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STATISTICAL ANALYSIS OF SPLIT-PLOT DESIGN USING SPECIAL TYPE OF GRAPHS

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Abstract

When all experimental runs cannot be done under homogeneous conditions, blocking can be utilized to increase the power for testing treatment effects. In many real-life environments, there is at least one factor that is hard to change, leading to a split-plot structure. This paper demonstrates how to generate certain graphs using main-plot and sub-plot analyses, as well as providing a catalog. As a result, during situations where the candidate set is too huge to be tractable, the design of split-plot experiments becomes computationally feasible. The designs were considered ideal because they were capable and efficient in estimating the fixed effects of the suitable statistical model given the split-plot design structure. The Split-Plot Design (SPD) is the complete block design which plays an important role in the fields of agriculture, medicine, and industries. This SPD is specifically suited for a two-factor experiment that has more treatments than can be accommodated by a complete block design. In an SPD, one factor is assigned to the main- plot. The assigned first factor is called the main - plot factor. The main- plot is then divided into subplots and the second factor is called the sub - plot factor. SPD is most used for (i) few experimental materials may be rare while the other experimental materials may be available in large quantity, (ii) the levels of one or more treatment factor or easy to change and the alteration of levels of other treatment factors are costly or time-consuming. Given the extensive study done in graph theory, it has developed to be a very broad subject in mathematics. Graphs are important because they are a visual way of expressing information. A graph shows data that is equivalent to many words. A graph can convey information that is difficult to express in words. A bipartite graph is a type of graph in which the entire graph may be divided into two bipartite sets, with edges connecting vertices in one set to vertices in the other. Vertex coloring is the procedure of assigning labels or colors to each vertex in a graph. The data set was also manually analyzed to validate the software-analyzed outcomes. R gave the same results as the manual analysis, showing that they were both correct. R is mainly command-based. The proposed approach is demonstrated using agricultural and industrial examples.

Key Word: *Split-plot design, complete bipartite graph, colored graph.*

1. Introduction

Tamil Nadu is one of the leading rice-growing states in India and has been successfully cultivating rice since ancient times as the state has all the favorable climatic conditions suitable for rice cultivation. Rice research was initiated in Madras State to increase rice production and productivity (mixed).1902 at Samalkota in East Godavari district was extended to 12 more places. This study uses primary data to determine the rice production of the Salem district of Tamil Nadu. The present study aims to analyze rice cultivation using different levels of nitrogen. The constraint analysis is applied to find out the problems of paddy cultivation.

A split plot design and some graphs were adopted for this present study. The split-plot design originated in the field of agriculture. Experimenters applied one treatment to a large area of land, called a whole plot, and other treatments to smaller areas of land within the whole plot called a subplot. Split plots have two types of factors Hard-to-change (HTC) applied to the whole plots and Easy-to-change (ETC) applied to the subplots. In such case different sizes of plots are required and the resulting design is known as Split Plot Design (*SPD*). In 1925, Fisher developed these designs for the purpose of agricultural experiments.

One of the fastest-expanding sciences in modern technology is graph theory. Graphs are commonly used in applications of many fields to represent different objects and their relationships. The declaration of an object vertex serves as the graph's visual representation, while an edge represents the relationships between objects. Graph theory has recently become established as a significant mathematical tool in a wide range of fields, including functional research, chemistry, genetics, and linguistics, as well as electrical engineering, geography, sociology, and architecture of themselves.

Wooding W M [1] has discussed split-plot designs characteristics and applications. To design the first section, models and least squares are reviewed. The main part shows how a fundamental split-plot design is created through a process of "evolution," starting with a completely random model and progressing through a randomized blocks design to a split-plot while using the same set of runs. George Box and Stephen Jones [2] have evaluated the applicability of split-plot designs for the experimental setting and have concentrated on the use of statistical experimental designs in designing goods that are robust to environmental factors. They conclude that the split-plot and strip-block designs are valuable for creating strong products. Peter Goos and Martina Vandebroek [3] have developed an exchange algorithm for constructing D-optimal split-plot designs and the resulting designs are analyzed. Natalino Calegario et al. [4] have analyzed the split-split-plot design and established the impact of fertilizer concentration on the establishment of Begonia and Petunia. Then they draw the conclusion that the pH values declined with fertilizer concentration over time and the EC values increased over time, resulting in values that limited nutrient availability and plant growth.

Bradley Jones and Peter Goos [5] have suggested a fresh technique for producing ideal split-plot designs. These split-plot designs are best when they are effective at estimating the fixed effects of the proper statistical model, given the structure of the design. Pwasong A D and Choji D N [6] have analyzed the rabbit feeds data obtained from the Department of Agricultural Science, Federal College of Education Pankshin and determined that there is any significant variation in the categories of feeds given. The result illustrates that there was no significant different between the various types of feed utilized to feed the rabbits.

