A TYPE I HALF LOGISTIC TOPP-LEONE INVERSE LOMAX DISTRIBUTION WITH APPLICATIONS IN SKINFOLDS ANALYSIS

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Abstract

This paper proposed a novel distribution parameterized by four parameters. This is achieved by compounding the potentials properties of the Type I half logistic topp-leone generalized distribution family with the properties of the inverse lomax distribution to form the novel Type I half logistic topp-leone inverse lomax distribution. The novel distribution is potentially capable of extending classical inverse lomax distribution. The potentiality of the shape of the probability density function of the novel distribution is worth recognizing since it produces right skewed, approximately normal, left skewed and a reverted J-shaped. Decreasing life failure shape is also observed. Distinctive features of the novel distribution such as moments, entropy, moment generating function, reliability and hazard function were derived. The estimation method explored in this study is maximum likelihood estimation. It is adopted to estimate the novel distribution unknown parameters. Real life data set was adopted to investigate the potentiality and applicability of the novel model. The type I half logistic topp-leone inverse lomax distribution outperform the recent models.

Keywords: Type I Half Logistic Topp-leone-G family, Inverse Lomax, Maximum Likelihood Estimation.

I. Introduction

Extension of the classical models have received tremendous attention, and the new extension is applicable to real life problems ranging from medical science, environmental, economics, demography, engineering, industrial statistics, biological sciences, and actuary science. There are several approaches to improve the classical distribution. However, the recent approaches provide the parents distributions with more shapes capacities and model flexibility through the generalized distribution families.

The type I half logistic Topp-leone –G (TIHLTL-G) distribution family was proposed by [1]. The family is characterized with two shapes, and the hazard rate shape which includes increasing, decreasing and bathtub shapes. The family is seen with potentiality capable of improve the classical model such as exponential model. On the other hand, the Lomax (L) distribution [2] sometimes refers to as Pareto type II distribution, coined from the second kind of generalized beta distribution. The L distribution is purposely applied to solve problems in insurance, biological sciences, economics, reliability modeling, lifetime and engineering and other areas [3]. According to [4], the L distribution is an excellent distribution with potential of modeling survival complexity, and life-experimentation (engineering) and survival analysis.

The Inverse Lomax (IL) distribution is an excellent replacement for some closely related distributions like Inverse Weibull, Lomax, Gamma, Weibull distribution. Reason being that IL distribution possesses decreasing and upside-down bathtub hazard rate shape. Researchers, analysts and statistician found the IL distribution has a viable model useful in modeling diverse data sets. The [5] illustrated that IL distribution is among the inverted distribution family with noticeable flexibility in modeling various data sets, especially the non-monotonic failure rate. The IL distribution has also witness diverse extension, as it can be seen in [5], [6], [7], [8], [9], [10], [11], [12] and [13].

The author, [14] study reliability data using generalized IL distribution. The breaking stress of carbon fibres data was investigated by [15], the statistical methods for reliability data was study by [16], the [17] analyzed competing risks survival data, the reliability assessment under extended Chen distribution by [18]

In a scenario where a random variable X emanated from IL distribution, with cumulative distribution function (cdf) and probability density function (pdf) is expressed as:

$$H(x)=\left[1+\frac{a}{x}\right]^{-b} \tag{1}$$

$$h(x)=abx^{-2}\left[1+\frac{a}{x}\right]^{-b-1} \tag{2}$$

The cdf likewise the pdf of the TIHLTL-G by [1] are distinctly stated below

$$F_{TIHLTL-G}\left(x;\theta,\beta,\lambda\right)=\frac{1-\left[1-\left[1-\left(1-\left(H\left(x;\lambda\right)\right)\right)^{2}\right]^{\beta}\right]^{\theta}}{1+\left[1-\left[1-\left(1-\left(H\left(x;\lambda\right)\right)\right)^{2}\right]^{\beta}\right]^{\theta}} \tag{3}$$

$$f_{TIHLTL-G}\left(x;\theta,\beta,\lambda\right)=\frac{4\theta\beta h\left(x;\lambda\right)\left[1-H\left(x;\lambda\right)\right]\left[1-\left[1-H\left(x;\lambda\right)\right]^{2}\right]^{\beta-1}\left[1-\left[1-\left[1-H\left(x;\lambda\right)\right]^{2}\right]^{\beta}\right]^{\theta-1}}{\left[1+\left[1-\left[1-\left(1-H\left(x;\lambda\right)\right)^{2}\right]^{\beta}\right]^{\theta}\right]^{2}} \tag{4}$$

The justification for this study lies in the fact that the IL distribution is noticed to have suffered from lack of pliability in the tail and peak features. This call for extensions of the IL distribution, diverse extensions has been witnessed. However, some of the extensions lack good flexibility. This motivates us to introduce a new attractive extension with TIHLTL-G with two shape parameters which can offer additional flexibility and improve the goodness of fit of the IL distribution.

