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RELIABILITY BASED DESIGN OF CIVIL AND INDUSTRIAL ENGINEERING STRUCTURES USING THE LIFE QUALITY INDEX CRITERION

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Abstract

The paper considers and analyses the current world practice of assigning acceptable values for the probability of failure of civil and industrial engineering structures based on monetary optimization using the life quality index. The analysis is illustrated by calculating the approximate target values of the threshold probability of failure for multi-storey residential buildings in the Sverdlovsk region of the Russian Federation using the LQI criterion and the social willingness to pay concept assessment of the effectiveness of the costs of safety measures.

Keywords: reliability, probability of failure, civil and industrial engineering structures, life quality index, social willingness to pay, monetary optimization

I. Introduction

The failure of a structure entails a risk to life, so the economically viable level of safety of the structure to which it is designed and operated must also include a social assessment (**by** evaluating the cost **effectiveness** of safety measures). This circumstance is not regulated **yet** in any way by the regulatory and technical documentation of the Russian Federation. The reliability assessment of civil and industrial engineering structures within the framework of the world standards in forc**e** (based on the limit states design metho**d**) is carried out by comparing some conditional reliability measures (failure probability P_f and associated reliability index β_f) with their allowable (target) values:

$$P_f \le P_t, \ \beta_f \ge \beta_t, \tag{1}$$

where P_t, β_t are the target values of the probability of failure (POF) and the reliability index (RI), respectively.

To estimate the POF P_f , a simple limit state function (LFS) is used:

$$g = R - S, \tag{2}$$

where S is the external load (effect of impacts) and R is the bearing capacity (resistance) of the structur**e**, **c**onsidered as random variables (RV).

As a measure of reliability, RI β is used, which is associated with the probability of failure P_f by the Laplace function, that is, under the assumption that the *load and resistance* obey *a normal or lognormal distribution laws*.

When the load and resistance are independent and normally distributed, the structure POF can be estimated by the formula:

$$P_f = P\left[g<0\right] = \frac{1}{\sigma_g\sqrt{2\pi}}\int_{-\infty}^{0}\exp\left[-\frac{(t-\mu_g)^2}{2\sigma_g^2}\right]dt = \Phi\left(\frac{0-\mu_g}{\sigma_g}\right), \tag{3}$$

where μ_g,σ_g^2 are the mathematical expectation and variance of RV g, respectively; $\Phi(x)$ is a function of the standard normal distribution law.

The RI (with independent R and S) is defined as the ratio of the mathematical expectation μ_g to the standard deviation σ_g :

$$\beta = \frac{\mu_g}{\sigma_g}. \tag{4}$$

From (4) it follows that

$$P_f = P(g<0) = \Phi(-\beta) = 1-\Phi(\beta). \tag{5}$$

Equation (5) relates the probability of failure to the reliability index, from which it is possible to determine the reliability index corresponding to the given POF:

$$\beta = -\Phi^{-1}\left(P_f\right). \tag{6}$$

For the case when R and S are lognormally distributed, the RI is calculated by the formula:

$$\beta = \frac{\mu_Y}{\sigma_Y} = \frac{\ln\left(\dfrac{E[R]}{E[S]}\sqrt{\dfrac{1+V_S^2}{1+V_R^2}}\right)}{\sqrt{\ln\left(\left[1+V_R^2\right]\left[1+V_S^2\right]\right)}}, \tag{7}$$

where $E[R],E[S]$ are, respectively, the mathematical expectations of resistance and load; V_R,V_S are, respectively, the coefficients of variation of resistance and load.

The obvious logical step in assigning acceptable/permissible values for the POF of civil and industrial engineering structures should be based on monetary optimization using a benefit-cost assessment of the effectiveness of safety measures, with considering the economic value of a statistical life.

In the current national standard GOST R ISO 2394-2016 of the Russian Federation the permissible level of structural reliability is related to the consequences of failure and the cost of safety measures formalistically and in a simple form. This GOST is based on the international standard ISO 2394:1998 *General principles on reliability for structures*, which has been replaced by ISO 2394:2015 [1].

The generally accepted approach to the assignment of permissible target reliability measures is discussed below, based on the ISO standard [1] and the standard developed by the Joint Committee for Structural Safety (JCSS)[2].

II. Monetary optimization (based on economic costs) of construction projects

ISO 2394:2015 [1] and PMC JCSS [2] are based on Rudiger Rackwitz' seminal works [3, 4]. These papers posit that design codes should be optimized in terms of costs, benefits, and failure effects. The objective function is based on the approach proposed by Rosenblueth and Mendoza (1971) [6] for optimizing the reliability of isostatic structures, using the Cox renewal theory (1967) [5].

