EVALUATION OF THE INFLUENCE OF LONGITUDINAL OSCILLATIONS ON DYNAMIC RESISTANCE OF NON-UNIFORM CORES

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Abstract

Mechanical systems, the calculation of vibrations of which is the content of many practical problems, are mostly complex elastic systems. At the same time, many structural elements can be represented by a combination of different rods; therefore, in various fields of modern technology, it is necessary to solve the problems of oscillations of complex rod systems, including those in a collision with an obstacle.
The relevance of the problem of calculating the vibrations of complex rod systems is due to the practical need to improve the technical characteristics of the designed machines and mechanisms and ensure their functioning under ever wider ranges of operational impacts, as well as to reduce the material consumption of machines and structures. To fully determine the strains and stresses that occur at any point in the system during vibrations, it is necessary to know the displacements at these points. This leads to the need to consider systems with an infinite number of degrees of freedom.

Keywords: heterogeneity, oscillations, instability, core, deflection

I. Introduction

The practical needs of calculating the dynamic characteristics of various machine-building and other structures have led to the complication of design schemes. In many cases, the study of oscillations of complex rod systems with an infinite number of degrees of freedom is associated with great difficulties. In some cases, the possibility of a mathematical interpretation of the problem of vibrations becomes feasible only if certain simplifications are introduced into the calculation. The problem of non-uniform cores taking into account mutual influence of longitudinal and cross-section oscillations is investigated and solved. The decision is carried out in the assumption that the core is made of elastic continuously non-uniform material and the module of elasticity depends on the average module of elasticity, parameter of heterogeneity and length of a core.

II. Methods

The problem of dynamic stability of inhomogeneous rods is considered taking into account the mutual influence of longitudinal and transverse vibrations.

Let us assume that the rod is made of an elastic continuously inhomogeneous material, and the modulus of elasticity changes according to the following law

$$E = E_0 \left(1 + \alpha \frac{\chi}{l}\right),$$

where E_0 - is the average modulus of elasticity, α - is the inhomogeneity parameter, l - is the rod length.

Let υ - deflection, u - longitudinal displacement of the rod section. Then the total longitudinal displacement, up to second-order values, is determined by the formula:

$$W = u + \frac{1}{2} \int_0^{\chi} \left(\frac{d\upsilon}{d\xi} \right)^2 d\xi \tag{1}$$

The longitudinal force in any section will be

$$N = P_0 + P_t \, Cos\ \theta - \int_{\chi}^{l} m \frac{d^2 W}{dt^2} d\xi \tag{2}$$

On the other hand, there is:

$$\frac{du}{d\chi} = \frac{N}{E_0 F \left(1 + \alpha \frac{\chi}{l}\right)} \tag{3}$$

where F - is the cross-sectional area of the rod.

III. Results

From relations (1)-(3) after some transformations we obtain

$$\begin{aligned} &E_0 F \left(1 + \alpha \frac{\chi}{l}\right) \frac{d^2 u}{d\chi^2} + \frac{E_0 F_\alpha}{l} \frac{du}{d\chi} - m \frac{d^2 u}{dt^2} = \\ &= m \int_0^{\chi} \left[\frac{d\upsilon}{d\xi} \cdot \frac{d^3 \upsilon}{d\xi \, dt^2} + \left(\frac{d^2 \upsilon}{d\xi \, dt} \right)^2 \right] d\xi \end{aligned} \tag{4}$$

The bending equation of the considered rod is obtained in the following form

$$\begin{aligned} &E_0 I \left(1 + \alpha \frac{\chi}{l}\right) \frac{d^4 \upsilon}{d\chi^4} + 2 E_0 I \frac{\alpha}{l} \frac{d^3 \upsilon}{d\chi^3} + E_0 I \left(1 + \alpha \frac{\chi}{l}\right) \times \\ &\times \left(\frac{du}{d\chi} \cdot \frac{d^2 \upsilon}{d\chi^2} + \frac{d\upsilon}{d\chi} \cdot \frac{d^2 u}{d\chi^2} \right) + E_0 F \frac{\alpha}{l} \frac{du}{d\chi} \cdot \frac{d\upsilon}{d\chi} + m \frac{d^2 \upsilon}{dt^2} = 0 \end{aligned} \tag{5}$$

The boundary conditions for $u\ (x, t)$ will have the form

$$u(x,t) = 0 \ , \ E_0 F (1 + \alpha) \frac{du(l,t)}{d\chi} = P_0 + P_t \cdot \cos \theta t \tag{6}$$

