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GENERALIZED X-EXPONENTIAL BATHTUB SHAPED FAILURE RATE DISTRIBUTION AND ESTIMATION OF RELIABILITY OF MULTICOMPONENT STRESS-STRENGTH

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Abstract

In an engineering setup, one is interested to know and determine the reliability of the system of different components. These components are usually subjected to different kinds of stress, and the reliability of the components needs to be estimated under stress. In this paper, we aim to estimate the reliability of a multicomponent stress-strength model assuming that the components of the system are working independently with a common life distribution. The system follows a comparatively new distribution named as; Generalized X-Exponential bathtub failure rate distribution. This paper studies the usefulness of this distribution in terms of estimating the maximum likelihood estimate of the reliability parameter and its asymptotic confidence intervals. Paper uses methods of parametric estimation and reliability estimation. Results are computed using Monte Carlo simulation for small samples. Real data set is presented to evaluate the performance of Generalized X Exponential Distribution (GXED) reliability estimator. Findings show that with the usage of proposed distribution, estimator of reliability parameter fits very well to the real-world situations

Key words: Generalized X -Exponential distribution, Multicomponent stress-strength, Reliability, *ML* estimation, Average variance, Confidence intervals.

I. Introduction

The X-Exponential distribution was introduced by Chacko [4], to add another model to the class of bathtub type failure rate distributions. When x is X-Exponential with parameters $\boldsymbol{\alpha}$ and λ. It has distribution function: $F(x) = \left(1 - (1 + \lambda x^2)e^{(-\lambda x)}\right)^{\alpha}$ with the corresponding density function: $f(x) = \alpha e^{-\lambda x}(\lambda^2 x^2 - 2\lambda x + 1)\left(1 - (1 + \lambda x^2)e^{(-\lambda x)}\right)^{\alpha - 1}$. Its properties and reliability applications were studied by the author. However, in order to get more flexibility to the model, Chacko and Deepthi [5] made a small change in the exponential part. The corresponding distribution is named as Generalized X-Exponential distribution. Basically, bathtub failure rate distribution's curve illustrates three phases of a product's life. First phase is known as early failure, next is a roughly prolonged intrinsic period and failure rate is approximately constant here. This stage is very important for reliability prediction of a product. And finally, there is a wear out failure phase, where failure rate increases. In the past several bathtub failure rate distributions have been studied by Kundu &Gupta, Srinivasa Rao [11] to carry out reliability testing by using single component stress strength, as well as multi- component stress strength models. Since no substantial work has been done on reliability estimation of multicomponent stress strength by using a flexible distribution i.e., *GXED*, hence there was a need to study the reliability estimator of newly introduced Generalized X-Exponential distribution having distribution function, $F(x) = \left(1-(1+\lambda x^2)e^{-\lambda(x^{2}+x)}\right)^{\alpha}, x>0, \lambda > 0$ and $\alpha > 0$.and the density function is:

$$f(x) = \alpha e^{-\lambda(x^{2}+x)}(\lambda(1+\lambda x^2)(2x+1)-2\lambda x)((1-(1+\lambda x^2)e^{-\lambda(x^{2}+x)})^{\alpha-1}\;;\;\alpha>0,\lambda>0 \tag{1}$$

$$\text{Failure rate}=\frac{\alpha e^{-\lambda(x^{2}+x)}(\lambda(1+\lambda x^2)(2x+1)-2\lambda x)((1-(1+\lambda x^2)e^{-\lambda(x^{2}+x)})^{\alpha-1}}{1-\left(1-(1+\lambda x^2)e^{-\lambda(x^{2}+x)}\right)^{\alpha}};\quad x>0,\alpha>0,\lambda>0 \tag{2}$$

The authors (Chacko and Deepthi) have investigated the properties and some reliability applications of the new model. Here we are interested in the reliability analysis of multicomponent system where the components are connected in parallel and function independently, with the same Generalized X -Exponential distribution *GXED* and stress too has the same distribution but with different parameters.

Let the random samples $Y, X_1, X_2, X_3, \dots X_K$ be independent, *G(y)* be the continuous distribution function of *Y*, and *F(x)* be the common distribution function of$Y, X_1, X_2, X_3, \dots X_K$.The reliability in a multi component stress-strength model developed by Bhattacharyya and Johnson [2] is given by.