II. Methods

2.1 Type I Half Logistic Toppleone Inverse Lomax (TIHLTL-IL) Distribution

This section introduces the novel TIHLTL-IL distribution cdf and pdf. The pdf plots, densities expansion, and statistical features of the novel distribution. This characterization of this new distribution will be revealed and evaluated. The cdf and pdf of the novel TIHLTL-IL distribution is obtained as:

$$F_{TIHLTL-IL}\left(x;\theta,\beta,a,b\right)=\frac{1-\left[1-\left[1-\left(1-\left[1+\frac{a}{x}\right]^{-b}\right)^{2}\right]^{\beta}\right]^{\theta}}{1+\left[1-\left[1-\left(1-\left[1+\frac{a}{x}\right]^{-b}\right)^{2}\right]^{\beta}\right]^{\theta}} \tag{5}$$

$$f_{TIHLTL-IL}\left(x;\theta,\beta,a,b\right)=\frac{4\theta\beta abx^{-2}\left[1+\frac{a}{x}\right]^{-b-1}\left[1-\left[1+\frac{a}{x}\right]^{-b}\right]\left[1-\left[1-\left[1+\frac{a}{x}\right]^{-b}\right]^{2}\right]^{\beta-1}\left[1-\left[1-\left[1-\left[1+\frac{a}{x}\right]^{-b}\right]^{2}\right]^{\beta}\right]^{\theta-1}}{\left[1+\left[1-\left[1-\left(1-\left[1+\frac{a}{x}\right]^{-b}\right)^{2}\right]^{\beta}\right]^{\theta}\right]^{2}} \tag{6}$$

where $x\geq 0$, $a>0$ scale and $b,\beta,\theta>0$ are shape parameters.

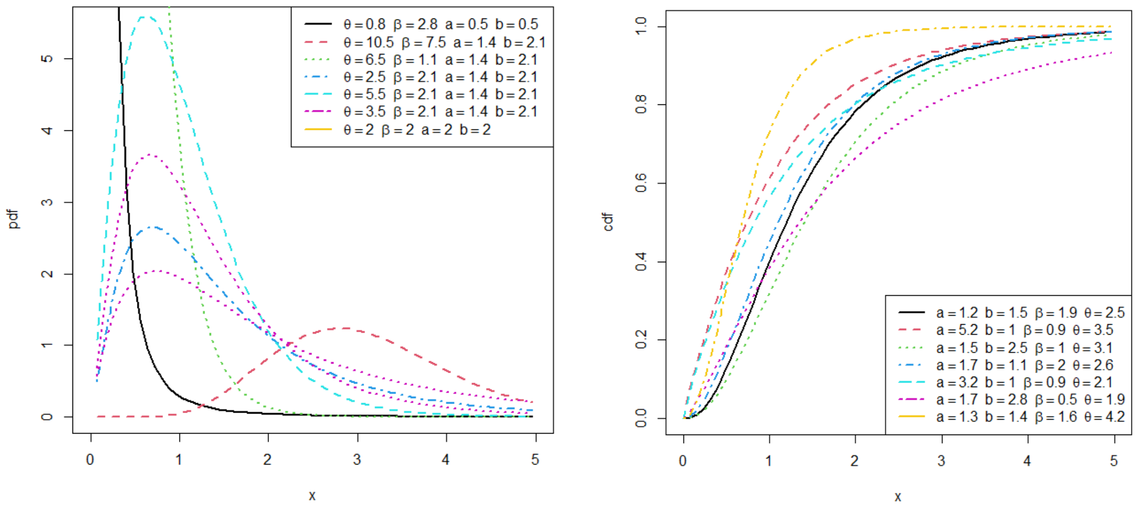

Figure 1: *Pdf and cdf plots of TIHLTL-IL distribution with various parameters' choices.*

From Figure 1, it is noticeable that the TIHLTL-IL distribution pdf can be seen as reversed J, approximately symmetric, right-skewed shaped. The additional plus observed in the new model is that, it revealed different forms of shapes with certain versatility in skewness, kurtosis and mode. The pdf is capable of modeling a heavy tailed and approximately symmetric data. The plus observed from the new model cannot be attributed to the IL distribution. The cdf plot of the novel TIHLTL-IL distribution converges to one. Its probability values range from zero to one. This implies that the novel TIHLTL-IL is a valid distribution.

