A LITERATURE SURVEY ON QUEUEING MODEL WITH WORKING VACATION

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Abstract

In 2002, the Working Vacation (WV) queues were implemented as an extension of standard queueing models with vacations. During the vacation period in WV queues, the server provides service at a slower pace as opposed to the typical busy period.The objective of this survey is to provide a concise overview of the latest scholarly investigations on queueing models for WVs. The concept of a queue with WV has been implemented across various domains, encompassing computer systems, communication networks, production management, computer communication, manufacturing, and inventory systems. Additionally, it has been applied to network service, web service, file transfer service, and mail service.

Keywords:
Working Vacation Queue(WVQ), $M/M/1$ and $M/G/1$ queue, $GI/M/1$ and $GI/G/1$ queue, Retrial queue, Discrete time $Geo/G/1$ queue, Multi-server queue, $M^{[X]}/M/1$ -Batch arrival queue, MAP queue.

1. Introduction

In the realm of service industries like healthcare and manufacturing, as well as computer systems, the queueing model plays a vital role. This mathematical concept, known as queuing theory, finds applications in predicting queue lengths and waiting durations when different types of customers are served by distinct servers following various queue disciplines.

One interesting aspect of queueing systems is the idea of a "working vacation" (WV). Traditionally, when there are no customers or the server experiences a failure, the system goes on vacation, and the server stops serving customers entirely. However, a WV introduces a more efficient approach where the server continues working with different service rates during vacation times, rather than coming to a complete halt. This way, the server can make better use of its idle time. Model for WV is shown in 1.

Our focus in this review paper is on the literature surrounding WV models. The idea of vacation, which involves utilising the idle time of a server for additional work in a secondary system, was first introduced by Levy and Yechiali in 1975 [36]. Subsequently, the concept of a WV was afterwards introduced by Servi and Finn [63]. Over the last three decades, WV queueing models have emerged as a prominent subject of interest within the field of queuing theory.

The objective of this paper is to present a comprehensive overview of the progress achieved in the examination of arrival and service operations in diverse WV models. We'll explore the application of WVs in the $M/M/1$ and $M/G/1$ queueing models in Section 2, while Section 3 will delve into the models for the $GI/M/1$ and $GI/G/1$ queues with WVs. Furthermore, Section 4 will cover recent research on retrial queueing models incorporating WVs. Finally, in Section 5, we'll discuss some of the most recent developments in WV models. The paper will conclude in Section 6, summarizing the key findings and offering concluding remarks to aid readers in understanding the field of WVQ.

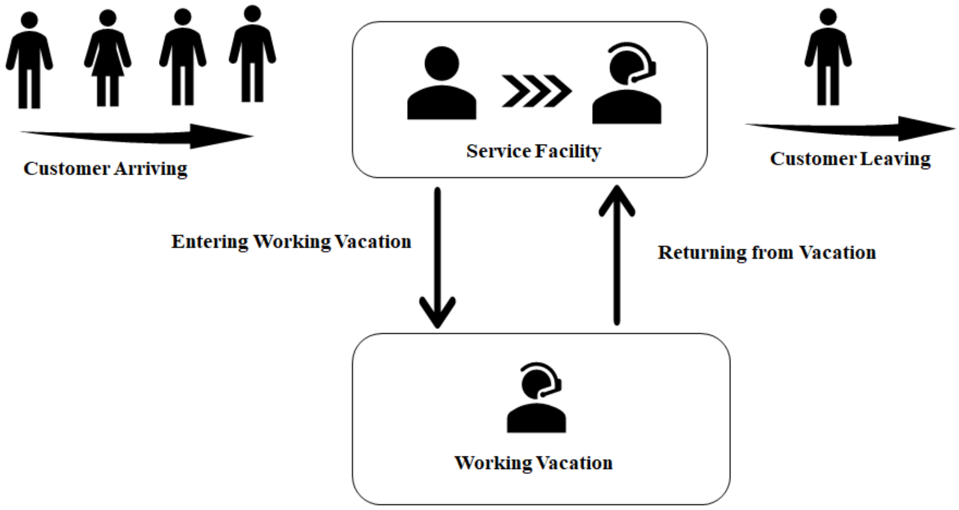

Figure 1: *Queueing Syetem with Working Vacation*

2. An $M/M/1$ and $M/G/1$ Queue Models with Working vacations

The concept of vacations in queueing models was first explored by Levy and Yechiali [36]. They utilized decomposition results to derive the optimal vacation size. Servi and Finn [63] introduced a semi-vacation policy and derived an $M/M/1$ queue with multiple WVs (MWV). They also provided explicit formulas for average, variance, and distribution of time and number of customers in the system. Wu and Takagi [78] extended Servi and Finn's [63] $M/M/1$ model to an $M/G/1/WV$ model, considering general distributions for both service times and WVs. They further obtained the Laplace-Stieltjes Transform (LST) for the distribution of vacation sizes.

