THE USE OF EXPERIMENTAL MODELLING IN THE PREDICTION OF PRODUCT RELIABILITY

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Abstract

When designing new systems and components, it is very important to correctly determine the degree and ability of the joint to withstand stress and load. Every new product that is intended for the market must meet the requirements for high safety and reliability during the entire life cycle. The presented article deals with the possibility of modelling the ability to withstand such a load, the principle of the interference method was used in the experimental modelling. The interference theory of reliability is based on the analysis of regularities and properties of two random variables that characterize reliability. Among these elementary properties from the point of view of reliability assessment, we can successfully use dependability and lifetime analysis. It originates from the concept of "safe life", which is determin-istic, based on determining and respecting the values of reliability factors. The described approach as-sumes that a malfunction or a faulty function occurs when the strength limit of the object is exceeded, i.e., ability to withstand stress.

Keywords: Reliability, Interference theory, Dependability, Load, Strength

1. Introduction

The interference theory of reliability is based on the analysis of the regularities and properties of two random variables that characterize the elementary properties of dependability and lifetime. Interference reliability theory offers reliability prediction in new product design because it can simulate the various loads and stresses that are applied to the product during its life cycle. The method uses the assessment of reliability properties in interesting interactions, which ultimately affect the resulting reliability. Such analyses are important precisely in the first stages of the product life cycle and are therefore successfully included in the process of creation and production of parts. The basic step of the analysis is the observation of two random variables, which we will describe in the following text.

Distribution of random variables

•The first random variable characterizes the operating mode and the resulting operating stress L (Load stress). Operating stress is caused by the sum of external stress and the conditions of the selected modes of use.

• The second random variable quantifies the strength S (carrying capacity). Strength to load S (Strength) is the ability to withstand physical, or chemical and biological loads, which, because of their action, result in changes causing element failures.

Both parameters of the model are random variables, characterized by random variables or processes. The form of their expression can be expressed by a histogram, or after statistical processing by probability distribution functions [1]. The literature presents many models of analytical quantification of dependability interference for the cases of exponential, normal, Weibull or gamma and log-normal distributions of load probability densities fL (L) and strength fS (S) [2].

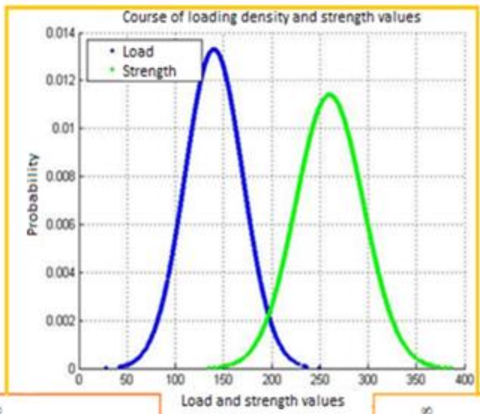
For the combination of different distributions of load and strength, the method of calculating the integrals of the two-dimensional joint function is complex, and the calculation of fault-freeness is difficult. Then it is advanta-geous to use mathematical or simulation modelling [3]. Today, the reliability of products is successfully predicted already when designing new systems and can effectively use mathematical modelling and simulation. From the point of view of partial reliability properties, in the presented article, the authors will focus on the prediction of dependability modelling. Therefore, we will deal with the calculation method of the interference theory of relia-bility in the present paper.

2. Definition of the model

The assessed system or object Mk, which is exposed to the load during the monitored time, will be reliable if the given operating stress L together with a certain probability does not exceed the strength S.

$$Mk = Pr\ (S > L) \tag{1}$$

Where:
Mk... System or object,
S…Stress [%/MPa],
L…Load Stress [%/MPa].



$$F = \int_0^\infty f_L(L).F_S(L)dL$$

$$F = \int_0^\infty f_S(S)[1 - F_L(S)dS]$$

Figure 1: *Representation of the range of interference*

This area (Figure.1) is proportional to the probability of a malfunction. It expresses the fact that due to the random properties of quantities (mainly their dispersion) there is a certain degree of possibility - the probability that a state will occur when the stress will be greater than the resistance to failure in each case. As a result, a malfunction will occur. When calculating the probability of trouble-free operation R, assuming that the random variables L, S are independent of each other, we can start from the well-known fact that the probability of the simultaneous occurrence of two independent phenomena is equal to the product of their probabilities.

