A HYBRID APPROACH TO SINGLE ACCEPTANCE SAMPLING PLANS FOR LIFETIMES MODELED WITH THE EXPONENTIAL RAYLEIGH DISTRIBUTION

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Abstract

This article explores into the examination of a novel compound distribution termed the "Exponential Rayleigh distribution" in the context of truncated life testing within a sampling plan. It introduces a hybrid single acceptance sampling plan tailored for truncated life testing scenarios where the item's lifespan adheres to the Exponential Rayleigh distribution. One of the primary segments within the domain of product quality control is referred to as "sampling inspection by variables". This category encompasses procedures that involve the selection of multiple individual units based on measurements taken from a sample to assess a specific quality attribute under scrutiny. These plans, used to assess whether to accept or reject a submitted batch of items based on their observed lifetimes, are commonly known as reliability test plans. The article also outlines the development of a test plan to determine when to conclude the experiment given specific parameters like sample size, producer's risk, consumer's risk, and termination criteria. Sampling inspection, or reliability sampling, plays a pivotal role in maintaining product quality. It involves subjecting items to testing, collecting data on their lifespans, and making acceptance or rejection decisions based on the test results. When assessing an item's quality primarily based on its lifespan, which can be suitably described using a continuous probability distribution; such a plan is termed a "life test sampling plan." This article explores the application of the Exponential Rayleigh distribution within the realm of reliability sampling plans, emphasizing the utilization of hybrid censoring for life checks and median lifetime evaluations. This approach is leveraged to formulate reliability single sampling plans applicable to the Exponential Rayleigh distribution. The article utilizes binomial probabilities to compute the parameters of these sampling plans, aiming to strike a balance between protecting the interests of both the producer and the consumer while minimizing producer risks. The study involves calculating the specified median lifetime and determining design parameters like sample size and acceptance thresholds to meet predefined quality standards. The flexibility of the Exponential Rayleigh distribution in analyzing various types of lifetime data is highlighted, owing to its scale and shape parameters. To illustrate the concepts related to sampling strategies, a numerical example is provided in the sampling strategies section of the article.

Keywords: Reliability Sampling, Median life-time, Hybrid Censoring, Exponential-Rayleigh Distribution.

I. Introduction

Based on the examination of a sample of goods, sampling inspection plans are used to determine the appropriateness of batches that contain finished products. In reliability sampling plans, the lifespan of the tested items is a crucial element when determining the outcome of the batch after the testing of life. As a result, it may be appropriate to conclude a life test by setting a time limit and counting the number of failures that occur before the time limit. This is because the length of inspections may be a significant constraint. In manufacturing industries, the reliability sampling plan serves as a statistical tool for determining the allocation of lots by information gathered through a life check. This method requires a greater amount of sampling cost and inspection time compared to regular sampling plans. To make the inspection cost-effective, censoring schemes, such as Time censoring (Type-I), Product censoring (Type-II), and hybrid censoring, are used frequently throughout the life test. It is appropriate to develop reliability sampling plans with censoring methods when inspection time is constrained and inspection costs are minimal.

