MANAGING THE RISK OF EMERGENCIES CAUSED BY GROUNDWATER FLOODING OF THE HISTORICAL BUILT-UP AREAS

Elena Arefyeva
All-Russian Research Institute for Civil Defense and Emergencies of the EMERCOM of Russia
elaref@mail.ru

Abstract

The article presents an approach to managing the risk of groundwater flooding emergencies, including those in historical built-up areas. The approach is based on the application of a set of methods, including the method of forecasting, the method of developing an optimal control action, and developing a compromise solution. The peculiarity of historical built-up areas is that there is a significant cultural layer that does not allow drainage, because watered soil preserves the cultural layer. At the same time, architectural monuments need to drain the foundations and not water the soils of the foundations of structures.

Keywords: groundwater flooding, optimization approach, Boussinesq equation, historical territories, architectural monuments

I. Introduction

Urbanized territories are a complex of interconnected systems of natural and man-made character. Their joint functioning determines the degree of quality of the urban infrastructure function. Unbalanced economic activity can initiate hidden and activate existing dangerous natural processes. Their timely tracking, forecasting and decision-making aimed at their prevention is the task of municipal structures and structures responsible for security.

It has been established that the hazards caused by geological and hydrological processes in cities are of a synergistic nature [1]. One realized danger causes a chain of subsequent dangerous processes. The damage from the total emergencies caused by synergistic chains exceeds the sum of individual damages in the separate development of such processes. And the measures taken are not adequate to the development of a sequence of negative events. The action of synergetic processes - consequences continues after the action of the initiating factor. Such initiating factors for the emergence of synergistic chains include flooding.

Flooding initiates dangerous natural processes, such as karst, landslides, suffosions, loess subsidence, soil swelling, and at the same time, the bearing capacity of soils decreases, the seismic intensity of the territory increases, etc. [2, 3]. Particularly affected by the negative impact of the underground hydrosphere are historical cities and architectural monuments that are not adapted to modern man-made impacts [2]. Almost half of Russian cities (425) are historical cities (79% in the European part, 11% in the Urals, 10% in Siberia and the Far East). 25,000 monuments of federal significance. This is a huge cultural historical heritage, which is being destroyed before our eyes when measures are not taken to protect against dangerous processes. The most defenseless from dangerous natural impacts were such cities as St. Petersburg, Novgorod, Pskov, Smolensk, Rostov the Great, Yaroslavl [4].

These processes can lead and lead to emergency situations. The negative impacts of groundwater on buildings and structures cause from 50 to 80% of all accidents and collapses [5, 6].

The operational reliability of buildings and structures is reduced by 30%. The damage from emergencies during flooding is up to 5.5 billion dollars annually [7].

Emergencies can potentially arise if the following conditions exist [1]:

the presence of a hydrogeological or engineering-geological hazard in a given area and the location of the object of protection within this hazard;

absence or unsatisfactory operation of the engineering protection system;

unsatisfactory state of structures of buildings.

The usual protective and preventive measures against undergroundwater flooding are various types of drainage, waterproofing systems, and surface runoff control systems [8]. Since emergency situations during flooding are usually caused by man-made causes, this allows us to conclude that the process of preventing these emergencies is manageable [1, 8]. A feature of historical cities is the presence of a cultural layer - a valuable archaeological material. Groundwater for the cultural layer is a preservative. The foundation of buildings in Russian northern cities was traditionally located on a wooden base. Soil moisture ensures the safety of the wooden base, and its drying leads to rotting and deformation and destruction of buildings. At the same time, basements and foundations of buildings must be drained. Therefore, in built-up areas, it is necessary to perform a soft regulation of the groundwater regime, without sharp fluctuations in the groundwater level, in order to prevent additional negative effects. Thus, drainage can cause suffusion, subsidence of the surface, overdrying of the soil, and further deformation of buildings [1, 8].

An important solution for soft regulation is the use of a criterion - the threshold of hydrogeological hazard, the critical level of groundwater, determined for each object and plot of a built-up area [5].

To determine the critical and limiting levels of groundwater for various buildings, a typology of objects was obtained depending on the properties of the foundation foundations. In historical territories, 4 main classes of objects were identified according to drainage requirements; these are objects that allow and do not allow a moist state of buried structures and basements, and among these groups are objects located on a stone or wooden base [1].

II. Methods

To develop solutions for flexible optimal regulation of the groundwater regime, including the regulation of groundwater regime in the historical built-up areas, a software-computer complex has been developed; mathematical modeling is carried out.

Calculation of the estimated groundwater level (geofiltration equation) is performed using the Boussinesq equation (it is a parabolic partial differential equation) with appropriate boundary and initial conditions). Coefficients of equations characterizing the properties of soils and additional infiltration nutrition [9].

