PREDICTIVE MAINTENANCE SCHEME FOR PHASED MISSION SYSTEMS

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Abstract

In both industrial and military fields, many systems are phase mission systems (PMSs) which execute mission composed of different phases in sequence. The structure, failure behaviour, and working condition of such a system may change from phase to phase. Maintenance actions comprising corrective and preventive maintenance schemes studied in the literature are aimed at retaining the maintained system in a proper condition and improving its availability and extending its life. The present paper deals with finding optimal periodic inspection time using multi-objective criteria comprising objectives of minimizing expected maintenance cost incurred due to predictive, breakdown and periodic maintenance of a PMS , and maximizing its expected residual lifetime. The predictive maintenance is condition-based preventive maintenance that anticipates system failures in order to plan timely interventions on the system and hence improve its performance. The dependency is modelled using Gumbel-Haugaard copula.

An aircraft flight PMS comprising Taxiing phase, Take-Off phase, Cruising phase and Landing phase has been used to illustrate the method developed.

Keywords: phased mission system, reliability , Gumbel-Haugaard copula, predictive maintenance, periodic maintenance, mean residual life, cost optimization, cumulative exposure model

1. Introduction

The reliability of a supply chain depends on the reliability of all the equipment involved including transportation vehicles, sophisticated machines and computer -based information systems in network of suppliers , manufacturers and distributors whose sole aim is to provide goods and services in a timely manner . The reliability of such equipment in turn depends on their design, maintenance and subsequent repairs. Reliability engineering is therefore part and parcel of operations management.

In real life, systems such as coal transportation systems [1][2], aircrafts [3], avionic parts of airborne weapon systems [4], machining line [5], and nuclear plants are required to execute missions sequentially . Such systems called phased mission systems (PMSs) are subject to multiple, consecutive, non-overlapping operation phases. Failures of these systems during the mission may cause great economic losses to enterprises, serious security threats to personnel, or extensive damage to the environment. Some maintenance activities need to be undertaken during the mission break to reduce the probability of system failure of a PMS in the succeeding mission.

Unlike a non-repairable PMS in a repairable PMS, the state of the system depends not only on failure characteristics of its components but also on maintenance conducted during the mission.

Further , the system reliability depends on its age and the maintenance policy applied. It usually decreases as components deteriorate. Performing proper maintenance actions is necessary to keep the reliability of a system at a desired level. Maintenance is classifie into two main categories: corrective maintenance (CM) and preventive maintenance (PM). Corrective maintenance is generally performed after the system breakdown. Preventive maintenance corresponds to the scheduled actions which are performed while the system is still operational. It aims at keeping the system in available state by improving the condition of its components. Usually, preventive maintenance is more advantageous as it may prevent catastrophic losses due to unpredicted failures [6][7][8][9][10][11][12][13]. The PM actions are usually performed at predetermined points in time to keep the reliability of the system at a desired level.

Predictive maintenance (PdM) also known as condition-based maintenance is meant to minimize unscheduled equipment failures, lost production, and maintenance costs. It involves the use of information such as maintenance logs and sensor data to predict maintenance needs in advance. PdM plays a very important role in the airline industry by helping in reducing delays and costs, while improving and maintaining aircraft operational reliability.
The aim of this paper is to determine optimal periodic inspection time using multi-objective criteria of minimizing the expected maintenance cost due to predictive, breakdown and periodic maintenances, of the PMS, and maximizing its mean residual lifetime. The decision variable is the length of the periodic interval, T, subject to the constraints that the reliability of each phase does not exceed the pre-specifie values.

The paper is organized as follows: Section 2 is a brief literature review. The model of Predictive maintenance cost is explained in Section 3. In Section 4 the phased mission system is explained. Traditional maintenance models involving periodic and breakdown maintenances, and integrated models involving predictive maintenance besides periodic and breakdown maintenances are discussed in Section 5. The concept of Remaining Useful Life (RUL) is highlighted in Section 6, and multi-objective optimization problem is formulated in Section 7. In Section 8, the proposed method is explained using an aircraft fligh PMS. The concluding remarks have been made in the last section.

2. Literature Review

The maintenance models used in the literature predict problems that can help timely replacement or repair of an equipment before it fails for a single system. The researchers have used knowledge about degradation state of the equipment for prediction purpose [14] out-of-control condition using statistical process control [15][16][17][18][19][20] and on-line sensors [21][22] for prediction purpose for a single system. Maintenance at system-level of a PMS without considering predictive maintenance has been studied by [23]. The present paper deals with maintenance of a PMS taking into account predictive, periodic and breakdown maintenances along with its mean residual lifetime. It is assumed that the components are dependent within a phase, and all the phases involved are dependent. The dependency is modelled using Gumbel-Hougaard copula.

3. Predictive Maintenance Model

Defin $f_{PMS}(t)$ as the density function that specifie the probability of failure of a PMS at time t and $g(s \mid t)$ as the conditional density function that specifie the probability that the signal of a potential failure is received at time s given that the actual failure would have occurred at time t. The conditional density, $g(s \mid t)$, define the capability (i.e., accuracy and precision) of the prediction system.

The choice of the distribution form for the prediction signal, conditional on the equipment failure, is based on the concept of "P-F curves" for prediction systems [24] as well as diagnosis of the sensor equipment by the concerned technician(s).
Thus

$$g(s \mid t) = \begin{cases} k(1-\beta)\, s^{k-1} t^{-k} & 0 \le s \le t \\ \beta & s > t \end{cases}$$

$$G(s \mid t) = \begin{cases} (1-\beta)\left(\frac{s}{t}\right)^{k} & 0 \le s \le t \\ 1 & s > t \end{cases},$$

where s is the time of the signal, t is the time of failure if no replacement is made, k is the prediction precision, $(1-\beta)$ is the prediction accuracy and $k \geq 1$, $0 \leq \beta \leq 1$, are respectively, the conditional density and distribution function used for the purpose. This form of the conditional distribution function characterizes the features of typical signal and failure times seen in industry [22]).