The two main distributions in life testing and reliability theory are the Exponential and Rayleigh distributions. They possess important structural properties and mathematical flexibility. One of the fundamental distributions in statistics theory and application is the exponential distribution. It has several important statistical characteristics, but its absence of memory property best describes it. But it shows excellent tractability in mathematics. As a result, the theory and uses of the exponential distribution are extensively covered in the literature [1]. When studying any lifespan data or skewed data, the three-parameter gamma and three-parameter Weibull distributions are frequently used. Both distributions have several favorable characteristics and intriguing physical explanations. Both have quite an amount of versatility for examining various forms of lifetime data because of the scale and shape factors [2]. A two-dimensional random vector of normal variables which has independent, identically distributed coordinates with mean zero is the basis of the Rayleigh distribution, which bears Lord Rayleigh's name. Numerous scenarios where the magnitudes of normal variables are crucial can be addressed using this distribution. A function that appears in the Maximum Likelihood equation is approximated using a hyperbolic approximation rather than a linear approximation in a Modified Maximum Likelihood Estimate of the scale parameter of the Rayleigh distribution [3]. Since they reduce the amount of time and resources needed for testing, these sample programs are very helpful to practitioners, the ability, at various phases of the experiment, to exclude functional test specimens from further testing [4]. Acceptance sampling, also known as sampling inspection is a crucial quality control technique that outlines the policies and steps for deciding whether to accept or reject a batch of goods based on the examination of one or more samples. Consideration is given to the Burr (XII) distribution's application in the reliability sampling plan. Utilizing a set of simulated observations from the Burr (XII) distribution, the evaluation of such plan was discussed [5]. In the industrial sector, reliability sampling plans are used to make disposition decisions for batches based on product life testing. These plans are created while taking into account pertinent probability distributions for the lifespans of the tested products [6]. A new single sampling plan based on ranked data scheme for generalized exponential distribution using median ranked set sampling [7].

The objective of this research is to establish dependable sampling plans based on exponential Rayleigh distribution employing a hybrid censoring scheme that corresponds to producer's and consumer's risk levels. A Lifetime of products follows a specific behavior that is described by a probability distribution. Estimation and inferential part of the developed theory of statistics is the key interest of the researcher and this is fulfilled with the help of these distributions [8]. According to the criteria of the exponential Rayleigh distribution, the study produces the Operating Characteristic (OC) function of the Reliability Single Sampling Plan (RSSP) in part 2. In part 3, it is explained how to create and use the sampling plans. Moreover, part 4 discusses the development of tables that provide optimal sampling plans for certain situations. An example is given to illustrate the selection of a sampling plan. Part 5 summarizes the outcomes of the study.

II. The Theoretical Perspective on the Rayleigh Distribution

The Rayleigh distribution is a continuous probability distribution widely used in probability theory and statistics, particularly for random variables with non-negative values. It has a connection with the chi distribution, specifically when having two degrees of freedom, albeit involving rescaling. This distribution is named after Lord Rayleigh. It frequently appears when analyzing the overall magnitude of a vector in a plane in relation to its directional components. For instance, in the two-dimensional analysis of wind velocity, the Rayleigh distribution naturally emerges when each component has zero mean, equal variance, and follows a normal distribution. Another scenario where the Rayleigh distribution is relevant is in the context of random complex numbers. When real and imaginary components are independently and identically distributed as Gaussian with equal variance and zero mean, the absolute value of the complex number follows a Rayleigh distribution.

In the field of Magnetic Resonance Imaging (MRI), Rayleigh distribution is applied. MRI images are often interpreted as magnitude images, although they are recorded as complex images. Consequently, the background data in MRI images follows a Rayleigh distribution, allowing for the estimation of noise variance in MRI images using this method. Furthermore, the Rayleigh distribution has found application in the field of nutrition. It has been employed to establish connections between dietary nutrient levels and the physiological responses of both humans and animals. This approach represents a method for computing nutritional response relationships through the utilization of Rayleigh distribution parameters."

III. Operating Characteristics of RSSPs under Exponential Rayleigh distribution

One technique is known as a single sampling plan for reliability to make decisions about submitted lots by testing randomly selected items from the lot. Mean life is used as a quality metric to calculate the probability of acceptance to determine design parameters like sample size 'n' and acceptance number 'c' [9]. This plan is characterized by four parameters (N, n, c, t), which include the lot size (N), sample size (n), acceptance number (c), and test termination time (t). The implementation of the sampling plan involves using these parameters to make decisions about the lot. Choose a random selection of n products from the submitted lot of size N.

(1) The supplied lot of size N should be randomly selected to yield a set of n products.
(2) Execute a life test on the chosen items with t as the test termination time. Count the number of things that failed, X=x.
(3) If X>C or time t, whichever occurs first, the life test should be terminated.
(4) Accept the lot, if x≤ c at time t; reject the lot if x>c either at time t or earlier.