For optimization based on the economic costs of civil and industrial engineering objects the following objective function is proposed [3, 4]:

$$\max_p\left\{Z(p) = B - C(p) - A(p) - D(p) - U(p) - M(p) - I(p)\right\}, \tag{8}$$

where p is the decision parameter, the so-called *global safety factor, GSF* (described below);
B is the benefit derived from the useful existence (revenue producing time) of the object;
$C(p)$ are the construction costs;
$A(p)$ are the obsolescence costs;
$D(p)$ are the damage costs due to a failure of the structure's bearing capacity (corresponding to its limit states);
$U(p)$ are the costs associated with operability failures (maintaining the serviceability of the structure (corresponding to its operability/serviceability limit states);
$M(p)$ are the costs associated with fatigue and other failures due to aging;
$I(p)$ are the diagnostics and maintenance costs.

Here it is appropriate to define the scope of this paper more precisely. In general, there are three types of hazards for a civil or an industrial engineering system: Structural (Geo- and industrial, man-made hazards); Operational and Black-Swan-type hazards. There is also a hazard, that is the system itself, as it produces its own carbon footprint that damages the environment. In this paper only the bearing capacity limit state type of failures and associated with it damages are considered.

The last two components of the objective function (8), costs $M(p)$ and $I(p)$ are not considered in the optimization scheme (see [4]). It is also assumed that the cost of failures during operation $U(p)$ can be neglected (see Appendix B in [4]) and the benefit B does not depend on the parameter p. Then the objective function (8) takes the simplified form:

$$\min_{p}\left\{Z(p)=C(p)+A(p)+D(p)\right\}. \tag{9}$$

The approach described in [4] posits that under a few simplifying assumptions, all cost components in (9) can be estimated as a function of construction costs $C(p)$. To achieve this, it is assumed that: (1) the construction is periodically rebuilt (renewed) after each failure or obsolescence, (2) obsolescence and failure are independent events and occur at random times [this is needed for estimating the $A(p)$ and $D(p)$]; (3) the times between failure (renewal) events have *the same distribution and are independent* (i.e., the loads and resistances are independent during successive renewal periods, and there are no changes in the design rules after the first and all subsequent failures); (4) even if the structure changes during renewal, the failure time distribution must remain the same; (5) the renewal (reconstruction) time is negligible compared to the average service life of the structure [4].

A structure that will eventually fail after some time must be optimized at the time of decision (design), that is, at time t = 0. Hence, it is necessary to discount all costs of the objective function (9).

In design optimization of structures, renewal theory is used to estimate the total costs associated with a series of structures to be built in the future. A renewal strategy, referring to structural design codes, is a systematic renewal (or repair) after a structure becomes obsolete or fails, as the need for structures persists beyond the life of individual objects, and even if the structure is not restored as an exact copy on the same location, new structures will always be based on the same or similar design standards. Hence, the time horizon T_u for making decisions based on design codes is usually very long and can be assumed to approach infinity ($T_u \to \infty$). *Thus, the objective function is not limited to a single structure; it relates to several similar ones that should be built in the future* [10].

Return to the objective function (9). The expected present value of obsolescence costs (with systematic renewal) is estimated as:

$$A(p)=\left(C(p)+A\right)\sum_{n=1}^{\infty}E\left[\exp\left(-\gamma T_n\right)\right]=\left(C(p)+A\right)\sum_{n=1}^{\infty}\int_0^{\infty}\exp\left(-\gamma\tau_n\right)f_n\left(\tau_n\right)d\tau_n. \quad (10)$$

where $C(p)$ denotes the expected (re)construction costs, A are the demolition costs (do not depend on the decision parameter); T_n is the time to *the n-th* reconstruction (obsolescence) event.

The mathematical expectation in equation (10) is, by definition, the Laplace transform of the function $f_n(t)$ (PDF of the RV T_n), hence:

$$\sum_{n=1}^{\infty}E\left[\exp\left(-\gamma T_n\right)\right]=\sum_{n=1}^{\infty}\left[f^*(\gamma)\right]^n. \quad (11)$$

The sum on the right in this expression is equal to the Laplace transform of the renewal density. Assuming a uniform Poisson process of obsolescence events occurring (resulting in an exponentially distributed time between renewals), then the expected present value of obsolescence costs will be a function of the obsolescence rate ω = 1/E[T], i.e.

$$A(p)=\left(C(p)+A\right)\frac{\omega}{\gamma}. \quad (12)$$

Usually, when calculating, $\omega=0.02$ (2%) is used. The obsolescence rate can be considered as the average time interval between the object reconstructions, and the average interval between reconstructions, in turn, as the expected service life of the structure (ω= 0.01 implies a reconstruction once every 100 years [4]).

Although the Poisson flow for obsolescence events is probably not very realistic, Equation (12) is used to study the effect of (average) service life on optimization results; the behavior will be, at least qualitatively, similar for other probabilistic models [10].

Ignoring the cost of obsolescence is unacceptable, since it is tantamount to assuming an infinite service life (until failure occurs), which is also unrealistic. Further, only the demolition costs *A are neglected*, since in any case they are assumed to be independent of the decision parameter *p*.

