LIMIT CYCLES OF LENGTH TWO IN THE RIKKER MODEL AND THEIR APPLICATION IN FISHING

[1] Gurami Tsitsiashvili, [2] Tatyana Shatilina, [1,3] Marina Osipova, [1] Tatyana Radchenkova

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[1] Russia, 690041, Vladivostok, Radio street 7, IAM FEB RAS,

[2]VNIRO (TINRO), 690091, Vladivostok, lane. Shevchenko, 4

[3] Russia, 690002, Vladivostok, FEFU Campus 10 Ajax Bay, Russian Island

guram@iam.dvo.ru, tatyana.shatilina@tinro-center.ru, mao1975@list.ru, tarad@yandex.ru

Abstract

The paper investigates the limit cycle of length two in the Rikker model. It is established that the dependence of the ratio of the maximum value of the cycle to the minimum depends monotonously and almost linearly on the growth coefficient of the Rikker model. Models of the parity shift of the limit cycle of length two are constructed, which is provided by a simultaneous sharp decrease/increase in the growth coefficient. On the example of the Amur salmon in 1994 It is shown that a decrease in the growth coefficient, leading to a shift in the parity of the cycle of length two, is accompanied by a low temperature during the life cycle of pink salmon, when the pink salmon population is in a state of spawn and when the young are rolling.

Keywords: the limit cycle of length two, the parity shift of the cycle and its conditions, the growth coefficient of the Rikker model.

1. Introduction

In chaos theory, and specifically in population dynamics, the Riker model is a population growth model [1]. In ichthyology research, it aroused the interest of mathematicians in determining the length of limit cycles depending on the value of the growth coefficient (Malthusian parameter) [2] - [5]. Despite numerous studies of the Rikker model, the analysis of limit cycles of length two may lead to new results.

In this paper, the ratio of the maximum value of the limit cycle of length two to the minimum is investigated. With the help of computational experiments, it is shown that this ratio depends monotonously and almost linearly on the growth coefficient for almost the entire range of values. This result may be applied to solving an important applied problem formulated by A.A. Goryainov, an employee of the Tinro Center [6], [7] on changing the parity of the limit cycle of length two in the Rikker model. If the maximum value of a cycle of length two is taken in even (odd) years, then a change in parity is understood as such a change in the cycle in which the maximum value begins to be taken in odd (even) years.

The task of changing the parity is important for predicting significant changes in populations, the dynamics of which is subject to the Rikker model. Such parity changes occur very rarely and are the result of a significant influence of external (hydrometeorological) conditions on population dynamics. A feature of the method for solving the problem of parity change is the consideration of the Rikker model at an extreme value of the growth coefficient, which makes it possible to determine the conditions for parity shift from hydrometeorological data. It is shown that the very shift of the cycle of length two at the minimum point of the cycle leads to a significant decrease in catches. On the contrary, at the maximum point of the cycle, a very high growth coefficient is required for its implementation. These circumstances allow us to point out an analogy between the parity shift (number/catch of pink salmon) and the failure of the technical system.

2. Methods

Consider the Rikker model $x_{n+1} = a_n x_n \exp(-b x_n)$, $n \geq 0$. Using the standard substitution $y_n = b x_n$, we arrive at a recurrent sequence

$$y_{n+1} = f(y_n) = \alpha_n y_n \exp(-y_n), \; n \geq 0. \tag{1}$$

Here $\alpha_n = a_n / b$ is the growth coefficient and $c_n = y_{n+1}/y_n$ is the return coefficient. It follows from the formula (1) that the equality $\alpha_n = c_n \exp(y_n)$ is fulfilled, linking the growth coefficient α_n with the return coefficient c_n and with y_n.

Let's focus on the case when $\alpha_n \equiv \alpha$. In this case, the following classification of stable limit modes in the Rikker model [1] - [4] is known. For $0 < \alpha < \beta_0 = 1$, the sequence y_n, $n \geq 0$, has a stable rest point $Y_1 = 0$. At $\beta_0 < \alpha < \beta_1 \approx e^2 \approx 7,39$ the sequence y_n, $n \geq 0$, has a stable rest point $Y_2 = Y_2(\alpha) > 0$. At $\beta_1 < \alpha < \beta_2 \approx 12.49$ the sequence y_n, $n \geq 0$, has a stable limit cycle of length two. At $\beta_2 < \alpha < \beta_3 \approx 14.68$ the sequence y_n, $n \geq 0$, has a stable limit cycle of length four, etc.