In this paper, the objective is to minimize the expected maintenance cost of a PMS per unit time. The maintenance costs include costs of periodic and predictive replacements and that of failures. It is assumed that the PMS will go for maintenance after completing the mission and restored to "**as good as new**" condition, therefore, using renewal reward process the expected maintenance cost per period is:

$$\frac{E[Predictive\ Maintenance\ cost + Breakdown\ cost + Periodic\ maintenance\ cost]}{E[Time\ until\ maintenance]}$$

(See for reference [25]).

4. Phased Mission System (PMS)

A phased mission system (PMS) is define as a system comprising multiple, consecutive, and non-overlapping phases. During each phase, a PMS needs to complete a specifie task without failure. In these phases, the system may be subject to different working conditions and environmental stresses, as well as different performance requirements. For example, in a twin-engine airplane with two phases, namely, taxiing phase and take-off phase, one engine is required in the former phase, and both the engines are necessary in the latter phase. In contrast to the other phases of the fligh profile the engines are more prone to failure during the take-off period due to enormous pressure they undergo during this period [26][27]. So in different phases, the system configurations and the components, failure rates and even failure criteria could be vastly different.

Let T_{mn} denote lifetime of component m of phase n with reliability $\overline{H}_{mn}(t)$. Let $\overline{F}_{m1}(t), \overline{F}_{m2}(t)$, ... and $\overline{F}_{mn}(t)$ be the reliability of phase 1, phase 2, ... and phase n, respectively. Then, reliability of PMS is:

$$\overline{F}_{PMS}(t) = \begin{cases} \overline{F}_{m1}(t), \ 0 \leq t \leq \tau_1 \\ \overline{F}_{m2}(t), \ \tau_1 \leq t \leq \tau_2 \\ \quad . \\ \quad . \\ \quad . \\ \overline{F}_{mn}(t), \ \tau_{n-1} \leq t \leq \tau_n, \end{cases} \tag{1}$$

where (τ_{n-1}, τ_n) represents time-duration of functioning of phase n of the phased mission system $n = 1, 2, 3, 4, \ldots, n$, $\tau_0 = 0$.

Since considering phase n has m dependent components and reliability of phase n dnoted by $\overline{F}_{mn}(t)$ so dependency is modelled using Gumbel-Haugaard copula [28] gives,

$$\overline{F}_{mn}(t) = C\left(\overline{H}_{1n}(t), \overline{H}_{2n}(t), \ldots, \overline{H}_{mn}(t)\right). \tag{2}$$

And , reliability of PMS is:

$$\overline{F}_{PMS}(t) = C\left(\overline{F}_{m1}(\tau_1), \overline{F}_{m2}(\tau_2), \overline{F}_{m3}(\tau_3), \ldots, \overline{F}_{mn}(\tau_n)\right). \tag{3}$$

The cumulative exposure model [29] is used in equation (2), to obtain the reliability of phase n at τ_n. We obtain,

$$\overline{F}_{mn}(\tau_n) = C\left(\overline{H}_{1n}(\tau_n - \tau_{n-1} + l_{1n}), \overline{H}_{2n}(\tau_n - \tau_{n-1} + l_{2n}), \ldots, \overline{H}_{mn}(\tau_n - \tau_{n-1} + l_{mn})\right) \tag{4}$$

l_{mn} , where m denotes the components and n denotes the phase of the system, $m = 1, .., m$, & $n = 1, .., n$, is determined in such a way that [30])

$\overline{H}_{mn}(l_{mn}) = \overline{H}_{mn-1}(\tau_{n-1} - \tau_{n-2} + l_{mn-1})$, and $l_{1n-1} = 0$,
where $C\left(\overline{H}_{1n}(t), \overline{H}_{2n}(t), \overline{H}_{3n}(t), \ldots, \overline{H}_{mn}(t)\right)$ is the m-dimensional Gumbel-Hougaard.
Thus,

$$C\left(\overline{H}_{1n}(t), \overline{H}_{2n}(t), \overline{H}_{3n}(t), \ldots, \overline{H}_{mn}(t)\right) = \\ exp\left[-\left(\left(-log\left(\overline{H}_{1n}(t)\right)\right)^{\theta} + \left(-log\left(\overline{H}_{2n}(t)\right)\right)^{\theta} + \left(-log\left(\overline{H}_{3n}(t)\right)\right)^{\theta} + \cdots + \left(-log\left(\overline{H}_{mn}(t)\right)\right)^{\theta}\right)^{1/\theta}\right].$$

5. Maintenance Model

The present section focuses on the traditional periodic maintenance model (TM) and integrated model.(IM)

5.1. Traditional Model

In TM no predictive maintenance is used, periodic maintenance is conducted if there has been no failure prior to time T, and breakdown maintenance is conducted if the equipment fails prior to time T.

For the TM, the decision variable is the periodic interval T and the objective function value is as follows:

$$C_{TM}(T) = \frac{E[C_{BP}]}{E[C_{T1}]}, \tag{5}$$

where

$$E[C_{BP}] = M_b\left[\int_0^T f_{PMS}(t)\,dt\right] + M_p\left[\int_T^{\infty} f_{PMS}(t)dt\right],$$

is sum of expectation of breakdown maintenance costs and periodic maintenance cost,
and

$$E[C_{T1}] = \left[\int_0^T t f_{PMS}(t)dt\right] + T\left[\int_T^{\infty} f_{PMS}(t)dt\right],$$

is mean time between failure (replacement).

