ESTIMATION OF DIFFERENT ENTROPIES OF INVERSE RAYLEIGH DISTRIBUTION UNDER MULTIPLE CENSORED DATA

Hemani Sharma and Parmil Kumar

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Department of Statistics, University of Jammu, J&K.
hemanistats@gmail.com, parmil@yahoo.com

Abstract

The inverse Rayleigh distribution finds widespread applications within life testing and reliability research. Particularly, it proves invaluable in scenarios involving multiple censored data points. In this context, the Renyi, Havrda, Charvat, and Tsallis entropies of the inverse Rayleigh distribution are efficiently calculated. The maximum likelihood approach is used to get the estimators, as well as the approximate confidence interval. The mean squared errors, approximate confidence interval, and their related average length are computed. To illuminate the behavior of estimates across varying sample sizes, a comprehensive simulation study is conducted. The outcomes of the simulation study consistently reveal a downward trend in mean squared errors and average lengths as the sample size increases. Additionally, an interesting finding emerges as the censoring level diminishes. The entropy estimators progressively converge towards their true values. For practical demonstration, the effectiveness of the approach is showcased through the analysis of two real-world datasets. These applications underscore the real-world relevance of the methodology, further validating its utility in addressing complex scenarios involving censored data and inverse Rayleigh distributions.

Keywords: inverse Rayleigh distribution, Renyi entropy, Havrda and Charvat entropy, Tsallis entropy, multiple censored.

1. Introduction

The concept of entropy measurement is essential in many fields, including statistics, economics, and physical, chemical, and biological phenomena. The concept of entropy was first proposed as a thermodynamic state variable in classical thermodynamics, and it is based on principles from probability theory and mathematical statistics. Although the term information theory does not have a precise meaning, it can be considered of as the study of problems involving any probabilistic system. Entropy is referred to as the amount of information found in the sample. One of the most important aspects of statistics is the study of probability distributions. Every probability distribution contains some element of uncertainty. Entropy is a phenomenon that can be utilised to provide a quantitative estimate of uncertainty. Entropy is also a measure of disorder or randomness in a probabilistic system having a large number of random states with equal probability, and is zero when the system is in a specified state with no uncertainty. In other terms, a random variable's entropy is a measure of the amount of information required to explain a random variable on average. Shannon [13] established the concept of entropy as a measure of information. Here, we focus our attention on three entropy measures- the Renyi [11], Havrad and Chavrat [7], Tsallis entropies [15]. The Renyi entropy [11] comes from information theory, whereas the Tsallis entropy [15] comes from statistical physics, and both have a wide range of applications in their respective fields. These three entropy measures are defined, accordingly, for an arbitrary variable X with the Probability Density Function (PDF) f(x;φ), where φ denotes the corresponding parameters.

$$R_\delta(X;\varphi) = \frac{1}{1-\delta}\log\left[\int_{-\infty}^{\infty} f(x;\varphi)^\delta dx\right] \tag{1}$$

where $\delta \neq 1$ and $\delta > 0$, and

$$HC_\delta(X;\varphi) = \frac{1}{2^{1-\delta}-1}\left[\int_{-\infty}^{\infty} f(x;\varphi)^\delta dx - 1\right] \tag{2}$$

where $\delta \neq 1$ and $\delta > 0$, and

$$T_\delta(X;\varphi) = \frac{1}{\delta-1}\left[1-\int_{-\infty}^{\infty} f(x;\varphi)^\delta dx\right] \tag{3}$$

where $\delta \neq 1$ and $\delta > 0$,

Lord Rayleigh [12] initially proposed the Rayleigh distribution in relation to an acoustic problem. Since then, a great deal of work has been done in numerous domains of science and technology to improve this distribution. The Rayleigh distribution's hazard function is an increasing function of time, which is an important property. If the random variable Y has a Rayleigh distribution, the random variable $X = \frac{1}{Y}$ has an inverse Rayleigh distribution (IRD). Trayer [14] proposed the Inverse Rayleigh distribution (IRD). The IRD is used in a variety of applications, including as life tests and reliability studies. A random variable X is said to have inverse Rayleigh distribution if its PDF and CDF has the following form:

$$f(x;\sigma) = \frac{2\sigma^2}{x^3}\exp\left[-\left(\frac{\sigma}{x}\right)^2\right];\ x > 0, \sigma > 0 \tag{4}$$

$$F(x;\sigma) = \exp\left[-\left(\frac{\sigma}{x}\right)^2\right];\ x > 0, \sigma > 0 \tag{5}$$

Wong and Chan [17] explored the entropy of ordered sequences and the order statistic. The entropy of upper record values was studied by Baratpour et al. [4], whereas the entropy of lower record values was proposed by Morabbi and Razmkhah [?]. Abo-Eleneen [1] discussed the entropy of progressively censored samples, Cho et al. [5] estimated the entropy for the Rayleigh distribution via doubly-generalized Type II hybrid censored samples using maximum likelihood and Bayes estimators, and Hassan and Zaky [6] investigated point and interval estimation of the Shannon entropy for the for the inverse Weibull distribution under multiple censored data. Bantan et al. [3] used multiple censored data to derive the Renyi and q-entropy for the inverse Lomax distribution. To measure the Lomax distribution's dynamic cumulative residual Renyi entropy, Al-Babtain et.al [2] explored the Bayesian and non-Bayesian techniques.

