RT&A, No 1 (77)
Volume 19, March 2024

A NEW ALGORITHM TO SOLVE MULTI-OBJECTIVE TRANSPORTATION PROBLEM WITH GENERALIZED TRAPEZOIDAL FUZZY NUMBERS

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Abstract

Transportation Problem is a specific type of linear programming problem (LPP). Today, in the real world, the decision maker handles the multi-objectives at the same time. Fuzzy Concepts are used in LPP to handle the uncertainty and vagueness of data. This paper presents a new algorithm to solve a special type of fuzzy transportation problem (FTP) with the generalized trapezoidal fuzzy numbers (GTpFN) in which the decision maker is not certain about the exact value of transportation charge and the availabilities and requirements are the real numbers. In this Proposed Algorithm first, the fuzzy multi-objective transportation problem (FMOTP) is converted into a Crisp multi-objective transportation problem (MOTP) by the Proposed ranking function, and then the Crisp MOTP is transformed into a single objective transportation problem using the sum of objective functions values. The proposed algorithm gives an efficient compromise solution of FMOTP. To elaborate the proposed algorithm, one numerical example is solved.

Keywords: Ranking function, Multi-Objective Transportation Problem, Generalized Trapezoidal fuzzy number.

1. Introduction

The transportation problem (TP) is a classical optimization problem in operations research and logistics. To satisfy requirements and availabilities, it involves determining the most cost-effective way to distribute a product from various providers to various consumers. TP aims to minimize the total transportation cost. Traditional methods, including the Vogel approximation method, the Matrix Minima approach, and the North West Corner method, are used to solve the TP. In the real-world scenario, nowadays the decision maker can handle multiple objectives at a single time in which the decision maker is unsure about the precise value of transportation cost, requirements, and availabilities. The multi-objective transportation problem (MOTP) is a linear optimization problem with several variable objectives and equality Constraints. Fuzzy concepts often deal with such types of uncertainty and vagueness in the exact cost of transportation, availabilities, and requirements. The Concept of fuzzy transportation problems (FTP) was developed to find the solution to the TP's unpredictable parameters, such as fuel prices, weather conditions, product supply, demands, etc. Trapezoidal fuzzy numbers (TpFN) are useful when modeling uncertain parameters in transportation problems, such as requirements and availabilities quantities or transportation costs, which are not precisely known but have a range of potential values.

The TP was developed by F.L. Hitchcock [1] originally in 1941. The TP was represented by a standard LPP form that can be solved by the simplex method. Lotfi A. Zadeh [2] was given the concept of fuzziness in 1965. Charnes and Cooper [3] developed The Stepping Stone approach offers an alternative approach for obtaining information from the Simplex Method. Zimmermann H.J. [4] was the first to use an appropriate membership function to solve an LP problem with multiple objectives. Ringuest et al. [5] gave two interactive algorithms for solving MOTP. Bit et al. [6] Solved TP problems with several criteria by using Fuzzy Programming. Chanas et al. [7] Proposed a model based on fuzzy linear programming to solve TPs in which cost coefficients are crisp values and supplies and demands are fuzzy values. Liu et al. [8] developed a method that is based on the extension principle to solve FTPs. Kiruthiga, M., et al. [9] used Interval arithmetic based on Alpha-cut to solve non-linear programming problems (NLP). M Afwat et al. [10] introduced the Product Approach to find an efficient solution for MOTP. Gani and Razak [11] presented a parametric approach for two-stage fuzzy cost-minimizing TP that has supplies and demands in the form of a trapezoidal fuzzy number. Bagheri. M. et al. [12] presented the DEA approach to solving FMOTP. Maity. G. et al. [13] studied the MOTP under uncertain environments. Dinagar and Palanivel [14] studied the FTP with trapezoidal fuzzy numbers. Pandian et.al [15] developed the zero-point method to find the fuzzy solution for the FTP. Hamiden Abd El-Waheed Khalifa et.al. [16] Presented a fuzzy geometric programming approach to find an optimal compromise solution for two-stage multi-objective TP. Srikanth Gupta et. al. [17] Investigated the MOTP with capacitated restrictions that have some linear objective functions and some that are fractional. Murshid Kamal et.al. [18] Studied the MOTP, where the objective function is type-2 TpFN in which supply and demand follow various types of probabilistic distributions. They used the fuzzy goal programming method to find an optimal solution. M.A. Sayed et.al. [19] Developed a novel approach to solve intuitionistic Fuzzy fractional MOTP. H. Adb E. Khalifa [20] proposed a signed distance ranking function method to obtain the set of efficient solution fuzzy MOTP. Yi-Mang et al. [21] adopted two fuzzy ranking methods based on their mean graded values and distance from the mean ranking function and proposed a novel ripple-spreading algorithm to solve FMOLPP. Y Kacher et al. [22] presented a novel two-step generalized parametric approach to solving different fuzzy parametric-based MOTP. SG Bodke [23] introduced a method to solve fuzzy MOTP after converting it into Crisp MOTP which is based on Zimmerman technique using the exponential membership function.