The initial data for modeling are distributed fields of numerical values of the coefficients of equations (water conductivity, water loss, infiltration, etc.). With the help of monitoring, by interpolation and extrapolation, calculated geofiltration schemes are compiled - the parameters of the geological environment [1, 5].

The control parameters are the water levels in the drains, the volume of pumped water. Optimality criterion: the volume of the drained prism of the cultural layer, the drained area of the buried premises.

Regulating parameters are set in the form of boundary conditions of the geofiltration equation or source function [1, 5].

The optimization criteria are formulated as a functional. Minimization of the functional determines the degree of closeness of the calculated reduced values of the groundwater level with the help of control actions to a given value [1, 5]. At the same time, taking into account the presence of an acceptable lower limit, it is ensured that the groundwater level does not go beyond its limits.

The gradient of the functional is determined through the solution of the adjoint problem to the direct problem[10, 11, 12]. The definition of the gradient of the optimizing functionality (and the construction of a sequence minimizing it is carried out according to the gradient projection method, which is included in the main mathematical method [5,10]. The result of calculations in the iterative process is the optimal solution that provides a minimum of optimizing functionality [10, 11] and is the desired optimal control parameter, which physically corresponds to the selected real drain position or the volume of water pumped out from drain [1, 5].

The process of groundwater flooding in general form will be described by a parabolic equation with a source function $f(x,t)$ [5, 9]:

$$\frac{\partial y(u)}{\partial t} + A(t) * y(u) = f(x,t) + u(t)$$
$$(x,t) \in Q; y(u) = 0, (x,t) \in \partial Q; y(x,0\$u) = y_0(0), x \in D, \partial Q = \partial D * (0,T); \quad (1)$$

Where is $A(t) * y(u) = -\sum_{i=1}^{n}\sum_{j=1}^{n} \frac{\partial}{\partial x_i a_{ij}(x,t)} \frac{\partial y}{\partial x_j}$; - a differential operator;

$u \in U = L_2(Q); u = (u_1, u_2, ...u_k);$ -- control vector; $y(t) = (y_1(t), y_2(t), ...y_k(t))$ - -phase trajectory; $Y = \{y\}$ --phase space; $z(u) = C * y(u)$ - - the observed state of the object; $C : L_2(0,T;V) \to H_1$ - linear operator; H_1 - Hilbert space.

The quality of control in general terms is described by a criterion - an optimizing functional:

$$J(u) = \|Cy(u) - z_0\|^2_{H_1} + (Nu,u)_U; \quad (Nu,u) \ge c\|u\|^2_U, \quad (2)$$

$c > 0, \forall u \in U$, *u*- and is the control vector,

Taking into account the accepted notation, the optimal control problem is formulated as follow.

Let the set of admissible controls be , it is required to find such a control u from , so that the solution of the original Boussinesq equation (1) with the initial and boundary conditions[9], the control would deliver a minimum to the functional J(u) [5]:

$$u \in U_D,$$
$$J(u) = \inf J(u), \quad (3)$$
$$u(t), 0 \le t \le T$$

From the point of view of solving a specific control optimization problem, the statement is formulated as follows: by controlling the water level in the drains (the depth of the drains) or the water flow in them, achieve the necessary drainage of the object with minimal drainage of the nearby territory (for example, the cultural layer, park zone) and maintain the specified range of GWL changes with the help of water discharges in drains. For the two-dimensional case (plane-parallel filtration flow), the following Boussinesq equation is considered, which is non-linear and can only be solved numerically using stable convergent difference schemes, for example, implicit Crank-Nicolson schemes.

Non-stationary filtration of groundwater in an area of arbitrary shape D, contoured by a curvilinear boundary *Г* with a single-layer non-pressure-pressure planned flow, is described by a system of two differential equations of parabolic type. In a non-pressure subdomain (D_1)-piecewise-heterogeneous in terms of porosity-capacitance, a single-layer planned flow on a non-horizontal aquiclude (in the presence of a hydraulic connection with a neighboring aquifer and the amount of infiltration supply entering the free surface of groundwater) is described by a nonlinear differential equation of the form [9]:

$$\mu\frac{\partial h}{\partial t}=\frac{\partial}{\partial x}\left[k(h-H^{B})\frac{\partial h}{\partial x}\right]+\frac{\partial}{\partial y}\left[k(h-H^{B})\frac{\partial h}{\partial y}\right]\pm\frac{k_0}{m_0}(\overline{H}-h)+w(x,y,t) \tag{4}$$