2.2.1 Density expansion of TIHLTL-IL distribution

Consider the generalized binomial expansion expressed below

$$\left(1+z\right)^{-p}=\sum_{j=0}^{\infty}\left(-1\right)^{j}\binom{p+j-1}{j}z^{j} \tag{7}$$

$$\left(1-z\right)^{r}=\sum_{k=0}^{\infty}\left(-1\right)^{k}\binom{r}{k}z^{k} \tag{8}$$

Now consider the pdf given in (6) for expansion.

$$f_{TIHLTL-IL}\left(x;\beta,\theta,a,b\right)=4\theta\beta abx^{-2}\left[1+\frac{a}{x}\right]^{-b-1}\left[1-\left[1+\frac{a}{x}\right]^{-b}\right]\left[1-\left[1-\left[1+\frac{a}{x}\right]^{-b}\right]^{2}\right]^{\beta-1}\left[1-\left[1-\left[1-\left[1+\frac{a}{x}\right]^{-b}\right]^{2}\right]^{\beta}\right]^{\theta-1}$$
$$\left[1+\left[1-\left[1-\left(1-\left[1+\frac{a}{x}\right]^{-b}\right)^{2}\right]^{\beta}\right]^{\theta}\right]^{-2}$$

now, consider this term for expansion using the generalized binomial expansion in (7) and (8)

$$\left[1+\left[1-\left[1-\left(1-\left[1+\frac{a}{x}\right]^{-b}\right)^{2}\right]^{\beta}\right]^{\theta}\right]^{-2}=\sum_{j=0}^{\infty}\left(-1\right)^{j}\binom{1+j}{j}\left[1-\left[1-\left(1-\left[1+\frac{a}{x}\right]^{-b}\right)^{2}\right]^{\beta}\right]^{j\theta}$$

$$\left[1-\left[1-\left(1-\left[1+\frac{a}{x}\right]^{-b}\right)^{2}\right]^{\beta}\right]^{j\theta+\theta-1}=\sum_{k=0}^{\infty}\left(-1\right)^{k}\binom{j\theta+\theta-1}{k}\left[1-\left(1-\left[1+\frac{a}{x}\right]^{-b}\right)^{2}\right]^{\beta k}$$

Following similar approach of expansion, we obtained the simplified version of the pdf.

Hence the pdf is rewritten as

$$f_{TIHLTL-IL}\left(x;\beta,\theta,a,b\right)=\sum_{j=0}^{\infty}\zeta_{\psi}x^{-2}\left[1+\frac{a}{x}\right]^{-b(1+m)-1} \tag{9}$$

where $\zeta_{\psi}=4\theta\beta ab\sum_{k,l,m=0}^{\infty}\left(-1\right)^{(k+l+m)}\binom{1+j}{j}\binom{j\theta+\theta-1}{k}\binom{\beta k+\beta-1}{l}\binom{2l+1}{m}$

Similarly, cdf expansion goes same way.

Consider the $\left[F_{TIHLTL-IL}\left(x;\theta,\beta,a,b\right)\right]^{k}$

$$F\left[F_{TIHLTL-IL}\left(x;\theta,\beta,a,b\right)\right]^{k}=\left[1-\left[1-\left[1-\left(1-\left[1+\frac{a}{x}\right]^{-b}\right)^{2}\right]^{\beta}\right]^{\theta}\right]^{k}\left[1+\left[1-\left[1-\left(1-\left[1+\frac{a}{x}\right]^{-b}\right)^{2}\right]^{\beta}\right]^{\theta}\right]^{-k}$$

Now, we expand this using the generalized binomial expansion in (8) and (9)

$$\left[1+\left[1-\left[1-\left(1-\left[1+\frac{a}{x}\right]^{-b}\right)^2\right]^{\beta}\right]^{\theta}\right]^{-w}=\sum_{j=0}^{\infty}(-1)^j\binom{w+j-1}{j}\left[1-\left[1-\left(1-\left[1+\frac{a}{x}\right]^{-b}\right)^2\right]^{\beta}\right]^{\theta j}$$

$$\left[1-\left[1-\left[1-\left(1-\left[1+\frac{a}{x}\right]^{-b}\right)^2\right]^{\beta}\right]^{\theta}\right]^{w}=\sum_{k=0}^{\infty}(-1)^k\binom{w}{k}\left[1-\left[1-\left(1-\left[1+\frac{a}{x}\right]^{-b}\right)^2\right]^{\beta}\right]^{\theta k}$$

Following similar approach of expansion, we obtained the simplified version of the cdf.

Hence the $\left[F_{TIHLTL-IL}\left(x;\theta,\beta,a,b\right)\right]^k=\sum_{j=0}^{\infty}\alpha_{\varsigma}\left[1+\frac{a}{x}\right]^{-bn}$ (10)

where $\zeta_{\psi}=\sum_{k,l,m,n=0}^{\infty}(-1)^{(k+l+m)}\binom{w+j-1}{j}\binom{w}{k}\binom{\theta(k+j)}{l}\binom{\beta l}{m}\binom{2m}{n}$

2.3 Properties of TIHLTL-IL distribution

In this section, derivation of the TIHLTL-IL distribution statistical properties is done. Properties explored are moments, probability weighted moment, entropy, reliability function, hazard function and quantile function.