A similar approach is used to estimate the expected present value o**f** D(p) resulting from a failure. For simplicity, it is assumed that failure can only occur during random disturbances (e.g., earthquakes, hurricanes, fires) that follow a Poisson process with intensity λ. Randomness in the intensity and occurrence of an external impact can be modeled as a marked Poisson process [7–9]. In this case, the cost associated with the failure of the structure is defined as:

$$D(p)=\left(C(p)+H\right)\frac{\lambda P_{f,ULS}(p)}{\gamma}, \quad (13)$$

where H are the costs that accrue in case of failure in addition to the costs of reconstruction (or construction of a new building); λ is the intensity of the process of random disturbances, which can lead to the bearing capacity type of failure wit**h** annual (unconditional frequency)/**p**robability $P_{f,ULS}(p)$.

Construction costs $C(p)$ are modeled as a function of the global safety factor *p*:

$$C(p)=C_0+C_1(p), \quad (14)$$

where C_0 refers to that part of the construction costs that does not depend on the decision parameter (some "fixed" construction cost); $C_1(p)$ are the marginal safety costs.

Global safety factor p as a decision parameter**, i**s defined [3,4] as the ratio of the mathematical expectations of resistance *R* and load *S*:

$$p=\frac{E[R]}{E[S]}. \quad (15)$$

According to [3, 4], the dependence of the POF on the decision-making parameter p is determined under the assumption of a log-normal distribution of resistance and impact effects. Then, from formula (7), it follows that the RI

$$\beta(p) = \frac{\ln\left(p\sqrt{\frac{1+V_S^2}{1+V_R^2}}\right)}{\sqrt{\ln\left(\left[1+V_R^2\right]\left[1+V_S^2\right]\right)}}. \tag{16}$$

In this case, the failure probability is modeled as:

$$P_f(p) = P\{g = p \cdot R - S \leq 0\}, \tag{17}$$

where resistance R and load S are modeled as VRs with coefficients of variation V_R and V_S respectively. Equation (17) assumes that both RVs are normalized with respect to their mathematical expectations, so that E[R] = E[S] = 1. In this case, the bearing capacity is:

$$g_{ULS} = R(p) - S, \tag{18}$$

where RV $R(p) = p \cdot R$ is modeled with an average equal to p, and the load S is modeled with an average equal to one.

Ultimately, the objective function (9) takes the form:

$$\min_p \left\{ Z(p) = C(p) + C(p)\frac{\omega}{\gamma} + \left(C(p) + H\right)\frac{\lambda P_{f,ULS}(p)}{\gamma} \right\}. \tag{19}$$

By relating all cost components to a fixed construction cost C_0, the optimal level of reliability can be determined as a function of safety and failure damage cost.

If the failure of the structure is not caused by random perturbations, then for failures due to time-varying loads, Eq. (19) is simplified (see [4] for details). In this case, to study the parameters, it is assumed that $\lambda = 1$. For simplicity, from now on the notation λ is omitted since the focus is on the normal/standard design case.

III. Monetary optimization and life safety criterion

The purpose of the analysis when setting the target values of reliability measures is to establish both some optimal value p^* of the decision-making parameter (the level of ensuring the safety of the structure) based on monetary optimization, and the minimum (threshold) value of this parameter p_{acc} from the condition of ensuring life safety (saving people's lives) using LQI criterion that determines the boundary of the admissible area for p, within which monetary optimization should be performed. Fig. 1 shows the interaction between the safety parameter p^* set on the basis of monetary optimization and the boundary (threshold) value p_{acc} set according to the criterion of ensuring human safety (**a**cceptable risk to life).

The permissible limit (threshold) value p_{acc} is determined according to [1] based on the Marginal Life Saving Costs Principle, according to which the decisions made that affect life safety are considered acceptable if the costs associated with possible measures to save one additional (anonymous) life, are balanced with the costs that society is economically capable of, or willing to bear for the sake of saving one statistical individual.

The estimate of the marginal safety costs is based on the Life Quality Index (LQI). This is a complex indicator is equal to (USD·year) [12-14]:

$$LQI = G^q e(0), \tag{20}$$

where G (USD) is the gross domestic product (GDP) per capita; $e(0)$ (years) is the average life expectancy (ALE) at birth; q is a complex coefficient depending on the duration of the active phase of life and the measure of compromise between the resources available for consumption and the value of healthy life time [14]:

$$q = \frac{w}{\beta(1-w)}, \tag{21}$$

where β is the ratio of income S (in USD per capita per *month*) to GDP:

$$\beta = \frac{S \cdot 12}{g}, \tag{22}$$

w is the duration of the phase of active life (the ratio of the duration of profitable work and leisure, including hobbies):

$$w = h \cdot M/P, \tag{23}$$

h is the ratio of the number of worked hours to the total hours, taking into account a 40-hour working week h = 0.24; M (in thousands of people) *is* the number of people employed in the economy; P (in thousands of people) is the total population.