where *k(x, y)* and *h(x, y, t)* are functions that determine, respectively, the filtration coefficient and the absolute elevation of the groundwater level of the main aquifer; H^B is the absolute elevation of the base of the aquifer; $k_0(x, y)$ and $m_0(x, y)$-functions, which determine, respectively, the filtration coefficient and the thickness of the weakly permeable interlayer separating aquifers; $\overline{H}$ *(x,y)* - absolute mark of the groundwater level of the underlying aquifer; *w(x, y, t)* - intensity of infiltration nutrition; *μ(x,y)* is the coefficient of water loss when the free surface is lowered ($\frac{\partial h}{\partial t}<0$) or the coefficient of undersaturation when the free surface is raised ($\frac{\partial h}{\partial t}>0$).

For a unique solution to (4) and (5), boundary conditions are set: boundary and initial. Let the outer contour be denoted by Γ, and the inner one, corresponding to the assignment of boundary conditions for wells, drains, canals, springs, denoted by p. Therefore, by boundary conditions we mean both "external" and "internal" boundary conditions (BC) [9].
Boundary conditions of the *1st* kind:

$$\begin{aligned}&h(x,y,t)\big|_{\Gamma+p}=h(s,t),\\&s\in\Gamma+p\end{aligned} \tag{6}$$

Boundary conditions of the second kind have the form:

$$\begin{aligned}&-T\frac{\partial h}{\partial n}\Big|_{\Gamma+p}=q(s,t),\\&s\in\Gamma+p\end{aligned} \tag{7}$$

where *p* is the normal to the contour *(Γ + p)*; in the case of an impenetrable boundary, we have
q(s,t)=0.

The initial conditions look like:

$$\begin{aligned}&h(x,y,0)=h_0(x,y);x,y\in D_1;\\&H(x,y,0)=H_0(x,y);x,y\in D_{2.}\end{aligned} \tag{8}$$

The initial conditions have the form: Boundary conditions of the first kind set the groundwater level at the boundary. In the case of laying drains (internal boundary conditions) - the area is divided into sub-areas bounded by drains, and the water level in them - are the boundary conditions of the first kind. Boundary conditions of the second kind, when the water flow in drains (both horizontal and vertical drainage) is set on the inner boundary. In the boundary conditions of the third kind, the relationship between surface and ground waters is specified. Equations (4) and (5) with boundary conditions (6-8) form the basis for solving the optimization problem of managing the groundwater regime in order to prevent the development of emergencies, which is generally given in formulas (1-3).

Thus, the problem of optimal management of the prevention of hydrogeological emergencies at facilities is a sequence of tasks [5, 9, 14-18]:

• solution of a direct predictive problem, including simulation of the filtration process (modeling of the geological and hydrogeological environment using schematization - drawing up a geofiltration model of the environment, modeling the process of flooding - the filtration process using equations of mathematical physics with the assignment of boundary and initial conditions) [5, 9, 14];
• construction of an optimization model for regulating the groundwater regime in order to prevent emergencies at facilities based on the formation of optimization criteria, methods for solving extreme problems (conjugate gradient method) [5, 14];
• development of an optimization modeling algorithm for generating a control action, based on the numerical implementation of the constructed algorithms.
Consider a one-dimensional case, it is applicable to a large number of flood protection objects that have a shape close to a square with a uniform aquiclude profile. Using the superposition principle, a two-dimensional problem can be approximated by one-dimensional problems [17].

Mathematically, the problem is formulated for hydrogeological sections in a one-dimensional region on the interval $x_l < x < x_r$.

In the difficult conditions of a built-up area, in the presence of simultaneously different requirements for drainage standards (objects, buildings with different bases of foundations, and other requirements from soils vulnerable to a sharp decrease in soils or the presence of a wooden base of structures on frozen and heaving soils), the functional was chosen as an optimization criterion (1) to be minimized [5, 17, 22]:

$$J^l(h,u) = J^l_n + J^l_k, \quad l=1,2,...; \tag{9}$$

where is $J^l_n = A^h \int_Q (h(x,T,u)-h_n(x))^2\,dx$, the functional-criterion of control responsible for the unsinkability of the object:; where is A^h -- some weight;
$h(x,T,u)$- the level obtained as a result of the calculated control actions;
T – the time by which it is required to carry out water reduction works;
u - control vector (control parameters of the drain—depth of laying, water flow, etc.;
$h_n(x)$- function of limiting groundwater level for an object;
$Q=(x_1\ ,\ x_2\ )$, region of integration,($x_1,\ x_2$ - drain location).

