2 *State Transition diagram of Model*

III. Formulation of the Mathematical Model

With regards to the probability and continuity arguments, the train set of difference differential equations are lump together with the present mathematical model.

$$\left(\frac{\partial}{\partial t}+\lambda_1+3\lambda_2+\lambda_3+\lambda_4+2\lambda_5\right)p_0(t)=\int_0^\infty \phi(y)p_2(y,t)dy+\int_0^\infty \mu_0(z)p_3(z,t)dz+\int_0^\infty \mu_0(m)p_4(m,t)dm+\int_0^\infty \phi(n)p_5(n,t)dn+\int_0^\infty \mu_0(x)p_1(x,t)dx+\int_0^\infty \mu_0(y)p_8(y,t)dy+\int_0^\infty \mu_0(n)p_9(n,t)dn \tag{1}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\mu_0(x)\right)p_1(x,t)=0 \tag{2}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\lambda_1+2\lambda_2+\lambda_3+\lambda_4+2\lambda_5+\phi(y)\right)p_2(y,t)=0 \tag{3}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+\mu_0(z)\right)p_3(z,t)=0 \tag{4}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial m}+\mu_0(m)\right)p_4(m,t)=0 \tag{5}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial n}+\lambda_5+\phi(n)\right)p_5(n,t)=0 \tag{6}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\lambda_2+\phi(y)\right)p_6(y,t)=0 \tag{7}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial n}+\lambda_5+\phi(n)\right)p_7(n,t)=0 \tag{8}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\mu_0(y)\right)p_8(y,t)=0 \tag{9}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial n}+\mu_0(n)\right)p_9(n,t)=0 \tag{10}$$

Boundary Conditions

$$p_1(0,t)=\lambda_1 p_0(t)+\lambda_1 p_2(0,t) \tag{11}$$

$$p_2(0,t)=3\lambda_2 p_0(t) \tag{12}$$

$$p_3(0,t)=\lambda_3 p_0(t)+\lambda_3 p_2(0,t) \tag{13}$$

$$p_4(0,t)=\lambda_4 p_0(t)+\lambda_4 p_2(0,t) \tag{14}$$

$$p_5(0,t)=2\lambda_5 p_0(t) \tag{15}$$

$$p_6(0,t)=2\lambda_2 p_2(0,t) \tag{16}$$

$$p_7(0,t)=2\lambda_5 p_2(0,t) \tag{17}$$

$$p_8(0,t)=\lambda_2 p_6(0,t) \tag{18}$$

$$p_9(0,t)=\lambda_5 p_5(0,t)+\lambda_5 p_7(0,t) \tag{19}$$

Solution of the Model:

Using Laplace transformation of equations on (1) to (10) together with the initial condition, P0 (0) =1, one can attain.

$$(s+\lambda_1+3\lambda_2+\lambda_3+\lambda_4+2\lambda_5)\overline{p_0}(s)=1+\int_0^\infty \mu_0(x)\overline{p_1}(x,s)dx+\int_0^\infty \phi(y)\overline{p_2}(y,s)dy+\int_0^\infty \mu_0(z)\overline{p_3}(z,s)dz+\int_0^\infty \mu_0(m)\overline{p_4}(m,s)dm+\int_0^\infty \phi(n)\overline{p_5}(n,s)dn+\int_0^\infty \mu_0(y)\overline{p_8}(y,s)dy+\int_0^\infty \mu_0(n)\overline{p_9}(n,s)dn \tag{20}$$

$$\left(s+\frac{\partial}{\partial x}+\mu_0(x)\right)\overline{p_1}(x,s)=0 \tag{21}$$

$$\left(s+\frac{\partial}{\partial y}+\lambda_1+2\lambda_2+\lambda_3+\lambda_4+2\lambda_5+\phi(y)\right)\overline{p_2}(y,s)=0 \tag{22}$$

$$\left(s+\frac{\partial}{\partial z}+\mu_0(z)\right)\overline{p_3}(z,s)=0 \tag{23}$$

