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A COMPARATIVE STUDY OF INVENTORY MODELLING: DETERMINISTIC OVER STOCHASTIC APPROACH

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Abstract

This research study provides a comprehensive comparison of two critical approaches to inventory modelling- deterministic and stochastic. The deterministic model employs traditional optimization techniques to optimize complex systems, while the stochastic model leverages Particle Swarm Optimization (PSO) simulations to tackle the challenges posed by uncertain dynamics. This approach enables us to develop effective strategies for optimizing complex systems. After conducting sensitivity analyses, it was found that the deterministic model oversimplifies demand dynamics, whereas the stochastic model more adeptly captures market uncertainties. As a result, this study suggests that businesses adopt stochastic approaches to inventory management to better engage in adaptive decision-making, contingency planning, optimal resource allocation, risk mitigation, and realistic performance metrics. The research provides valuable insights for businesses seeking to navigate the complexities of modern supply chains.

Keywords: Inventory, Deterioration, Stochastic optimization, Risk analysis, Particle swarm optimization (PSO).

MSC Classification: 90B05, 90B30, 90B50, 91B70, 93E20

I. Introduction

Inventory modelling plays an integral role in contemporary supply chain management. A comprehensive understanding of the relationship between inventory dynamics and market uncertainties is essential, prompting a comprehensive exploration of deterministic and stochastic approaches. This research delves into the core of this dichotomy, aiming to provide invaluable insights for businesses grappling with the challenges of unpredictable market conditions. In the global marketplace, businesses encounter continuous volatility and uncertainty. The traditional deterministic approach to inventory modelling, relying on fixed parameters and constant demand assumptions, has limitations in capturing the fluidity of real-world markets. A sudden surge in market trends or an unforeseen external event, such as a pandemic, can disrupt this equilibrium, leaving the inventory misaligned with actual demand. This mismatch results in potential revenue loss due to stockouts or excessive holding costs and underscores the urgency for a more adaptive and resilient approach. On the other hand, the stochastic paradigm acknowledges the inherent variability in market dynamics. In a world where demand fluctuations are normal, businesses cannot afford to disregard the impact of uncertainty on inventory management. For example, a manufacturing company that utilizes stochastic modelling may adjust its production levels dynamically based on probabilistic demand forecasts. This approach allows for real-time responsiveness to market shifts, minimizing the risks associated with stockouts, excess inventory, and subsequent financial repercussions.

To address this problem, a meticulous methodology has been crafted, leveraging both classical optimization techniques for deterministic modelling and Particle Swarm Optimization (PSO) simulations for the stochastic scenario. The deterministic approach involves traditional optimization algorithms, aiming to find the optimal solution based on fixed parameters. While this method is widely used, it often must account for the inherent uncertainties in dynamic markets. In contrast, the stochastic approach utilizes PSO, a nature-inspired optimization algorithm that mimics the social behavior of particles to search for optimal solutions in a multidimensional space. In the context of inventory modelling, PSO enables the exploration of diverse demand scenarios, considering the stochastic nature of the objective function. By simulating multiple scenarios, PSO provides a more realistic representation of the potential outcomes in uncertain market conditions. The application of PSO in stochastic modelling is particularly relevant when dealing with complex and dynamic objective functions influenced by stochastic variables, such as fluctuating demand patterns. This approach allows the model to adapt and evolve as market conditions change, providing decision-makers with a versatile tool for strategic inventory planning.

This study holds significant importance as it has the potential to revolutionize the way businesses approach inventory management. It presents a paradigm shift from rigid and deterministic methods to adaptable and stochastic techniques. Given supply chains' growing interconnectedness and susceptibility to global disruptions, the need for a responsive and agile inventory modelling framework is increasingly crucial. From a managerial perspective, this study empowers decision-makers to make informed decisions amidst uncertainty. By highlighting the limitations of deterministic models and the benefits of stochastic approaches, it encourages managers to adopt adaptive strategies that align with the ever-evolving nature of modern markets. For example, a retail manager equipped with insights from stochastic modelling can proactively adjust inventory levels based on probabilistic demand forecasts, thereby minimizing the impact of unforeseen events such as stockouts or excess inventory. Furthermore, the study contributes to academic discourse by comparing deterministic and stochastic inventory modelling comprehensively. By contrasting classical optimization techniques with advanced optimization algorithms such as PSO, it provides a holistic understanding of the strengths and weaknesses of each approach. This nuanced understanding is essential for researchers and academicians seeking to advance the theoretical foundations of inventory management. The study's real-world applicability extends beyond conventional industries to emerging sectors like e-commerce, where demand patterns are subject to rapid and unpredictable changes. By highlighting the adaptability and effectiveness of stochastic modelling, the study provides a roadmap for businesses navigating the complexities of a digital economy.

Altogether, this research aims to redefine the contours of inventory management, transcending the limitations of deterministic paradigms. By combining real-world context, meticulous methodologies, and a profound understanding of the problem at hand, this study aims to guide businesses in navigating the uncertainty of modern supply chains.

II. Literature Review

The recent literature encompasses diverse studies, highlighting the ongoing debate between stochastic and deterministic approaches in various operations research and management domains. The work on crude oil price forecasting ([17]) emphasizes the advantages of their stochastic pruning DE-DL method and shows superior results compared to deterministic counterparts. Using a two-stage stochastic programming model, [6] explores the ability of Industrial Symbiosis networks to withstand fluctuations in demand, revealing their resilience. Adopting a stochastic perspective [2], this article demonstrates the efficacy of their modified particle swarm optimization algorithm within a rolling horizon framework for contributing to aggregate production planning under uncertainty. A two-stage stochastic programming model is presented for disaster preparedness [9], which considers uncertainties in emergency demand and road network congestion. Proposing a two-stage stochastic programming model, [14] advocates for smaller initial networks to adapt to future uncertainties in district cooling network design. A study on forecasting intermittent demand ([20]) uses genetic algorithms and particle swarm optimization, highlighting the stochastic nature of these optimization methods.