$R_{s,k}$=P [at least *s* of the $X_1, X_2, X_3, \dots X_K$exceed *Y*]

$$=\sum_{i=s}^{k}\binom{k}{i}\int_{-\infty}^{+\infty}[1-F(y)]^{i}\,[F(y)^{k-i}]dG(y) \tag{3}$$

Where$X_1, X_2, X_3, \dots X_K$identically and independently distributed (iid) are with common distribution function *F(x)* and subjected to random stress *Y*. The probability in (3) is called 'Reliability in a multicomponent stress –strength model' Bhattacharyya and Johnson [2]. The survival probabilities of single component stress- strength version was considered by several authors for different distributions. Some of them are: Enis and Geisser [9], Downtown [8], Awad and Gharraf [1], McCool [18], Hanagal [12], Nandi and Aich [19], Surles and Padgett [27], Kundu and Gupta [15,16], Raqab et al. [26] and Kundu and Raqab [17]. More over Kotz & Pensky [14] studied the generalizations of stress strength model.

Reliability in a multicomponent stress-strength model was developed by Bhattacharyya and Johnson [2]. Pandey & Burhan [21] computed the estimation of reliability for a multicomponent model using Burr distribution. Zimmer et al [29] studied the reliability analysis for Burr X11 distribution. Estimation of reliability in models with correlated stress and strength has been studied by Balakrishnan &Lai [3]. Rao and Kantam [24] studied the estimation of reliability in a multicomponent stress- strength model for logistic distribution, Rao [23] also developed the procedure for the estimation of reliability in multicomponent stress-strength model based on Generalized exponential distribution. Ghitany et al. [10] studied the estimation of reliability of multicomponent model using Power Lindley distribution. Burr-X11 distribution for parametric and reliability estimation in a multicomponent stress-strength environment has been analyzed by Rao et al. [25]. Dey, S. et al [6] considered Bayesian and non-Bayesian estimation of multicomponent stress-strength reliability using Kumaraswami distribution.

Dey, Raheem & Mukherjee [7] derived the form of stress-strength reliability parameter for transmuted Rayleigh distribution. Hassan [13] developed the procedure for the estimation of stress-strength model using Lindley distribution. Estimation on Reliability in a multicomponent Stress-strength model with Power Lindley distribution is carried out by Abbas Pak et al [22]. Similarly, a recent study has been conducted on the estimation of stress strength reliability for Akash distribution by Akhila. K. Varghese & V. M. Chacko [28].

The aim of this paper is to estimate the reliability in a multi component stress-strength model based on X, Y being two independent random variables, where X~$GXED$, (α_1, λ) and Y~$GXED$ (α_2, λ). We use parametric estimation and estimation reliability. Suppose a system with k identical components, functions if at least s $(1 \leq s \leq k)$ components operate simultaneously. In its operating environment, the system is subjected to stress Y which is a random variable with distribution function G (.). The strengths of the components, that is the minimum stresses causing failure, are independently and identically distributed random variables with distribution function F(.).The reliability of the system can be obtained by (3). An attempt has been made here to study the estimation of reliability in a multicomponent stress-strength model with reference to two parameter *GXED*.

The remainder of the paper is organized as follows. In section 2, research methodology and procedure for expression of $R_{s,k}$. The asymptotic distribution and confidence interval of (3) are calculated using *MLE*. The results of small sample comparisons derived from Monte Carlo simulations and analysis of real data sets are described in section 3. Findings are discussed in section 4.

