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CONFIDENCE INTERVAL USING MAXIMUM LIKELIHOOD ESTIMATION FOR THE PARAMETERS OF POISSON TYPE RAYLEIGH CLASS MODEL

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Abstract

In this research paper, confidence interval using maximum likelihood estimation is obtained for Poisson type Rayleigh class for the parameters. The failure intensity function, mean time to failure function and likelihood function for the parameter is derived. Confidence interval has been obtained for the parameters using maximum likelihood estimation. To study the performance of proposed Confidence interval, average length and coverage probability are calculated by using Monte Carlo simulation technique. From the obtained intervals, it is concluded that Confidence interval for the parameters perform better for appropriate choice of execution time and certain values of parameters.

Keywords: Rayleigh distribution, Software reliability growth model, Maximum likelihood estimation (MLE), Average length and coverage probability.

1. Introduction

Software reliability is the quality characteristic of operation system which can measure, predict and estimate quality of software system. In last several decades various model have been proposed to assess software reliability. Most of them are probabilistic models. Software modeling techniques can be divided into two categories: Prediction and estimation models. Estimation models determines the current software reliability by applying statistical inference techniques to failure data while the prediction models determines future software reliability based upon available software metrics and measures. The parameters involved in software reliability models can be estimated by using some basic procedures like maximum likelihood, least square estimation and Bayesian point estimation, etc. Among all software reliability models, software reliability growth models are very useful to assess software reliability. With the help of software reliability approach customer fulfill their requirements.

Most of the past research work in software reliability modeling has concentrated on the point estimation of the parameters. The uncertainty of the estimates by using interval estimation has not been fully discussed. The most commonly applied interval estimation technique is based on the central limit theorem assuming large sample size. In real world testing the number of software failures observed is usually large. Maximum likelihood Estimation is most preferable because of its easy computation and it is suitable for large sample size. The Confidence interval provides two pairs of values based on sample (say an interval) within which the parameter will lie with certain probability

In this research paper Poisson type Rayleigh class model is considered according to Musa and Okumoto [10] classification scheme. The Rayleigh distribution is widely used for communications, physical sciences, medical imaging and engineering, applied statistics and clinical trials. Sinha and Howlader [16] have computed credible an HPD intervals of the parameters of Rayleigh distribution and also estimated credible interval using reliability function. Sinha [17] has estimated Bayesian interval for the parameters of Rayleigh distribution and its reliability function. Hirano [5] has described origin and properties of Rayleigh distribution. Merovci and Elbatal [9] have defined and studied Weibull Rayleigh distribution and its mathematical properties. Roy [13] proposed Discrete Rayleigh distribution for univariate and bivariate situations. Dey et al [2] have described different approaches for the estimation of two parameters Rayleigh distribution and also computed credible intervals. Rao et al [12] have proposed software reliability growth model of inverse Rayleigh distribution to assess the failure process of developed software and estimated the model parameters by maximum likelihood estimation. Fang and Yeh [3] have proposed a software reliability estimation that uses Stochastic differential equations i.e. SDEs with the fault detection function.

Rao and Cunha [1] have estimated credible intervals and confidence intervals through maximum likelihood estimators for lognormal distribution and also compared average length and coverage probability of the calculated interval. Jeske et al [6] have developed the confidence intervals of average failure rate on the basis of asymptotic theory. Fang and Yeh [4] have proposed confidence interval of the software fault detection process of software reliability growth models using stochastic differential equations. Saroj et al [14] have proposed transformed distribution called inverse Muth distribution and obtained asymptotic confidence interval for parameters of distribution in case of maximum likelihood estimation and maximum product spacing estimation (MPSE) is alternative method for MLE. Lalitha and Mishra [7] have obtained modified maximum likelihood estimate of the Rayleigh distribution using hyperbolic approximation. Lee et al [8] used Obha's model to build the SRGM with confidence intervals that can help the software developers to determine the optimal release time in practice. The Rayleigh distribution is widely used for communications, physical sciences, medical imaging and engineering, applied statistics and clinical trials. Seo et al [15] have obtained the exact confidence intervals for unknown parameters and predictive intervals for future upper record values by considering some pivotal quantities in the two parameter Rayleigh distribution. Yamada et al. [18] have proposed software reliability growth models incorporating the quantity of test-effort exhausted on software testing described by the Rayleigh curve and Rayleigh function used to estimate the detection rate of defects as a function of time during the software development process.

The frame of this paper is such that section 2 presents derivation of failure intensity and expected number of failures using Rayleigh distribution. Section 3 presents Likelihood function and construction of Confidence interval of parameters for Rayleigh distribution. Results are discussed in the section 4 while concluding remarks are provided in section 5.

