PROFIT ANALYSIS OF REPAIRABLE JUICE PLANT

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Abstract

Juice is a non-fermented beverage that is obtained by squeezing fruits to increase immunity. Generally, juice contains calcium, vitamin, iron, etc. to give the refresh tests. There are multiple steps to store the juice at large levels such as storing, grinding pasteurization, etc. In this paper, the performance and reliability measures of a juice plant are discussed. The juice plant has three distinct units. Unit A has washing and storage tank, unit B has grinding, blending, evaporation and pasteurization, and unit C has bottling, labeling and packing units. If any unit partially fails then the system works to a limited extent. A technician is always available to repair the failed unit. The system fails when one unit completely fails. In this paper, the failure time and repair time follow general distributions. The regenerative point graphical technique is used to explore the reliability measures.

Keywords: Reliability measures, juice plant, evaporation and pasteurization.

I. Introduction

Manufacturers must constantly innovate their products in order to keep up with the rising demand for their products, which is made feasible by optimizing their manufacturing processes. The MTSF, availability and profitability of a juice factory with priority in repair are discussed in this study by utilizing the regenerating point graphical technique under specific circumstances.

Barlow *et al.* [2] investigated the reliability theory with redundancy and system availability while taking into account the significance of individual system components. The reliability study of a single unit system with non-repairable spare units and its optimization applications was covered by Nakagawa and Osaki [12]. Balagurusamy [1] described the terms related to the system's meantime, failure, repair, redundancy, maintainability, availability, etc. Tuteja and Malik [16] examined the dependability of two distinct single-unit models with three operating modes and various repair procedures applied to the repairman. Malik [11] examined a single-unit system with a server under inspection. Pawar *et al.* [13] threw light on an operating system under different climates having repair at varying levels of damages subject to inspection.

Gupta [4] talked about employing a base state to analyze a single-unit system. The reliability analysis of a one-unit system with finite vacations was examined by Liu and Liu [10]. The dependability metrics of a repairable stochastic model on the production of printed circuit boards were given by Kumar and Batra [9]. Chaudhary *et al.* [3] studied the valuable parameters for the nature of the distillery system having three distinct units and a single server facility using the

regenerative point graphical technique. Kumar *et al.* [8] threw light on the preventive maintenance of a sustainable one unit system under degradation facilities. Sharma and Goel [15] described the nature of whole-grain flour mills having two units using base state and regenerative point techniques. Kumar and Saini [5] described the fault detection concept in stochastic computing device under repair and replacement by an expert repairman. A redundant system with a first come, first served repair policy was examined by Kumar *et al.* [7] under different weather. Sengar and Mangey [14] analyzed the reliability measures of a complex manufacturing system with an inspection facility. Kumar *et al.* [6] analyzed the reliability and performance of two unit system under inspection facility.

II. System Assumptions

To describe the juice plant, there are following assumptions

- The juice plant consists of three distinct units *A*, *B* and *C*.
- It is considered that units *A* and *B* may be in a complete failed state through partial failure mode but unit *C* is in only partially failed state.
- Unit *A* has washing and storage tank.
- Unit *B* has grinding, blending, evaporation and pasteurization.
- Unit *C* has bottling, labeling and packing units.
- Failure rate and repair rate are generally distributed and are independent.
- The repaired unit functions just like a brand-new one.

III. System Notations

To explain the juice plant, there are following notations

$i \xrightarrow{Sr} j$	r^{th} directed simple path from state ' <i>i</i> ' to state ' <i>j</i> ' where ' <i>r</i> ' takes the positive integral values for different directions from state ' <i>i</i> ' to state ' <i>j</i> '.
$\xi \xrightarrow{sff} i$	A directed simple failure free path from state ξ to state ' <i>i</i> '.
m – cycle	A circuit (may be formed through regenerative or non regenerative / failed state) whose terminals are at the regenerative state ' m '.
$m-\overline{cycle}$	A circuit (may be formed through the unfailed regenerative or non regenerative state) whose terminals are at the regenerative ' m ' state.
$U_{k,k}$	Probability factor of the state 'k' reachable from the terminal state 'k' of 'k' cycle.
$U_{\overline{k,k}}$	The probability factor of state 'k' reachable from the terminal state 'k' of $k \ \overline{cycle}$.
μ_i	Mean sojourn time spent in the state ' i ' before visiting any other states.
μ_i'	Total unconditional time spent before transiting to any other regenerative state while the system entered regenerative state ' i ' at t=0.
η_i	Expected waiting time spent while doing a job given that the system entered to the regenerative state ' i ' at t=0.
$A/\overline{A}/a$	First unit is in the operative state/reduced state/failed state.
$B / \overline{B} / b$	Second unit is in the operative state/reduced state/failed state.
$C/\overline{C}/c$	Third unit is in the operative state/reduced state/failed state.
λ_1 , λ_2 , λ_3	Fixed partial failure rate of the unit A/B/C respectively.
λ_4,λ_5	Fixed complete failure rate of the unit A/B respectively.