This paper presents a new algorithm for solving fuzzy MOTP with cost values as generalized trapezoidal fuzzy numbers and requirements availabilities are the real numbers. In this algorithm, firstly, the fuzzy MOTP is converted into Crisp MOTP by the proposed Ranking function. After converting the fuzzy MOTP into Crisp MOTP, the Crisp MOTP is changed into the single objective crisp transportation problem. The algorithm is based on row/column maximum and minimum. Our proposed method directly obtained a unique, efficient solution, which leads to a Compromise Solution of crisp and fuzzy MOTP.

2. Abbreviations

1. Linear Programming Problem - LPP
2. Transportation Problem - TP
3. Fuzzy Transportation Problem - FTP
4. Trapezoidal Fuzzy Number - TpFN
5. Generalized Trapezoidal Fuzzy Number - GTpFN

6. Multi-Objective Transportation Problem - MOTP
7. Fuzzy Multi-Objective Transportation Problem - FMOTP
8. Decision Maker- DM
9. Fuzzy Transportation Cost - FTC
10. Minimum Transportation Cost - MTC
11. Single objective transportation problem - SOTP

3. Basic Definitions

1. Fuzzy Number: A fuzzy set $\tilde{A}$ is said to be fuzzy number if its membership function $\tilde{A}$: $\mathbb{R}$ $\rightarrow [0, 1]$ has satisfy the following conditions:
   $\tilde{A}(\lambda x_1 + (1-\lambda)x_2) \geq \min\{A(\tilde{x_1})\}, A(\tilde{x_2})$
   there exist a x $\in \mathbb{R}$ such that $A(\tilde{x})$ = 1
   $\tilde{A}$ is piece-wise continuous

2. Generalized Trapezoidal Fuzzy Numbers (GTpFN): A fuzzy number $\tilde{A} = (p_1, p_2, p_3, p_4; w)$ where $p_1 < p_2 < p_3 < p_4$ and $0 < w \leq 1$ with membership function defined as:

$$\mu_{A(\tilde{x})} = \begin{cases} w\{1 - \frac{p_2 - x}{p_2 - p_1}\} & \text{if } p_1 \leq x \leq p_2 \\ w & \text{if } p_2 \leq x \leq p_3 \\ w\{1 - \frac{x - p_3}{p_4 - p_3}\} & \text{if } p_3 \leq x \leq p_4 \\ 0 & \text{Otherwise} \end{cases}$$

3. Properties of Trapezoidal Fuzzy Numbers (TpFN): let $\tilde{A} = (p_1, p_2, p_3, p_4; w_1)$ and $\tilde{B} = (q_1, q_2, q_3, q_4; w_2)$ be any two GTpFNs. then
   $\tilde{A} + \tilde{B} = (p_1, p_2, p_3, p_4; w_1) + (q_1, q_2, q_3, q_4; w_2) = (p_1 + q_1, p_2 + q_2, p_3 + q_3, p_4 + q_4; min(w_1, w_2))$
   $\tilde{A} - \tilde{B} = (p_1, p_2, p_3, p_4; w_1) - (q_1, q_2, q_3, q_4; w_2) = (p_1 - q_1, p_2 - q_2, p_3 - q_3, p_4 - q_4; min(w_1, w_2))$
   $\tilde{A} \times \tilde{B} = (p_1, p_2, p_3, p_4; w_1) \times (q_1, q_2, q_3, q_4; w_2) = \{min(p_1q_1, p_1q_4, p_4q_1, p_4q_4), min(p_2q_2, p_2q_3, p_3q_2, p_3q_3),$
   $max(p_2q_2, p_2q_3, p_3q_2, p_3q_3), max(p_1q_1, p_1q_4, p_4q_1, p_4q_4)\}$
   $\sigma\tilde{A} = (\sigma p_1, \sigma p_2, \sigma p_3, \sigma p_4)$, where σ is any constant.