$$\left(s+\frac{\partial}{\partial m}+\mu_0(m)\right)\overline{p_4}(m,s)=0 \tag{24}$$

$$\left(s+\frac{\partial}{\partial n}+\lambda_5+\phi(n)\right)\overline{p_5}(n,s)=0 \tag{25}$$

$$\left(s+\frac{\partial}{\partial y}+\lambda_2+\phi(y)\right)\overline{p_6}(y,s)=0 \tag{26}$$

$$\left(s+\frac{\partial}{\partial n}+\lambda_5+\phi(n)\right)\overline{p_7}(n,s)=0 \tag{27}$$

$$\left(s+\frac{\partial}{\partial y}+\mu_0(y)\right)\overline{p_8}(y,s)=0 \tag{28}$$

$$\left(s+\frac{\partial}{\partial n}+\mu_0(n)\right)\overline{p_9}(n,s)=0 \tag{29}$$

Laplace transform of boundary conditions

$$\overline{p_1}(0,s)=\lambda_1\overline{p_0}(s)+\lambda_1\overline{p_2}(0,s) \tag{30}$$

$$\overline{p_2}(0,s)=3\lambda_2\overline{p_0}(s) \tag{31}$$

$$\overline{p_3}(0,s)=\lambda_3\overline{p_0}(s)+\lambda_3\overline{p_2}(0,s) \tag{32}$$

$$\overline{p_4}(0,s)=\lambda_4\overline{p_0}(s)+\lambda_4\overline{p_2}(0,s) \tag{33}$$

$$\overline{p_5}(0,s)=2\lambda_5\overline{p_0}(s) \tag{34}$$

$$\overline{p_6}(0,s)=2\lambda_2\overline{p_2}(0,s) \tag{35}$$

$$\overline{p_7}(0,s)=2\lambda_5\overline{p_2}(0,s) \tag{36}$$

$$\overline{p_8}(0,s)=\lambda_2\overline{p_6}(0,s) \tag{37}$$

$$\overline{p_9}(0,s)=\lambda_5\overline{p_5}(0,s)+\lambda_5\overline{p_7}(0,s) \tag{38}$$

Solving (20) to (29), together with equations (30) to (38) one may attain;

$$\overline{P_0}(s)=\frac{1}{D(s)} \tag{39}$$

$$\bar{P}_1(s)=\frac{(\lambda_1+3\lambda_1\lambda_2)}{D(s)}\left\{\frac{1-\bar{S}_{\mu_0}(s)}{s}\right\} \tag{40}$$

$$\overline{P_2}(s)=\frac{3\lambda_2}{D(s)}\left\{\frac{1-\bar{S}_{\phi}(s+\lambda_1+2\lambda_2+\lambda_3+\lambda_4+2\lambda_5)}{s+\lambda_1+2\lambda_2+\lambda_3+\lambda_4+2\lambda_5}\right\} \tag{41}$$

$$\overline{P_3}(s)\frac{=(\lambda_3+3\lambda_2\lambda_3)}{D(s)}\left\{\frac{1-\bar{S}_{\mu_0}(s)}{s}\right\} \tag{42}$$

$$\bar{P}_4(s)=\frac{(\lambda_4+3\lambda_2\lambda_4)}{D(s)}\left\{\frac{1-\bar{S}_{\mu_0}(s)}{s}\right\} \tag{43}$$

$$\overline{P_5}(s)=\frac{2\lambda_5}{D(s)}\left\{\frac{1-\bar{S}_{\phi}(s+\lambda_5)}{s+\lambda_5}\right\} \tag{44}$$

$$\overline{P_6}(s)=\frac{6{\lambda_2}^2}{D(s)}\left\{\frac{1-\bar{S}_{\phi}(s+\lambda_2)}{s+\lambda_2}\right\} \tag{45}$$

$$\overline{P_7}(s)=\frac{2\lambda_2\lambda_5}{D(s)}\left\{\frac{1-\bar{S}_{\phi}(s+\lambda_5)}{s+\lambda_5}\right\} \tag{46}$$

$$\overline{P_8}(s) = \frac{6\lambda_2{}^3}{D(s)}\left\{\frac{1-\bar{S}_{\mu_0}(s)}{s}\right\} \tag{47}$$

$$\bar{P}_9(s) = \frac{(2\lambda_5{}^2+2\lambda_2\lambda_5{}^2)}{D(s)}\left\{\frac{1-\bar{S}_{\mu_0}(s)}{s}\right\} \tag{48}$$

