REGULARITY OF ALTERNATE QUADRA SUBMERGING POLAR FUZZY GRAPH AND ITS APPLICATION

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Abstract

Fuzzy soft sets and graphs are invented to solve uncertain problems in the field of Applied mathematics. It is a general mathematical tool introduced with many parameters to model the vagueness of the changing world. The insight learning of the AQSP fuzzy soft graphs paved the way to discover the extension of the AQSP fuzzy soft graph. In this research article we introduce the Regularity of AQSP fuzzy soft graph with definitions, theorems, properties, and real-life applications. The aim of this invention is mainly to obtain the parametric values in submerging level of confidence [-0.5, 0.5] $\subset$ [-1,1]. The scope of this new AQSP fuzzy soft graph is to solve the imprecise problems in the field of Mathematical Engineering, Bio Mathematics, Economics, Medical Science, Artificial Intelligence and Machine learning. The regularity of AQSP fuzzy soft graph is combined with the concepts of regular, totally regular, and perfectly regular. The application of this new graph is developed for governing of the women safety vehicle network in different spots with membership submerging values. The future extension can be applied in Approximate reasoning, Mathematical psychology, Decision making for medical diagnosis.

Keywords: Regular AQSP fuzzy soft graph, Totally regular, Perfectly regular AQSP fuzzy soft graph, Alternate Quadra Submerging level of confidence.

1. Introduction

The concept of graph theory was introduced by Euler in 1736. He concreted the way to find the solution of Konigsberg bridge problem. In 1965 Zadeh[20] invented Fuzzy set theory as a mathematical fuzzy tool for handling uncertainties like vagueness, ambiguity, and imprecision in linguistic variables. Fuzzy set has resulted as a potential area of interdisciplinary exploration and the fuzzy graph theory is of modern inducement. The first definition of fuzzy graph was determined by Kaufmann[10] in 1973, based on Zadeh's fuzzy relation in 1971. In 1975, Rosenfeld[16] introduced the concept of fuzzy graph. The structure of fuzzy graphs, using fuzzy relations, obtaining contrasts of several graph hypothetical concepts are the masterpiece of Rosenfeld. Operations on fuzzy graphs were exposed by J.N.Moderson[14] and C.S.Peng. A.Nagoorgani[8] and K.Radha[9] invented the concept of regular fuzzy graphs in 2008.

In 1999, D.Molodtsov[12] intended the notion of soft set theory to solve complicated uncertain problems in Applied Mathematics, Engineering and Environmental studies. In 2001, P.K.Maji[11], initiated the concept of fuzzy soft sets. Zou and Xio discussed the application of the fuzzy soft sets in an imprecise scenario. Later, Akram[4] and Nawaz[15] presented new ideas known as fuzzy soft graphs. A.Pouhassani[24] and H.Doostie studied degree, total degree, regularity and total regularity of fuzzy soft graph and its properties. Regular fuzzy soft graphs and its related properties are investigated by B.Akhilandeswari. The concepts of fuzzy bipolar soft sets and bipolar fuzzy soft sets have been introduced by Naz and Shabir. Aslam et al studied some basic operations on bipolar fuzzy soft sets.

In this article, we portray a new mathematical fuzzy graph model AQSP Fuzzy Soft graph for dealing imprecise information by integrating the concepts of fuzzy graph and fuzzy soft graphs. We estimate the regularities of AQSP fuzzy soft graphs and some of their characteristics and properties. Here Regular AQSP fuzzy soft graphs, and totally regular AQSP fuzzy soft graphs and perfectly regular AQSP fuzzy soft graphs are examined. Total degree of an AQSP fuzzy soft graph is designed. Theorems for regular AQSP fuzzy soft graphs and totally regular AQSP fuzzy soft graphs are presented. A necessary condition under which they are equivalent is provided. Some properties of regular AQSP fuzzy soft graphs, perfectly regular AQSP fuzzy soft graphs are reviewed with real life applications.The perception of AQSP fuzzy soft graph membership values with submerging level of confidence is applicable in Machine learning and medical psychology.We explored the AQSP fuzzy soft graph module in Governing of women safety police vehicle network with membership score functions.

2. Preliminaries

2.1. Fuzzy Graph [16]

Let Ũ is a non-empty set. A fuzzy graph is a set of two of functions $G:(\sigma,\mu)$ where σ is a fuzzy subset of Ũ, μ is a symmetric fuzzy relation on σ, where $\sigma: \tilde{U} \rightarrow [0,1]$ and the edge set $\mu: \tilde{U} \times \tilde{U} \rightarrow [0,1]$ such that, $\mu(x,y) \leq \min(\mu(x), \mu(y)) \; \forall x,y \in \tilde{U}$. The underlying crisp graph of fuzzy graph $G:(\sigma,\mu)$ is with the notion $G^*:(\sigma^*,\mu^*)$ where σ^* is denoted as the non-empty set Ũ of vertices and $\mu^* = E \in V \times V$.

2.2. Fuzzy Soft graph [13]

A fuzzy soft graph $G = (G^*, F, K, A)$ is a four tuple such that

1. $G^* = (V, E)$ is a simple graph.
2. A is a non empty set of parameters.
3. (F, A) is a fuzzy soft vertex set V.
4. (K, A) is a fuzzy soft edge set E.
5. $F(a), K(a)$ is a fuzzy soft graph of $G^* \forall\, a \in A$.

Then it satisfies the condition, $K(a)(x,y) \leq F(a)(x) \wedge F(a)(Y) \; \forall a \in A$ and $(x,y) \in V$.

2.3. Fuzzy soft graph degree of a vertex [4]

Let $G = (G^*, F, K, A)$ be a fuzzy soft graph on G^*. The fuzzy soft graph degree of a vertex a is defined as $deg_G(a) = \sum_{e \in A} \sum_{x \neq y} K(e)(x,y) \forall a \in A$ and $(x,y) \in V$.

2.4. Regular Fuzzy soft graph [4]

Let $G = (G^*, F, K, A)$ be a regular fuzzy soft graph if $(F(e), K(e))$ is regular fuzzy graph of degree k for all $e_i \in A$ then G is a k- regular fuzzy soft graph.

2.5. Order of fuzzy soft graph [4]

Let $G_{A,V} = ((A, \sigma,), (A, \mu,))$ be a fuzzy soft graph. Then the order of fuzzy soft graph $G_{A,V} = \sum_{e \in A} \sum_{x \in A} \sigma_e(x)$.

2.6. AQSP Fuzzy Soft Graph [18]

Let $V = ((\sigma_1^P(x), \sigma_1^N(x), \rho_1^P(x), \rho_1^N(x)), (\sigma_2^P(x), \sigma_2^N(x), \rho_2^P(x), \rho_2^N(x))...(\sigma_n^P(x), \sigma_n^N(x), \rho_n^P(x), \rho_n^N(x)))$ be a nonempty AQSP fuzzy set. E (Parameters set) and $A_{AQSP} \subset E$. Also let,

(i) $\sigma^P : A_{AQSP} \longrightarrow F_{AQSP}(V)$(Collection of all AQSP fuzzy subsets in V), $e \longmapsto \sigma_e^P$, and $\sigma_e^P : V \longrightarrow [0,1]$, $v_i \longmapsto \sigma_e^P$ then (A_{AQSP}, σ^P) : AQSP fuzzy soft vertex set.

(ii) $\sigma^N : A_{AQSP} \longrightarrow F_{AQSP}(V)$(Collection of all AQSP fuzzy subsets in V), $e \longmapsto \sigma_e^N$, and $\sigma_e^N : V \longrightarrow [-1,0]$, $v_i \longmapsto \sigma_e^N$ then (A_{AQSP}, σ^N) : AQSP fuzzy soft vertex set.

(iii) $\rho^P : A_{AQSP} \longrightarrow F_{AQSP}(V)$(Collection of all AQSP fuzzy submerge subsets in V), $e \longmapsto \rho_e^P$, and $\rho_e^P : V \longrightarrow [0, 0.5]$, $v_i \longmapsto \rho_e^P$ then (A_{AQSP}, ρ^P) : AQSP fuzzy soft vertex set.

(iv) $\rho^N : A_{AQSP} \longrightarrow F_{AQSP}(V)$ (Collection of all fuzzy submerge subsets in V), $e \longmapsto \rho_e^N$, and $\rho_e^N : V \longrightarrow [-0.5, 0]$, $v_i \longmapsto \rho_e^N$ then (A_{AQSP}, ρ^N) : AQSP fuzzy soft vertex set.