Numerous studies followed, exploring different aspects of WV models. Liu et al. [50] analyzed the stochastic decomposition structures of the number of customers and sojourn time in $M/M/1/WV$ queues. Zhang and Xu [89] investigated an $M/M/1$ queue with MWV and N-policy. Li et al. [39] studied an $M/G/1$ queue with exponential WVs using matrix analytic methods. Xu et al. [80] examined $M/M/1$ queue with SWV, utilizing quasi birth and death (QBD) process and matrix-geometric solution (MGS) method.

The research expanded to consider various scenarios, such as server breakdowns and disasters. Kim et al. [30] explored the $M/G/1$ queue with disasters and working breakdown services. Additionally, WV models were studied with different impatient behaviors, multiple types of WVs, and variant service interruptions [83, 66, 76].

Vacation interruption (VI) models emerged, where vacation and VI are interconnected, and the server may interrupt vacation based on specific system indices. Jihong Li and Naishuo Tian [41] introduced VI, analyzing the $M/M/1$ queue using QBD process and MGS method. Zhang and Hou [84] extended this to an $M/G/1$ queue with WV and VI, obtaining queue length distribution and service status.

The integration of WVs and service interruption due to server breakdowns added strength to queueing models. Various analytical methods, such as generating functions, were employed [24, 14, 35, 88]. Imbalanced behavior of servers was also considered [51, 40, 21, 17].

Overall, extensive research has been conducted to understand the dynamics of queueing models with WVs and vacation interruptions, offering valuable insights into optimizing system performance and resource utilization.

3. An $GI/M/1$ and $GI/G/1$ Queue Models with Working vacations

In the context of general input (GI) queue models with WVs (WV), several studies have been conducted. Baba [5] explored a GI/M/1 queue with WV, extending Servi and Finn's M/M/1/WV system to a GI/M/1/WV model. Building on this, Banik et al. [7] analyzed the GI/M/1/N queue with a MWV policy. Li and Tian [42] delved into the details of a GI/M/1 queue with SWV, where the server can continue working at a reduced rate during the vacation period.

Zhang and Hou [86] studied the GI/M/1/N queue with a variant of MWV and obtained the queue length distribution at different time periods using the supplementary variable technique (SVT) and embedded Markov chain (EMC) method. Goswami et al. [19] developed the GI/M(n)/1 queue model with finite buffer, considering state-dependent services and state-dependent MWV. Vijayalaxmi et al. [34] focused on a limited buffer come-back arrival single server queueing system with multiple state-dependent exponential WV.

Ye and Liu [82] presented the GI/M/1 queue with SWV and derived the stationary distribution of the system size at arrival time using the matrix-geometric solution (MGS) method. They also found the stationary distribution of the system size at arbitrary time using the semi-Markov process (SMP) method. Panda et al. [56] explored an infinite buffer come-back arrival queue with MWV policy, considering general bulk service (a,b)-rule.

In the context of general input and vacation interruption models, where the server goes on vacation when there are no customers, several studies have been conducted. Li and Tian [38] presented WV and VI in a discrete-time GI/Geo/1 queue using the MGS approach. Ji-hong et al. [25] studied a GI/M/1 queue with WVs and vacation interruptions. Zhao et al. [90] introduced setup time with VI policy and investigated a single server general input queue with set-up period, WV, and VI, obtaining the distribution of the number of customers in the system and waiting time.

Chen et al. [11] analyzed PH (Phase-type) WVs and vacation interruptions in GI/M/1 queues. They obtained steady-state distributions for the queue length and waiting time of customers and revealed stochastic decomposition structures of the queue length and waiting time using the method of matrix analytic method (MAM).