The quantities characterizing the operating stress L (load) and the strength of the structure S are expressed by distribution functions and probability densities. Let us denote the probability density for the random stress variable L by fL(L) and the probability density for the random variable S against failure by fS(S). Let's denote the distribution function for the random stress variable L by FL(L) and the distribution function for the random stress strength variable S by FS(S). The quantities L and S are random, they have a specific probability distribution law, most often continuous or discrete. They can influence each other, which means to interfere, and this property can therefore be successfully used when assessing the reliability of a technical system or object in general. The extreme points of penetration, which arise during the analysis itself, define the area of mutual influence of both quantities his area is proportional to the probability of a malfunction [4]. The overlapping area defines the area of mutual influence of both quantities. It is proportional to the probability of failure and expresses the fact that due to the random properties of quantities (primarily their dispersion) there is a certain degree (probability) of the possibility that a state will occur when the stress will be greater than the strength to failure in each case and as a result a failure will occur. The area expresses the fact that due to the random properties of the quantities (especially their dispersion) there is a certain degree of possibility - the probability that a state will occur where the stress will be greater than the strength to failure in the given case. As a result, a malfunction will occur [5]. The curves are shown in Fig. 1. When calculating the probability of trouble-free operation R, we can assume that the random variables L, S are independent of each other, based on the known fact that the probability of the simultaneous occurrence of two independent phenomena is equal to the product of their probabilities. In accordance with the introduction of labels for the probability densities of quantities L and S, the following will apply to the probability of dependability operation R:

$$F = \int_0^\infty f_L(L).F_s(L)dL \tag{2}$$

Or

$$F = \int_0^\infty f_S(S)\,.\,[1 - F_L(S)dS] \tag{3}$$

The mentioned relationships are the methodological basis for modelling the failure rate or failure-freeness of elements using the SSI interference method [6,7].

3. SSI simulation model

The input quantities of load L and strength S have a random character obtained from experimental measurement. The result is a non-parametric distribution of the obtained data, which we can statistically process in the form of a histogram or convert to a usable parametric distribution, as illustrated in Fig. 2. [8]. Both cases provide us with the possibility of generating input quantities and assessing the occurrence of decisive events for the statistical expression of failure rate or failure-freeness of elements using the interference method.

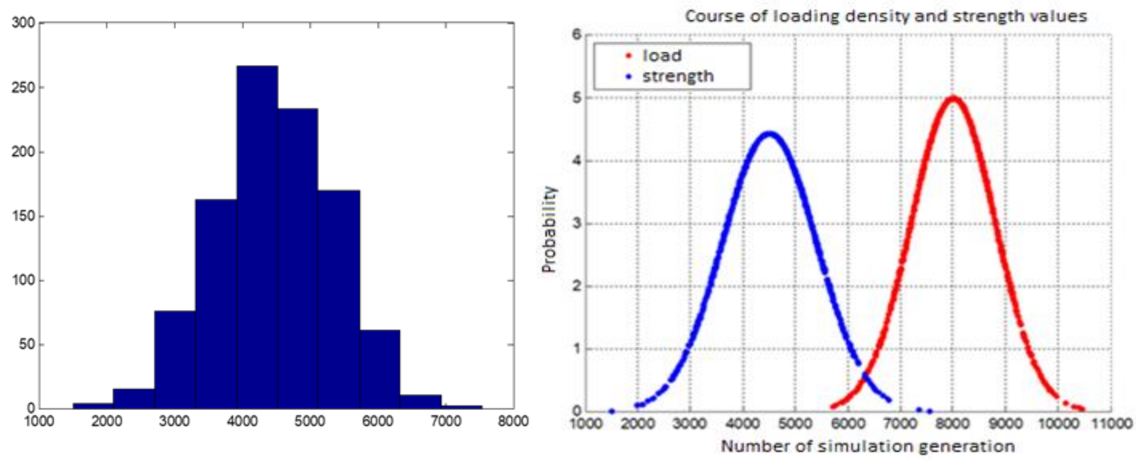


Figure 2: *Expression of random variables L and S by histogram of relative abundances and probability distribution density*

For the range of experimental or generated values, we determine the size of the values of the distribution functions $F_L(S)$ and $F_S(S)$ for different strength values S. For the range of values of both functions, stress L and S contribute to failure. If we plot the values of L and S in the interdependence graph, the intersection represents the product of two independent phenomena. The area below the line of the graph represents the probability of fault-free operation expressed, and the area above the line of the graph represents the probability of the occurrence of a fault. A probability distribution model is characterized by a density function and a distribution function based on precisely specified parameters that need to be estimated from the data using a likelihood function. We also test hypotheses in statistical models, which often represent models of causal dependence of dependent variables on predictors. In the experiment, graphic tools are used and serve for a quick and illustrative presentation of the results, especially when it comes to more comprehensive data and mutual comparison of several files. By graphically representing the frequency distribution, we get a clear idea of the nature of the frequency distribution of the observed character.