However, the estimation of entropy measures for the inverse Rayleigh distribution (IR), such as the Renyi, Havrad, and Chavrat, Tsallis entropies, still an unresolved subject . The problem is examined in the context of multiple censored data in this study, which fills the gap. This is a common scenario in which many censoring levels are logically present, as it is in many situations for life assessment and survival analysis. Renyi, Havrad and Chavrat, Tsallis entropies are derived in our study after analysing the maximum likelihood estimator of σ . A comprehensive numerical analysis is carried out, demonstrating that the derived estimates behave well across a range of sample sizes. The mean squared errors, estimated confidence intervals, and associated average lengths are considered as benchmarks. The values of the mean squared errors and average lengths decreases as the sample size rises, according to our numerical findings. Furthermore, as the censoring level is reduced, the Renyi, Havrad and Chavrat, Tsallis entropies estimates approaches the real value. The findings are illustrated using a real-life data set.

The next is how the rest of the article is organised: Section 2 gives the Renyi, Havrad, and Chavrat, Tsallis entropies for the inverse Rayleigh (IR) distribution. Section 3 focuses at how they can be estimated using multiple censored data. Section 4 contains the simulation and numerical results. Section 5 demonstrates how the method can be used to real-world data sets. Section 6 ends with some summing comments.

2. Expressions of the Renyi, Havrad and Chavrat, Tsallis entropies

Let X be an arbitrary variable with parameter σ that follows the IR distribution. The Renyi entropy of X with $\varphi = (\sigma)$ by using (1)and (3) is given as

$$R_\delta(X;\sigma) = \frac{1}{1-\delta} \log \left[\int_0^\infty \frac{2\sigma^2}{x^3} \exp\left[-\left(\frac{\sigma}{x}\right)^2 \right] \right]^\delta dx \tag{6}$$

Put $\frac{\sigma}{x} = y \Rightarrow x = \frac{\sigma}{y} \Rightarrow dx = \sigma\left(-\frac{1}{y^2}\right) dy.$

$$R_\delta(X;\sigma) = \frac{1}{1-\delta} \log \int_0^\infty 2^\delta y^{2\delta} \left(\frac{y}{\sigma}\right)^\delta \exp(-\delta y^2)\sigma \left(\frac{-1}{y^2}\right) dy$$

$$= \frac{1}{1-\delta} \log \int_0^\infty y^{3\delta-2} \frac{(-2^\delta)}{\sigma^{\delta-1}} \exp\left(-\delta y^2\right) dy$$

$$= \frac{1}{1-\delta} \log \left[\frac{-2^\delta}{\sigma^{\delta-1}} \int_0^\infty y^{3\delta-2} \exp\left(-\delta y^2\right) dy \right]$$

Put $y^2 = t \Rightarrow y = \sqrt{t} \Rightarrow dy = \frac{1}{2\sqrt{t}} dt$

$$R_\delta(X;\sigma) = \frac{1}{1-\delta} \log \left[\frac{-2^\delta}{\sigma^{\delta-1}} \int_0^\infty t^{\frac{3\delta-2}{2}} \exp\left(-\delta t\right) \frac{1}{2\sqrt{t}} dt \right]$$

$$= \frac{1}{1-\delta} \log \left[\frac{-2^\delta}{\sigma^{\delta-1}} . \frac{1}{2} \int_0^\infty t^{\frac{3\delta}{2}-\frac{1}{2}-1} \exp\left(-\delta t\right) dt \right]$$

$$= \frac{1}{1-\delta} \log \left[\frac{-2^{\delta-1}}{\sigma^{\delta-1}} \int_0^\infty \exp\left(-\delta t\right) * t^{\frac{3\delta}{2}-\frac{1}{2}-1} dt \right]$$

$$R_\delta(X;\sigma) = \frac{1}{1-\delta} \log \left[-\left(\frac{2}{\sigma}\right)^{\delta-1} \int_0^\infty \exp\left(-\delta t\right) * t^{\frac{3\delta}{2}-\frac{1}{2}-1} dt \right]$$

$$R_\delta(X;\sigma) = \frac{1}{1-\delta} \log \left[-\left(\frac{2}{\sigma}\right)^{\delta-1} \frac{\Gamma\frac{3\delta-1}{2}}{\delta^{\frac{3\delta-1}{2}}} \right] \tag{7}$$

with $\delta \neq 1$, $\delta > 0$ and $3\delta - 1 > 0$.