w_1, w_2, w_3	Fixed repair rate of the unit A/B/C after partial failure respectively.
w_4, w_5	Fixed repair rate of unit A/B after the complete failure respectively.
$\bigcirc \bigcirc \square$	Upstate/ reduced state/ failed state.

IV. Circuits Descriptions

Primary, secondary and tertiary circuits are used to find the base state such that

i	(C1)	(C2)	(C3)
0	(0,1,0), (0,2,0), (0,3,0)	Nil	Nil
	(0,1,4,0), (0,2,5,0)		
1	(1,0,1)	(0,2,0), (0,3,0)	Nil
2	(2,0,2)	(0,1,0), (0,3,0)	Nil
3	(3,0,3)	(0,1,0), (0,2,0)	Nil
4	(4,0,1,4)	(0,1,0), (0,2,0)	(2,0,2), (3,0,3)
		(0,3,0), (1,0,1)	
5	(5,0,2,5)	(0,1,0), (0,2,0)	(1,0,1), (3,0,3)
		(0,3,0), (2,0,2)	

Table 1: Circuit Descriptions

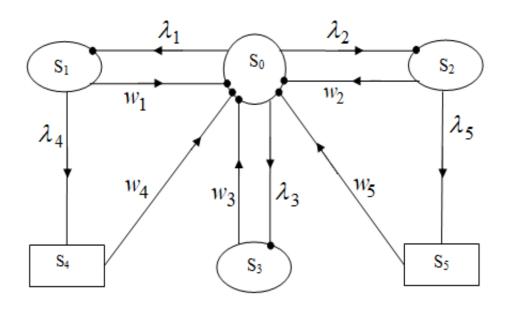


Figure 1 State Transition Diagram

where,
$$S_0 = ABC$$
, $S_1 = \overline{ABC}$, $S_2 = A\overline{BC}$
 $S_3 = AB\overline{C}$, $S_4 = aBC$, $S_5 = AbC$

V. Transition Probabilities

There are following transition probabilities

$$p_{0,1} = \lambda_1 / (\lambda_1 + \lambda_2 + \lambda_3), p_{0,2} = \lambda_2 / (\lambda_1 + \lambda_2 + \lambda_3), p_{0,3} = \lambda_3 / (\lambda_1 + \lambda_2 + \lambda_3)$$

$$p_{1,0} = w_1 / (w_1 + \lambda_4), p_{1,4} = \lambda_4 / (w_1 + \lambda_4), p_{2,0} = w_2 / (w_2 + \lambda_5)$$

$$p_{2,5} = \lambda_5 / (w_2 + \lambda_5), p_{3,0} = p_{4,0} = p_{5,0} = 1$$
(1)
It has been conclusively established that

$$p_{01} + p_{03} = 1, p_{10} + p_{12} + p_{14} = 1, p_{21} + p_{27} = 1, p_{31} + p_{38} = 1$$

$$p_{41} + p_{45} = 1, p_{56} = p_{76} = p_{86} = 1, p_{31} + p_{31.8(65)}^{n} = 1$$

$$p_{10} + p_{12} + p_{11.4} + p_{11.4(56)}^{n} = 1, p_{21} + p_{21.7(65)}^{n} = 1$$
(2)

VI. Mean Sojourn Time

Time taken by a system in a particular state becomes, $\mu_i = \sum_j m_{i,j} = \int_0^\infty P(T > t) dt$.

$$\mu_{0} = 1/(\lambda_{1} + \lambda_{2} + \lambda_{3})$$

$$\mu_{1} = 1/(w_{1} + \lambda_{4}), \quad \mu_{2} = 1/(w_{2} + \lambda_{5})$$

$$\mu_{3}(t) = 1/(w_{3}), \quad \mu_{4} = 1/(w_{4}), \quad \mu_{5} = 1/(w_{5})$$
(3)

