ESTIMATION OF THE PROBABILITY OF ELIMINATING CRITICAL FAILURES OF DISTANTLY CONTROLLED OBJECTS

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Abstract

The problem of determining the probability of timely elimination of critical failures under conditions of incomplete data represented by small samples of hardware dwell time in failure state and critical hardware time is considered. This probability is the probability of stochastic dominance of the critical time of the hardware over the time of the hardware in the state of failure, i.e. the probability that the random time of the hardware in the state of failure is less than the critical time of the hardware. Critical time is the time during which the elimination of the occurred failures does not lead to the occurrence of an unrecoverable failure of the object or to a significant reduction in the efficiency of its functioning. The problem of determining the probability of timely elimination of critical failures is formulated as a problem of finding guaranteed (lower and upper) probability estimates on the set of available initial data, represented by small samples of the time of stay of the equipment in the state of failure and critical time of the equipment. The guaranteed estimates of the probability of timely elimination of critical failures are found using the results of solving the problem of Markov moments.

Keywords: probability, critical failures, distantly controlled objects

I. Introduction

Remotely Operated Controlled Objects (ROOs) are complex and expensive objects designed to solve important national economic, scientific and applied tasks. RCEs include artificial satellites, spacecraft, probes, pipelines and oil pipelines. In the process of their target functioning they are exposed to a large number of external factors. [1] This impact in most cases is the cause of failures of their hardware. Hardware failures lead to different consequences for DKO. In the case of some hardware failures, the recovery of its performance is automatic with the built-in system of control, diagnostics and recovery, in the case of other failures for their detection, diagnosis and recovery requires the involvement of ground control complex facilities, including ground control and management facilities [1]. Among the failures of the second group there are such failures, the untimely elimination of which leads (may lead) to the occurrence of unrecoverable hardware failure, and further to the failure of RSC or to a significant reduction in the efficiency of its operation. Let us call such hardware failures critical failures; they include failures of the power supply system, failures of the satellite navigation system, failures of the orientation and stabilization system, etc.

Let at a random moment of time there is a hardware failure, which after some random time ξ manifests itself, as a rule, is detected during the telemetry control session following the failure. The detected failure is diagnosed and eliminated within the period of time γ.

Currently, only the task of technical condition monitoring is solved on board some ROVs. The task of diagnostics and technical condition management in most cases is realized remotely. [2].

The random dwell time of the hardware in a failure state σ will consist of the time of failure occurrence ξ and the time of diagnostics and restoration of the equipment operability γ. The value of time σ will be determined by the ratio:

$$\sigma = \xi + \gamma, \quad (1)$$

where σ - is the random dwell time of the equipment in the state of failure,

ξ - random time of hardware failure manifestation,

γ - random time of diagnostics and restoration of equipment operability.

Let η - a random time during which the elimination of critical failures does not lead to an unrecoverable hardware failure or to a significant reduction in the efficiency of its operation. Let us call this time the critical time.

The task is to determine the probability of timely elimination of DKO equipment failures - the probability that the random dwell time of the equipment in the state of failure will be less than the critical time, i.e. to determine the probability of$P = P < \eta)$, the probability of stochastic dominance of the critical time of the hardware over the time of the hardware stay in the state of failure.

At known functions of distribution of time of stay of the equipment in a state of failure and critical time this problem is solved. Ratios for determining the probability $P = P < \eta)$ with respect to the basic distribution functions of random variables σ, η are given, for example, in [3]. In a real situation, the distribution functions of the time of the equipment stay in the state of failure and the critical time are not known. The present paper is devoted to the consideration of this case.

II. Main results

Let $\sigma v = (\sigma v_1, \sigma v_2, \dots, \sigma v_{n\sigma}) R^{n\sigma}$ - a sample of values of the time of the equipment staying in the failure state, $\eta v = (\eta v_1, \eta v_2, \dots, \eta v_{n\gamma}) R^{n\eta}$ - sample of values of critical time of the equipment. The components of the sample $\sigma v_i > 0$ are independent equally distributed values from some unknown distribution H(t), $\eta v_i > 0$ are independent equally distributed values from some unknown distribution G(t). The samples $\sigma v, \eta v$ are finite samples of small volume, for which it is impossible to recover the original distributions F(t) and G(t).

