EJAZ DISTRIBUTION A NEW TWO PARAMETRIC DISTRIBUTION FOR MODELLING DATA

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Abstract

This paper introduces a novel probability distribution known as the Ejaz distribution (ED), which is characterized by two parameters. The study offers a comprehensive analysis of this distribution, including an examination of key properties such as moments, moment-generating functions, order statistics, and reliability functions. Additionally, the paper explores the graphical representation of essential functions like the probability density function, cumulative distribution function, and hazard rate function, enhancing our visual understanding of their behavior. The distribution's parameters are estimated using the widely accepted method of maximum likelihood estimation. Through real-world examples, the paper highlights the practical applicability of the Ejaz distribution, demonstrating its performance and relevance in diverse scenarios.

Keywords: Moments, Reliability analysis, oder statistics, maximum likelihood estimation, Data analysis.

1. Introduction

In numerous fields such as economics, engineering, finance, insurance, demography, biology, and environmental and medical sciences, various statistical distributions have been widely utilized to describe and predict observed phenomena. However, the data encountered in these disciplines often exhibit complex behaviors and diverse shapes, characterized by varying degrees of skewness and kurtosis. Consequently, many of the conventional standard distributions have limitations when it comes to accurately representing these data. As a result, the application of these classical distributions may not yield satisfactory fits. Hence, numerous researchers have endeavored to enhance these established classical distributions to achieve greater adaptability in modeling data from a wide array of academic domains. In recent times, researchers have been actively engaged in the development of new families of continuous probability distributions known for their remarkable flexibility. This innovation involves the incorporation of extra parameters into the foundational distributions. These novel families of lifetime distributions have gained prominence, particularly in fields like economics, engineering, finance, insurance, demography, biology, and environmental and medical sciences, where data frequently exhibit intricate behaviors, diverse shapes, skewness, and kurtosis variations. The integration of additional parameters empowers these distributions to offer a more adaptable and versatile framework for modeling complex data. By doing so, they overcome the limitations of traditional standard distributions, enabling researchers to better capture and predict real-world phenomena with precision. Thus, these newly proposed lifetime distribution families have become invaluable tools for data analysis and modeling in a wide range of disciplines. In recent years, researchers have introduced modifications to enhance the adaptability of conventional distributions when interpreting diverse datasets. These changes aim to improve the accuracy of data analysis across different fields by tailoring distribution characteristics to specific dataset requirements. For reference Aijaz et al. [1-3], Terna Godfrey Ieren [18], Albert Luguterah [4], Topp-Leone Rayleigh distribution by Fatoki olayode [9],Amal S. Hassan et al. [5], Frank Gomes-silva et al. [10], Brito et al.[7], Morad Alizadeh et al. [15], Shanker et al. [17],Lindley [14], Flaih, A et al. [11], Akhter, Z et al. [6], G.M. Corderio et al. [13]. The formulated distribution is versatile and suitable for modeling various data types, including left-skewed, right-skewed, and symmetric datasets. This versatility is evident when examining probability density function (PDF) plots, as they demonstrate that this distribution can offer the most optimal fit for complex datasets. Whether the data exhibits a pronounced tail on the left, a tail on the right, or a balanced symmetry, this distribution's flexibility allows it to adapt and provide a robust representation. Its ability to accommodate a wide range of data patterns makes it a valuable tool for statistical modeling and analysis, ensuring accurate and meaningful insights across diverse data scenarios.

Let us suposse $F(x;\alpha,\beta)$ be cdf of a random variable x with α,β parameters, then the cumulative distribution function of Ejaz distribution is described as.

$$F(x;\alpha,\beta) = 1 - e^{-\alpha(e^{\beta x}-1)}\left(2 - e^{-\alpha(e^{\beta x}-1)}\right) \quad ; \quad x > 0, \alpha, \beta > 0 \tag{1}$$