J^l_k - the functional-criterion "responsible" for the requirement that the water level "arrange" the adjacent soil, in other words, при $J^l_k=0$ cultural layer or park zone, soil unstable in terms of suffusion in some vicinity of the protected object is not strongly overdried.
The form of this term-functional (10) is as follows (for two filtration areas separated by the protection object):

$$J^l_к = A_{к1}{}^h \int_{Q1}(h(x,T,u)-h_к(x))^2\,dx + A_{к2}{}^h \int_{Q2}(h(x,T,u)-h_к(x))^2\,dx, \tag{10}$$

Where are $A_{к1}, A_{к2}$ -- some weights;
$h_к(x)$ - limit level function for the adjacent cultural layer, soil.

Limiting and critical levels for objects and other structures are given in the work [5].

Let us now formulate the control optimization problem:

it is required during the iterative process to minimize the functional $J^l(h,u)$ provided that the groundwater profile $h(x,T,u)$- satisfies the Boussinesq equations (4) or (5) with boundary conditions (6, 7), in general, the parabolic equation is given in expression (1). For many practical problems, a two-dimensional equation can be reduced to a series of one-dimensional equation and, using the principle of superposition, combine the resulting solutions. Levels in the vicinity of drains h_1 and h_2 are control actions, $u=(h_1, h_2)$-control vector, equation in the first region $x_l < x < x_1$:

$$\frac{\partial h}{\partial t} = a^2 \frac{\partial^2 h}{\partial x^2} + f(x,t), \quad 0 < t < T \quad ; \tag{11}$$

$$\frac{\partial h}{\partial x} = 0, x = xl$$
$$h = h1, x = x1$$

Where is h_1 - level in the first drain, control parameter.
Equation in the second - central region $x_1 < x < x_2$:

$$\frac{\partial h}{\partial t} = a^2 \frac{\partial^2 h}{\partial x^2} + f(x,t), \quad 0 < t < T; \tag{12}$$

$$h = h1, x = x1$$
$$h = h2, x = x2$$

Where is h_2 - level in the first drain, control parameter.
Equation in the third region $x_2 < x < x_r$:

$$\frac{\partial h}{\partial t} = a^2 \frac{\partial^2 h}{\partial x^2} + f(x,t), \quad 0 < t < T: \tag{13}$$

$$\frac{\partial h}{\partial x} = 0, x = x_r$$
$$h = h2, x = x_2.$$

Initial distribution $h(x,0)=h_0(x)$, $f(x,t)$ is a function of additional infiltration (precipitation, pipe leaks, pumping water from the ground).
To minimize the functional $J(u)$ - goal criterion, it is required to calculate its gradient, for which the conjugate gradient method is used.
Let us write the adjoint equations for the region $x_l \langle x \langle x_1$ for formula (11):

$$\psi_t = -a^2 \psi_{xx} - 2A_l^{h_k} \max(\underline{h_k} - h(x,t,u),0)$$
$$\psi_x = 0, x = x_l$$
$$\psi = 0, x = x_1 \tag{14}$$
$$\psi(x,t=T) = 2A^{hk_{k(}}(h(x,T,u) - h_0(x));$$

For the domain $x_1 \langle x \langle x_2$, for formula (12), and for formula (13), similar equations are written.

Adjoint equations are solved similarly to the main ones. The formal replacement of the direction of time from "+" to "-" leads to the fact that the value of the function is set not at the final, but at the initial moment of time. The variables are changed $t`=T-t$. Finally, the gradient of the functional is determined by the formula:

$$J' = (a^2(\psi'(x_2^+,t) - \psi'(x_2^-,t)), a^2(\psi'(x_{1,}^-,t) - \psi'(x_1^-,t))),$$
$$если.$$
$$h_1 = h_1(t), h_2 = h_2(t), \tag{15}$$
$$и$$
$$J' = (a^2\int_0^T(\psi'(x_2^+) - \psi'(x_2^-))dt, a^2\int_0^T(\psi'(x_1^+) - \psi'(x_1^-)))dt,$$

The last expression is given for the gradient if groundwater levels are assumed to be time independent.

The derivatives Ψ′ of the solution of adjoint problems in the neighborhoods of the points x_1 and x_2 are denoted by the derivatives $\psi'(x_{1,2}^{+-})$, while "+" means the right neighborhood, and "-" corresponds to the left neighborhood.

Hence $\psi'(x_1^+,t)$ и $\psi'(x_2^-,t)$ -- solution of the adjoint problem in the domain $x_1<x<x_2$.

Accordingly -- $\psi'(x_1^-,t)$ - solution of the adjoint problem in the domain $x_l<x<x_1$.

And - $\psi'(x_2^+,t)$ - solution of the adjoint problem in the domain $x_2<x<x_K$.