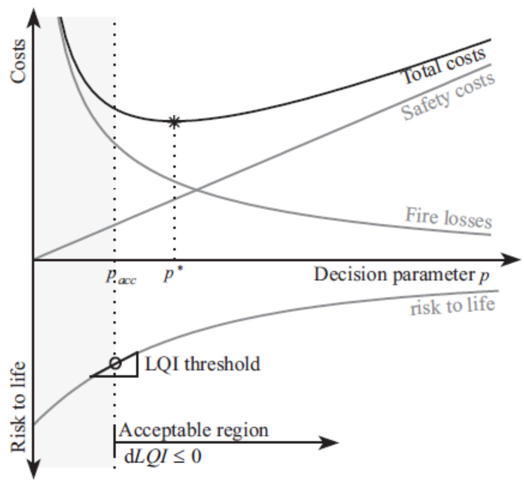

Fig. 1: *Interaction of the monetary optimization criteria* p^* *and the social criterion* p_{acc} *related to society's investment in life safety [11]*

Using this concept, ISO 2394 [1] defines a basic requirement according to which a target POF can be set based on monetary optimization only when the risk of loss of human life because of failure is minimized (which is equivalent to defining the limit or threshold value of the reliability measure). Otherwise, the LQI-criterion is used to determine the limit target values of POF.

Costs H, which are charged in case of failure in addition to the costs of reconstruction, include the costs of dismantling, as well as human losses, expressed in monetary units:

$$H = C_d + N_F \cdot H_c, \tag{24}$$

where C_d is the demolition cost; N_F is the estimated number of human casualties and injuries; Hc is the compensation payment in case of human death or permanent injury (loss of limb or cognition).

Failure costs $D(p)$ are not included in the acceptance criteria, since the reduction in these cost components can be seen as a monetary benefit from improved safety. Thus, the following rule can be formulated:

When establishing a criterion based on LQI, it is sufficient to quantify the marginal increase in construction and obsolescence costs with a corresponding change (decrease) in risk to life (see Fig. 1) as a function of the decision parameter:

$$\frac{d\left[C(p)+A(p)\right]}{dp} \geq -\frac{1}{\gamma_s}\frac{dN_F(p)}{dp} \cdot SWTP. \tag{25}$$

In criterion (25), the risk to human life is determined by: (1) the expected number of victims $N_F(p)$, (2) the social discount rate γ_s, and (3) the social willingness to pay (SWTP). The latter is the amount of money a society is willing to pay to reduce mortality by one unit. The concept of SWTP is based on the assumption that investments in life saving measures will lead to a change in the level (intensity) of mortality. From the change in mortality intensity $d\mu(a)$ the change in the remaining life expectancy $de(a)$ for each age group can be calculated.

In general, SWTP for saving one additional life of a statistically average individual is defined as follows [10, 20-23]:

$$SWTP = -dG = \frac{G}{q} E_A \left[\frac{de_d(a)}{e_d(a)} \right], \tag{26}$$

where dG is the loss of disposable income (reduction in GDP per capita) due to investment in life saving measures (hence, dG is negative), $E_A \left[\frac{de_d(a)}{e_d(a)} \right]$ is the age-averaged relative change of discounted life expectancy $e_d(a)$.

Age-averaging is introduced to account for the fact that different life saving strategies may have different effects on different age groups. Discounting is introduced to take into account the effect that lives saved are saved in the future [1].

Discounted life expectancy for age a is determined by the formula [10, 23]

$$e_d(a) = \int_a^{a_u} \exp\left[-\int_a^t \left[\mu(\tau) + \gamma \right] d\tau \right] dt, \tag{27}$$

where $\mu(a)$ is the intensity of mortality at the age a (determined from mortality tables), γ is the discount rate, a_u is the maximum age to which people live in the country, region, industry, company, or the cohort under consideration.

For a small reduction in mortality, the SWTP for saving one life can be obtained using the *demographic constant* C_x for a certain mortality regimen x [1, 20-23]:

$$SWTP \approx \frac{G}{q} C_x, \tag{28}$$

There are three main mortality regimens (see details in [21, 23]). Here we consider the so-called Δ-regimen when the change of mortality intensity is the same for all ages: $\mu_\Delta(a) = \mu(a) + \Delta$. This regimen is suitable for most cases related to structural safety, since the risk reduction measures usually have the same effect on all people, regardless of their age [1].

According to [20-23], for the Δ-regimen:

$$C_x = \int_0^{a_u} \frac{\int_a^{a_u} (t-a) \exp\left[-\int_a^t \left[\mu(\tau) + \gamma \right] d\tau \right] dt}{\int_a^{a_u} \exp\left[-\int_a^t \left[\mu(\tau) + \gamma \right] d\tau \right] dt} h(a)\, da, \tag{29}$$

where $h(a)$ is the PDF of the age distribution of the considered population.