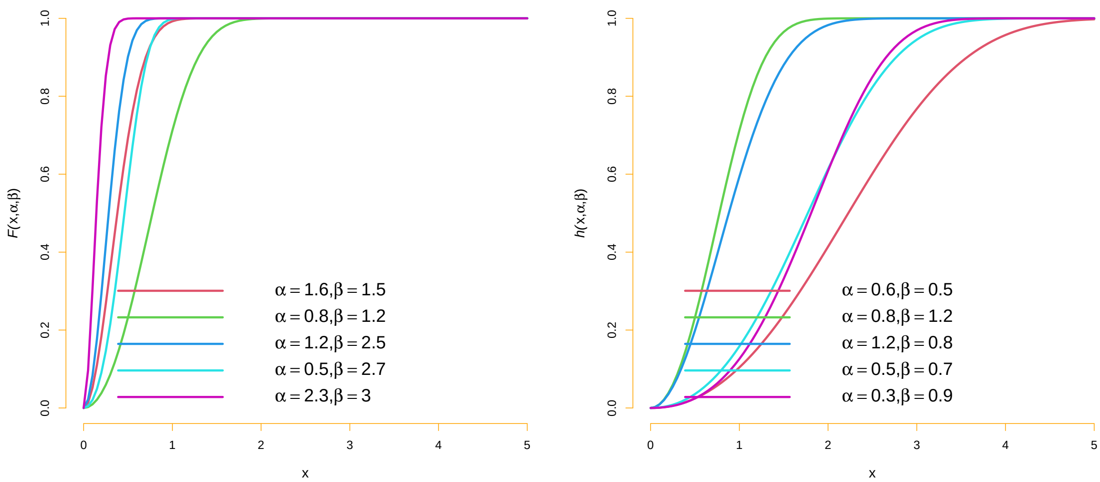

Figure 1: *The cdf plots of Ejaz distribution for distinct parameter values.*

The corresponding probability density function is described as

$$f(x;\alpha,\beta) = 2\alpha\beta e^{-\alpha(e^{\beta x}-1)+\beta x}\left(1 - e^{-\alpha(e^{\beta x}-1)}\right) \quad ; \quad x > 0, \alpha, \beta > 0 \tag{2}$$

Here we examine the validity of pdf

$$\int_0^\infty f(x;\alpha,\beta)dx = 1$$
$$= \int_0^\infty 2\alpha\beta e^{-\alpha(e^{\beta x}-1)+\beta x}\left(1-e^{-\alpha(e^{\beta x}-1)}\right)dx$$

On substituting $e^{\beta x}-1=z$, so that $o<z<\infty$ we have

$$=2\alpha\int_0^\infty e^{-\alpha z}\left(1-e^{-\alpha z}\right)dz$$
$$=2\alpha\left(\frac{1}{2\alpha}\right)=1$$

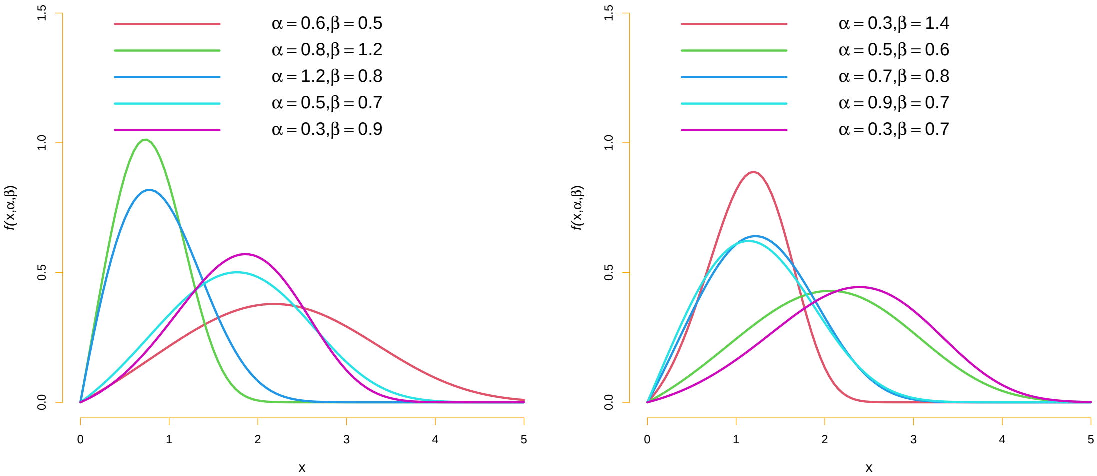

Figure 2: *The pdf plots of Ejaz distribution for distinct parameter values.*

2. Moments

To understand and characterize the properties of the formulated distribution, we perform a moment analysis about the origin. This analysis allows us to derive essential statistical measures such as skewness, kurtosis, and other relevant properties. By examining these moments, we gain valuable insights into the distribution's shape, central tendency, and the presence of any outliers or heavy tails, aiding in its comprehensive statistical characterization and interpretation.
Suppose x denotes a random variable follows Ejaz distribution. Then k^{th} moment about origin denoted as $\mu_k^{'}$ can be obtained as

$$\mu_k^{'} = E(x^k) = \int_0^\infty x^k f(x;\alpha\beta)dx$$
$$=2\alpha\beta\int_0^\infty x^k e^{-\alpha(e^{\beta x}-1)+\beta x}\left(1-e^{-\alpha(e^{\beta x}-1)}\right)dx$$