2. Model Formulation and Evaluation

Considering that software system has experienced m_e failures at times t_i, where i = 1,2,…………m_e during the execution time t_e. Let the parameter γ_0 be the total number of failures and the second parameter is γ_1. Let 't' be the positive random variable having Rayleigh distribution then its probability density function in terms of 't' is given as

$$f(t) = \begin{cases} t\gamma_1^{-2} e^{-\frac{1}{2}\left[\frac{t}{\gamma_1}\right]^2} & , \; t > 0, \gamma_1 > 0 \\ 0 & , \; Otherwise \end{cases} \tag{1}$$

The failure intensity function of the above model can be given as:

$$\lambda(t) = \gamma_0 t \gamma_1^{-2} e^{-\frac{1}{2}[t\gamma_1^{-1}]^2} \qquad , \; t > 0, \gamma_1 > 0, \gamma_0 > 0 \tag{2}$$

Also, the mean failures function i.e. expected number of failures at time t_e is given by:

$$\mu(t_e) = \gamma_0 \gamma_1^{-2} \int_0^{t_e} t\gamma_1^{-2} e^{-\frac{1}{2}[t\gamma_1^{-1}]^2} \, dx$$

After some algebraic simplification, the above equation can be given as:

$$\mu(t_e) = \gamma_0 \left[1 - e^{-\frac{1}{2}(t_e\gamma_1^{-1})^2}\right] \tag{3}$$

3. Likelihood function and Confidence Interval

Confidence interval is one of the estimation techniques to draw statistical inference. Confidence interval can construct with several different methods. The method of confidence interval constructed through maximum likelihood estimation is discussed here. Likelihood function is significant part of frequentist and Bayesian analysis. It can be used to compare probability of various parameter values. The likelihood function of (γ_0, γ_1) is obtained with the help of failure intensity (2) and expected number of failures (3) the likelihood function (see for details Musa et al [11]) and can be expressed as follows:

$$L(\gamma_0, \gamma_1) = \gamma_0^{m_e} \gamma_1^{-2m_e} \left[\prod_{i=1}^{m_e} t_i\right] e^{-\frac{1}{2}T\gamma_1^{-2}} \, \mathrm{e}^{-\gamma_0} \exp\left\{\gamma_0 e^{-\frac{1}{2}\left(\frac{t_e}{\gamma_1}\right)^2}\right\} \tag{4}$$

Where, $\sum_{i=1}^{m_e} t_i^2 = T$

Maximum likelihood estimators for the parameters γ_0 and γ_1 are given by:

$$\hat{\gamma}_{m0} = m_e \left[1 - e^{-\frac{1}{2}\left(\frac{t_e}{\hat{\hat{\gamma}}_{m1}}\right)^2}\right]^{-1} \tag{5}$$

$$\hat{\gamma}_{m1} = \frac{1}{2}\left\{\frac{T}{m_e} - t_e^2 e^{-\frac{1}{2}\left(\frac{t_e}{\hat{\gamma}_1}\right)^2}\left[1 - e^{-\frac{1}{2}\left(\frac{t_e}{\hat{\gamma}_1}\right)^2}\right]^{-1}\right\} \tag{6}$$

To obtain confidence limits for the parameters γ_0 and γ_1 asymptotic variances of the maximum likelihood estimator of the parameters γ_0 and γ_1 are derived, which is the inverse of Fisher information matrix. The negative second order partial derivative of log likelihood function is obtained as follows:

$$\frac{\partial^2 logL}{\partial {\gamma_0}^2} = -\frac{m_e}{\hat{\gamma}_0^2} \tag{7}$$

$$\frac{\partial^2 logL}{\partial {\gamma_1}^2} = \frac{1}{[(-\frac{2me}{\hat{\gamma}_1^2} + 3\sum_{i=1}^{me}\frac{t_i}{\hat{\gamma}_1^4} + 3\hat{\gamma}_0 e^{-\frac{t_e^2}{2\hat{\gamma}_1}}\frac{t_e^2}{\hat{\gamma}_1^4} + \hat{\gamma}_0 e^{-\frac{t_e}{2\hat{\gamma}_1}}\frac{t_e^4}{\hat{\gamma}_1^6})]} \tag{8}$$