(v) $\mu^P : A_{AQSP} \longrightarrow F_{AQSP}(V \times V)$ (Collection of all AQSPfuzzy subsets in $V \times V$), $e \longmapsto \mu_e^P$, $\mu_e^P : V \times V \longrightarrow [0,1]$, $(v_i, v_j) \longmapsto \mu_e^P(v_i, v_j)$ then (A_{AQSP}, μ^P) : AQSP fuzzy soft membership edge set.

(vi) $\mu^N : A_{AQSP} \longrightarrow F_{AQSP}(V \times V)$(Collection of all AQSPfuzzy subsets in $V \times V$), $e \longmapsto \mu_e^N$, and $\mu_e^N : V \times V \longrightarrow [-1,0]$, $(v_i, v_j) \longmapsto \mu_e^N(v_i, v_j)$ then (A_{AQSP}, μ^N) : AQSP fuzzy soft non - membership edge set.

(vii) $\gamma^P : A_{AQSP} \longrightarrow F_{AQSP}(V \times V)$(Collection of all AQSPfuzzy subsets in $V \times V$), $e \longmapsto \gamma_e^P$, and $\gamma_e^P : V \times V \longrightarrow [0, 0.5]$, $(v_i, v_j) \longmapsto \gamma_e^P(v_i, v_j)$ then (A_{AQSP}, γ^P) : AQSP fuzzy soft submerge membership edge set.

(viii) $\gamma^N : A_{AQSP} \longrightarrow F_{AQSP}(V \times V)$(Collection of all AQSPfuzzy subsets in $V \times V$), $e \longmapsto \gamma_e^N$, and $\gamma_e^N : V \times V \longrightarrow [-0.5, 0]$, $(v_i, v_j) \longmapsto \gamma_e^N(v_i, v_j)$ then (A_{AQSP}, γ^N) : AQSP fuzzy soft submerge membership edge set. Then the AQSP fuzzy soft graph is, $((A_{AQSP}), (\sigma^P, \sigma^N, \rho^P, \rho^N)), ((A_{AQSP}), (\mu^P, \mu^N, \gamma^P, \gamma^N))$ if the conditions are satisfied

(a) $\mu_e^P(x,y) \leq \sigma_e^P(x) \wedge \sigma_e^P(y)$, (b) $\mu_e^N(x,y) \geq \sigma_e^N(x) \vee \sigma_e^N(y)$,
(c) $\gamma_e^P(x,y) \leq \rho_e^P(x) \wedge \rho_e^P(y)$, (d) $\gamma_e^N(x,y) \geq \rho_e^N(x) \vee \rho_e^N(y)$, for all $e \in A_{AQSP}$ and for all values of $x, y = 1, 2, 3, ..., n$ and this AQSP fuzzy soft graph is denoted as $G_{AQSP}(A, V)$.

3. Method

The essential definition of AQSP fuzzy soft graph method is deliberated with an examples.

3.1. Alternate Quadra Sub - merging Polar(AQSP) Fuzzy Graph

An Alternate Quadra - Submerging Polar (AQSP) Fuzzy Graph $G = (\sigma_{AQSP}, \mu_{AQSP})$ is a fuzzy graph with crisp graph $G^* = (\sigma^*_{AQSP}, \mu^*_{AQSP})$ is given as $V = (\sigma^P(x), \sigma^N(x), \rho^P(x), \rho^N(x))$ which is the membership value of vertices along with the uncertain membership value of edges is given as, $E = V \times V = (\mu^P(x,y), \mu^N(x,y), \gamma^P(x,y), \gamma^N(x,y))$.

Here the vertex set V is defined with the given condition in a unique method which is an alternate contrast submerging polarized uncertain transformation.Here $\sigma^P = V \to [0,1]$, $\sigma^N = V \to [-1,0]$, $\rho^P = d\ |\ 0.5, \sigma^P\ (x)\ |$ and $\rho^N = -d\ |-0.5,\ \sigma^N\ (x)|$. Here (-0.5, 0.5) is the fixation of uncertain alternate contrast polarized
submerging transformation into certain consistent preferable position. And the edge set E satisfies the following sufficient conditions.

$$(i)\quad \mu^P(x,y) \leq \min(\sigma^P(x), \sigma^P(y)), \quad (ii)\ \mu^N(x,y) \geq \max(\sigma^N(x), \sigma^N(y))$$

$$(iii)\ \gamma^P(x,y) \leq \min(\rho^P(x), \rho^P(y)) \quad (iv)\ \gamma^N(x,y) \geq \max(\rho^N(x), \rho^N(y)),$$

$\forall (x,y) \in E$. By definition, $\mu^P = V \times V \to [0,1] \times [1,0]$, $\mu^N = V \times V \to [-1,0] \times [0,-1]$ and the submerging mappings, $\gamma^P = V \times V \to [0, 0.5] \times [0.5, 0]$,

$\gamma^N = V \times V \rightarrow [-0.5, 0] \times [0, -0.5]$, which denotes the impact of the alternate quadrant polarized fuzzy mapping.

The maximum of submerging presumption to be at the level of confidence $[0, 0.5] \subseteq [0, 1]$ and the minimum of submerging presumption level of confidence is $[-0.5, 0] \subseteq [-1, 0]$ extension of the graph with its membership and non - membership values portrait the unique level of submerging destination in an AQSP fuzzy graph.

Also it must satisfy the condition, $-1 \leq \sigma^P\ (x) + \sigma^N\ (x) \leq 1\ \ and\ \ |\rho^P\ (x) + \rho^N\ (x)| \leq 1$ with constrains $0 \leq\ \sigma^P\ (x) + \sigma^N\ (x) +\ \left|\rho^P\ (x) + \rho^N\ (x)\right|\ \leq 2$ such that the uncertain status of submerging presumption, transform into its precise consistent level with fixation mid - value 0.5, which implies that level of confidence 0.5 in an AQSP as the valuable membership of its position which is real and valid in the fuzzification. The example of AQSP fuzzy graph is given in Figure.1.

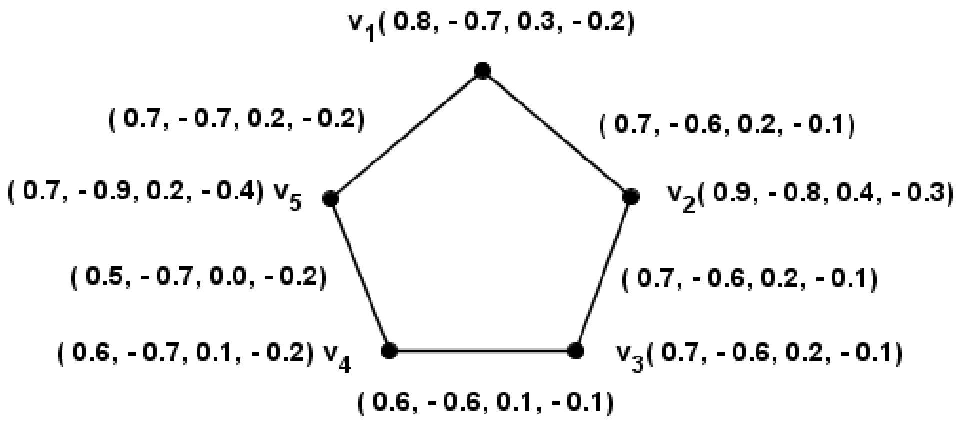

Figure 1: *AQSP Fuzzy Graph $G = (\sigma_{AQSP}, \mu_{AQSP})$*

3.2. Example of AQSP Fuzzy Soft Graph

Consider an AQSP fuzzy soft graph $G_{AQSP}(A, V)$, where $V = (v_1, v_2, v_3, v_4)$ and $E = (e_1, e_2, e_3)$. Here $G_{AQSP}(A, V)$ is described in Table.1. and $\mu_e(v_i, v_j) = 0,\ \forall (v_i, v_j) \in V \times V\ \{(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_1, v_4), (v_1, v_3)\}$ for all $e \in E$.