Making substitution $e^{\beta x}$=z so that $1<z<\infty$, we have

$$\mu_k^{'} = \frac{2\alpha}{\beta^k}\left\{e^{\alpha}\int_1^\infty (log(z))^k e^{-\alpha z}dz - e^{2\alpha}\int_1^\infty (log(z))^k e^{-2\alpha z}dz\right\}$$

Applying integro-Exponential function by Milgram [16].

$$E_s^j(\lambda) = \frac{1}{j+1}\int_1^\infty (log(t))^j t^{-s}e^{-\lambda t}dt$$

$$\mu_k' = \frac{2\alpha e^{\alpha}\Gamma(k+1)}{\beta^k}\left(E_0^k(\alpha) - e^{\alpha}E_0^k(2\alpha)\right)$$

Substituting $k = 1, 2, 3, 4$ we obtain first four moments of the distribution about origin. The variance σ^2, skewness $\sqrt{\beta_1}$, kurtosis β_2 , coefficient of variation (C.V) and index of dispersion γ.

Let x be a random variable follows Ejaz distribution. Then the moment generating function of the distribution denoted by $M_X(t)$ is given by

$$\begin{aligned} M_X(t) =& E(e^{tx}) = \int_0^{\infty} e^{tx} f(x;\alpha,\beta)dx \\ =& \sum_{k=0}^{\infty} \frac{t^k}{k!} x^k f(x;\alpha,\beta) = \sum_{k=0}^{\infty} \frac{t^k}{k!} E(x^k) \\ =& \sum_{k=0}^{\infty} \frac{t^k}{k!} \frac{2\alpha e^{\alpha}\Gamma(k+1)}{\beta^k}\left(E_0^k(\alpha) - e^{\alpha}E_0^k(2\alpha)\right) \end{aligned}$$

3. Reliability indicators

This section is focused on researching and developing distinct ageing indicators for the formulated distribution.

3.1. Survival function

Let us suppose x be a continuous random variable with cdf $F(x)$.Then its Survival function which is also known as reliability function is stated as

$$S(x) = p_r(X > x) = \int_x^{\infty} f(x)dx = 1 - F(x)$$

Therefore, the survival function for Ejaz distribution is given by

$$\begin{aligned} S(x;\alpha,\beta) =& 1 - F(x;\alpha,\beta) \\ =& e^{-\alpha(e^{\beta x}-1)}\left(2 - e^{-\alpha(e^{\beta x}-1)}\right) \end{aligned} \tag{3}$$

3.2. Hazard rate function

The hazard rate function of a random variable x is denoted as

$$h(x;\alpha,\beta) = \frac{f(x;\alpha,\beta)}{S(x;\alpha,\beta)} \tag{4}$$

using equation (1) and (3) in equation (4), then the hazard rate function of Ejaz distribution is given as

$$h(x;\alpha,\beta) = \frac{2\alpha\beta e^{\beta x}\left(1 - e^{-\alpha(e^{\beta x}-1)}\right)}{\left(2 - e^{-\alpha(e^{\beta x}-1)}\right)}$$