2.3.1 Moments

Moments of any distributions avails researcher the chance to investigate and reveal some important properties such as kurtosis, skewness, dispersion and central tendency. Assuming z is a random variable.

$$E\left(Z^r\right)=\int_0^{\infty}z^r f\left(z\right)dz \tag{11}$$

To obtain the Ms of the TIHLTL-IL distribution, we substitute (9) and (10) in (11). Then we have

$$E\left(X^r\right)=\zeta_{\psi}\int_0^{\infty}\left[1+\frac{a}{x}\right]^{-b(1+m)-1}x^{r-2}dx$$

where $\int_0^{\infty}\left[1+\frac{a}{x}\right]^{-b(1+m)-1}x^{r-2}dx=a^rB\left[\left(1-r\right),\left(b(1+m)+r\right)\right]$

$$E\left(X^r\right)=\sum^{\infty}\zeta_{\psi}a^rB\left[\left(1-r\right),\left(b(1+m)+r\right)\right] \tag{12}$$

2.3.2 Probability Weighted Moments (PWMs)

The PWMs generally represented mathematically as:

$$\omega_{r,s} = E\left\lfloor z^r F(z)^s \right\rfloor = \int_0^\infty z^r f(z) F(z)^s \, dz \tag{13}$$

In order to obtained the PWMs of the TIHLTL-IL distribution, we substitute (9) and (10) in (13) and make $k = s$. Then we have,

$$\omega_{r,s} = \sum_{j=0}^{\infty} \zeta_{\psi} \alpha_{\varsigma} a^r B\left[(1-r),(b(1+m+n)+r)\right]$$

now,

$$\int_0^\infty \left[1+\frac{a}{x}\right]^{-b(1+m+n)-1} x^{r-2} dx = a^r B\left[(1-r),(b(1+m+n)+r)\right]$$

$$\omega_{r,s} = \sum_{j=0}^{\infty} \zeta_{\psi} \alpha_{\varsigma} a^r B\left[(1-r),(b(1+m+n)+r)\right] \tag{14}$$

2.3.3 Entropy

Entropy is applied as a metric of uncertainty or randomness, which exists in a random observation of its real population composition. A larger value of entropy signifies greater uncertainty in the data. It follows that continuous random variable X under the Shannon entropy is expressed as:

$$H_\alpha(x) = \frac{1}{1-\alpha} \log \int_{-\infty}^{\infty} f_{TIHLTL-IL}(x;\beta,\theta,a,b)^\alpha \, dx \tag{15}$$

$$f_{TIHLTL-IL}(x;\beta,\theta,a,b)^\alpha = \left[\sum_{j=0}^{\infty} \zeta_{\psi} x^{-2} \left[1+\frac{a}{x}\right]^{-b(1+m)-1}\right]^\alpha$$

$$= \left[\sum_{j=0}^{\infty} \zeta_{\psi}\right]^\alpha \left[\left[1+\frac{a}{x}\right]^{-b(1+m)-1} x^{-2}\right]^\alpha$$

$$let \left[\left[1+\frac{a}{x}\right]^{-b(1+m)-1} x^{-2}\right]^\alpha = \Phi^\alpha$$

then, $$H_\alpha(x) = \frac{1}{1-\alpha} \left[\sum_{j=0}^{\infty} \zeta_{\psi}\right]^\alpha \log \int_{-\infty}^{\infty} \Phi^\alpha dx \tag{16}$$

2.3.4 Reliability Function

The reliability function generally represented mathematically as:

$$R(z) = 1 - F(z) \tag{17}$$

Now, the reliability function for TIHLTL-IL distribution can be obtained from (17) as

$$R(x)=1-\frac{1-\left[1-\left[1-\left(1-\left[1+\frac{a}{x}\right]^{-b}\right)^{2}\right]^{\beta}\right]^{\theta}}{1+\left[1-\left[1-\left(1-\left[1+\frac{a}{x}\right]^{-b}\right)^{2}\right]^{\beta}\right]^{\theta}}$$

$$=\frac{2\left[1-\left[1-\left(1-\left[1+\frac{a}{x}\right]^{-b}\right)^{2}\right]^{\beta}\right]^{\theta}}{1+\left[1-\left[1-\left(1-\left[1+\frac{a}{x}\right]^{-b}\right)^{2}\right]^{\beta}\right]^{\theta}} \tag{18}$$

