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# PROCESSİNG OF DATA BASED ON FUZZY RULES

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**Abstract**

*After establishing a fuzzy rule base using the interpolation result mechanism, arranging a fuzzy inference system is performed. The established system will infer about the level of risk using a fuzzy rule base and input statistics. In this work, the construction of the inference system has been determined using fuzzy rules interpolation. The construction determines the distances between input signals and relation functions based on if-then rules. The cross section α used here reduces the distance between input variables. In this work, interpolation is used to measure the distance between α-section fuzzy sets.*
*Such economics issues as forecasting, planning, calculation of expected income are considered in uncertainty conditions, incomplete information conditions using interpolation.*

**Keywords:** fuzzy sets, Z numbers, fuzzy rules, judgments, interpolation, difference of fuzzy sets, forecasting, expected income

## I. Introduction

The presentation and processing of data is a class of mathematical problems having an obvious practical direction, primarily in the field of information science, radio engineering, control and communication theory, radar and navigation equipment. In most cases, in solving mathematical problems, the initial data is represented by a number of points of arbitrary dependence of form y (x). In the free state, this dependence may be unknown. The interpolation apparatus is used to calculate the intermediate values of a function.

Non-fuzzy interpolation methods allowing to solve specific problems of approximate calculations and obtain stable solutions are used to illustrate the uncertainties and unmanageable errors occuring in the development of various models used in technical applications, to reduce the impact of uncertainties. The application of fuzzy set theory methods in the above-mentioned problems provides simple and effective algorithms. One way to filter data with uncontrollable distortions is fuzzy interpolation, which is a natural generalization of the corresponding distinct analog. Distinct and fuzzy interpolation algorithms are based on the use of Newton and Lagrange polynomials. Spline interpolation is widely used in computational mathematics with the further development of variation methods to solve difference problems.

Significant advancement has also been achieved in solving various problems using fuzzy linear systems in computational mathematics, control theory, and other fields. In this paper, based on fuzzy calculations and the theory of fuzzy linear systems, Newton's interpolation problem for fuzzy data is solved and conditions are provided for the presence or absence of strong, weak interpolation [1].

In the paper, the model of fuzzy data interpolation in the production process is developed. A polynomial with a fuzzy coefficient is constructed as a model function. Moreover, the concept of gravity point used to determine the distances between data is presented. The optimization problem is solved by the minimum of the squares sum of the distances between the initial and model data and the equality of fuzzy functions.

## II. Inclusion of fuzzy interpolation

Being a part of classical sets, fuzzy sets are always perceived as an extension of classical sets. The elements of these sets have a degree of relation. Let us give definitions of fuzzy sets and fuzzy triangle relation functions used in this paper.

**Defintion 1**. Suppose, $U = \{x_1, x_2, \dots, x_n\}$ is a space of judgments, universe. The fuzzy set $A$ taken from the set $U$ is a relation function of (A $\subset U$) $\mu_A: U \to [0,1]$ $A$ set and indicates the relation function of $\mu_A(x) \in [0,1]$ x- to A[0,1].

**Defintion 2**. $R$ is a fuzzy subset $A$ defined by the elements of a set of real numbers $(a_1, a_2, a_3)$. The triangular fuzzy relation function of the set $A$ means the following function:

$$\mu_A(x) = \begin{cases} 0 \;, & -\infty \le x \le a_1 \\ \frac{x-a_1}{a_2-a_1}, & a_1 \le x \le a_2 \\ \frac{a_2-x}{a_3-a_2}, & a_2 \le x \le a_3 \\ 0 \;, & -\infty \le x \le \infty \end{cases} \tag{1}$$

The fuzzy triangular relation function is often used to represent linguistic terms numerically. Here, triangular relation functions were used to represent the constraints and reliability of Z-number. Systems based on fuzzy regulations cover the fuzzy sets in the previous and subsequent parts of the rules, as well as determine the relationship between the input and output of the system. In addition, using fuzzy rules base and inputs, fuzzy inference is performed to make decision. Various fuzzy information mechanisms are proposed for the development of fuzzy data. These mechanisms are mainly based on analogy, similarity, composition, interpolation and distance approach. In these approaches, the performance, speed and complexity of the inference mechanisms are considered to be important issues [2-4].