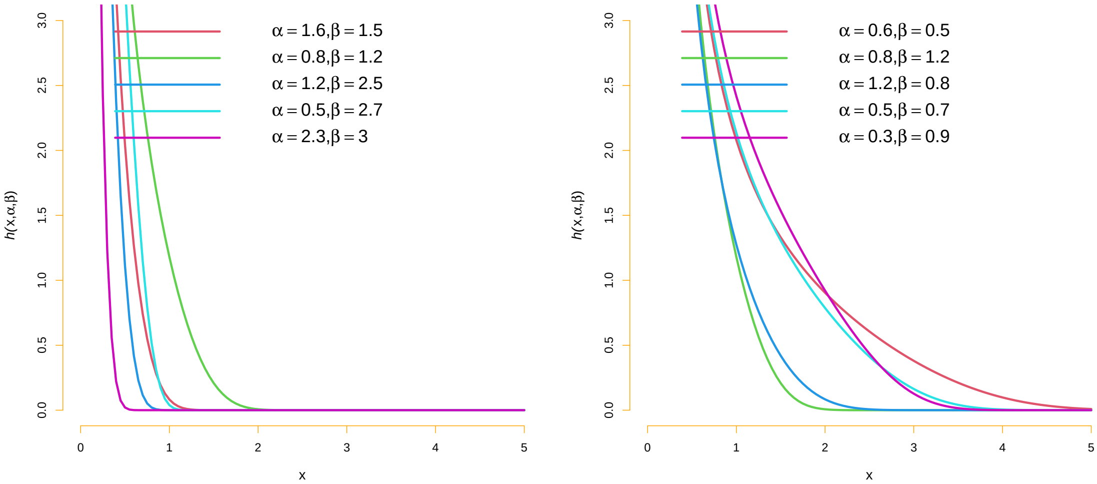

Figure 3: *The hrf plots of Ejaz distribution for distinct parameter values.*

3.3. Cumulative hazard rate function

The cumulative hazard rate function of a random variable x is given as

$$H(x,\alpha,\beta) = -\ln[\bar{F}(x;\alpha,\beta)] \tag{5}$$

using equation (1) in equation (5), then we obtain cumulative hazard rate function of Ejaz distribution as

$$H(x;\alpha,\beta) = \alpha\left(e^{\beta x}-1\right) - \log\left(2-e^{-\alpha(e^{\beta x}-1)}\right)$$

3.4. Reverse Hazard rate function

The reverse hazard rate function of random variable x is described as

$$r(x;\alpha,\beta) = \frac{f(x;\alpha,\beta)}{F(x;\alpha,\beta)} \tag{6}$$

using equation (1) and (2) in equation (6), then the reverse hazard rate function of Ejaz distribution is given as

$$r(x;\alpha,\beta) = \frac{2\alpha\beta e^{-\alpha(e^{\beta x}-1)+\beta x}\left(1-e^{-\alpha(e^{\beta x}-1)}\right)}{1-e^{-\alpha(e^{\beta x}-1)}\left(2-e^{-\alpha(e^{\beta x}-1)}\right)}$$

4. Order Statistics

Let us suppose $x_1, x_2, ..., x_n$ be random samples of size n from Ejaz distribution with pdf $f(x)$ and cdf F(x). Then the probability density function of the k^{th} order statistics is given as

$$f_x(k) = \frac{n!}{(k-1)!(n-1)!} f(x)\,[F(x)]^{k-1}\,[1-F(x)]^{n-k} \tag{7}$$

Using equation (1) and (2) in equation (7), we have

$$f_x(k) = \frac{n!}{(k-1)!(n-1)!} 2\alpha\beta e^{-\alpha(e^{\beta x}-1)+\beta x}\left(1-e^{-\alpha(e^{\beta x}-1)}\right)\left[1-e^{-\alpha(e^{\beta x}-1)}\left(2-e^{-\alpha(e^{\beta x}-1)}\right)\right]^{k-1}$$
$$\times\left[e^{-\alpha(e^{\beta x}-1)}\left(2-e^{-\alpha(e^{\beta x}-1)}\right)\right]^{n-k}$$

The pdf of the first order statistics X_1 of Ejaz distribution is given by

$$f_x(1) = 2n\alpha\beta e^{-\alpha(e^{\beta x}-1)+\beta x}\left(1-e^{-\alpha(e^{\beta x}-1)}\right)\left[e^{-\alpha(e^{\beta x}-1)}\left(2-e^{-\alpha(e^{\beta x}-1)}\right)\right]^{n-1}$$

The pdf of the first order statistics X_n of Ejaz distribution is given by

$$f_x(n) = 2n\alpha\beta e^{-\alpha(e^{\beta x}-1)+\beta x}\left(1-e^{-\alpha(e^{\beta x}-1)}\right)\left[1-e^{-\alpha(e^{\beta x}-1)}\left(2-e^{-\alpha(e^{\beta x}-1)}\right)\right]^{n-1}$$

5. Maximum Likelihood Estimation

Let the random samples $x_1, x_2, x_3, ..., x_n$ are drawn from Ejaz distribution. The likelihood function of n observations is given as

$$L = \prod_{i=1}^{n}\left(2\alpha\beta e^{-\alpha(e^{\beta x}-1)+\beta x}\left(1-e^{-\alpha(e^{\beta x}-1)}\right)\right)$$