Let T be the product's lifespan; it will be distributed using an exponential Rayleigh distribution with a probability density function (PDF)

$$f(x) = \lambda\beta x e^{\frac{\beta}{2}x^2} . e^{-\lambda(e^{\frac{\beta}{2}x^2-1)}} \qquad x\varepsilon R;\ \lambda,\beta > 0 \tag{1}$$

The specifications for the scale and shape are indicated here by λ and β. Following are the formulas for the exponential Rayleigh distribution's cumulative distribution function.

$$F\,(x) = 1 - \lambda e^{-\lambda(e^{\frac{\beta}{2}x^2-1)}} \qquad x\varepsilon R;\ \lambda,\beta > 0 \tag{2}$$

With median respectively,

$$m=\sqrt{\frac{2}{\beta} log\ (1+(\frac{-log\ (1-\mu)}{\lambda}))}$$

Estimate to a parameter β respectively

$$\beta=\frac{2}{m^2} log\ (\frac{1-log\ (\frac{1}{2})}{\lambda})$$

From each value of 1/m, the lot fraction non-conforming, p may be computed.

$$F(X)=F\left(\frac{1}{m}\right)=p$$

Utilizing their OC functions, a sampling plan's effectiveness can be evaluated. A sample plan's OC function is defined by

$$P_a=P\ (x\leq c)=\sum_{x=0}^{c} P\ (X=x)$$

It is reasonable that the probability distribution for X follows a hyper geometric distribution. The probability distribution of X can be assumed approximately as hyper geometric distribution. When N is large, the sampling distribution of X can be approximated by the Binomial (n, p) distribution [10]. Under these circumstances, here, it is proposed that

$$P_a\ (p)=\sum_{x=0}^{c} nc_x\, p^x q^{n-x}$$

IV. Plan Parameter determination under the conditions of the Exponential Rayleigh Distribution

Utilizing the OC function stipulated by the Binomial probability distribution, the best reliability single sampling plans are identified under the circumstances of the ER (λ, θ) distribution. A modified maximum likelihood estimate for Rayleigh distribution using hyperbolic approximation [11]. A sample strategy is typically developed so that it simultaneously protects the manufacturer and the customer. By designating two points on the OC curve, namely (p_1, 1-α) and (p_2, β), the protection of the producer and the customer is guaranteed. In this case, p_1 stands for the acceptable quality level, for producer risk, p_2 for restricting quality level, and for consumer risk. It is possible to determine an ideal RSSP for points meeting the following criteria.

$$P_a(p_1)\geq 1-\alpha$$

and

$$P_a(p_2)\leq \beta$$

These conditions may be written as

$$\sum_{x=0}^{c} nc_x\, {p_1}^x {q_1}^{n-x} \geq 1-\alpha \quad (3)$$

and

$$\sum_{x=0}^{c} nc_x\, {p_2}^x {q_2}^{n-x} \leq \beta \quad (4)$$

To find the best values of n and c subject to (3) and (4), various techniques may be used. Finding the plan parameters involve using the iterative process outlined below. Therefore, the ideal values of the plan parameters n and c for given, λ, t, m_1, m_2, α, β, and may be found as follows:.

(1) When m_1>m_2 with the required values of m_1 and m_2, calculate
$\beta_1 = \frac{2}{m_1^2} \log\left(\frac{1-\log\left(\frac{1}{2}\right)}{\lambda}\right)$ and $\beta_2 = \frac{2}{m_2^2} \log\left(\frac{1-\log\left(\frac{1}{2}\right)}{\lambda}\right)$

(2) Corresponding to t, β_1 and β_2, determine $p_1=F_T(1/m_1)$ and $p_2=F_T(1/m_2)$

(3) Set c=0

(4) Find the largest n, say n_L, such that $P_a(p_1) \geq 1-\alpha$

(5) Find the smallest n, say n_S, such that $P_a(p_2) \leq \beta$

(6) If $n_S \leq n_L$, then the optimum plan is (n_S, c); otherwise increase c by 1.

(7) Repeat Steps 4 through 6 until optimum values of n and c are obtained.