This paper discusses an inference mechanism based on fuzzy interpolation considerations proposed by Cauchy and Hirota. Fuzzy interpolation can be used effectively to develop a database of rules in conditions of uncertainty. In this case, some conditions are required to be carried out. The fuzzy sets used under such conditions must be continuous, convex, and limited normal. Interpolation is based on measuring the distance between two fuzzy sets. In this work, interpolation is used to measure the distance between $\alpha$-section fuzzy sets. Consider two fuzzy sets $A_1$ and $A_2$. $\alpha$-section of fuzzy sets $A_1$ and $A_2$ will be marked with $A_1^\alpha$ and $A_2^\alpha$. Suppose, fuzzy set $A_1$ is smaller than $A_2$, i.e. $A_1 < A_2$, if

$$inf\{A_1^\alpha\} < inf\{A_2^\alpha\} \text{ və } sup\{A_1^\alpha\} < sup\{A_2^\alpha\}\,. \tag{2}$$

Where $inf\{A_1^\alpha\}$ and $inf\{A_2^\alpha\}$ are lower boundries for $A_1$ və $A_2$, $sup\{A_1^\alpha\}$ *and* $sup\{A_2^\alpha\}$ are upper boundaries for $A_1$ və $\underline{A_2}$ (1). Consider the mechanism of interpolation justification. Let's consider a fuzzy controller based on a fuzzy rule base with one input and one output. Suppose that the current input variable $X$ is taken equal to $A^*$ as a result of the observation. Let's determine the value of output $Y$ of the system based on the corresponding rules database $A^*$. Suppose, $A^*$ is smaller than fuzzy set $A_1$ and greater than fuzzy set $A_2$, i.e., $A_1 < A^* < A_2$ and at the same time $B_1 < B_2$. Let's define a system output for $A^*$ input. The problem can be expressed as follows [1]:

It is provided:

$X$ is taken equal to $A^*$, and $Y$ equal to $B^*$.

if $X$ *is* $A_1$, then $Y$ *is* $B_1$, (3)

if $X$ *is* $A_2$, then $Y$ *is* $B_2$,

Where,

$$\frac{d(A^*,A_1)}{d(A^*,A_2)} = \frac{d(B^*,B_1)}{d(B^*B_2)} \tag{4}$$

Where, $d(\cdot)$ is the distance between two fuzzy sets. Using $\alpha$-sections based metrics, Cauchy and Hirota finally determined fuzzy sets. The distance between the two fuzzy sets intersected by $d^L$ at the bottom and $d^U$ at the top can be calculated as follows by using $\alpha$ [2,3]:

$$d_{ij}^L = d^L(A_{ij}^\alpha, X_j^\alpha) = d(inf\{A_{ij}^\alpha\}, inf\{X_j^\alpha\}) = inf\{A_{ij}^\alpha\} - inf\{X_j^\alpha\}, \tag{5}$$

$$d_{ij}^U = d^U(A_{ij}^\alpha, X_j^\alpha) = d(sup\{A_{ij}^\alpha\}, sup\{X_j^\alpha\}) = sup\{A_{ij}^\alpha\} - sup\{X_j^\alpha\} d_j^L =$$

$$= d^L(B_j^\alpha, Y_j^\alpha) = d(inf\{B_j^\alpha\}, inf\{Y_j^\alpha\}) = inf\{B_j^\alpha\} - inf\{Y_j^\alpha\} d_j^U = d^U(B_j^\alpha, Y_j^\alpha) == d(sup\{B_j^\alpha\}, sup\{Y_j^\alpha\})$$

$$= sup\{B_j^\alpha\} - sup\{Y_j^\alpha\}$$

Homming or Euclidean formula can be used to measure distances in (5). Using the distance, Cauchy and Hirota put forward a proposal based an interpolation for the rule of *2k*.