2.3.5 Hazard function

The hazard function is generally represented mathematically as:

$$H(z)=\frac{f(z)}{R(z)}$$

Then, the hazard function for TIHLTL-IL distribution can be obtained as,

$$H(x)=\frac{4\theta\beta abx^{-2}\left[1+\frac{a}{x}\right]^{-b-1}\left[1-\left[1+\frac{a}{x}\right]^{-b}\right]\left[1-\left[1-\left[1+\frac{a}{x}\right]^{-b}\right]^{2}\right]^{\beta-1}\left[1-\left[1-\left[1-\left[1+\frac{a}{x}\right]^{-b}\right]^{2}\right]^{\beta}\right]^{\theta-1}}{\left[1+\left[1-\left[1-\left(1-\left[1+\frac{a}{x}\right]^{-b}\right)^{2}\right]^{\beta}\right]^{\theta}\right]\left[1-\left[1-\left[1-\left(1-\left[1+\frac{a}{x}\right]^{-b}\right)^{2}\right]^{\beta}\right]^{\theta}\right]} \tag{19}$$

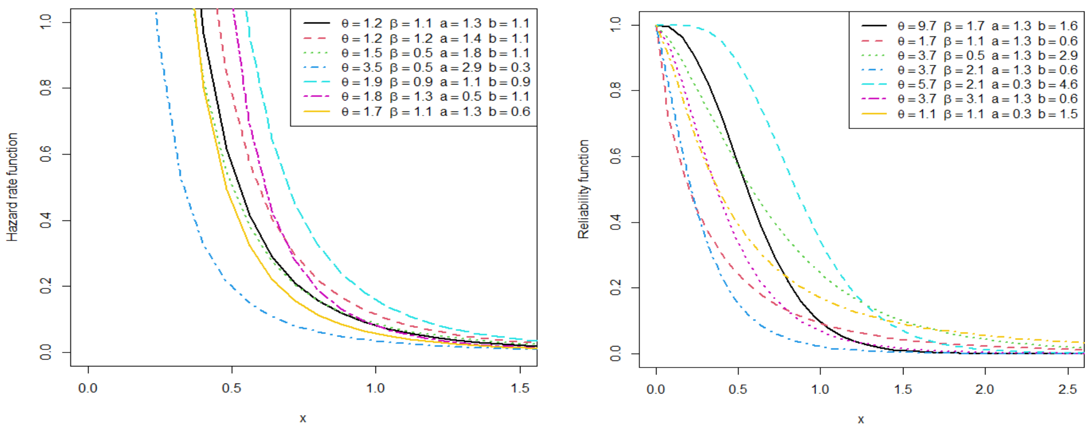

Figure 2: *Plots of hazard rate and reliability function of TIHLTL-IL distribution with various parameter choices*

2.3.6 Quantile Function

The quantile function of the TIHLTL-IL distribution is expressed below

$$F_{TIHLTL-IL}\left(x;\theta,\beta,a,b\right)=u$$

$$\frac{1-\left[1-\left[1-\left(1-\left[1+\frac{a}{x}\right]^{-b}\right)^{2}\right]^{\beta}\right]^{\theta}}{1+\left[1-\left[1-\left(1-\left[1+\frac{a}{x}\right]^{-b}\right)^{2}\right]^{\beta}\right]^{\theta}}=u \tag{20}$$

$$x=\frac{a}{\left[1+\left[1-\left[1-\left[1-\left[\frac{1-u}{u+1}\right]^{\frac{1}{\theta}}\right]^{\frac{1}{\beta}}\right]^{2}\right]^{\frac{1}{-b}}\right]} \tag{21}$$

2.3.7 Maximum Likelihood Estimation (MLE)

MLE is an approach channeled towards parameter estimation which has gained spread in terms of usage. This article adopted this method to estimate the parameters of the TIHLTL-IL model. Consider a randomly sampled X_i from the TIHLTL-IL distribution with parameter $\Psi=(\theta,\beta,a,b)$ β , where $i=1,,,n$. The log-likelihood function for TIHLTL-IL model $L(\Psi)$ is obtained as

$$\begin{aligned}L\left(\Psi\right)&=n\log 4+n\log\theta+n\log\beta+n\log a+n\log b-2\sum_{i=0}^{n}\log x_i-\left(b+1\right)\sum_{i=0}^{n}\log\left[1+\frac{a}{x_i}\right]\\&+\sum_{i=0}^{n}\log\left[1-\left[1+\frac{a}{x_i}\right]^{-b}\right]+\left(\beta-1\right)\sum_{i=0}^{n}\log\left[1-\left[1-\left[1+\frac{a}{x_i}\right]^{-b}\right]^{2}\right]\\&+\left(\theta-1\right)\sum_{i=0}^{n}\log\left[1-\left[1-\left[1-\left[1+\frac{a}{x_i}\right]^{-b}\right]^{2}\right]^{\beta}\right]\\&-2\sum_{i=0}^{n}\log\left[1+\left[1-\left[1-\left(1-\left[1+\frac{a}{x_i}\right]^{-b}\right)^{2}\right]^{\beta}\right]^{\theta}\right]\end{aligned} \tag{22}$$

By differentiating $L(\Psi)$ in (22) with respect to θ,β,a and b, the resulting equation is set to zero will produce the MLE estimates.