Table 1: *Tabular representation of AQSP Fuzzy Soft Graph parameter vertex set.*

(σ, ρ)	v_1	v_2	v_3	v_4
e_1	(0.6, - 0.7, 0.1,- 0.2)	(0.7, - 0.8, 0.2, -0.3)	(0.8, - 0.9, 0.3, - 0.4)	(0.6, - 0.7, 0.1,- 0.2)
e_2	(0.7, - 0.6, 0.2,- 0.1)	(0.8, - 0.7, 0.3, -0.2)	(0.9, - 0.8, 0.4, - 0.3)	(0.8, - 0.8, 0.3,- 0.3)
e_3	(0.8, - 0.6, 0.3,- 0.1)	(0.9, - 0.7, 0.4, -0.2)	(0.8, - 0.8, 0.3, - 0.3)	(0.9, - 0.9, 0.4,- 0.4)

Table 2: *Tabular representation of AQSP Fuzzy Soft Graph parameter edge set.*

(μ, γ)	v_1, v_2	v, v_3	v_3, v_4	v_4, v_1	v_1, v_3
e_1	(0.6, - 0.7, 0.1,- 0.2)	(0.7, - 0.8, 0.2, -0.3)	(0.6, -0.7, 0.1, - 0.2)	(0.6, - 0.7, 0.1,- 0.2)	(0.6, - 0.7, 0.1,- 0.2)
e_2	(0.7, - 0.6, 0.2,- 0.1)	(0.7, - 0.7, 0.2, -0.2)	(0.8, - 0.8, 0.3, - 0.3)	(0.7, - 0.6, 0.2,- 0.1)	(0.6, - 0.6, 0.1,- 0.1)
e_3	(0.8, - 0.6, 0.3,- 0.1)	(0.8, - 0.7, 0.3, -0.2)	(0.8, - 0.7, 0.3, - 0.2)	(0.7, - 0.6, 0.2,- 0.1)	(0.8, - 0.6, 0.3,- 0.1)

Table. 2. represents the AQSP fuzzy graph with parametric membership and non - membership with submerge values.

4. Descriptions of the regularity of AQSP fuzzy soft graph

4.1. Regular AQSP Fuzzy Ssoft Graph

Let $G^* = (\sigma^*, \mu^*)$ be a crisp graph and $G_{AQSP}(A, V)$ be an regular AQSP fuzzy soft graph of G^*. Then $G_{AQSP}(A, V)$ is said to be an regular AQSP soft graph, if $R_{AQSP}(e_i)$ is an regular AQSP fuzzy soft graph of degree k for all $e_i \in A_{AQSP}$, then $G_{AQSP}(A, V)$ is a k - regular AQSP fuzzy soft graph.

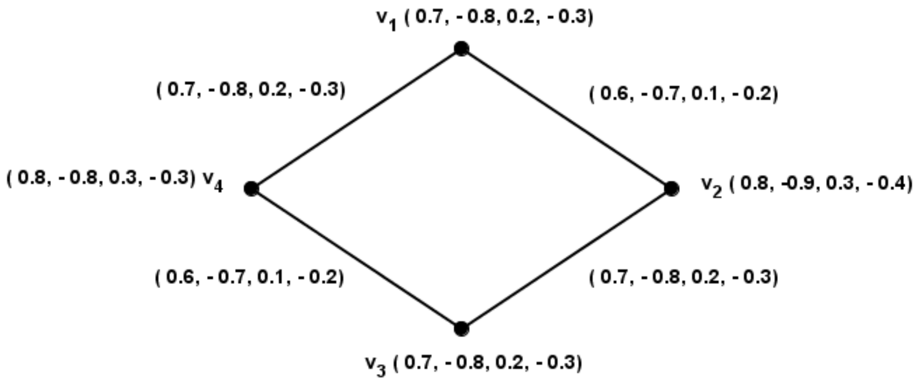

Figure 2: *$G_{AQSP}(A, V)$ - Corresponding to the parameter e_1*

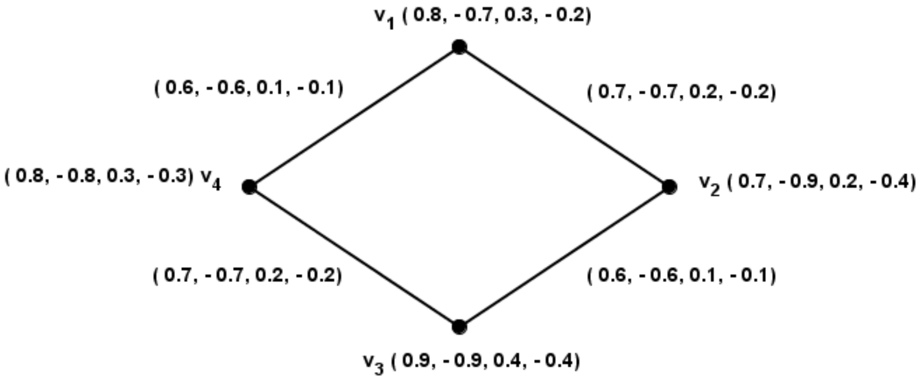

Figure 3: *$G_{AQSP}(A, V)$ - Corresponding to the parameter e_2*

4.2. Example of an AQSP Fuzzy Soft Graph

Consider, an AQSP fuzzy soft graph, $G_{AQSP}(A,V)$, the vertex set $V = (v_1, v_2, v_3, v_4)$ and let the corresponding parameters $E = (e_1, e_2)$.
Here $G_{AQSP}(A,V) = ((A_{AQSP}),\ (\sigma^P, \sigma^N, \rho^P, \rho^N)),\ ((A_{AQSP}), (\mu^P, \mu^N, \gamma^P, \gamma^N))$ is described by Table.3 and Table. 4 $(v_1, v_2, v_3, v_4))$.

4.3. Remark on Regular AQSP Fuzzy Graph

Fron Figure.2 and Figure.3 we get the result that the regular AQSP fuzzy graph which can not be a totally regular AQSP fuzzy graph. Table 3. represents the AQSP fuzzy soft graph vertex set.

Table 3: *Tabular representation of AQSP Fuzzy Soft Graph parameter vertex set.*

(μ,γ)	v_1	v_2	v_3	v_4
e_1	(0.7, - 0.8, 0.2,- 0.3)	(0.8, - 0.9, 0.3, - 0.4)	(0.7, - 0.7, 0.2, - 0.2)	(0.8, - 0.8, 0.3,- 0.3)
e_2	(0.8, - 0.7, 0.3,- 0.2)	(0.7, - 0.9, 0.2, - 0.4)	(0.9, -0.9, 0.4, - 0.4)	(0.8, - 0.8, 0.3 ,- 0.3)

Table 4: *Tabular representation of AQSP Fuzzy Soft Graph parameter edge set.*

(μ,γ)	v_1v_2	v_2v_3	v_3v_4	v_4v_1
e_1	(0.6, - 0.7, 0.1,- 0.2)	(0.7, - 0.8, 0.2, - 0.3)	(0.6, - 0.7, 0.1, - 0.2)	(0.7, - 0.8, 0.2,- 0.3)
e_2	(0.7, - 0.7, 0.2,- 0.2)	(0.6, - 0.6, 0.1, - 0.1)	(0.7, - 0.7, 0.2, - 0.2)	(0.6, - 0.6, 0.1,- 0.1)

Table. 4 represents the corresponding edges, $(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_1)$, for all values of $e \in A_{AQSP}$.

4.4. Totally Regular AQSP Fuzzy Soft Graph

Let $G^* = (\sigma, \mu)$ be a simple graph and $G_{AQSP}(A,V)$ be an AQSP fuzzy soft graph of G^*. Then $G_{AQSP}(A,V)$ is said to be a totally regular AQSP fuzzy soft graph if $R_{AQSP}(A,V)$ is totally regular fuzzy soft graph for all values of $e_i \in A_{AQSP}$, then $G_{AQSP}(A,V)$ is called k totally regular AQSP fuzzy soft graph.

Theorem 1. If $G_{AQSP}(A,V)$ satifies the condition of regular and totally regular AQSP fuzzy soft graph, then we prove that $((A_{AQSP}),\ (\sigma^P, \sigma^N, \rho^P, \rho^N))$ is a constant AQSP fuzzy soft function in $H_{AQSP}(A,V)$ of G^* for all values of $e \in A_{AQSP}$.

Proof. Let $G_{AQSP}(A,V)$ satifies the condition of regular and totally regular AQSP fuzzy soft graph. Then we have the degree of vertices as,

(i) $deg\sigma_e^P(a) = k_1,\quad deg\sigma_e^N(a) = k_2,\quad deg\rho_e^P(a) = k_3,\quad deg\rho_e^N(a) = k_4$ and

(ii) $tdeg\sigma_e^P(a) = l_1,\quad tdeg\sigma_e^N(a) = l_2,\quad tdeg\rho_e^P(a) = l_3,\quad tdeg\rho_e^N(a) = l_4$.