By the hybrid censoring systems covered in part 2 and after figuring out n and c, a submitted lot may undergo sample inspection.

V. Construction of Tables

Binomial probabilities are used to calculate the values of n and c for the best reliability sampling plans for various combinations of λ, t, m_1, m_2, α, and β. Plans for acceptance sampling from exponential populations that use the lifetime-performance index both with and without censoring [12]. Both the producer's risk and the consumer's risk are taken into account at two distinct levels, such as α=0.05, 0.05 and β=0.05, 0.10 respectively. The producer's expectations for the mean lifetime of the products are considered as m_1=6000, 7000, 8000, 9000, and 10000 hours respectively. Assumed values for the shape parameter λ and the test termination times t are 300, 450, and 600 hours and λ=1 correspondingly. The consumer's projected mean product lifespan is taken as m_2= 1000, 1500, 2000, 2500, 3000, 3500, and 4000 hours respectively. Tables 1 through Table 3 give the n and c values for the best reliability sampling strategies. Each cell entry (n, c) in every table reflects the ideal value of the pair (n, c) that corresponds to the given values of λ, t, m_1, m_2, α, and β. choosing a plan from these for certain requirements is illustrated in the following example.

Illustration

Let ER (1, β) is distributing the lifetime of the products that have been submitted for inspection. The average lifespan of products that live up to producer and customer expectations is, respectively, m_1=6000 hours and m_2=4000 hours. Let's say the quality inspector instructs the life test to be censored at t=300 hours. The values of the limiting quality level and the acceptable quality level can therefore be calculated as p_1=0.0013 and p_2=0.0029, respectively. The plan parameters can be calculated using the binomial probabilities from Table 1 as n=8200 and c=16 if the producer's risk and the consumer's risk are α=0.05 and β=0.05, respectively.

Now, the inspection of the lot-by-lot sampling based on the life test can be done as follows: The submitted lot may have up to 8200 products randomly chosen as a sample. All of the sampled goods are eligible for life testing. The life test may be stopped if there have been 16 failures or fewer after 300 hours. The lot might be taken. However, if the seventeenth failure happens before t=300 hours, the life test should be stopped. The lot could be disregarded.

Table 1: *Parameters of RSSPs under the conditions of ER (β, λ=1) Distribution with α=0.05, λ=1 and t=300 hours.*

t=300, λ=1		m_1	6000	7000	8000	9000	10000
		t/m_1	0.05	0.0428	0.0375	0.0333	0.03
m_2	t/m_2	P1 P2	0.0013	0.0009	0.0007	0.0005	0.0004
1000	0.3	0.0473	(81,1) (99,1)	(48,0) (99,1)	(48,0) (62,0)	(48,0) (62,0)	(48,0) (62,0)
1500	0.2	0.0210	(184,1) (224,1)	(184,1) (224,1)	(184,1) (224,1)	(184,1) (224,1)	(184,1) (224,1)
2000	0.15	0.0118	(448,2) (530,2)	(327,1) (530,2)	(327,1) (399,1)	(327,1) (399,1)	(327,1) (399,1)
2500	0.12	0.0075	(880,3) (1021,3)	(701,2) (829,2)	(701,2) (829,2)	(512,1) (829,2)	(512,1) (624,1)
3000	0.1	0.0052	(1760,5) (2246,6)	(1267,3) (1736,4)	(1010,2) (1471,3)	(1010,2) (1194,2)	(738,1) (1194,2)
3500	0.0857	0.0038	(3357,8) (4057,9)	(2396,5) (3059,6)	(1725,3) (2364,4)	(1375,2) (2002,3)	(1375,2) (1626,2)
4000	0.075	0.0029	(6398,13) (8200,16)	(3972,7) (5299,9)	(3130,5) (3996,6)	(2254,3) (3088,4)	(2254,3) (2616,3)

In each cell, the first pair is the value of (n, c) corresponding to (α=0.05, β=0.10) and the Second pair corresponding to (α=0.05, β=0.05).