Modelling the hot deformation ([16]) of multiphase steels requires advanced numerical models for deterministic and stochastic approaches. A stochastic inventory model that incorporates quadratic price-sensitive demand ([12]). The effects of different probability distributions are compared. Genetic Algorithms is advocated for optimizing inventory management, favoring their efficiency over traditional deterministic systems ([4]). In conversion processes, [11] explore homogeneous and heterogeneous scenarios, acknowledging the stochastic nature of optimal conversion timing, quantity, cost, and time considerations and optimizes using a metaheuristic algorithm. The collective findings suggest a growing preference for stochastic approaches, recognizing their ability to capture and address uncertainties inherent in real-world operational scenarios. The contributions to the comparative study between stochastic and deterministic approaches in inventory control ([4]), particularly in a pharmaceutical distribution setting. Their conceptual model, rooted in modern control theory, addresses practical supply chain constraints. The dynamic mathematical model considers multiple products, variable lead time, deterministic and stochastic demand, and various ordering policies. Objective functions maximize planned versus realized inventory levels and minimize stock-out situations. Real-life data validate the model, providing a comprehensive solution to pharmaceutical supply chain inventory challenges. Exploring the intricate relationship between inventory and demand, [19] proposes a logistic growth model for inventory-dependent demand rates. The study begins with a deterministic optimal control problem, optimizing the present value of total net profit over an infinite horizon. It then extends to the stochastic version, solving the associated Hamilton-Jacobi-Bellman equation and demonstrating optimal inventory levels in a stochastic context. A study ([3]) investigates how prices and production are jointly determined over multiple periods in the face of non-stationary stochastic demand. Their study considers limited production capacity and discretionary sales, comparing partial planning or delayed strategies. The analysis, incorporating deterministic approximations, provides insights into the effectiveness of delayed production versus delayed pricing, with heuristics achieving a high percentage of the corresponding optimal strategy.

A deterministic inventory model is presented for a single item with two storage facilities ([10]). The model addresses linearly time-dependent demand over a fixed and finite time horizon. The model, applicable to scenarios like food grain production, offers a general solution through the gradient method, highlighting its versatility in products with periodic production and linearly increasing demand. Tackling the inventory control problem of nonstationary stochastic demand by incorporating a certainty-equivalent mixed integer linear programming model using the (R, S) policy ([18]). The study provides numerical examples and demonstrates the model's application through a piecewise linear approximation to handle non-linear cost functions. The focus is on inventory planning in closed-loop supply chains ([8]), specifically in equipment-intensive service industries. The planning approach is tactical, which is concerned with short-term decisions rather than long-term strategy. Their mixed-integer programming model, addressing conflicting business objectives, is accompanied by a metaheuristic approach to solution. Experimental evaluations demonstrate the model's effectiveness, emphasizing the impact of cost weightings on different planning strategies. A stochastic inventory model is presented ([13]), considering price-dependent demand, probabilistic lead time, and allowances for shortages in a finite time horizon. The study emphasizes the financial implications of advance payment on unit prices, deriving an expected average profit expression. Numerical examples and solution techniques, such as the generalized reduced gradient technique and stochastic search genetic algorithm, show the model's applicability and benefits.

Inspired by these studies, the current research endeavors to conduct a comprehensive comparative analysis between deterministic and stochastic approaches in inventory management. The aim is to derive insights into each approach's trade-offs, advantages, and practical implications, contributing to the ongoing discourse in the field.

III. Notations

The following notations are used in subsequent discussions, in accordance with usual tradition:

Parameters

$I(t)$	:	Instantaneous inventory level
a	:	Demand potential
b	:	Time dependent parameter
c	:	Time sensitive parameter
θ	:	Constant deterioration rate per unit per unit of time
Q	:	Stock replenishment quantity
h_c	:	Per unit holding cost
C_s	:	Per unit shortage cost
C_0	:	Per unit purchasing cost

Decision Variable

t'	:	Stock ending time
T	:	Inventory cycle time
p	:	Per unit price
$\pi(T, t', p)$	:	Total profit per cycle

IV. Model Formulation with Deterministic Approach

Consider the initial stock size at time $t = 0$ is Q. As the business begins, the stock experiences depletion over time. The demand is price and time-dependent, expressed as $D(p, t) = a - bt - cp$, where $a, b, c > 0$. Here, a represents the base demand, b is time dependent parameter, and c reflects the price sensitivity parameter of demand. After the time t' stock will be end and then shortage begins. It is assumed that the shortage is fully backlogged during stock out time till the time T.

Based on this condition, the rate of the declining of the inventory level $\big(I(t)\big)$, due to demand and per unit deterioration rate θ, is given as:

$$\frac{dI}{dt} = \begin{cases} -(a - bt - c\,p) - \theta\, I(t), & 0 \le t \le t' \\ -\big(a - bt - c_p\big) & t' \le t \le T \end{cases} \tag{1}$$

with the conditions $I(t') = 0,\ I(0) = Q$.

Solving the differential equation we have,

$$I(t) = \begin{cases} \frac{1}{\theta}\left[e^{\theta} t'\left(a - bt' - c_p\right) - e^{\theta t}\left(a - bt - c_p\right)\right] & 0 \le t \le t' \\ \left(at' - \frac{bt'^2}{2} - c\,p\,t'\right) - \left(at - \frac{bt^2}{2} - c\,p\,t\right) & t' \le t \le T \end{cases} \tag{2}$$

Based on these inventory equations, there are several costs associated with the profit function. The cost incurred in holding the products with per unit holding cost h_c is given as:

$$THC = h_c \int_0^{t'} I(t)dt \tag{3}$$

Or,

$$THC = \frac{1}{2\theta^3}\left[h\left(-2(a - c\,p)\theta\left(1 - e^{t'\theta} + t'\theta\right) + b\left(-2 + t'^2\theta^2 + e^{t'\theta}(2 - 2t'\theta)\right)\right)\right] \tag{4}$$

Total shortage cost with per unit shortage cost c_s, during the stock-out period is given as:

$$TSC = c_s \int_{t'}^{T} I(t)dt \tag{5}$$

Or,

$$TSC = c_s\left(-\frac{aT^2}{2} + \frac{1}{2}c\,p\,T^2 + \frac{b\,T^3}{6} + a\,T\,t' - c\,p\,T\,t' - \frac{at'^2}{2} + \frac{1}{2}c\,p\,t'^2 - \frac{1}{2}b\,T\,t'^2 + \frac{b\,t'^3}{3}\right) \tag{6}$$