4. Proposed Ranking Method

The ranking method is used to compare the fuzzy numbers.Assuming that the natural order is preserved, The ranking function $\mathfrak{R}$: $T(\mathbb{R}) \rightarrow \mathbb{R}$ defined on set of real numbers maps each fuzzy number into a real number where $T(\mathbb{R})$ is set of the fuzzy numbers.
the proposed ranking function for the Trapezoidal number $\tilde{A} = (p_1, p_2, p_3, p_4; w)$ is given as

$$\mathfrak{R}(\tilde{A}) = \frac{2p_1 + 5w(p_2 + p_3) + 2p_4}{14}$$

5. Properties of Ranking functions

$\tilde{A} = (p_1, p_2, p_3, p_4; w_1)$ and $\tilde{B} = (q_1, q_2, q_3, q_4; w_2)$ be any two GTpFNs. then the properties of the ranking function is given as:
$\tilde{A} \leq \tilde{B}$ iff $\mathfrak{R}(\tilde{A}) \leq \mathfrak{R}(\tilde{B})$

$\tilde{A} \equiv \tilde{B}$ iff $\Re(\tilde{A}) \equiv \Re(\tilde{B})$
$\tilde{A} \geq \tilde{B}$ iff $\Re(\tilde{A}) \geq \Re(\tilde{B})$

6. Mathematical model for FMOTP with GTpFN

The FMOTP with k objectives in mathematical form is given as:

Min $\tilde{Z}_k(x) = \Sigma_{i=1}^{m}\Sigma_{j=1}^{n}\tilde{a}_{ij}^{(k)}x_{ij}$ for k = 1,2,.....

Subject to
$\Sigma_{i=1}^{m}x_{ij} = d_j$: for fixed j=1,2,......,n
$\Sigma_{j=1}^{n}x_{ij} = s_i$: for fixed i=1,2,......,m
$x_{ij} \geq 0$

Where,
s_i = the product's availability at the i-th source
d_j = the product's requirements at the j-th destinations
$\tilde{a}_{ij}^{(k)}$ = the fuzzy cost for transporting one unit of the given product from i-th source to j-th destination of k-th objective
x_{ij} = Product's quantity transported from i-th source to j-th destination.
$\tilde{a}_{ij}^{(k)}$ are the GTpFNs.

7. Efficient Solution

A feasible solution $X^0 = \{x_{ij}^0, i = 1,2,....,m, j = 1,2,.....,n\}$ is called an efficient solution to the problem (T) is there does not exist any feasible solution Y of MOTP such that $Z_1(X) \leq Z_1(X^0)$ and $Z_2(X) \leq Z_2(X^0)$.

8. Our Proposed Algorithm

The Compromise efficient fuzzy solution of fuzzy MOTP is obtained by the proposed algorithm. The proposed algorithm's steps are as follows:

Step I: In this step first, the fuzzy MOTP is converted into crisp MOTP by the proposed Ranking Method. The proposed ranking method converted the fuzzy quantities into crisp quantities. The crisp MOTP in mathematical form can be given as:

Min $Z_k(x) = \Sigma_{i=1}^{m}\Sigma_{j=1}^{n}a_{ij}^{(k)}x_{ij}$ for k = 1,2,.....

Subject to
$\Sigma_{i=1}^{m}x_{ij} = d_j$: for fixed j=1,2,......,n
$\Sigma_{j=1}^{n}x_{ij} = s_i$: for fixed i=1,2,......,m
$x_{ij} \geq 0$

Where
s_i = the product availability at the i-th source
d_j = the product requirements at the j-th destinations
$a_{ij}^{(k)}$ = the crisp cost for transporting one unit quantity of product from i-th source to j-th destination of k-th objective,

x_{ij} = quantity of product transported from i-th source to j-th destination.

the crisp MOTP is represented in tabular form in Table 1.

Table 1: *Tabular representation of Crisp MOTP*

Destination → source ↓	A_1	A_2	......	A_n	Availability $\{s_i\}$
B_1	$a_{11}^{(1)}$	$a_{12}^{(1)}$	......	$a_{1n}^{(1)}$	s_1
	$a_{11}^{(2)}$	$a_{12}^{(2)}$	......	$a_{1n}^{(2)}$	
	:	:	......	:	
	$a_{11}^{(k)}$	$a_{12}^{(k)}$	......	$a_{1n}^{(k)}$	
B_2	$a_{21}^{(1)}$	$a_{22}^{(1)}$	......	$a_{2n}^{(1)}$	s_2
	$a_{21}^{(2)}$	$a_{22}^{(2)}$	......	$a_{2n}^{(2)}$	
	:	:	......	:	
	$a_{21}^{(k)}$	$a_{22}^{(k)}$	......	$a_{2n}^{(k)}$	
:	:	:	:	:	:
B_m	$a_{m1}^{(1)}$	$a_{m2}^{(1)}$	......	$a_{mn}^{(1)}$	s_m
	$a_{m1}^{(2)}$	$a_{m2}^{(2)}$	......	$a_{mn}^{(2)}$	
	:	:	......	:	
	$a_{m1}^{(k)}$	$a_{m2}^{(k)}$	......	$a_{mn}^{(k)}$	
Requirement (d_j)	d_1	d_2	......	d_n	

Step II: In this step the sum of the objectives is calculated.

$t_{ij} = \Sigma_{v=1}^{k} C_{ij}^{(r)}$ for $1 \leq i \leq m$ and $1 \leq v \leq k$.

then the crisp Single-Objective Transportation Problem (SOTP) in tabular form is represented in table 2.

