RT&A, No 1 (77)
Volume 19, March 2024

STUDIES ON A NEW MANPOWER MODEL WITH NON-HOMOGENEOUS POISSON RECRUITMENT, PROMOTION AND LEAVING PROCESSES

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Abstract

For proper utilization of manpower in any organization manpower modeling is needed. This paper addresses the two graded manpower model with non-stationary recruitment, promotion and leaving processes. Here it is assumed that the recruitment process in the first grade follows a NHP process which is further assumed that the promotion and leaving processes are also NHP processes. Using the difference-differential equations, the joint p.g.f of the number of employees in the organization at any time 't' is derived. The characteristics of the model such as the average number of employees in each grade, the average waiting time of an employee in each grade, the variance of the number of employees in each grade and the C.V of an employee in each grade are derived explicitly. The sensitivity analysis of the model with respect to the changes in parameter is also studied through numerical illustration. The comparative study between homogeneous Poisson recruitment and NHP recruitment is also discussed. This model also improves some of the earlier models as particular cases.

Keywords: NHP process, two-graded manpower model, duration of stays any grade, performance of the model.

1. Introduction

An optimal utilization of Human Resources planning of manpower structure is a prerequisite for any organization. Hence, several works have been reported in literature regarding manpower models with various assumptions on the constituent processes. Graded manpower systems and its analysis are more important in order to develop policies of the organization with respect to manpower. Starting with the pioneering work by Seal [1] with manpower modeling of human resources much work has been reported in literature regarding graded manpower systems (Srinivasa Rao et al. [2]). The different approaches in manpower modeling are explained by Ugwuowo [3] and Wang [4]. Parthasarathy et al. [5] have analyzed the two grade system and tried to use to represent the threshold as a specific case of the exponentiated exponential distribution (EE distribution). Jeeva and Geetha [6], Gulzarul Hasan [7, 8] studied the manpower models governed by a fuzzy environment. Kannan Nilakantan [9] analyzed the manpower models with staffing policies. Maijamma [10] is approach has the benefit of being the first to use linear programming and determined the ideal number of hires and promotions to make in order to reduce the overall cost of the manpower planning system, particularly the cost of hiring and promoting people. This study specifically examined how applying the linear programming model can result in lower recruitment and promotion costs. In terms of dependability and attainability, the actionable model has been found to be effective and reliable. Sathiyamoorthi and Elangovan [11], Lalithadevi and Srinivasan [12] have utilized geometrical process and shock models for analyzing single graded manpower models. Parameswari [13] studied the estimation of the variability of the time to recruitment for a two-graded personnel system. Ravichandran [14], Sendhamilzselvi et al. [15, 16] studied on calculating the mean and variance of the time to recruitment in a two graded manpower system with two continuous thresholds for depletion.

The Poisson process is extensively utilized in manpower models for analyzing the manpower system with respect to various organization by Srinivasa Rao et al. [17], Kondababu and Srinivasa Rao [18], Srinivasa Rao and Kondababu [19], Govinda Rao et al. [20, 21]. Srinivasa Rao and Mallikharjuna Rao [22] have studied two graded manpower models with NHP recruitments. NHP processes can be used to incorporate time-varying complexity. In order to reflect potential recruitment patterns over time, one can use this method. The time spent on trial recruitment modelling has many advantages. Saral et al. [23] has studied manpower models with two graded systems with respect to recruitment policy and thresholds. Jayanthi [24] studied and analyzed the single graded system by considering time to recruitment with breakdown thresholds. Thilaka et al. [25] studied a method by deriving the characteristics of a two-grade human resource system under the conditions that (a) personnel can move from one grade to the next for training and skill improvement, and (b) people who previously left the system can be hired in both grades. The steady state and transient behaviors are discussed. Srinivasa Rao and Ganapathi Swamy [26, 27] studied the manpower models with Duane recruitment processes. They considered that the leaving or promotion processes are stationary and independent of time. But in many practical situations it is observed that the employee leaving and promotion is dependent on time for example in corporate and public sector offices having the graded system employee promotions or leaving is done based on the time and duration of their stay in the organization. Hence, in analyzing the manpower models ignoring the non-stationary influence off promotion or leaving process may lead to falsification in the model and may not estimate the characteristic of the model accurately if the system is governed by non-stationary.

To have an accurate analysis one has to consider the non homogeneity of the recruitment/promotion/leaving processes of the models. Very little work has been reported in literature regarding manpower models with non-homogeneous recruitment/promotion/leaving processes in graded systems. Therefore in this paper, the model with NHP recruitment, promotion and leaving processes is developed and analyzed. The rest of the paper is arranged as follows: Section 2 deals with the development of the two graded manpower model using the difference differential equations. Section 3 deals with the derivation of the characteristics of the model such as probability of extinction, probability of at least one employee in grade 1 and grade 2, average number of employees in each grade, the variance of the number of employees in the organization and the variance of the number of employees in the organization. Section 4 deals with numerical illustration and discussion on the characteristics of the model. Section 5 deals with sensitivity analysis of the model. Section6 is to compare the proposed model with that of the manpower model with homogeneous poison recruitment and promotion/leaving processes. Section 7 deals with conclusions.

2. Two graded manpower model

Consider a two graded manpower model in which the organization is having two grades namely, grade-1 and grade-2. The recruitment process of grade-1 is assumed to follows a NHP process with mean recruitment rate is $\lambda(t) = \lambda_1 + \lambda_2 t$. The promotion process from grade-1 to grade-2 follows a NHP process with mean promotion rate $\alpha(t) = a_1 + a_2 t$. The leaving processes in grade-2 follow a NHP process with mean leaving rate $\beta(t) = b_1 + b_2 t$.

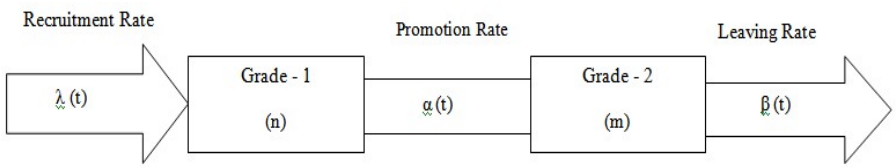

Figure 1: *Manpower model*

With these suppositions, the model postulates are:

- The probability that an employee will be recruited in grade-1 at random intervals of time h is [λ(t) h + o(h)].
- When there are 'n' employees in grade 1, the probability of a promotion from grade-1 to grade-2 during an random interval of time 'h' is [n α(t) h + o(h)].
- When there are 'm' employees in grade 2, the probability of an employee quitting the company from grade-2 during an random interval of time 'h' is [m β(t) h + o(h)].
- When there are 'n' employees in grade 1 and 'm' employees in grade-2, the probability that no employee will join or leave the company during an tiny interval of time 'h' is [1 - λ(t)h – n μ(t)h – m β(t) h + o(h)].
- The probability that an event other than those listed above took place within a tiny period of time 'h' is o (h).