2. Maximum Likelihood Estimator of $R_{s,k}$

Let X~$GXED$ (α_1, λ) and Y~$GXED$ (α_2, λ) be independently distributed with unknown shape parameters (α_2, λ)while common scale parameterλ. Using (3) the reliability in multicomponent stress-strength for two- parameter *GXED* distribution is as follows:

$$R_{s,k} = \sum_{i=s}^{k} \binom{k}{i} \int_{0}^{+\infty} [1 - F(y)]^i \, [F(y)^{k-i}] dG(y)$$

$$F(y) = \left(1 - (1 + \lambda x^2) e^{-\lambda(x^2+x)}\right)^{\alpha} ; \; x > 0, \alpha > 0, \lambda > 0$$

$$1 - F(y) = 1 - \left(1 - (1 + \lambda x^2) e^{-\lambda(x^2+x)}\right)^{\alpha}$$

$$dG(y) = \alpha e^{-\lambda(y^2+y)} (\lambda(1 + \lambda y^2)(2y + 1) - 2\lambda y)(\left(1 - (1 + \lambda y^2) e^{-\lambda(y^2+y)}\right)^{\alpha-1} \, dy$$

$$R_{s,k} = \sum_{i=s}^{k} \binom{k}{i} \, v \int_0^1 (1 - t)^i \, t^{k-1+v-1} dt$$

where $t = 1 - \left(1 - (1 + \lambda x^2) e^{-\lambda(y^2+y)}\right)^{\alpha}$ and $v = \frac{\alpha_2}{\alpha_1}$

After simplification we get

$$R_{s,k} = \sum_{i=s}^{k} \binom{k}{i} \, vB(i + 1, k - i + v) \tag{4}$$

The probability in (4) is termed reliability in a multicomponent stress-strength model. It is important to mention here that MLE of $R_{s,k}$depends on that of α_1& α_2. Hence, we need to calculate MLE of the latter to derive that of the former. Similarly, to find the MLE of α_1& $\alpha_2$and we need to find the MLE of λ as well. Here we assume that $X_{1,}X_2, X_3 \dots \dots X_n$ is a random sample from *GXED* (α_1, λ) and $Y_1, Y_{,2} Y_3, \dots \dots . Y_m$ is a random sample from *GXED* (α_2, λ).

The loglikelihood function *LLF* of these samples is expressed as:

$$L(\alpha_1, \alpha_2, \lambda) = m\ln\alpha_1 + n\ln\alpha_2 - (m+n)\lambda\left(x_i^2 + x_i + y_j^2 + y_j\right) + m\ln\sum(\lambda(1+\lambda x_i^2)(2x_i+1) - 2\lambda x_i) + (\alpha_1 - 1)\sum\left(\ln\left(1-(1+\lambda x_i^2)e^{-\lambda(x^2+x)}\right)\right) + n\ln\sum\left(\lambda\left(1+\lambda y_j^2\right)(2y_j+1) - 2\lambda y_j\right) + (\alpha_2 - 1)\sum\left(\ln\left(1-(1+\lambda y_i^2)e^{-\lambda(y^2+y)}\right)\right) \tag{5}$$

Thus, the *MLE* of λ is the solution of

$$\frac{\partial logL(\alpha_1,\alpha_2,\lambda)}{\partial\lambda} = 0 \Rightarrow -\Sigma_{i=1}^m(x_i^2+x_i) + \Sigma_{i=1}^m \frac{\left((2x_i+1)(1+2\lambda x_i^2)-2x_i\right)}{(\lambda(1+\lambda x_i^2)(2x_i+1)-2\lambda x_i)} + (\alpha_1 - 1)\Sigma_{i=1}^m \frac{(1+\lambda x_i^2)e^{-\lambda(x_i^2+x_i)}(x_i^2+x_i) - e^{-\lambda(x_i^2+x_i)}x_i^2}{(1-(1+\lambda x_i^2)e^{-\lambda\left(x_i^2+x_i\right)})} - \Sigma_{j=1}^n\left(y_j^2+y_j\right)$$
$$+\Sigma_{j=1}^n \frac{\left((2y_j+1)\left(1+2\lambda y_j^2\right)-2y_j\right)}{\left(\lambda\left(1+\lambda y_j^2\right)(2y_j+1)-2\lambda y_j\right)} + (\alpha_2-1)\Sigma_{j=1}^n \frac{\left(1+\lambda y_j^2\right)e^{-\lambda\left(y_j^2+y_j\right)}\left(y_j^2+y_j\right) - e^{-\lambda\left(y_j^2+y_j\right)}y_j^2}{\left(1-\left(1+\lambda y_j^2\right)e^{-\lambda\left(y_j^2+y_j\right)}\right)} = 0 \tag{6}$$

