OPTIMIZATION OF THE COMPOSITION OF SPARE PARTS FOR THE AGING TYPE OBJECTS

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Abstract

In the field of the Russian Federation nuclear power facilities operation, special attention is paid to ensuring the safety of nuclear power plants and improving their economic efficiency. The development and implementation of methods for the optimal organization of the NPP operation is carried out. To solve these tasks, the application of the methodology of "reliability-centered maintenance" and "risk-oriented maintenance" is carried out at NPP power units. The implementation of these methodologies involves the operational reliability of equipment analysis and, based on the results of the analysis, planning of preventive maintenance and optimization of spare parts sets. The solution of these tasks is complicated by the fact that a significant part of the equipment has large operating time, and is on the verge of running out of resources.

The paper solves the problem of optimizing the composition of spare parts for objects of aging type. A model for calculating the profit from the operation of equipment with spare elements is proposed. The Kijima-Sumita model is used as a model to describe the process of exhaustion of the object's operability. The developed model is investigated on simulation data.

Keywords: preventive maintenance, risk-based maintenance, objects of aging type, inventory management model, Kijima-Sumita model, distribution density, average profit

I. Introduction

The nuclear power industry of the Russian Federation is currently on the rise. New facilities are being built both in Russia and abroad. New Russian-designed nuclear power plants (NPP) are being commissioned in Belarus, China, India, Turkey and other countries. In the Russian Federation, power plants of the VVER-1200 project have been commissioned at the Leningrad and Novovoronezh nuclear power plants, and a number of other facilities are under construction.

Special attention is paid to ensuring safe operation and increasing the economic efficiency of previously commissioned nuclear power facilities. In this regard, the issues of optimal operation organization of the nuclear power plants are being solved. Modern methods of organizing and conducting scheduled preventive maintenance of equipment are being introduced. Thus, the methodology of "reliability-centered maintenance" and "risk-oriented maintenance" is being implemented at NPP power units.

The Risk-based Maintenance (RBM) is a maintenance strategy based on the equipment failure risk analysis, taking into account the predicted losses and their significance for achieving the established indicators of the efficiency of electricity production based on an assessment of the current equipment's technical condition [1].

The Risk-based approach is aimed at optimizing the processes of operation and maintenance, as well as the process of managing the equipment's operability [2].

The purpose of applying the risk-based maintenance and repair strategy (MRS) is to reduce operating (operational and repair) expenses while unconditionally complying with the safety requirements of the NPP.

The main objective of the risk-based maintenance strategy is to optimize the MRS of equipment by selecting the most effective processes aimed at ensuring the safety and reliability of the operation of the NPP, upon condition that the associated costs are minimized.

The method of monitoring the technical condition and maintenance of equipment based on risk assessment is aimed at:

- ensuring control and maintenance of equipment;
- coverage of inspections and maintenance of all types of equipment;
- taking into account the technical and organizational aspects of planning control measures and maintenance;
- implementation of asset management related to inspections, maintenance and service life assessment for equipment;
- stock management and optimization of the composition of spare parts and devices for the entire range of equipment.

The formation, storage, use, replenishment and accounting of an irreducible stock of equipment, components, spare parts and materials for the repair and operational needs of nuclear power plants is one of the important tasks of MRS management.

The issues of determining the optimal composition of spare parts, as well as calculating the reliability characteristics of systems, taking into account spare elements, were considered in the works of both Russian and foreign authors.

Thus, in [3], the problem of optimizing the composition of spare elements is solved, taking into account the complex strategy of their functioning, assuming that the failed product is being repaired. In the papers [4, 5], the solution of the problem of optimizing the composition of spare parts is presented in the presence of restrictions on the cost of acquisition, delivery and storage of spare elements. In the publications [6, 7], dynamic models of managing the composition of spare elements at the enterprise are considered. In articles [8, 9], the problem of optimizing a set of spare elements is solved, taking into account the depletion of a part of the resource by these elements. Based on the methods of restoration theory, the issue of calculating the reliability characteristics of the system is considered, in the case when spare elements can be in cold, hot and spinning reserve. In [10], the problem of optimizing a set of spare parts for elements of an aging type is considered, taking into account restrictions on the cost of spare elements. Based on the methods of the restoration theory, the readiness coefficient of the system, which includes elements of the aging type, is obtained. In [11, 12], the task of calculating the reliability characteristics of systems whose elements, in case of failure, are replaced by workable objects from the spare parts, and the failed element is restored by the repair team and further replenishes the spare parts. These works take into account the aging of objects during operation. Geometric processes are used as a model that takes into account the aging of elements.