Finally, to determine the optimal control-control action, one can use the iterative gradient projection method

$$u_{l+1} = P_U(u_l - \alpha_l J'(u_l)), l = 1,2,3... \tag{16}$$

for some initial u_0. The operator P_U is the projection operator on the space of admissible controls.

The desired control provides a minimum for the functional *J(u).*

III. Results

As an example of calculations in Fig.1. the drainage of the protection facility of the Faceted Chamber on the territory of the Novgorod Kremlin is given - the optimal position of the drains was obtained with complete drainage of the basement [17]. Variants of calculations for the recommended optimal drainage to ensure protection from the negative impact of groundwater protection objects. The results obtained are the basis for the expert to make the final control decision on the prevention of emergencies at protected objects and allow making recommendations on engineering measures (Fig. 1) [17- 20].

The flooding process is described by a mathematical model representing the geofiltration equation for describing non-stationary planned filtration (the Boussinesq equation) with the corresponding boundary conditions [5]. Such equations do not have an analytical solution, so numerical simulation is used [10, 11, 12]. Explicit or implicit difference schemes are used, such as the Dufort-Frankel scheme and the method of alternating directions. A grid is superimposed on the filtration area, and the initial data necessary for the calculations are "entered" into each grid cell. For the first time, the original input of initial information was implemented in the model, which allows taking into account all archival, stock information about the engineering-geological and hydrogeological conditions of the built-up area, given on paper [22].

Data input includes a layer-by-layer graphical setting of the calculated parameters (in the form of maps of the corresponding isolines), boundary conditions, critical and initial levels of groundwater, terrain, which allows them to be converted into a monochromatic format in which the numerical values of the parameters correspond to shades of the selected color, which allows the software the complex to perceive this format as numerical data continuously distributed in the study area.

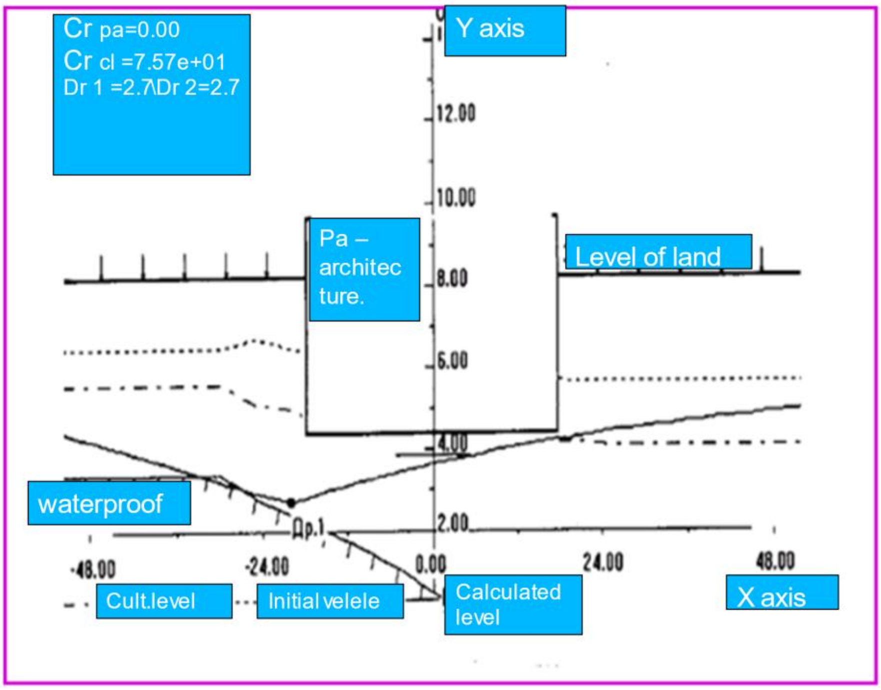
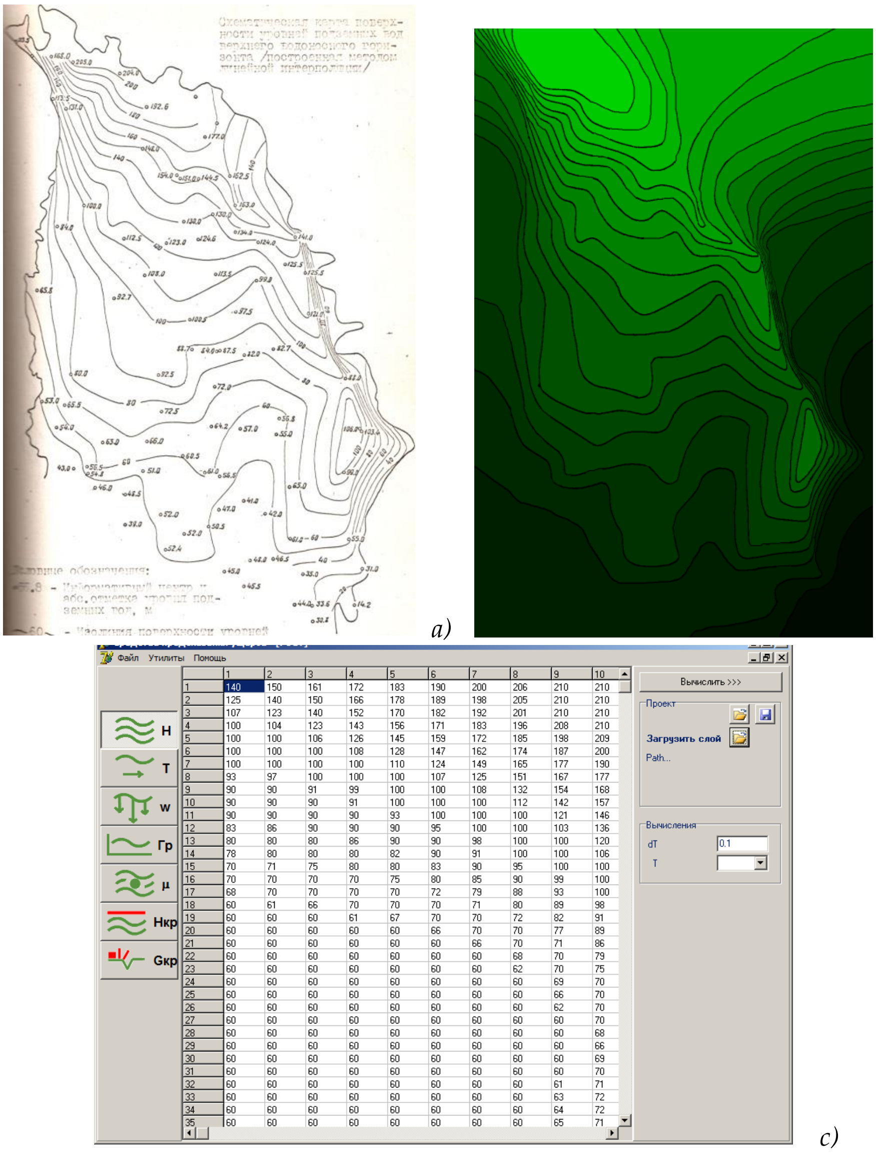