Along with SWTP (corresponds to the amount that society can afford to invest to save one life), there is also the concept of Social Value of a Statistical Life (SVSL), which corresponds to the amount that a society can pay for each death (compensation payment) [1]:

$$SVSL \approx \frac{G}{q} E_A \left[e_d(a) \right], \tag{30}$$

where $E_A\left[e_d(a) \right]$ is the age-averaged discounted life expectancy:

$$\bar{e}_d = E_A\left[e_d(a) \right] = \int_0^{a_u} e_d(a) h(a)\, da. \tag{31}$$

According to [23], the age-averaged life expectancy without discounting $\bar{e}_d \approx \frac{e(0)}{2} = 40$ years for industrialized countries. Using a typical average discount rate (3%-4%) one gets

$\bar{e}_d \approx \frac{e(0)}{4} = 20$ years.

Using the results of section II, criterion (26) can be transformed to the form:

$$C_1\left(1+\frac{\omega}{\gamma_s}\right) \geq \frac{1}{\gamma_s} SWTP \cdot N_F \frac{dP_f(p)}{dp}. \quad (32)$$

In criterion (32), the risk to life is expressed in terms of the probability of failure $P_f(p)$ and the estimated (expected) number of victims N_F. Since the LQI-criterion is a society determined /established boundary condition, the social discount rate γ_s is used in criterion (32).

For a combination of an LQI-criterion with functions used in monetary optimization, Rackwitz and Streicher [16] relate the LQI-criterion to the relative cost of safety measures C_1/C_0, assuming a fixed value of construction costs C_0. As a result of applying this assumption, the criterion becomes dependent on the scale of C_1/C_0.

The transformation of inequality (32) leads to the following criterion:

$$-\frac{dP_f(p)}{dp} \leq \frac{C_1(\gamma_s+\omega)}{SWTP \cdot N_F} = k_1. \quad (33)$$

The numerator $C_1(\gamma_s+\omega)$ on the right side of the inequality shows how much the annual costs associated with ensuring safety measures unit increase with an increase in the value of the decision parameter p (global safety factor). In the denominator of the right side of inequality (33), the consequences of the loss of human lives N_F because of the failure are given in monetary units. Thus, the coefficient k_1 is the ratio of the costs of ensuring the safety measures to the costs associated with the marginal cost of saving N_F human lives (SWTP). Reliability targets according to LQI can now be obtained as a function of the constant k_1.

Table 1 according to ISO 2394 [1] presents the tentative minimum target reliability measures obtained using the LQI criterion and monetary optimization, for various classes of relative life saving costs. These classes are defined in terms of the range of the constant k_1 (second column of Table 1). The values in Table 1 are derived from log-normal distributions of load and resistance effects with coefficients of variation $0.1 \leq V \leq 0.3$.

Table 1: *Tentative minimum target reliability measures (ULS, $T_{ref} = 1$ year) based on the LQI-criterion and monetary optimization according to ISO 2394 [1]*

Relative life saving costs	$k_1 = \frac{C_1(\gamma+\omega)}{SWTP \cdot N_F}$	LQI-objective measure of reliability
Large	$10^{-2}...10^{-3}$	$\beta = 3.1\left(P_f = 10^{-3}\right)$
Medium	$10^{-3}...10^{-4}$	$\beta = 3.7\left(P_f = 10^{-4}\right)$
Small	$10^{-4}...10^{-5}$	$\beta = 4.2\left(P_f = 10^{-5}\right)$

Note: the values in the table were obtained with $0.1 \leq V \leq 0.3$. The target probability of failure may be increased by a factor of 5 for higher coefficients of variation of the basic RVs. For low variability, on the other hand, it should be reduced by a factor of 2, $\gamma_s = 0.04, \omega = 0.02$.

It was shown in [17] that combinations of different distributions for load and resistance with their values of coefficients of variation $V \leq 0.3$ have little effect on optimization results. The distribution of resistance is important only when $V_R \geq 0.3$. At the same time, it is noted [17] that this is an extremely rare case for resistance (a Black Swan event). At the same time, these estimates are given without an in-depth analysis that takes into account the uncertainties of the calculation models of resistances and loads.

For the case of small variation coefficients of resistance and load effects, $(0.1 \le V \le 0.3)$ it is proposed to determine the target value of the threshold probability of failure p_{acc} as a measure of reliability based on the LQI criterion using the following simplified formula [17]:

$$p_{acc} \cong \frac{1}{5} k_1 = \frac{1}{5} \frac{C_1(\gamma + \omega)}{SWTP \cdot N_F}. \quad (34)$$

At the same time, it is noted that if the consequences associated with the loss of human life in case of failure are very high (constant $k_1 > 10^{-2}$), criterion (34) should not be applied. The threshold (boundary) value p_{acc}, set using the LQI -criterion, determines a certain range of acceptable values within which monetary optimization should be performed.