Table 2: *Tabular representation of Crisp SOTP*

Destination → source ↓	A_1	A_2	......	A_n	Availability $\{s_i\}$
B_1	t_{11}	t_{12}	......	t_{1n}	s_1
B_2	t_{21}	t_{22}	......	t_{2n}	s_2
:	:	:	......	:	
B_m	t_{m1}	t_{m2}	......	t_{mn}	s_m
Requirement (d_j)	d_1	d_2	......	d_n	

Step III: Penalties of each row and columns

The Penalties for each row $B_i : 1 \leq i \leq m$ is calculated as:
Row penalties μ_i = [maximum (t_{1r}) - minimum (t_{1r})] for $1 \leq r \leq n, \forall i : 1 \leq i \leq m$

Similarly, the penalties for each column $A_p : 1 \leq p \leq n$ is calculated as:
Column Penalties ρ_j = [maximum (t_{s1}) - minimum (t_{s1})] for $1 \leq s \leq m, \forall j : 1 \leq j \leq n$.

The Crisp MOTP with penalties in given in table 3.

Table 3: *Tabular representation of Crisp SOTP with penalties*

Destination → source ↓	A_1	A_2	......	A_n	Availability (s_i)	Row penalties (μ_i)
B_1	t_{11}	t_{12}	......	t_{1n}	s_1	μ_1
B_2	t_{21}	t_{22}	......	t_{2n}	s_2	μ_2
:	:	:	......	:	:	:
B_m	t_{m1}	t_{m2}	......	t_{mn}	s_m	μ_m
Requirement (d_j)	d_1	d_2	......	d_n		
Column Penalties (ρ_j)	ρ_1	ρ_2	......	ρ_n		

Step IV: In this step, the maximum penalty (δ) is calculated as:

δ = max $\{\mu_i, \rho_j; 1 \leq i \leq m, 1 \leq j \leq n\}$
Select that Row/column which has a maximum penalty δ

Step V: In this step, select the cell that has minimum objective value in the row/column, that is selected in step IV.

Step VI: The maximum requirement/availability is allocated to the cell that is selected in Step V, and ignore that row/column that requirement/availability is satisfied.

Step VII: Repeat the process until all the requirements/availabilities are not fulfilled.

The flowchart of the proposed algorithm is given in figure 1.

To elaborate the proposed algorithm, a numerical example is considered.

Example: A fuzzy MOTP with three objective functions is considered.
Coe-efficient Matrix for a first objective function

$$C_1 = \begin{bmatrix} \tilde{a}_{11}^{(1)} & \tilde{a}_{12}^{(1)} & \tilde{a}_{13}^{(1)} & \tilde{a}_{14}^{(1)} & \tilde{a}_{15}^{(1)} \\ \tilde{a}_{21}^{(1)} & \tilde{a}_{22}^{(1)} & \tilde{a}_{23}^{(1)} & \tilde{a}_{24}^{(1)} & \tilde{a}_{25}^{(1)} \\ \tilde{a}_{31}^{(1)} & \tilde{a}_{32}^{(1)} & \tilde{a}_{33}^{(1)} & \tilde{a}_{34}^{(1)} & \tilde{a}_{35}^{(1)} \\ \tilde{a}_{41}^{(1)} & \tilde{a}_{42}^{(1)} & \tilde{a}_{43}^{(1)} & \tilde{a}_{44}^{(1)} & \tilde{a}_{45}^{(1)} \end{bmatrix}$$

$$= \begin{bmatrix} (9,10,11,12;0.8) & (12,13,14,18;0.8) & (8,9.5,11.5,13;0.8) & (3,7,9,8;0.8) & (5,9,12,16;0.8) \\ (6,7,8,13;0.8) & (2,3,3.5,6;0.8) & (5,6.5,8.5,14;0.8) & (5,6,9,14;0.8) & (4,5,6,9;0.8) \\ (4,6,7,12;0.8) & (3,4,7,10;0.8) & (7,9,12,14;0.8) & (3,14,15,16;0.8) & (2,2.5,4,6;0.8) \\ (2,6,9,10;0.8) & (6,9,10,12;0.8) & (3.5,13,16,16.5;0.8) & (1,1.5,2.5,5;0.8) & (1,2,2.5,4;0.8) \end{bmatrix}$$