Total purchasing cost with per unit purchasing cost C_0, is given as:

$$TPC = C_0 Q, \quad where\ Q = I(0) \tag{7}$$

Or,

$$TPC = \frac{1}{2\theta^2}\left(c\,\theta\left(-b + (-a + c\,p)\theta + e^{t'\theta}(b + (a - c\,p - bt')\theta\ )\right)\right) \tag{8}$$

Total cost incurred in terms of deterioration is as follows:

$$TDC = c_d\theta \int_0^{t'} I(t)dt \tag{9}$$

Or,

$$TDC = \frac{1}{2\theta^3}\left(c_d\left(-2(a - c\,p)\theta\left(1 - e^{t'\theta} + t'\theta\right) + b\left(-2 + t'^2\theta^2 + 2e^{t'\theta}(1 - t'\theta)\right)\right)\right) \tag{10}$$

Total revenue generated during the selling period is as follows:

$$TRV = p\int_0^T D(t,p)dt \tag{11}$$

Or,

$$TRV = P\left((a - c\,p)T - \frac{b\,T^2}{2}\right) \tag{12}$$

Combining these costs, we have formulated the profit function given as:

$$\pi(t',T,p) = TRV - TDC - TPC - TSC - THC \tag{13}$$

Or,

$$\pi(t',T,p) = P\left((a-c\,p)T - \frac{b\,T^2}{2}\right) - \frac{1}{2\theta^3}\left(c_d\left(-2(a-c\,p)\theta\left(1-e^{t'\theta}+t'\theta\right)+b\left(-2+t'^2\theta^2+2e^{t'\theta}(1-t'\theta)\right)\right)\right) - \frac{1}{2\theta^2}\left(c\,\theta\left(-b+(-a+c\,p)\theta+e^{t'\theta}(b+(a-c\,p-bt')\theta\ )\right)\right) - \frac{1}{2\theta^3}\left[h_c\left(-2(a-c\,p)\theta\left(1-e^{t'\theta}+t'\theta\right)+b\left(-2+t'^2\theta^2+e^{t'\theta}(2-2t'\theta)\right)\right)\right] - \frac{1}{2\theta^3}\left[h_c\left(-2(a-c\,p)\theta\left(1-e^{t'\theta}+t'\theta\right)+b\left(-2+t'^2\theta^2+e^{t'\theta}(2-2t'\theta)\right)\right)\right] \quad (14)$$

For the optimization of this model, we have utilized the classical optimization approach given in the following theorem.

Theorem 1: *For the positive values parameters, the proposed profit function is concave with respect to the holding time t' and replenish time T.*

Proof: *Using the objective function, we have formulated the Hessian matrix, given as,*

$$H = \begin{bmatrix} \frac{\partial^2 \Pi}{\partial t'^2} & \frac{\partial^2 \Pi}{\partial t' \partial T} \\ \frac{\partial^2 \Pi}{\partial t' \partial T} & \frac{\partial^2 \Pi}{\partial T^2} \end{bmatrix} \quad (15)$$

Where,

$$\frac{\partial^2\pi}{\partial t'^2} = \frac{1}{6\theta^3}\Bigg(3(a-c\,p)\theta\left(-2e^{\theta t'}\,h_c\,\theta^2+\theta\left(2\,c_s\,\theta-2C_0\,e^{\theta t'}\theta^2-2\,c_d\,e^{\theta t'}\theta^2\right)\right)+ b\left(h_c\left(-6\theta^2+12\,e^{\theta t'}\theta^2+6e^{\theta t'}\theta^2(-1+\theta t')\right)-\theta(\theta^2\left(-8\,c\,s\,(T-t')+2\,c\,s\,(T+t')\right)+ c_d\,\left(6\,\theta^2-12\,e^{\theta t'}\theta^2+e^{\theta t'}\theta^2(6-6\theta t')\right)-6c_0\left(2\,e^{\theta t'}\theta^2+e^{\theta t'}\theta^2(-1+\theta t')\ \right)\right)\Bigg)\ ;$$

$$\frac{\partial^2\,\pi}{\partial T\partial t'} = \frac{\partial^2 \Pi}{\partial t' \partial T} = 0\ ;$$

and,

$$\frac{\partial^2\pi}{\partial T^2} = \frac{1}{6\,\theta^3}\left(6\,c_s\,(a-c\,p)\theta^3 - b\,\theta^3(6\,p+4c_s(T-t')+2\,c_s(T+2t')\ )\right)$$

From the evaluation, we have, $\frac{\partial^2\pi}{\partial t'^2}, \frac{\partial^2\pi}{\partial T^2} > 0$, *and* $\frac{\partial^2\pi}{\partial t'^2}\frac{\partial^2\pi}{\partial T^2} - \frac{\partial^2\pi}{\partial t'\partial T} < 0.$

Thus, the objective function is concave with respect to replenish time T and t'. The figure 1 and 2, shows the concavity of the profit function plotted on the values provided in example 1.