Let $P_{n,m}(t)$ represent the probability that the organization will have 'n' employees in grade-1 and 'm' employees in grade-2 at time t. The difference-differential equations of the model with this structure are:

$$\frac{\partial P_{n,m(t)}}{\partial t} = -[\lambda(t) + n\,\alpha(t) + m\beta(t)]p_{n,m}(t) + \lambda(t)P_{n-1,m}(t) + (n+1)\alpha(t)P_{n+1,m-1}\,(t) + (m+1)\beta(t)P_{n,m+1}\,(t)\forall\, n,m\, \geq 0 \tag{1}$$

$$\frac{\partial P_{n,0(t)}}{\partial t} = -[\lambda(t) + n\,\alpha(t)]P_{n,0}\,(t) + \lambda(t)P_{n-1,0}\,(t) + \beta(t)P_{n,1}\,(t)\forall\, n > 0, m = 0 \tag{2}$$

$$\frac{\partial P_{0,m(t)}}{\partial t} = -[\lambda(t) + m\,\beta(t)\,]P_{0,m}\,(t) + \alpha(t)P_{1,m-1}\,(t) + (m+1)\beta(t)P_{0,m+1}(t)\forall\, n = 0, m > 0 \tag{3}$$

$$\frac{\partial P_{0,0(t)}}{\partial t} = -[\lambda(t)]P_{0,0}(t) + \beta(t)\,P_{0,1}(t)\forall n = 0, m = 0 \tag{4}$$

$P(z_1, z_2; t)$ be the joint p.g.f of $P_{n,m}(t)$. Then

$$P(z_1, z_2; t) = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} P_{n,m}(t){z_1}^n {z_2}^m \tag{5}$$

This implies

$$\frac{\partial P_{n,m}(t)}{\partial t} = -\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}[\lambda(t) + n\,\alpha(t) + \; m\,\beta(t)\,]p_{n,m}(t){z_1}^n{z_2}^m$$

$$+\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\lambda(t)p_{n-1,m}(t){z_1}^n{z_2}^m + \sum_{n=0}^{\infty}\sum_{m=0}^{\infty}(n+1)\alpha(t)\,p_{n+1,m-1}(t){z_1}^n{z_2}^m$$

$$+\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}(m+1)\beta(t)p_{n,m+1}\,(t)z_1^n z_2^m \tag{6}$$

This implies

$$\frac{\partial p(\,z_1,z_2\,;t)}{\partial t} = [\alpha(t)(z_2 - z_1)]\frac{\partial p}{\partial z_1} + [\beta(t)(\,1 - z_2)]\frac{\partial p}{\partial z_2} + \lambda(t)\,(z-1)\,P(\,z_1, z_2\,;\,t) \tag{7}$$

Solving the equation (7) by Lagrangian's method, the auxiliary equation is

$$\frac{dt}{1} = \frac{dz_1}{-\alpha(t)\,(z_2 - z_1)} = \frac{dz_2}{-\beta(t)\,(\,1 - z_2)} = \frac{dP}{-\lambda(t)(\,1 - z_1)P(z_1, z_2\,,t)} \tag{8}$$

Consider the recruitment rate, promotion rate and leaving rates are linear and time dependent and is of the form.

$\lambda(t) = \lambda_1 + \lambda_2 t$

$\alpha(t) = a_1 + a_2 t$, Where $a_1 > 0$, $a_2 > 0$

$\beta(t) = b_1 + b_2 t$, Where $b_1 > 0$, $b_2 > 0$

First and third terms in equation (8), will give

$$A = (z_2 - 1)e^{-\int \beta(t)dt} \tag{9}$$

$$B = z_1 e^{-\int \alpha(t)dt} + (z_2 - 1)e^{-\int \beta(t)dt}\left(\int \alpha(t)e^{\int[\beta(t)-\alpha(t)]dt}\,dt\right) + \int \alpha(t)e^{-\int \alpha(t)dt}\,dt \tag{10}$$

First and fourth terms in equation (8), will give

$$C = P(z_1, z_2; t)\,exp(-[z_1\;e^{-\int \alpha(t)dt} + \;(z_2\;-1)e^{-\int \beta(t)dt}\left(\int \alpha(t)e^{\int[\beta(t)-\alpha(t)]dt}\,dt\right)$$

$$+\int \alpha(t).\,e^{-\int \alpha(t)dt}dt]\left[\int \lambda(t).\,e^{\int \alpha(t)dt}dt\right])$$

$$+\left[(z_2\;-1)e^{-\int \beta(t)dt}\int \lambda(t).\,e^{\int \alpha(t)dt}\left(\int \alpha(t)e^{\int[\beta(t)-\alpha(t)]dt}\,dt\right)dt\right]$$

$$+\left[\int \lambda(t).\,e^{\int \alpha(t)dt}\left(\int \alpha(t)e^{-\int \alpha(t)dt}\,dt\right)dt\right] + \int \lambda(t)dt \tag{11}$$

Where A, B &C are arbitrary constants. With the initial conditions $P_{00}(0)$ =1,$P_{00}(t)$ = 0, $\forall t$ >0. We have the joint p.g.fof the number of employees in the grade-1 and the number of employees in the grade-2 at time 't' is

$$P(z_1,z_2;t) = exp[\lambda_1[(z_1-1)e^{-\left(a_1t+a_2\frac{t^2}{2}\right)\left(\frac{\int_0^t(\lambda_1+\lambda_2v)e^{\left(a_1v+a_2\frac{v^2}{2}\right)}dv}{\lambda_1}-\frac{1}{a_1}\right)}$$

$$+(z_2-1)e^{-\left(b_1t+b_2\frac{t^2}{2}\right)}\left(\frac{1}{b_1-a_1}-\frac{\int_0^t(a_1+a_2v)\,e^{(b_1-a_1)v+(b_2-a_2)\frac{v^2}{2}}dv}{a_1}\right)$$

$$-(z_2-1)e^{-\left(b_1t+b_2\frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_1+\lambda_2v)e^{\left(a_1v+a_2\frac{v^2}{2}\right)}dv\int_0^t(a_1+a_2v)e^{(b_1-a_1)v+(b_2-a_2)\frac{v^2}{2}}dv}{\lambda_1}\right.$$

$$\left.-\frac{\int_0^t(\lambda_1+\lambda_2v)e^{\left(a_1v+a_2\frac{v^2}{2}\right)}dv\left(\int_0^t(a_1+a_2v)e^{(b_1-a_1)v+(b_2-a_2)\frac{v^2}{2}}dv\right)dv}{\lambda_1}-\frac{1}{b_1}\right)]] \qquad (12)$$

3. Characteristics of the model

Expanding $P(z_1,z_2,t)$, we obtain the probability that there are no employee in the organization as.

$$P_{0,0}(t) = exp[-\lambda_1[e^{-\left(a_1t+a_2\frac{t^2}{2}\right)\left(\frac{\int_0^t(\lambda_1+\lambda_2v)e^{\left(a_1v+a_2\frac{v^2}{2}\right)}dv}{\lambda_1}-\frac{1}{a_1}\right)}$$

$$+e^{-\left(b_1t+b_2\frac{t^2}{2}\right)}\left(\frac{1}{b_1-a_1}-\frac{\int_0^t(a_1+a_2v)\,e^{(b_1-a_1)v+(b_2-a_2)\frac{v^2}{2}}dv}{a_1}\right)$$

$$+e^{-\left(b_1t+b_2\frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_1+\lambda_2v)e^{\left(a_1v+a_2\frac{v^2}{2}\right)}dv\int_0^t(a_1+a_2v)e^{(b_1-a_1)v+(b_2-a_2)\frac{v^2}{2}}dv}{\lambda_1}\right.$$

$$\left.-\frac{\int_0^t(\lambda_1+\lambda_2v)e^{\left(a_1v+a_2\frac{v^2}{2}\right)}\left(\int_0^t(a_1+a_2v)e^{(b_1-a_1)v+(b_2-a_2)\frac{v^2}{2}}dv\right)dv}{\lambda_1}-\frac{1}{b_1}\right)]] \qquad (13)$$