Let $K=\left[1-\left[1-\left[1-\left[1+\frac{a}{x_i}\right]^{-b}\right]^2\right]^\beta\right]$, $P=\left[1-\left[1-\left[1+\frac{a}{x_i}\right]^{-b}\right]^2\right]$ $Q=\left[1-\left[1+\frac{a}{x_i}\right]^{-b}\right]$ and $L=\left[1+\frac{a}{x_i}\right]$

Thus,

$$\frac{\delta L(\Psi)}{\delta\theta}=\frac{n}{\theta}+\sum_{i=0}^{n}\log K-2\sum_{i=0}^{n}\left[1+K^{\theta}\right]\times\frac{\log K}{\left[1+K^{\theta}\right]}=0 \tag{23}$$

$$\frac{\delta L(\Psi)}{\delta\beta}=\frac{n}{\beta}-\sum_{i=0}^{n}\log P+(\theta-1)\sum_{i=0}^{n}P^{\beta}\frac{\log P}{K}-2\theta\sum_{i=0}^{n}\frac{K^{\theta-1}}{\left[1+K^{\theta}\right]}\left[K\right]^{\theta}\log K=0 \tag{24}$$

$$\begin{aligned}\frac{\delta L(\Psi)}{\delta b}&=\frac{n}{b}-\sum_{i=0}^{n}\log M+\sum_{i=0}^{n}M^{-b}\frac{\log M}{Q}+2(\beta-1)\sum_{i=0}^{n}\frac{QM^{-b}\log M}{Q}\\&+2\beta(\theta-1)\sum_{i=0}^{n}\frac{\left[P\right]^{\beta-1}QM^{-b}\log M}{K}-4\theta\beta\sum_{i=0}^{n}\frac{K^{\theta-1}P^{\beta-1}Q}{\left[1+K^{\theta}\right]}=0\end{aligned} \tag{25}$$

$$\begin{aligned}\frac{\delta L(\Psi)}{\delta a}&=\frac{n}{a}-(b+1)\sum_{i=0}^{n}\frac{x_i^{-1}}{M}+b\sum_{i=0}^{n}\frac{x_i^{-1}M^{-b-1}}{Q}-2b(\beta-1)\sum_{i=0}^{n}\frac{x_i^{-1}QM^{-b-1}}{P}\\&-2\beta(\theta-1)\sum_{i=0}^{n}\frac{x_i^{-1}P^{\beta-1}QM^{-b-1}}{K}+4\theta\beta\sum_{i=0}^{n}\frac{x_i^{-1}K^{\theta-1}P^{\beta-1}QM^{-b-1}}{\left[1+K^{\theta}\right]}=0\end{aligned} \tag{26}$$

2.3.8 Information Criterion

The information criteria considered in this study include Bayesian (BIC), Akaike's (AIC), Hannan-Quinn (HQIC) and lastly, Consistent Akaike's (CAIC) Information Criterion. Their statistics are expressed mathematically as follow;

$$\begin{cases} AIC=-2\ell+2q\\ CAIC=-2\ell+\dfrac{2qn}{n-1-q}\\ BIC=-2\ell+q\log(n)\\ HQIC=-2\ell+2q\log(\log(n))\end{cases} \tag{27}$$

Where q and n are the number of distribution's estimated parameters, and the observations size, while ℓ represents the log-likelihood (maximized) of the parameter vector $\Psi=(\theta,\beta,a,b)$. The preferred model according to this criterion is the one with least values estimated from the model

III. Results

3.1 Application

This section provides application to real-life data sets, demonstrating the applicability and flexibility of the TIHLTL-IL distribution against its comparators such as exponentiated generalized inverse lomax (EGIL) distribution [14] and half logistic inverse lomax (HLIL) distribution [9]. The choice of the distribution with most applicability and flexibility is determined by the distribution with the large likelihood's values and the lowest information criteria's values.