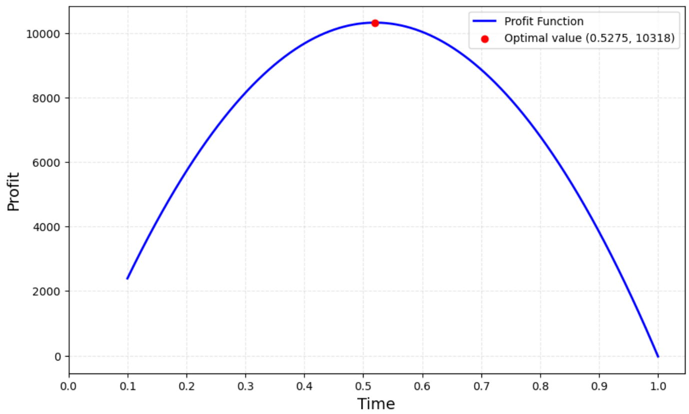

Figure 1: *Concavity of the profit function with respect to replenish time*

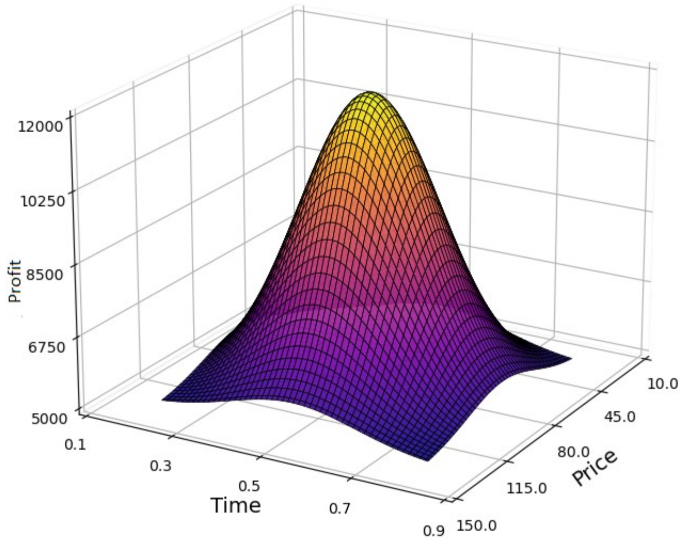

Figure 2: *Concavity of the profit function with respect to replenish time and per unit selling price*

Theorem 2: *For the optimal value of* p*, the objective function (eq. 14) is concave with respect to the selling price.*

Proof: *From objective function eq. 14, we have*

$$\frac{\partial \pi(.)}{\partial p} = \frac{1}{2\theta^2}\Big(T(2a - bT)\theta^2 + 2h_c\left(-1 + \mathrm{e}^{t'\theta} - t'\theta\right) - c_\theta\left(-2c_0\left(-1 + \mathrm{e}^{t'\theta}\right) + (4pT + c_s(T - t')^2)\theta + c_d\left(2 - 2\mathrm{e}^{t'\theta} + 2t'\theta\right)\right)\Big) \tag{16}$$

Putting $\frac{\partial \pi(.)}{\partial p} = 0$, *we yield the optimal value of per unit selling price as,*

$$p^* = -\frac{1}{4cT\theta^2}\left(2c\,h_c - 2ce^{t'\theta}h_c + 2cc_0\theta + 2cc_d\theta - 2cc_0e^{t'\theta}\theta - 2cc_de^{t'\theta}\theta + 2h_ct'\theta - 2aT\theta^2 + bT^2\theta^2 + cc_sT^2\theta^2 + 2cc_dt'\theta^2 - 2cc_sTt'\theta^2 + c\,c_st'^2\theta^2\right)$$

Again differentiating eq. 14, we have,

$$\frac{\partial^2 \pi(.)}{\partial p^2} = -2\,c\,T \tag{17}$$

Thus, for $c > 0$, *the proposed profit function is concave. The concavity of the profit function with respect to the price can also be seen in figure 3.*

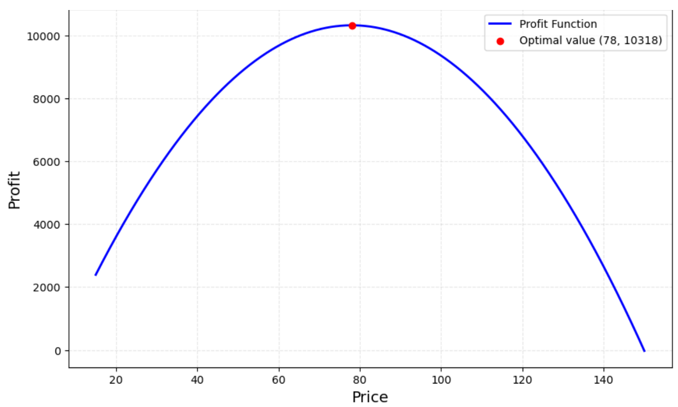

Figure 3: *Concavity of the profit function with respect to per unit selling price*

Following an in-depth exploration of the deterministic approach, the subsequent section delves into the stochastic approach. This approach intricately considers and integrates uncertain factors associated with demand, acknowledging the dynamic and unpredictable nature of variables. In contrast to the deterministic approach, which assumes a fixed and known demand, the stochastic approach takes into account the inherent variability and unpredictability in demand, providing a more comprehensive and realistic perspective in decision-making processes.

V. Model Formulation with Stochastic Approach

Let consider the general form where the demand $D(p,t)$, is the function of price and time, and $f(p)$ is the probability distribution function over price. To be more specific, consider the pre-defined relation of the demand and uncertain factor, i.e., $D(p,t) = a - b\,t - c\,p + \epsilon$, where b and c are time dependent and price sensitive parameters, and ϵ can be determined with the specific distribution, such as uniform, normal etc. depends on the characteristic of the demand fluctuations.

For choosing specific ϵ, follows the normal distribution with mean μ, and standard deviation σ, the $\epsilon \sim N(0,\sigma^2)$, the pdf for the stochastic demand can be experessed as

$$f(p) = e^{-\frac{(D(p,t)-\mu)^2}{2\,\sigma^2}} \tag{18}$$

The uniform distribution is characterized by the constant probability density within a specific range. Consider the price range as $[p_{max}, p_{min}]$. In this case the probability distributon over this range will be as follows:

$$F(p) = \frac{1}{p_{max}-p_{min}} \tag{19}$$

Therefore, we can express the demand with uniform distribution as $D(p,t) = a - b\,t - cp + \epsilon$ where $\epsilon \sim U\,(p_{min}, p_{max})$.

Taking these stochastic demand values and the inventory equation (1, 2), we have reworked for all the costs using these equations:

Expected holding cost with per unit holding cost h_c is as follows:

$$EHC = E\left(\int_{p_{min}}^{p_{max}} \left(h_c\left[\int_0^{t'} I(t)dt\right]\right)\ f(p)dp\right) \tag{20}$$

Expected shortage cost with per unit shortage cost C_s is as follows:

$$ESC = E\left(\int_{p_{min}}^{p_{max}} \left(c_s\left[\int_{t'}^{T} I(t)dt\right]\right)\ f(p)dp\right) \tag{21}$$

Expected shortage cost with per unit shortage cost C_s is as follows:

$$EPC = E\left(\int_{p_{min}}^{p_{max}} C_0 Q\ \ f(p)dp\right) \tag{22}$$

Expected deteriorating cost with per unit deterioration cost C_s is as follows:

$$EDC = E\left(\int_{p_{min}}^{p_{max}} \left(c_d \theta \int_0^{t'} I(t)dt\right)\ f(p)dp\right) \tag{23}$$