Similarly, on using equation equation (7), Havrad and Chavrat entropy and Tsallis entropy of X is given by

$$HC_\delta(X;\sigma) = \frac{1}{2^{1-\delta}-1} \left[\left(\int_0^\infty \frac{2\sigma^2}{x^3} \exp\left[-\left(\frac{\sigma}{x}\right)^2 \right] dx \right)^\delta - 1 \right]$$

$$= \frac{1}{2^{1-\delta}-1} \left[-\left(\frac{2}{\sigma}\right)^{\delta-1} \frac{\Gamma\frac{3q-1}{2}}{\delta^{\frac{3q-1}{2}}} - 1 \right] \tag{8}$$

with $\delta \neq 1$, $\delta > 0$ and $3\delta - 1 > 0$.

$$T_\delta(X;\sigma) = \frac{1}{\delta-1} \left[1 - \left(\int_0^\infty \frac{2\sigma^2}{x^3} \exp\left[-\left(\frac{\sigma}{x}\right)^2 \right] dx \right)^\delta \right]$$

$$= \frac{1}{\delta - 1}\left[1 + \left(\frac{2}{\sigma}\right)^{\delta-1} \frac{\Gamma\frac{3q-1}{2}}{\delta^{\frac{3q-1}{2}}}\right] \tag{9}$$

with $\delta \neq 1$, $\delta > 0$ and $3\delta - 1 > 0$.

The appropriate expressions of Renyi, Havrad, and Chavrat and Tsallis entropies of X, simply stated as functions of parameter σ, are represented by Equations (7), (8), and (9) respectively.

3. Entropy Estimation

Let X be a random variable with cdf and pdf equal to f(x;φ) and F(x;φ), respectively. We acquire n values $x_1, x_2, ..., x_n$ based on n units under a given test, where n_f and n_m are the number of failed and censored units, respectively. The Likelihood function for φ is as follows:

$$L(\varphi) = K\prod_{i=1}^{n}[f(x_i;\varphi)]^{\varepsilon_{i,f}}[1 - F(x_i;\varphi)]^{\varepsilon_{i,m}} \tag{10}$$

where K is a constant.

$\varepsilon_{i,f}$=1 if the ith unit failed, and 0 otherwise (so $\sum_{i=1}^{n} \varepsilon_{i,f} = n_f$)

$\varepsilon_{i,m}$=1 if the ith unit censored, and 0 otherwise (so $\sum_{i=1}^{n} \varepsilon_{i,m} = n_m$).

By inserting (4) and (5) in (10), we can get the likelihood function of the IR distribution based on multiple censored samples is given by

$$L(\sigma) = K\prod_{i=1}^{n}\left[\frac{2\sigma^2}{x^3}\exp\left[-\left(\frac{\sigma}{x}\right)^2\right]\right]^{\varepsilon_{i,f}}\left[1 - \exp\left[-\left(\frac{\sigma}{x}\right)^2\right]\right]^{\varepsilon_{i,m}} \tag{11}$$

The log-likelihood function is given by

$$\log l(\sigma) = \log K + 2\sum \varepsilon_{i,f}\log(\sigma^2) - \sum_{i=1}^{n}\varepsilon_{i,f}\log(x_i^3) - \sum_{i=1}^{n}\varepsilon_{i,f}\left(\frac{\sigma}{x_i}\right)^2 + \sum_{i=1}^{n}\varepsilon_{i,m}\log\left[1 - \exp\left(-\frac{\sigma}{x_i}\right)^2\right]$$

The MlE is obtained by maximizing L(σ) with respect to σ, and is given by

$$\frac{\partial \log l(\sigma)}{\partial\sigma} = \frac{2n_f}{\sigma^2}.2\sigma - \sum_{i=1}^{n}\varepsilon_{i,f}\frac{2\sigma}{x_i^2} + \sum_{i=1}^{n}\varepsilon_{i,m}\left(\frac{1}{1 - \exp\left(-\frac{\sigma}{x_i}\right)^2}\right)\left[-\exp\left(\frac{-\sigma}{x_i}\right)^2\right]\left(\frac{-2\sigma}{x_i^2}\right)$$

$$= \frac{4n_f}{\sigma} - \sum_{i=1}^{n}\varepsilon_{i,f}\left(\frac{2\sigma}{x_i^2}\right) + \frac{\sum_{i=1}^{n}\varepsilon_{i,m}\exp\left(\frac{-\sigma}{x_i}\right)^2}{\left(1 - \exp\left(\frac{-\sigma}{x_i}\right)^2\right)}.\left(\frac{2\sigma}{x_i^2}\right) \tag{12}$$

The above equation is in closed form therefore, cannot be solved manually. So the MLE estimate of σ is obtained with the help of matlab.