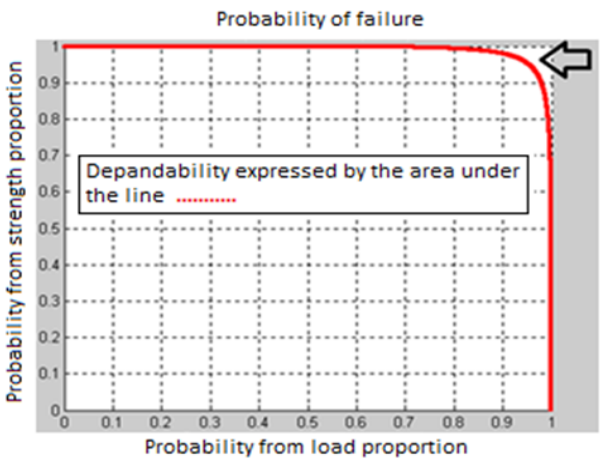


Figure 3: *Probability of failure*

The modelling procedure was designed as follows. The first step is to obtain the input data of histogram parameters, or the probability distribution of operating load L and strength S of the investigated system element. Subsequently, a random level of operating load and strength is generated, thus creating a point of realization of the phenomenon. The next analysis will assess which area it falls into and show it graphically. The boundary between fault-free and fault-free areas is given by the condition S > = L and expressed by the red line in the fig. 3. If the condition S > = L is not met, this is a fault condition. We record the number of simulation steps N and the number of failure states n. We will statistically process the generated data into the form of values of probability density functions and distribution functions, and by plotting them we will get an idea of their interference. In the proposed procedure, we will successfully use the MATLAB simulation language, because its graphics allow the creation of interactive programs, the environment of which allows the user to dialogically change the parameters of the distributions and judge what load and strength values are acceptable for the structural design application. The program in the basic window offers the option of choosing the type of load distribution and strength of the investigated element, distribution parameters and the number of simulations. If, from the input data used, the simulation results indicate that the required fault-free parameters do not meet, the simulations can be carried out by changing the load and strength parameters until an acceptable level of interference is reached.

4. Steps of the experimental simulation model

The first steps of the analysis require the determination of the number of simulations and the loading of the necessary input data of the parameters of the distributions of the probability density functions of the operating load and the strength of the investigated element. The verification analysis is shown in fig.4.

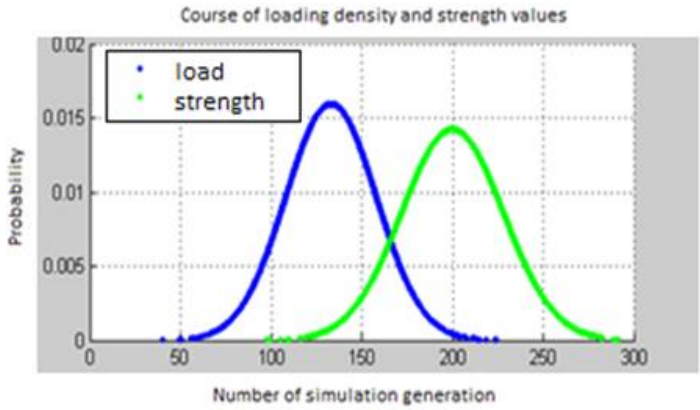


Figure 4: *Determination of the number of simulations in the mathematical model*

The level of operational load and strength is generated and statistically processed into values of probability density functions and distribution functions. The curves of the distribution functions are shown in fig. 5.