Let's calculate the components of the limit cycle of length two Y, $f(Y)$, defined for $Y > 0$ by the relations

$$Y = f(f(Y)) \; \Rightarrow \; 1 = \alpha^2 \exp\left(-Y(1 + \alpha e^{-Y})\right) \; \Rightarrow \; \varphi(Y) = \psi(Y), \tag{2}$$

$$\varphi(Y) = 2 \ln \alpha, \; \psi(Y) = Y(1 + \alpha e^{-Y}).$$

Numerical calculations of the roots of the equation (2) show that for $\beta_1 < \alpha < \beta_2$ this equation has three roots, because the function $\psi(Y)$ has both a minimum and a maximum. Moreover, the minimum root Y_{min} and the maximum root Y_{max} are related by the relations $Y_{max} = f(Y_{min})$, $Y_{min} = f(Y_{max})$. The root $Y_{mid} = \ln \alpha$, contained between the minimum and maximum roots, corresponds to the unstable rest point of the sequence y_n, $n \geq 0$, (see Fig. 1).

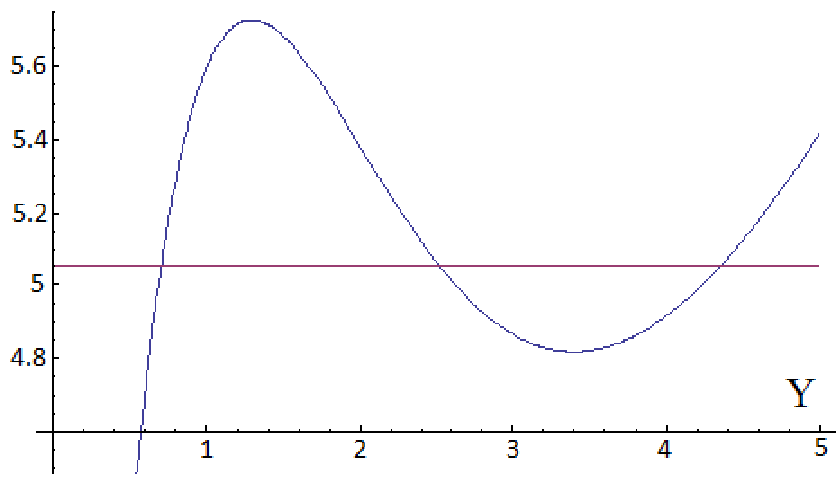

Figure 1. Graphs of functions $\varphi(Y)$ (red line), $\psi(Y)$ (blue line) at $\alpha = \beta_2$.

The results of calculating the roots of the equation (2) are presented in Table 1.

Table 1. The values of the roots Y_{min}, Y_{min}, Y_{max} and the ratio Y_{max}/Y_{min} depending on the growth coefficient α, $\beta_1 < \alpha < \beta_2$.

α	Y_{max}/Y_{min}	Y_{min}	Y_{mid}	Y_{max}
7.39	1.02807	1.97244	2.00013	2.02781
8	2	1.38629	2.07944	2.77259
9	3	1.09861	2.19722	3.29584
10	3.92745	0.934596	2.30259	3.67057
11	4.83672	0.821659	2.3979	3.97413
12	5.74133	0.737215	2.48491	4.2326
12.15	5.87697	0.726287	2.49733	4.26837
12.3	6.01263	0.715737	2.5096	4.30346
12.45	6.14831	0.705543	2.52172	4.3379
12.49	6.1845	0.702882	2.52493	4.34697

From the table 1 it can be seen that the ratio Y_{max}/Y_{min} increases with the growth coefficient α increases from a value close to one at $\alpha \approx \beta_1$ to a significantly larger unit (≈ 6.1845) value at $\alpha \approx \beta_2$. This is shown in more detail in Fig. 2.