The optimal solution p^* is determined when the first derivative of the objective function (19) at point p^* is equal to zero

$$\frac{dZ(p)}{dp} = \frac{dC(p^*)}{dp}\left(1 + \frac{\omega}{\gamma}\right) + \frac{dC(p^*)}{dp} \frac{P_f(p^*)}{\gamma} + \frac{dP_f(p^*)}{dp} \frac{C(p^*) + H}{\gamma} = 0. \quad (35)$$

Considering that $P_f(p^*) \ll \gamma$, the second term in (36) can be neglected, then

$$\frac{dC(p^*)}{dp} = \frac{C(p^*) + H}{(\gamma + \omega)} \cdot \frac{dP_f(p^*)}{dp}. \quad (36)$$

Substituting (36) into criterion (33) and considering that $\frac{dP_f(p)}{dp} < 0$ for all p, we obtain:

$$\frac{SWTP \cdot N_F}{\left(C(p^*) + H\right)} \le \frac{\gamma_s + \omega}{\gamma + \omega}. \quad (37)$$

Criterion (37), considering compensation payments $H_c N_F$ can be rewritten as follows:

$$\frac{N_F \cdot H_c}{C(p^*) + H} \le \frac{(\gamma_s + \omega)}{(\gamma + \omega)} \cdot \frac{H_c}{SWTP}. \quad (38)$$

Criterion (38) includes three components: (1) the ratio of death compensation payments $H_c N_F$ resulting from a failure to the full costs associated with the failure $C(p^*) + H$. Obviously, this ratio is always less than unity; (2) the ratio of the social discount rate γ_s to the discount rate γ set by a private decision maker and performing monetary optimization. In the case of applying the LQI-criterion from a social point of view $\gamma_s = \gamma$ and then $\frac{\gamma_s + \omega}{\gamma + \omega} = 1$. In monetary optimization, when the decision is made by an individual, usually $\gamma > \gamma_s$; (3) the ratio of how the loss of life translates into monetary optimization (compensation payments H_c) and a LQI criterion for the threshold value of SWTP, respectively.

The amount of compensation payments H_c depends on several different factors. For a private decision maker, most likely it is impossible to accept one fixed value of H_c regardless of the specific situation. Hence, in the general case, the ratio $H_c / SWTP$ should be considered as some variable.

As shown in [17], for many computational cases it can be assumed that $\gamma_s / \gamma = \frac{\gamma_s + \omega}{\gamma + \omega}$. Hence, it suffices to check whether the ratio of the discount rates is γ_s / γ greater than the ratio of the cost of SWTP for saving N_F human life to the total damage cost from structure failure $C(p^*) + H$. For a social decision maker, $\gamma_s / \gamma = 1$ and criterion (39) is simplified to checking whether $SWTP \cdot N_F$ is greater than $C\left(p^{*}\right)+H$.

Target reliability measures associated with the consequences of failure and based on monetary optimization according to ISO 2394 [1] are given in Table. 2. Consequence classes should be taken according to Table F.1 of ISO 2394 [1], which contains not only a description of the expected consequences of failure, but also the predicted number of victims (for example, for Class 2 - no more than 5; Class 3 - no more than 50; a Class 4 - no more than 500).

Table 2: *Tentative target reliabilities (ULS, $T_{ref}=1$ year), based on monetary optimization according to ISO 2394 [1]*

Relative cost of safety measures	Failure consequences (classes according to Table F.1 [15])		
	Class 2	Class 3	Class 4
Large	$3.1\left(P_f\approx 10^{-3}\right)$	$3.3\left(P_f\approx 5\cdot 10^{-4}\right)$	$3.7\left(P_f\approx 10^{-4}\right)$
Medium	$3.7\left(P_f\approx 10^{-4}\right)$	$4.2\left(P_f\approx 10^{-5}\right)$	$4.4\left(P_f\approx 5\cdot 10^{-6}\right)$
Small	$4.2\left(P_f\approx 10^{-5}\right)$	$4.4\left(P_f\approx 5\cdot 10^{-5}\right)$	$4.7\left(P_f\approx 10^{-6}\right)$

It should be noted that the same values of failure probabilities and reliability indexes (see Table 3) are presented in the PMC JCSS code [2], which is also based o**n** **[**3]. The target values of POF depend not only on the failure consequences, but also on the relative cost of the safety measures.

Table 3: *Target values of reliability index β_{tag} (failure probabilities) for $T_{\text{ref}}=1$ year and ULS according to [2]*

Relative cost of security measures C_1/C_0	Small Consequences $\rho<2$	Medium Consequences $2\le\rho<5$	Large (significant) consequences $5\le\rho<10$
Large (A)	$3.1\left(P_{\text{tag}}\approx 10^{-3}\right)$	$3.3\left(P_{\text{tag}}\approx 5\cdot 10^{-4}\right)$	$3.7\left(P_{\text{tag}}\approx 10^{-4}\right)$
Medium (B)	$3.7\left(P_{\text{tag}}\approx 10^{-4}\right)$	$4.2\left(P_{\text{tag}}\approx 10^{-5}\right)$	$4.4\left(P_{\text{tag}}\approx 5\cdot 10^{-6}\right)$
Small (C)	$4.2\left(P_{\text{tag}}\approx 10^{-5}\right)$	$4.4\left(P_{\text{tag}}\approx 5\cdot 10^{-5}\right)$	$4.7\left(P_{\text{tag}}\approx 10^{-6}\right)$