Let us transform the expression for the probability of timely elimination of critical failures. We have

$$P = P(\sigma < \eta) = P\left(\frac{\sigma}{\eta} < 1\right) = P(\sigma\lambda < 1) = P(\beta < 1). \quad (2)$$

where$\lambda = \frac{1}{\eta}; \beta = \sigma\lambda$.

Let's find on the basis of samples $\sigma v, \eta v$ sample initial moments (hereinafter - moments) of random variables σ, λ, β:

$$\mu_{j\sigma} = \frac{1}{n\sigma} \sum_{i=1}^{n\sigma} \sigma v_i^j ; \mu_{j\lambda} = \frac{1}{n\eta} \sum_{i=1}^{n\eta} \left(\frac{1}{\eta v_i}\right)^j ; j = \overline{1,k}; k > 0. \quad (3)$$

Random variables σ, λ are independent quantities, therefore, the random variables σ^j, λ^j are also independent. The mathematical expectation of the product of random variables σ^j, λ^j is equal to the product of mathematical expectations of these quantities

$$M(\sigma^j \lambda^j) = M) \text{ M}(\lambda^j; j = \overline{1,k}; k > 0. \quad (4)$$

Then the moments of the random variable β will be defined by the expression:

$$\mu_{j\beta} = \mu_{j\sigma}\mu_{j\lambda}\,; j = \overline{1,k}; k > 0. \tag{5}$$

Let us define the set F_0 - the set of distribution functions $F(t)$ of the random variable β with moments equal to the moments defined by the relation (8)

$$F_0 = \left\{F(t)\colon \int_0^\infty t^j dF(t) = \mu_{j\beta};\ j = \overline{1,k}; k > 0\right\}. \tag{6}$$

The problem of determining the probability of timely elimination of critical failures of DKO equipment due to incompleteness of initial data represented by small samples of the time of equipment stay in the state of failure and critical time of the equipment should be formulated as a problem of determining the guaranteed (lower and upper) estimates. It is necessary to determine the lower and upper (guaranteed) estimates of probability $P = P(\beta < 1)$ on the set of distributions F_0, i.e. to find

$$P_n = \min_{F(t)\in F_0} P(\beta < 1); P_v = m\,\underset{F(t)\in F_0}{ax} P(\beta < 1). \tag{7}$$

Ratio $P = P(\beta < 1)$ can be written in the form:

$$P = P(\beta < 1) = \int_0^1 dF(t). \tag{8}$$

The solution of the latter problem, defined by relation (10), is obtained in [5-10] and it is summarized as follows.

Highest (lowest) value of the integral

$$J(F) = \int_0^\tau c(t)dF(t). \tag{9}$$

at $F(t)$ F_0 is achieved on the only stepwise distribution F(t) which among the growth points t_1 t_2 …,t_v has a point ;

at odd *k the* number of growth points *v of the* distribution function F(t) is determined by the relation $v = (k + 3)\ 2$ and $t_0 = 0 < t < t_{12} < \ldots < t <_v$

at even *k the* number of growth points *v of the* distribution function F(t) is determined by the relation $v = k\ 2 + 1$ with $0 < t_1 < t_2 < \ldots < t <_v$

numbers p_j 0 t_j $j = 1\ 2$ v satisfy the system of equations:

$$\mu_{j\beta} = \sum_{i=1}^{v} t_i^j p_i\,; j = \overline{1,k}; k > 0. \tag{10}$$

The function $c(t)$ must have a non-negative k+1th derivative.

Let us consider for certainty the problem of determining the lower estimate of probability $P = P(\beta < 1)$ on the set of distribution functions F_0.

Let two moments of a random variable be defined β - μ_1иμ_2, (further, where it does not cause discrepancies, the index β is omitted) then using the above result we find the lower estimate of probability $P = P(\beta < 1)$which is defined by the following expression [6]:

$$P_n = \frac{(\mu_1 - 1)^2}{(\mu_1 - 1)^2 + \mu_2 - \mu_1^2}, 1 > \mu_{1.} \tag{11}$$

In the case when three moments of a random variable are defined β - $\mu_{1,}\mu_{2,}\mu_{3,}$ we find the lower estimate of the probability $P = P(\beta < 1)$which is determined by the following expression [6]:

$$P_n = \frac{\mu_1 1^3 + \mu_3 1 - 2\mu_2 1^2 - \mu_3\mu_1 + \mu_2^2}{\mu_1 1^3 + \mu_3 1 - 2\mu_2 1^2}, 1 > \mu_{1.} \tag{12}$$

In the last ratio, the symbol "1" (τ=1 - growth point of the distribution function F(t)) is used to confirm that the numerator and denominator dimensions are the same.