Where $D(s) = s + \lambda_1 + 3\lambda_2 + \lambda_3 + \lambda_4 + 2\lambda_5 - \Big\{(\lambda_1 + 3\lambda_1\lambda_2)\{\bar{S}_{\mu_0}(s)\} + 3\lambda_2\{\bar{S}_\phi(s + \lambda_1 + 2\lambda_2 + \lambda_3 + \lambda_4 + 2\lambda_5)\} + (\lambda_3 + 3\lambda_2\lambda_3)\{\bar{S}_{\mu_0}(s)\} + (\lambda_4 + 3\lambda_2\lambda_4)\{\bar{S}_{\mu_0}(s)\} + 2\lambda_5\{\bar{S}_\phi(s + \lambda_5)\} + 6\lambda_2{}^3\{\bar{S}_{\mu_0}(s)\} + (2\lambda_5{}^2 + 2\lambda_2\lambda_5{}^2)\{\bar{S}_{\mu_0}(s)\}\Big\}$

The Laplace transformations of the state transition probabilities that the system is in operative condition and failed condition at any time is as follows:

$$\overline{P_{up}}(s) = \overline{P_0}(s) + \overline{P_2}(s) + \overline{P_5}(s) + \overline{P_6}(s) + \overline{P_7}(s)$$

$$\overline{P_{up}}(s) = \overline{P_0}(s)\left(1 + 3\lambda_2\left\{\frac{1-\bar{S}_\phi(s+\lambda_1+2\lambda_2+\lambda_3+\lambda_4+2\lambda_5)}{s+\lambda_1+2\lambda_2+\lambda_3+\lambda_4+2\lambda_5}\right\} + 2\lambda_5\left\{\frac{1-\bar{S}_\phi(s+\lambda_5)}{s+\lambda_5}\right\} + 6\lambda_2{}^2\left\{\frac{1-\bar{S}_\phi(s+\lambda_2)}{s+\lambda_2}\right\} + 2\lambda_2\lambda_5\left\{\frac{1-\bar{S}_\phi(s+\lambda_5)}{s+\lambda_5}\right\}\right) \tag{49}$$

$$\bar{P}_{down}(s) = 1 - \bar{P}_{up}(s)$$

IV. Analytical Computations

Availability Analysis

Applying $\bar{S}_\phi(s) = \frac{\phi}{s+\phi}$, $\bar{S}_{\mu_0}(s) = \frac{\mu_0}{s+\mu_0}$, $\frac{1-\bar{S}_\phi(s)}{s} = \frac{1}{s+\phi}$, $\frac{1-\bar{S}_{\mu_0}(s)}{s} = \frac{1}{s+\mu_0}$ and considering the values of different parameters as $\lambda_1 = 0.01, \lambda_2 = 0.02, \lambda_3 = 0.03, \lambda_4 = 0.04, \lambda_5 = 0.05, \phi = \mu_0 = 1$ in (49), then with the inverse Laplace transform, the availability may be, obtained as:

$$\bar{P}_{up}(t) = 0.03439655083e^{-2.816787262t} - 0.01746442940e^{-1.293636565t} - 0.002630732952e^{-1.113039421t} + 0.9862252387e^{-0.004836752497t} - 0.0005266271282e^{-1.020000000t} \tag{50}$$

For different values of time variable t= 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, units of time, different values of Pup(t) with the help of (50) may be attained as shown in Table 1 and Figure 3.