In AQSP fuzzy subgraphs $H_{AQSP}(A,V)$ for all values of $e \in A_{AQSP},\ a \in V$. This implies that,
$deg\sigma_e^P(a) + A_{AQSP}\ \sigma_e^P(a) = l_1,$
$deg\sigma_e^N(a) + A_{AQSP}\ \sigma_e^N(a) = l_2,$
$deg\rho_e^P(a) + A_{AQSP}\ \rho_e^P(a) = l_3,$
$deg\rho_e^N(a) + A_{AQSP}\ \rho_e^N(a) = l_4 \in H_{AQSP}(A,V), \forall\ e \in A_{AQSP},\ a \in V.$

$A_{AQSP}\ \sigma_e^P(a) = l_1 - k_1$
$A_{AQSP}\ \sigma_e^N(a) = l_2 - k_2$
$A_{AQSP}\ \rho_e^P(a) = l_3 - k_3$
$A_{AQSP}\ \rho_e^N(a) = l_4 - k_4 \in H_{AQSP}(A, V)$,
$\forall\ e \in A_{AQSP},\ a \in V$.
Hence, $((A_{AQSP}),\ (\sigma^P, \sigma^N, \rho^P, \rho^N))$ is a constant AQSP fuzzy soft function in $H_{AQSP}(A, V)$ of G^* for all values of $e \in A_{AQSP}$.

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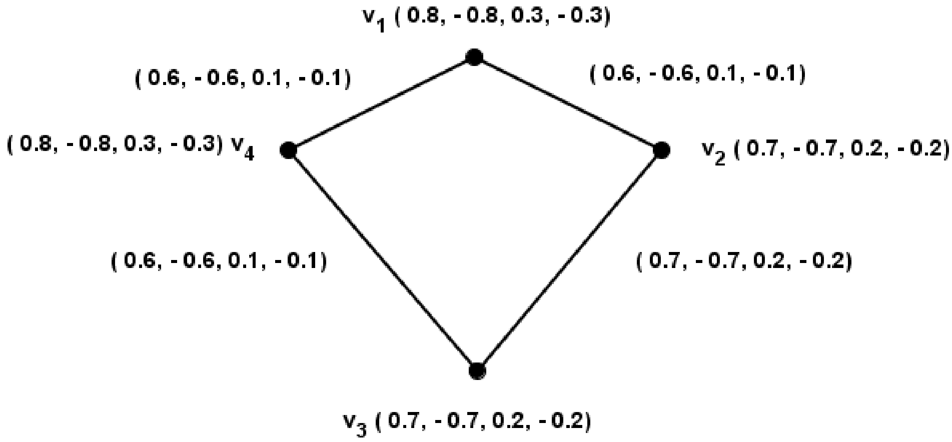

Figure 4: *$G_{AQSP}(A, V)$ - Corresponding to the parameter e_1*

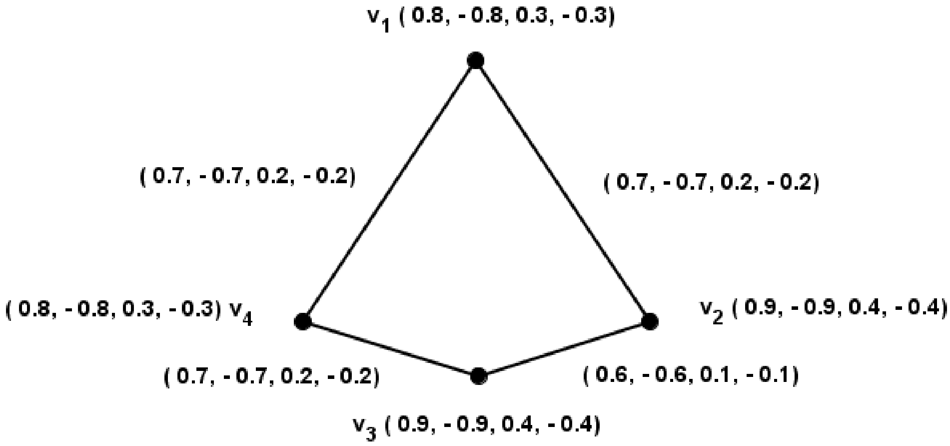

Figure 5: *$G_{AQSP}(A, V)$ - Corresponding to the parameter e_2*

4.5. Example Totally Regular AQSP Fuzzy Soft Graph

Consider, an AQSP fuzzy soft graph, $G_{AQSP}(A, V)$, the vertex set $V = (v_1, v_2, v_3, v_4)$ and let the corresponding parameters $E = (e_1, e_2)$ is shown in the Figure.4 and Figure.5.

Here $G_{AQSP}(A, V) = ((A_{AQSP}),\ (\sigma^P, \sigma^N, \rho^P, \rho^N)),\ ((A_{AQSP}), (\mu^P, \mu^N, \gamma^P, \gamma^N))$ is described Figure.6 $(v_1, v_2, v_3, v_4))$. Figure. 7 represents the corresponding edges, $(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_1)$, for all values of $e \in A_{AQSP}$.

Table 5: *Tabular representation of AQSP Fuzzy Soft Graph parameter vertex set.*

(μ,γ)	v_1	v_2	v_3	v_4
e_1	(0.8, - 0.8, 0.3,- 0.3)	(0.7, - 0.7, 0.2, - 0.2)	(0.7, - 0.7, 0.2, - 0.2)	(0.8, - 0.8, 0.3,- 0.3)
e_2	(0.8, - 0.8, 0.3,- 0.3)	(0.9, - 0.9, 0.4, - 0.4)	(0.9, -0.9, 0.4, - 0.4)	(0.8, - 0.8, 0.3 ,- 0.3)

Table 6: *Tabular representation of AQSP Fuzzy Soft Graph parameter edge set.*

(μ,γ)	v_1v_2	v_2v_3	v_3v_4	v_4v_1
e_1	(0.6, - 0.6, 0.1,- 0.1)	(0.7, - 0.7, 0.2, - 0.2)	(0.6, - 0.6, 0.1, - 0.1)	(0.6, - 0.6, 0.1,- 0.1)
e_2	(0.7, - 0.7, 0.2,- 0.2)	(0.6, - 0.6, 0.1, - 0.1)	(0.7, - 0.7, 0.2, - 0.2)	(0.7, - 0.7, 0.2,- 0.2)

4.6. Example of AQSP Fuzzy Soft Graph

Consider, an AQSP fuzzy soft graph, $G_{AQSP}(A,V)$, the vertex set $V = (v_1, v_2, v_3, v_4)$ and let the corresponding parameters $E = (e_1, e_2)$.

Here $G_{AQSP}(A,V) = ((A_{AQSP}), (\sigma^P, \sigma^N, \rho^P, \rho^N)), ((A_{AQSP}), (\mu^P, \mu^N, \gamma^P, \gamma^N))$ is described by Table.5 and Table. 5 such as, $(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_1, v_3), (v_1, v_4), (v_4, v_1), (v_1, v_1)$, for all values of $e \in A_{AQSP}$.

4.7. Remark on Regular AQSP Fuzzy Soft Graph

From Theorem.5.7. we get the result if $G_{AQSP}(A,V)$ is a regular AQSP fuzzy soft graph and $((A_{AQSP}), (\sigma^P, \sigma^N, \rho^P, \rho^N))$ is a constant AQSP fuzzy soft function, then $G^C_{AQSP}(A,V)$ is a regular AQSP fuzzy soft graph.

4.8. Remark on Totally Regular AQSP Fuzzy Soft Graph

From Theorem.5.7. similarly we get the result if $G_{AQSP}(A,V)$ is a totally regular AQSP fuzzy soft graph and $((A_{AQSP}), (\sigma^P, \sigma^N, \rho^P, \rho^N))$ is a constant AQSP fuzzy soft function, then $G^C_{AQSP}(A,V)$ is a totally regular AQSP fuzzy soft graph.

Theorem 2. Let $G_{AQSP}(A,V) = ((A_{AQSP}), (\sigma^P, \sigma^N, \rho^P, \rho^N)), ((A_{AQSP}), (\mu^P, \mu^N, \gamma^P, \gamma^N))$, for all values of $e \in A_{AQSP}$. be an AQSP fuzzy soft graph with the vertex and edge membership and non - membership submerging values. Then we prove that,

(i) $\sum_{a\in A} tdeg_{G_{AQSP}(A,V)}\ (\sigma^P_e(a) = 2S(G_{AQSP}(A,V)) + O(G_{AQSP}(A,V))$

(ii) $\sum_{a\in A} tdeg_{G_{AQSP}(A,V)}\ (\sigma^N_e(a) = 2S(G_{AQSP}(A,V)) + O(G_{AQSP}(A,V))$

(iii) $\sum_{a\in A} tdeg_{G_{AQSP}(A,V)}\ (\rho^P_e(a) = 2S(G_{AQSP}(A,V)) + O(G_{AQSP}(A,V))$

(iv) $\sum_{a\in A} tdeg_{G_{AQSP}(A,V)}\ (\rho^N_e(a) = 2S(G_{AQSP}(A,V)) + O(G_{AQSP}(A,V))$

Proof. (i) $tdeg_{G_{AQSP}(A,V)}\ (\sigma^P_e(a)) = \sum_{e\in A_{AQSP}}(\sum_{a\in V}(\mu^P_e(a,b) + \sigma^P_e(a),$