Bradley Jones [7] has suggested for the use of split-plot designs in industrial applications are provided after an examination of current developments. Johannes Ledolter [8] has reviewed the factorial split-plot design and fractional factorial split-plot designs experiments and uses several illustrative examples to illustrate why they frequently occur in industrial investigations. Abhishek K. Shrivastava [9] has presented an effective method for constructing split-plot design catalogues by transforming the design isomorphism problem to a graph isomorphism problem utilizing a new graph representation. Derya Dogan and Pinar Dundar [10] have introduced the new concept of average covering number of a graph and establish the brief relationship between the average covering number and some other graph parameters.

David I J and Adehi M U [11] have utilized a 21×52 split-plot experiment with three replicates for comparison. Here they review the sorghum thresher's improved threshing efficiency.Vahide Hajihassani and Yadollah Rajaei [12] have used five agricultural machinery companies that have existed accepted in the Tehran Stock Exchange since (1388-1390) and a sample that is representative of society, conduct a split-plot design model study on the factors impacting liquidity accepted in stock exchange Agricultural Machinery companies. David I J et al. [13] have presented the steps for the estimated generalized least square (EGLS) technique, which estimates the parameters of a nonlinear split-plot design (SPD) model utilizing theoretical iterative Gauss Newton via Taylor Series expansion. Yoshimi Egawaa et al. [14] have discussed the 4-connected graph in triangles and let G be a 4-connected graph, and let $E^{\sim}(G)$ denote the set of those edges of G which are not contained in a triangle, and let $E_c(G)$ denote the set of 4-contractible edges of G. We show that if $3 \leq |E^{\sim}(G)| \leq 4$ or $|E^{\sim}(G)| \geq 7$, then $|E_c(G)| \geq (|E^{\sim}(G)| + 8)/4$ unless G has one of the three specified configurations.

Table 1: *Background of this research*

Review	The related articles of Split-plot design, application of split-plot design, graph theory, vertex coloring and split-plot design with colored graph are given.
Example 1	Yield of paddy in different level of nitrogen and the given data are collected from the agriculture filed of salem district.
Example 2	Application method of paint in different mixing level and the given data are collected from different hardware’s in salem district.

2. Preliminaries

2.1 Split-plot Design

A randomized complete block design with two factors is no longer a randomized complete block design because the order of experiments is controlled to obtain observations in each treatment under each block. Splitting the randomization of an experiment to obtain observations under the treatment of one factor is called a split-plot design.

2.2 Complete Bipartite Graph

A complete bipartite graph is a graph whose vertex set V can be divided into two subsets V_1 and V_2 such that no edge has both endpoints in the same subset and every edge is connected to every vertex of the first subset and every vertex of the second subset.

2.3 Colored Graph

In a graph, the procedure for assigning the labels (colors) to the nodes or edges or areas is known as graph coloring. In this assignment no two adjacent vertices or adjacent edges or adjacent areas are getting the same color.

3. Statistical Analysis of Split Plot Design

The liner model for Split Plot Design is.

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Y_{ijk} = μ + r_i + t_j + s_k + ts_{jk} + ∂_{ij} + ε_{ijk}, ∀ *i=1,2,…r; j=1,2,…v; k=1,2…n.*
Where, Y_{ijk} is the observation corresponding to k^{th} level of sub plot factor *(B)*, j^{th} level of main plot factor *(A)* and i^{th} replication.
μ is general mean effect.
r_i is i^{th} replication effect.
t_j is j^{th} main - plot treatment effect.
s_k is k^{th} sub - plot treatment effect.
ts_{jk} is interaction effect.
The error components ∂_{ij} and ε_{ijk} are independently and normally distributed with mean zero and respective variance $\sigma^2\,\partial$ and $\sigma^2\varepsilon$.

3.1 Main-Plot Analysis

This analysis part is based on the comparisons of main plot totals:
The levels of *A* are assigned to the main plots within blocks based on *RBD* and the sum of squares are given below,

- Correction factor (*CF*) = $\frac{G^2}{rvn}$
- Total sum of square *(SST)* = $\Sigma X^2 - CF$
- Replication sum of square *(SSR)* = $\frac{\Sigma R^2}{vn} - CF$
- Main-plot sum of square *(SS (MP))* = $\frac{\Sigma A^2}{rn}$ - CF
- Main-plot error sum of square *(SSE₁)* = $\frac{\Sigma(AR)^2}{n} - CF - SSR - SS(MP)$

3.2 Sub-Plot Analysis

This analysis part is based on the comparisons of sub plot totals:

- Sub-plot sum of square *(SS (SP))* = $\frac{\Sigma B^2}{rv} - CF$
- Interaction effect sum of square (*A*×*B*) = $\frac{\Sigma(AB)^2}{r} - CF - SSA - SSB$
- Sub-plot error sum of square *(SSE₂)* = *SST*- (*SSR* + *SS (MP)* + *SSE₁* + *SS (SP)* + (*A*×*B*))