In the case when more than three moments of a random variable are defined $\beta$the estimate of the probability $P = P(\beta < 1)$ should be calculated numerically.

Let us consider the case when four moments of the random variable are defined β - $\mu_1,\mu_2,\mu_3,\mu_4.$ In this case, the number of growth points of the distribution function *F(t)* is equal to three, the system of equations (10) will have the form:

$$\begin{cases} p_1+p_2+p_3=1, \\ p_1t_1+p_2t_2+p_3t_3=\mu_1, \\ p_1t_1^2+p_2t_2^2+p_3t_3^2=\mu_2, \\ p_1t_1^3+p_2t_2^3+p_3t_3^3=\mu_3, \\ p_1t_1^4+p_2t_2^4+p_3t_3^4=\mu_4. \end{cases} \quad (13)$$

Since the distribution function *F(t)* is a left continuous function, the growth point τ=1 can be either t_2 or $t_3$then we should consider two systems of equations

$$\begin{cases} p_1+p_2+p_3=1, \\ p_1t_1+p_2\tau+p_3t_3=\mu_1, \\ p_1t_1^2+p_2\tau^2+p_3t_3^2=\mu_2, \\ p_1t_1^3+p_2\tau^3+p_3t_3^3=\mu_3, \\ p_1t_1^4+p_2\tau^4+p_3t_3^4=\mu_4, \end{cases} \quad (14)$$

$$\begin{cases} p_1+p_2+p_3=1, \\ p_1t_1+p_2t_2+p_3\tau=\mu_1, \\ p_1t_1^2+p_2t_2^2+p_3\tau^2=\mu_2, \\ p_1t_1^3+p_2t_2^3+p_3\tau^3=\mu_3, \\ p_1t_1^4+p_2t_2^4+p_3\tau^4=\mu_4. \end{cases} \quad (15)$$

To solve the systems of equations (14), (15) we used Microsoft Excel package (item) "Solution Search" sub-item "Search for the solution of nonlinear problems by OPG method[1] ". In this case, the first equation of the systems of equations (17), (18) was considered as a target function of the following form

$$CF=p_1+p_2+p_3-1,$$

the value of which should be equal to zero, the other equations were considered as constraints in the optimization problem.

The results of numerical solution of the problem of determining the lower estimate of the probability of $P=P(\beta<1)$ - of the probability of timely elimination of failures of DKO equipment on the set of distribution functions F_0 at different values of the first four moments are given in Table 1.

Table 1: Lower estimate of the probability of timely elimination of failures space craft hardware

μ_1	μ_2	μ_3	μ_4	P_n
0,200	0,080	0,048	0,038	0,959
0,300	0,180	0,162	0,194	0,832
0,400	0,320	0,384	0,614	0,648
0,500	0,500	0,750	1,500	0,579
0,600	0,720	1,296	3,110	0,488
0,700	0,980	2,058	5,762	0,429
0,800	1,280	3,072	9,830	0,330

[1] The OPG method is the generalized reduced gradient method.

III. Conclusion

The problem of determining the probability of timely elimination of critical failures of DKO equipment in the conditions of incomplete data represented by small samples of the time of equipment stay in the state of failure and critical time of the equipment is considered. This problem is formulated as the problem of determining guaranteed (lower and upper) estimates of the probability that the random dwell time of the equipment in the failure state is less than the random critical time of the equipment, i.e., the probability of stochastic dominance of the critical time of the equipment over the dwell time of the equipment in the failure state. To determine the probability of timely elimination of critical failures of DKO equipment, the results of solving the problem of Markov moments are used, with their help we obtained relations for determining the lower estimate of this probability at two and three moments of a random variable equal to the ratio of the time of stay of the equipment in the state of failure to the critical time of the equipment. At four moments of the last random variable, the lower estimates of the probability of timely elimination of critical failures of DKO equipment were determined numerically using Microsoft Excel.

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