Table1. *Variation of Availability with respect to time*

Time	0	1	2	3	4	5	6	7	8	9
Availability	1.0000	0.9772	0.9752	0.9715	0.9672	0.9626	0.9580	0.9534	0.9488	0.9442

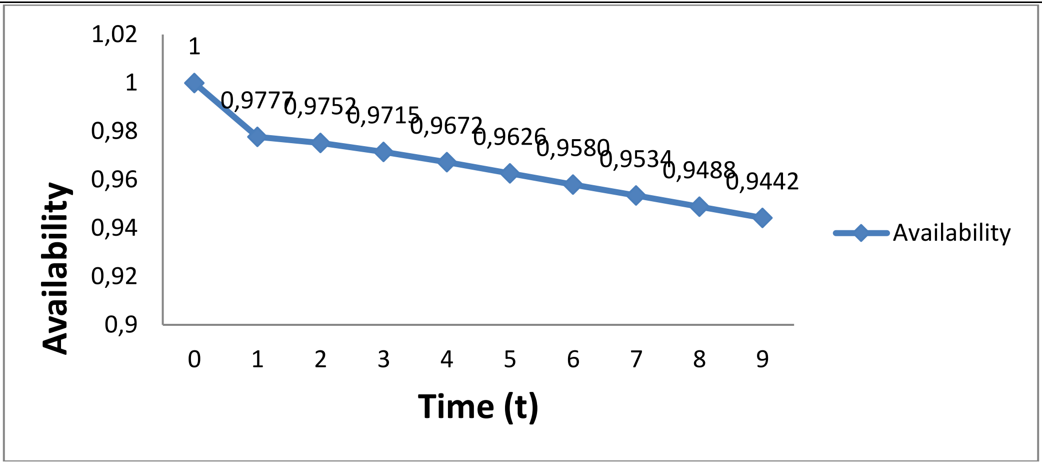

Figure 3 *Availability as function of time*

Reliability Analysis

Using all repair rates, ϕ, μ_0, in equation (49) to zero and for same values of failure rates as

$\lambda_1 = 0.01, \lambda_2 = 0.02, \lambda_3 = 0.03, \lambda_4 = 0.04, \lambda_5 = 0.05$ And then computing inverse Laplace transform, the reliability for the system may be expressed as;

$$R(t) = 3.e^{-0.2200000000t} + 0.5368421053e^{-0.05000000000t} - 2.547751196e^{-0.2400000000t} + 0.01090909091e^{-0.02000000000t} \quad (51)$$

For, different values of time t= 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.., units of time, different values of Reliability may be attained as seen in Table 2 and graphical display in Figure 4.

Table 2. Computation of reliability for different values of time

Time	0	1	2	3	4	5	6	7	8	9
Reliability	1.0000	0.9248	0.8518	0.7828	0.7184	0.6592	0.6052	0.5561	0.5118	0.4718