The analysis of the variance table is outlined as follows

Table 2: *ANOVA for split-plot designs*

Sv	Df	Ss	Mss	F-Ratio
Replication	(r-1)	SSR	$S_R^2 = \frac{SSR}{(r-1)}$	$F_R = \frac{S_R^2}{S_{E_1}^2 \sim F_{(r-1),(r-1)(v-1)}}$
MP(*A*)	(v-1)	SSA	$S_A^2 = \frac{SSA}{(v-1)}$	$F_A = \frac{S_A^2}{S_{E_1}^2 \sim F_{(v-1),(r-1)(v-1)}}$
MPE (*E₁*)	(r-1)(v-1)	SSE₁	$S_E^2 = \frac{SSE_1}{(r-1)(v-1)}$	-
SP (*B*)	(n-1)	SSB	$S_B^2 = \frac{SSB}{(n-1)}$	$F_B = \frac{S_B^2}{S_{E_2}^2 \sim F_{(v-1),(r-1)(n-1)}}$
IE(AB)	(v-1)(n-1)	SSAB	$S_{AB}^2 = \frac{SSAB}{(v-1)(n-1)}$	$F_{AB} = \frac{S_{AB}^2}{S_{E_2}^2 \sim F_{(v-1),(r-1)(n-1)}}$

SPE(E_2)	v(r-1)(n-1)	SSE2	$S_A^2 = \frac{SSE_2}{v(r-1)(n-1)}$	-
Total	rvn-1	SST	-	-

3.3 Flow – Chart

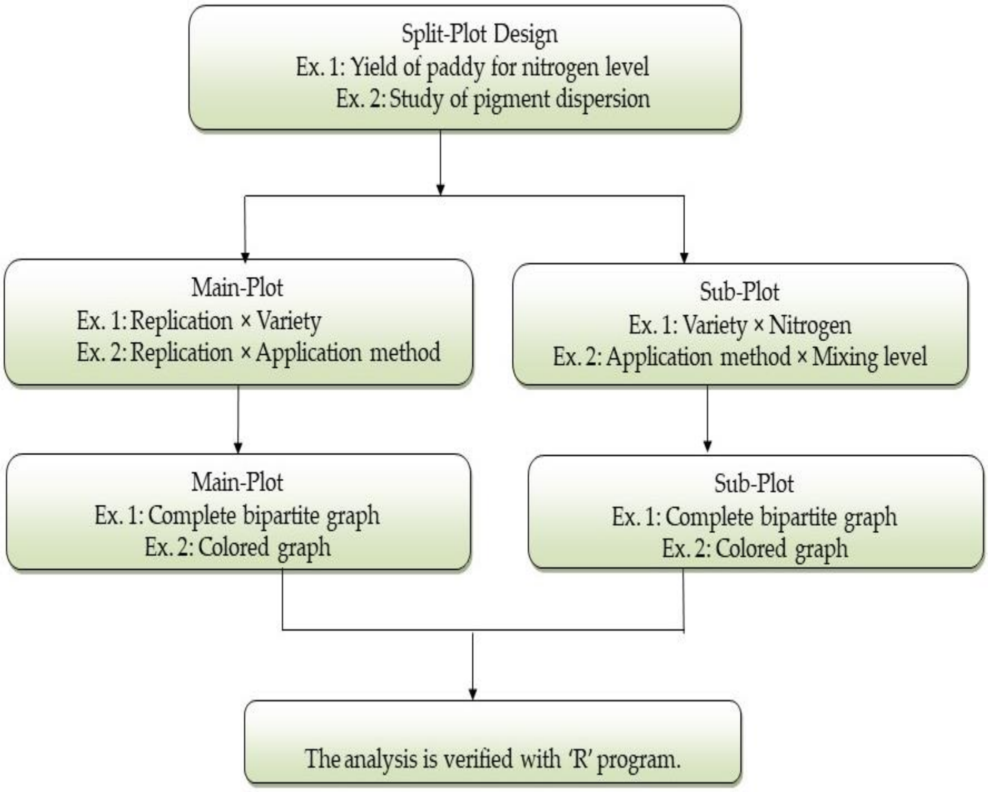

Figure 1: *Flow chart*

4. Construction of Split-Plot Design using Complete Bipartite Graph

4.1 Method for Construction of Complete Bipartite Graphs

- Let us consider the main-plot and sub-plot as vertex set S . This vertex set can be divided into subsets of S_1 and S_2.
- In main-plot, the replication is considered as first subset S_1 and variety as second subset S_2.
- Now consider the first vertex of first subset and then R_1 is connected to all the vertices of second subset through edges.
- Next consider the second vertex and it is connected to all the vertices of the second subset through the edges.
- Similarly, all the remaining vertices of the first subset are connected to all the vertices of second subset through the corresponding edges.
- Finally, we get the complete bipartite graph for main-plot and sub-plot.