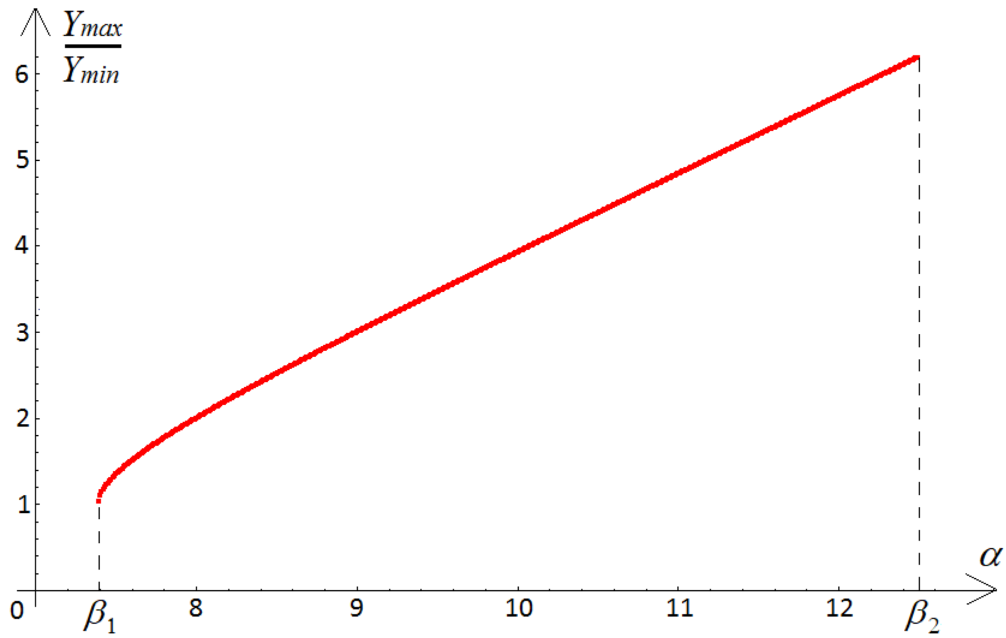

Figure 2. Graph of the dependence of Y_{max}/Y_{min} on the growth coefficient of α at $\beta_1 < \alpha < \beta_2$.

As a result, it has been empirically established that on the segment $\beta_1 < \alpha < \beta_2$ the ratio Y_{max}/Y_{min} depends on the growth coefficient α almost linearly (except for a small tail on the left). Moreover, this fact cannot be established analytically .

Using these estimates and formulas (1), (2), we give one example of the parity shift of a stable cycle at the minimum point (see Fig. 3). Let $\alpha_n = \beta_2$, $0 \leq n \leq 9$, $n \neq 4$, $\alpha_4 = \beta_2^* = \exp(0.702882) \approx 2.01956$, and the sequence y_n, $0 \leq n \leq 9$, $n \neq 4$ coincides with a stable cycle of length two $y_0 = y_2 = y_4 = y_5 = y_7 = y_9 = 0,702882$, $y_1 = y_3 = y_6 = y_8 = 4,34697$. However, at $n = 4$, there is a shift in the parity of the stable cycle.

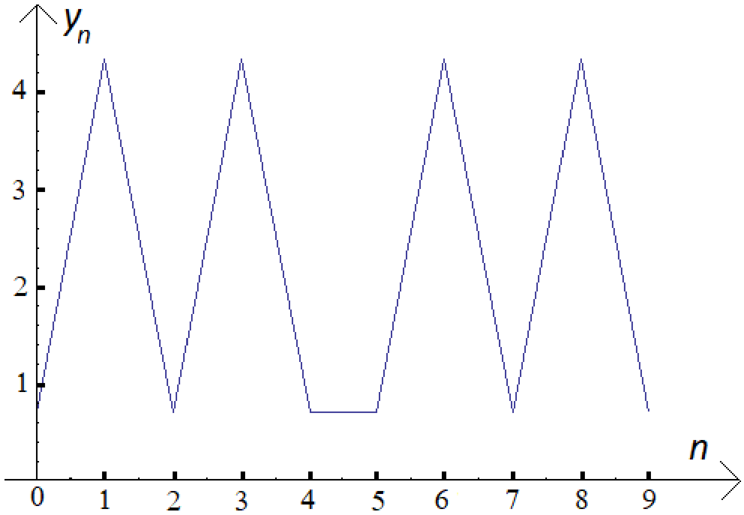

Figure 3. The graph of the parity shift in the Rikker model at the minimum point.

This calculation gives some idealized example of the parity shift of a cycle of length two. To achieve such a shift, it is necessary at the moment $n = 4$ to significantly reduce the growth coefficient, namely, by $\beta_2/\ \beta_2^* \approx .6,184$ times.