$\Longrightarrow \sum_{a\in V} tdeg_{G_{AQSP}(A,V)}\ (\sigma^P_e(a)) = \sum_{a\in V}(\sum_{e\in A_{AQSP}}(\sum_{a\in V}(\mu^P_e(a,b) + \sigma^P_e(a),$

$= \sum_{a \in A} tdeg_{G_{AQSP}(A,V)} \ (\sigma_e^P(a)) = 2S(G_{AQSP}(A,V)) + O(G_{AQSP}(A,V)).$

For non - membership AQSP fuzzy soft graph values are,

(ii) $tdeg_{G_{AQSP}(A,V)} \ (\sigma_e^N(a)) = \sum_{e \in A_{AQSP}} (\sum_{a \in V} (\mu_e^N(a,b) + \sigma_e^N(a),$

$\Longrightarrow \sum_{a \in V} tdeg_{G_{AQSP}(A,V)} \ (\sigma_e^N(a)) = \sum_{a \in V} (\sum_{e \in A_{AQSP}} (\sum_{a \in V} (\mu_e^N(a,b) + \sigma_e^N(a),$

$= \sum_{a \in A} tdeg_{G_{AQSP}(A,V)} \ (\sigma_e^N(a)) = 2S(G_{AQSP}(A,V)) + O(G_{AQSP}(A,V).$

Now, the Submerging membership values are,

(iii) $tdeg_{G_{AQSP}(A,V)} \ (\rho_e^P(a)) = \sum_{e \in A_{AQSP}} (\sum_{a \in V} (\gamma_e^P(a,b) +_e^P (a)),$

$\Longrightarrow \sum_{a \in V} tdeg_{G_{AQSP}(A,V)} \ (\rho_e^P(a)) = \sum_{a \in V} \sum_{e \in A_{AQSP}} (\sum_{a \in V} (\gamma_e^P(a,b) + \rho_e^P(a)),$

$= \sum_{a \in A} tdeg_{G_{AQSP}(A,V)} \ (\rho_e^P(a)) = 2S(G_{AQSP}(A,V)) + O(G_{AQSP}(A,V)).$

For the Submerging non - membership values are,

(iv) $tdeg_{G_{AQSP}(A,V)} \ (\rho_e^N(a)) = \sum_{e \in A_{AQSP}} (\sum_{a \in V} (\gamma_e^N(a,b) +_e^N (a)),$

$\Longrightarrow \sum_{a \in V} tdeg_{G_{AQSP}(A,V)} \ (\rho_e^N(a)) = \sum_{a \in V} \sum_{e \in A_{AQSP}} (\sum_{a \in V} (\gamma_e^N(a,b) + \rho_e^N(a)),$

$= \sum_{a \in A} tdeg_{G_{AQSP}(A,V)} \ (\rho_e^N(a)) = 2S(G_{AQSP}(A,V)) + O(G_{AQSP}(A,V)).$ ■

5. Properties of Regular and Totally Regular AQSP Fuzzy Soft Graph

Theorem 3. The size of the (k_1, k_2, k_3, k_4) regular AQSP fuzzy soft graph $(G_{AQSP}(A,V)$ on $G^* = (V,E)$ is (i) $\frac{pk_1}{2}$, (ii) $\frac{pk_2}{2}$, (iii) $\frac{pk_3}{2}$ and (iv) $\frac{pk_4}{2}$ where p = $|V|$ and $deg\ \sigma_e^P(a) = k_1$, $deg\ \sigma_e^N(a) = k_2$, $deg\ \rho_e^P(a) = k_3$ and $deg\ \rho_e^N(a) = k_3$

Proof.
(i) $S(G_{AQSP}(A,V) = \sum_{e \in A_{AQSP}} (\sum_{a \neq b} \mu_e^P(a,b))$
since $G_{AQSP}(A,V)$ is a k_1 regular AQSP fuzzy soft graph we get,
$deg\sigma_e^P(a) = k_1,\ \forall a \in V$,
Now, $S(G_{AQSP}(A,V)) = \sum_{e \in A_{AQSP}} (\sum_{a \neq b} \mu_e^P(a,b))$
$\sum_{a \in V} \ \frac{deg\ \sigma_e^P(a)}{2},$
$\sum_{a \in V} \ \frac{deg\ \sigma_e^P(a)}{2} = \sum_{a \in V} \frac{k_1}{2}.$
(ii) $S(G_{AQSP}(A,V)) = \sum_{e \in A_{AQSP}} (\sum_{a \neq b} \mu_e^N(a,b))$
since $(G_{AQSP}(A,V))$ is a (k_1, k_2, k_3, k_4) regular AQSP fuzzy soft graph we get,
$deg\sigma_e^N(a) = k_2,\ \forall a \in V$,
Now, $S(G_{AQSP}(A,V)) = \sum_{e \in A_{AQSP}} (\sum_{a \neq b} \mu_e^N(a,b))$

$\sum_{a \in V} \ \frac{deg\ \sigma_e^N(a)}{2}$
$\sum_{a \in V} \ \frac{deg\ \sigma_e^N(a)}{2} = \sum_{a \in V} \frac{k_2}{2}$
(iii) $S(G_{AQSP}(A,V)) = \sum_{e \in A_{AQSP}} (\sum_{a \neq b} \gamma_e^P(a,b))$
since $(G_{AQSP}(A,V))$ is a k_3 regular AQSP fuzzy soft graph we get,
$deg\rho_e^P(a) = k_3,\ \forall a \in V$,
Now, $S(G_{AQSP}(A,V)) = \sum_{e \in A_{AQSP}} (\sum_{a \neq b} \gamma_e^P(a,b))$
$\sum_{a \in V} \ \frac{deg\ \rho_e^P(a)}{2},$

$\sum_{a\in V} \frac{deg\ \rho_e^P(a)}{2} = \sum_{a\in V} \frac{k_3}{2}$.
(iv) $S(G_{AQSP}(A,V)) = \sum_{e\in A_{AQSP}}(\sum_{a\neq b} \gamma_e^N(a,b))$
since $(G_{AQSP}(A,V))$ is a k_4 regular AQSP fuzzy soft graph we get,
$deg\rho_e^P(a) = k_4,\ \forall a \in V$,
Now, $S(G_{AQSP}(A,V)) = \sum_{e\in A_{AQSP}}(\sum_{a\neq b} \gamma_e^N(a,b))$
$\sum_{a\in V} \frac{deg\ \rho_e^N(a)}{2}$
$\sum_{a\in V} \frac{deg\ \rho_e^N(a)}{2} = \sum_{a\in V} \frac{k_4}{2}$
Hence, The size of the (k_1,k_2,k_3,k_4) regular AQSP fuzzy soft graph $(G_{AQSP}(A,V)$ on $G^* = (V,E)$ is $\frac{pk_1}{2}$, $\frac{pk_2}{2}$, $\frac{pk_3}{2}$ and $\frac{pk_4}{2}$ where p = $|V|$ ■

Theorem 4. If $G_{AQSP}(A,V) = ((A_{AQSP}),\ (\sigma^P,\sigma^N,\rho^P,\rho^N)),\ ((A_{AQSP}),(\mu^P,\mu^N,\gamma^P,\gamma^N))$ be an regular AQSP fuzzy on $G^* = (\sigma^*,\mu^*)$ is a k-totally regular AQSP fuzzy soft graph. Then, $2S(G_{AQSP}(A,V) + O(G_{AQSP}(A,V)) = (\sigma^P pk, \sigma^N pk, \gamma^P pk, \gamma^N pk)$ where, $(\sigma^P p, \sigma^N p, \gamma^P p, \gamma^N p) = |V|$.

Proof. Since, $G_{AQSP}(A,V)$ is a k-totally regular AQSP fuzzy soft graph,
$tdeg_{G_{AQSP}(A,V)}\sigma^P a = k_1$, $tdeg_{G_{AQSP}(A,V)}\sigma^N a = k_2$, $tdeg_{G_{AQSP}(A,V)}\rho^P a = k_3$
and $tdeg_{G_{AQSP}(A,V)}\rho^N a = k_2$, $\forall a \in V$.
$\implies deg_{G_{AQSP}(A,V)}\sigma^P a + \sum_{e\in A} \sigma_e^P(a)$, $deg_{G_{AQSP}(A,V)}\sigma^N a + \sum_{e\in A} \sigma_e^N(a)$,
$deg_{G_{AQSP}(A,V)}\rho^P a + \sum_{e\in A} \rho_e^P(a)$ and $deg_{G_{AQSP}(A,V)}\rho^N a + \sum_{e\in A} \rho_e^N(a), \forall a \in V$.