VII. Evaluation of Parameters

Using the circuit table, '0' is used as the base state to calculate the reliability using the regenerative point graphical technique. The probability factors of all the reachable states from the base state '0' are given below

$$\begin{split} U_{0,0} &= (0,1,0) + (0,2,0) + (0,3,0) = 1, \ U_{0,1} = \frac{\lambda_1}{\lambda_1 + \lambda_2 + \lambda_3}, \\ U_{0,2} &= \frac{\lambda_2}{\lambda_1 + \lambda_2 + \lambda_3}, \ U_{0,3} = \frac{\lambda_3}{(\lambda_1 + \lambda_2 + \lambda_3)} \\ U_{0,4} &= \frac{\lambda_1 \lambda_4}{(\lambda_1 + \lambda_2 + \lambda_3)(w_1 + \lambda_4)}, \ U_{0,5} = \frac{\lambda_2 \lambda_5}{(\lambda_1 + \lambda_2 + \lambda_3)(w_2 + \lambda_5)} \end{split}$$

I. Mean Time to System Failure

The regenerative un-failed states (*i*=0, 1, 2, 3) to which the system can transit (with initial state 0) before entering to any failed state (using base state ξ =0) then MTSF becomes

$$T_{0} = \begin{bmatrix} 3\\ \sum \\ i = 0 \end{bmatrix} Sr \left\{ \frac{\left\{ pr(0 \quad Sr(sff) \\ 0 \\ i = 0 \end{bmatrix}, \mu_{i} \right\}}{\prod_{k_{1} \neq 0} \left\{ 1 - V_{\overline{k_{1}k_{1}}} \right\}} \right\} \\ \div \left[1 - \sum_{Sr} \left\{ \frac{\left\{ pr(0 \quad Sr(sff) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 - V_{\overline{k_{2}k_{2}}} \right\} \right\} \right]$$

$$T_{0} = \frac{\begin{bmatrix} (w_{1} + \lambda_{4})(w_{2} + \lambda_{5})(w_{3} + \lambda_{3}) \\ + w_{3}[\lambda_{1}(w_{2} + \lambda_{5}) + \lambda_{2}(w_{1} + \lambda_{4})] \end{bmatrix}}{\begin{bmatrix} w_{3}[(\lambda_{1} + \lambda_{2} + \lambda_{3})(w_{1} + \lambda_{4})(w_{2} + \lambda_{5})] \\ - \lambda_{1}w_{1}(w_{2} + \lambda_{5}) - \lambda_{2}w_{2}(w_{1} + \lambda_{4})] \end{bmatrix}}$$
(4)

II. Availability of the system

The system is available for use at regenerative states *j*=0, 1, 2, 3 with ξ =0 then the availability of system is defined as

$$A_{0} = \begin{bmatrix} 3\\ \sum \\ j = 0 \end{bmatrix} Sr \left\{ \frac{\left\{ pr(0 \longrightarrow Sr \rightarrow j) \right\} \cdot f_{j} \cdot \mu_{j}}{\prod_{k_{1} \neq 0} \left\{ 1 - V_{\overline{k_{1}k_{1}}} \right\}} \right\} \\ \stackrel{:}{=} \left[\sum_{i=0}^{5} Sr \left\{ \frac{\left\{ pr(0 \longrightarrow Sr \rightarrow i) \right\} \cdot \mu_{i}}{\prod_{k_{2} \neq 0} \left\{ 1 - V_{\overline{k_{2}k_{2}}} \right\}} \right\} \right] \\ A_{0} = \frac{\begin{bmatrix} w_{4}w_{5}[(w_{1} + \lambda_{4})(w_{2} + \lambda_{5})(w_{3} + \lambda_{3})] \\ + w_{3}\{\lambda_{1}(w_{2} + \lambda_{5}) + \lambda_{2}(w_{1} + \lambda_{4})\}] \\ \frac{1}{2} \begin{bmatrix} w_{4}w_{5}(w_{1} + \lambda_{4})(w_{2} + \lambda_{5})(w_{3} + \lambda_{3}) \\ + \lambda_{1}w_{3}w_{5}(w_{2} + \lambda_{5})(w_{4} + \lambda_{4}) \\ + \lambda_{2}w_{3}w_{4}(w_{1} + \lambda_{4})(w_{5} + \lambda_{5}) \end{bmatrix} \right]$$
(5)