On substituting the MLE of σ in (7), (8) and (9), estimates for the entropies $R_\delta(X;\sigma)$, $HC_\delta(X;\sigma)$ and $T_\delta(X;\sigma)$, are, respectively, given by

$$R_\delta(X;\sigma) = \frac{1}{1-\delta}\log\left[-\left(\frac{2}{\hat{\sigma}}\right)^{\delta-1}\frac{\Gamma\frac{3\delta-1}{2}}{\delta^{\frac{3\delta-1}{2}}}\right] \tag{13}$$

with $\delta \neq 1$, $\delta > 0$ and $3\delta - 1 > 0$.

$$HC_{\delta}(X;\sigma) = \frac{1}{2^{1-\delta}-1}\left[-\left(\frac{2}{\hat{\sigma}}\right)^{\delta-1}\frac{\Gamma\frac{3q-1}{2}}{\delta^{\frac{3q-1}{2}}}-1\right] \tag{14}$$

with $\delta \neq 1$, $\delta > 0$ and $3\delta - 1 > 0$.

$$T_{\delta}(X;\sigma) = \frac{1}{\delta-1}\left[1+\left(\frac{2}{\hat{\sigma}}\right)^{\delta-1}\frac{\Gamma\frac{3q-1}{2}}{\delta^{\frac{3q-1}{2}}}\right] \tag{15}$$

with $\delta \neq 1$, $\delta > 0$ and $3\delta - 1 > 0$.

Under sufficient regularity requirements, the MLE estimators are consistent and asymptotically normal distributed for large sample sizes. At the confidence level 100(1-α) with $\alpha = (0, 1)$, the estimated confidence interval for the Renyi entropy can be calculated as follows:

$$P\left[-z_{\frac{\alpha}{2}} \leq \frac{\hat{R}_{\delta}(X) - R_{\delta}(X)}{\sigma_{\hat{R}_{\delta}(X)}} \leq z_{\frac{\alpha}{2}}\right] = 1-\alpha \tag{16}$$

where $z_{\frac{\alpha}{2}}$ is $100(1-\frac{\alpha}{2})$ the standard normal percentile and ν is the significant level. As a result, approximate Renyi entropy confidence bounds can be determined, such that

$$P[\hat{R}_{\delta}(X) - z_{\frac{\alpha}{2}}\sigma_{(R_{\delta}\hat{(}X))} \leq R_{\delta}(X) \leq \hat{R}_{\delta}(X) + z_{\frac{\alpha}{2}}\sigma_{(R_{\delta}\hat{(}X))}] \cong 1-\alpha \tag{17}$$

where $L_H = \hat{R}_{\delta}(X) - z_{\frac{\alpha}{2}}\sigma_{(R_{\delta}\hat{(}X))}$, $U_H = \hat{R}_{\delta}(X) + z_{\frac{\alpha}{2}}\sigma_{(R_{\delta}\hat{(}X))}$ are the lower and upper confidence limits for $R_{\delta}(X)$ and σ is the standard deviation and $\alpha = 0.05$, the approximate confidence limits for Renyi entropy will be constructed with confidence levels 95%. A similar result holds for $\hat{HC}_{\delta}$(X) and $\hat{T}_{\delta}$(X) .

4. Simulation Study

The procedure adopted to examine the performance of the Proposed estimators given by (13), (14) and (15) are as:

• 1000 random samples of sizes $n = 50, 100, 150, 200, 300, 400$ are obtained from the IR distribution based on multiple censored samples, Using the method described in [16].

• The values of parameters are selected as $\delta = 0.4, 1.2, 1.5$ and $\sigma = 1.2$. We chose $CL = 0.5$ and 0.7 at random for failures at the censoring level (CL).

• The estimated value for σ, true values for $R_{\delta}(X;\sigma)$, $HC_{\delta}(X;\sigma)$ and $T_{\delta}(X;\sigma)$ are obtained by (12), (7), (8) and (9), and the estimates $\hat{R}_{\delta}(X;\sigma)$, $\hat{HC}_{\delta}(X;\sigma)$ and $\hat{T}_{\delta}(X;\sigma)$ given by (13), (14)and (15) are calculated, respectively.

• At last, the average of the derived estimates, MSEs, and ALs are computed with a threshold of 95% All the calculations are done by the use of the software Matlab and R. From the tables,the following conclusions have been made:

- As the sample size grows, the bias and MSEs of entropy estimates fall.
- Additionally, as the sample size grows, the ALs of estimates diminish.
- As the sample size expands, the entropy estimations approach their true values.
- The MSE of entropy estimates at $CL = 0.5$ is usually less than the MSE of estimates at $CL = 0.7$.

These findings demonstrate the high precision of our entropy estimates.