4.1.1 Application

This example is to determine the yield response in N fertilization between different paddy varieties, three varieties of Paddy (V_1 = ADT 36, V_2=ASD 16, V_3 = IR50) are the treatments of main plot, nitrogen rates such as 0, 30 and 60 Kg/ha are the sub- plot treatments. The study was replicated four times and the primary data gathered for this experiment from the agricultural field of salem district of tamil nadu in India and shown in table 3.

Table 3: *Replication wise data for yield of paddy (Kg/ha)*

Replication	R_1	R_2	R_3	R_4
Variety		Nitrogen (N_1)		
V_1	15.8	19.2	13.2	13.2
V_2	20.8	15.3	20.5	13.8
V_3	15.9	16.3	16.2	12.8
		Nitrogen (N_2)		
V_1	17.8	20.5	14.8	13.8
V_2	24.8	20.8	18.8	17.8
V_3	18.5	16.1	20.8	12.2
		Nitrogen (N_3)		
V_1	21.1	24.8	13.8	18.8
V_2	30.5	19.2	25.7	15.2
V_3	18.3	18.2	22.8	10.8

Table 4: *Replication × variety (R×V) for main – plot*

	V_1	V_2	V_3	Replication Total
R_1	54.7	76.1	52.7	183.5
R_2	64.5	55.3	51	170.8
R_3	41.8	65	59.8	166.6
R_4	45.8	46.8	35.8	128.4
Variety Total	206.8	243.2	199.3	649.3

The procedure for constructing the complete bipartite graph mentioned in section 4.1 is followed for the main-plot and sub-plot for the above experiments and then the finalized complete bipartite graph.

From the above table 4 as vertex is fixed as S, which is divided into two subsets, figure 2 shows that S_1 (replication) and S_2 (variety). Figure 3 shows that the first replication vertex (R_1) and it is connected to all the vertices of variety (V_1, V_2 and V_3) through the edge values 54.7(Y_1), 76.1(Y_2) and 54.7(Y_3).

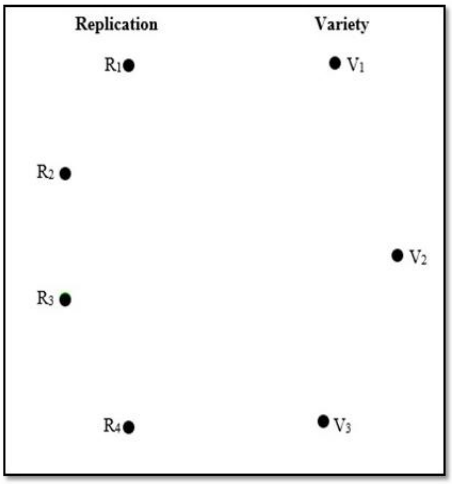

Figure 2: *Graph of subsets*

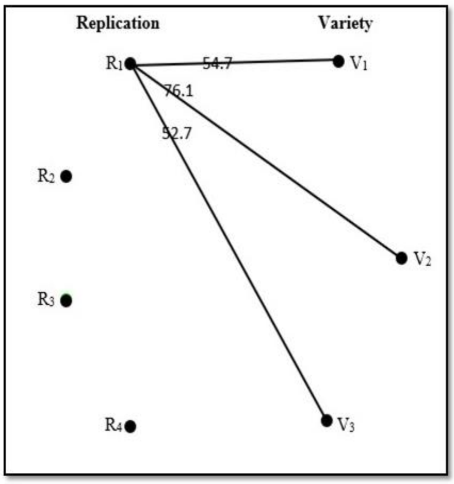

Figure 3: *Graph for first replication (R_1)*

Figure 4 shows that the second replication vertex (R_2) and it is connected to all the vertices of variety *(V_1, V_2 and V_3)* through the edge values 64.5*(Y_1)* 55.3*(Y_2)* and 51*(Y_3)*. Similarly, figure 5 shows that the third and fourth replication vertices R_3 and R_4 are connected to all the vertices of variety *($V_1, V_2$and V_3)* through the corresponding edge values *($Y_1, Y_2$and Y_3)* 41.8,65, and 59.8 *($Y_1, Y_2$and Y_3)* 45.8, 46.8 and 35.8.