At the same time, PMC JCSS [2] proposes to consider following additional aspects when differentiating reliability and classifying structural designs:

a) qualify the failure consequences using the coefficient ρ, which is the ratio of the total cost of damage $C_0+C_1(p)+H$ associated with the same structure failure, to the cost $C_0+C_1(p)$ of its (re) construction:

$$\rho=\frac{C_0+C_1(p)+H}{C_0+C_1(p)}=1+\frac{H}{C_0+C_1(p)}\approx 1+\frac{H}{C_0}. \tag{39}$$

The construction cost includes a fixed part C_0 and a cost $C_1(p)$ depending on the decision parameter. The cost H denotes the direct damage associated with the failure, including the actual cost of damage and subsequent total or partial dismantling of the structural system as a whole or part of it, environmental consequences (carbon footprint), damage to infrastructure, communications, etc., as well as human casualties expressed in monetary units. When $\rho>10$ the target values of the probability of failure and the reliability index should be set, based on the results of a systematic risk analysis for the design object;

b) Reliability target values given in Table 3 refer to the structural system or, with some allowance, for the dominant form of system failure. The same values apply to a key structural element (that dominates the failure of the system). Based on this, it is assumed that structural systems with several equally significant forms (modes) of failure should be designed with a higher level of reliability.

The values given are, of course, tentative. At the same time, the methods of their determination contain quite significant uncertainties. This refers, in the first place, to the division into appropriate classes of the relative cost of safety measures (C_1 / C_0), mainly adopted from [17, 18], in which the following relative cost intervals are established:

- large cost (A): $10^{-2} \le C_1 / C_0 \le 10^{-1}$;
- normal/medium (B): $10^{-3} \le C_1 / C_0 \le 10^{-2}$;
- small (C): $10^{-4} \le C_1 / C_0 \le 10^{-3}$.

These cost classes were derived in [17] from dependencies $[(C_1 / C_0)-(H / C_0)]$when compared with the optimal values of target reliability measures according to PMC JCSS [2]. They are derived following [3] at a discount rate $\gamma = 0,03$, coefficient $\omega = 0,02$, variation coefficients $V_R = V_S = 0,3$ for a log-normal distributions of resistance and load effects.

In general, ISO 2394:2015 [1] and PMC JCSS [2], which are based o**n** [3, 4], provide a more appropriate reliability differentiation for existing designs than Eurocodes [19, 20], as they consider the cost of safety measures.

IV. Determination of acceptable reliability targets for residential buildings in the Sverdlovsk region of the Russian Federation based on the LQI criterion

To assess the threshold probability of failure p_{acc}, as a measure of reliability based on the LQI criterion, for residential multi-storey buildings in the Sverdlovsk region, the initial data for 2021 were used, presented in Table. 4. All monetary components were converted into USD at the average exchange rate in 2021: 1 USD = 73.65 rubles.

Table 4: *Initial data for assessing the threshold probability of failure of residential multi-storey buildings in the Sverdlovsk region (for 2021)*

Parameter	Meaning
GDP per capita, *G*	710.381 thousand rubles/person 9645.36 USD/person
Life expectancy at birth, e_0	68.79 years
Cash income per capita/mo, *S*	40.275 thousand roubles $ 546.84 USD
Population, *P*	4277.203 thousand people
Number of people employed in the economy, *M*	2034.600 thousand people
The average cost of construction of 1m² of residential multi-storey buildings, C_0	44.800 thousand rubles/m² 608.28 USD/m²
Average key rate of the Central Bank of Russia	6%

For further analysis, when estimating the (discounted) life expectancy and the demographic constant C_x, it is necessary to know the type of mortality intensity function $\mu(a)$ and, therefore, the law of mortality. Assume that the mathematical model of mortality is the Gompertz-Makham law (G–M), according to which:

$$\mu(a) = M + \alpha e^{\beta a},$$

where M is the Makeham coefficient characterizing the contribution to accidental mortality of impacts (the effect of which does not depend on age), α is the level of initial mortality, β is the rate of increase in mortality. The G–M law is most suitable for studying the process of human mortality [24], since it contains both the exponentially growing component of mortality due to aging, and the age-independent component associated with extreme situations.

The G–M law parameters are calculated according to the mortality (survival) tables. Mortality tables (MT) are built using age-specific mortality rates (MR). The Russian database on fertility and mortality in Russian regions is located on the website of the Center for Demographic Research of the Russian Economic School [25]. A MT was **b**uilt based on the MR of the Sverdlovsk region for 2021, according to the methodology of the Federal State Statistics Service [26]. The G–M model parameters are calculated according to the method [24] on the basis of the obtained MT:

$$M = 7.4 \cdot 10^{-4}, \ \alpha = 2.1 \cdot 10^{-4}, \ \beta = 7.6 \cdot 10^{-2}.$$

The mean absolute percentage error between the life expectancy calculated according to the G–M law and the life expectancy from the mortality table is 6.5%. Fig.2 shows the values of the ALE obtained on the basis of the G–M law and their discounted values.