Expected revenue is as follows:

$$ERV = \mathrm{E}\left(\int_{p_{min}}^{p_{max}} \left(p\int_0^{T} D(t,p)dt\right)\ f(p)dp\right) \tag{24}$$

Combining all the above cost, the profit function governs as:

$$\pi(t', T, p) = ERV - EHC - ESC - EPC - EDC \tag{25}$$

Or,

$$\pi(t',T,p) = E\left(\int_{p_{min}}^{p_{max}}\left(p\int_0^T D(t,p)dt\right)\ f(p)dp\right) - E\left(\int_{p_{min}}^{p_{max}}\left(h_c\left[\int_0^{t'} I(t)dt\right]\right)\ f(p)dp\right) - E\left(\int_{p_{min}}^{p_{max}}\left(c_s\left[\int_{t'}^{T} I(t)dt\right]\right)\ f(p)dp\right) - E\left(\int_{p_{min}}^{p_{max}} C_0 Q\ \ f(p)dp\right) - E\left(\int_{p_{min}}^{p_{max}}\left(c_d\theta\int_0^{t'} I(t)dt\right)\ f(p)dp\right) \tag{26}$$

subject to the conditions $c_0 \le p,\ \ p_{min} \le p^* \le p_{max},$ and $t' < T$.

As the objective function is probabilistic, we have utilized Particle Swarm Optimization (PSO) to maximize the profit function. PSO is advantageous in optimization processes and excels in navigating complex solution spaces by simulating the social behavior of particles. PSO facilitates swift convergence to optimal outcomes by continuously adapting individual positions guided by personal and global best solutions. This collaborative, swarm-based approach is particularly effective in tackling intricate profit optimization challenges, especially when confronted with uncertainties such as stochastic demand. The algorithm's capacity to balance exploration and exploitation makes it an adaptable and powerful tool, contributing to improved decision-making in scenarios where traditional optimization methods may fall short. Here is the algorithm (Algorithm 1), inspired by [11] that optimizes the profit function effectively.

Algorithm 1: *Algorithm to maximize the profit function using PSO.*

Input: *Parametric values, objective function, constraints.*
Output: *Global best values for $t'.T$, and p.*

1. *Define module 1 taking argument (μ, σ)*
2. *Evaluate the ϵ using desired probability distribution from eq. 18*
3. *Return ϵ*
4. *Define module 2 taking the distribution function for desired probability distribution calling module 1, and objective function and return the value of objective function*
5. *Initialize the parameters associated with PSO and identify the decision variables*
6. *Initialize the maximum number of iterations for PSO*
7. *Initialize the random position and velocities*
8. *for i=1, 2,…, maximum iterations do*
9. *Evaluate the fitness for each particle by calling module 2*
10. *Identify the global best position for each particle*
11. *Update the particle's position and velocity equations*
12. *Check the convergence criteria for each iteration and find the global best value among them*
13. *If maximum value found from existing value, replace the value and set new position*
14. *Plot the iteration and function's value*
15. *end for*
16. *Return the optimal decision variables and corresponding maximum profit based on the best particle's position.*

Utilizing the above algorithm, the following plot (figure 4) illustrates the global convergence of the profit function with respect to their decision variables.

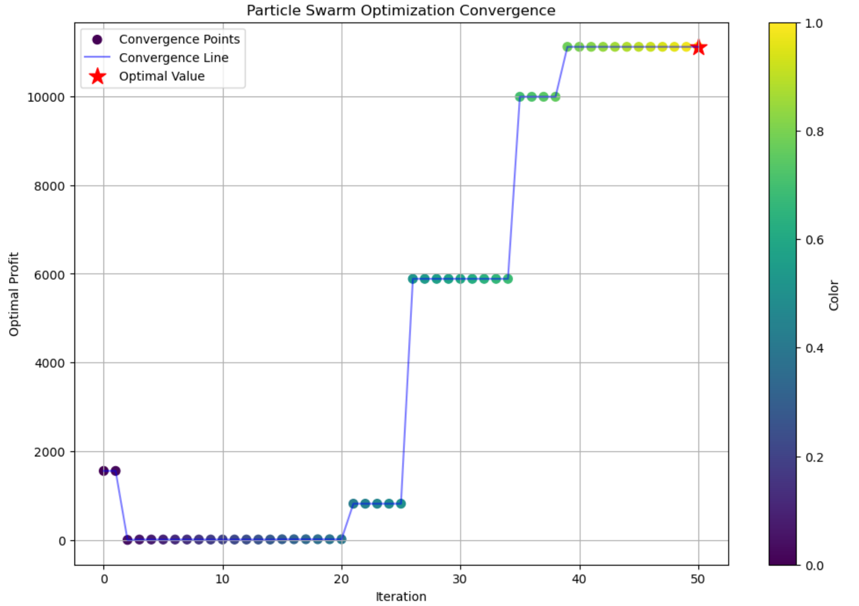

Figure 4: *Iterative convergence of the probabilistic profit function*

Now, in further section we will generalize the result difference for both the approaches, i.e., deterministic over stochastic and will find the superiority of the approaches through the numerical simulations.

VI. Results

Effective inventory management is a critical aspect of supply chain optimization. Two primary approaches have been developed to address uncertainties: deterministic and stochastic. Each of these approaches offers distinct strategies for handling uncertainties in inventory management. In this study, we aim to explore the dynamics of both approaches by utilizing numerical formulations to visualize their impact on demand and profitability. The findings of this study will provide valuable insights for improving inventory management practices, which can ultimately contribute to the overall efficiency of the supply chain. Example 1 illustrates the deterministic method, which relies on known variables and minimizes uncertainties. In the following example, we compare the outcomes of a stochastic approach to a deterministic one. This exploration aims to understand better how different methods influence inventory management and decision-making for businesses seeking stability and precision in stock management.