In turn, the graph of the parity shift of the limit cycle of length two at the maximum point is possible at the point $n = 3$ at $\alpha_3 = \beta_2^{**} = \exp(4,34697) \approx 77,244$. Let $\alpha_n = \beta_2,\ 0 \leq n \leq 9,\ n \neq 3,\ \alpha_3 = \beta_2^{**}$, and the sequence $y_n,\ 0 \leq n \leq 9,\ n \neq 3$ coincides with a stable cycle of length two: $y_0 = y_2 = y_5 = y_5 = y_7 = y_9 = 0.702882,\ y_1 = y_3 = y_4 = y_6 = y_8 = 4.34697$. However, at $n = 3$, a steady cycle parity shift occurs. To achieve such a shift, we need to greatly increase the growth coefficient, namely, by $\beta_2^{**}/\ \beta_2 \approx .6{,}184$ times.

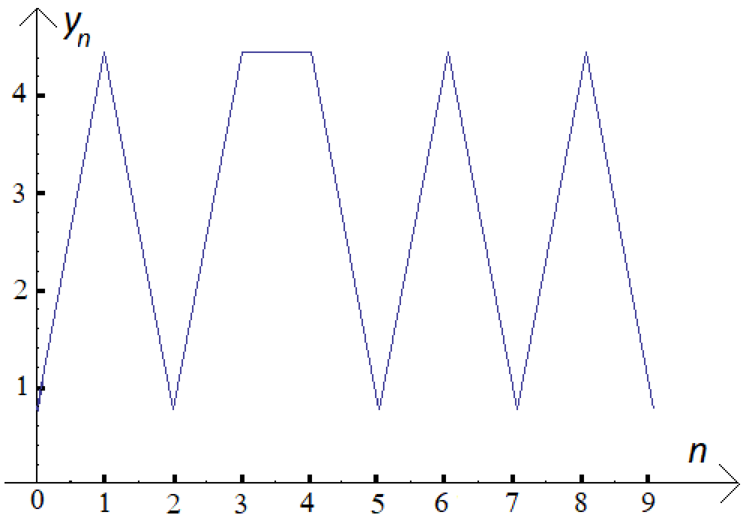

Figure 4. The graph of the parity shift in the Rikker model at the maximum point.

Thus, in the idealized parity shift model of the limit cycle of length two, it is necessary to significantly (6, 184 times) simultaneously reduce/increase the growth coefficient. Therefore, a more realistic procedure is to reduce the growth coefficient and the corresponding shift of the limit cycle of length two at the minimum point.

3. Results

Let's focus on the meteorological conditions that ensure a parity shift at the minimum point of the limit cycle for the dynamics of the Amur pink salmon population. Similar studies have previously been conducted for the Seaside pink salmon [8]. Detailed ichthyology studies show that negative meteorological effects leading to a parity shift are possible in two periods of the pink salmon life cycle. The first period is incubation: in January, when the pink salmon population is in a state of caviar. The second period is in June-July during the decline of young pink salmon. Table 2 shows data on the air temperature in Nikolaevsk on Amur in the first period and in the second period on the water temperature in the Tatar Strait. These data show that the parity shift of the length two cycle occurs at sufficiently low temperatures during the specified periods of the Amur pink salmon life cycle.

Table 2. Temperature data for 1994.

Month	HMS Air Temperature Nikolaevsk-on-Amur	Water Temperature in Tatar Strait
January	-6.4	0.5
February	3.3	0.3
March	0.1	0.3
April	-0.8	0.1
May	-2.2	-0.6
June	-1.5	-0.9
July	-0.9	0.1
August	1.2	0.9
September	0.5	0.2
October	0.9	0.2
November	-0.8	0.1
December	-2.8	0.5

4. Discussion

The problem of shifting the parity of a cycle of length two on the one hand is of serious theoretical and practical interest. When solving it, we have to limit ourselves to cycles of length two in order to compress the initial biological and hydrometeorological information. Of course, this approach to analyzing the source information allows for a certain approximation. But such an approximation can be justified by setting the problem of shifting the parity of a cycle of length two. Moreover, when solving this problem, it is necessary to analyze in detail the life cycle of the Amur pink salmon and identify critical moments in it.

5. Conclusion

In conclusion, it should be noted that the idealized model of the parity shift of a cycle of length two is not always implemented in practice. The parity shift can occur in states where the ratio of the maximum cycle value to the minimum value is close to one. Nevertheless, even in this case, there is a decrease in the growth coefficient caused by adverse meteorological conditions.

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