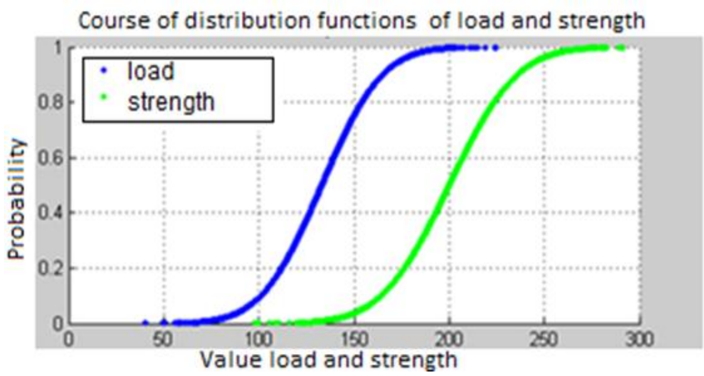


Figure 5: *Determining the operating load and strength of the part*

The next step of the analysis is to plot the curve of probability density functions and distribution functions, shown in Fig. 6. Determines the minimum value of the strength function and the maximum value of the load function. Plots the interdependence of L and S values. Calculates the size of the area under the graph line.

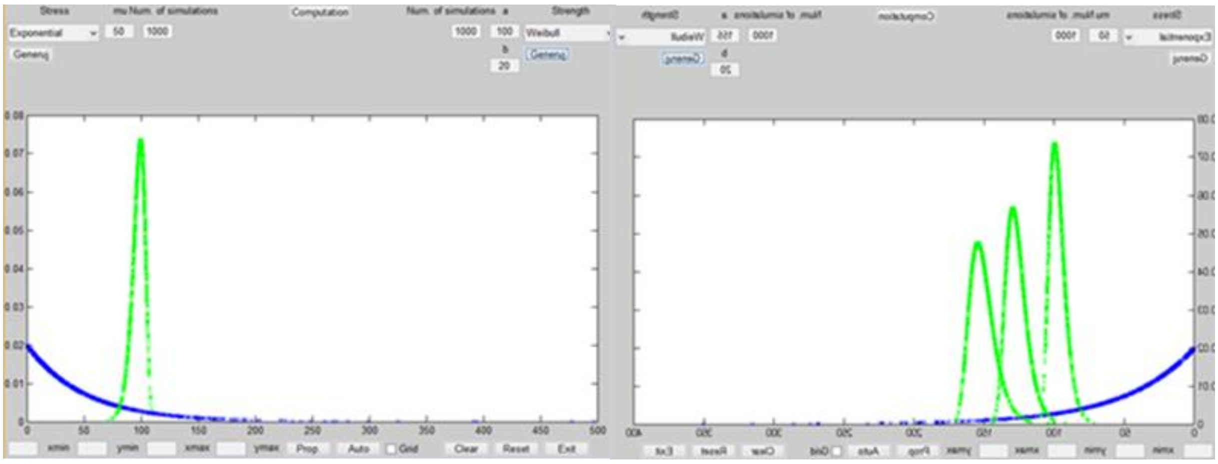


Figure 6: *Probability density functions and distribution functions*

The next step of the experiment was to analyse the impact of changes in the values of standard deviations.

5. Outputs of experimental simulation of parameters of dependability

Load L is a stochastic quantity with properties as in the previous case. It has its distribution of the probability of occurrence at individual levels, which do not change its character (type and parameters of the distribution) with time (period of operation). The resistance of the structure to failure S with time does not change its type (law) of distribution but changes its position relative to the origin of the coordinates. A change in position occurs when the stress repeatedly exceeds a certain threshold limit Sc of the sensitivity (resistance) of the structure. The application of the dynamic model requires the clarification of some important concepts and properties of the random variables used in the model. Above all, the clarification of the stochastic nature of the quantities S and L, especially their possible change with the time of stress exposure, and further the concept of "accumulation of damage". The possibilities of variations in how the system will react to different strength need to be verified by repeated modelling. ongoing analyses can be evaluated in Fig.7.

Procedure for processing the analysis experiment:

- In the first step, we summarize the input data that evaluates the parameters of histograms, distribution probabilities
- Simulation of random variable operating load and resistance.
- Modelling the point of realization of the phenomenon, assessing which area it falls into and graphically representing it.
- Graphically determine the boundary between the fault-free and fault-free areas, represented by a red line in the picture. 3.
- If the condition S > = L is not met, this is a fault condition.
- Control of the number of simulation steps N to the ratio of the number of failure states n.
- We statistically process the generated data into the form of values of probability density functions and distribution functions.
- Generation of mutual interference of phenomena.
- We calculate probability of failure.