Example 1: *Consider a scenario where demand stability is critical and uncertainties are minimized through a deterministic approach. We assume a fixed potential demand of 150 units and factors like price sensitivity parameter* $(c) = 0.1$ *and time sensitivity parameter* $(b) = 0.1$ *to tackle the optimization process. We considered holding costs* $(h) = \$2$ *per unit per unit of time, shortage costs* (c_s) *is* $\$2$ *per unit of time, purchasing costs* (c_0) *is 10 per unit, deteriorating rate* $(\theta) = 0.001$ *per unit per unit of time and deterioration cost* $(c_d) = \$0.1$.
The analysis finds insightful metrics: the optimal replenishment time is t' *is 1.69719 units, a shortage duration of 0.29951 units, a streamlined inventory cycle time* $(T) = 1.9967$ *units, the optimal selling price per unit* $(p) = \$78.01$, *the optimal profit is \$10318.*

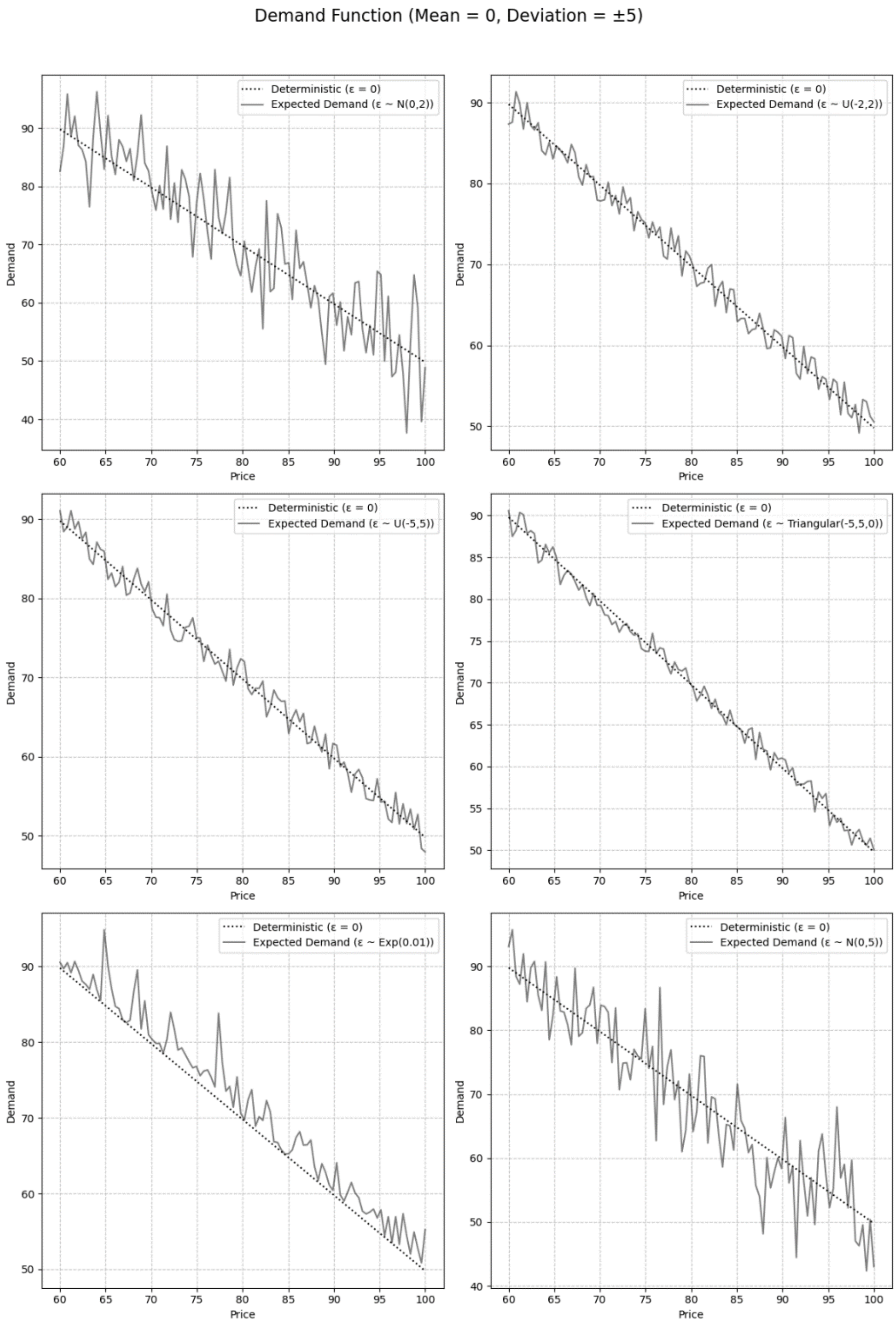

Figure 5: *Demand under the uncertainty*

The data set from example 1 has been utilized to formulate numerical results for the stochastic approach to compare results better. Using Figures 5 and 6, we have illustrated the disparities between deterministic and stochastic models and their implications for managerial decision-making in uncertain situations.

In the initial exploration, Figure 5 captures the essence of demand dynamics under deterministic and stochastic circumstances. The deterministic line, depicted by a dotted black line, represents a scenario where demand is predictable and follows a predefined pattern. In contrast, stochastic scenarios introduce variability, depicted through fluctuating demand graphs under various distributions. The numerical formulation of demand incorporates distributions such as normal, uniform, triangular, and exponential, simulating market conditions with different levels of unpredictability. This formulation allows us to visually notice how demand evolves when subject to varying degrees of uncertainty. The result is a series of demand scenarios that reflect the potential variability inherent in real-world markets.