Using equation (7) and (8) equations, the variance for the parameters γ_0 and γ_1 can be given as follows:

$$\text{Var}(\hat{\gamma}_0) = \frac{{\hat{\gamma}_0}^2}{m_e} \tag{9}$$

$$\text{Var}(\hat{\gamma}_1) = \frac{1}{[(-\frac{2me}{\hat{\gamma}_1^2} + 3\sum_{i=1}^{me}\frac{t_i}{\hat{\gamma}_1^4} + 3\hat{\gamma}_0 e^{-\frac{t_e^2}{2\hat{\gamma}_1}}\frac{t_e^2}{\hat{\gamma}_1^4} + \hat{\gamma}_0 e^{-\frac{t_e}{2\hat{\gamma}_1}}\frac{t_e^4}{\hat{\gamma}_1^6})]} \tag{10}$$

Using equation (9) and (10) the 100(1-α) % confidence interval for parameters γ_0 i.e. ($\tilde{\gamma}_{0L}\tilde{\gamma}_{0U}$) and γ_1 i.e. ($\tilde{\gamma}_{1L},\tilde{\gamma}_{1U}$) are given by

$$\tilde{\gamma}_{0L} = \hat{\gamma}_0 + z_{\alpha/2}\sqrt{\frac{\hat{\gamma}_0}{m_e}} \tag{11}$$

$$\tilde{\gamma}_{0U} = \hat{\gamma}_0 - z_{\alpha/2}\sqrt{\frac{\hat{\gamma}_0}{m_e}} \tag{12}$$

$$\tilde{\gamma}_{1L} = \hat{\gamma}_1 - z_{\alpha/2}\sqrt{\frac{1}{[(-\frac{2me}{\hat{\gamma}_1^2} + 3\sum_{i=1}^{me}\frac{t_i}{\hat{\gamma}_1^4} + 3\hat{\gamma}_0 e^{-\frac{t_e^2}{2\hat{\gamma}_1}}\frac{t_e^2}{\hat{\gamma}_1^4} + \hat{\gamma}_0 e^{-\frac{t_e}{2\hat{\gamma}_1}}\frac{t_e^4}{\hat{\gamma}_1^6})]}} \tag{13}$$

$$\gamma_{1U} = \hat{\gamma}_1 + z_{\alpha/2}\sqrt{\frac{1}{[(-\frac{2me}{\hat{\gamma}_1^2} + 3\sum_{i=1}^{me}\frac{t_i}{\hat{\gamma}_1^4} + 3\hat{\gamma}_0 e^{-\frac{t_e^2}{2\hat{\gamma}_1}}\frac{t_e^2}{\hat{\gamma}_1^4} + \hat{\gamma}_0 e^{-\frac{t_e}{2\hat{\gamma}_1}}\frac{t_e^4}{\hat{\gamma}_1^6})]}} \tag{14}$$

Where, $z_{\alpha/2}$ is the $[100(1+\alpha)/2]^{th}$ standard normal percentile.

Substituting the tabulated values of $Z_{\alpha/2}$, 95% confidence intervals ($\tilde{\gamma}_{0L}$, $\tilde{\gamma}_{0U}$) and ($\tilde{\gamma}_{1L},\tilde{\gamma}_{1U}$) can be obtained.

4. Results and Discussion

Here, two sided interval confidence interval at 95% confidence level using maximum likelihood estimation are obtained for the parameter γ_0 and γ_1. A sample size m_e was generated up to execution time t_e and it was repeated 1000 times from the Rayleigh distribution by considering different values of γ_0 and γ_1 to study the performance of proposed confidence interval. 95% confidence intervals have been obtained by using Monte Carlo simulation technique. The values of average length and coverage probability have been obtained by assuming execution time $t_e(= 5,6,7,8)$, and parameters$\gamma_0(= 10(1)14)$, and$\gamma_1(= 0.75(0.25)1.75)$. Average length and coverage probability obtained for confidence interval has been summarized in the tables 1 to 8.

Tables 1 to 4 summarize average length and coverage probability for parameter γ_0.From these tables, it is seen that as the value of $\gamma_0$increases, average length calculated for parameter $\gamma_0$decreases and as value of $\gamma_1$increases,average length also increases. As the value of$\gamma_0$increases coverage probability also decreases. And coverage probability increases as $\gamma_1$increases. It was also found that as execution time increases average length also increases and coverage probability also increase.

From the tables 5 to 8 it is noticed that the average length calculated for parameter γ_1 increases as the value of scale parameter γ_0 and shape parameter$\gamma_1$increase. Coverage probability for γ_1 parameter is increasing as value of γ_0 increasing and coverage probability increases slightly as γ_1 increases. It can also be observed that varying execution time, average length and coverage probability increases.