Table 1: *Renyi Entropy Estimates at CL=0.5($\sigma = 1.2, \delta = 0.4$)*

n	Actual Value	Estimate	Bias	MSE	AL
50	0.9930	1.1060	0.1130	$6.33 * e^{-05}$	0.0395
100		0.9089	0.0841	$5.40 * e^{-05}$	0.0190
150		0.9465	0.0495	$4.71 * e^{-05}$	0.0121
200		0.9506	0.0424	$1.79 * e^{-05}$	0.0079
300		0.9560	0.0370	$4.56 * e^{-06}$	0.0064
400		0.9943	0.0013	$9.52 * e^{-09}$	0.0047

Table 2: *Renyi Entropy Estimates at CL=0.7($\sigma = 1.2, \delta = 0.4$)*

n	Actual Value	Estimates	Bias	MSE	AL
50	0.9930	0.8798	0.1132	$3.19 * e^{-04}$	0.0382
100		1.0934	0.1004	$1.01 * e^{-04}$	0.0219
150		1.0694	0.0764	$3.19 * e^{-04}$	0.0140
200		1.0533	0.0603	$2.42 * e^{-05}$	0.0099
300		0.9561	0.0369	$1.95 * e^{-05}$	0.0071
400		0.9864	0.0065	$7.26 * e^{-07}$	0.0044

Table 3: *HC Entropy Estimates at CL=0.5($\sigma = 1.2, \delta = 1.5$)*

n	Actual Value	Estimate	Bias	MSE	AL
50	6.5486	6.0175	0.5311	0.0056	0.2407
100		6.0733	0.4753	0.0023	0.1215
150		6.2092	0.3394	$7.67 * -04$	0.0828
200		6.3493	0.1993	$1.98 * e^{-04}$	0.0635
300		6.6832	0.1346	$6.03 * e^{-05}$	0.0446
400		6.5667	0.0181	$8.16 * e^{-07}$	0.0328

Table 4: *HC Entropy Estimates at CL=0.7($\sigma = 1.2, \delta = 1.5$)*

n	Actual Value	Estimate	Bias	MSE	AL
50	6.5486	5.8458	0.7028	0.0099	0.2338
100		5.9364	0.6122	0.0037	0.1187
150		6.1541	0.3945	0.0010	0.0821
200		6.2186	0.3300	$5.44 * e^{-05}$	0.0622
300		6.4073	0.1413	$6.65 * e^{-05}$	0.0427
400		6.5249	0.0237	$1.40 * e^{-07}$	0.0326

Table 5: *Tsallis Entropy Estimates at CL=0.5($\sigma = 1.2, \delta = 1.2$)*

n	Actual Value	Estimate	Bias	MSE	AL
50	11.9156	12.5135	0.5979	0.0036	0.4883
100		12.2072	0.2916	0.0017	0.2434
150		12.0878	0.1722	$9.88 * e^{-05}$	0.1607
200		12.0497	0.1341	$1.19 * e^{-05}$	0.1198
300		11.9792	0.0636	$2.02 * e^{-05}$	0.0806
400		11.9027	0.0130	$4.19 * e^{-07}$	0.0595

Table 6: *Tsallis Entropy Estimates at CL=0.7($\sigma = 1.2, \delta = 1.2$)*

n	Actual Value	Estimate	Bias	MSE	AL
50	11.9156	12.6254	0.7098	0.0101	0.5050
100		12.3534	0.4377	0.0019	0.2471
150		12.1353	0.2197	$3.21 * e^{-04}$	0.1618
200		12.0573	0.1417	$1.00 * e^{-04}$	0.1206
300		11.8533	0.0623	$1.29 * e^{-05}$	0.0790
400		11.9585	0.0429	$4.60 * e^{-07}$	0.0598

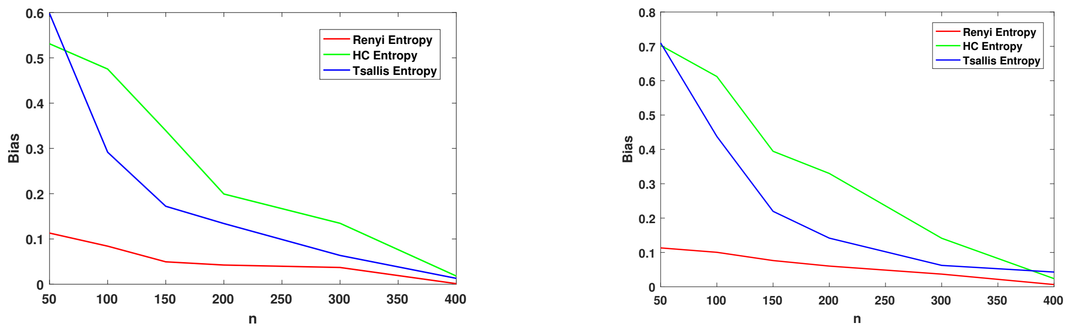

Figure 1: *(a) Bias of Renyi, Havrda and Charvat, Tsallis entropy at CL=0.5 and (b) Bias of Renyi, Havrda and Charvat, Tsallis entropy at CL=0.7*