Taking $z_2 = 1$ in $P(z_1,z_2;t)$, we obtain thep.g.f of employees in the grade-1 in the organization as

$$P(z_1,t)= exp\left[\lambda_1(z_1-1)e^{-\left(a_1t+a_2\frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_1+\lambda_2v)e^{\left(a_1v+a_2\frac{v^2}{2}\right)}dv}{\lambda_1}-\frac{1}{a_1}\right)\right] \qquad (14)$$

Expanding $P(z_1,t)$ and collecting the constant terms, we obtain the probability that there is no employee in grade -1 of the organization as

$$P_{0\,.}(t) = exp\left[-\lambda_1e^{-\left(a_1t+a_2\frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_1+\lambda_2v)e^{\left(a_1v+a_2\frac{v^2}{2}\right)}dv}{\lambda_1}-\frac{1}{a_1}\right)\right] \qquad (15)$$

In grade-1the average number of employees in organization is

$$L_1(t) = \lambda_1 e^{-\left(a_1 t+a_2 \frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_1+\lambda_2 v)e^{\left(a_1 v+a_2 \frac{v^2}{2}\right)}dv}{\lambda_1}-\frac{1}{a_1}\right) \tag{16}$$

The probability that there the existence of employees in grade-1 of the organization is

$$U_1(t) = 1 - exp\left[-\lambda_1 e^{-\left(a_1 t+a_2 \frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_1+\lambda_2 v)e^{\left(a_1 v+a_2 \frac{v^2}{2}\right)}dv}{\lambda_1}-\frac{1}{a_1}\right)\right] \tag{17}$$

The average waiting time of an employee in grade-1of the organization is

$$W_1(t) = \frac{L_1(t)}{\alpha(t)[1-P_{0.}(t)]}$$

$$W_1(t) = \frac{\lambda_1 e^{-\left(a_1 t+a_2 \frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_1+\lambda_2 v)e^{\left(a_1 v+a_2 \frac{v^2}{2}\right)}dv}{\lambda_1}-\frac{1}{a_1}\right)}{(a_1+a_2 t)\left[1-exp\left[-\lambda_1 e^{-\left(a_1 t+a_2 \frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_1+\lambda_2 v)e^{\left(a_1 v+a_2 \frac{v^2}{2}\right)}dv}{\lambda_1}-\frac{1}{a_1}\right)\right]\right]} \tag{18}$$

The variance of the number of employees in grade-1of the organization is

$$V_1(t) = \lambda_1 e^{-\left(a_1 t+a_2 \frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_1+\lambda_2 v)e^{\left(a_1 v+a_2 \frac{v^2}{2}\right)}dv}{\lambda_1}-\frac{1}{a_1}\right) \tag{19}$$

The C.V of the number of employees in grade-1of the organization is

$$CV_1(t) = \left[\lambda_1 e^{-\left(a_1 t+a_2 \frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_1+\lambda_2 v)e^{\left(a_1 v+a_2 \frac{v^2}{2}\right)}dv}{\lambda_1}-\frac{1}{a_1}\right)\right]^{\frac{-1}{2}} \tag{20}$$

Similarly, taking $z_1 = 1$ in$P(z_1, z_2; t)$, we obtain the p.g.fof the number of employees in grade-2 of the organization as

$$\begin{aligned} P(z_2,t) = exp[\lambda_1[(z_2-1)e^{-\left(b_1 t+b_2 \frac{t^2}{2}\right)}\left(\frac{1}{b_1-a_1}-\frac{\int_0^t(a_1+a_2 v).\ e^{(b_1-a_1)v+(b_2-a_2)\frac{v^2}{2}}dv}{a_1}\right) \\ +(z_2-1)e^{-\left(b_1 t+b_2 \frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_1+\lambda_2 v)e^{\left(a_1 v+a_2 \frac{v^2}{2}\right)}dv \int_0^t(a_1+a_2 v)e^{(b_1-a_1)v+(b_2-a_2)\frac{v^2}{2}}dv}{\lambda_1}\right. \\ \left.-\frac{\int_0^t(\lambda_1+\lambda_2 v)e^{\left(a_1 v+a_2\frac{v^2}{2}\right)}dv\left(\int_0^t(a_1+a_2 v)e^{(b_1-a_1)v+(b_2-a_2)\frac{v^2}{2}}dv\right)dv}{\lambda_1}-\frac{1}{b_1}\right)]] \end{aligned} \tag{21}$$

Expanding $P(z_2, t)$and collecting the constant terms, we obtain the probability that there is no employee in grade-2 of the organization as

$$P_{.0}(t) = exp[-\lambda_1[e^{-\left(b_1 t+b_2\frac{t^2}{2}\right)}\left(\frac{1}{b_1-a_1}-\frac{\int_0^t (a_1+a_2 v).\ e^{(b_1-a_1)v+(b_2-a_2)\frac{v^2}{2}}dv}{a_1}\right)$$

$$+e^{-\left(b_1 t+b_2\frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_1+\lambda_2 v)e^{\left(a_1 v+a_2\frac{v^2}{2}\right)}dv\int_0^t(a_1+a_2 v)e^{(b_1-a_1)v+(b_2-a_2)\frac{v^2}{2}}dv}{\lambda_1}\right.$$

$$\left.-\frac{\int_0^t(\lambda_1+\lambda_2 v)e^{\left(a_1 v+a_2\frac{v^2}{2}\right)}dv\left(\int_0^t(a_1+a_2 v)e^{(b_1-a_1)v+(b_2-a_2)\frac{v^2}{2}}dv\right)dv}{\lambda_1}-\frac{1}{b_1}\right)]] \tag{22}$$

In grade-2 the average number of employees in organization is

$$L_2(t) = \lambda_1 e^{-\left(b_1 t+b_2\frac{t^2}{2}\right)}\left(\frac{1}{b_1-a_1}-\frac{\int_0^t (a_1+a_2 v).\ e^{(b_1-a_1)v+(b_2-a_2)\frac{v^2}{2}}dv}{a_1}\right)$$

$$+\lambda_1 e^{-\left(b_1 t+b_2\frac{t^2}{2}\right)}\left[\frac{\int_0^t(\lambda_1+\lambda_2 v)e^{\left(a_1 v+a_2\frac{v^2}{2}\right)}dv\int_0^t(a_1+a_2 v)e^{(b_1-a_1)v+(b_2-a_2)\frac{v^2}{2}}dv}{\lambda_1}\right.$$

$$\left.-\frac{\int_0^t(\lambda_1+\lambda_2 v)e^{\left(a_1 v+a_2\frac{v^2}{2}\right)}dv\left(\int_0^t(a_1+a_2 v)e^{(b_1-a_1)v+(b_2-a_2)\frac{v^2}{2}}dv\right)dv}{\lambda_1}-\frac{1}{b_1}\right] \tag{23}$$

The probability that there the existence of employees in grade-2 of the organization is

$$U_2(t) = 1 - exp[-\lambda_1[e^{-\left(b_1 t+b_2\frac{t^2}{2}\right)}\left(\frac{1}{b_1-a_1}-\frac{\int_0^t (a_1+a_2 v).\ e^{(b_1-a_1)v+(b_2-a_2)\frac{v^2}{2}}dv}{a_1}\right)$$

$$+\ e^{-\left(b_1 t+b_2\frac{t^2}{2}\right)}\left[\frac{\int_0^t(\lambda_1+\lambda_2 v)e^{\left(a_1 v+a_2\frac{v^2}{2}\right)}dv\int_0^t(a_1+a_2 v)e^{(b_1-a_1)v+(b_2-a_2)\frac{v^2}{2}}dv}{\lambda_1}\right.$$

$$\left.-\frac{\int_0^t(\lambda_1+\lambda_2 v)e^{\left(a_1 v+a_2\frac{v^2}{2}\right)}\left(\int_0^t(a_1+a_2 v)e^{(b_1-a_1)v+(b_2-a_2)\frac{v^2}{2}}dv\right)dv}{\lambda_1}-\frac{1}{b_1}\right)]] \tag{24}$$

The average waiting time of an employee in grade-2 of the organization is

$$W_2(t) = \frac{L_2(t)}{(b_1+b_2 t)[\,U_2(t)]}$$

Where $L_2(t)$ and $\mathrm{U}_2(\mathrm{t})$ are given in equation (23) and (24) respectively.