5.2. Integrated Model (IM)

The second model utilizes both predictive and periodic maintenance and is referred to as the Integrated Model. For IM, the decision variable is the periodic interval, T, and the objective function is:

$$C_{IM}(T) = \frac{E[C_{PdBP}]}{E[C_{T2}]}, \tag{6}$$

where,

$$E[C_{PdBP}] = M_{pd}\left[(1-\beta)\int_0^T f_{PMS}(t)dt + \int_T^{\infty} G(T \mid t) f_{PMS}(t)dt\right] \\ + M_b\left[\beta\int_0^T f_{PMS}(t)dt\right] + M_p\left[\int_T^{\infty}\left[1 - G(T \mid t)\right] f_{PMS}(t)dt\right],$$

is sum of expectation of predictive maintenance cost, breakdown maintenance costs and periodic maintenance cost, M_{pd} is predictive maintenance cost, M_b is breakdown maintenance and M_p is periodic maintenance,
and

$$E\left[\, C_{T2}\, \right] = \left[\int_0^T \int_0^t sg\left(s \mid t\right) f_{PMS}(t) ds dt + \int_T^\infty \int_0^T sg\left(s \mid t\right) f_{PMS}(t) ds dt\right] + \left[\beta \int_0^T t f_{PMS}(t) dt\right] + \left[T \int_T^\infty \left[1 - G\left(T \mid t\right)\right] f_{PMS}(t) dt\right],$$

is sum of expected time between replacement with signal and without signal.

6. Remaining Useful Life (RUL)

RUL is the residual life time of a system used to perform its functional capabilities before failure. It is a key metric and critical for predicting the failure of a machine in the production line, and is used by engineers to decide whether to do maintenance or delay it due to production requirements [31].

Let T_{PMS} be the time to failure of the phased mission system, and suppose the phased mission system has survived until time t. Then the "conditional" random variable

$$X_{PMS} = T_{PMS} - t(T_{PMS} > t),$$

i.e., the remaining time to failure, is called "RUL" of the phased mission system.
The conditional reliability function

$$\overline{F}_{PMS}\left(t\right) = P_{PMS}\left(X_{PMS} > x\right) = P\left(T_{PMS} - t > x | T_{PMS} - t\right), x \geq 0,$$

incorporates all the information relevant for prediction and future planning. The mean residual life (MRL) used as a point estimate of RUL or a prediction interval for RUL is define as:

$$\mu_{PMS}\left(t\right) = E_{PMS}[X_{PMS}] = E\left[\left[T_{PMS} - t | T_{PMS} > t\right]\right].$$

Then, $\mu_{PMS}\left(0\right) = \mu_{PMS} = E[T]$ and

$$\mu_{PMS}\left(t\right) = \int_0^\infty \overline{F}_{PMS}\left(x\right) dx = \frac{\int_t^\infty \overline{F}_{PMS}\left(x\right) dx}{\overline{F}_{PMS}\left(t\right)}. \tag{7}$$

7. Optimization Problems

Amongst various approaches used to solve a multi-objective optimization problem, one of the commonly used approach is to combine the objectives involved into one single composite objective so that the traditional mathematical programming method can be used for the propose.

In this paper, the weighted sum multi-objective optimization problem is used to minimize the expected maintenance cost per unit time and maximize mean residual lifetime function for the PMS subject to the constraints that the reliability of the each phase does not exceed the pre-specifie values, R_i, $i = 1, 2, \ldots, n$.

Let $T1$ be the periodic inspection time for the traditional model and $T2$ be that for the Integrated Model.

The optimization problem is formulated as:

7.1. Optimizing C_{TM}

$$Min\ Z_1 = w1C_{TM}(T1) + w2(-\mu_{PMS}(T1))$$

subject to, $T1 \geq \tau_n$,

$$1 \geq \overline{F}_{mi}(T1) \geq R_i, i = 1, 2, \ldots, nC_{TM} \geq 0.$$

7.2. Optimizing C_{IM}

$$Min\ Z_2 = w1C_{IM}(T2) + w2(-\mu_{PMS}(T2))$$

subject to, $T2 \geq \tau_n$,

$$1 \geq \overline{F}_{mi}(T2) \geq R_i, i = 1, 2, \ldots, nC_{IM} \geq 0.$$

Mathematica 11.0 has been used for solve the optimization problem.

8. Numerical Illustrations

In this section aircraft fligh PMS used for the illustrative purpose, Figure 1(a)-(d), shows reliability block diagrams for the four-phase aircraft flight comprises The firs phase is taxiing in which the navigation system, one out of the four engines and all three landing gears are needed, the second phase is take-off where in all four engines, the navigation system and all three landing gears are needed, the third phase is cruising in which the navigation system and three of the four engines are required. Finally, the fourth phase is landing comprising the navigation system, two of the four engines and all three landing gears.