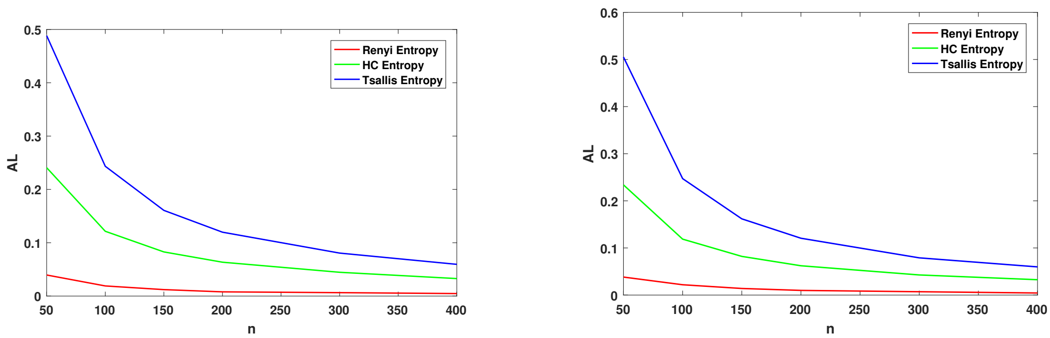

Figure 2: *(a) Average Length of Renyi, Havrda and Charvat, Tsallis entropy at CL=0.5 and (b) Average Length of Renyi, Havrda and Charvat, Tsallis entropy at CL=0.7*

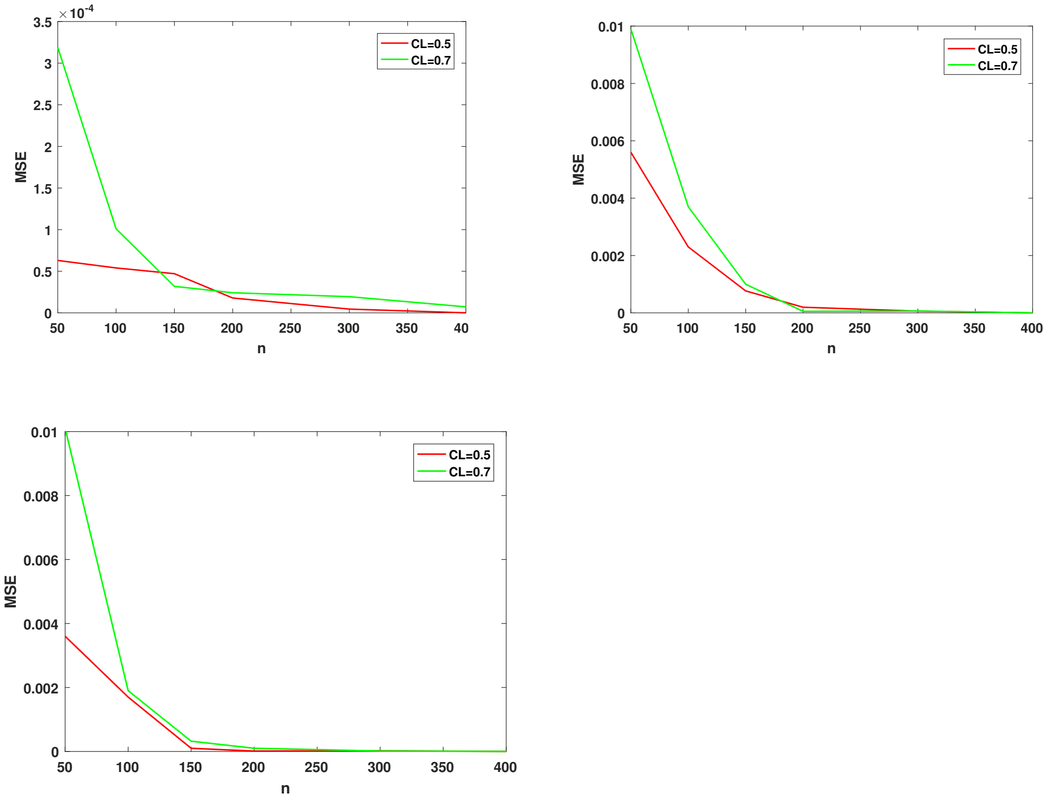

Figure 3: *(a) MSE of Renyi entropy at CL=0.5 and CL=0.7, (b) MSE of Havrda and Charvat entropy at CL=0.5 and CL=0.7 and (c) MSE of Tsallis entropy at CL=0.5 and CL=0.7*

5. Data Analysis

To demonstrate the effectiveness of our estimation methods, we utilize the dataset pertaining to fatigue failure times of twenty-three ball bearings as documented in [8]. This dataset has been extensively employed in various research investigations.