The variance of the number of employees in grade-2 of the organization is

$$V_2(t)=\lambda_1 e^{-\left(b_1 t+b_2\frac{t^2}{2}\right)}\left(\frac{1}{b_1-a_1}-\frac{\int_0^t (a_1+a_2 v).\ e^{(b_1-a_1)v+(b_2-a_2)\frac{v^2}{2}}dv}{a_1}\right)$$

$$+\lambda_1 e^{-\left(b_1 t+b_2\frac{t^2}{2}\right)}\left[\frac{\int_0^t(\lambda_1+\lambda_2 v)e^{\left(a_1 v+a_2\frac{v^2}{2}\right)}dv\int_0^t(a_1+a_2 v)e^{(b_1-a_1)v+(b_2-a_2)\frac{v^2}{2}}dv}{\lambda_1}\right.$$

$$\left.-\frac{\int_0^t(\lambda_1+\lambda_2 v)e^{\left(a_1 v+a_2\frac{v^2}{2}\right)}dv\left(\int_0^t(a_1+a_2 v)e^{(b_1-a_1)v+(b_2-a_2)\frac{v^2}{2}}dv\right)dv}{\lambda_1}-\frac{1}{b_1}\right] \tag{25}$$

The C.V of the number of employees in grade-2 of the organization is

$$CV_2(t)=\left[\lambda_1 e^{-\left(b_1 t+b_2\frac{t^2}{2}\right)}\left(\frac{1}{b_1-a_1}-\frac{\int_0^t (a_1+a_2 v).\ e^{(b_1-a_1)v+(b_2-a_2)\frac{v^2}{2}}dv}{a_1}\right)\right.$$

$$+\lambda_1 e^{-\left(b_1 t+b_2\frac{t^2}{2}\right)}\left[\frac{\int_0^t(\lambda_1+\lambda_2 v)e^{\left(a_1 v+a_2\frac{v^2}{2}\right)}dv\int_0^t(a_1+a_2 v)e^{(b_1-a_1)v+(b_2-a_2)\frac{v^2}{2}}dv}{\lambda_1}\right.$$

$$\left.\left.-\frac{\int_0^t(\lambda_1+\lambda_2 v)e^{\left(a_1 v+a_2\frac{v^2}{2}\right)}dv\left(\int_0^t(a_1+a_2 v)e^{(b_1-a_1)v+(b_2-a_2)\frac{v^2}{2}}dv\right)dv}{\lambda_1}-\frac{1}{b_1}\right]\right]^{\frac{-1}{2}} \tag{26}$$

The average number of employees in the organization is

$$L(t)=\lambda_1 e^{-\left(a_1 t+a_2\frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_1+\lambda_2 v)e^{\left(a_1 v+a_2\frac{v^2}{2}\right)}dv}{\lambda_1}-\frac{1}{a_1}\right)$$

$$+\lambda_1 e^{-\left(b_1 t+b_2\frac{t^2}{2}\right)}\left(\frac{1}{b_1-a_1}-\frac{\int_0^t (a_1+a_2 v).\ e^{(b_1-a_1)v+(b_2-a_2)\frac{v^2}{2}}dv}{a_1}\right)$$

$$+\lambda_1 e^{-\left(b_1 t+b_2\frac{t^2}{2}\right)}\left[\frac{\int_0^t(\lambda_1+\lambda_2 v)e^{\left(a_1 v+a_2\frac{v^2}{2}\right)}dv\int_0^t(a_1+a_2 v)e^{(b_1-a_1)v+(b_2-a_2)\frac{v^2}{2}}dv}{\lambda_1}\right.$$

$$\left.-\frac{\int_0^t(\lambda_1+\lambda_2 v)e^{\left(a_1 v+a_2\frac{v^2}{2}\right)}dv\left(\int_0^t(a_1+a_2 v)e^{(b_1-a_1)v+(b_2-a_2)\frac{v^2}{2}}dv\right)dv}{\lambda_1}-\frac{1}{b_1}\right] \tag{27}$$

The variance of the number of employees in the organization is

$$V(t) = \lambda_1 e^{-\left(a_1 t + a_2 \frac{t^2}{2}\right)} \left(\frac{\int_0^t (\lambda_1 + \lambda_2 v) e^{\left(a_1 v + a_2 \frac{v^2}{2}\right)} dv}{\lambda_1} - \frac{1}{a_1} \right)$$

$$+ \lambda_1 e^{-\left(b_1 t + b_2 \frac{t^2}{2}\right)} \left(\frac{1}{b_1 - a_1} - \frac{\int_0^t (a_1 + a_2 v).\ e^{(b_1 - a_1)v + (b_2 - a_2)\frac{v^2}{2}} dv}{a_1} \right)$$

$$+ \lambda_1 e^{-\left(b_1 t + b_2 \frac{t^2}{2}\right)} \left[\frac{\int_0^t (\lambda_1 + \lambda_2 v) e^{\left(a_1 v + a_2 \frac{v^2}{2}\right)} dv \int_0^t (a_1 + a_2 v) e^{(b_1 - a_1)v + (b_2 - a_2)\frac{v^2}{2}} dv}{\lambda_1} \right.$$

$$\left. - \frac{\int_0^t (\lambda_1 + \lambda_2 v) e^{\left(a_1 v + a_2 \frac{v^2}{2}\right)} dv \left(\int_0^t (a_1 + a_2 v) e^{(b_1 - a_1)v + (b_2 - a_2)\frac{v^2}{2}} dv \right) dv}{\lambda_1} - \frac{1}{b_1} \right] \qquad (28)$$

4. Numerical illustration and results

The behavior of the proposed manpower model is discussed through a numerical illustration. Since the performance characteristics of the manpower model are highly sensitive with respect to time; the transient behavior of the model is studied through computing the performance measures with the following set of values for the model parameters:

t = 0.13, 0.14, 0.15, 0.16; λ_1 = 2, 3, 4, 5, 6; λ_2 = 3, 4, 5, 6, 7; a_1 = 7, 7.4, 7.8, 8.2, 8.6
a_2 = 5, 7, 9, 11, 13; b_1 = 9, 9.4, 9.8, 10.2, 10.6; b_2 = 9, 12, 15, 17, 20

For different values of parameters t, λ_1, λ_2, a_1, a_2, b_1, b_2 and using the equations, the performance measures such as the average number of employees in grade-1 and in grade-2, the average waiting time of an employee in grade-1 and in grade-2, the variance of the number of employees in grade-1and in grade-2 and the C.V of the number of employees in both grade-1and in grade-2 are computed and presented in Table 1 and Table 2. The relationship between the parameters and performance measures are represented in the Figure 1 and Figure 2.