In the papers [13, 14], the issues of the formation of the composition of spare parts for objects with two types of failures are considered. The first type of failure leads the object to an inoperable maintainable condition. The second type of failures is more catastrophic, leads the object to an inoperable non-repairable state. In the first type of failure, an inoperable, but maintainable object is replaced with a workable one from the spare parts. The inoperable unit is transferred to the repair authority for restoration. In the second type of failure, a workable unit from a spare parts kit is installed in place of the failed object, and an inoperable and non-repairable unit is sent to an industrial enterprise in accordance with the established procedure for repair.

And finally, we note the fundamental work [15], which on the one hand provides a fairly detailed analysis of publications on the topic of calculating the reliability of systems taking into account spare parts, calculations of the required number of spare parts, and on the other hand describes various models for calculating reliability indicators taking into account spare parts, including in conditions of periodic and continuous replenishment stocks in the spare parts kit, with periodic replenishment of stocks with emergency deliveries, the issues of optimizing the spare parts kit are considered.

II. Stock management model for aging type objects

Let's consider a model of inventory management at a nuclear power industry enterprise.

The operation of the power unit is designed for a long service life. The considered service life is divided into n equal intervals of duration Δ. Let's assume that the parameter A denotes profit from the operation of equipment per unit of time, and B – losses from equipment downtime in case of a shortage of spare parts. At the beginning of each step in the system the orders are made of an arbitrary number of spare parts $l \geq 0$ with a total cost of cl. These l sets of spare parts are added to the amount that could have remained unused in the previous steps.

Let L_j – number of spare parts ordered at the beginning of the j- th step ($j = 1, \ldots, n$), δ_j – is the duration of the system operation at the j- th step, τ_j – duration of the system downtime at the j- th step.

The task is to determine the optimal procedure that maximizes the average profit of the enterprise

$$M\left[\sum_{j-1}^{n}\left(A\delta_j - B\tau_j - cL_j\right)\right] \rightarrow max \quad (1)$$

Tasks of this type were considered in [16].

Consider a random failure process formed by the moments of failures: $\tau_1 = \xi_1; \tau_2 = \xi_1 + \xi_2; \tau_k = \sum_{i=1}^{k}\xi_i$, where ξ_i is the duration of the system operation between the $(i-1)$-th and i-th failures.

Let y denote the number of spare parts sets available in stock, before ordering a new set of spare parts is planned, through $V_n(y)$ – denote the average profit in the n- step process. The duration of one step is equal to Δ, the duration of the step has the dimensionality of time.

Suppose that at the beginning of each step, $t \geq 0$ units of spare elements will be ordered, the number of observed failures is $N(\Delta) = x$. If $x \leq y + l$, then the profit at this step will be $A\Delta$, the number of spare elements decreases to $y + l - x$ and the average profit from following the optimal procedure at the remaining $n - 1$ steps is $V_{n-1}(y + l - x)$. If $x > y+l$, then the profit is $A\tau_{y+l+1} - B(\Delta - \tau_{y+l+1})$, the stock level is reduced to zero, and the average profit on the remaining $n - 1$ steps is $V_{n-1}(0) - d(x - y - l)$, where d the cost of a set of spare parts purchased in an extreme situation.

Thus, the average profit at step n is equal to:

$$V_n(y) = sup_{l\geq 0}\left\{\int_0^{y+l}[A\Delta + V_{n-1}(y + l - x)]f_{N(\Delta)}(x)dx + \int_0^{\Delta}[A\tau - B(\Delta - \tau)]g_{\tau_{y+l+1}}(\tau)d\tau + \int_{y+l}^{\infty}[V_{n-1}(0) - d(x - y - l)]f_{N(\Delta)}(x)dx - cl\right\}, \quad (2)$$

where $f_{N(\Delta)}(x)$ – is the generalized density distribution of the value x, g_{τ_k} – is the generalized density function of the moment of the k- th failure.