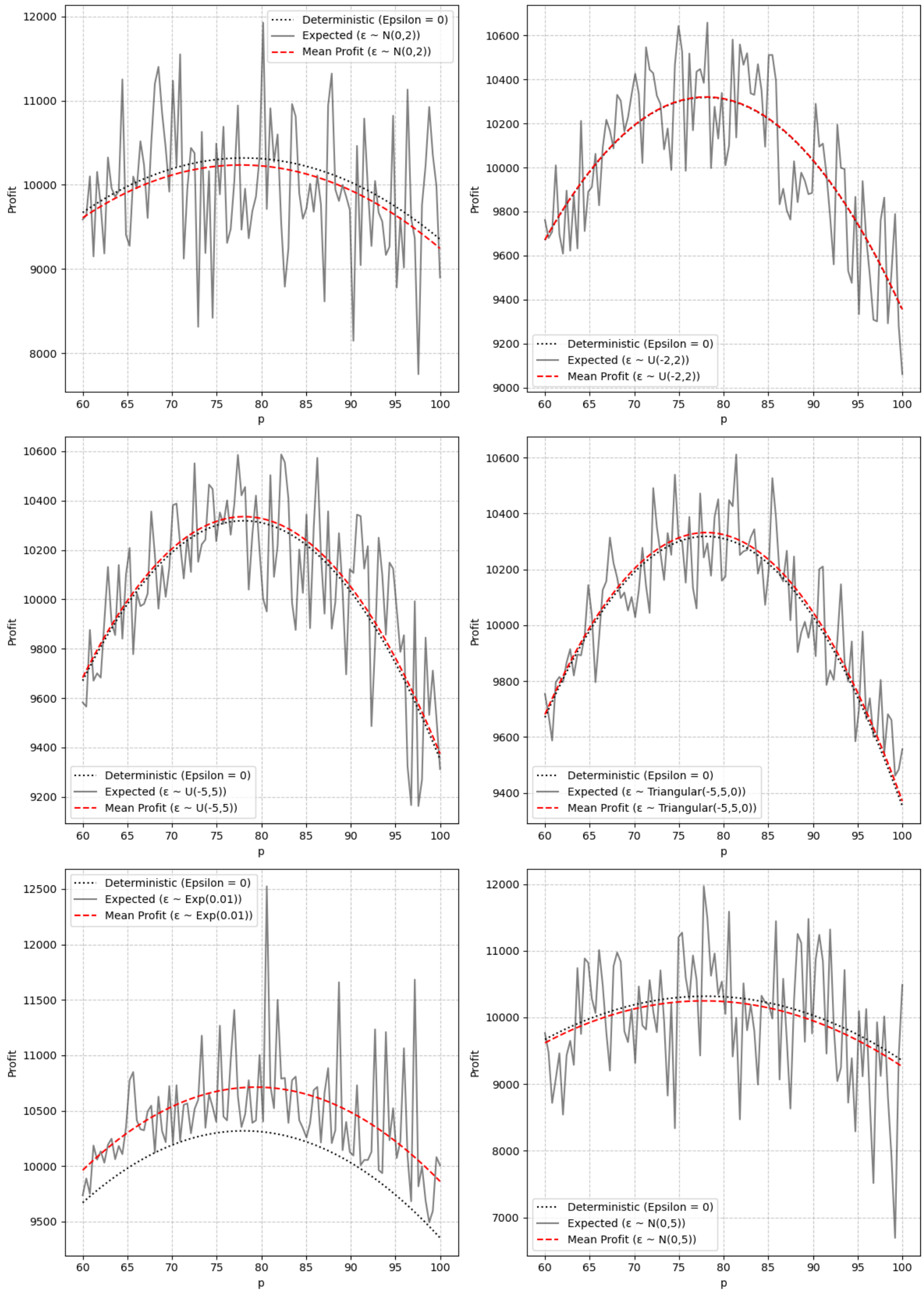

Figure 6: *Profit under the uncertainty*

The profit formulation integrates normal, uniform, triangular, and exponential distributions to simulate the impact of unpredictable market dynamics on profitability. The accompanying mean profit lines offer a glimpse into the expected profitability under stochastic conditions. Here, the interplay between deterministic and stochastic trends becomes apparent, illustrating how market uncertainties can significantly affect overall profitability.

The numerical exploration provides a foundation for understanding the managerial implications of deterministic and stochastic models in uncertain environments. The deterministic approach, while straightforward and easy to implement, may need to be revised when faced with the unpredictable nature of real-world markets. The illustrated figures provide evidence that deterministic models have the potential to oversimplify demand and profit scenarios, thereby leading to erroneous decisions. In contrast, the stochastic models offer a more nuanced perspective, acknowledging and embracing uncertainty. This acknowledgement is crucial for managerial decision-making in unpredictable environments. Managers armed with stochastic insights can anticipate a spectrum of possible outcomes and strategically plan for contingencies. The superiority of stochastic models logically unfolds through the comparison of deterministic and stochastic trends. In Figure 5, the deterministic line represents a singular path, unable to capture the diverse and fluctuating nature of market demand. The stochastic demand scenarios, on the other hand, reflect the inherent variability in market dynamics, allowing for a more comprehensive understanding.

Figure 6 reinforces this logic by illustrating the rigid nature of deterministic profit trends contrasted with the dynamic and adaptable nature of stochastic profitability. The mean profit lines in stochastic scenarios serve as beacons, guiding managers toward a more informed and resilient decision-making process. In uncertain environments, where market conditions are subject to change, the deterministic approach may lead to missed opportunities or unexpected challenges. Stochastic models, by accommodating variability, empower managers to make decisions that align with the complex reality of supply and demand fluctuations. The numerical exploration of deterministic and stochastic models in inventory management provides valuable insights for managerial decision-making. The visual representations in Figures 5 and 6 underscore the limitations of deterministic approaches in handling uncertainties compared to stochastic models' more adaptable and realistic nature. Managerial implications highlight the importance of embracing uncertainty and leveraging stochastic insights to navigate unpredictable market conditions effectively. The logical illustration of the superiority of stochastic models emphasizes their capacity to capture the dynamic nature of demand and profitability, offering a strategic advantage in decision-making.

As businesses operate in an increasingly complex and uncertain global landscape, adopting stochastic models becomes imperative for those seeking resilience, adaptability, and optimized decision outcomes. The numerical results presented here guide managers, encouraging them to explore and implement stochastic approaches in their quest for effective and agile inventory management strategies.

VII. Conclusion

The study conducted a comparative study between deterministic and stochastic approaches in inventory modelling. The deterministic model was subjected to classical optimization techniques, while the stochastic optimizations were addressed using particle swarm optimization (PSO). The analysis presented above sheds light on the intricacies and implications of these approaches, unveiling valuable insights for inventory management strategies when dealing with uncertainty. The sensitivity analyses conducted on deterministic and stochastic models emphasize the significance of acknowledging uncertainty in inventory dynamics. The deterministic paradigm assumes that demand and other parameters remain constant, resulting in robust predictability. However, this approach needs to be more balanced with the complex nature of real-world markets and may lead to suboptimal decision-making. On the other hand, the stochastic model, which embraces variability in demand, offers a more realistic depiction of market dynamics. There are several insights into this study are given below:

1. Stochastic modelling enables managers to make adaptive decisions and respond to changing market conditions in real time. By contrast, deterministic methods may need to pay more attention to the dynamic nature of demand, putting businesses at a disadvantage.