III. Busy Period of the Technician

The Technician is busy due to repair of the failed unit at regenerative states j= 1, 2, 3, 4, 5 with ξ = 0 then the fraction of time for which the server remains busy is defined as

$$B_{0} = \begin{bmatrix} 5 \\ 2 \\ j = 1 \end{bmatrix} Sr \left\{ \frac{\left\{ pr(0 \xrightarrow{Sr} j) \right\} \eta_{j}}{\prod_{k_{1} \neq 0} \left\{ 1 - V_{\overline{k_{1}k_{1}}} \right\}} \right\} \stackrel{!}{=} \left[5 \\ i = 0 \end{bmatrix} Sr \left\{ \frac{\left\{ pr(0 \xrightarrow{Sr} i) \right\} \mu_{i}}{\prod_{k_{2} \neq 0} \left\{ 1 - V_{\overline{k_{2}k_{2}}} \right\}} \right\} \right]$$

$$B_{0} = \frac{\begin{bmatrix} w_{4}w_{5}\lambda_{3}(w_{1} + \lambda_{4})(w_{2} + \lambda_{5}) \\ + \lambda_{1}w_{3}w_{5}(w_{2} + \lambda_{5})(w_{4} + \lambda_{4}) \\ + \lambda_{2}w_{3}w_{4}(w_{1} + \lambda_{4})(w_{5} + \lambda_{5}) \end{bmatrix}}{\begin{bmatrix} w_{4}w_{5}(w_{1} + \lambda_{4})(w_{2} + \lambda_{5})(w_{3} + \lambda_{3}) \\ + \lambda_{1}w_{3}w_{5}(w_{2} + \lambda_{5})(w_{4} + \lambda_{4}) \\ + \lambda_{2}w_{3}w_{4}(w_{1} + \lambda_{4})(w_{5} + \lambda_{5}) \end{bmatrix}}$$

$$(6)$$

IV. Estimated number of visits made by the Technician

The technician visits at regenerative states j=1, 2, 3 with $\xi=0$ then the number of visits by the repairman is defined as

$$V_{0} = \begin{bmatrix} \frac{3}{2} \\ j = 1 \end{bmatrix} Sr \left\{ \frac{\left\{ pr(0 \longrightarrow Sr \rightarrow j) \right\}}{\prod_{k_{1} \neq 0} \left\{ 1 - V_{\overline{k_{1}k_{1}}} \right\}} \right\} \\ \div \begin{bmatrix} \frac{5}{2} \\ i = 0 \end{bmatrix} Sr \left\{ \frac{\left\{ pr(0 \longrightarrow Sr \rightarrow i) \right\} \cdot \mu_{i}}{\prod_{k_{2} \neq 0} \left\{ 1 - V_{\overline{k_{2}k_{2}}} \right\}} \right\} \end{bmatrix}$$

$$V_{0} = \begin{bmatrix} w_{4}w_{5}\lambda_{3}(w_{1} + \lambda_{4})(w_{2} + \lambda_{5})) \\ + \lambda_{1}w_{3}w_{4}w_{5}(w_{2} + \lambda_{5}) \\ + \lambda_{2}w_{3}w_{4}w_{5}(w_{1} + \lambda_{4}) \end{bmatrix}$$

$$V_{0} = \begin{bmatrix} w_{4}w_{5}(w_{1} + \lambda_{4})(w_{2} + \lambda_{5})(w_{3} + \lambda_{3}) \\ + \lambda_{1}w_{3}w_{5}(w_{2} + \lambda_{5})(w_{4} + \lambda_{4}) \\ + \lambda_{2}w_{3}w_{4}(w_{1} + \lambda_{4})(w_{5} + \lambda_{5}) \end{bmatrix}$$

$$(7)$$

V. Profit Analysis

The profit function may be used to do a profit analysis of the system and it is given by

$$P = E_0 A_0 - E_1 B_0 - E_2 V_0$$

(8)

where, $E_0 = 25000$ (Revenue per unit uptime of the system)

 $E_1 = 500$ (Cost per unit time for which technician is busy due to repair)

 $E_2 = 200$ (Cost per visit of the technician)