$$inf\{B_j^{*\alpha}\} = \frac{\sum_{i=1}^{2k}\left(1/d^L\left(A_{ij}^\alpha, A_i^{*\alpha}\right)\right) inf\{B_j^\alpha\}}{\sum_{i=1}^{2k} 1/d^L\left(A_{ij}^\alpha, A_i^{*\alpha}\right)}, \tag{6}$$

$$sup\{B_j^{*\alpha}\} = \frac{\sum_{i=1}^{2k}\left(1/d^U\left(A_{ij}^\alpha, A_i^{*\alpha}\right)\right) sup\{B_j^\alpha\}}{\sum_{i=1}^{2k} 1/d^U\left(A_{ij}^\alpha, A_i^{*\alpha}\right)}$$

Where, $B_j^{*\alpha} = [inf\{B_j^{*\alpha}\}, sup\{B_j^{*\alpha}\}]$. In order to find the result of the fuzzy system Z, consider the justification using interpolation.

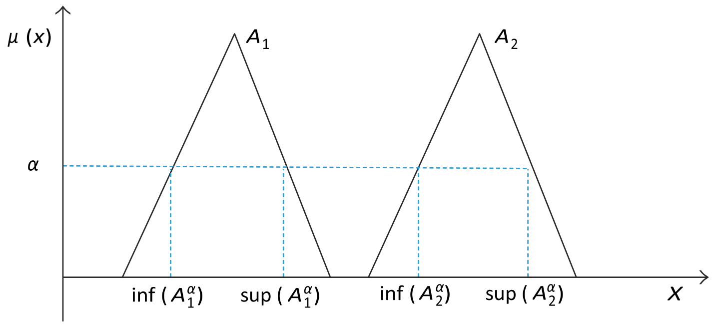


**Fig.** 1: *$\alpha$ is the cross section of the relation function: infimum and supremum.*

## III. Fuzzy inference system based on Z-numbers

**Defintion 1**. Number $Z$ is an ordered pair of fuzzy numbers denoted by $Z\ (A,\ B)$, where the first component $A$ is a constraint for the values of fuzzy variable $X$, and the second component $B$ is a measure of reliability for $A$ components.

Figure 1 uses the triangular relation functions to illustrate the components of number $Z$. Where, $A$ will express the fuzzy value of variable $X$, and $B$ will express the accuracy degree or reliability measure, or accuracy probability of $A$; for example, $X$ is $A$ with possible constraints, i.e.

$$B(X) : X \subset A \longrightarrow Poss(X = u) = \mu_A(x) \tag{7}$$

Where, $\mu_A(x)$ is the relation function of fuzzy set $A$ constrained by $B(X)$. $u$ is the total value of $X$. $\mu_A(x)$ is a relation degree that provides $u$. One of the most effective ways to present fuzzy data is to use number $Z$. For example, let's consider the forecast of grain harvest in our country. As you know, this parameter depends on a number of factors. "If we say that the grain harvest will be higher this year, it is considered an event that can happen 100%. More precisely, this event can be expressed as follows: "This year the grain harvest will be higher" [5-7].

As it is shown, the event can be illustrated by a number Z, i.e., *"X = Z (A, B)"*. Where, fuzzy variable *X* is the "*grain harvest*". The values of variable *X* are described using the pair (*A, B*). *A* is "*higher*" constraint and *B* is the reliability of *A*, which is illustrated as "*most probable*".