From the Table 1, As time (t) varies from 0.13 to 0.16, the average number of employees in grade-1 increases from 0.07505 to 0.12475 and in grade 2 decreases from 0.13306 to 0.07818, the average waiting time of an employee in grade-1 increases from 0.13569 to 0.13637 and grade-2 decreases from 0.10502 to 0.09958, when all the other parameters are fixed.

As the recruitment rate (λ_1) varies from 3 to 6, the average number of employees in grade1 and in grade-2 raises from 0.17384 to 0.32109 and 0.11727 to 0.23454 respectively, the average waiting time of an employee in grade-1and in grade-2 raises from 0.13967 to 0.14989 and 0.10151 to 0.10746 respectively, when all the other parameters are fixed.

As the recruitment rate (λ_2) varies from 4 to 7, the average number of employees in grade-1 increases from 0.32995 to 0.35654 and in grade-2 it remains constant , the average waiting time of an employee in grade-1increases from 0.15052 to 0.15242 and in grade-2 it remains constant, when all the other parameters are fixed.

As the promotion rate parameter (a_1) varies from 7.4 to 8.6, the average number of employees in grade-1 and in grade-2 increases from 0.37094 to 0.39615 and 0.40239 to 2.80333 respectively, the average waiting time of an employee in grade-1 decreases from 0.14596 to 0.12884 and in grade-2 increases from 0.11635 to 0.28584, when all the other parameters are fixed.

Table 1 : *Value of $L_1(t)$,$L_2(t)$,$W_1(t)$ and $W_2(t)$ for different value of parameters*

t	λ_1	λ_2	a_1	a_2	b_1	b_2	$L_1(t)$	$L_2(t)$	$W_1(t)$	$W_2(t)$
0.13	2	3	7	5	9	9	0.07505	0.13306	0.13569	0.10502
0.14	2	3	7	5	9	9	0.09266	0.11241	0.13598	0.10305
0.15	2	3	7	5	9	9	0.10921	0.09419	0.13621	0.10124
0.16	2	3	7	5	9	9	0.12475	0.07818	0.13637	0.09958
0.16	**3**	3	7	5	9	9	0.17384	0.11727	0.13967	0.10151
0.16	**4**	3	7	5	9	9	0.22292	0.15636	0.14303	0.10347
0.16	**5**	3	7	5	9	9	0.27200	0.19545	0.14643	0.10545
0.16	**6**	3	7	5	9	9	0.32109	0.23454	0.14989	0.10746
0.16	6	**4**	7	5	9	9	0.32995	0.23454	0.15052	0.10746
0.16	6	**5**	7	5	9	9	0.33881	0.23454	0.15115	0.10746
0.16	6	**6**	7	5	9	9	0.34767	0.23454	0.15178	0.10746
0.16	6	**7**	7	5	9	9	0.35654	0.23454	0.15242	0.10746
0.16	6	7	**7.4**	5	9	9	0.37094	0.40239	0.14596	0.11635
0.16	6	7	**7.8**	5	9	9	0.38196	0.67530	0.13990	0.13174
0.16	6	7	**8.2**	5	9	9	0.39020	1.21167	0.13419	0.16526
0.16	6	7	**8.6**	5	9	9	0.39615	2.80333	0.12884	0.28584
0.16	6	7	8.6	**7**	9	9	0.39277	2.80133	0.12440	0.28568
0.16	6	7	8.6	**9**	9	9	0.38942	2.79940	0.12025	0.28551
0.16	6	7	8.6	**11**	9	9	0.38611	2.79754	0.11636	0.28536
0.16	6	7	8.6	**13**	9	9	0.38282	2.79575	0.11270	0.28521
0.16	6	7	8.6	13	**9.4**	9	0.38282	1.13528	0.11270	0.15432
0.16	6	7	8.6	13	**9.8**	9	0.38282	0.59809	0.1127	0.11821
0.16	6	7	8.6	13	**10.2**	9	0.38282	0.34032	0.11270	0.10136
0.16	6	7	8.6	13	**10.6**	9	0.38282	0.19334	0.11270	0.09134
0.16	6	7	8.6	13	10.6	**12**	0.38282	0.18320	0.11270	0.08741
0.16	6	7	8.6	13	10.6	**15**	0.38282	0.17348	0.11270	0.08379
0.16	6	7	8.6	13	10.6	**18**	0.38282	0.16417	0.11270	0.08044
0.16	6	7	8.6	13	10.6	**21**	0.38282	0.15524	0.11270	0.07734