Fig. 1: *The result of the calculation of the drain for the object of protection - the Faceted Chamber (Novgorod Kremlin).*

The change in the intensity of the hue of the selected color corresponds to the change in the numerical values of the parameters of the source data. The input data can be simultaneously presented in three formats: in a tabular, numerical task (in each cell); in a graphic in a monocolor image (Fig. 2) [22]. The use of such data input makes it possible to divide the filtering area into any number of cells, limited by the computer memory, greatly simplifying and reducing the time of data entry, increasing the accuracy of calculations (Fig. 3) [22].

The parameters entered for calculations are shown in Figure 3b) (column on the left). The parameters must be set in the entire computational domain: groundwater level, water permeability, infiltration, water loss parameters, water level in the reservoir, critical level for the entire domain and individual objects. Parameters can be set both using maps and tables, while dividing the area into cells, the size of which is adjusted depending on the accuracy of the required forecast. The division of the computational domain into cells is specified for the main parameter (H), for all other parameters this division is preserved. All other parameters are "attached" to each cell of the grid, i.e. each cell of the grid contains all the specified layer-by-layer information (Fig. 3). Layers (calculation parameters) can be added by defining a new layer [22].

Unlike existing models of flooding, the developed software and computing system implemented in a common software shell allows, in a single iterative process, to determine the excess of the critical level of groundwater for various objects, evaluate the effectiveness of control actions, take into account and enter all archival and stock information, evaluate the impact rivers and reservoirs, accidents at hydraulic structures, graphically present the results of calculations in three formats (graphical: two-dimensional, three-dimensional, and also numerical). The original algorithm for entering initial information significantly improves the quality and speed of entering initial data, allows you to enter data on the same topographic basis layer by layer, impose additional restrictions and conditions for predictive calculations [21, 22].



a) b) c)

Fig. 2: *An example of transferring data from archival maps (a) to the electronic color range (b) and their numerical values in the area under consideration (c)*