Similarly, the *MLE* of α_1 can be obtained as the solution of

$$\frac{\partial logL(\alpha_1,\alpha_2,\lambda)}{\partial\alpha_1} = 0 \Rightarrow \frac{m}{\alpha_1} + \Sigma_{i=1}^m \log\left(1-(1+\lambda x_i^2)e^{-\lambda\left(x_i^2+x_i\right)}\right) = 0 \tag{7}$$

Also, for α_2

$$\frac{\partial logL(\alpha_1,\alpha_2,\lambda)}{\partial\alpha_2} = 0 \Rightarrow \frac{n}{\alpha_2} + \Sigma_{j=1}^n \log\left(1-\left(1+\lambda y_j^2\right)e^{-\lambda\left(y_j^2+y_j\right)}\right) = 0 \tag{8}$$

From (7) and (8) we obtain:

$$\alpha_1^{\wedge}(\lambda) = \frac{-m}{\Sigma_{i=1}^m \log\left(1-(1+\lambda x_i^2)e^{-\lambda\left(x_i^2+x_i\right)}\right)} \; and \; \alpha_2^{\wedge}(\lambda) = \frac{-n}{\Sigma_{i=1}^n \log\left(1-(1+\lambda x_i^2)e^{-\lambda\left(y_i^2+y_i\right)}\right)} \tag{9}$$

Putting the values of $\alpha_1^{\wedge}(\lambda)$ *and* $\alpha_2^{\wedge}(\lambda)$into equation (6), we got a function of λ which is nonlinear.

$$`h(\lambda) = \lambda \tag{10}$$

$$\frac{\Sigma_{i=1}^m \frac{4\lambda\chi_i^3 + 2\lambda x_i^2 + 1}{2\lambda x_i^2 + 2x_i^2 + 1} + \Sigma_{j=1}^n \frac{4\lambda y_j^3 + 2\lambda y_j^2 + 1}{2\lambda y_j^2 + 2y_j^2 + 1}}{\Sigma_{i=1}^m(x_i^2+x_i) + \Sigma_{j=1}^n\left(y_j^2+y_j\right) + \frac{m}{\sum_{i=1}^m log\left(1-(1+\lambda x_i^2)e^{-\lambda(x_i^2+x_i)}\right)}\Sigma_{i=1}^m \frac{x_i(1+\lambda x_i^3+\lambda x_i^2).e^{-\lambda(x_i^2+x_i)})}{\left(1-(1+\lambda x_i^2)e^{-\lambda(x_i^2+x_i)}\right)} + \frac{n}{\sum_{j=1}^n log\left(1-\left(1+\lambda y_j^2\right)e^{-\lambda\left(y_j^2+y_j\right)}\right)}\Sigma_{j=1}^n \frac{y_j\left(1+\lambda y_j^3+\lambda y_j^2\right).e^{-\lambda(y_i^2+y_j)}}{\left(1-\left(1+\lambda y_j^2\right)e^{-\lambda\left(y_j^2+y_j\right)}\right)} + \frac{n}{\sum_{j=1}^n log\left(1-(1+\lambda x_i^2)e^{-\lambda\left(x_i^2+x_i\right)}\right)}\Sigma_{i=1}^m \frac{x_i\left(1+\lambda x_i^3+\lambda x_i^2\right).e^{-\lambda\left(x_i^2+x_i\right)})}{\left(1-(1+\lambda x_i^2)e^{-\lambda\left(x_i^2+x_i\right)}\right)}} \tag{11}$$

Here $\lambda^{\wedge}$is a fixed-point solution of nonlinear equation (10). It can be obtained using a simple iterative procedure:

$$h_{\lambda(j)} = \lambda(j+1) \quad (12)$$

Where λ_j is the j^{th} iteration of $\lambda^{\wedge}$.During the simulation process, when the difference between λ_j and $\lambda(j+1)$ becomes sufficiently small; then we stop the iterative process. Once we obtain $\lambda^{\wedge}$, the parameters $\alpha_1^{\wedge}$ *and* $\alpha_2^{\wedge}$can be obtained from (9) as respectively. To obtain the asymptotic confidence interval for $R_{s,k}$ we proceed as follows.