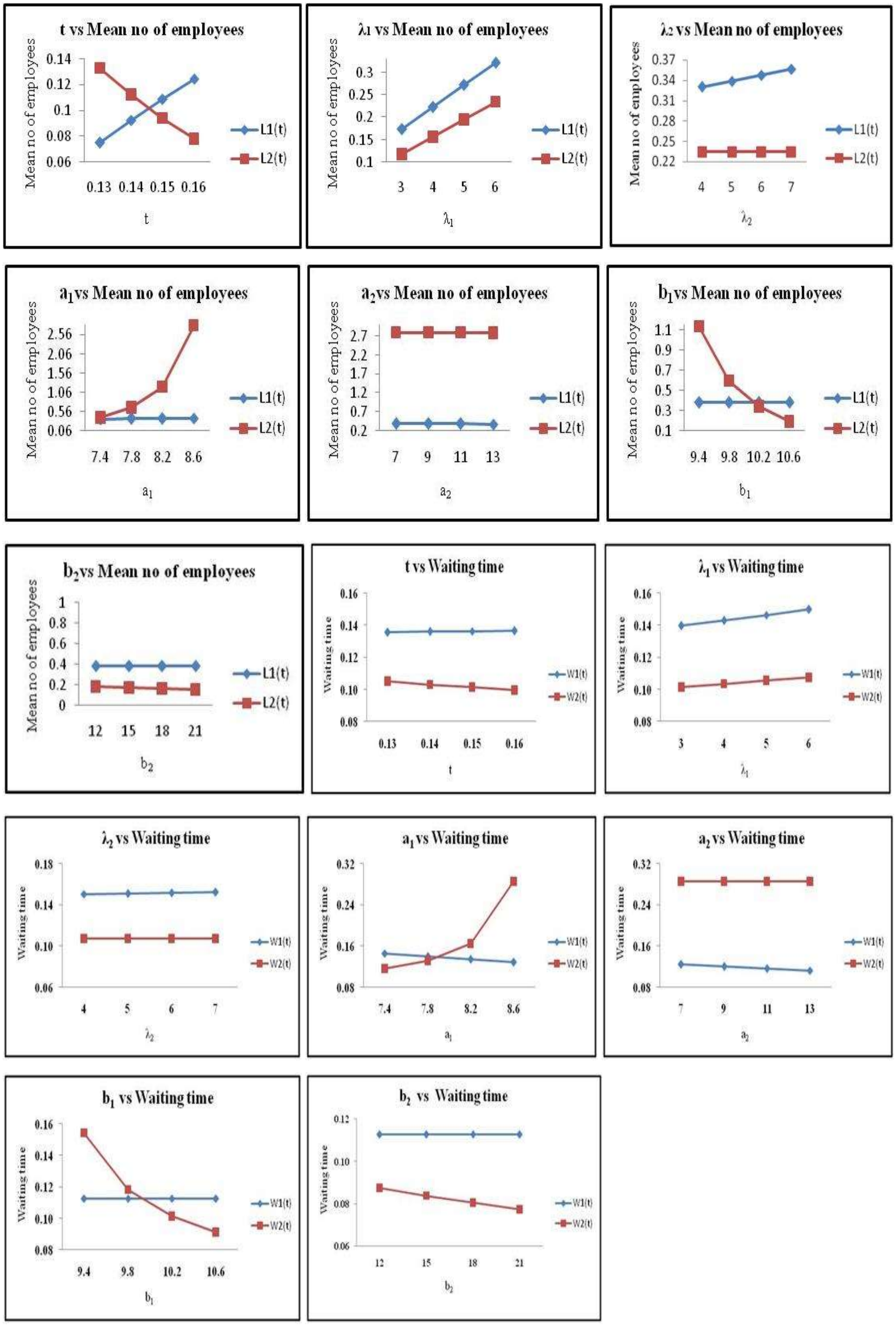

Figure 2: *Relation between the parameters and performance measures*

As the promotion rate parameter (a_2) varies from 7 to 13, the average number of employees in grade-1 and in grade-2 reduces from 0.39277 to 0.38282 and 2.80133 to 2.79575 respectively, the average waiting time of an employee in grade-1and in grade-2 reduces from 0.12440 to 0.11270 and 0.28568 to 0.28521 respectively, when all the other parameters are fixed.

As the leaving rate parameter (b_1) varies from 9.4 to 10.6, the average number of employees in grade-1 remains constant and in grade-2 decreases from 1.13528 to 0.19334, the average waiting time of an employee in grade-1 is not affected and in grade-2 it is decreasing from 0.15432 to 0.09134, when all the other parameters are fixed.

As the promotion rate parameter (b_2) varies from 12 to 21, the average number of employees in grade-1 remains constant and in grade-2 decreases from 0.18320 to 0.15524, the average waiting time of an employee in grade-1 is not affected and in grade-2 it is decreasing from 0.08741 to 0.07734,when all the other parameters are fixed.

From Table 2, As time (t) varies from 0.13 to 0.16, the variance of the number of employees in grade-1 increases from 0.07505 to 0.12475 and in grade-2 decreases from 0.13306 to 0.07818, C.V of the number employees in grade-1 decreases from 4.21372 to 2.83122 and in grade-2 increases from 2.74138 to 3.57642, When all the other parameters are fixed.

As the recruitment rate parameter (λ_1) varies from 3 to 6, the variance of the number of employees in grade-1 and in grade-2 raises from 0.17384 to 0.32109 and 0.11727 to 0.23454 respectively, the C.V of the number of employees in grade-1 and in grade-2 reduces from 2.39844 to 1.76477 and 2.92013 to 2.06485 respectively, when all the other parameters are fixed.

As the recruitment rate parameter (λ_2) varies from 4 to7, the variance of the number of employees in grade-1 increases from 0.32995 to 0.35654 and in grade-2 remains constant, the C.V of the number employees in grade-1 decreases from 1.74091 to 1.67474 and in grade-2 remains constant, when all the other parameters are fixed.

As the promotion rate parameter (a_1) varies from 7.4 to 8.6, the variance of the number of employees in grade-1 and in grade-2 raises from 0.37094 to 0.39615 and 0.40239 to 2.80333 respectively, the C.V of the number of employees in grade-1 and in grade-2 reduces from 1.64191 to 1.58881 and 1.57643 to 0.59726 respectively, when all the other parameters are fixed.

As the promotion rate parameter (a_2) variation from 7 to 13, the variance of the number of employees in grade-1 and in grade-2 raises from 0.39277 to 0.38282 and 2.80133 to 2.79575 respectively, the C.V of the number of employees in grade-1 and in grade-2 raises from 1.59563 to 1.61623 and 1.59747 to 0.59807 respectively, when all the other parameters are fixed.

As the promotion rate parameter (b_1) varies from 9.4 to 10.6, the variance of the number of employees in grade-1 remains constant and in grade-2 decreases from 1.13528 to 0.19334, the C.V of the number of employees in grade-1 remains constant and in grade-2 increases from 0.93853 to 2.27427, when all the other parameters are fixed.

As the promotion rate parameter (b_2) varies from 12 to 21, the variance of the number of employees in grade-1 remains constant and in grade-2 decreases from 0.18320 to 0.15524, the C.V of the number of employees in grade-1 remains constant and in grade-2 increases from 2.33637 to 2.53801, when all the other parameters are fixed.

Table 2: *Values of $V_1(t)$, $V_2(t)$, $CV_1(t)$ and $CV_2(t)$ for different values of parameters*

t	λ_1	λ_2	a_1	a_2	b_1	b_2	$V_1(t)$	$V_2(t)$	$CV_1(t)$	$CV_2(t)$
0.13	2	3	7	5	9	9	0.07505	0.13306	3.65017	2.74138
0.14	2	3	7	5	9	9	0.09266	0.11241	3.28514	2.98268
0.15	2	3	7	5	9	9	0.10921	0.09419	3.02606	3.25833
0.16	2	3	7	5	9	9	0.12475	0.07818	2.83122	3.57642
0.16	**3**	3	7	5	9	9	0.17384	0.11727	2.39844	2.92013
0.16	**4**	3	7	5	9	9	0.22292	0.15636	2.11799	2.52891
0.16	**5**	3	7	5	9	9	0.27200	0.19545	1.91740	2.26193
0.16	**6**	3	7	5	9	9	0.32109	0.23454	1.76477	2.06485
0.16	6	**4**	7	5	9	9	0.32995	0.23454	1.74091	2.06485
0.16	6	**5**	7	5	9	9	0.33881	0.23454	1.71799	2.06485
0.16	6	**6**	7	5	9	9	0.34767	0.23454	1.69595	2.06485
0.16	6	**7**	7	5	9	9	0.35654	0.23454	1.67474	2.06485
0.16	6	7	**7.4**	5	9	9	0.37094	0.40239	1.64191	1.57643
0.16	6	7	**7.8**	5	9	9	0.38196	0.67530	1.61806	1.21689
0.16	6	7	**8.2**	5	9	9	0.39020	1.21167	1.60088	0.90847
0.16	6	7	**8.6**	5	9	9	0.39615	2.80333	1.58881	0.59726
0.16	6	7	8.6	**7**	9	9	0.39277	2.80133	1.59563	0.59747
0.16	6	7	8.6	**9**	9	9	0.38942	2.79940	1.60247	0.59768
0.16	6	7	8.6	**11**	9	9	0.38611	2.79754	1.60934	0.59788
0.16	6	7	8.6	**13**	9	9	0.38282	2.79575	1.61623	0.59807
0.16	6	7	8.6	13	**9.4**	9	0.38282	1.13528	1.61623	0.93853
0.16	6	7	8.6	13	**9.8**	9	0.38282	0.59809	1.61623	1.29305
0.16	6	7	8.6	13	**10.2**	9	0.38282	0.34032	1.61623	1.71419
0.16	6	7	8.6	13	**10.6**	9	0.38282	0.19334	1.61623	2.27427
0.16	6	7	8.6	13	10.6	**12**	0.38282	0.18320	1.61623	2.33637
0.16	6	7	8.6	13	10.6	**15**	0.38282	0.17348	1.61623	2.40092
0.16	6	7	8.6	13	10.6	**18**	0.38282	0.16417	1.61623	2.46808
0.16	6	7	8.6	13	10.6	**21**	0.38282	0.15524	1.61623	2.53801