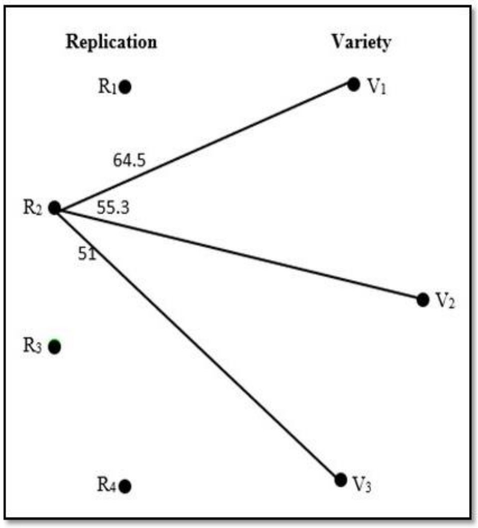

Figure 4: *Graph for second replication (R_2)*

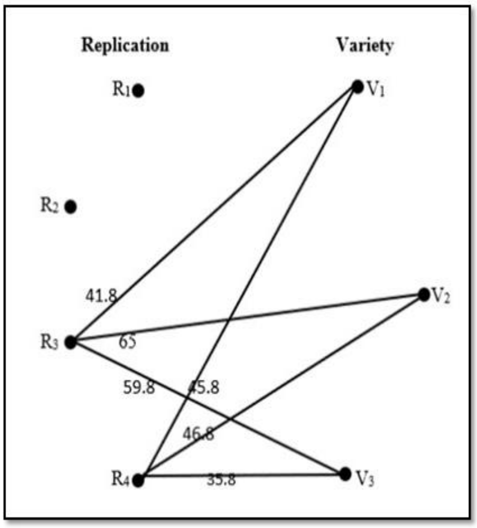

Figure 5: *Graph for third and fourth replication (R_3 and R_4)*

Finally, figure 6 shows that the complete bipartite graph of variety and replication for main - plot.

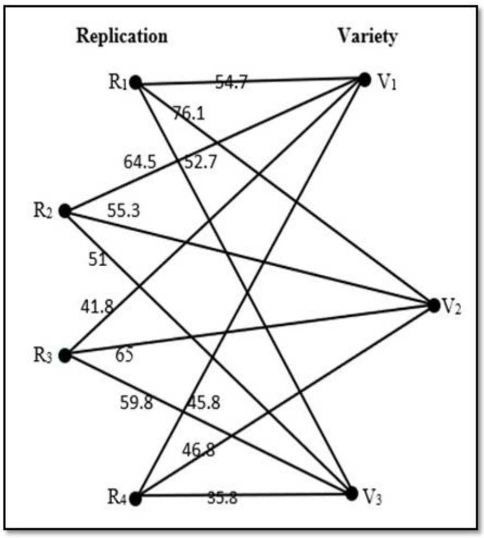

Fig. 6: *Graph for complete bipartite graph of main - plot*

Table 5: *Variety × nitrogen (V × N) for sub-plot*

	N_1	N_2	N_3	Variety Total
V_1	61.4	66.9	78.5	206.8
V_2	70.4	82.2	90.6	243.2
V_3	61.2	67.6	70.5	199.2
Nitrogen Total	193	216.7	239.6	649.3

The construction of complete bipartite graph for the sub-plot are given below.
From the above table 5 as vertex is fixed as G, which is divided into two subsets, figure 7 shows that G_1(variety) and G_2 (nitrogen). Figure 8 shows that the first variety (V_1) is connected to all nitrogen ($N_1, N_2 and\ N_3$) through the values 61.4(Y_1), 66.9(Y_1) and 78.5(Y_1).

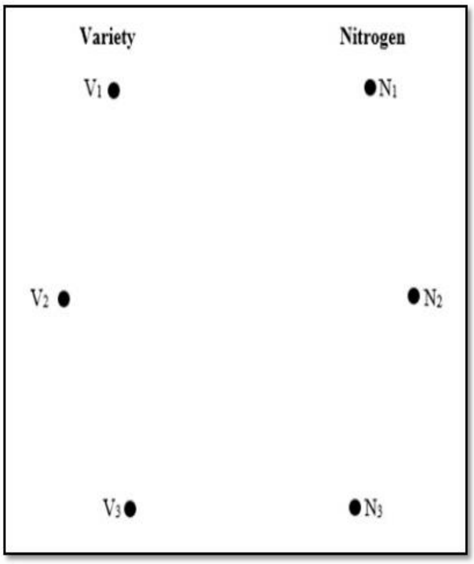

Figure 7: *Graph for vertex subset*

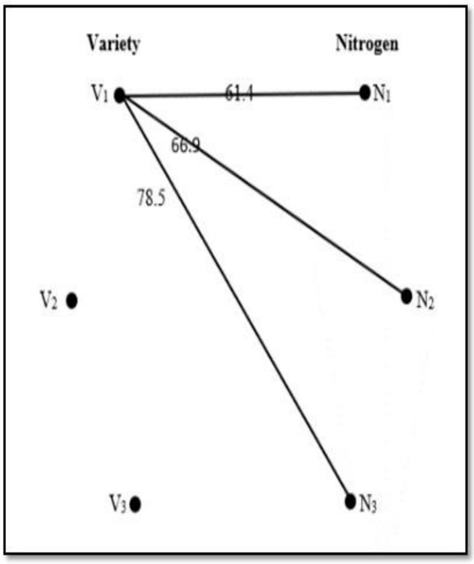

Figure 8: *Graph for first variety (V_1)*

Similarly, figure shows that the second and third variety V_2 *and* V_3 is connected to all the nitrogen (N_1, N_2 *and* N_3) through the corresponding values 70.4, 82.2 and 90.6 (Y_1, Y_2 *and* Y_3), 61.2, 67.6 and 70.7 (Y_1, Y_2 *and* Y_3).Finally, figure 10 shows the complete bipartite graph for variety and nitrogen.