The joint solution of system (4), (5) is associated with great difficulties. Let us consider the case when undamped transverse oscillations occur. In this case, the nonlinear terms on the right side of equation (4) can be neglected. Then equation (4) contains only $u\ (x, t)$ and it is resolved independently of equation (5). The solution of equation (4) is represented as

$$u = A \cdot \sin \nu x \cdot \cos \theta t \tag{7}$$

where indicated

$$A = \frac{P_t}{\upsilon E_0 F (1 + \alpha) \cos \upsilon l} \ , \qquad \upsilon^2 = \frac{m}{E_0 F} \cdot \theta^2 \tag{8}$$

IV. Discussion

As can be seen from (8), at $\cos \upsilon l = 0$ the amplitude of the longitudinal oscillations goes to infinity. This corresponds to the resonance of longitudinal vibrations. In this case, resonance with respect to the lowest natural frequency occurs at

$$\omega_i = \frac{\pi}{2l}\sqrt{\frac{E_0 F}{m}} \tag{9}$$

We represent the deflection in the following form

$$\upsilon(\chi, t) = f(t)\cdot \sin\frac{\pi\chi}{l} \tag{10}$$

Taking into account (7), (10) from (5) using the Bubnov-Galerkin method, we find

$$\frac{d^2 f}{dt^2} + \Omega_1^2\left[1 - 2\,\mu_1\cdot\psi_1\cdot\cos\theta t\right] f = 0 \tag{11}$$

where

$$\begin{gathered}\mu_1 = \frac{P_t}{2P^*_{\text{кр}}},\; P^*_{\text{кр}} = \frac{\pi^2}{l^2}E_0 I\left(1+\frac{\alpha}{2}\right),\\ \Omega_1^2 = \frac{\pi^4}{l^4}\frac{E_0 I}{m}\left(1+\frac{\alpha}{2}\right),\end{gathered} \tag{12}$$

$$\psi_1 = \frac{l^2}{\pi^2(1+\alpha)}\cdot\frac{1}{\cos\nu l}\cdot\frac{2}{l}\left[\frac{\pi^2}{l^2}\left(a_0 + \frac{\alpha}{l}a_1\right) + \right.$$
$$\left. +\nu\frac{\pi}{l}\left(a_2 + \frac{\alpha}{l}a_3\right) + \frac{\alpha}{l}\frac{\pi}{l}a_4\right]$$

The following notations are introduced in these formulas:

$$\begin{aligned} a_0 &= \int_0^l \cos\nu\chi\cdot\sin^2\frac{\pi\chi}{l}\,d\chi,\\ a_1 &= \int_0^l \chi\cos\nu\chi\cdot\sin^2\frac{\pi\chi}{l}\,d\chi,\\ a_2 &= \frac{1}{2}\int_0^l \sin\nu\chi\cdot\sin\frac{2\pi\chi}{l}\,d\chi,\\ a_3 &= \frac{1}{2}\int_0^l \chi\sin\nu\chi\cdot\sin\frac{2\pi\chi}{l}\,d\chi,\\ a_4 &= \frac{1}{2}\int_0^l \cos\nu\chi\cdot\sin\frac{2\pi\chi}{l}\,d\chi. \end{aligned} \tag{13}$$

It should be noted that the solution of equation (11) is constructed similarly to [1]. The analysis shows that the boundary of the main region of instability in the first approximation is determined from the condition

$$1 \pm \frac{\mu_1}{1 - \frac{\theta^2}{\omega_L^2}} - \frac{\theta^2}{4\Omega_1^2} = 0 \tag{14}$$

Let us transform equation (14) to the form:

$$1 \pm \frac{\mu_1}{1 - \beta_1 n_1^2} - n_1^2 = 0 \tag{15}$$

$$\mu_1 = \frac{\mu}{1+\alpha/2},\ n_1 = \frac{n}{1+\alpha/2},\ \beta_1 = \beta\left(1+\frac{\alpha}{2}\right),$$
$$n = \frac{\theta}{2\Omega_1},\ \beta = \frac{4\Omega_1^2}{\omega_L^2} \tag{16}$$

Taking into account (15), (16), an asymptotic analysis is carried out and the region of dynamic instability is constructed.

Note that for, the obtained solution coincides with the known classical solutions [1].
In this case, from (12), (13) we find:

$$\mu_1 = \mu,\ \Omega_1 = \Omega,$$
$$\psi_1 = \frac{\mathrm{tg}\,\nu l}{\nu l} \cdot \frac{1 - \frac{\nu^2 l^2}{2\pi^2}}{1 - \frac{\nu^2 l^2}{4\pi^2}} \tag{17}$$

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