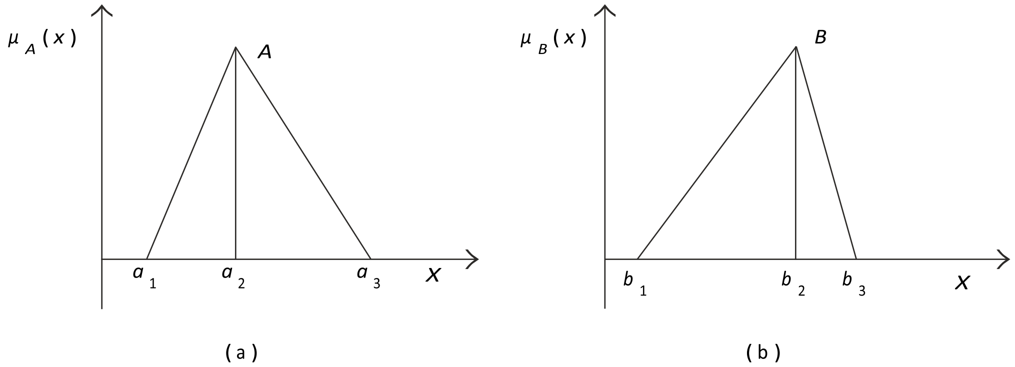


**Fig. 2:** *Z, Z (A, B): a) constraint; b) reliability*

**Definition 2**. Suppose, $Z_1(A_1,R_1)$ and $Z_2(A_2,R_2)$ are two Z numbers. The $\alpha$-cross-sectional distance between numbers $Z_1$ and $Z_2$ is defined as follows:

$$D(Z_1^\alpha, Z_2^\alpha) = |a_1^{L\alpha} - a_2^{L\alpha}| + |a_1^{R\alpha} - a_2^{R\alpha}| + |r_1^{L\alpha} - r_2^{L\alpha}| + |r_1^{R\alpha} - r_2^{R\alpha}| \quad (8)$$

where, $a_1^{L\alpha}, a_2^{L\alpha}, r_1^{L\alpha}, r_1^{L\alpha}, a_1^{R\alpha}, a_2^{R\alpha}, r_1^{R\alpha}$ *və* $r_2^{R\alpha}$ are respectively $\alpha$-cross-sectional fuzzy triangular numbers of fuzzy sets $A_1$, $R_1$, $A_2$ *və* $R_2$ from right and left sides.

Consider the development of Z-number based interpolation mechanism. Suppose that *if-then* fuzzy rules with multiple inputs and one output, have the following form:

$$\begin{gathered} if \quad x_1 = (A_{11}, R_{11}) \dots, və \quad x_m = (A_{1m}, R_{1m}), then \quad y = (B_1, R_1), \\ if \quad x_1 = (A_{21}, R_{21}) \dots, və \quad x_m = (A_{2m}, R_{2m}), then \quad y = (B_2, R_2), \\ \dots . \\ if \quad x_1 = (A_{n1}, R_{n1}) \dots, və \quad x_m = (A_{nm}, R_{nm}), then \quad y = (B_n, R_n), \end{gathered} \quad (9)$$

where $A_{ij}$ and $R_{ij}$ are fuzzy limiting and reliability parameters, respectively, and $x_i$ and $y$ are system inputs and outputs. Here $j = 1, \dots, m$, $m$ are the number of input signals; $i = 1, \dots, n$, $n$ are the number of rules. Let's use the fuzzy interpolation given above for fuzzy rules based on *Z* numbers in Fig.2. Suppose that these input signals are included in a fuzzy system. In the first iteration, the distance between the input signal included and the fuzzy values of the variables according to the rules used in the previous section will be calculated. The distance is calculated using the $\alpha$-section using Hamming or Euclidean distances.

In the next section, the metrics will be calculated separately for each constraint and reliability variable [1-4].

Formula (5) is used in the previous part of the rules to find the metrics of the relation function at the bottom and top. $\alpha$ sections of *A* are used to determine the lower and upper distances [8]:

$$d_{ij}(A_{ij}^\alpha, X_j^\alpha) = d_{ij}^\alpha = |A_{ij}^\alpha - X_{ij}^\alpha|, \quad (10)$$

$$d_i^\alpha = \sum_j^m d_{ij}(A_{ij}^\alpha, X_j^\alpha),$$

here, $d_j(A_{ij}^\alpha, X_j^\alpha)$ is the metrics between two fuzzy sets defined for rule *i*; *j*=1, ..., *m* and *m* are the number of input signals; *i*=1, ..., *n* and *n* are the number of the rules. Distances are described as lower and upper metrics:

$$d_{ij}^{\alpha}(A_{ij}^{\alpha}, X_i^{\alpha}) = \{d_{ij}^{L}(A_{ij}^{\alpha}, X_i^{\alpha}), d_{ij}^{U}(A_{ij}^{\alpha}, X_i^{\alpha})\} \text{ and } d_i^{\alpha}(d_i^L, d_i^U).$$