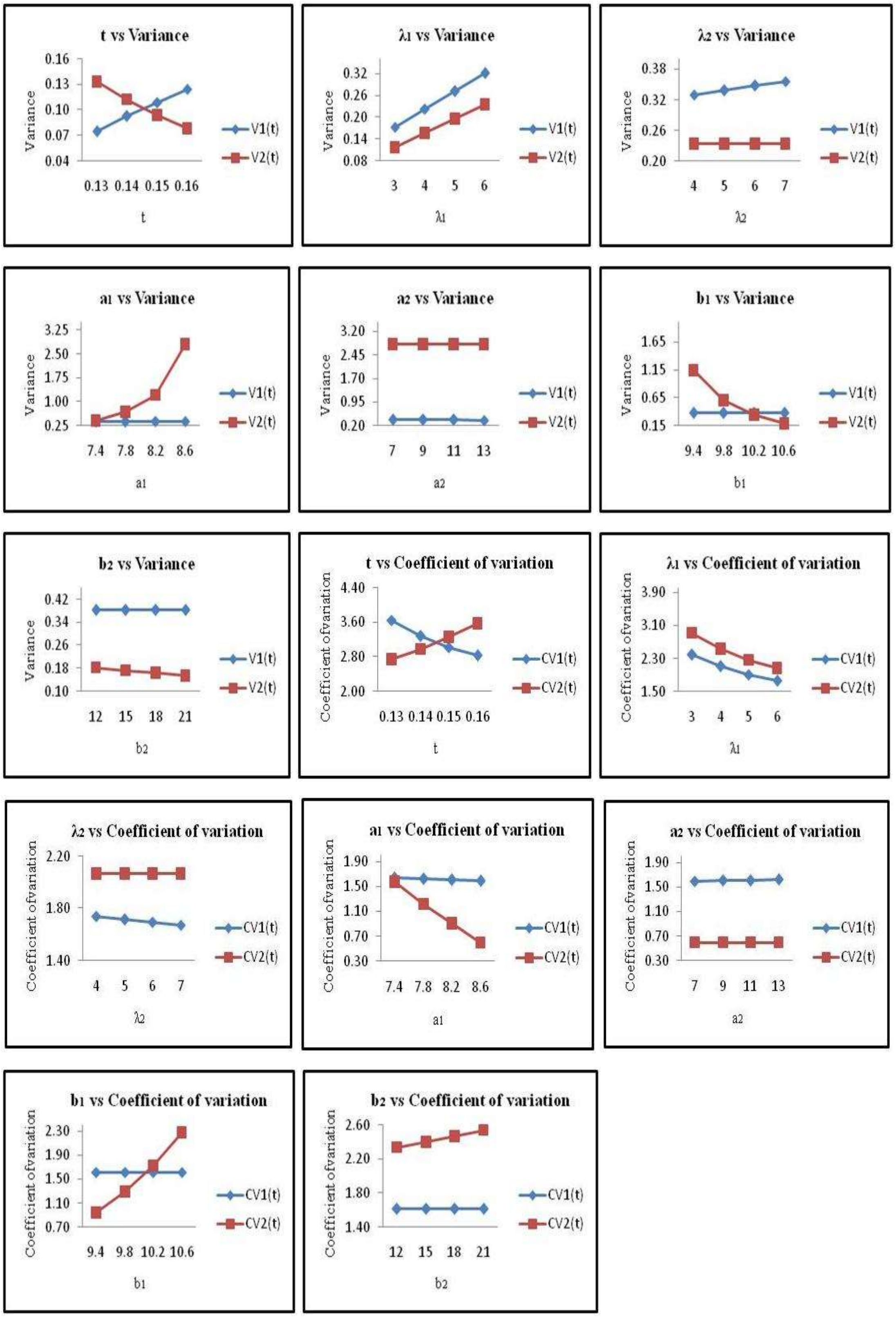

Figure 3: *Relation between the parameters and performance measures*

5. Sensitivity analysis of the model

Table3: *The values of $L_1(t)$,$L_2(t)$,$W_1(t)$,$W_2(t)$,$V_1(t)$ and $V_2(t)$ for different values of t,λ_1,λ_2,a_1,a_2,$b_1$and b_2*

Parameter	Performance Measures	-15%	-10%	-5%	0%	5%	10%	15%
t=0.2	$L_1(t)$	0.20768	0.22948	0.25000	0.26931	0.28747	0.30456	0.32064
	$L_2(t)$	0.33263	0.29217	0.25585	0.22330	0.19420	0.16825	0.14516
	$W_1(t)$	0.14345	0.14385	0.14416	0.14439	0.14455	0.14464	0.14466
	$W_2(t)$	0.12032	0.11672	0.11345	0.11048	0.10776	0.10528	0.10300
	$V_1(t)$	0.20768	0.22948	0.25000	0.26931	0.28747	0.30456	0.32064
	$V_2(t)$	0.33263	0.29217	0.25585	0.22330	0.19420	0.16825	0.14516
λ_1=3	$L_1(t)$	0.23848	0.24876	0.25903	0.26931	0.27958	0.28986	0.30013
	$L_2(t)$	0.18980	0.20097	0.21213	0.22330	0.23446	0.24563	0.25679
	$W_1(t)$	0.14228	0.14298	0.14368	0.14439	0.14510	0.14581	0.14653
	$W_2(t)$	0.10870	0.10929	0.10988	0.11048	0.11107	0.11167	0.11227
	$V_1(t)$	0.23848	0.24876	0.25903	0.26931	0.27958	0.28986	0.30013
	$V_2(t)$	0.18980	0.20097	0.21213	0.22330	0.23446	0.24563	0.25679
λ_2=5	$L_1(t)$	0.25974	0.26293	0.26612	0.26931	0.27250	0.27569	0.27888
	$L_2(t)$	0.22330	0.22330	0.22330	0.22330	0.22330	0.22330	0.22330
	$W_1(t)$	0.14373	0.14395	0.14417	0.14439	0.14461	0.14483	0.14505
	$W_2(t)$	0.11048	0.11048	0.11048	0.11048	0.11048	0.11048	0.11048
	$V_1(t)$	0.25974	0.26293	0.26612	0.26931	0.27250	0.27569	0.27888
	$V_2(t)$	0.22330	0.22330	0.22330	0.22330	0.22330	0.22330	0.22330
a_1=6.7	$L_1(t)$	0.25225	0.25993	0.26537	0.26931	0.27180	0.27323	0.27373
	$L_2(t)$	0.01724	0.06421	0.12665	0.22330	0.38499	0.75544	2.30366
	$W_1(t)$	0.16421	0.15707	0.15060	0.14439	0.13877	0.13335	0.12844
	$W_2(t)$	0.09987	0.10222	0.10541	0.11048	0.11929	0.14107	0.25340
	$V_1(t)$	0.25225	0.25993	0.26537	0.26931	0.27180	0.27323	0.27373
	$V_2(t)$	0.01724	0.06421	0.12665	0.22330	0.38499	0.75544	2.30366
a_2=6	$L_1(t)$	0.27093	0.27039	0.26985	0.26931	0.26877	0.26823	0.26770
	$L_2(t)$	0.22410	0.22383	0.22356	0.22330	0.22304	0.22278	0.22252
	$W_1(t)$	0.14787	0.14669	0.14553	0.14439	0.14327	0.14216	0.14107
	$W_2(t)$	0.11052	0.11050	0.11049	0.11048	0.11046	0.11045	0.11043
	$V_1(t)$	0.27093	0.27039	0.26985	0.26931	0.26877	0.26823	0.26770
	$V_2(t)$	0.22410	0.22383	0.22356	0.22330	0.22304	0.22278	0.22252
b_1=7.9	$L_1(t)$	0.26931	0.26931	0.26931	0.26931	0.26931	0.26931	0.26931
	$L_2(t)$	62.67895	1.19799	0.46792	0.2233	0.11041	0.04563	0.00701
	$W_1(t)$	0.14439	0.14439	0.14439	0.14439	0.14439	0.14439	0.14439
	$W_2(t)$	7.03467	0.1843	0.12909	0.11048	0.10069	0.09394	0.08896
	$V_1(t)$	0.26931	0.26931	0.26931	0.26931	0.26931	0.26931	0.26931
	$V_2(t)$	62.67895	1.19799	0.46792	0.2233	0.11041	0.04563	0.00701
b_2=11	$L_1(t)$	0.26931	0.26931	0.26931	0.26931	0.26931	0.26931	0.26931
	$L_2(t)$	0.23241	0.22934	0.22630	0.22330	0.22033	0.21740	0.21451
	$W_1(t)$	0.14439	0.14439	0.14439	0.14439	0.14439	0.14439	0.14439
	$W_2(t)$	0.11471	0.11326	0.11185	0.11048	0.10913	0.10781	0.10653
	$V_1(t)$	0.26931	0.26931	0.26931	0.26931	0.26931	0.26931	0.26931
	$V_2(t)$	0.23241	0.22934	0.22630	0.22330	0.22033	0.21740	0.21451