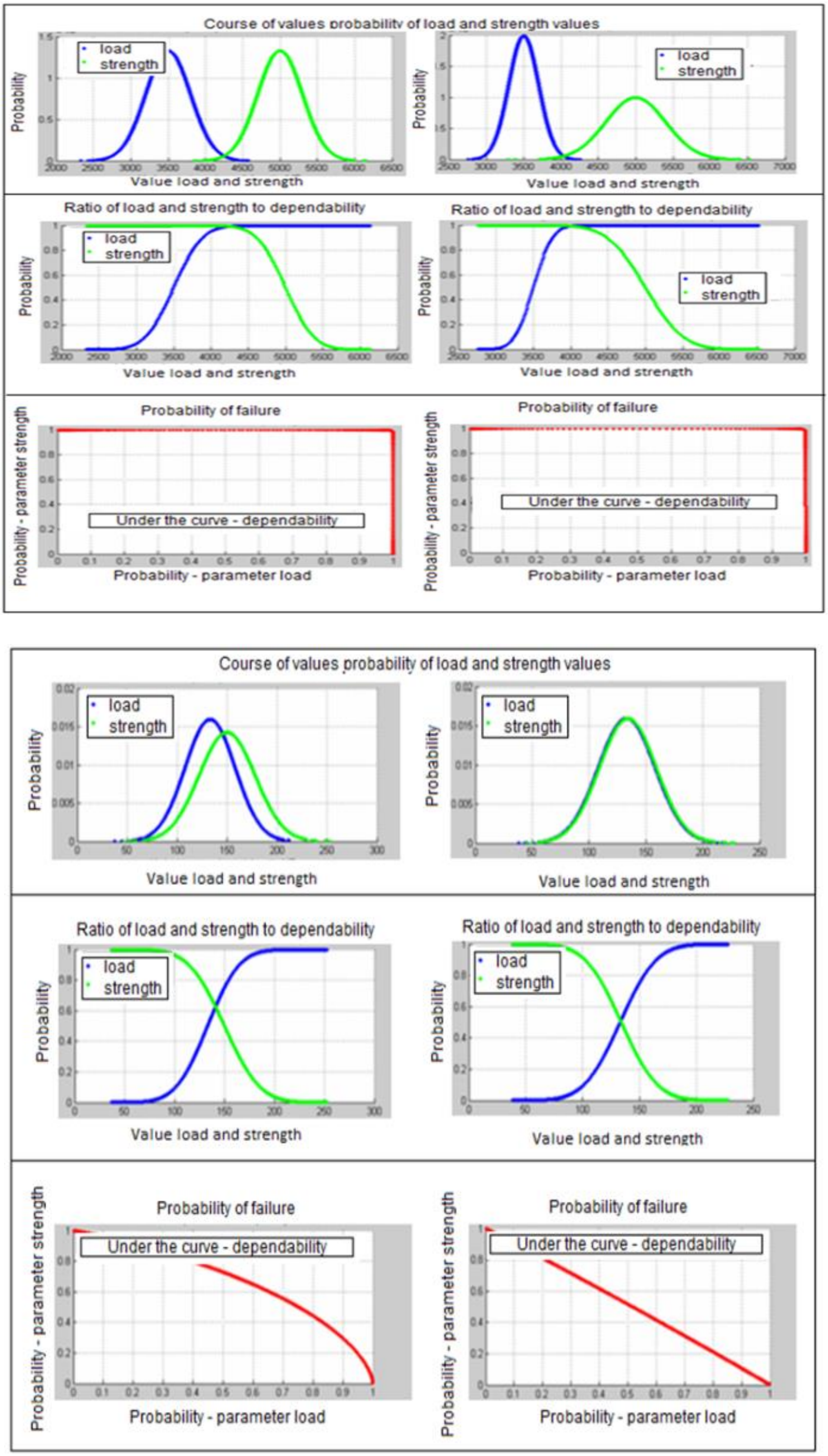


Figure 7: *Principle of the impact of changes in the strength parameter*

The method of creating a statistical probability model and graphical representation of the share of load and strength in the form of histograms of the generated values can be expressed with the scaatherhist function. The next step is the analysis of the probability of failure achieved in a simulation experiment with parameters according to Table 1. Again, we choose a smaller number of simulations, for indicative results. With a higher number of simulations, the accuracy of the result increases, but the graphical representation of the results deteriorates.