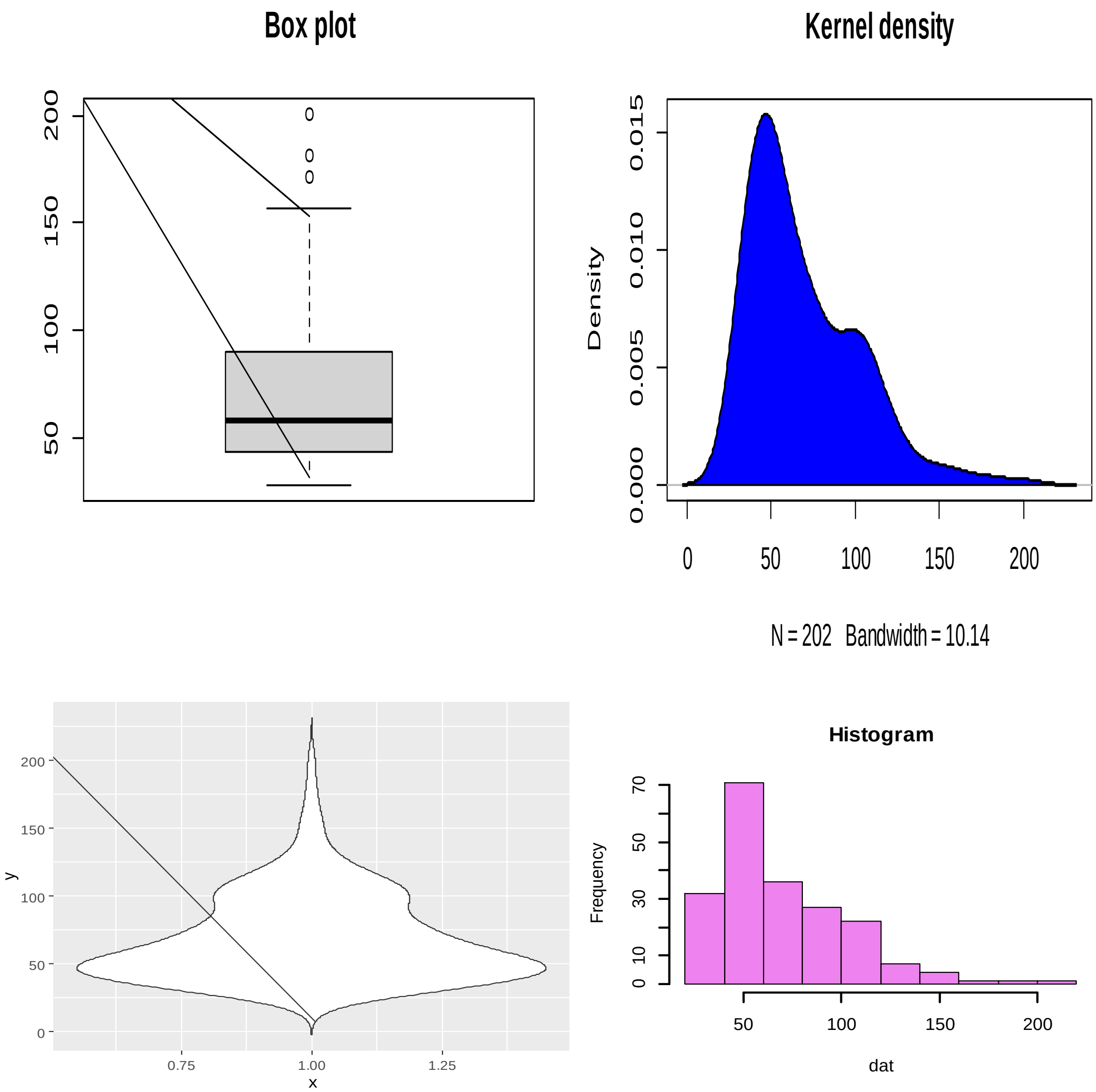

Figure 3: *Boxplot, kernel density, violin and histogram of the data set.*

The boxplot reveals information on the data set given below. It provides us with the necessary overview of the dispersion and the location of the data set. First of all, three outliers (171.1 181.7 200.8) are revealed from the data set, same is seen from the histogram, the minimum and maximum (28.0, 156.6) without outliers), first and third quartiles (43.80, 88.95) and the median (57.9) of the distribution. The kernel density and the histogram revealed that the data set is positively skewed, meaning that, bulk number of the observations is concentrated in left side of the distribution.