Coe-efficient Matrix for the second objective function

$$C_2 = \begin{bmatrix} \tilde{a}_{11}^{(2)} & \tilde{a}_{12}^{(2)} & \tilde{a}_{13}^{(2)} & \tilde{a}_{14}^{(2)} & \tilde{a}_{15}^{(2)} \\ \tilde{a}_{21}^{(2)} & \tilde{a}_{22}^{(2)} & \tilde{a}_{23}^{(2)} & \tilde{a}_{24}^{(2)} & \tilde{a}_{25}^{(2)} \\ \tilde{a}_{31}^{(2)} & \tilde{a}_{32}^{(2)} & \tilde{a}_{33}^{(2)} & \tilde{a}_{34}^{(2)} & \tilde{a}_{35}^{(2)} \\ \tilde{a}_{41}^{(2)} & \tilde{a}_{42}^{(2)} & \tilde{a}_{43}^{(2)} & \tilde{a}_{44}^{(2)} & \tilde{a}_{45}^{(2)} \end{bmatrix}$$

$$= \begin{bmatrix} (1,2.5,3.5,4;0.6) & (10,12,13,15.5;0.6) & (9.5,10,11,15;0.6) & (0.5,1,1.5,2.75;0.6) & (2.5,5,6,9;0.6) \\ (0.25,1,1.5,3) & (9,11,14,16.5;0.6) & (8.5,10,15,17;0.6) & (2.5,7,8,10;0.6) & (1.5,2,3,5;0.6) \\ (4.5,12,13,14;0.6) & (0.75,1,1.5,2.5;0.6) & (4,11,14,14.5;0.6) & (2,4,7,9.5;0.6) & (3,6,9,9.5;0.6) \\ (1.5,2,3,5;0.6) & (7,8,13,17;0.6) & (2.5,9,10,11;0.6) & (8,10,15,17.5;0.6) & (6,8,13,18.5;0.6) \end{bmatrix}$$