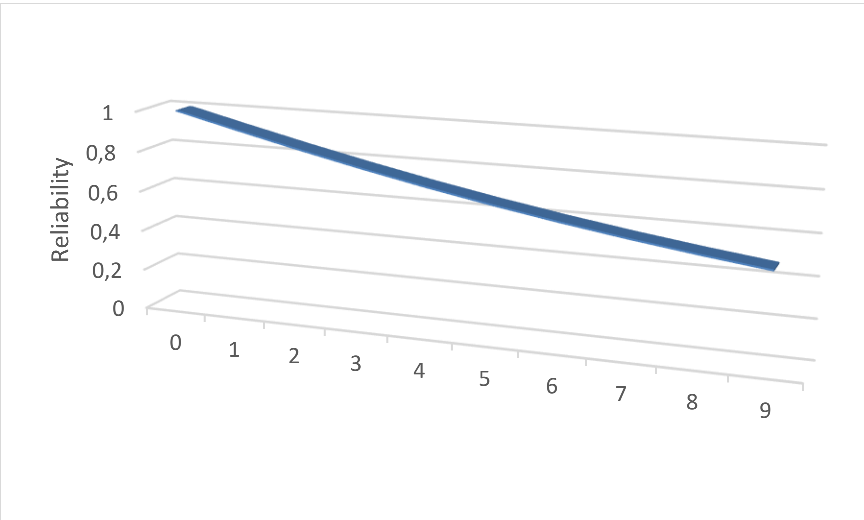

Figure 4 *Reliability as function of Time*

Mean Time to Failure (MTTF) Analysis

Mean time to failure (MTTF) analysis is an important tool in system reliability theory. It provides a measure of the expected time between failures of a system or component, and is often used to assess the reliability and performance of various systems, including mechanical, electrical, and software systems. MTTF analysis plays a crucial role in system reliability theory. It provides a quantitative measure of a system's expected performance, which can be used to guide design improvements, maintenance activities, and safety procedures.

There are several reasons why MTTF analysis is important and necessary in system reliability theory:

1. Predictive Maintenance: MTTF analysis allows us to predict when a system or component is likely to fail. This information can be used to schedule maintenance activities and prevent costly and unexpected downtime.
2. Design Improvement: MTTF analysis can be used to identify weaknesses in a system's design or components, and guide design improvements to increase reliability.
3. Cost-Effective: MTTF analysis can help companies identify the most cost-effective approach to maintaining and repairing their systems. By prioritizing maintenance activities based on the expected MTTF, companies can optimize their maintenance budget and reduce overall costs.
4. Safety: MTTF analysis is crucial for ensuring the safety of critical systems, such as those used in aviation, healthcare, and nuclear power. By understanding the expected failure rate of these systems, we can design appropriate safety protocols and procedures.

Making all repairs to zero in equation (49), and then considering limit as s tends to zero, the MTTF may be expressed as:

$$MTTF = \lim_{s\to 0}\overline{P_{up}}(s) = \frac{1}{\lambda_1+3\lambda_2+\lambda_3+\lambda_4+2\lambda_5}\left(3 + 6\lambda_2 + \frac{3\lambda_2}{\lambda_1+2\lambda_2+\lambda_3+\lambda_4+2\lambda_5}\right) \quad (52)$$

Setting $\lambda_2 = 0.02, \lambda_3 = 0.03, \lambda_4 = 0.04, \lambda_5 = 0.05$ and varying λ_1 one by one respectively as $0.01, 0.02, 0.03, 0.04, 005, 0.06, 0.07, 0.08, 0.09$, $\lambda_1 = 0.01, \lambda_3 = 0.03, \lambda_4 = 0.04, \lambda_5 = 0.05$ and varying λ_2 one by one respectively as $0.01, 0.02, 0.03, 0.04, 005, 0.06, 0.07, 0.08, 0.09$, $\lambda_1 = 0.01, \lambda_2 = 0.02, \lambda_4 = 0.04, \lambda_5 = 0.05$ and varying λ_3 one by one respectively as $0.01, 0.02, 0.03, 0.04, 005, 0.06, 0.07, 0.08, 0.09$, $\lambda_1 = 0.01, \lambda_2 = 0.02, \lambda_3 = 0.03, \lambda_5 = 0.05$ and varying λ_4 one by one respectively as $0.01, 0.02, 0.03, 0.04, 005, 0.06, 0.07, 0.08, 0.09$, and $\lambda_1 = 0.01, \lambda_2 = 0.02, \lambda_3 = 0.03, \lambda_4 = 0.04$ and varying λ_5 one by one respectively as $0.01, 0.02, 0.03, 0.04, 005, 0.06, 0.07, 0.08, 0.09$ in (52), the variation of MTTF with respect to failure rates may be attained as seen in Table 3 and corresponding Figure 5

Table 3. *Computation of MTTF corresponding to the various values of failure rates*