In the special case, equation $\alpha$ = {0,1}.(2.10) is applied to the constraint $A$ and reliability $R$. The total distance will form the sum of the two distances:

$$d_i^{\alpha} = dc_i^{\alpha} + dr_i^{\alpha}\ , \tag{11}$$

where, $dc_i^{\alpha}$ and $dr_i^{\alpha}$ are the distances calculated sequentially with (8) for the constraint and reliability parameters. $d_i^{\alpha}$ is the calculated distance for this rule. Each of the calculated distances is characterized by two left and right values [9]:

$$\begin{aligned} dc_i^{\alpha} &= (dc_i^L, dc_i^U) \\ dr_i^{\alpha} &= (dr_i^L, dr_i^U) \\ d_i^{\alpha} &= \ d_i^{\alpha} + d_i^{\alpha} \end{aligned} \tag{12}$$

Output data for output fuzzy set is calculated as:

$$Y^{\alpha} = \frac{\sum_{i=1}^{n}(1/d_i^{\alpha})B_{Y_i}^{\alpha}}{1/\sum_{i=1}^{n} 1/d_i^{\alpha}}$$

$$R_Y^{\alpha} = \frac{\sum_{i=1}^{n}(1/d_i^{\alpha})B_{Y_i}^{\alpha}}{1/\sum_{i=1}^{n} 1/d_i^{\alpha}} \tag{13}$$

Formula (13) is used to find the values of constraints and reliability of fuzzy sets. If we combine (10) and (13), the following equations can be obtained:

$$Y^{\alpha} = \frac{\sum_{i=1}^{n}\left(1/\sum_{j}^{m} d_L\left(A_{X_{i,j}}^{\alpha}, X_j^{\alpha}\right)\right)B_{i,j}^{\alpha}}{1/\left(\sum_{i=1}^{n}\left(1/\left(\sum_{j}^{m} d_L\left(A_{X_{i,j}}^{\alpha}, X_j^{\alpha}\right)\right)\right)\right)} \tag{14}$$

$$R_Y^{\alpha} = \frac{\sum_{i=1}^{n}\left(1/\sum_{j}^{m} d_L\left(A_{X_{i,j}}^{\alpha}, X_j^{\alpha}\right)\right)B_{i,j}^{\alpha}}{1/\left(\sum_{i=1}^{n}\left(1/\left(\sum_{j}^{m} d_L\left(A_{X_{i,j}}^{\alpha}, X_j^{\alpha}\right)\right)\right)\right)}$$

The fuzzy signal $Z$ of the system can be obtained using (10) - (13).

It should be noted that $B_{Y_i}$ is used to find the output signal, and variables $R_{Y_i}$ are used to find reliability on the right of formula (13) or (14). After obtaining the output signals, the number Z is converted into a non-fuzzy number. To do this, a formula is used to calculate the middle of two fuzzy numbers.

$$Y = \left((Y_1 + 4 * Y_m + Y_r)/6\right) * \left((R_{Y_1} + 4 * R_{Y_m} + R_{Y_r})/6\right) \tag{15}$$

Formula (15) is used to obtain the exact value of the output signal. Here $Y$ is the fuzzy output value and $R_Y$ is the reliability. It should be noted that in the previous and next parts of the rules base we use triangular fuzzy sets for input and output parameters. Given the levels $\alpha$ = 0 and $\alpha$ = 1, we can obtain the left value of the output signal $Y_l$, the average $Y_m$ and the right $Y_r$. Left $(Y_l, R_{Y_l})$ and right $(Y_r, R_{Y_r})$ values correspond to the level $\alpha = 0$, the average value $(Y_m, R_{Y_m})$ corresponds to the level $\alpha = 1$. First, the left and right values are determined according to the level $\alpha$ = 0, then the highest value of the triangle corresponding to the *level $\alpha$ = 1* is determined. These values are applied to find the output triangle relation function.

## IV. Conclusions

In this work, the construction of the result system was determined using interpolation of fuzzy rules. The distances between the input signals and relation functions based on *if-then* rules are determined in the construction. The cross section $\alpha$ used here reduces the distance between the input variables. In this work, interpolation is used to measure the distance between $\alpha$-section fuzzy sets.

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