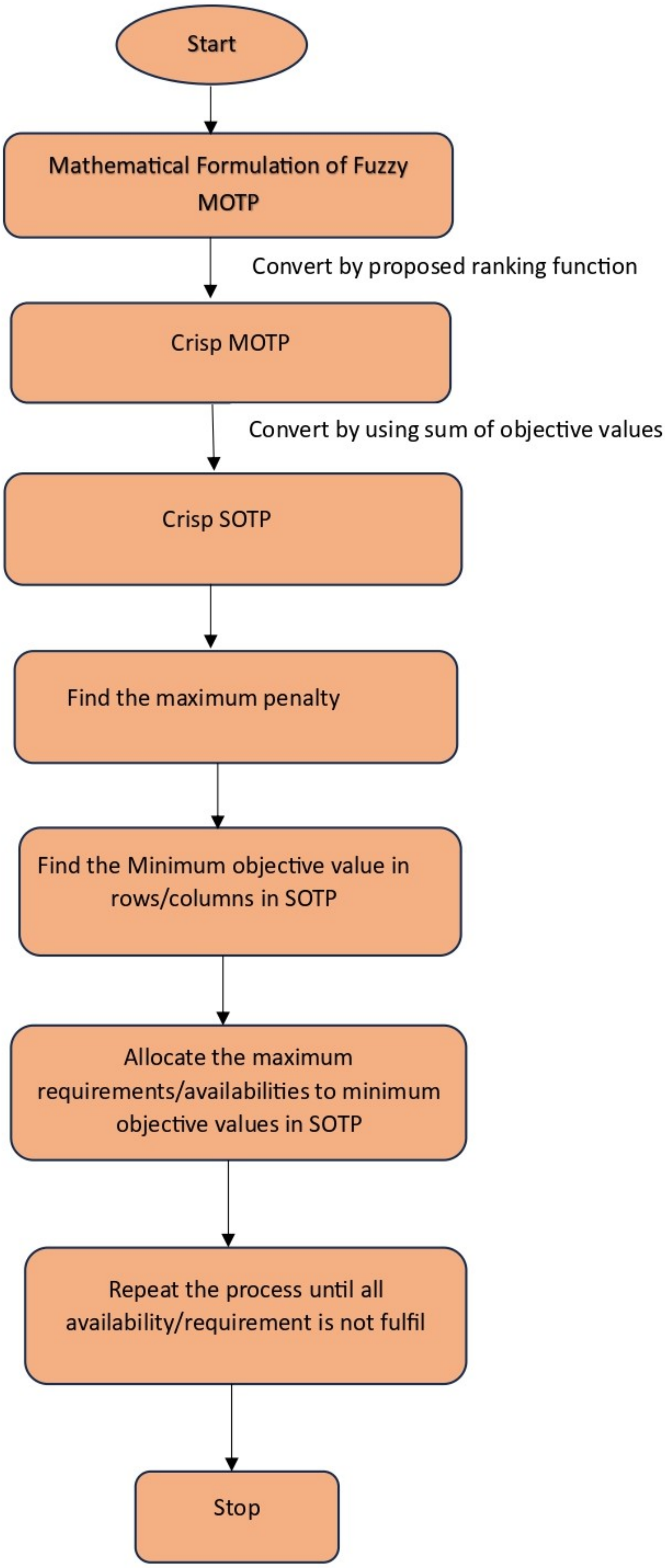

Figure 1: *Flowchart for the Proposed Algorithm*

Coe-efficient Matrix for the third objective function

$$C_3 = \begin{bmatrix} \tilde{a}_{11}^{(3)} & \tilde{a}_{12}^{(3)} & \tilde{a}_{13}^{(3)} & \tilde{a}_{14}^{(3)} & \tilde{a}_{15}^{(3)} \\ \tilde{a}_{21}^{(3)} & \tilde{a}_{22}^{(3)} & \tilde{a}_{23}^{(3)} & \tilde{a}_{24}^{(3)} & \tilde{a}_{25}^{(3)} \\ \tilde{a}_{31}^{(3)} & \tilde{a}_{32}^{(3)} & \tilde{a}_{33}^{(3)} & \tilde{a}_{34}^{(3)} & \tilde{a}_{35}^{(3)} \\ \tilde{a}_{41}^{(3)} & \tilde{a}_{42}^{(3)} & \tilde{a}_{43}^{(3)} & \tilde{a}_{44}^{(3)} & \tilde{a}_{45}^{(3)} \end{bmatrix}$$

$$= \begin{bmatrix} (0.5,1,2,6;1) & (2.5,3,4,8;1) & (4.5,6,10;1) & (1,2,4,5;1) & (3,4,7,11.5;1) \\ (2,2.5,4.5,8.5;1) & (6,7,8,12.5;1) & (1,2,5,9.5;1) & (7.5,8,9,13;1) & (0.25,1,2,6.25;1) \\ (3.5,4,5,9;1) & (1,2.5,3.5,5;1) & (3,4,5,9.5;1) & (2,2.5,3.5,4;1) & (4,4.5,6.5,10.5;1) \\ (3.5,5,6,11;1) & (7,7.5,9.5,13.5;1) & (2.5,4,7,12;1) & (1.5,2.5,3.5,4.5;1) & (0.25,0.5,1.5,1.75;1) \end{bmatrix}$$

Availabilities: $s_1 = 5$, $s_2 = 4$, $s_3 = 2$, $s_4 = 9$.

Requirements: $d_1 = 4$, $d_2 = 4$, $d_3 = 6$, $d_4 = 2$ $d_5 = 4$

Step I: In this step, the fuzzy MOTP is transformed into crisp MOTP using the Ranking function given above:
$\tilde{a}_{11}^{(1)} = (9, 10, 11, 12; 0.8)$
Here $p_1 = 9$, $p_2 = 10$, $p_3 = 11$, $p_4 = 12$ w = 0.8

$$\Re(\tilde{a}_{11}^{(1)}) = \frac{2\times9+5\times0.8(10+11)+2\times12}{14} = 9 = a_{11}^{(1)}$$

Similarly, all the fuzzy values $\tilde{a}_{ij}^{(v)}$ for $1 \leq j \leq 5, 1 \leq i \leq 4$ and $1 \leq v \leq 3$. can be converted in crisp values by using ranking function.

The crisp MOTP is represented in tabular form in table 4.

Table 4: *Tabular representation of Crisp MOTP*

Destination → source ↓	A_1	A_2	A_3	A_4	A_5	Availability $\{s_i\}$
B_1	9	12	9	6	9	5
	2	9	8	1	4	
	2	4	6	3	6	
B_2	7	3	7	7	5	4
	1	9	9	5	2	
	4	8	4	9	2	
B_3	6	5	9	11	3	2
	8	1	8	4	5	
	5	3	5	3	6	
B_4	6	8	11	2	2	9
	2	8	6	9	8	
	6	9	6	3	1	
Requirement (d_j)	4	4	6	2	4	

Step II: In this step, the sum of objectives value is calculated.
$t_{11} = \Sigma_{v=1}^{3} a_{11}^{(v)} = a_{11}^{1} + a_{11}^{2} + a_{11}^{3}$ = 9+2+3 = 13
Similarly,
$t_{12} = \Sigma_{v=1}^{3} a_{12}^{(v)}$ =25, $t_{13} = \Sigma_{v=1}^{3} a_{13}^{(v)}$ =23,$t_{14} = \Sigma_{v=1}^{3} a_{14}^{(v)}$ =10, $t_{15} = \Sigma_{v=1}^{3} a_{15}^{(v)}$ = 19
$t_{21} = \Sigma_{v=1}^{3} a_{21}^{(v)}$ =12, $t_{22} = \Sigma_{v=1}^{3} a_{22}^{(v)}$ =20,$t_{23} = \Sigma_{v=1}^{3} a_{23}^{(v)}$ =20, $t_{24} = \Sigma_{v=1}^{3} a_{24}^{(v)}$ = 21, t_{25} =$\Sigma_{v=1}^{3} a_{25}^{(v)}$ = 9