IV. Discussion

The developed set of methods and models served as the basis for creating a situational model - a special software-computer complex. The model represents elements of local GIS and a block of mathematical modeling of geofiltration. It was performed for local objects where there was no need to use expensive software products and there was a requirement for the openness of program codes to correct predictive models. The software-computer complex allows solving the following tasks in the simulation mode:

predict and evaluate the potential and actual flooding of technosphere objects (by exceeding their critical levels);

predict and evaluate the achievement and excess of the critical level of groundwater during the development of flooding throughout the territory under consideration;

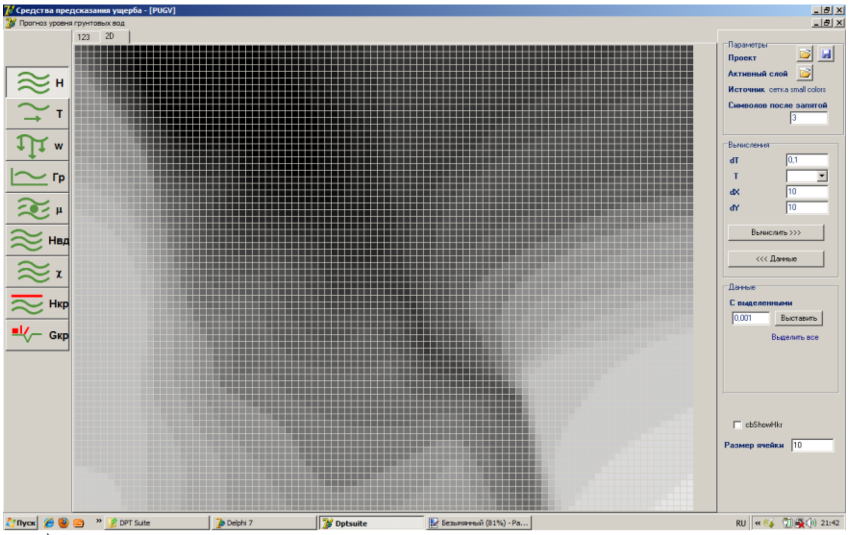
predict and evaluate the possibility of development of hazardous processes induced by flooding;

to carry out the forecast of flooding of the territory in case of flooding of rivers, breakthrough of dams, dams;

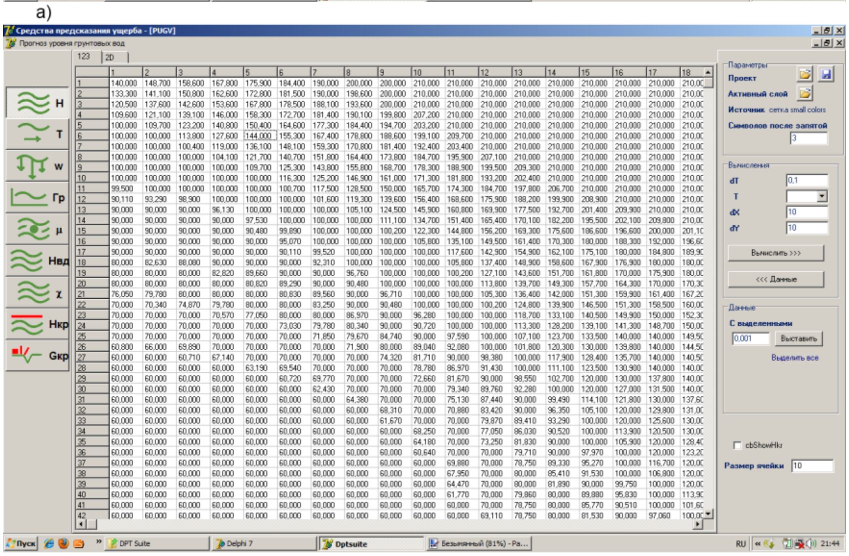
develop and evaluate in a simulation mode options for the operation of drainage systems;

assessment of the degree of groundwater level reduction as a result of the operation of drainage systems;

assessment the impact of control actions on the natural and man-made environment.



a)



b)

Fig. 3: *Formation of initial data (on the example of groundwater levels): a) conversion of the map into a graphical monochromatic format; b) digital format for setting the initial data, corresponding to the graphic one.*

Managing the risk of emergencies in case of flooding of built-up areas using the developed methods and technologies can reduce the risk of collapses and accidents of buildings and structures. The approaches have been tested in some built-up historical areas [23].

References

[1] Arefyeva E.V., Mukhin V.I. Assessment of territorial security in case of flooding. - M., APGS EMERCOM of Russia, 2008. – 101p.

[2] Dzektser E.S. Patterns of formation of flooding of built-up areas, principles of forecasting and engineering protection.-M.: 1987.- 77 p.

[3] Khomenko V.P. Suffosion processes in the territories of cities.// Problems of engineering geology and engineering seismology of cities and urbanized territories. Materials of scientific-practical. Sem. 1990-T.1.- M., 1990.- S.118-124.