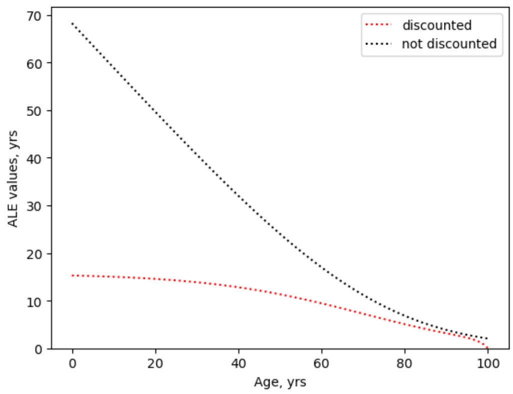

Fig. 2: *ALE obtained on the basis of the G–M law and their discounted values*

The demographic constant calculated by formula (29), at the discount rate $\gamma = 0.06$, is equal to C_x = 10.4 (years). For comparison, according to [23], $\gamma = 0.03$ for industrialized countries, C_x ranges from 13.9 to 16.9. In our case, when $\gamma = 0.03$, the constant C_x = 14.2, which fits into the overall picture. Using formulas (21)-(23) and Table 4, w = 0.11, β = 0.68, q = 0.19. Then for the Sverdlovsk region LQI = 383 USD·year and at $\gamma = 0.06$ SWTP = 535,748 USD per year. At $\gamma = 0.03$ SWTP = 732,298 USD

To estimate SVSL, it is necessary to have the density $h(a)$ of the age distribution of the Sverdlovsk region population. Since at the time of writing this paper, these statistics were not available in the public domain, the age distribution for the whole of Russia was used, shown in Fig. 3.

Thus, we got that compensation payments for the death of one person SVSL = 609,941 USD, i.e. costs that are economically feasible to invest in saving lives. Using dependencies (33) and (34), the target values of POF of residential multi-storey buildings in the Sverdlovsk region of Russia were calculated and presented in Table 5. To obtain the threshold (limit) values of POF p_{acc} it is necessary to divide the data of Table 5 by 5 (see formula (34)).

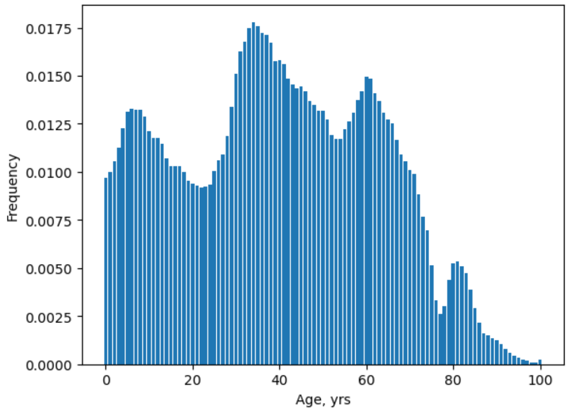

Fig. 3: *Age distribution of the Russian population*

Table 5: *k_1 values for residential multi-storey buildings in the Sverdlovsk region depending on the relative cost of safety measures and human losses in case of structural failure*

N_f/m^2	Residential multi-storey buildings construction cost C_0 = 608.28 USD/m ^{2}		
	Relative cost of safety measures C_0/C_1		
	0.001 (small)	0.01 (normal)	0.1 (large)
0.0001	$9{\cdot}10^{-4}$	$9{\cdot}10^{-3}$	$9{\cdot}10^{-2}$
0.001	$9{\cdot}10^{-5}$	$9{\cdot}10^{-4}$	$9{\cdot}10^{-3}$
0.01	$9{\cdot}10^{-6}$	$9{\cdot}10^{-5}$	$9{\cdot}10^{-4}$
0.1	$9{\cdot}10^{-7}$	$9{\cdot}10^{-6}$	$9{\cdot}10^{-5}$

The residential norm in Russia is 10 m^2 per person, hence, $N_F / m^2 = 0.1$ and K_1 changes from $9{\cdot}10^{-7}$ to $9{\cdot}10^{-5}$ (depending on the relative cost of safety measures). Based on the obtained values, for the normal relative cost of safety measures, a tentative target value of the threshold POF can be taken as $p_{acc} \approx \frac{1}{5}k_1 = 2\cdot10^{-6}$, set using the LQI -criterion, which determines a certain range of acceptable values within which monetary optimization should be performed.

V. Conclusion

The generally accepted approaches to the assignment of acceptable target reliability measures [1,2] are considered and analyzed. These standards reflect the main results of [3,4], but have several important uncertainties left unresolved. Future research should consider the effect of different diagnostic, monitoring, maintenance strategies and the sub/supra resilience effect [27, 28] on the target values of reliability of civil and industrial engineering infrastructures and systems.

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