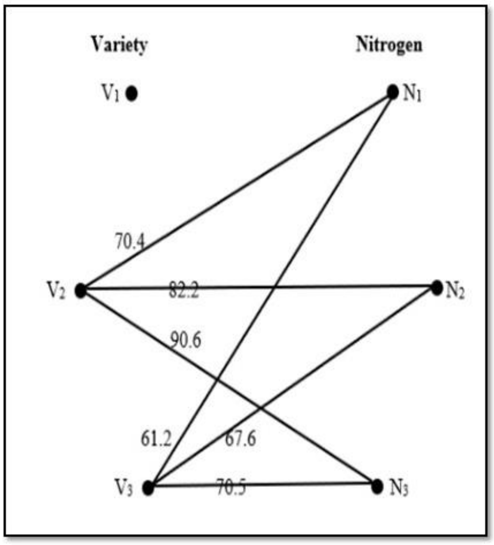

Figure 9: *Graph for second and third variety (V2 and V3)*

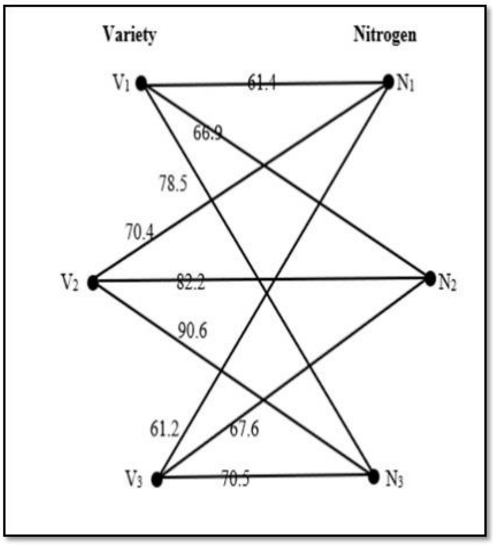

Figure 10: *Complete bipartite graph for sub – plot*

The sum of squares for main- plot:

- Correction factor (CF) = 17910.8469
- Total sum of square (SST) = 637.46
- Replication sum of square (SSR) = 187.709
- Variety sum of square (SS*V*) = 91.9006
- Main-plot error sum of square (SSE_1) = 175.4527

The sum of squares for sub-plot:

- Nitrogen sum of square (SSN) = 90.4906
- Interaction effect sum of square (V×N) = 10.4194
- Sub-plot error sum of square (SSE_2) = 81.4869

The ANOVA table for split-plot design is shown in below table:

Table 6: *ANOVA table for split-plot design*

Sv	Df	Ss	Mss	F-Ratio	P-Value
Replication	3	187.7098	62.570	2.1397	0.196473
Variety *(A)*	2	91.9006	45.950	1.5714	0.282634
Main - plot error*(E_1)*	6	175.4527	29.242	-	-
Nitrogen*(B)*	2	90.4906	45.245	9.9945	0.001204**
Interaction *(AB)*	4	10.4194	2.605	0.5754	0.684090
Sub - plot error*(E_2)*	18	81.49	4.527	-	-
Total	35	-	-	-	-

The table value of replication and variety are greater than the calculated values. So, the null hypothesis is accepted. There is no significant difference between the four replications and three varieties. The table value of nitrogen level is greater than the calculated value. So, the null hypothesis is accepted. There is no significant difference between the three nitrogen levels. The table value of the interaction effect is also greater than the calculated value. So, the null hypothesis is accepted.

There is no significant difference between the interaction effects. The P-value of the above experiment is greater than the 5% level of significant. Therefore, the null hypothesis is accepted. There is no significant difference that occurred in the above experiment.

4.2 Method for Construction of Colored Graph

- Let us consider the main-plot and sub-plot factors as the vertex set $S_1$and S_2 . Here the common factor assigned as S_1 and the other factor assigned S_2.
- The vertices of set $S_1$and S_2 are colored using the vertex coloring and the vertices are differentiate with different colors. Now consider the first vertex of S_1 and it is connected to the corresponding vertices of S_2 through edges.
- Next the second vertex of S_1which is connected to the corresponding vertices of S_2 through edges.
- Similarly, all the remaining vertices of the first set are connected to the corresponding vertices of second set through the corresponding edges.
- Finally, we get the colored graph (vertex coloring graph) for main-plot and sub-plot.