Dataset I: 0.1788, 0.2892, 0.3300, 0.4152, 0.4212, 0.4560, 0.4840, 0.5184, 0.5196, 0.5412, 0.5556, 0.6780, 0.6864, 0.6888, 0.8412, 0.9312, 0.9864, 1.0512, 1.0584, 1.2792, 1.2804, 1.7340. The Kolmogorov-Smirnov (K-S) distance and its corresponding p-value for the actual dataset are calculated as 0.1440 and 0.6988 respectively. These values suggest that the observed dataset aligns well with the inverse Rayleigh distribution. This assertion gains further validation through the visualization of the empirical Cumulative Distribution Function (ECDF) plot, the quantile-quantile (Q-Q) plot, and the Histogram, showcased in figures 4 and 5. Derived from the complete sample, the maximum likelihood estimate of the parameter sigma is determined as 0.4681, with a standard error of 0.0499.

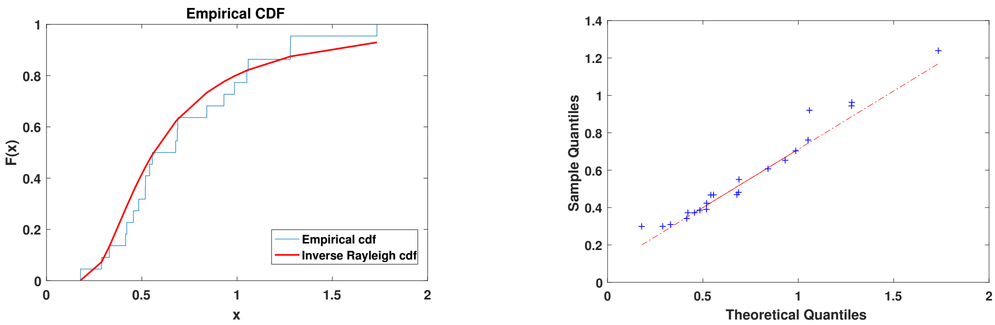

Figure 4: *(a) Ecdf plot for the dataset I (b) Q-Q plot for the dataset I*

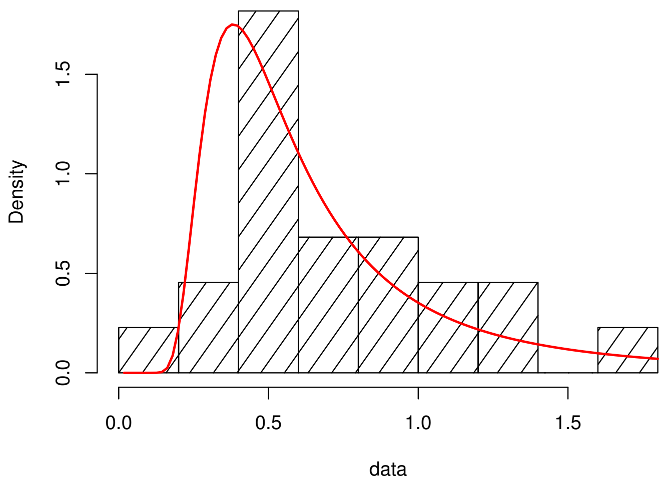

Figure 5: *Plot of the fitted density for dataset I*

Dataset II: The second dataset, sourced from [9], encompasses monthly actual tax revenues in Egypt spanning from January 2006 to November 2010. These data points are measured in 1000 million Egyptian pounds and exhibit the following sequence: 5.9, 20.4, 14.9, 16.2, 17.2, 7.8, 6.1, 9.2, 10.2, 9.6, 13.3, 8.5, 21.6, 18.5, 5.1, 6.7, 17, 8.6, 9.7, 39.2, 35.7, 15.7, 9.7, 10, 4.1, 36, 8.5, 8, 9.2, 26.2, 21.9, 16.7, 21.3, 35.4, 14.3, 8.5, 10.6, 19.1, 20.5, 7.1, 7.7, 18.1, 16.5, 11.9, 7, 8.6, 12.5, 10.3, 11.2, 6.1, 8.4, 11, 11.6, 11.9, 5.2, 6.8, 8.9, 7.1, 10.8. The Kolmogorov-Smirnov (K-S) distance and its corresponding p-value for this dataset stand at 0.08219 and 0.8203, respectively. These results suggest a fitting match with the inverse Rayleigh distribution. This assertion gains further support from the visual analyses, including the Empirical CDF plot, Quantile-Quantile (Q-Q) plot, and Histogram are depicted in figures 6 and 7. The maximum likelihood estimate for the parameter sigma, obtained from the complete dataset is 9.3595, with a standard error of 0.6092. Table 7 and 8 present estimates for different entropy measures in both datasets. These tables reveal as the parameter δ increases, Renyi entropy demonstrates an ascending trend, whereas Tsallis and HC entropies exhibit a descending trend with the increase of δ. Additionally, the estimates are notably influenced by the level of censoring.