If ξ_1, ξ_2,…– are independent identically distributed random variables having an exponential distribution law with intensity λ, then $f_{N(\Delta)}(x) = \frac{(\lambda\Delta)^x}{x!}e^{-\lambda\Delta}$ – is Poisson's law, and $g_{\tau_k}(\tau) = \frac{\lambda^k\tau^{k-1}}{\Gamma(k)}e^{-\lambda\tau}$ – is the gamma distribution. This type of distribution reflects the case when a new spare set is installed in place of the failed element. The described model is presented in [6].

Note that most of the approaches presented in the literature to optimize the composition of spare elements are based on the assumption that the failed element is replaced by an identical new object.

However, it should be said that a number of NPP power units that are in operation have been operating for a long time (70% of Russian NPP power units are operating in the operation life extension mode). The equipment operating as part of these power units is exposed to internal and external factors, which leads to a gradual loss of their operability. In other words, the effects of aging are observed in the operation of the equipment. In this regard, the reliability of the equipment is reduced, this may lead to the fact that failures may occur more often than it was at the initial stage of operation. As a result, there is a need to re-evaluate the composition of spare parts and devices.

Let's consider an approach to optimizing the composition of spare parts for objects of an aging type. To account for the effects of aging in the operation of equipment, we will use the Kijima-Sumita models [17]. Let's outline the main points of these models. The Kijima-Sumita models use the concepts of the real and virtual age of the object.

For the case of instant recovery, the real age of the element at the time of the n-th failure is represented as the sum of its developments:

$$S_n = \sum_{i=1}^{n} X_i \,, S_0 = 0,$$

where X_i – is the i-th operating time to failure.

For the models, a constant value q is introduced – the recovery coefficient. Certain function $v = v(\{X\}, q)$, where $\{X\}$ is a sample of operating time to failure, determines the virtual age of the element. Let v_{i-1} – be the virtual age of the element at the time of the $(i-1)$- th restoration. Then X_i as the following conditional distribution function [18]:

$$F_i(x|v_{i-1}) = \frac{F_i(x + v_{i-1}) - F_i(v_{i-1})}{1 - F_i(v_{i-1})},$$

from where we can find the probability of trouble-free operation:

$$P_i(x|v_{i-1}) = \frac{P_i(x + v_{i-1})}{P_i(v_{i-1})}.$$

Here $F(x)$ – is the operating time to-first-failure distribution function for a completely new element.

There are 2 Kijima models. The feature the Kijima-1 model is that the n-th recovery affects only the damage received by the element between the (n-1)-th and the n-th failure, reducing the increase in the virtual age of the element from X_i to qX_i. The virtual age of the element after the n -th restoration is written as follows:

$$v_n = v_{n-1} + qX_n = q \sum_{i=1}^{n} X_i = qS_n; v_0 = 0.$$

According to the Kijima-2 model, each recovery affects the total damage, reducing the total virtual age:

$$v_n = qv_{n-1} + qX_n = q(q^{n-1}X_1 + q^{n-2}X_2 + \cdots + X_n); v_0 = 0.$$

In this paper, we will consider the first Kijima model.

In case of incomplete recovery according to the Kijima-Sumita model, the leading flow function looks like this:

$$\Omega(t) = \int_0^t (g(\tau|0) + \int_0^\tau \omega(x) g(\tau - x|x) dx) d\tau \,,$$

where:

$$g(\tau|x) = \frac{f(t+qx)}{1-F(qx)}; \omega(t) = \frac{d\Omega(t)}{dt}; f(t) = \frac{dF(t)}{dt}.$$

Here $\omega(t)$ – is the failure flow parameter.