t_{31} = $\Sigma_{v=1}^{3} a_{31}^{(v)}$ = 18, t_{32} = $\Sigma_{v=1}^{3} a_{32}^{(v)}$ =9,t_{33} = $\Sigma_{v=1}^{3} a_{33}^{(v)}$ =22, t_{34} = $\Sigma_{v=1}^{3} a_{34}^{(v)}$ = 18, t_{35} =$\Sigma_{v=1}^{3} a_{35}^{(v)}$ = 14
t_{41} = $\Sigma_{v=1}^{3} a_{41}^{(v)}$ = 14, t_{42} = $\Sigma_{v=1}^{3} a_{42}^{(v)}$ =25,t_{43} = $\Sigma_{v=1}^{3} a_{43}^{(v)}$ =23, t_{44} = $\Sigma_{v=1}^{3} a_{44}^{(v)}$ = 14, t_{45} =$\Sigma_{v=1}^{3} a_{45}^{(v)}$ = 11
Then, Crisp SOTP is given in Table 5.

Table 5: *Tabular representation of Crisp SOTP*

Destination $\rightarrow$ source $\downarrow$	A_1	A_2	A_3	A_4	A_5	Availability $\{s_i\}$
B_1	13	25	23	10	19	5
B_2	12	20	20	21	9	4
B_3	18	9	22	18	14	2
B_4	14	25	23	14	11	9
Requirement (d_j)	4	4	6	2	4	

Step III: the penalties for each Row and Columns is calculated as:

Rows Penalties:
μ_1 = [maximum(t_{1r}) - minimum(t_{1r})] for $1 \leq r \leq 5$
= [maximum$(t_{11}, t_{12}, t_{13}, t_{14}, t_{15})$ - minimum$(t_{11}, t_{12}, t_{13}, t_{14}, t_{15})$]
= [maximum$(13, 25, 23, 10, 19)$ - minimum$(13, 25, 23, 10, 19)$]
= 25-10 =15
Similarly, the remaining row penalties is
μ_2 =12, μ_3 =13, μ_4 =14

Columns Penalties:
ρ_1 = [maximum(t_{s1}) - minimum(t_{s1})] for $1 \leq s \leq 4$
= [maximum$(t_{11}, t_{21}, t_{31}, t_{41})$ - minimum$(t_{11}, t_{21}, t_{31}, t_{41})$]
= [maximum$(13, 12, 18, 14)$ - minimum$(13, 12, 18, 14)$]
= 18-12 =6
Similarly, the remaining column penalties is
ρ_2 =16, ρ_3 =3, ρ_4 =11,ρ_5 = 10
The tabular representation of SOTP with row and column penalties is given in Table 6.

Table 6: *Tabular representation of Crisp SOTP With Penalties*

Destination $\rightarrow$ source $\downarrow$	A_1	A_2	A_3	A_4	A_5	Availability $\{s_i\}$	row penalties μ_i
B_1	13	25	23	10	19	5	15
B_2	12	20	20	21	9	4	12
B_3	18	9	22	18	14	2	13
B_4	14	25	23	14	11	9	14
Requirement (d_j)	4	4	6	2	4		
Column penalties (ρ_j)	6	16	3	11	10		

Step IV: In this Step, the maximum penalty δ is calculated.
δ = max $\{\mu_i, \rho_j; 1 \leq i \leq 4, 1 \leq j \leq 5\}$
= max $\{\mu_1, \mu_2, \mu_3, \mu_4, \rho_1, \rho_2, \rho_3, \rho_4, \rho_5\}$
= max$\{15, 12, 13, 14, 6, 16, 3, 11, 10\}$
= 16 = ρ_2
Here, Column A_2 has a maximum penalty.

Step V: In the table 6, in Column A_2, the cell a_{32} (= 9) has the minimum objective value.

Step VI: Now, allocate min (2,4) = 2 to cell t_{32} and delete the Row B_3 whose availability is fulfilled.

Step VII : Apply the same procedure from Step II to Step VIII for making the possible allocation in the remaining rows and columns, hence the 2^{nd}, 3^{rd}, 4^{th}, 5^{th},6^{th}, 7^{th}, and,8^{th} allocations as 2,4,3,1,2,2,4 at cells $t_{14}, t_{45}, t_{11}, t_{41}, t_{22}, t_{23}, t_{43}$ positions respectively. the optimum allocation of MOTP is given in Table 7.