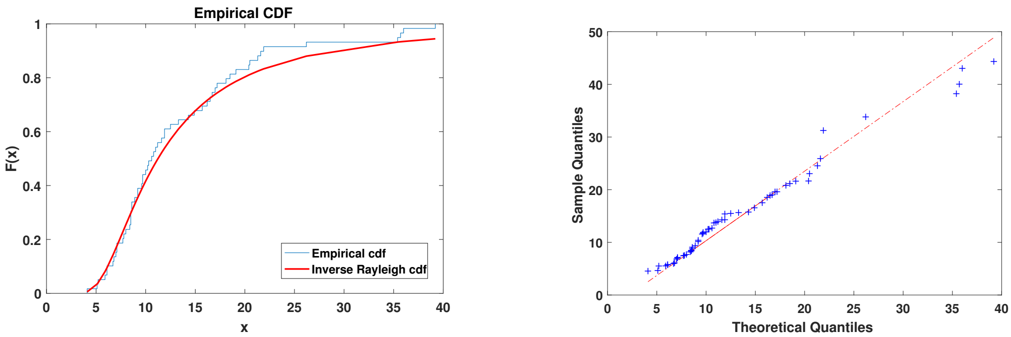

Figure 6: *(a) Ecdf plot for the dataset II (b) Q-Q plot for the dataset II*

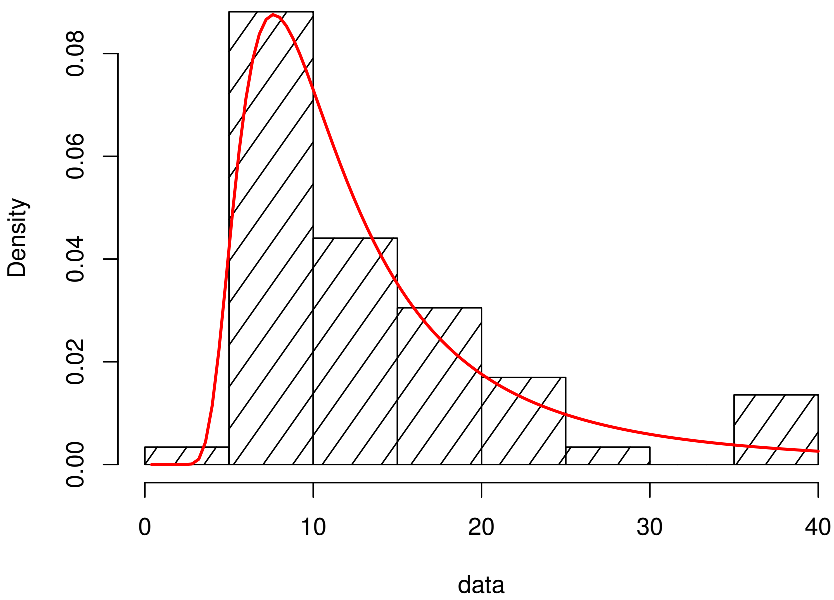

Figure 7: *Plot of the fitted density for dataset II*

Table 7: *Estimated of Renyi entropy, Tsallis entropy and HC entropy at CL=0.5, 0.7 for Dataset I.*

	CL=0.5			CL=0.7		
δ	$R_\delta(X)$	$T_\delta(X)$	$HC_\delta(X)$	$R_\delta(X)$	$T_\delta(X)$	$HC_\delta(X)$
1.2	-2.6986	13.5778	14.7472	-2.2308	12.8115	15.4360
2	-1.7333	6.6593	2.4727	-1.2654	4.5447	2.7547

Table 8: *Estimated of Renyi entropy, Tsallis entropy and HC entropy at CL=0.5, 0.7 for Dataset II.*

	CL=0.5			CL=0.7		
δ	$R_\delta(X)$	$HC_\delta(X)$	$T_\delta(X)$	$R_\delta(X)$	$HC_\delta(X)$	$T_\delta(X)$
1.2	0.3961	9.6191	20.7654	0.4187	9.5983	20.8244
2	1.3615	1.2562	12.4407	1.3841	1.2505	12.6790

6. Conclusion

In this article, the Renyi, Havrda and Charvat, Tsallis entropies of the inverse Rayleigh distribution are estimated using multiple censored data. Using maximum likelihood and plugging approach, we present an efficient estimation strategy. The Renyi, Havrda and Charvat, and Tsallis entropies estimates' behaviour is measured in terms of mean squared errors and average lengths. According to numerical results, the bias and mean squared errors of our estimators decreases as the sample size grows. It's also worth noting that as the sample size grows, the average length of our estimators shrinks. As a result, the proposed estimates show to be efficient, giving new valuable tools with potential relevance in a wide range of applications involving the inverse Rayleigh distribution's entropy. The paper concludes with an applications to a real-world data sets. In upcoming research endeavors, one could explore the assessment of entropies using both Bayesian and E-Bayesian methodologies across various censoring scenarios.

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