2.1 Asymptotic Variance and Confidence Intervals

$$V(\alpha_1^{\wedge}) = [E(-\partial^2 L/\partial\alpha_1^2)]^{-1} = \frac{\alpha_1^2}{m} \text{ and } V(\alpha_2^{\wedge}) = [E(-\partial^2 L/\partial\alpha_2^2)]^{-1} = \frac{\alpha_2^2}{n} \quad (13)$$

The asymptotic variance AV of an estimate of $R_{s,k}$ which is a function of two independent statistics $\alpha_1^{\wedge}, \alpha_2^{\wedge}$ is established by Rao (1973):

$$AV(R_{s,k}^{\wedge}) = V(\alpha_1^{\wedge})(\frac{\partial R_{s,k}}{\partial\alpha_1})^2 + V(\alpha_2^{\wedge})(\frac{\partial R_{s,k}}{\partial\alpha_2})^2 \quad (14)$$

Thus from (14), asymptotic variance in $R_{s,k}$can be obtained for *GXED*.

We obtain $R_{s,k}$ and their derivatives for (s, k) = (1, 3) and (2, 4) separately:

$$R_{1,3}^{\wedge} = \frac{3v^2+9v+6}{(v+1)(v+2)(v+3)} \text{ and } R_{2,4}^{\wedge} = \frac{12(v^2+3v+2)}{(v+1)(v+2)(v+3)(v+4)}$$

$$\frac{\partial R_{1,3}^{\wedge}}{\partial\alpha_1} = \frac{3v(v^4+6v^3+13v^2+12v+4)}{\alpha_1[(v+1)(v+2)(v+3)]^2}$$

$$\frac{\partial R_{1,3}^{\wedge}}{\partial\alpha_2} = \frac{-3v(v^4+6v^3+13v^2+12v+4)}{\alpha_1[(v+1)(v+2)(v+3)]^2}$$

$$\frac{\partial R_{2,4}^{\wedge}}{\partial\alpha_1} = \frac{12v(2v^5+19v^4+68v^3+115v^2+92v+28)}{\alpha_1[(v+1)(v+2)(v+3)(v+4)]^2} \text{ and}$$

$$\frac{\partial R_{2,4}^{\wedge}}{\partial\alpha_2} = \frac{-12(2v^5+19v^4+68v^3+115v^2+92v+28)}{\alpha_1[(v+1)(v+2)(v+3)(v+4)]^2}$$

Therefore as $n \to \infty$ *and* $m \to \infty$, $(R_{s,k}^{\wedge} - R_{sk})/AV(R_{s,k}^{\wedge})\, N\,(0,1)$

$$AV(R_{1,3}^{\wedge}) = \frac{9v^2(v^4+6v^3+13v^2+12v+4)^2(1/m+1/n)}{[(v+1)(v+2)(v+3)]^4}$$

$$\text{and } AV\,(R_{2,4}^{\wedge}) = \frac{144v^2(2v^5+19v^4+68v^3+115v^2+92v+28)^2(1/m+1/n)}{[(v+1)(v+2)(v+3)(v+4)]^4}$$

Where $R_{s,k}^{\wedge} \mp 1.96\sqrt{AV(R_{s,k})}$is the asymptotic 95%confidence interval (C.I) of system reliability $R_{s,k}$ and asymptotic 95% C.I for $R_{1,3}$ is given by:

$$R_{1,3}^{\wedge} \mp 1.96\frac{3v(v^4+6v^3+13v^2+12v+4)\sqrt{1/m+1/n}}{[(v+1)(v+2)(v+3)]^2}$$

and the asymptotic 95% confidence interval (C.I) for $R_{2,4}$ is given by:

$$R_{2,4}^{\wedge} \mp 1.96\frac{12v(2v^5+19v^4+68v^3+115v^2+92v+28)\sqrt{1/m+1/n}}{[(v+1)(v+2)(v+3)(v+4)]^2}$$

3. Simulation Study

3.1 Results

5000 random samples are generated each of size 10(5)30 from stress and strength populations for different values of α_1 *and* α_2: (2.0,2.5), (2.0,3.0), (2.0,3.5), (3.0,2.0), (3.0,2.5), (3.0,3.0). The MLE of scale parameter λ is estimated by the iterative method and using λ the shape parameters α_1 and α_2 are estimated from eq (8).