The sensitivity of the model is performed with respect to the value of time, recruitment rate, promotion rate and leaving rate of the both grade-1 and grade-2.

For different values of t, λ_1, λ_2, a_1, a_2, a_2, b_1 and $b_2$the average number of employees in grade-1 and in grade-2, average waiting time of an employee in grade-1 and in grade-2, the variance of the number of employees in grade-1 and in grade-2 are computed and presented in Table-3 with variation of -15%,-10%,-5% 0%,5%,10%,15% of the model parameters.

The performance measures are highly influenced by time (t). As t increases from -15% to +15%, the average number of employees along with the average waiting time of employees, the variance of the number of employees increases in grade-1. The average number of employees along with the average waiting time of employees, the variance of the number of employees decreases in grade-2.

As the recruitment rate parameter $\lambda_1$increases from -15% to +15%, the average number of employees, average waiting time of employees and the variance of the number of employees increasing in grade-1 and in grade-2.

As the recruitment rate parameter λ_2 increasesfrom -15% to +15%, the average number of employees along with the average waiting time of employees, the variance of the number of employees are increases in grade-1 and there is no change with respect to the performance measures in grade-2.

When the promotion rate parameter a_1 increases from -15% to +15%, the average number of employees along with the variance of the number of employees increases, the average waiting time of employees decreases in grade-1 and the average number of employees along with average waiting time of employees, the variance of the number of employees increases in grade-2.

When the promotion rate parameter a_2 increases from -15% to +15%, the average number of employees, average waiting time of employees and the variance of the number of employees decreasing in grade-1 and in grade-2.

When the leaving rate parameter b_1 increases from -15% to +15%, the average number of employees, average waiting time of employees and the variance of the number of employees in grade-1 remain constant and in grade-2 are decreasing.

When the leaving rate parameter b_2 increases from -15% to +15%, the average number of employee, average waiting time of employee and the variance of the number of employees in grade-1 are not influenced and in grade-2 are decreasing.

6. Comparative study of the models

The comparative study of the developed model with that of homogeneous Poisson recruitment is presented in this section. The performance measures of both the models are presented in Table 4 for different values of t =0.18, 0.19, 0.20, 0.21, and 0.22.

From the Table 4, As time (t) increases the percentage variation of the performance measures between two models also increasing. The model with NHP recruitment can predict the performance measure more accurately than the model with homogeneous Poisson recruitment. It is also observe that the assumption of NHP recruitment has a significant influence on all the performance measure of the model. Time also has a significant effect on the system performance measures.

Table-4: *Comparative study of models with non-homogeneous and homogeneous recruitment*

t	Parameter Measure	Non-Homogeneous recruitment	Homogeneous recruitment	Difference	Percentage of Variation
t=0.18	$L_1(t)$	0.62162	0.62475	0.00313	0.50100
	$L_2(t)$	0.09084	0.20145	0.11061	54.90692
	$W_1(t)$	0.10049	0.12807	0.02758	21.53510
	$W_2(t)$	0.04519	0.08833	0.04314	48.83958
	$V_1(t)$	0.62162	0.62475	0.00313	0.50100
	$V_2(t)$	0.09084	0.20145	0.11061	54.90692
t=0.19	$L_1(t)$	0.64298	0.65171	0.00873	1.33955
	$L_2(t)$	0.06385	0.16513	0.10128	61.33349
	$W_1(t)$	0.10027	0.12962	0.02935	22.64311
	$W_2(t)$	0.04188	0.08679	0.04491	51.74559
	$V_1(t)$	0.64298	0.65171	0.00873	1.33955
	$V_2(t)$	0.06385	0.16513	0.10128	61.33349
t=0.20	$L_1(t)$	0.66132	0.67598	0.01466	2.16870
	$L_2(t)$	0.04256	0.13433	0.09177	68.31683
	$W_1(t)$	0.09992	0.13103	0.03111	23.74265
	$W_2(t)$	0.03812	0.08549	0.04737	55.40999
	$V_1(t)$	0.66132	0.67598	0.01466	2.16870
	$V_2(t)$	0.04256	0.13433	0.09177	68.31683
t=0.21	$L_1(t)$	0.67695	0.69784	0.02089	2.99352
	$L_2(t)$	0.02599	0.10828	0.08229	75.99741
	$W_1(t)$	0.09946	0.13230	0.03284	24.82237
	$W_2(t)$	0.03335	0.08441	0.05106	60.49046
	$V_1(t)$	0.67695	0.69784	0.02089	2.99352
	$V_2(t)$	0.02599	0.10828	0.08229	75.99741
t=0.22	$L_1(t)$	0.69019	0.71751	0.02732	3.80761
	$L_2(t)$	0.01330	0.08632	0.07302	84.59222
	$W_1(t)$	0.09891	0.13346	0.03455	25.88791
	$W_2(t)$	0.02629	0.08350	0.05721	68.51497
	$V_1(t)$	0.69019	0.71751	0.02732	3.80761
	$V_2(t)$	0.01330	0.08632	0.07302	84.59222

7. Conclusion

In this paper, a novel model with two grades of manpower is developed and examined. This procedure has the ability to describe time-dependent recruiting. The model's characteristics, such as the average number of employees in each grade, the average waiting time for an employee in each grade, the number of employees in each grade's variance and the number of employees in each grade's C.V in the organization are explicitly derived. The sensitivity analysis of the model revealed that the system performance metrics are significantly influenced by non-homogeneous recruitment rate.

When recruiting is done in a time-dependent manner, the performance measures can be predicted more correctly and realistically by employing the developing model. This model also incorporates few of the prior models as special instances for particular values of parameters. This model can also be improved by taking cost factors into account and determining the ideal values for the model's parameters, which will be considered later. This model can be utilized to predict the human resource characteristics of the organization at defense and IT sectors as the recruitment, promotion and leaving processes in these organizations are time dependent.

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