$\implies \sum_{a\in V} deg_{G_{AQSP}(A,V)}\sigma^P a + \sum_{a\in V} \sum_{e\in A_{AQSP}} \sigma^P a = \sum_{a\in V}$,
$\sum_{a\in V} deg_{G_{AQSP}(A,V)}\sigma^N a + \sum_{a\in V} \sum_{e\in A_{AQSP}} \sigma^N a = \sum_{a\in V}$.

For submerging AQSP fuzzy soft graph values are,

$\sum_{a\in V} deg_{G_{AQSP}(A,V)}\rho^P a + \sum_{a\in V} \sum_{e\in A_{AQSP}} \rho^P a = \sum_{a\in V}$,
$\sum_{a\in V} deg_{G_{AQSP}(A,V)}\rho^N a + \sum_{a\in V} \sum_{e\in A_{AQSP}} \rho^N a = \sum_{a\in V}$.
$\implies tdeg_{G_{AQSP}(A,V)}\sigma^P a = k_1$, $tdeg_{G_{AQSP}(A,V)}\sigma^N a = k_2$,
$tdeg_{G_{AQSP}(A,V)}\rho^P a = k_3$ and $tdeg_{G_{AQSP}(A,V)}\rho^N a = k_2$, $\forall a \in V$.
Hence, $2S(G_{A,V}(AQSP) + O(G_{A,V}(AQSP)) = (\sigma^P pk, \sigma^N pk, \gamma^P pk, \gamma^N pk)$.

■

Theorem 5. If $(G_{AQSP}(A,V)) = ((A_{AQSP}),\ (\sigma^P,\sigma^N,\rho^P,\rho^N)),\ ((A_{AQSP}),(\mu^P,\mu^N,\gamma^P,\gamma^N))$ be an AQSP fuzzy soft graph on $G^* = (\sigma^*,\mu^*)$ is a k-regular AQSP fuzzy soft graph.Then (i)$O(G_{AQSP}(A,V)) = n(l_1 - k_1)$, (ii)$O(G_{AQSP}(A,V)) = n(l_2 - k_2)$,
(iii) $O(G_{AQSP}(A,V))= n(l_3 - k_3)$ and (iv) $O(G_{AQSP}(A,V)) = n(l_4 - k_4)$ where $n = |V|$.

Proof. Since $(G_{AQSP}(A,V))$ is an k-regular AQSP fuzzy soft graph, then we have
$deg_{G_{AQSP}(A,V)}\sigma^P a = k_1$, $deg_{G_{AQSP}(A,V)}\sigma^N a = k_2$, $deg_{G_{AQSP}(A,V)}\rho^P a = k_3$ and
$deg_{G_{AQSP}(A,V)}\rho^N a = k_4, \forall a \in V$. Here, $(G_{AQSP}(A,V))$ is totally regular
AQSP fuzzy soft graph, then we consider,
$tdeg_{G_{AQSP}(A,V)}\sigma^P a = l_1$, $tdeg_{G_{AQSP}(A,V)}\sigma^N a = l_2$, $tdeg_{G_{AQSP}(A,V)}\rho^P a = l_3$
and $tdeg_{G_{AQSP}(A,V)}\rho^N a = l_4, \forall a \in V$. Now we have,
$\sum_{a\in A} tdeg_{G_{AQSP}(A,V)}\sigma^P a = \sigma^P\ pk_1$,
$\sum_{a\in A} tdeg_{G_{AQSP}(A,V)}\sigma^N a = \sigma^N\ pk_2$,
$\sum_{a\in A} tdeg_{G_{AQSP}(A,V)}\rho^P a = \rho^P\ pk_3$,
$\sum_{a\in A} tdeg_{G_{AQSP}(A,V)}\sigma^N a = \sigma^N\ pk_4$.
(i) The AQSP fuzzy soft graph membership value is,
$\implies \sum_{a\in V} l_1 = \sum_{a\in V} deg_{G_{AQSP}(A,V)}\sigma^P a$ + $O(G_{AQSP}(A,V))$
$\implies \sum_{a\in V} l_1 = \sum_{a\in V} k_1$ + $O(G_{AQSP}(A,V))$

$\Longrightarrow nl_1 = nk_1 + O(G_{AQSP}(A,V))$
$\Longrightarrow O(G_{AQSP}(A,V)) = nk_1 - nl_1$
$\Longrightarrow O(G_{AQSP}(A,V)) = n(k_1 - l_1)$
$O(G_{AQSP}(A,V)) = n(l_1 - k_1)$.
(ii) The AQSP fuzzy soft graph non-membership value is,
$\Longrightarrow \sum_{a\in V} l_2 = \sum_{a\in V} deg_{G_{AQSP}(A,V)}\sigma^N a + O(G_{AQSP}(A,V))$
$\Longrightarrow \sum_{a\in V} l_2 = \sum_{a\in V} k_2 + O(G_{AQSP}(A,V))$
$\Longrightarrow nl_2 = nk_2 + O(G_{AQSP}(A,V))$
$\Longrightarrow O(G_{AQSP}(A,V)) = nk_2 - nl_2$
$\Longrightarrow O(G_{AQSP}(A,V)) = n(k_2 - l_2)$
$O(G_{AQSP}(A,V)) = n(l_2 - k_2)$.
(iii) The AQSP fuzzy soft graph submerrging membership value is,
$\Longrightarrow \sum_{a\in V} l_3 = \sum_{a\in V} deg_{G_{AQSP}(A,V)}\rho^P a + O(G_{AQSP}(A,V))$
$\Longrightarrow \sum_{a\in V} l_3 = \sum_{a\in V} k_3 + O(G_{AQSP}(A,V))$
$\Longrightarrow nl_3 = nk_3 + O(G_{AQSP}(A,V))$
$\Longrightarrow O(G_{AQSP}(A,V)) = nk_3 - nl_3$
$\Longrightarrow O(G_{AQSP}(A,V)) = n(k_3 - l_3)$
$O(G_{AQSP}(A,V)) = n(l_3 - k_3)$.
(iv) The AQSP fuzzy soft graph submerrging non- -membership value is,
$\Longrightarrow \sum_{a\in V} l_3 = \sum_{a\in V} deg_{G_{AQSP}(A,V)}\rho^N a + O(G_{AQSP}(A,V))$
$\Longrightarrow \sum_{a\in V} l_3 = \sum_{a\in V} k_3 + O(G_{AQSP}(A,V))$
$\Longrightarrow nl_3 = nk_3 + O(G_{AQSP}(A,V))$
$\Longrightarrow O(G_{AQSP}(A,V)) = nk_3 - nl_3$
$\Longrightarrow O(G_{AQSP}(A,V)) = n(k_3 - l_3)$
$O(G_{AQSP}(A,V)) = n(l_3 - k_3)$. Hence the result.

■

6. Perfectly regular AQSP fuzzy soft graph

Let $G_{AQSP}(A,V)$ be an AQSP fuzzy soft graph on V. Then $G_{AQSP}(A,V)$ is called as perfectly regular AQSP fuzzy soft graph if $G_{AQSP}(A,V) = ((A_{AQSP}),\ (\sigma^P,\sigma^N,\rho^P,\rho^N))$, $((A_{AQSP}),(\mu^P,\mu^N,\gamma^P,\gamma^N))$ is a regular and totally regular AQSP fuzzy soft graph $\forall e_i \in A_{AQSP}$.

Table 7: *Tabular representation of AQSP Fuzzy Soft Graph parameter vertex set.*

(μ,γ)	v_1	v_2	v_3	v_4
e_1	(0.8, - 0.8, 0.3,- 0.3)	(0.8, - 0.8, 0.3,- 0.3)	(0.8, - 0.8, 0.3,- 0.3)	(0.8, - 0.8, 0.3,- 0.3)
e_2	(0.9, - 0.9, 0.4,- 0.4)	(0.9, - 0.9, 0.4,- 0.4)	(0.9, - 0.9, 0.4,- 0.4)	(0.9, - 0.9, 0.4,- 0.4)

Table 7. represent the AQSP Fuzzy Soft Graph corresponding parameteric vertex set

Table 8: *Tabular representation of AQSP Fuzzy Soft Graph parameter edge set.*

(μ,γ)	v_1v_2	v_2v_3	v_3v_4	v_4v_1
e_1	(0.7, - 0.7, 0.2, - 0.2)	(0.7, - 0.7, 0.2, - 0.2)	(0.7, - 0.7, 0.2, - 0.2)	(0.7, - 0.7, 0.2, - 0.2)
e_2	(0.8, - 0.7, 0.3,- 0.2)	(0.8, - 0.7, 0.3, - 0.2)	(0.8, - 0.7, 0.3, - 0.2)	(0.8, - 0.7, 0.3,- 0.2)

Table 8. explains the AQSP Fuzzy Soft Graph corresponding parameteric edge set

6.1. Example of AQSP Fuzzy Soft Graph

From the Figure.8 we get the result of AQSP fuzzy soft graph with the condition, $tdeg_{G_{AQSP}}(A,V)$ = $2S(G_{AQSP}(A,V) + O(G_{AQSP}(A,V)) : 5.6 + 3.2 = 8.8$
where, $2S(G_{AQSP}(A,V) = 5.6$ and $O(G_{AQSP}(A,V)) = 3.2$, then, $tdeg_{G_{AQSP}}(A,V)$ = 8.8. Using Figure.6 and Figure.7 we can get the same result of AQSP fuzzy soft graph.