8.1. Reliability of Aircraft Flight PMS system

Let T_1denote lifetimes of navigation with reliability $\overline{H}_{1n}(t)$. T_2, T_3, T_4 and T_5 denote lifetimes of the four engines E_1, E_2, E_3 and E_4 with reliabilities $\overline{H}_{2n}(t), \overline{H}_{3n}(t), \overline{H}_{4n}(t)$ and $\overline{H}_{5n}(t)$, respectively, and T_6, T_7 and T_8 denote lifetimes of landing gear 1 (G_1), landing gear 2 (G_2) and landing gear 3 (G_3) with reliabilities $\overline{H}_{6n}(t), \overline{H}_{7n}(t)$and $\overline{H}_{8n}(t)$, respectively. Let $\overline{F}_{p1}(t), \overline{F}_{p2}(t), \overline{F}_{p3}(t)$, and $\overline{F}_{p4}(t)$ be the reliability of subsystems in phase 1, phase 2, phase 3 and phase 4, respectively. Then, reliability of 4-PMS is:

$$\overline{F}_{PMS}(t) = \begin{cases} \overline{F}_{p1}(t), & 0 \leq t \leq \tau_1 \\ \overline{F}_{p2}(t), & \tau_1 \leq t \leq \tau_2 \\ \overline{F}_{p3}(t), & \tau_2 \leq t \leq \tau_3 \\ \overline{F}_{p4}(t), & \tau_3 \leq t \leq \tau_4. \end{cases} \tag{8}$$

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PHASE-1(Taxiing Phase)

Let $H_{11}(t)$ be life distribution of navigation, $H_{21}(t)$, $H_{31}(t)$, $H_{41}(t)$ & $H_{51}(t)$ be life distribution of components 'E_1', 'E_2', 'E_3' & 'E_4', respectively further let $H_{61}(t)$, $H_{71}(t)$ & $H_{81}(t)$ be life distribution of components 'G_1', 'G_2' & 'G_3', respectively.

Reliability of navigation,

$$\overline{F}_{11}(t) = p[T_1 > t].$$

Reliability of engines,

$\overline{F}_{21}(t) =$

$$p[T_2 > t, T_3 \leq t, T_4 \leq t, T_5 \leq t] + p[T_2 \leq t, T_3 > t, T_4 \leq t, T_5 \leq t] + p[T_2 \leq t, T_3 \leq t, T_4 > t, T_5 \leq t] +$$
$$p[T_2 \leq t, T_3 \leq t, T_4 \leq t, T_5 > t] + p[T_2 > t, T_3 > t, T_4 \leq t, T_5 \leq t] + p[T_2 > t, T_3 \leq t, T_4 > t, T_5 \leq t] +$$
$$p[T_2 > t, T_3 \leq t, T_4 \leq t, T_5 > t] + p[T_2 \leq t, T_3 > t, T_4 > t, T_5 \leq t] + p[T_2 \leq t, T_3 > t, T_4 \leq t, T_5 > t] +$$
$$p[T_2 \leq t, T_3 \leq t, T_4 > t, T_5 > t] + p[T_2 > t, T_3 > t, T_4 > t, T_5 \leq t] + p[T_2 > t, T_3 > t, T_4 \leq t, T_5 > t] +$$
$$p[T_2 > t, T_3 \leq t, T_4 > t, T_5 > t] + p[T_2 \leq t, T_3 > t, T_4 > t, T_5 > t] + p[T_2 > t, T_3 > t, T_4 > t, T_5 > t].$$