The data set represents the represents the sum of skin folds in 202 athletes collected at the Australian Institute of Sports, it was previously studied by [19].
The data set is:
28.0, 98, 89.0, 68.9, 69.9, 109.0, 52.3, 52.8, 46.7, 82.7, 42.3, 109.1, 96.8, 98.3, 103.6, 110.2, 98.1, 57.0, 43.1, 71.1, 29.7, 96.3, 102.8, 80.3, 122.1, 71.3, 200.8, 80.6, 65.3, 78.0, 65.9, 38.9, 56.5, 104.6, 74.9, 90.4, 54.6, 131.9, 68.3, 52.0, 40.8, 34.3, 44.8, 105.7, 126.4, 83.0, 106.9, 88.2, 33.8, 47.6, 42.7, 41.5, 34.6, 30.9, 100.7, 80.3, 91.0, 156.6, 95.4, 43.5, 61.9, 35.2, 50.9, 31.8, 44.0, 56.8, 75.2, 76.2,101.1, 47.5, 46.2, 38.2, 49.2, 49.6, 34.5, 37.5, 75.9, 87.2, 52.6, 126.4, 55.6, 73.9, 43.5, 61.8, 88.9, 31.0, 37.6,52.8, 97.9, 111.1, 114.0, 62.9, 36.8, 56.8, 46.5, 48.3, 32.6, 31.7, 47.8, 75.1, 110.7, 70.0, 52.5, 67, 41.6, 34.8, 61.8, 31.5, 36.6, 76.0, 65.1, 74.7, 77.0, 62.6, 41.1, 58.9, 60.2, 43.0, 32.6, 48, 61.2, 171.1, 113.5, 148.9, 49.9, 59.4, 44.5, 48.1, 61.1, 31.0, 41.9, 75.6, 76.8, 99.8, 80.1, 57.9, 48.4, 41.8, 44.5, 43.8, 33.7, 30.9, 43.3, 117.8, 80.3, 156.6, 109.6, 50.0, 33.7, 54.0, 54.2, 30.3, 52.8, 49.5, 90.2, 109.5, 115.9, 98.5, 54.6, 50.9, 44.7, 41.8, 38.0, 43.2, 70.0, 97.2, 123.6, 181.7, 136.3, 42.3, 40.5, 64.9, 34.1, 55.7, 113.5, 75.7, 99.9, 91.2, 71.6, 103.6, 46.1, 51.2, 43.8, 30.5, 37.5, 96.9, 57.7, 125.9, 49.0, 143.5, 102.8, 46.3, 54.4, 58.3, 34.0, 112.5, 49.3, 67.2, 56.5, 47.6, 60.4, 34.9.

Table 1: The descriptive statistics of the data set

N	Min	Max	median	Mean	Var	Skewness	Kurtosis
202	28	200.8	58.6	69.0218	106.0501	1.1659	1.3220

Table 2: The Estimates of the MLE based on data set

Models	θ	β	a	b
TIHLTL-IL	4.7115	26.0005	1.1422	2.7533
EG-IL	12.1005	17.9792	1.2394	2.6083
TIHL-IL	-	0.0401	7.5925	12.5563

Table 3: The Performance evaluation based on data set

Models	ℓ	**AIC**	**AICC**	**BIC**	**HQIC**
TIHLTL-IL	-954.0944	1916.1819	1916. 392	1929.422	1921.543
EG-IL	-1094.129	2196.258	2196.461	2209.491	2201.612
TIHL-IL	-963.0500	1932.100	1932.251	1942.025	1936.116

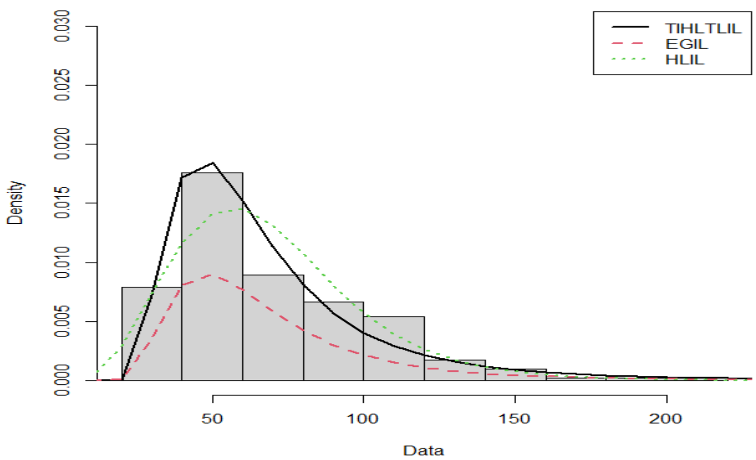

Figure 4: *Fitted pdfs plot for the TIHLTL-IL, EGIL and HLIL distribution to the data set*

IV. Discussion

This paper introduced a novel model called the Type I Half Logistic Topp-leone Inverse Lomax TIHLTL-IL model. The model's properties was defined and studied. We explored some useful statistical features of this novel model, including probability weighted moment, moments, moment generating functions, entropy, reliability functions, hazard function and the quantile functions. In order to have an insight of the model and to buttress our study, different plots were constructed such as the Pdf and cdf plots of TIHLTL-IL model, one can deduced that the model displays capability of handling data set with left and right skewed shape, reverse J-shape and approximately symmetric shape while the cdf plot confirmed the validity of the model. Investigation was conducted to visualize the data set using boxplot kernel density, violin and histogram, mainwhile, the boxplot suggest that there are three outliers in the data set, violin and histogram show a direction of the extreme values, indicating a positively skewed data set. The hazard shape reveals that the model can handle data set with monotonic decreasing life failure. We present MLE method, in order to estimate the unknown model's parameters. We delve in the applicability and flexibility of the novel model using a real life data set. The information criterion reveals that the proposed TIHLTL-IL model outclassed the related models. This claim is also supported by the fitted pdf plot.

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