The log-likelihood function is given as

$$l = nlog(2) + nlog(\alpha) + nlog(\beta) - \alpha\left(e^{\beta x}-1\right) + \beta x + \sum_{i=n}^{n} log\left(1-e^{-\alpha\left(e^{\beta x_i}-1\right)}\right) \tag{8}$$

The partial derivatives of the log-likelihood function with respect to α and β are given as

$$\frac{\partial l}{\partial \alpha} = \frac{1}{n} - e^{\beta x_i} + 1 + \sum_{i=1}^{n}\frac{\left(e^{\beta x_i}-1\right)e^{-\alpha\left(e^{\beta x_i}-1\right)}}{1-e^{-\alpha\left(e^{\beta x_i}-1\right)}} \tag{9}$$

$$\frac{\partial l}{\partial \beta} = \frac{n}{\beta} - \alpha x_i e^{\beta x_i} + x_i - \alpha\sum_{i=1}^{n}\frac{x_i e^{\beta x_i} e^{-\alpha\left(e^{\beta x_i}-1\right)}}{1-e^{-\alpha\left(e^{\beta x_i}-1\right)}} \tag{10}$$

For interval estimation and hypothesis tests on the model parameters, an information matrix is required. The 2 by 2 observed matrix is

$$I(\psi) = \frac{-1}{n}\begin{bmatrix} E\left(\frac{\partial^2 logl}{\partial\alpha^2}\right) & E\left(\frac{\partial^2 logl}{\partial\alpha\partial\beta}\right) \\ E\left(\frac{\partial^2 logl}{\partial\beta\partial\alpha}\right) & E\left(\frac{\partial^2 logl}{\partial\beta^2}\right)\end{bmatrix}$$

The elements of above information matrix can be obtain by differentiating equations (9) and (10) again partially. Under standard regularity conditions when $n \to \infty$ the distribution of $\hat{\psi}$ can be approximated by a multivariate normal $N(0, I(\hat{\psi})^{-1})$ distribution to construct approximate confidence interval for the parameters. Hence the approximate $100(1-\zeta)\%$ confidence interval for α and β are respectively given by

$$\hat{\alpha} \pm Z_{\frac{\zeta}{2}}\sqrt{I_{\alpha\alpha}^{-1}(\hat{\psi})} and \quad \hat{\beta} \pm Z_{\frac{\zeta}{2}}\sqrt{I_{\beta\beta}^{-1}(\hat{\psi})}$$

6. Simulation Analysis

The bias, variance and MSE were all addressed to simulation analysis. From Ejaz distribution taking N=500 with samples of size n=25, 50, 150, 200, 250 and 400. For various parameter combinations, simulation results have been achieved. The bias, variance and MSE values are calculated and presented in table 1 and 2. As the sample size increases, this becomes apparent that these estimates are relatively consistent and approximate the actual values of parameters. Interestingly, with all parameter combinations, the bias and MSE reduce as the sample size increases.

Table 1: *Bias, variance and their corresponding MSE's for different parameter values* $\alpha = 1.2, \beta = 0.8$

Sample size	Parameters	Bias	Variance	MSE
25	α	0.01130	0.00432	0.01473
	β	0.01251	0.00154	0.00165
50	α	0.00314	0.00413	0.00514
	β	0.00103	0.00071	0.00061
150	α	-0.00021	0.00301	0.00201
	β	0.00406	0.00049	0.00051
200	α	-0.00201	0.00156	0.00205
	β	0.00237	0.00027	0.00028
250	α	0.00120	0.00206	0.00203
	β	0.00255	0.00025	0.00022
300	α	0.00177	0.00203	0.00201
	β	0.00066	0.00021	0.00020

Table 2: *Bias, variance and their corresponding MSE's for different parameter values* $\alpha = 2.2, \beta = 1.5$

Sample size	Parameters	Bias	Variance	MSE
25	α	0.01230	0.03553	0.03573
	β	0.02003	0.01832	0.01031
50	α	0.01214	0.01105	0.01132
	β	0.01121	0.00506	0.00420
150	α	0.00672	0.00668	0.00607
	β	0.00146	0.00224	0.00216
200	α	0.00265	0.00416	0.00506
	β	0.01076	0.00232	0.00214
250	α	0.0027	0.00360	0.00361
	β	0.00208	0.00145	0.00145
300	α	0.00150	0.00301	0.00211
	β	0.00063	0.00130	0.00130