Let's define the generalized distribution density $f_{N(\Delta)}(x)$. For the generalized density of the failures number distribution, the following equality holds:

$$f_{N(\Delta)}(x) = P(N(\Delta) = x),$$

where $N(\Delta)$ - is the number of flow points on an interval of length Δ, i.e. $\xi[t, t + \Delta]$ for any t, hence:

$$P(N(\Delta) = x) = P(\tau_x \leq \Delta \leq \tau_{x+1}) = F_{\tau_x}(\Delta) - F_{\tau_{x+1}}(\Delta),$$

where $F_{\tau_k}(\Delta)$ – is the distribution function of the time moment of the *k*-th failure. For the distribution function of the time moment of the *k*-th failure, the expression representing the convolution is valid:

$$F_k(x) = F(x) * F_{k-1}(x),$$

The generalized density of the distribution of the moment of time of the *k*-th failure $g_{\tau_k}(\tau)$ is obtained by taking the derivative of the higher statement.

Let's perform the calculations of the resulting model on conditional data. In [19], a variety of models for describing the time-to-failure distribution functions of the aging type objects of are considered. Let's take as a function of the time distribution one of the widely used distributions with a linear intensity function, which has the following form:

$$F(x) = 1 - e^{-\lambda_1 x - \frac{\lambda_2 x^2}{2}}.$$

The conditional distribution function, respectively, will be written as:

$$F_i(x|\nu_{i-1}) = \frac{F_i(x + \nu_{i-1}) - F_i(\nu_{i-1})}{1 - F_i(\nu_{i-1})}.$$

Then the distribution function of the moment of time of *k*-th failure will be represented as follows:

$$F_{x_i}(x) = \frac{1 - e^{-\lambda_1 (x+\nu_{i-1}) - \frac{\lambda_2 (x+\nu_{i-1})^2}{2}} - 1 + e^{-\lambda_1 \nu_{i-1} - \frac{\lambda_2 {\nu_{i-1}}^2}{2}}}{e^{-\lambda_1 \nu_{i-1} - \frac{\lambda_2 {\nu_{i-1}}^2}{2}}} = 1 - e^{-x\left(\lambda_1 + \frac{\lambda_2 x}{2} + \lambda_2 \nu_{i-1}\right)}.$$

To obtain a generalized distribution density, it is necessary to take the derivative:

$$f_{x_i}(x) = F'_{x_i}(x) = (\lambda_1 + \lambda_2 x + \lambda_2 \nu_{i-1}) e^{-x\left(\lambda_1 + \frac{\lambda_2 x}{2} + \lambda_2 \nu_{i-1}\right)}.$$

Thus, we obtained the necessary formulas to represent the average profit (2) in the formula.

III. Model research

The study of the model will be provided for the following data.
Let the service life of the power unit be 30 years of operation,
the number of steps of the process $n = 10$;
the time interval after which the spare parts are replenished $\Delta = 3$;
the economic effect of a timely replacement $A = 10$;
losses from equipment downtime in case of shortage of spare parts B=15;
the cost of a spare parts set $c = 8$;
the cost of a spare parts elements purchased in an extreme situation is d = 10.
Statistics on failures of the research object are presented in the table, which shows the number of the object failures that took place at consecutive intervals of operation equal to three years.

Table 1: *Statistical information on object failures during its operation*

j	1	2	3	4	5	6	7	8	9	10
n_j	2	16	7	11	21	5	5	4	2	2

This statistical information was processed by nonparametric estimation methods, namely Kernel density estimation. By omitting the results of intermediate calculations, here is a table that presents data characterizing the optimal strategy for replenishing a set of spare elements.

Table 2: *Optimal strategy for replenishing a set of spare parts*

j	1	2	3	4	5	6	7	8	9	10
$l+y$	12	11	10	10	9	7	7	6	5	3

Thus, a model for optimizing the composition of spare elements for a system that has been in operation for a long time is considered. The effects of aging begin to affect the operation of the system equipment. The reliability characteristics of the system elements deteriorate over time, as the process of damage accumulation due to the influence of external and internal factors affects. The Kijima-1 model was used as a model for accounting the aging effect. As a function of the distribution of operating time to the first failure, the linear failure rate function is selected. The calculation results showed that for the given failure statistics and the selected model parameters, the optimal replenishment strategy is a decreasing function. This suggests that when the equipment is functioning at the intervals of replenishment of the spare parts, there is no complete consumption of a spare parts set. Thus, the remainder of the elements from the previous period is added to the ordered batch of spare elements.

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