Failure rate	MTTF $\boldsymbol{\lambda_1}$	MTTF $\boldsymbol{\lambda_2}$	MTTF $\boldsymbol{\lambda_3}$	MTTF $\boldsymbol{\lambda_4}$	MTTF $\boldsymbol{\lambda_5}$
0.01	13.7576	15.0476	15.0909	15.8596	21.2857
0.02	13.1757	13.7576	14.3934	15.0909	18.7222
0.03	12.641	12.7037	13.7578	14.3934	16.7111
0.04	12.1481	11.8256	13.1757	13.7576	15.0909
0.05	11.6923	11.0823	12.641	13.1757	13.7576
0.06	11.2695	10.4444	12.1481	12.641	12.641
0.07	10.8762	9.891	11.6923	12.1481	11.6923
0.08	10.5095	9.4062	11.2695	11.6923	10.8762
0.09	10.1667	8.9778	10.8762	11.2695	10.1667

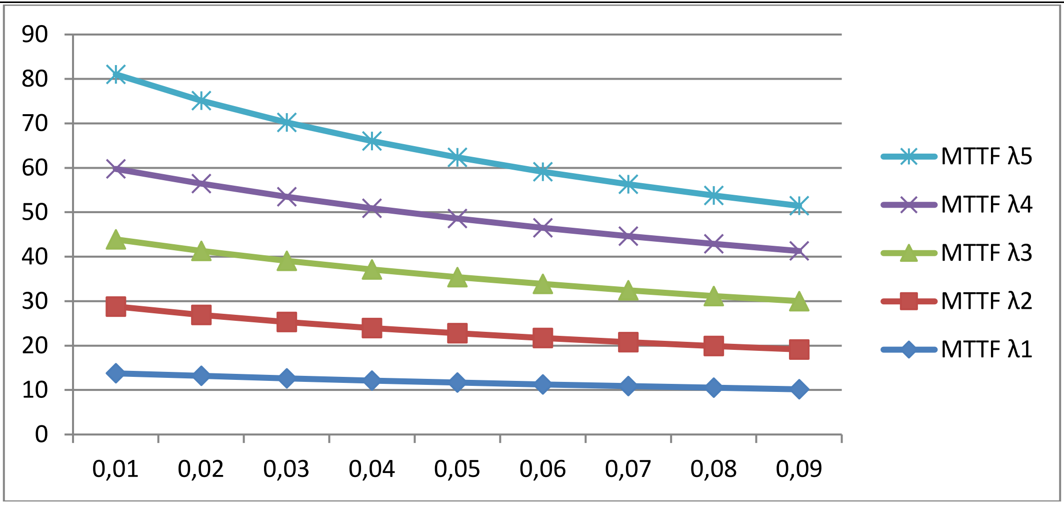

Figure 5 *MTTF as function of Failure rate*

Sensitivity Analysis corresponding to (MTTF)

Sensitivity analysis is an important tool in system reliability theory, as it can help identify critical components, optimize maintenance schedules, quantify uncertainty, and assess risk. Sensitivity analysis is an essential tool in system reliability theory for evaluating the impact of uncertainty and variability in the inputs of a system on the outputs. It involves varying the values of the inputs within a range and analyzing the corresponding changes in the outputs to determine how sensitive the outputs are to the inputs. Sensitivity analysis can help in several ways:

1. Identifying critical components: Sensitivity analysis can help identify the most critical components in a system, those whose failure has the most significant impact on the system's overall reliability. By varying the parameters associated with each component, sensitivity analysis can help determine which components are most sensitive to changes in their input values.
2. Optimal maintenance: Sensitivity analysis can also be used to determine the optimal maintenance schedule for a system. By varying the maintenance parameters and observing the corresponding changes in the system's reliability, it is possible to determine the maintenance schedule that maximizes the system's reliability while minimizing maintenance costs.
3. Uncertainty quantification: Sensitivity analysis can help quantify the uncertainty associated with a system's reliability estimates. By varying the input parameters and observing the corresponding changes in the output, it is possible to determine the range of variability in the system's reliability estimates.
4. Risk assessment: Sensitivity analysis can be used for risk assessment by identifying the most critical inputs in a system and quantifying their impact on the system's reliability. This information can be used to assess the risk associated with different scenarios and identify strategies to mitigate the risk.