7. Data Aanalysis

This subsection evaluates a real-world data sets to demonstrate the Ejaz distribution's applicability and effectiveness. The Ejaz distribution (ED) adaptability is determined by comparing its efficacy to the following conventional distributions.
1:- Weibull distribution having pdf

$$f(x;\alpha,\beta) = \alpha\beta x^{\beta-1} e^{-\alpha x^{\beta}}; \quad x > 0, \alpha, \beta > 0$$

2:- Frećhet distribution having pdf

$$f(x;\alpha,\beta) = \alpha\beta x^{-\beta-1} e^{-\alpha x^{-\beta}}; \quad x > 0, \alpha, \beta > 0$$

3:- Inverse Burr distribution having pdf

$$f(x;\alpha,\beta) = \alpha\beta \left(1 - x^{-\alpha}\right)^{-\beta-1}; \quad x > 0, \alpha, \beta > 0$$

4:- Lomax distribution having pdf

$$f(x;\alpha,\beta) = \alpha\beta \left(1 + \alpha x\right)^{-\beta-1}; \quad x > 0, \alpha, \beta > 0$$

5:- Exponentiated Rayleigh distribution having pdf

$$f(x;\alpha,\beta) = 2\alpha\beta x e^{-\alpha x^2}\left(1-e^{-\alpha x^2}\right)^{\beta-1}; \quad x>0,\alpha,\beta>0$$

6:- Lindley distribution having pdf

$$f(x;\alpha) = \frac{\alpha^2}{(1+\alpha)}(1+x)\,e^{-\alpha x}; \quad x>0,\alpha>0$$

7:- Inverse Rayleigh distribution having pdf

$$f(x;\alpha,\beta) = \frac{2\alpha}{x^3}e^{-\alpha x^{-2}}; \quad x>0,\alpha>0$$

To compare the versatility of the explored distribution, we consider the criteria like AIC (Akaike information criterion), CAIC (Consistent Akaike information criterion), BIC (Bayesian information criterion) and HQIC (Hannan-Quinn information criterion). Distribution having lesser AIC, CAIC, BIC and HQIC values is considered better.

$$AIC = -2l + 2p, \qquad AICC = -2l + 2pm/(m-p-1), \qquad BIC = -2l + p(log(m))$$
$$HQIC = -2l + 2plog(log(m)), \qquad K.S = max_{1\le j\le m}\left(F(x_j) - \frac{j-1}{m}, \frac{j}{m} - F(x_j)\right)$$

Where $'l'$ denotes the log-likelihood function,'p'is the number of parameters and'm'is the sample size.

Data set 1: The followig observation are due to Caramanis et al and Mazmumdar and Gaver [12], where they compare the two distinct algorithms called SC16 and P3 for estimating unit capacity factors. The values resulted from the algorith SC16 are 2.01, 6.32, 3.52, 2.15, 5.42, 2.04, 2.77, 2.26, 1.95, 1.00, 2.45, 0.74, 0.98, 1.27, 2.77, 3.68, 1.18, 1.09,1.60, 0.57, 3.33, 0.91, 7.14, 2.08, 3.85, 1.99, 7.76, 2.52, 1.57, 4.67, 4.22, 1.92, 1.59, 4.08, 2.02, 0.84,6.85, 2.18, 2.04, 1.05, 2.91, 1.37, 2.43, 2.28, 3.74, 1.30, 1.59, 1.83, 3.85, 6.30, 4.83, 0.50, 3.40, 2.33,4.25, 3.49, 2.12, 0.83, 0.54, 3.23, 4.50, 0.71, 0.48, 2.30, 7.73.

Data set 2: The followig observation are due to Caramanis et al and Mazmumdar and Gaver [12], where they compare the two distinct algorithms called SC16 and P3 for estimating unit capacity factors. The values resulted from the algorith SC16 are 0.1, 0.33, 0.44, 0.56, 0.59, 0.59, 0.72, 0.74, 0.92, 0.93,0.96, 1, 1, 1.02, 1.05, 1.07, 1.07, 1.08, 1.08, 1.08, 1.09, 1.12, 1.13,1.15, 1.16, 1.2, 1.21, 1.22, 1.22, 1.24, 1.3, 1.34, 1.36, 1.39, 1.44, 1.46,1.53, 1.59, 1.6, 1.63, 1.68, 1.71, 1.72, 1.76, 1.83, 1.95, 1.96, 1.97,2.02, 2.13, 2.15, 2.16, 2.22, 2.3, 2.31, 2.4, 2.45, 2.51, 2.53, 2.54, 2.78,2.93, 3.27, 3.42, 3.47, 3.61, 4.02, 4.32, 4.58, 5.55, 2.54, 0.77.