2. Stochastic modelling captures a wide range of potential outcomes, making it an effective tool for robust contingency planning. Managers can anticipate and plan for uncertainties, reducing the impact of unexpected disruptions on inventory management.
3. Sensitivity analyses have demonstrated the superiority of the stochastic model in optimizing resource allocation. This helps managers efficiently use resources, minimize holding costs, and increase profitability.
4. By quantifying uncertainties, the stochastic model becomes a powerful tool for risk mitigation. Managers can use it to implement proactive risk management strategies, ensuring resilience in the face of unforeseen market fluctuations.
5. The stochastic model provides more realistic performance metrics, enabling managers to evaluate inventory management strategies against dynamic market conditions. This provides a comprehensive understanding of operational effectiveness.

The following are some potential avenues for extending this work, which may help to develop further and advance the research:

1. Integrate machine learning algorithms for a data-driven approach.
2. Incorporate multi-objective optimization techniques.
3. Incorporate real-time market feedback to enhance accuracy.
4. Explore cross-functional collaboration between inventory management and other business units.
5. Investigate the potential of leveraging blockchain for improved inventory visibility and risk management.

This research emphasizes the differences between deterministic and stochastic inventory modelling and offers practical suggestions for managerial decision-making. Businesses dealing with modern supply chains' complex and uncertain landscape would benefit from implementing stochastic approaches, particularly when combined with advanced optimization methods like PSO. The identified managerial implications and proposed future extensions pave the way for a more adaptive, resilient, and technologically advanced approach to inventory management.

Declarations

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- The authors declare no competing interests

Data Availability

All data supporting this study are available from the corresponding author upon reasonable request

References

[1] Antic S., Djordjevic M.L., Lisec A., Dynamic discrete inventory control model with deterministic and stochastic demand in pharmaceutical distribution. Applied Sciences, 2022, v. 12, no. 3, p. 15-36.

[2] Boujnah I., Tlili M., Korbaa O., A modified particle swarm optimization algorithm in a rolling horizon framework for the aggregate production planning problem: pharmaceutical industry case. Annals of Operations Research, 2024, p. 1-18.

[3] Chan L. M., Simchi-Levi D., & Swann J., Pricing, Production, and Inventory Policies for Manufacturing with Stochastic Demand and Discretionary Sales. Manufacturing & Service Operations Management, 2006, v. 8, no. 2, p. 149-168.

[4] Chaudhari R.H., Gor A.S., Narsingani F.J., Statistical model for inventory optimization using genetic approach. AIP Conference Proceedings, 2023, v. 2728, no. 1.

[5] Darmawan A., Evaluating proactive and reactive strategies in supply chain network design

with coordinated inventory control in the presence of disruptions. Journal of Industrial and Production Engineering, 2024, p. 1-17.

[6] Daş G.S., Yeşilkaya M., Birgören B., A two-stage stochastic model for an industrial symbiosis network under uncertain demand. Applied Mathematical Modelling, 2024, v. 125, p. 444-462.

[7] Datta A., Sarkar B., Dey B.K., Sangal I., Yang L., Fan S.K., Sardar S.K., Thangavelu L., The impact of sales effort on a dual-channel dynamical system under a price-sensitive stochastic demand. Journal of Retailing and Consumer Services, 2024, v. 76, 103561.

[8] Desport P., Lardeux F., Lesaint D., Cairano-Gilfedder C. D., Liret A., Owusu G., A combinatorial optimisation approach for closed-loop supply chain inventory planning with deterministic demand. European journal of industrial engineering, 2017, v. 11, no. 3, p. 303-327.

[9] Huatian G., Xiaoguang, Y., A two-stage stochastic programming for the integrated emergency mobility facility allocation and road network design under uncertainty. Research Square, 2024.

[10] Kar S., Bhunia A. K., Maiti M., Deterministic inventory model with two levels of storage, a linear trend in demand and a fixed time horizon. Computers & Operations Research, 2001, v. 28, no. 13, p. 1315-1331.

[11] Khedlekar U.K., Kumar L., Mathematical modelling for convertible items with rework using particle swarm optimization. International Journal of Systems Science: Operations & Logistics, 2024, v. 11, no. 1, 2306222.

[12] Khedlekar U.K., Kumar L., Keswani M., A Stochastic Inventory Model with Price-Sensitive Demand, Restricted Shortage and Promotional Efforts. Yugoslav Journal of Operations Research. 2023, v. 33, no. 4, p. 613-642.

[13] Maiti A.K., Maiti M.K., Maiti M., Inventory model with stochastic lead-time and price dependent demand incorporating advance payment. Applied Mathematical Modelling, 2009, v. 33, no. 5, p. 2433-2443.

[14] Neri M., Guelpa E., Verda V., Two-stage stochastic programming for the design optimization of district cooling networks under demand and cost uncertainty. Applied Thermal Engineering, 2024, v. 236, 121594.

[15] Olivares-Nadal A.V., Constructing decision rules for multiproduct newsvendors: An integrated estimation-and-optimization framework. European Journal of Operational Research, 2024.

[16] Oprocha P., Czyżewska N., Klimczak K., Kusiak J., Morkisz P., Pietrzyk M., Szeliga D., A Comparative Study of Deterministic and Stochastic Models of Microstructure Evolution during Multi-Step Hot Deformation of Steels. Materials, 2023, v. 16, no. 9, 3316.

[17] Purohit S.K., Panigrahi S., Novel deterministic and probabilistic forecasting methods for crude oil price employing optimized deep learning, statistical and hybrid models. Information Sciences, 2024, v. 658, 120021.

[18] Tarim S.A., Kingsman B.G., Modelling and computing (R_n, S_n) policies for inventory systems with non-stationary stochastic demand. European Journal of Operational Research, 2006, v. 174, no. 1, p. 581-599.

[19] Tsoularis A., Deterministic and stochastic optimal inventory control with logistic stock-dependent demand rate. International Journal of Mathematics in Operational Research, 2014, v. 6, no. 1, p. 41-69.

[20] Yuna F., Erkayman B., Yılmaz M., Inventory control model for intermittent demand: a comparison of metaheuristics. Soft Computing, 2023, v. 27, no. 10, p. 6487-6505.