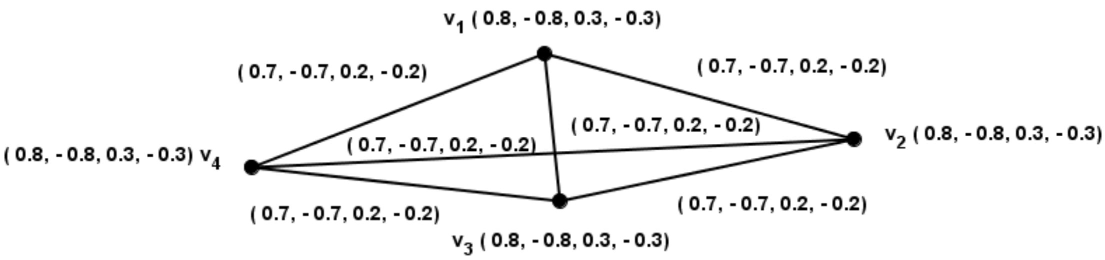

Figure 6: *Perfectly regular AQSP fuzzy soft graph Corresponding to the parameter* e_1

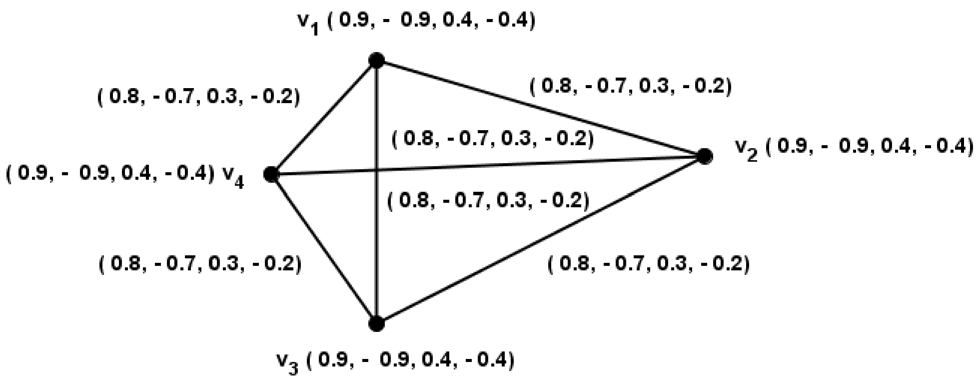

Figure 7: *Perfectly regular AQSP fuzzy soft graph Corresponding to the parameter* e_1

Theorem 6. For a perfectly regular AQSP fuzzy soft graph $G_{AQSP}(A,V)$ we have $((A_{AQSP}), (\sigma^P, \sigma^N, \rho^P, \rho^N))$ is a constant function.

Proof. From Theorem .4 and Theorem. 5 we prove that $G_{AQSP}(A,V) = ((A_{AQSP}),\ (\sigma^P, \sigma^N, \rho^P, \rho^N)),\ ((A_{AQSP}), (\mu^P, \mu^N, \gamma^P, \gamma^N))$ is perfectly regular AQSP fuzzy soft graph. ■

Theorem 7. Let $G_{AQSP}(A,V)$ be an AQSP fuzzy soft graph. Then we prove that $G_{AQSP}(A,V)$ is perfectly regular AQSP fuzzy soft graph if and only if the given conditions are satisfied for edges and vertices with membership values.

(i) $\sum_{x \neq y} \mu_e^P(x,y) = \sum_{z \neq y} \mu_e^P(z,y)$
(ii) $\sum_{x \neq y} \mu_e^N(x,y) = \sum_{z \neq y} \mu_e^N(z,y)$
(iii) $\sum_{x \neq y} \gamma_e^P(x,y) = \sum_{z \neq y} \gamma_e^P(z,y)$
(iv) $\sum_{x \neq y} \gamma_e^N(x,y) = \sum_{z \neq y} \gamma_e^N(z,y)\ \ \forall x,y \in V, e_i \in A_{AQSP}$.
(v) $\sigma_e^P(x) = \sigma_e^P(z)$, (vi) $\sigma_e^N(x) = \sigma_e^N(z)$
(vii) $\rho_e^P(x) = \rho_e^P(z)$, (viii) $\rho_e^N(x) = \rho_e^N(z), \forall x,y \in V, e_i \in A_{AQSP}$.

Proof. Consider, $G_{AQSP}(A,V)$ is perfectly regular AQSP fuzzy soft graph. By definition $G_{AQSP}(A,V)$ is regular AQSP fuzzy soft graph, hence it trivially satifies (i), (ii), (iii) and (iv). Therfore we have the following,

$deg_{G_{AQSP}(A,V)}\sigma^P(x) = deg_{G_{AQSP}(A,V)}\sigma^P(z)$,
$deg_{G_{AQSP}(A,V)}\sigma^N(x) = deg_{G_{AQSP}(A,V)}\sigma^N(z)$,
$deg_{G_{AQSP}(A,V)}\rho^P(x) = deg_{G_{AQSP}(A,V)}\rho^P(z)$,
$deg_{G_{AQSP}(A,V)}\rho^N(x) = deg_{G_{AQSP}(A,V)}\rho^N(z), \forall x, z \in V, e_i \in A_{AQSP}$.
Thus implies the results by proposition 8.2.in the following,
$\sum_{x\neq y} \mu_e^P(x, y) = \sum_{z\neq y} \mu_e^P(z, y)$
$\sum_{x\neq y} \mu_e^N(x, y) = \sum_{z\neq y} \mu_e^N(z, y)$
$\sum_{x\neq y} \gamma_e^P(x, y) = \sum_{z\neq y} \gamma_e^P(z, y)$
$\sum_{x\neq y} \gamma_e^N(x, y) = \sum_{z\neq y} \gamma_e^N(z, y) \ \ \forall x, y \in V, e_i \in A_{AQSP}$.
by Theorem.6, (v), (vi), (vii) and (viii) also holds.
Conversely, suppose that $G_{AQSP}(A, V)$ is an AQSP fuzzy soft graph such that it satisfies the conditions from (i), (ii), (iii) and (iv).
$deg_{G_{AQSP}(A,V)}\sigma^P(x) = deg_{G_{AQSP}(A,V)}\sigma^P(z) = r_1$,
$deg_{G_{AQSP}(A,V)}\sigma^N(x) = deg_{G_{AQSP}(A,V)}\sigma^N(z), r_2$
$deg_{G_{AQSP}(A,V)}\rho^P(x) = deg_{G_{AQSP}(A,V)}\rho^P(z), r_3$
$deg_{G_{AQSP}(A,V)}\rho^N(x) = deg_{G_{AQSP}(A,V)}\rho^N(z) = r_4, \forall x, z \in V, e_i \in A_{AQSP}$.
This implies that $G_{AQSP}(A, V)$ is a regular AQSP fuzzy soft graph.
From, (v), (vi), (vii) and (viii) we get the result,
(v) $\sigma_e^P(x) = \sigma_e^P(z) = k_1$,
(vi) $\sigma_e^N(x) = \sigma_e^N(z) = k_2$
(vii) $\rho_e^P(x) = \rho_e^P(z) = k_3$,
(viii) $\rho_e^N(x) = \rho_e^N(z) = k_4$, $\forall x, z \in V, e_i \in A_{AQSP}$.
Thus, $((A_{AQSP}),\ (\sigma^P, \sigma^N, \rho^P, \rho^N))$ is a constant AQSP fuzzy soft function.
$tdeg_{G_{AQSP}(A,V)}\sigma^P(z) = deg_{G_{AQSP}(A,V)}\sigma^P(z) + \sigma^P(z) = r_1 + k_1$,
$tdeg_{G_{AQSP}(A,V)}\sigma^P(w) = deg_{G_{AQSP}(A,V)}\sigma^P(w) + \sigma^P(w) = r_1 + k_1$,
$tdeg_{G_{AQSP}(A,V)}\sigma^N(z) = deg_{G_{AQSP}(A,V)}\sigma^N(z) + \sigma^N(z) = r_2 + k_2$,
$tdeg_{G_{AQSP}(A,V)}\sigma^N(w) = deg_{G_{AQSP}(A,V)}\sigma^N(w) + \sigma^N(w) = r_2 + k_2$,
$tdeg_{G_{AQSP}(A,V)}\rho^P(z) = deg_{G_{AQSP}(A,V)}\rho^P(z) + \rho^P(z) = r_3 + k_3$,
$tdeg_{G_{AQSP}(A,V)}\rho^P(w) = deg_{G_{AQSP}(A,V)}\rho^P(w) + \rho^P(w) = r_3 + k_3$,
$tdeg_{G_{AQSP}(A,V)}\rho^N(z) = deg_{G_{AQSP}(A,V)}\rho^N(z) + \rho^N(z) = r_4 + k_4$,
$tdeg_{G_{AQSP}(A,V)}\rho^N(w) = deg_{G_{AQSP}(A,V)}\rho^N(w) + \rho^N(w) = r_4 + k_4$, $\forall x, z \in V, e_i \in A_{AQSP}$.
The toally regular AQSP fuzzy soft graph is,
$tdeg_{G_{AQSP}(A,V)}\sigma^P(z) = tdeg_{G_{AQSP}(A,V)}\sigma^P(w) = k_1$,
$tdeg_{G_{AQSP}(A,V)}\sigma^N(z) = tdeg_{G_{AQSP}(A,V)}\sigma^N(w) = k_2$,
$tdeg_{G_{AQSP}(A,V)}\rho^P(z) = tdeg_{G_{AQSP}(A,V)}\rho^P(w) = k_3$,
$tdeg_{G_{AQSP}(A,V)}\rho^N(z) = tdeg_{G_{AQSP}(A,V)}\rho^N(w). = k_4 \ \forall x, z \in V, e_i \in A_{AQSP}$.
Hence $G_{AQSP}(A, V)$ is toally regular AQSP fuzzy soft graph. This implies that $G_{AQSP}(A, V) = ((A_{AQSP}),\ (\sigma^P, \sigma^N, \rho^P, \rho^N)),\ ((A_{AQSP}), (\mu^P, \mu^N, \gamma^P, \gamma^N))$ is perfectly regular AQSP fuzzy soft graph and $((A_{AQSP}), (\sigma^P, \sigma^N, \rho^P, \rho^N))$ is a constant function. therefore, $tdeg_{G_{AQSP}(A,V)}\rho^P(z) = tdeg_{G_{AQSP}(A,V)}\rho^P(w) = k_1, k_2, k_3,\ and\ k_4, \forall x, z \in V, e_i \in A_{AQSP}$.