Table 2: *Parameters of RSSPs under the conditions of ER (β, λ=1) Distribution with α=0.05, λ=1 and t=450 hours.*

t=450, λ=1		m_1	6000	7000	8000	9000	10000
		t/m_1	0.075	0.0642	0.0562	0.05	0.045
m_2	t/m_2	P1 P2	0.0029	0.0021	0.0016	0.0013	0.0010
1000	0.45	0.1064	(36,1) (43,1)	(21,0) (43,1)	(21,0) (27,0)	(21,0) (27,0)	(21,0) (27,0)
1500	0.3	0.0473	(81,1) (99,1)	(81,1) (99,1)	(81,1) (99,1)	(81,1) (99,1)	(48,0) (99,1)
2000	0.225	0.0266	(199,2) (235,2)	(145,1) (235,2)	(145,1) (177,1)	(145,1) (177,1)	(145,1) (177,1)
2500	0.18	0.0170	(390,3) (453,3)	(311,2) (367,2)	(311,2) (367,2)	(227,1) (367,2)	(227,1) (277,1)
3000	0.15	0.0118	(781,5) (997,6)	(563,3) (770,4)	(448,2) (653,3)	(448,2) (530,2)	(327,1) (530,2)
3500	0.1285	0.0087	(1491,8) (1801,9)	(1064,5) (1358,6)	(766,3) (1049,4)	(610,2) (889,3)	(610,2) (722,2)
4000	0.1125	0.0066	(2842,13) (3643,16)	(1764,7) (2354,9)	(1390,5) (1774,6)	(1001,3) (1371,4)	(1001,3) (1162,3)

In each cell, the first pair is the value of (n, c) corresponding to (α=0.05, β=0.10) and the Second pair corresponding to (α=0.05, β=0.05).

Table3: *Parameters of RSSPs under the conditions of ER (β, λ=1) Distribution with α=0.05, λ=1 and t=600 hours.*

t=600, λ=1		m_1	6000	7000	8000	9000	10000
		t/m_1	0.1	0.0857	0.075	0.0667	0.06
m_2	t/m_2	P1 / P2	0.0052	0.0386	0.0029	0.0023	0.0018
1000	0.6	0.1883	(20,1) (24,1)	(12,0) (24,1)	(12,0) (15,0)	(12,0) (15,0)	(12,0) (15,0)
1500	0.4	0.0841	(45,1) (55,1)	(45,1) (55,1)	(45,1) (55,1)	(45,1) (55,1)	(27,0) (55,1)
2000	0.3	0.0473	(111,2) (131,2)	(81,1) (131,2)	(81,1) (99,1)	(81,1) (99,1)	(81,1) (99,1)
2500	0.24	0.0303	(219,3) (254,3)	(174,2) (206,2)	(174,2) (206,2)	(127,1) (206,2)	(127,1) (155,1)
3000	0.2	0.0210	(439,5) (497,5)	(316,3) (433,4)	(252,2) (366,3)	(252,2) (297,2)	(184,1) (297,2)
3500	0.1714	0.0154	(838,8) (1012,9)	(598,5) (763,6)	(430,3) (589,4)	(343,2) (499,3)	(343,2) (405,2)
4000	0.15	0.0118	(1598,13) (2047,16)	(992,7) (1323,9)	(781,5) (997,6)	(563,3) (770,4)	(563,3) (653,3)

In each cell, the first pair is the value of (n, c) corresponding to (α=0.05, β=0.10) and the second pair corresponding to (α=0.05, β=0.05).

VI. Conclusion

In this article, a new sampling distribution is introduced for testing product quality when conducting acceptance sampling for life tests that follow the Exponential Rayleigh distribution. The paper also outlines reliability sampling plans for conducting life tests through hybrid censoring, specifically for products that follow the Exponential Rayleigh distribution. These plans have been designed to protect the interests of both the producer and consumer and the use of hybrid censoring helps to reduce the amount of time required for implementation. The article also includes tables that provide optimal plans for certain specified strengths.

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