4.2.1 Application

The test is designed to examine pigment dispersion in paint. Three different mixing levels of a particular pigment are studied. The procedure consists of three application methods (brushing, sparing, and rolling) and measured the percentage reflectance of a pigment. Four days required running the experiment from hardware shops in salem district and the data obtained below.

Table 7: *Replication wise data form mixes level and application method of paint*

Replication	R_1	R_2	R_3	R_4
Application Method		Mixing level (M_1)		
A_1	65.8	70.2	65.2	69.2
A_2	69.8	65.3	70.5	63.8
A_3	70.8	67.3	68.2	69.8
		Mixing level (M_2)		
A_1	68.7	73.5	69.9	66.8
A_2	74.8	70.8	68.8	67.8
A_3	50.8	69.1	71.8	63.2
		Mixing level (M_3)		
A_1	72.2	77.8	71.6	70.8
A_2	81.5	69.2	75.7	65.2
A_3	69.3	71.6	77.8	60.8

Table 8: *Replication × application method (R×A) for main - plot*

	A_1	A_2	A_3	Replication Total
R_1	206.8	226.1	190.9	623.8

R_2	221.5	205.3	208	634.8
R_3	206.7	215	217.8	639.5
R_4	206.8	196.8	193.8	597.4
Application Total	841.8	843.2	810.5	2495.5

Table 9: *Application method × mixing (A×M) for sub-plot*

	M_1	M_2	M_3	Application Total
A_1	270.4	219	292.4	841.8
A_2	269.4	282.2	291.6	843.2
A_3	276.1	254.9	279.5	810.5
Mixing Total	815.9	816.1	863.5	2495.5

The procedure for constructing the colored graph mentioned in section 4.2 for main-plot and sub-plot for the above experiments and then the finalize.

Here we take main-plot factors (replication and application) and sub-plot factors (application method and mixing level) as the set $S_1$and S_2. Figure 11 shows that the Here S_1 consists of the common factor which is application and S_2 consists of factors such as replication and mixing level. Next, figure 12 shows that the first vertex A_1 of first set and then A_1 is connected to the corresponding (replication and mixing) vertices of the second set through edges.

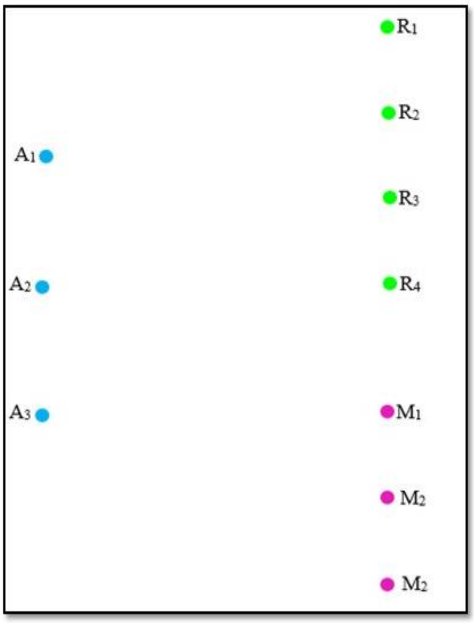

Figure 11: *Colored graph of vertex set*

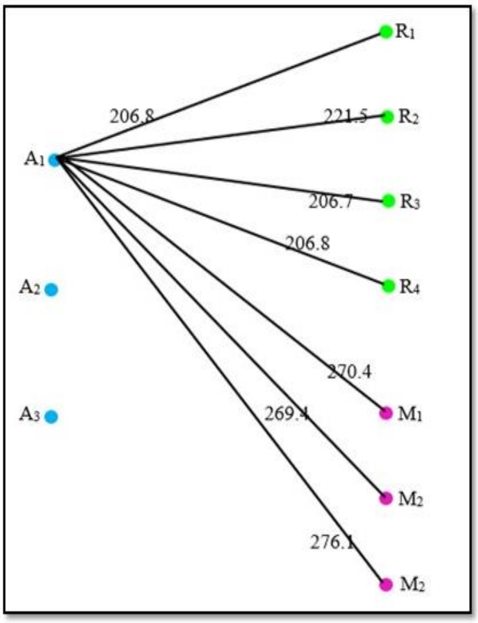

Figure 12: *Colored graph for first application method*

Next figure 13 shows that the second vertex A_2 of first set and it is connected to the corresponding vertices of second set. Similarly, figure 14 shows that the third vertex A_3 of first set are connected to the corresponding vertices of second set and finally, we get the colored graph (vertex coloring graph) for main-plot and sub-plot.