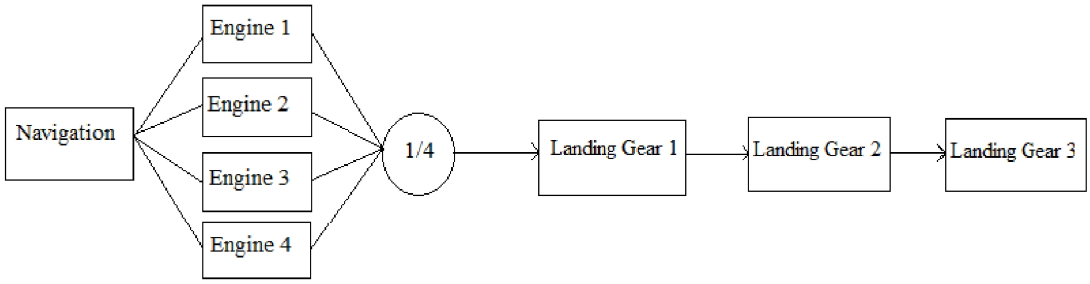

Figure 1(a): *Taxiing Phase*

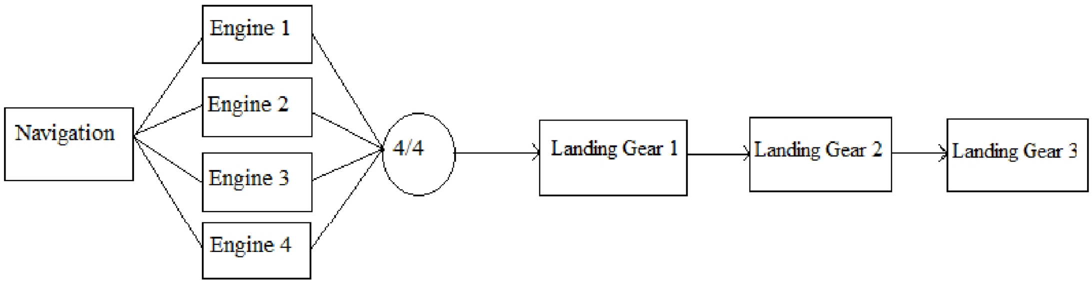

Figure 1(b): *Take-Off Phase*

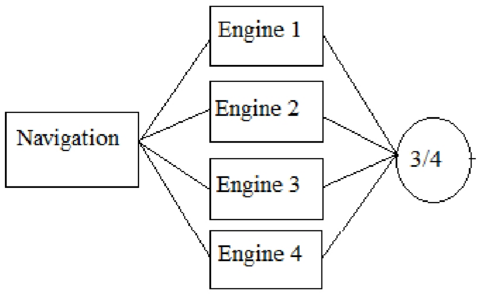

Figure 1(c): *Cruising Phase*

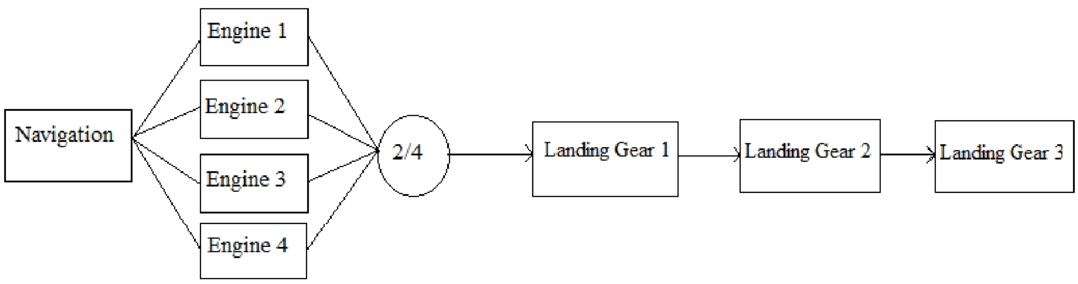

Figure 1(d): *Landing Phase*

Figure 1: *1(a)-(d) Reliability Block Diagrams for the four- phase aircraft flight[32]*

Reliability of landing gear,

$$\overline{F}_{31}(t) = p[T_6 > t, T_7 > t, T_8 > t].$$

Thus, Reliability of phase-1,

$$\overline{F}_{p1}(t) = \overline{F}_{11}(t).\overline{F}_{21}(t).\overline{F}_{31}(t). \tag{9}$$

PHASE-2 (Take-Off Phase)

Let $H_{12}(t)$ be life distribution of navigation, $H_{22}(t)$, $H_{32}(t)$, $H_{42}(t)$ & $H_{52}(t)$ be life distribution of components 'E_1', 'E_2', 'E_3' & 'E_4', respectively further let $H_{62}(t)$, $H_{72}(t)$ & $H_{82}(t)$ be life distribution of components 'G_1', 'G_2' & 'G_3', respectively.

Reliability of navigation,

$$\overline{F}_{12}(t) = p[T_1 > t].$$

Reliability of engines,

$$\overline{F}_{22}(t) = p[T_2 > t, T_3 > t, T_4 > t, T_5 > t].$$

Reliability of landing gear,

$$\overline{F}_{32}(t) = p[T_6 > t, T_7 > t, T_8 > t].$$

Thus, Reliability of phase-2,

$$\overline{F}_{p2}(t) = \overline{F}_{12}(t).\overline{F}_{22}(t).\overline{F}_{32}(t). \tag{10}$$

PHASE-3 (Cruising Phase)

Let $H_{13}(t)$ be life distribution of navigation, $H_{23}(t)$, $H_{33}(t)$, $H_{43}(t)$ & $H_{53}(t)$ be life distribution of components 'E_1', 'E_2', 'E_3' & 'E_4', respectively further let $H_{63}(t)$, $H_{32}(t)$ & $H_{83}(t)$ be life distribution of components 'G_1', 'G_2' & 'G_3', respectively.

Reliability of navigation,

$$\overline{F}_{13}(t) = p[T_1 > t].$$

Reliability of engines,

$$\overline{F}_{23}(t) = p[T_2 > t, T_3 > t, T_4 > t, T_5 \le t] + p[T_2 > t, T_3 > t, T_4 \le t, T_5 > t] +$$
$$p[T_2 > t, T_3 \le t, T_4 > t, T_5 > t] + p[T_2 \le t, T_3 > t, T_4 > t, T_5 > t] + p[T_2 > t, T_3 > t, T_4 > t, T_5 > t].$$

Thus, Reliability of phase-3,

$$\overline{F}_{p3}(t) = \overline{F}_{13}(t).\overline{F}_{23}(t). \tag{11}$$

PHASE-4 (Landing Phase)

Let $H_{14}(t)$ be life distribution of navigation, $H_{24}(t)$, $H_{34}(t)$, $H_{44}(t)$ & $H_{54}(t)$ be life distribution of components 'E_1', 'E_2', 'E_3' & 'E_4', respectively further let $H_{64}(t)$, $H_{74}(t)$ & $H_{84}(t)$ be life distribution of components 'G_1', 'G_2' & 'G_3', respectively.

Reliability of navigation,

$$\overline{F}_{14}(t) = p[T_1 > t].$$

Reliability of engines,

$$\overline{F}_{24}(t) = p[T_2 > t, T_3 > t, T_4 \le t, T_5 \le t] + p[T_2 > t, T_3 \le t, T_4 > t, T_5 \le t]$$
$$+ p[T_2 > t, T_3 \le t, T_4 \le t, T_5 > t] + p[T_2 \le t, T_3 > t, T_4 > t, T_5 \le t] + p[T_2 \le t, T_3 > t, T_4 \le t, T_5 > t] +$$
$$p[T_2 \le t, T_3 \le t, T_4 > t, T_5 > t] + p[T_2 > t, T_3 > t, T_4 > t, T_5 \le t] + p[T_2 > t, T_3 > t, T_4 \le t, T_5 > t] +$$
$$p[T_2 > t, T_3 \le t, T_4 > t, T_5 > t] + p[T_2 \le t, T_3 > t, T_4 > t, T_5 > t] + p[T_2 > t, T_3 > t, T_4 > t, T_5 > t].$$

Reliability of landing gear,

$$\overline{F}_{34}(t) = p[T_6 > t, T_7 > t, T_8 > t].$$

Thus, Reliability of phase-2,

$$\overline{F}_{p4}(t) = \overline{F}_{14}(t).\overline{F}_{24}(t).\overline{F}_{34}(t). \tag{12}$$

Reliability of Aircraft fligh PMS system using equation (3), we have

$$\overline{F}(t) = C\left(\overline{F}_{p1}(t), \overline{F}_{p2}(t), \overline{F}_{p3}(t), \overline{F}_{p4}(t)\right) \tag{13}$$

equations(9), (10), (11) and (12) give reliability of the four phases in PMS.