Sensitivity in MTTF of the system may be calculated through the partial differentiation of MTTF with respect to the failure rates of the system. By executing the set of parameters as $\lambda_1 = 0.01, \lambda_2 = 0.02, \lambda_3 = 0.03, \lambda_4 = 0.04, \lambda_5 = 0.05$ in the partial differentiation of MTTF, one may obtain the MTTF sensitivity as seen in Table 4 and corresponding graphs seen in Figure 6

Table 4 *MTTF sensitivity as function of time*

Failure rate	$\frac{\delta(MTTF)}{\lambda_1}$	$\frac{\delta(MTTF)}{\lambda_2}$	$\frac{\delta(MTTF)}{\lambda_3}$	$\frac{\delta(MTTF)}{\lambda_4}$	$\frac{\delta(MTTF)}{\lambda_5}$
0.01	-60.7668	-143.537	-73.1405	-80.7985	-291.581
0.02	-55.7272	-115.978	-66.5235	-73.1405	-225.386
0.03	-51.2903	-95.7819	-60.7668	-66.5235	-179.457

0.04	-47.3635	-80.5049	-55.7272	-60.7668	-146.281
0.05	-43.8715	-68.6513	-51.2902	-55.7272	-121.534
0.06	-40.7523	-59.2593	-47.365	-51.2903	-102.581
0.07	-37.9546	-51.6858	-43.8715	-47.3635	-87.7430
0.08	-35.4357	-45.4864	-40.7523	-43.8715	-75.9093
0.09	-33.1598	-40.3457	-37.9546	-40.7523	-66.3194

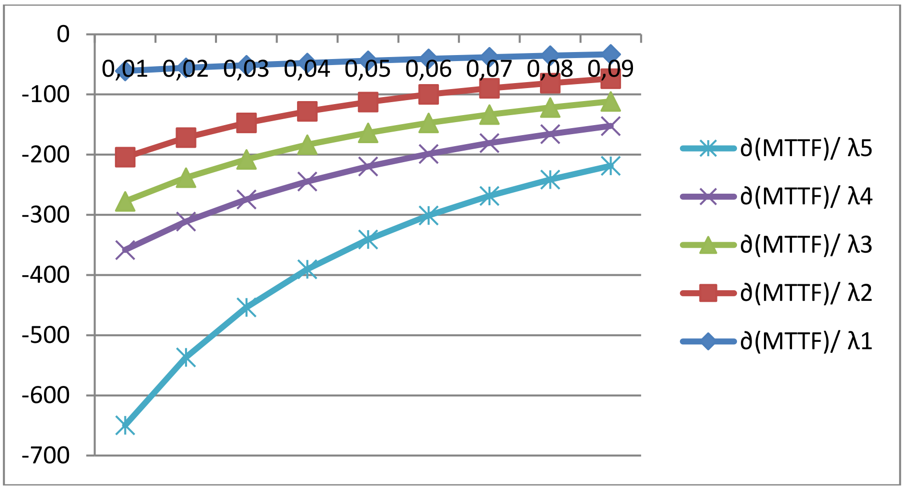

Figure 6 *MTTF Sensitivity with respect to time*

Cost Analysis

Conceding that the service facility be all the time available, then expected profit during the interval [0, t) of the system may be attained by the formula.

$$E_P(t) = K_1 \int_0^t P_{up}(t)dt - K_2 t \quad (53)$$

For the same set of a parameter of (49), one may attain (53). Therefore

$$E_P(t) = K_1(-0.01221127037e^{-2.816787262t} + 0.01350025956e^{-1.293636565t} + 0.002363557752e^{-1.113039421t} - 203.9023579e^{-0.004836752497t} + 0.0005163011061e^{-1.020000000t} + 203.898) - K_2 t \quad (54)$$

Setting $K_1 = 1$and $K_2 = 0.6, 0.5, 0.4, 0.3, 0.2$ and 0.1 respectively and varying t =0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. Units of time, the results for expected profit may be attained as seen in Figure 7.