Li et al. [68] considered Bernoulli schedule rule and studied the start-up period, SWV, and vacation interruption in the GI/M/1 queue. Goswami and Mund [18] dealt with impatient customers in a single server renewal arrival batch service queue with MWV and balking. They determined the probability distribution of queue length at pre-arrival epoch using the EMC method.

4. Retrial Queue Models with Working Vacations

Retrial queues are mathematical models used in queueing theory to describe systems with finite capacity where arriving jobs that find the system busy will wait for a while before attempting to enter again. Which is shown in Fig. 2. Such systems can be found in various real-world scenarios like restaurant reservations, telecommunication networks, and packet switching networks. Recently, the combination of retrial queues with WVs (WV) has become a subject of thorough investigation.

Studies have been conducted on different types of retrial queues incorporating WVs. For instance, T. Van Do [74] analyzed the stability of the $M/M/1$ retrial queue with WV. Tao et al. [69] used the matrix analytic method to propose conditions for stability in the $M/M/1$ retrial queue with WV interruption under N-policy. Several researchers, such as Li et al. [45], Gao et al. [16], and Aissani et al. [2], explored various aspects of single server retrial queues with WVs and vacation interruptions. Further research delved into specific aspects of retrial queues with

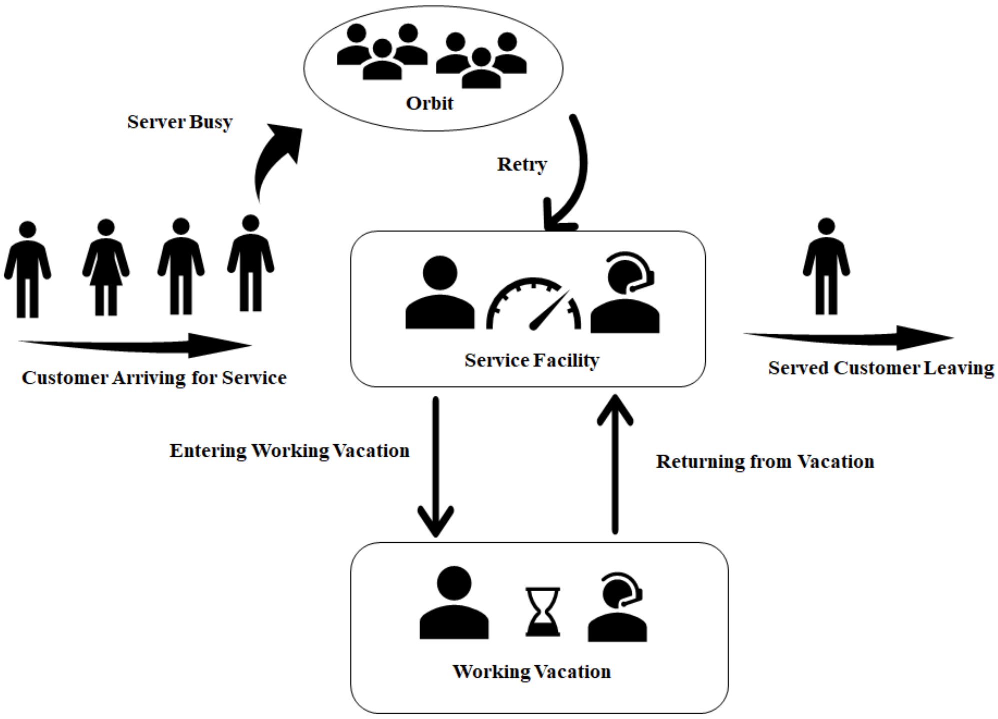

Figure 2: *Retrial Queueing Model with Working Vacation*

WV. For example, Upadhyaya [73] examined a discrete-time $Geo^{[X]}/Geo/1$ retrial queue with WV and derived various performance measures using the matrix-geometric method. Rajadurai et al. [61, 60] addressed RQ systems with general retrial times, feedback, balking, multiple WVs, and vacation interruptions using the supplementary variable technique.

Other studies considered specific features of retrial queues, such as starting failure, preemptive priority, balking customers, and Bernoulli feedback, in the presence of WVs and vacation interruptions [20, 59, 43, 46]. The effects of bulk arrivals, constant retrial rates, and socially optimal balking strategies were also investigated [53, 54, 12].