After using $p[AB] + p[AB^c] = p[A]$ and Gumbel-Hougaard copula equation (2) in above equations we get,

$$\overline{F}_{11}(t) = \overline{H}_{11}(t),$$

$$\begin{aligned}\overline{F}_{21}(t) &= C\left(\overline{H}_{21}(t),1,1,1\right) + C\left(1,\overline{H}_{31}(t),1,1\right) + C\left(1,1,\overline{H}_{41}(t),1\right) + C\left(1,1,1,\overline{H}_{51}(t)\right)\\ &- C\left(\overline{H}_{21}(t),\overline{H}_{31}(t),1,1\right) - C\left(\overline{H}_{21}(t),1,\overline{H}_{41}(t),1\right) - C\left(\overline{H}_{21}(t),1,1,\overline{H}_{51}(t)\right) - C\left(1,1,\overline{H}_{41}(t),\overline{H}_{51}(t)\right)\\ &- C\left(1,\overline{H}_{31}(t),1,\overline{H}_{51}(t)\right) - C\left(1,\overline{H}_{31}(t),\overline{H}_{41}(t),1\right) + C\left(\overline{H}_{21}(t),\overline{H}_{31}(t),\overline{H}_{41}(t),1\right)\\ &+ C\left(\overline{H}_{21}(t),\overline{H}_{31}(t),1,\overline{H}_{51}(t)\right) + C\left(\overline{H}_{21}(t),1,\overline{H}_{41}(t),\overline{H}_{51}(t)\right) + C\left(1,\overline{H}_{31}(t),\overline{H}_{41}(t),\overline{H}_{51}(t)\right)\\ &- C\left(\overline{H}_{21}(t),\overline{H}_{31}(t),\overline{H}_{41}(t),\overline{H}_{51}(t)\right),\end{aligned}$$

$$\overline{F}_{31}(t) = C\left(\overline{H}_{61}(t),\overline{H}_{71}(t),\overline{H}_{81}(t)\right),$$

$$\overline{F}_{12}(t) = \overline{H}_{12}(t),$$

$$\overline{F}_{22}(t) = C\left(\overline{H}_{22}(t),\overline{H}_{32}(t),\overline{H}_{42}(t),\overline{H}_{52}(t)\right),$$

$$\overline{F}_{32}(t) = C\left(\overline{H}_{62}(t),\overline{H}_{72}(t),\overline{H}_{82}(t)\right),$$

$$\overline{F}_{13}(t) = \overline{H}_{13}(t),$$

$$\begin{aligned}\overline{F}_{23}(t) &= C\left(\overline{H}_{23}(t),\overline{H}_{33}(t),\overline{H}_{43}(t),1\right) + C\left(\overline{H}_{23}(t),\overline{H}_{33}(t),1,\overline{H}_{53}(t)\right)\\ &+ C\left(\overline{H}_{23}(t),1,\overline{H}_{43}(t),\overline{H}_{53}(t)\right) + C\left(1,\overline{H}_{33}(t),\overline{H}_{43}(t),\overline{H}_{53}(t)\right) - 3C\left(\overline{H}_{23}(t),\overline{H}_{33}(t),\overline{H}_{43}(t),\overline{H}_{53}(t)\right),\end{aligned}$$

$$\overline{F}_{14}(t) = \overline{H}_{14}(t),$$

$$\begin{aligned}\overline{F}_{24}(t) &= C\left(\overline{H}_{24}(t),\overline{H}_{34}(t),1,1\right) + C\left(\overline{H}_{24}(t),1,\overline{H}_{44}(t),1\right) + C\left(\overline{H}_{24}(t),1,1,\overline{H}_{54}(t)\right)\\ &+ C\left(1,1,\overline{H}_{44}(t),\overline{H}_{54}(t)\right) + C\left(1,\overline{H}_{34}(t),1,\overline{H}_{54}(t)\right) + C\left(1,\overline{H}_{34}(t),\overline{H}_{44}(t),1\right)\\ &- 2C\left(\overline{H}_{24}(t),\overline{H}_{34}(t),\overline{H}_{44}(t),1\right) - 2C\left(\overline{H}_{24}(t),\overline{H}_{34}(t),1,\overline{H}_{54}(t)\right) - 2C\left(\overline{H}_{24}(t),1,\overline{H}_{44}(t),\overline{H}_{54}(t)\right)\\ &- 2C\left(1,\overline{H}_{34}(t),\overline{H}_{44}(t),\overline{H}_{54}(t)\right) + 3C\left(\overline{H}_{24}(t),\overline{H}_{34}(t),\overline{H}_{44}(t),\overline{H}_{54}(t)\right),\end{aligned}$$

$$\overline{F}_{34}(t) = p[T_6 > t, T_7 > t, T_8 > t] = C\left(\overline{H}_{64}(t),\overline{H}_{74}(t),\overline{H}_{84}(t)\right).$$