Table 1: *Average length and coverage probability of 95% Confidence interval of $\hat{\gamma}_{M0}$ calculated for different values of the parameters γ_0 = (10:1:14),γ_1= (0.75:0.25:1.75) and t_e= 5*

γ_1 \ γ_0	**10**	**11**	**12**	**13**	**14**
0.75	8.58030 (0.994)	7.80447 (0.993)	6.94918 (0.992)	6.61624 (0.991)	6.36012 (0.991)
1	8.58031 (0.994)	8.01258 (0.993)	7.06641 (0.992)	6.92884 (0.992)	6.36018 (0.991)
1.25	8.67284 (0.994)	8.19769 (0.993)	7.19420 (0.992)	7.30324 (0.992)	6.53328 (0.991)
1.50	8.67286 (0.994)	8.38281 (0.993)	7.27498 (0.992)	7.58629 (0.992)	6.73742 (0.991)
1.75	8.76538 (0.995)	8.39524 (0.994)	7.60509 (0.993)	7.71259 (0.993)	6.79844 (0.991)

*The values in the parenthesis is coverage probability.

Table 2: *Average length and coverage probability of 95% Confidence interval of $\hat{\gamma}_{M0}$ calculated for different values of the parameters γ_0 = (10:1:14),γ_1= (0.75:0.25:1.75) and t_e= 6*

γ_1 \ γ_0	**10**	**11**	**12**	**13**	**14**
0.75	8.58030 (0.994)	8.083131 (0.993)	7.55347 (0.993)	7.17032 (0.992)	6.38972 (0.991)
1	8.72548 (0.994)	8.382828 (0.993)	7.59294 (0.993)	7.28211 (0.992)	6.77412 (0.991)
1.25	8.76538 (0.995)	8.395234 (0.994)	7.68555 (0.993)	7.28413 (0.992)	6.93309 (0.0.992)
1.50	8.78338 (0.995)	8.395235 (0.994)	7.89537 (0.993)	7.36196 (0.992)	6.95633 (0.992)
1.75	8.86543 (0.995)	8.487783 (0.994)	8.092692 (0.993)	7.56139 (0.993)	7.30408 (0.992)

*The values in the parenthesis is coverage probability.

Table 3: *Average length and coverage probability of 95% Confidence interval of $\hat{\gamma}_{M0}$ calculated for different values of the parameters γ_0 = (10:1:14),γ_1= (0.75:0.25:1.75) and t_e= 7*

γ_1 \ γ_0	**10**	**11**	**12**	**13**	**14**
0.75	8.65738 (0.994)	8.48777 (0.994)	8.21017 (0.993)	7.39136 (0.993)	7.26634 (0.992)
1	8.68386 (0.994)	8.55324 (0.994)	8.39537 (0.993)	8.10580 (0.993)	7.48448 (0.992)
1.25	8.72529 (0.994)	8.60538 (0.994)	8.47527 (0.994)	8.19886 (0.993)	7.48785 (0.992)
1.50	8.74578 (0.994)	8.65581 (0.994)	8.48778 (0.994)	8.28175 (0.993)	7.51383 (0.992)
1.75	8.76538 (0.995)	8.70539 (0.994)	8.70251 (0.994)	8.29022 (0.993)	7.81726 (0.993)

*The values in the parenthesis is coverage probability.

Table 4: *Average length and coverage probability of 95% Confidence interval of $\hat{\gamma}_{M0}$ calculated for different values of the parameters γ_0 = (10:1:14),γ_1= (0.75:0.25:1.75) and t_e= 8*

γ_1 \ γ_0	10	11	12	13	14
0.75	8.75526 (0.995)	8.72563 (0.994)	8.48777 (0.993)	8.37329 (0.993)	7.70533 (0.992)
1	8.76530 (0.995)	8.72443 (0.994)	8.56786 (0.993)	8.38274 (0.993)	7.79001 (0.992)
1.25	8.77386 (0.995)	8.73284 (0.994)	8.67284 (0.993)	8.38875 (0.993)	7.79140 (0.992)
1.50	8.78628 (0.995)	8.76645 (0.995)	8.73713 (0.993)	8.58031 (0.993)	7.89511 (0.992)
1.75	8.79404 (0.996)	8.76852 (0.995)	8.74653 (0.993)	8.58157 (0.993)	8.080868 (0.993)

*The values in the parenthesis is coverage probability.