In conclusion, the combination of retrial queues with WVs has attracted significant attention in recent research, leading to a better understanding of system behaviors and performance measures in various queueing scenarios.

5. Other Working Vacation Models

5.1. Discrete time Queue Models with Working Vacations

Discrete-time (DT) queues with vacations have been extensively investigated by various researchers, owing to their wide range of applications in digital communication systems and telecommunication networks, such as B-ISDN, ATM, and related technologies.

Li [37] studied a discrete-time $Geo/G/1$ queueing system with multiple WVs, where the server operates at a reduced rate during vacation periods. Li and Tian [38] introduced a discrete-time queue model, where customer arrivals and service completions occur at discrete-time instants, in the $GI/Geo/1$ framework. Li and Zhang [44] examined a discrete-time $Geo/Geo/1$ queue with server breakdowns and repairs. Yang et al. [81] investigated the equilibrium joining/balking behavior in the discrete-time $Geo/Geo/1$ queuing model with multiple WVs.

5.2. Multi Server Queue Models with Working Vacations

Krishnamoorthy and Shreenivasan [31] investigated a two-server $M/M/2$ queueing system, where one server remains idle while the other goes on vacation if there are no customers waiting for service. Vijayashree and Janani [77] conducted a transient analysis of an $M/M/c$ queue subjected to multiple exponential WVs.

Bouchentouf et al. [9] studied a heterogeneous two-server queuing system with Bernoulli feedback and multiple WVs, considering impatient customers. They obtained performance measures and the steady-state probability of the queueing model. Sharma and Kumar [64] analyzed a multi-server queuing system with essential two-phase repair and multiple WVs. They employed the Runge-Kutta method to find the time-dependent probability.

5.3. Batch Arrival Queue Models with Working Vacations

Xu et al. [79] examined a batch arrival $M^{[X]}/M/1$ queue with single working vacation (SWV), using the matrix analytic method (MAM) to derive the probability generating function (PGF) of the stationary system length. Baba [6] investigated a batch arrival $M^{[X]}/M/1$ queue with multiple working vacations (MWV) and obtained the exact Laplace-Stieltjes Transform (LST) of the stationary waiting time distribution.

Gao and Yao [15] demonstrated a batch arrival $M^{[X]}/G/1$ queue with randomized WVs, allowing for at most J vacations. Laxmi and Rajesh [32] extended Baba's work [6] by incorporating the concept of variant WVs. They analyzed a single-server batch arrival infinite-buffer queueing system with various types of WVs. Laxmi and Rajesh [33] further expanded on their previous research and explored the effects of different WVs on a batch arrival queue with reneging and server breakdowns.

Thangaraj and Rajendran [70] discussed a batch arrival queueing system with two types of service and vacations. Niranjan et al. [55] analyzed a bulk arrival queueing model with batch size-dependent service and WVs.

5.4. Markovian Arrival Process Queue Models With Working Vacations

AThe Markov Arrival Process (MAP) system represents another significant advancement in the research of WV models. Zhang and Hou [85] conducted a study on a $MAP/G/1$ queue with N-policy WVs and vacation interruptions. They successfully determined the distribution of the system size at the pre-arrival epoch and the Laplace-Stieltjes Transform (LST) of waiting time using the supplementary variable technique (SVT) and matrix analytic method (MAM).

Sreenivasan et al. [65] expanded on the work of Li and Tian [41] by incorporating MAP arrivals, Phase-type (PH) services, and N-policy vacation queue models. Liu et al. [49] examined a cold standby repairable system with WVs and interruptions, utilizing the MAP arrival queueing model. Chakravarthy and Kulshrestha [10] investigated the $MAP/PH/1$ type queueing model with WVs, server breakdowns, and repairs.

6. Conclusion

In conclusion, this survey provides an in-depth exploration of the development of working vacation (WV) queueing models from their early stages to the present. The pioneering researchers who have contributed to the field of WV queueing policies are presented. Readers gain a comprehensive understanding of the current state of WV queueing models through this survey. A wide array of research papers have been reviewed, and proper citations have been included.

This survey offers readers a holistic view of the diverse applications of WV queueing models in various scenarios. It highlights the significance of WV models in predicting queue lengths, waiting durations, and other essential performance measures in queueing systems.

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