Table 1: *Input and output parameters of the simulation experiment*

Input:		Output:	
Number of simulations	6 1000	Probability of dependability	0.919
Load parameters, Exponencial	Median 50		
	-	Probability of failure	0.081
Strength parameters, Weibull	Median 130		
	Standard deviation 20		

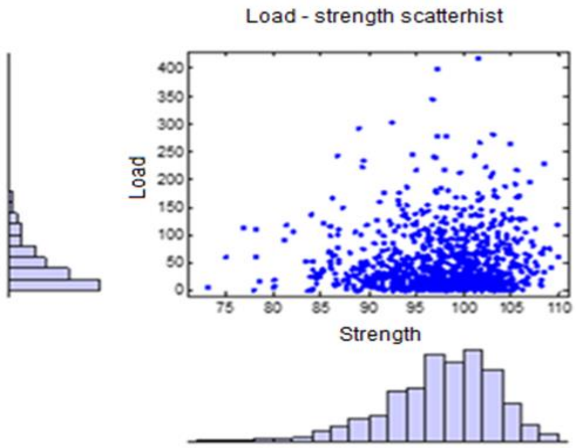


Figure 8: *Graphic representation of load and strength ratio*

The results of the simulation and their graphic representation point to a high degree of influence of individual parameters of strength and load. The principle of the approach is shown in Fig. 8 Graphical analysis of the impact of changes in the strength parameter. When assessing the results of the simulation, it must be remembered that the engineering object (element) has the structural and material properties to withstand stress. Load and strength are expressed by quantities that can be characterized as dynamic and stochastic. During operation, the engineering object (element) is stressed by combined effects, namely: Operational stress: operational load, environmental effects, and the human factor. This is represented by the quantity L - load. And resistance to physical stress, chemical stress, and biological stress, represented by the component S – strength. In Fig. 9. an analysis is shown, which provides a graphical output describing the state when we can identify failure. The red area represents a high load that the system is no longer able to withstand. Below the critical line is the permissible area. It is an area that characterizes dependability.

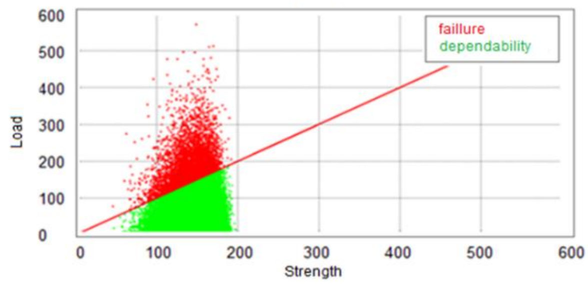


Figure 9: *Graphical representation of the 2D failure and failure-free set of the realization of 100,000 simulations*

6. Conclusion

An equivalent stress is a stress that has one constant level (level, amplitude) which, if applied to a component with frequency Sometimes, will cause a failure after the same lifetime of the component as would cause the complete spectrum of stress acting in service at all levels. So, the damage after a certain period of operation (life) caused by this equivalent stress is the same as the damage caused during the same period of operation by the complete spectrum of operational stress [9]. Thus, we can assume that any arbitrary operating stress spectrum can be "converted" to a single level equivalent spectrum of the described properties. The dynamic model is applied primarily to such processes when the strength S against failure due to repeated exposure to Load L of different (randomly variable) levels changes with the duration of operation (time). These are e.g., typical cases of element damage due to phenomena associated with material fatigue, exceeding the set parameters limits [10,11].

The results of experimental simulation using the reliability interference method can be summarized in the following advantages:

- The construction, component will be reliable if the operating load L does not exceed the strength S with a certain probability.
- The quantities L and S are random, and we assume that they have a specific probability distribution law.
- The operational load and strength of the structure will be expressed by probability densities and distribution functions.
- Load and strength are quantities that can influence each other (interfere).
- The extreme points of penetration delimit the area of mutual interference, which is proportional to the probability of the occurrence of a fault.

The simulation model makes it possible to eliminate the shortcomings of classical calculation methods and to use the results of few experimental measurements, to determine the interference of different probability density distributions of randomly variable functions of permitted operating loads and strength, to apply the results to determine the reliability of elements of diverse systems and, last but not least, to use graphic outputs for didactic support of the method explanation SSI and the behaviour of random variables of different probability distributions.

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