The cumulative exposure model is used in above equations, to obtain the reliability of subsystems in phase 1, phase 2, phase 3 and phase 4 at τ_1, τ_2, τ_3 and τ_4, respectively.
Thus,

$$\overline{F}_{11}(\tau_1) = \overline{H}_{11}(\tau_1),$$

$$\begin{aligned}\overline{F}_{21}(\tau_1) &= C\left(\overline{H}_{21}(\tau_1),1,1,1\right) + C\left(1,\overline{H}_{31}(\tau_1),1,1\right) + C\left(1,1,\overline{H}_{41}(\tau_1),1\right) + C\left(1,1,1,\overline{H}_{51}(\tau_1)\right)\\ &- C\left(\overline{H}_{21}(\tau_1),\overline{H}_{31}(\tau_1),1,1\right) - C\left(\overline{H}_{21}(\tau_1),1,\overline{H}_{41}(\tau_1),1\right) - C\left(\overline{H}_{21}(\tau_1),1,1,\overline{H}_{51}(\tau_1)\right)\\ &- C\left(1,1,\overline{H}_{41}(\tau_1),\overline{H}_{51}(\tau_1)\right) - C\left(1,\overline{H}_{31}(\tau_1),1,\overline{H}_{51}(\tau_1)\right) - C\left(1,\overline{H}_{31}(\tau_1),\overline{H}_{41}(\tau_1),1\right)\\ &+ C\left(\overline{H}_{21}(\tau_1),\overline{H}_{31}(\tau_1),\overline{H}_{41}(\tau_1),1\right) + C\left(\overline{H}_{21}(\tau_1),\overline{H}_{31}(\tau_1),1,\overline{H}_{51}(\tau_1)\right) + C\left(\overline{H}_{21}(\tau_1),1,\overline{H}_{41}(\tau_1),\overline{H}_{51}(\tau_1)\right)\\ &+ C\left(1,\overline{H}_{31}(\tau_1),\overline{H}_{41}(\tau_1),\overline{H}_{51}(\tau_1)\right) - C\left(\overline{H}_{21}(\tau_1),\overline{H}_{31}(\tau_1),\overline{H}_{41}(\tau_1),\overline{H}_{51}(\tau_1)\right),\end{aligned}$$

$$\overline{F}_{31}(\tau_1) = C\left(\overline{H}_{61}(\tau_1),\overline{H}_{71}(\tau_1),\overline{H}_{81}(\tau_1)\right),$$

$$\overline{F}_{12}(\tau_2) = \overline{H}_{12}(\tau_2-\tau_1+l_{12}),$$

$$\overline{F}_{22}(\tau_2) = C\left(\overline{H}_{22}(\tau_2-\tau_1+l_{22}),\overline{H}_{32}(\tau_2-\tau_1+l_{32}),\overline{H}_{42}(\tau_2-\tau_1+l_{42}),\overline{H}_{52}(\tau_2-\tau_1+l_{52})\right),$$

$$\overline{F}_{32}(\tau_2) = C\left(\overline{H}_{62}(\tau_2-\tau_1+l_{62}),\overline{H}_{72}(\tau_2-\tau_1+l_{72}),\overline{H}_{82}(\tau_2-\tau_1+l_{82})\right),$$

$$\overline{F}_{13}(\tau_3) = \overline{H}_{13}(\tau_3-\tau_2+l_{13}),$$

$$\begin{aligned}\overline{F}_{23}(\tau_3) &= C\left(\overline{H}_{23}(\tau_3-\tau_2+l_{23}),\overline{H}_{33}(\tau_3-\tau_2+l_{33}),\overline{H}_{43}(\tau_3-\tau_2+l_{43}),1\right)\\ &+ C\left(\overline{H}_{23}(\tau_3-\tau_2+l_{23}),\overline{H}_{33}(\tau_3-\tau_2+l_{33}),1,\overline{H}_{53}(\tau_3-\tau_2+l_{53})\right)\\ &+ C\left(\overline{H}_{23}(\tau_3-\tau_2+l_{23}),1,\overline{H}_{43}(\tau_3-\tau_2+l_{43}),\overline{H}_{53}(\tau_3-\tau_2+l_{53})\right)\\ &+ C\left(1,\overline{H}_{33}(\tau_3-\tau_2+l_{33}),\overline{H}_{43}(\tau_3-\tau_2+l_{43}),\overline{H}_{53}(\tau_3-\tau_2+l_{53})\right)\\ &- 3C\left(\overline{H}_{23}(\tau_3-\tau_2+l_{23}),\overline{H}_{33}(\tau_3-\tau_2+l_{33}),\overline{H}_{43}(\tau_3-\tau_2+l_{43}),\overline{H}_{53}(\tau_3-\tau_2+l_{53})\right),\end{aligned}$$

$$\overline{F}_{14}(\tau_4) = \overline{H}_{14}(\tau_4-\tau_3+l_{14}),$$

$$\begin{aligned}\overline{F}_{24}(\tau_4) &= C\left(\overline{H}_{24}(\tau_4-\tau_3+l_{24}),\overline{H}_{34}(\tau_4-\tau_3+l_{34}),1,1\right) + C\left(\overline{H}_{24}(\tau_4-\tau_3+l_{24}),1,\overline{H}_{44}(\tau_4-\tau_3+l_{44}),1\right)\\ &+ C\left(\overline{H}_{24}(\tau_4-\tau_3+l_{24}),1,1,\overline{H}_{54}(\tau_4-\tau_3+l_{54})\right) + C\left(1,1,\overline{H}_{44}(\tau_4-\tau_3+l_{44}),\overline{H}_{54}(\tau_4-\tau_3+l_{14})\right)\\ &+ C\left(1,\overline{H}_{34}(t),1,\overline{H}_{54}(t)\right) + C\left(1,\overline{H}_{34}(t),\overline{H}_{44}(t),1\right) - 2C\left(\overline{H}_{24}(t),\overline{H}_{34}(t),\overline{H}_{44}(t),1\right)\\ &- 2C\left(\overline{H}_{24}(t),\overline{H}_{34}(t),1,\overline{H}_{54}(t)\right) - 2C\left(\overline{H}_{24}(t),1,\overline{H}_{44}(t),\overline{H}_{54}(t)\right) - 2C\left(1,\overline{H}_{34}(t),\overline{H}_{44}(t),\overline{H}_{54}(t)\right)\\ &+ 3C\left(\overline{H}_{24}(t),\overline{H}_{34}(t),\overline{H}_{44}(t),\overline{H}_{54}(t)\right),\end{aligned}$$

$$\overline{F}_{34}(\tau_4) = p\left[T_6>t,T_7>t,T_8>t\right] = C\left(\overline{H}_{64}(t),\overline{H}_{74}(t),\overline{H}_{84}(t)\right).$$