VIII. Discussion

Tables 2, 3 and 4 described the nature of mean time to system failure, availability and profit values

 Table 2: MTSF vs. Repair Rate (w2)

W_2	λ1=0.3, λ2=0.4	λ1=0.4	λ2=0.5	λ3=0.3
Ī	λ3=0.25, λ4=0.35			
♦	$\lambda_5=0.5, w_1=0.4$			
	<i>w</i> ₃ =0.5, <i>w</i> ₄ =0.5			
	w5=0.6			
0.4	3.628692	3.333333	3.090278	3.037974
0.45	3.675035	3.368794	3.132184	3.062553
0.5	3.720609	3.403509	3.173516	3.086409
0.55	3.765432	3.4375	3.214286	3.109568
0.6	3.809524	3.47079	3.254505	3.197278
0.65	3.852901	3.503401	3.294183	3.153901
0.7	3.895582	3.535354	3.333333	3.175126
0.75	3.937583	3.566667	3.371965	3.195751
0.8	3.97892	3.59736	3.410088	3.2158
0.85	4.019608	3.627451	3.447712	3.235294

<i>W</i> ₂	λ1=0.3, λ2=0.4	λ1=0.4	λ2=0.5	λ3=0.3
	λ3=0.25, λ4=0.35			
↓	λ ₅ =0.5, <i>w</i> ₁ =0.4			
	<i>w</i> ₃ =0.5, <i>w</i> ₄ =0.5			
	w5=0.6			
0.4	0.623324	0.604782	0.542904	0.58624
0.45	0.628307	0.609813	0.547959	0.591319
0.5	0.633159	0.614717	0.552904	0.596275
0.55	0.637887	0.619499	0.557741	0.601111
0.6	0.642494	0.624164	0.562476	0.605834
0.65	0.646985	0.628716	0.56711	0.610447
0.7	0.651365	0.633159	0.571646	0.614953
0.75	0.655637	0.637497	0.576089	0.619357
0.8	0.659806	0.641734	0.58044	0.623662
0.85	0.663876	0.645873	0.584703	0.62787

Table 3: Availability vs. Repair Rate (w2)

Table 4: Profit vs. Repair Rate (w2)

W_2	λ1=0.3, λ2=0.4	λ1=0.4	λ2=0.5	λ3=0.3
Ĩ	λ3=0.25, λ4=0.35			
•	$\lambda_5=0.5, w_1=0.4$			
	<i>w</i> ₃ =0.5, <i>w</i> ₄ =0.5			
	w5=0.6			
0.4	2438.338	2386.076	2019.52	2333.814
0.45	2467.262	2415.876	2049.467	2364.49
0.5	2495.431	2444.927	2078.759	2394.423
0.55	2522.874	2473.257	2107.417	2423.64
0.6	2549.618	2500.892	2135.461	2452.166
0.65	2575.691	2527.857	2162.912	2480.023
0.7	2601.117	2554.178	2189.787	2507.239
0.75	2625.919	2579.875	2216.104	2533.831
0.8	2650.121	2604.972	2241.881	2559.823
0.85	2673.744	2629.49	2267.135	2584.836

of the juice plant having an increasing trend corresponding to repair rate (w_2). In these tables, the values of parameters $\lambda_1=0.3$, $\lambda_2=0.4$, $\lambda_3=0.25$, $\lambda_4=0.35$, $\lambda_5=0.5$, $w_1=0.4$, $w_3=0.5$, $w_4=0.5$, $w_5=0.6$ respectively taking as constant for the simplicity. When $\lambda_1=0.3$ changing into $\lambda_1=0.4$; $\lambda_2=0.4$ changing into $\lambda_2=0.5$ and $\lambda_3=0.25$ changing into $\lambda_3=0.3$ then MTSF, availability and profit values have decreasing trends.

IX. Conclusion

The performance of the juice plant is discussed using the regenerative point graphical technique. The above tables explore that when the repair rate increases then the MTSF, system's availability and profit values also increase but when the failure rate increases then the MTSF, availability and profit values decrease. It is clear that RPGT is helpful for industries to analyze the behaviour of the products and components of a system.

X. Future Scope

It is analyzed that the role of the regenerative point graphical technique for the juice plant will be beneficial and also used by the management, manufacturers and the persons engaged in reliability engineering and working on analyzing the nature and performance analysis of the system.

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