Table 5. *Expected profit as function of time*

Time(t)	K2=0.6	K2=0.5	K2=0.4	K2=0.3	K2=0.2	K2=0.1
0	0	0	0	0	0	0
1	0.3834	0.4834	0.5835	0.6834	0.7834	0.8834
2	0.7599	0.9599	1.1599	1.3599	1.5599	1.7599
3	1.1333	1.4333	1.7333	2.0333	2.3333	2.6333
4	1.5027	1.9027	2.3027	2.7027	3.1027	3.5027
5	1.8677	2.3677	2.8677	3.3677	3.8677	4.3677
6	2.2280	2.8280	3.4280	4.0280	4.6280	5.2280
7	2.5837	3.2835	3.9837	4.6837	5.3837	6.0837
8	2.9348	3.7348	4.5348	5.3348	6.1348	6.9348

9	3.2813	4.1813	5.0813	5.9813	6.8813	7.7813

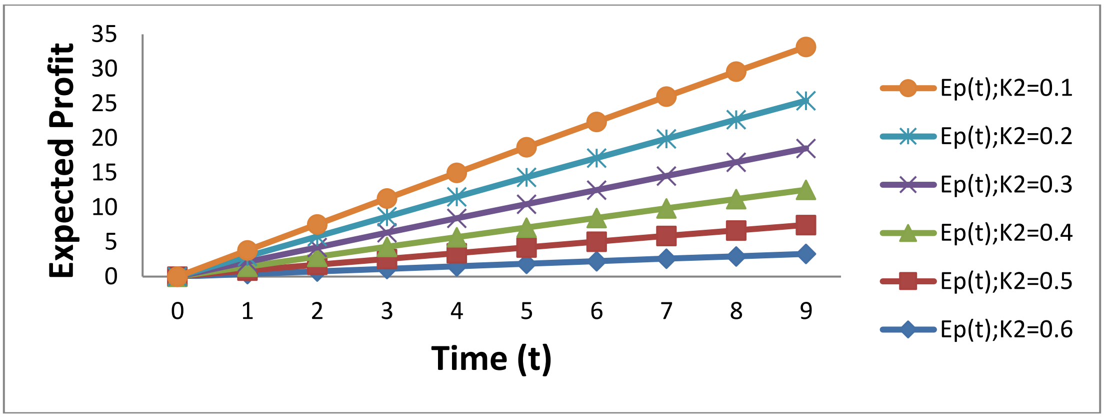

Figure 7 *Expected profit as function of time*

V. Discussion and Concluding Remark

The study aims to investigate and understand the behavior of the complex repairable system under different values of failure and repair rates. Figures 3 and 4 illustrate how the availability of the system changes over time with fixed failure rates at different values. When the failure rates are relatively low (e.g., $\lambda_1 = 0.01, \lambda_2 = 0.02, \lambda_3 = 0.03, \lambda_4 = 0.04, \lambda_5 = 0.05,$), both the availability and reliability of the system decrease gradually, while the probability of failure increases with time until it eventually stabilizes at zero after a sufficiently long period. By analyzing the graphical representation of the model, it becomes evident that the future behavior of the complex system can be reliably predicted for any given set of parametric values. Furthermore, from the observations in Table 3 and Table 4, it is evident that providing repair is more desirable for the system's performance compared to replacement, given the other parameters are held constant. Figure 5 presents the mean-time-to-failure (MTTF) of the system concerning variations in failure rates (. The reciprocal relationship between MTTF and failure rates indicates that these rates significantly influence the system's performance. The sensitivity analysis, as depicted in Figure 4, highlights how the system's sensitivity varies with changes in parameter values. In terms of profit analysis, the study considers a fixed revenue cost per unit (K1=1) and varying service costs (K2= 0.6, 0.5, 0.4, 0.3, 0.2, and 0.1). Figure 7 shows that as the service cost increases, the expected profit decreases.

In conclusion, this comprehensive investigation sheds light on the behavior of the complex repairable system under different conditions and parameter values. It provides valuable insights into the system's reliability, availability, MTTF, sensitivity, and profit optimization, offering practical guidance for decision-making and system performance enhancement.

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