■

7. Application of AQSP Fuzzy Soft Graph

AQSP fuzzy soft graph can be used in the governing of women safety police network (WSPN) of a city or a district or any Non safety area region. The WSPN can be utilized using AQSP fuzzy soft graph, where the police vehicle depots are the vertices $(v_1, v_2, v_3, ...v_n)$ and the route connecting two police vehicles are considered as corresponding edges.

For women safety police inspectors are positioned and the objective of the Governing problem is to find the minimum number of women inspectors required who will inspect the police vehicle for a particular time and particular bus stop or any region. The following description of AQSP fuzzy soft graph will help to find the solution of Patrolling of Police vehicle Network.

7.1. Method of AQSP Fuzzy Soft Graph Women Safety Police Vehicle Network

1. Let $V = (v_1, v_2, v_3 ... v_n)$ be the vertices of AQSP fuzzy soft graph police vehicle depots in a particular women safety vehicle network corresponding to the women Institutions, Companies, Colleges and Working places especially in bus stops.

2. We consider the edges as women working regions E = $(v_1v_2, v_1v_3, v_2v_4, ... v_mv_n)$. The vertices membership and non-membership values of the police vehicle V_i is determine as
$V_i \in A_{AQSP}\ for\ i = 1, 2, ...n$.

3. Now, define a term safety of women work is satisfied, which is the minimum number of women saved from particular people who distubs them while they stay or travel or work in different places.It is denoted as S vertices.The vehicle route is denoted by edges $R = v_iv_j$ in Alternate quadra submerging polar fuzzy soft graph.

4. Find the membership values of the women safety vehicle route v_iv_j between the range [-1,1] using AQSP fuzzy graph soft graph with the given conditions if
(i) $S > R$,for AQSP membership values (ii) $S < R$,for AQSP non-membership values.

5. (a) $\mu_e^P(x,y) \leq \sigma_e^P(x) \wedge \sigma_e^P(y)$, (b) $\mu_e^N(x,y) \geq \sigma_e^N(x) \vee \sigma_e^N(y)$,

(c) $\gamma_e^P(x,y) \leq \rho_e^P(x) \wedge \rho_e^P(y)$, (d) $\gamma_e^N(x,y) \geq \rho_e^N(x) \vee \rho_e^N(y)$, for all $e \in A_{AQSP}$ and for all values of $x, y = 1, 2, 3, ..., n$.

6. Let the capacity of five women police vehicle depots as vertices $v_1 = 4, v_2 = 3, v_1 = 5, v_4 = 4$,, number of women exist in the spot facing dangerous situation denoted as edges, v_1, v_2 = 55 , $v_2, v_3 = 95$, v_3, v_4 = 100 , v_4, v_1 =92 are tabulated below.

7. The score values are measured by the AQSP score formula which gives the result of low and high self-esteem influential person, $\frac{1}{n}\ (\frac{1}{l_d^P}\ \sum \varphi_x^P - \frac{1}{l_d^N}\ \sum \varphi_x^N\ )$

Table 9: *Tabular representation of AQSP Fuzzy Soft Graph parameter vertex set.*

(σ, ρ)	v_1	v_2	v_3	v_4
e_1	(0.6, - 0.8, 0.1,- 0.3)	(0.7, - 0.7, 0.2, -0.2)	(0.8, - 0.9, 0.3, - 0.4)	(0.6, - 0.9, 0.1,- 0.4)
e_2	(0.7, - 0.9, 0.2,- 0.4)	(0.8, - 0.6, 0.3, -0.1)	(0.9, - 0.8, 0.4, - 0.3)	(0.8, - 0.8, 0.3,- 0.3)
Score	0.500	0.900	0.925	0.900

The score values of the women needed safety in different spots are given with membership and non membership values of the edges are v_1, v_2 = 0.550 , v_2, v_3 = 0.950, v_3, v_4 = 1.000 , $v_4, v_1 = 0.925$. The police vehicle $v_3 = 3$ is the important vehicle to be in the spot $v_3, v_4 = 1.000$ where women in that area need safety. The bar diagram given below shows the result.

Table 10: *Tabular representation of AQSP Fuzzy Soft Graph parameter edge set.*

(μ, γ)	v_1v_2	v_2v_3	v_3v_4	v_4v_1
e_1	(0.7, - 0.7, 0.2, - 0.2)	(0.7, - 0.7, 0.2, - 0.2)	(1.0, - 0.7, 0.5, - 0.2)	(0.7, - 0.7, 0.2, - 0.2)
e_2	(0.8, - 0.7, 0.3,- 0.2)	(0.8, - 0.7, 0.3, - 0.2)	(0.9, - 1.0, 0.4, - 0.5)	(0.8, - 0.7, 0.3,- 0.2)
Score	0.550	0.950	1.000	0.925

8. Conclusion

The Alternate Quadra Submerging Polar (AQSP) fuzzy graph is introduced with the basic perception of Fuzzy soft graphs. In this article, we introduce the new module AQSP fuzzy soft graphs with suitable definitions, theorems, examples, and properties. The membership and non-membership values of AQSP fuzzy soft graph is introduced with submerging level of confidence [-0.5,0.5]. The introduction of this new module AQSP fuzzy soft graph is an indispensable concept that can be rather developed into interdisciplinary subjects. The main purpose of this new graph is to find the reliable corresponding parametric membership values. The regular, totally regular, and perfectly regular AQSP fuzzy soft graph combinatoric concepts and properties can be applied in Combinatoric subjects, Applied Mathematics, Statistics, Probability, Artificial intelligence, Approximate reasoning, Teaching learning projects and Mathematical psychology. Different types of AQSP fuzzy soft graphs and the Network method of Governing the women safety vehicle in different spots are presented specifically. Finding the important police vehicle, connected routes and spots are the extent of the AQSP fuzzy soft graph. In future the extension of the AQSP fuzzy soft graph can be developed in Decision making analysis, medical diagnosis, and machine learning. The regularity of AQSP fuzzy soft sets and graphs are applicable in real life situations which are uncertain. The combinatoric membership and non-membership submerging values can be found using corresponding parameters in different fuzzy fields.

Declarations

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