[4] Arefyeva E. V., Considering emergency hazards in construction and operation of infrastructures. IOP Conf. Ser.: Mater. Sci. Eng. 687 066023/ E V Arefyeva, E V Muraveva,T Yu Frose. International Conference on Construction, Architecture and Technosphere Safety (**2019**) 066023 IOP Publishing doi:10.1088/1757-899X/687/6/066023 c.

[5] Arefyeva E.V. Regulation of the groundwater regime during flooding of objects and built-up areas // Industrial and civil construction. - 2007. - No. 11. - S. 47-48.

[6] Arefyeva, E.V.; The issues of sustainability of historical and cultural areas associated with their periodic underflooding and solutions. / E. V. Arefyeva, Muraveva, E.V., Alekseeva, E.I. 2019 IOP Conf. Ser.: Mater. sci. Eng. 687 066031./ International Conference on Construction, Architecture and Technosphere Safety IOP Conf. Series: Materials Science and Engineering 687 (2019) 066031 IOP Publishing doi:10.1088/1757-899X/687/6/066031

[7] Roitman A.G. Deformations and damage to buildings. - M .: Stroyizdat, 1987. - 159 p.

[8] Dzektser E.S. Engineering protection of built-up areas from flooding // Design and engineering surveys. - 1986. -№5. – P.27-29.

[9] Gavich I.K. Theory and practice of modeling in hydrogeology. – M.: Nedra, 1980.–358 p.

[10] Vasiliev F.P. Methods for solving extreme problems. - M.: Nauka, 1987. - 397 p.

[11] Yevtushenko Yu.G. Methods for solving extreme problems and their application in optimization systems. - M.: Nauka, 1982. - 432 p.

[12] Lavrentiev M.M., Incorrect problems of mathematical physics and analysis./ Lavrentiev M.M., Romanov V.G. - M.: Nauka, 1980. - 285 p.

[13] Lyons J.-L. Optimal control of systems described by partial differential equations. - M.: Mir, 1972. - 293 p.

[14] Galeati G. Optimal dewatering schemes in the foundation desigh of an electronuclear plant // Galeati G., Gambolatti G. Water Resour. Res, 1988. - Vol. 24. - No. 4. - P. 541-552.

[15] Ahlfeld D., Combining physical containment with optimal withdrawal for contaminated groundwater remediation // Ahlfeld D., Mulvey J., Pinder G. Proc. of the VI Intl. Conf. on Finite Elem. in Water resources. - Lisboa: Portugal, 1986. - R. 205-214.

[16] Willis R., Water resources manager. In North China plain // Willis R., Finney B., Zhang Daosual /J. of Water res. Plann. and Manag. - 1989. - Vol.115. - No. 5. - 3. - R. 598-615.

[17] Arefyeva E.V. Flooding of objects and built-up territories as a potential source of emergency situations // Industrial and civil construction. - 2007. - No. 10. - S. 33-34.

[18] Arefyeva E., Assessment of the vulnerability of architectural monuments to dangerous natural process./ Arefyeva E., Alekseeva E., Gorina L.. Lecture Notes in Civil Engineering, 2022.T.180.p.159-170.

[19] Muravyeva, E.V. Improving the sustainability of cultural heritage sites using the UNFORM.// Muravyeva, E.V., Arefyeva, E.V., Danilina, N.E. IOP Conference Series: Materials Science and Engineering 2020 | Conference paper DOI: 10.1088/1757-899X/962/4/042018 EID: 2-s2.0-85097056033 Part of ISBN: 1757899X 17578981

[20] Arefyeva E.V. On some issues related to forecasting emergency situations caused by hydrological hazards and their consequences // Aref'eva E.V. , Bolgov M.V. Scientific and educational problems of civil protection. AGZ EMERCOM of Russia, No. 2017’4 (35), p. 102-110.

[21] Arefyeva E.V. Information support in the problems of modeling and forecasting emergency situations related to the underground hydrosphere of a built-up area. // Civil security technologies. –. Civil Security Technologies, No. 1, 2016, pp. 28-34.

[22] Arefyeva E.V. Situational-optimization model for determining hazards for built-up areas / Arefieva E.V. , Ziganshin A.I., Rybakov A.V. Scientific and educational problems of civil protection, scientific journal, - No. 1, 2012. P. 31-37.

[23] Alekseeva E.I., Arefyeva E.V. Assessment of exposure of objects of cultural heritage of the Republic of Tatarstan to hazardous exogenous geological processes // Vestnik NTSBDZhD. 2021. No. 3 (49) S. 73-80/.