These *ML* estimators of α_1 *and* α_2 are then substituted in υ to obtain the multicomponent reliability for *(s, k)* = (1,3) and (2,4). The average bias and average MSE of reliability estimate over 5000 replications are presented in Table 1 and Table 2. Average length of confidence interval and coverage probability of the simulated 95% CIs of $R_{s,k}$ are given in Table 3 and Table 4. The true values of reliability in multicomponent stress -strength with given combinations of $\hat{\alpha_1}, \hat{\alpha_2}$ for *(s, k)* = (1,3) are 0.7058824, 0.6666667, 0.6315789, 0.8181074, 0.7826768, 0.75, 0.7142857 and for (*s, k*) = (2,4) are 0.5378151, 0.4848485, 0.4393593, 0.7011849, 0.6477772, 0.6, 0.5494505.

Here it is seen that the true value of reliability in multicomponent stress-strength decreases as α_2 is increased for a fixed value of α_1,whereas reliability in multicomponent stress-strength also decreases as $\alpha_1$is increased for a fixed value ofα_2 . Thus, the true value of reliability increases as υ decreases and vice versa.

Table 1: Average bias of the simulated estimates of $R_{s,k}(\alpha_1, \alpha_2)$

s, k	n, m	2.0,2.5	2.0,3.0	2.0,3.5	3.0,2.0	3.0,2.5	3.0,3.0
1,3	**10,10**	-.006581	-.0016484	-.007107	-.010848	-.017399	-.061099
	15,15	-.006425	-.0041552	-.003801	-.005277	-.005233	-.056924
	20,20	-.005644	-.0009021	-.002751	-.003377	-.003877	-.055159
	25,25	-.004301	-.0035697	-.002016	-.003189	-.003199	-.055473
	30,30	-.003075	-.0032642	-.001574	-.003726	-.002866	-.055240
2,4	**10,10**	-.003129	-.0009622	0.000633	-.011485	-.007666	-.008216
	15,15	-.005138	-.0022865	-.001908	-.003719	-.008930	-.005695
	20,20	0.000287	0.0005562	-.000367	-.006453	-005077	-.004446
	25,25	-.000917	-.0003678	-.001444	-.001709	-.004725	-.005074
	30,30	-.000523	-.0020488	-.003957	-.002097	-.004196	-.003842

Results of Table 1 and Table 2 depicts that average bias and MSE decrease as sample size increases for both the cases of estimation of reliability. Bias is negative in all the combinations of parameters in both situations of (*s, k*). This shows the consistency of MSE. Also, absolute bias increases as $\alpha_1$increases for a fixed value of α_2.While MSE decreases as α_1 increases for a fixed value of α_2 for both the cases of (*s, k*). Also, for fixed α_1 and increasing α_2 MSE increases for same sample.

Table 2: Average MSE of the simulated estimates of $R_{s,k}(\alpha_1, \alpha_2)$

s, k	n, m	2.0,2.5	2.0,3.0	2.0,3.5	3.0,2.0	3.0,2.5	3.0,3.0
1,3	**10,10**	.008420	.008347	.0105324	.005115	.005915	.010999
	15,15	.005872	.006666	.0071572	.002870	.004008	.008089
	20,20.	.0049068	.004907	.0054370	.002320	.003011	.006478
	25,25	.0036479	.004291	.0045213	.001871	.002486	.005984
	30,30	.002805	.003195	.0037037	.001478	.002033	.005489
2,4	10,10	.015654	.0154285	.0165471	.010602	.012716	.014210
	15,15	.010693	.010985	.0111963	.004762	.008500	.009423
	20,20.	.0075629	.008428	.008489	.004305	.006470	.007210
	25,25	.006857	.006696	.0069927	.004016	.004969	.005663
	30,30	.005238	.005696	.0052105	.003354	.004067	.005053