Table 5: *Average length and coverage probability of 95% Confidence interval of $\hat{\gamma}_{M1}$ calculated for different values of the parameters γ_0 = (10:1:14),γ_1= (0.75:0.25:1.75) and t_e=5*

γ_1 \ γ_0	10	11	12	13	14
0.75	0.452752 (0.992)	0.865715 (0.992)	1.027517 (0.993)	1.23690 (0.994)	1.61596 (0.994)
1	0.456343 (0.992)	0.866659 (0.992)	1.246347 (0.994)	1.59739 (0.994)	1.689033 (0.994)
1.25	0.558101 (0.992)	1.002341 (0.992)	1.319541 (0.994)	1.796597 (0.994)	1.705186 (0.994)
1.50	0.57393 (0.992)	1.02732 (0.993)	1.419113 (0.994)	1.923769 (0.994)	1.927146 (0.994)
1.75	0.606502 (0.992)	1.34733 (0.993)	1.426714 (0.994)	1.991414 (0.994)	2.031833 (0.995)

*The values in the parenthesis is coverage probability.

Table 6: *Average length and coverage probability of 95% Confidence interval of $\hat{\gamma}_{M1}$ calculated for different values of the parameters γ_0 = (10:1:14),γ_1= (0.75:0.25:1.75) and t_e=6*

γ_1 \ γ_0	**10**	**11**	**12**	**13**	**14**
0.75	0.45221 (0.992)	0.77637 (0.993)	0.85234 (0.993)	1.02542 (0.993)	1.19246 (0.994)
1	0.46395 (0.992)	0.77946 (0.993)	1.06090 (0.993)	1.22735 (0.994)	1.59603 (0.994)
1.25	0.53536 (0.992)	0.92815 (0.993)	1.07249 (0.993)	1.32042 (0.994)	1.71065 (0.994)
1.50	0.61473 (0.992)	1.03821 (0.993)	1.44207 (0.994)	1.46521 (0.994)	1.97013 (0.994)
1.75	0.72554 (0.993)	1.36680 (0.994)	1.47128 (0.994)	1.62992 (0.994)	2.22514 (0.995)

*The values in the parenthesis is coverage probability.

Table 7: *Average length and coverage probability of 95% Confidence interval of $\hat{\gamma}_{M1}$ calculated for different values of the parameters γ_0 = (10:1:14),γ_1= (0.75:0.25:1.75) and t_e=7*

γ_1 \ γ_0	**10**	**11**	**12**	**13**	**14**
0.75	0.49438 (0.992)	0.83019 (0.992)	1.02628 (0.993)	1.50549 (0.994)	2.09446 (0.994)
1	0.49828 (0.992)	1.02850 (0.993)	1.19715 (0.993)	1.94524 (0.994)	2.45315 (0.994)
1.25	0.59144 (0.992)	1.10866 (0.993)	1.67258 (0.993)	2.09082 (0.994)	2.54534 (0.994)
1.50	0.59262 (0.992)	1.13891 (0.993)	1.70941 (0.993)	2.16158 (0.994)	2.83801 (0.994)
1.75	0.63329 (0.992)	1.16842 (0.993)	1.88625 (0.993)	3.37754 (0.994)	3.46204 (0.994)

*The values in the parenthesis is coverage probability.

Table 8: *Average length and coverage probability of 95% Confidence interval of $\hat{\gamma}_{M1}$ calculated for different values of the parameters γ_0 = (10:1:14),γ_1= (0.75:0.25:1.75) and t_e=8*

γ_1 \ γ_0	**10**	**11**	**12**	**13**	**14**
0.75	0.47184 (0.992)	0.78585 (0.993)	1.30901 (0.993)	1.44314 (0.993)	1.57704 (0.993)
1	0.51061 (0.992)	0.95328 (0.993)	1.36087 (0.993)	1.74261 (0.993)	2.70185 (0.993)
1.25	0.54229 (0.992)	1.03187 (0.993)	1.39260 (0.993)	2.15898 (0.993)	2.93696 (0.993)
1.50	0.56675 (0.992)	1.07622 (0.993)	1.51162 (0.993)	2.34460 (0.993)	4.63501 (0.993)
1.75	0.56918 (0.992)	1.09768 (0.993)	1.62715 (0.993)	2.96881 (0.993)	5.06561 (0.993)

*The values in the parenthesis is coverage probability.

5. Conclusion

In this research paper, confidence intervals are proposed for the parameters of Poisson type Rayleigh class model. For the proposed model confidence intervals, average length and coverage probability is calculated with the help of simulated data. From the above discussion it is observed that confidence interval for the parameters $\gamma_0$and γ_1 gives higher coverage probability for particular values of both the parameters for fixed execution time. It is concluded that confidence interval can be preferred for certain values of parameters for fixed execution time.

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