The ML estimates with corresponding standard errors in parenthesis of the unknown parameters are presented in Table 3 and Table 5. Also the comparison statistics, AIC, BIC, CAIC, HQIC and the goodness-of-fit statistic for the data sets are displayed in Table 4 and Table 6.

It is observed from the findings that ED provides best fit than other competative models based on the measures of statistics, AIC, BIC, AICC, HQIC and K-S statistic. Along with p-values of each model.

Table 3: *The ML Estimates (standard error in parenthesis) for data set 1*

Model	$\hat{\alpha}$	$\hat{\beta}$
ED	3.45342	0.19311
	(1.92236)	(0.08456)
WD	0.16787	1.59666
	(0.04105)	(0.15017)
FD	1.82550	1.42975
	(0.22717)	(0.12938)
IBD	1.79634	2.85966
	(0.15843)	(0.36236)
LXD	0.00769	48.2182
	(0.00464)	(29.232)
ERD	0.07499	0.73015
	(0.01347)	(0.11406)
LD	0.59651	...
	(0.05424)	....
IRD	1.78914	...
	(0.22191)	

Table 4: *Comparison criterion and goodness-of-fit statistics for data set 1*

Model	-2l	AIC	AICC	BIC	HQIC	K.S statistic	p-value
ED	**239.11**	**243.11**	**243.30**	**247.45**	**244.82**	**0.07732**	**0.8319**
WD	240.85	244.85	245.04	249.20	246.56	0.0955	0.5927
FD	250.87	254.87	255.07	259.22	256.59	0.1491	0.1111
IBD	245.36	249.36	249.56	253.71	251.08	0.12373	0.2726
LXD	130.57	265.14	265.33	269.49	266.86	1.00	2.2e-16
ERD	243.000	247.00	247.19	251.34	248.71	0.12333	0.2762
LD	249.59	251.59	251.65	253.77	252.45	0.11653	0.3406
IRD	267.49	269.49	269.56	271.67	270.35	0.27703	9.293e-05

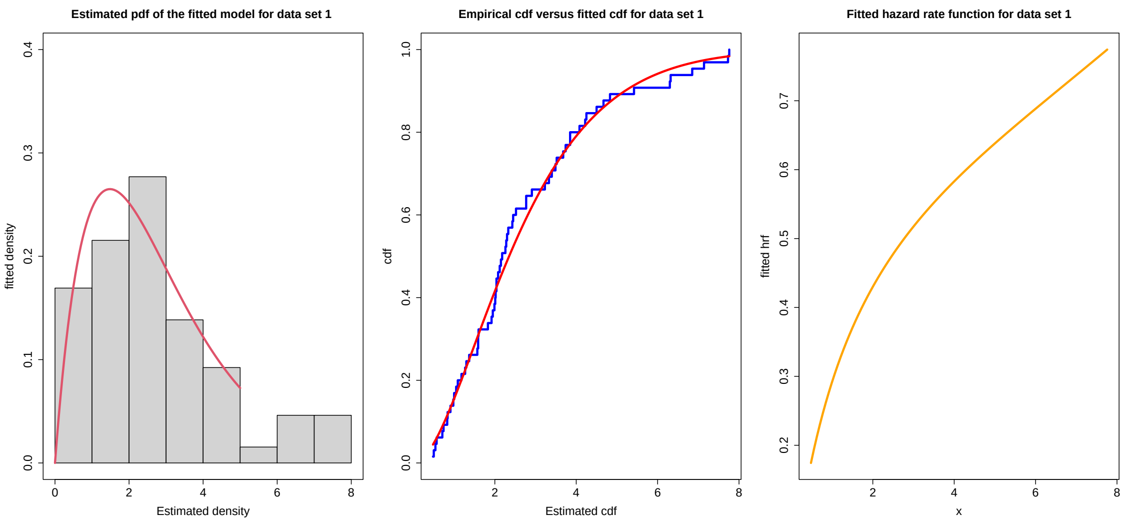

Figure 4: *Fitted pdf, cdf and hrf for data set 1.*

Table 5: *The ML Estimates (standard error in parenthesis) for data set 2*

Model	$\hat{\alpha}$	$\hat{\beta}$
ED	3.45342	0.19311
	(1.92236)	(0.08456)
WD	0.29347	1.796236
	(0.05540)	(0.15662)
FD	1.04750	1.17538
	(0.13022)	(0.08496)
IBD	2.31897	1.85769
	(0.21444)	(0.21925)
LXD	0.00841	68.4375
	(0.00579)	(47.3451)
ERD	0.22565	0.90717
	(0.03649)	(0.14049)
LD	0.87441	....
	(0.07718)	...
IRD	0.45560	...
	(0.05369)	...