Table 3: Average Length of the simulated 95% confidence intervals of $R_{s,k}(\alpha_1,\alpha_2)$

s, k	n, m	2.0,2.5	2.0,3.0	2.0,3.5	3.0,2.0	3.0,2.5	3.0,3.0
1,3	**10,10**	.350894	.378572	.390008	.263240	.299103	.322061
	15,15	.290262	.311732	.323802	.214849	.242167	0.26764
	20,20.	.253883	.272129	.323815	.186494	.210426	.230430
	25,25	.228945	.243448	.254189	.166324	.188198	.206166
	30,30	.208672	.223215	.232022	.150368	.170951	.189016
2,4	10,10	.475256	.483035	.485837	.392269	.428761	.453479
	15,15	.395910	.404009	.405260	.322269	.351963	.373558
	20,20.	.346274	.357539	.354290	.280074	.308866	.327652
	25,25	.309512	.318022	.318480	.250861	.275602	.291288
	30,30	.285038	.291335	.292680	.230350	.252819	.269990

Table 3 and Table 4 findings show that as the sample size increases, length of CI also decreases and coverage probability in most the cases crossing 0.95 and for few it is 0.98, which shows the performance of CI using Generalized X- Exponential Distribution *GXED* is excellent and it covers most of the cases. Among the parameters, it is observed that length of CI increases for fixed value of α_1 for (1,3) while for fixed value of α_2 length of CI decreases.

Table 4: Average Coverage Probability of simulated 95% confidence intervals of $R_{s,k}(\alpha_1,\alpha_2)$

s, k	n, m	2.0,2.5	2.0,3.0	2.0,3.5	3.0,2.0	3.0,2.5	3.0,3.0
1,3	**10,10**	.891333	.968000	.936667	.910667	.969333	.987333
	15,15	.972667	.944444	.880000	.950000	.925084	.905333
	20,20.	.905333	.980810	.914000	..968667	.912052	.956667
	25,25	.951333	.912300	.949333	.969333	.946000	.966667
	30,30	.953815	.965333	.926000	.952667	.896360	.936667
2,4	10,10	.964667	.957333	.9743178	.984000	.934667	.966677
	15,15	.962667	.966600	.953000	.970000	.942000	.967333
	20,20.	.963333	.955746	.952667	.969425	.954000	.953333
	25,25	.946000	.936667	.953333	.983333	.970883	.948007
	30,30	.902000	.946666	.937333	.956000	.960667	.960667

3.2 Data Analysis

In this section, we will deal with two real data sets, will show how reliability in a multicomponent stress-strength model can be applied for *GXED*. Both data sets were discussed by Zimmer et al. (1998) and Lio et al. (2010) for Burr-X11 reliability analysis. They showed that Burr-X11 distribution fits quite well. For both the data sets, here we are using *GXED*.

(X):0.19 ,0.78, 0.96, 0.31, 2.78, 3.16, 4.15, 4.67, 4.85, 6.50, 7.35, 8.01, 8.27, 12.06, 31.75, 32.52, 33.91, 36.71 and 72.89

(Y):0.9, 1.5, 2.3, 3.2, 3.9, 5.0, 6.2, 7.5, 8.3, 10.4, 11.1, 12.6, 15.0, 16.3, 19.3, 22.6,

24.8, 31.5 And 53.0. Iterative procedure was used to calculate the value of λ using (8) and then α_1 *and* α_2 were obtained by substituting the *MLE* of λ in (10).

The final estimates of $\alpha_1 = 0.844798$, $\alpha_2 = 1.551717$ and $\lambda = 0.04642891$. Based on these estimates the MLE of $R_{1,3}$ turned out to be 0.620246 and 95% CI (.4704636, .770028) while for $R_{2,4}$, came out to be 0.4250596; CI (.2752773,0.5748419).

4. Discussion

In this paper, we analyzed the behavior of Generalized X-Exponential Distribution (GXED) in calculating the multicomponent stress-strength reliability estimates. We also calculated 95% CI & coverage probability for reliability estimates and results were excellent. Coverage probability touched up to 0.98, which shows GXED estimates, very accurately.

The simulation results indicated that average bias and MSE decreased as the sample size increased for both the cases of $R_{s,k}$.The real data sets also revealed GXED fits very well and provides quite close results. Hence, GXED can be used readily to calculate the reliability in a multicomponent stress- strength environment.

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