It is assumed that a component's life distribution in a phase is Weibull with reliability function:

$$\overline{H}_{mn}(t) = exp\left[-(t/\alpha_{mn})^{\gamma}\right], t>0; \alpha_{mn}>0; \gamma>0; n=1,2,3,4, \quad m=1,2,3,4,5,6,7,8.$$

To illustrate the above model, assume that each of the phase- Taxiing and Take-Off has duration of 15 minutes, cruising phase has duration of 130 minutes and landing phase has duration of 20 minutes. Components of the aircraft follow weibull distribution with $\gamma = 1.8$ with $\alpha_{mn} = 1000$ hours for navigation system, $\alpha_{mn} = 950$ hours for engines and $\alpha_{mn} = 925$ hours for the landing gear. The value of M_p=10000, M_{pd}=M_p, $M_b = 5.500 * M_p, \beta = 0.260, k = 2.00$ [22].

Tables 1.1- 1.4 are obtained using these data for both the optimization problems formulated in Section 7, with $R_i = 0.995$, $i = 1, 2, \ldots, 4$

Table 1.1: *Values of Multi-objective functions and T (in minutes) for $\theta = 1.0$ with different weights*

w1	w2	$C_{TM}(T1)$	RUL1	$C_{IM}(T2)$	RUL2	T1	T2
1/2	1/2	22309.9	123841	7091.89	123841	4033.45	4033.45
1/3	2/3	22309.9	123841	7729.81	124301	4033.45	3894.98
1/4	3/4	22309.9	123841	8005.13	124416	4033.45	3839.89
2/3	1/3	22309.9	123841	7091.89	123841	4033.45	4033.45
3/4	1/4	22309.9	123841	7091.89	123841	4033.45	4033.45

Table 1.2: *Values of Multi-objective functions and T (in minutes) for $\theta = 1.182$ with different weights*

w1	w2	$C_{TM}(T1)$	RUL1	$C_{IM}(T2)$	RUL2	T1	T2
1/2	1/2	12647.6	121420	4352.71	123822	4832.62	3917
1/3	2/3	15018.8	123150	5135.21	124046	4085.97	3817.32
1/4	3/4	16155.5	123620	5250.38	124094	3977.96	3781.49
2/3	1/3	12082.4	120810	4360.13	123125	4426.73	4091.04
3/4	1/4	12082.4	120810	3998.01	122226	4426.73	4242.69

Table 1.3: *Values of Multi-objective functions and T (in minutes) for $\theta = 2.182$ with different weights*

w1	w2	$C_{TM}(T1)$	RUL1	$C_{IM}(T2)$	RUL2	T1	T2
1/2	1/2	4207.26	121771	2847.66	123458	4207.26	3818.37
1/3	2/3	11723.3	122956	3755.75	123547	3986.95	3755.75
1/4	3/4	12465.1	123263	3012.93	123566	3899.44	3733.48
2/3	1/3	8159.79	118946	2652.79	123170	4546.32	3929
3/4	1/4	6953.79	115971	2499.36	122789	4818.2	4025.64

Table 1.4: *Values of Multi-objective functions and T (in minutes) for $\theta = 3.182$ with different weights*

w1	w2	$C_{TM}(T1)$	RUL1	$C_{IM}(T2)$	RUL2	T1	T2
1/2	1/2	9877.32	121727	2649.01	123346	3814.62	4192.79
1/3	2/3	11438.3	122870	2769.67	123436	3976.54	3751.94
1/4	3/4	12150.9	123165	2814.46	123165	3890.79	3729.71
2/3	1/3	7992.92	118990	2453.59	123057	4526.04	3925.37
3/4	1/4	6821.02	116098	2299.76	122675	4793.23	4022.06

Table 1.1- Table 1.4 gives optimal cost and optimal residual useful life for the two models using different weight combinations. It is observed that the integrated model in almost all the cases yields lower cost and higher RUL with smaller periodic inspection time. Table 1.1 shows that for IM the minimum cost is obtained when $w1 = 1/4$ and $w2 = 3/4$ and the optimal periodic insepection time is $T2 = 3839.89$ implying that four-phase aircraft fligh needs to be send for maintenance after every 21 cycles. Similar interpretation holds for data depicted in Table- 1.2 to Table 1.4

9. Conclusion

In this paper predictive maintenance framework is proposed for a phased mission system. The multi-objective problem is used wherein weighted sum of expected maintenance cost and mean residual life function of the PMS is minimized subject to the constraints that the reliability of each phase doesn't exceed the pre-specifie values. The decision variable is the length of the periodic interval. The optimal solution obtained using IM model is compared with traditional model (TM). For illustrative purpose aircraft fligh PMS composed of four phases, namely; taxiing, take-off, cruising, and landing is used with dependency between components of each phase modelled using Gumbel-Haugaard copula. The cumulative exposure model is used to determine the reliability of the PMS. It is found that the integrated model yields lower cost and higher RUL with smaller periodic inspection time. Thus, the use of predictive tools with periodic maintenance reduces overall equipment maintenance costs with higher mean residual life.

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DECLARATION OF CONFLICT INTEREST
The authors have declared that no conflic interests exist.

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