Table 6: *Comparison criterion and goodness-of-fit statistics for data set 2*

Model	-2l	AIC	AICC	BIC	HQIC	K.S statistic	p-value
ED	**191.80**	**195.80**	**195.97**	**200.35**	**197.61**	**0.06414**	**0.6053**
WD	192.05	196.05	196.22	200.60	197.86	0.098266	0.4902
FD	234.65	238.65	238.82	243.20	240.46	0.18994	0.01109
IBD	195.21	199.21	199.38	201.02	203.76	0.10925	0.3565
LXD	225.58	229.58	229.75	234.13	231.39	0.28959	1.139e-05
ERD	193.26	197.26	197.44	201.82	199.08	0.10202	0.4419
LD	213.05	215.05	215.10	217.32	215.95	0.2356	0.000671
IRD	323.71	325.71	325.77	327.99	326.61	0.4674	4.352e-14

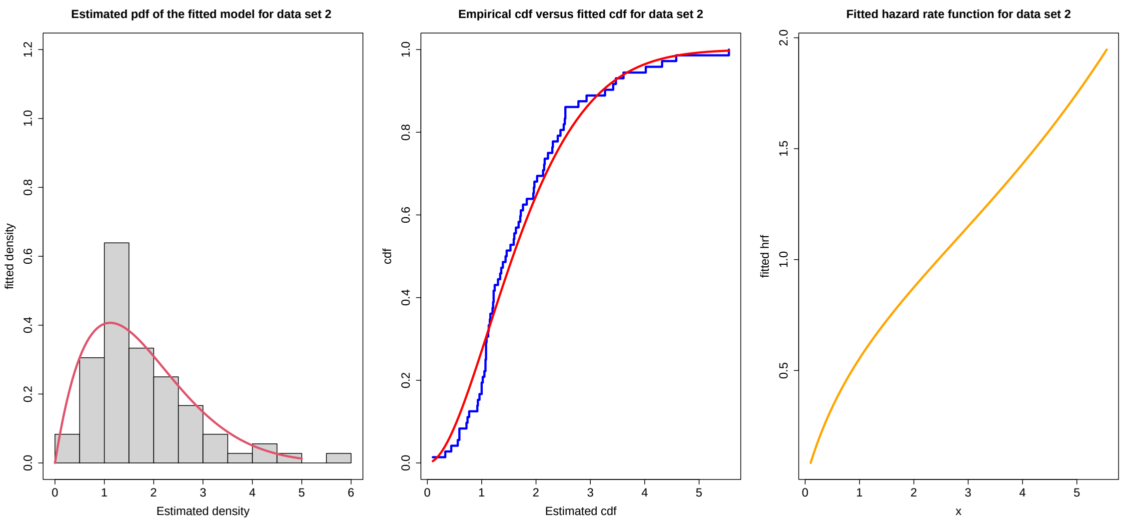

Figure 5: *Fitted pdf, cdf and hrf for data set 2.*

8. Conclusion

In this research paper, we introduce a novel two-parameter lifetime distribution, named the "Ejaz distribution." We delve into various mathematical properties associated with this distribution, including its shape, moments, hazard rate, and order statistics. Furthermore, we discuss the utilization of the maximum likelihood estimation method for estimating the distribution's parameters. To illustrate the practical effectiveness and superiority of the Ejaz distribution in comparison to existing alternatives such as the Weibull, Frećhet, Inverse Burr, Lomax, Exponentiated Rayleigh, Lindley, and inverse Rayleigh distributions, we conduct goodness-of-fit tests employing criteria such as the Akaike Information Criterion (AIC), Consistent Akaike Information Criterion (CAIC), and Bayesian Information Criterion (BIC) on real-life lifetime datasets. Additionally, we perform a simulation analysis, which reveals an intriguing trend: as the sample size increases, there is a reduction in bias and mean squared error (MSE) across all parameter combinations.

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