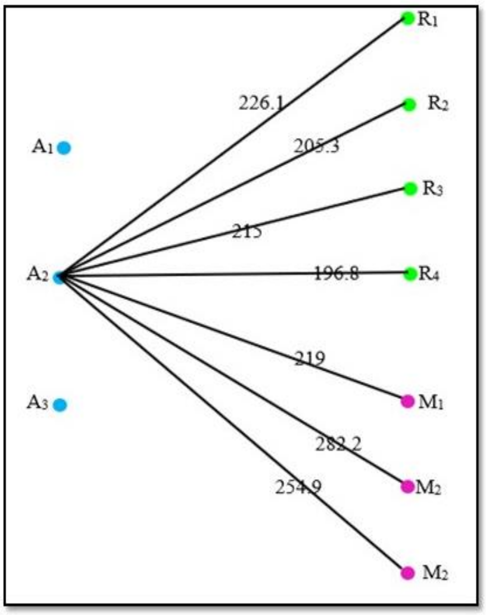

Figure 13: *Colored graph for second application method*

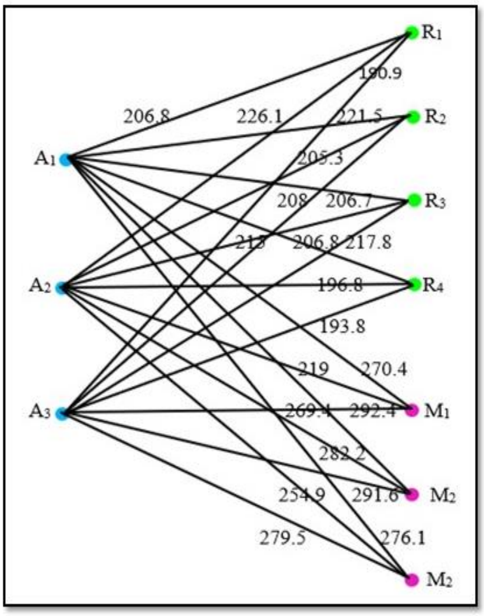

Figure 14: *Colored graph for main – plot and sub - plot*

The sum of squares for main-plot:

- Correction factor (CF) = 172986.6738
- Total sum of square (SST) = 975.3762
- Replication sum of square (SSR) = 118.269533
- Application method sum of square (SS*A*) = 56.970367
- Main-plot error sum of square (SSE_1) = 173161.9137

The sum of squares for main-plot:

- Sub-plot sum of square (SSM) = 125.348
- Interaction effect sum of square (A×M) = 87.0449
- Sub-plot error sum of square (SSE_2) = 334.2397

Table 10: *ANOVA table for split- plot design*

Sv	Df	Ss	Mss	F-Ratio	P-Value
Replication	3	118.2695	39.4217	0.9330	0.48062
Application method*(A)*	2	56.97037	28.4852	0.6742	0.54435
Main Plot Error*(E_1)*	6	253.503	42.2505	-	-
Mixing*(M)*	2	125.3487	62.674	3.3752	0.05691
Interaction *(AM)*	4	87.0449	21.7612	1.1719	0.35616
Sub-plot Error*(E_2)*	18	334.2397	18.5688	-	-
Total	35	-	-	-	-

The table values of replication and application method are greater than the calculated values. So, the null hypothesis is accepted. There is no significant difference between the four replications and three application methods. The table value of mixing level is greater than the calculated value. So, the null hypothesis is accepted. There is no significant difference between the three application methods. The table value of the interaction effect is also greater than the calculated value. So, the null hypothesis is accepted. There is no significant difference between the interaction effects.

The P-value of the above experiment is greater than the 5% level of significant. Therefore, the null hypothesis is accepted. There is no significant difference that occurred in the above experiment.

Table 11: *Table for comparative study*

ANOVA	Example 1	Example 2
Traditional method	Null hypothesis is accepted	Null hypothesis is accepted
R-Software method	Null hypothesis is accepted	Null hypothesis is accepted

5. Conclusion

Many of these real-world agricultural and industrial experiments involve factors called HTC. In these situations, experimenters have realized that the most efficient way to conduct an experiment is to fix the level of the hard-to-change factor and then run all or some combination of the easily changeable factors. This is repeated a few times. As we have seen, this leads to a split-plot design. Accounting for the split-plot nature of the design is equally important in the analysis of the data because the split-plot test contains two error terms. The present paper is classified into three parts namely rice production, nitrogen level and variety of rice in salem district. Constraint analysis is used to increase rice production. To construct and analyze the SPD using some special type of graphs through numerical esxamples from different field and the hypothesis testing is compared by the split-plot ANOVA method with software using method. When comparing the results of these methods, they produce the same results. Here some special type of graphs is used to construct the SPD. In future, there is an idea to expanding this procedure to other experimental designs such as strip-plot design and incomplete block designs etc.

List of Abbreviation

Sv - Sources of variance
Df - Degrees of freedom
Ss - Sum of squares
Mss - Mean sum of squares
MP(A) – Main -Plot(A)
MPE(E_1) – Main – Plot Error (E_1)
SP(B) – Sup -Plot(B)
SPE(E_2) - Sup – Plot Error (E_2)
IE(AB) - Interaction Effect (AB)

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