Table 7: *Final allocation table*

Destination → source ↓	A_1	A_2	A_3	A_4	A_5	Availability $\{s_i\}$
B_1	13(3)	25	23	10(2)	19	5
B_2	12	20(2)	20(2)	21	9	4
B_3	18	9(2)	22	18	14	2
B_4	14	25	23(4)	14	11(4)	9
Requirement (d_j)	4	4	6	2	4	

9. Result Analysis

In this section, the results obtained from the example are analyzed. The final Solution table for the example is shown in Table 8

Table 8: *Final Solution table*

Obtained Allocations	$x_{11} = 3, x_{12} = 0, x_{13} = 0, x_{14} = 2, x_{15} = 0$ $, x_{21} = 0, x_{22} = 2, x_{23} = 2, x_{24} = 0, x_{25} = 0, x_{31} = 0$ $,x_{32} = 2, x_{33} = 0, x_{34} = 0, x_{35} = 0, x_{41} = 1,$ $x_{42} = 0, x_{43} = 4, x_{44} = 0, x_{45} = 4$
Fuzzy Compromise efficient solution of MOTP	$[(71, 137, 170, 206; 0.8), (76, 123.5, 169.5, 212.5; 0.6),$ $(34, 53, 87, 148 : 1)]$
Crisp Compromise efficient solution of MOTP	$(127, 104, 76)$
Nature of Crisp Compromise Crisp Solution	Non-Degenerate

Physical Interpretation of the results: The obtained solution, as presented in Table 8, can be physically interpreted as follows:

(I) For First Objective Function: using the proposed algorithm the minimum fuzzy transportation cost (FTC) is [(71,137,170,206;0.8)]. It has the following physical interpretation:

(i) In the decision-maker's estimation, The minimum transportation cost (MTC) will be greater than Rs. 71 and less than Rs. 206 units.

(ii) The decision-maker is 80% satisfied overall with the statement that transportation costs will be 137-180.

(iii) The following values of the remaining minimum transportation cost can be used to determine the decision-maker's overall level of satisfaction:
If x is the MTC, then the overall decision-maker satisfaction level for x = $\mu_{\tilde{A}}(x) \times 100$

Where

$$\mu_{A(\tilde{x})} = \begin{cases} 0.8\{1-\frac{137-x}{66}\} & \text{if } 71 \leq x \leq 137 \\ 0.8 & \text{if } 137 \leq x \leq 170 \\ 0.8\{1-\frac{x-170}{36}\} & \text{if } 170 \leq x \leq 206 \\ 0 & \text{Otherwise} \end{cases}$$

(II) For Second Objective Function: using the proposed algorithm the minimum FTP is [(76,123.5,169.5,212.5;0.6)] It has the following physical interpretation:

(i) The MTC, in the decision-maker's estimation, will be greater than Rs. 76 and less than Rs. 212.5 units.

(ii) The decision-maker is 60% satisfied overall with the statement that transportation costs will be 123.5-169.5.

(iii) The following values of the remaining minimum transportation cost can be used to determine the decision-makers overall level of satisfaction:
If x is the MTC, then the overall decision-maker satisfaction level for x = $\mu_{\tilde{A}}\ (x)\ \times 100$
Where

$$\mu_{A(\tilde{x})} = \begin{cases} 0.6\{1-\frac{123.5-x}{47.5}\} & \text{if } 76 \leq x \leq 123.5 \\ 0.6 & \text{if } 123.5 \leq x \leq 169.5 \\ 0.6\{1-\frac{x-169.5}{43}\} & \text{if } 169.5 \leq x \leq 212.5 \\ 0 & \text{Otherwise} \end{cases}$$

(III) For Third Objective Function: using the proposed algorithm the minimum FTC is [(34,53,87,148;1)]. It has the following physical interpretation:

(i)The MTC in the decision-makers estimation, will be greater than Rs. 34 and less than Rs. 148 units.

(ii) The decision-maker is 100% satisfied overall with the statement that transportation costs will be 53-87.

(iii) The following values of the remaining minimum transportation cost can be used to determine the decision-makers overall level of satisfaction:
If x is the MTC, then the overall decision-maker satisfaction level for x = $\mu_{\tilde{A}}\ (x)\ \times 100$
Where

$$\mu_{A(\tilde{x})} = \begin{cases} 1.0\{1-\frac{53-x}{19}\} & \text{if } 34 \leq x \leq 53 \\ 1.0 & \text{if } 53 \leq x \leq 87 \\ 1.0\{1-\frac{x-87}{61}\} & \text{if } 87 \leq x \leq 148 \\ 0 & \text{Otherwise} \end{cases}$$

10. Conclusion

This paper presents a new algorithm for solving the fuzzy multi-objective transportation problem (FMOTP) with objective function values as generalized trapezoidal fuzzy numbers, availabilities, and requirements are given as real numbers. The proposed algorithm first converts the fuzzy

MOTP into Crisp MOTP by ranking function after converting, the multi-objective MOTP is converted into Single objective TP. The proposed algorithm gives an efficient compromise solution and also provides a satisfaction level to the decision-maker in real-life situations. the proposed algorithm is less time-consuming and simple to use.

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