COUNTER-TERRORISM: PROTECTION RESOURCES ALLOCATION. PART II. BRANCHING SYSTEM

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Part II. Branching System

Abstract

A concept of optimal resources allocation to protect countrie's objects against terrorists attack is presented. Under assumption of uncertainty of terrorists' intentions, minmax criterion is suggested. The Goal functions for cost-effectiveness analysis of country terrorism measurements are given. This work is a from of [*Ushakov*, 2006]

study

I. INTRODUCTION

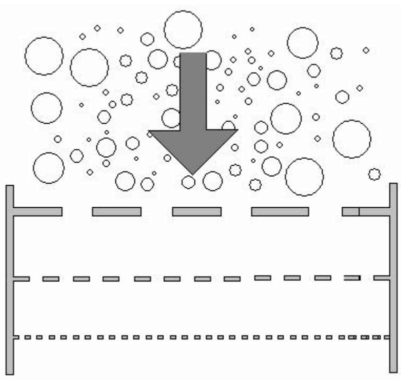
Protecting the country against terrorists' attacks cannot be solved without cost-effectiveness analysis because of the natural limitations on possible defending resources. The main problem for mathematical modeling of the phenomenon is its huge dimension.

Fortunately, the nature of the problem allows us to loosing the understanding of the problem loss of the sense of the problem. The system of the country defending objects can be presented as a system with a special type of a branching structure with additive type of global objective function. The proposed approach is based on [*Ushakov*, 2005; *Gnedenko & Ushakov*, 1995; *Ushakov*, 1994).

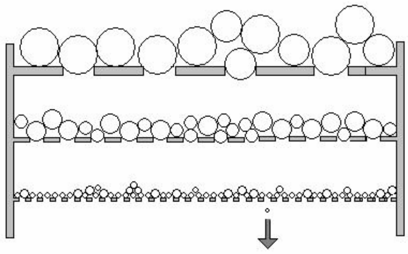
The proposed approach assumes that input data is delivered by counter-terrorism experts.

II. DESCRIPTION OF LEVELS OF SAFETY PROTECTION

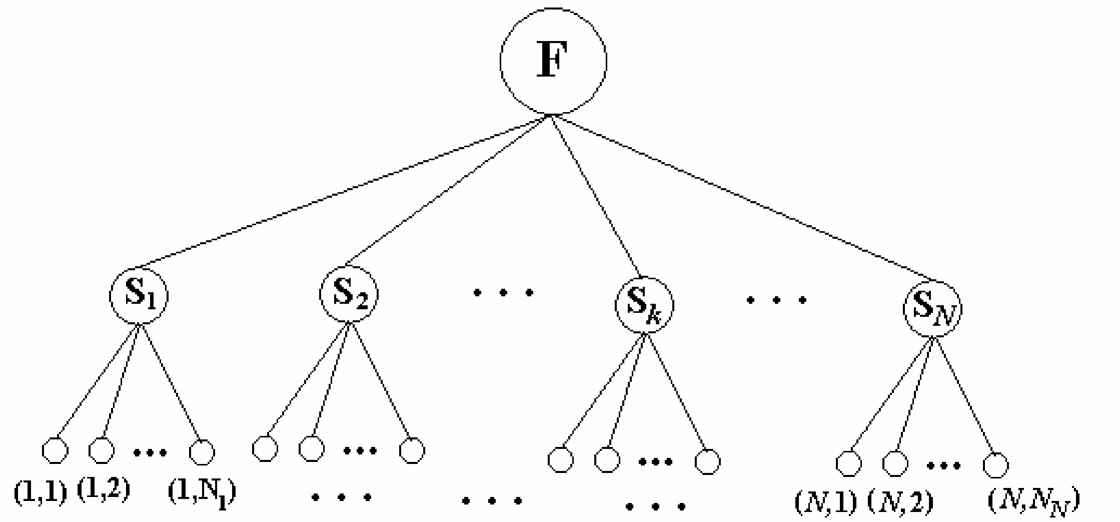
As it was emphasized in Part I (*Minmax Criterion*) the counter-terrorism measures can be divided into three relatively independent levels in such a way that each level presents a kind of a sieve: the lower level is, the higher its "recognition" will be.



After such sifting, chances to penetrate the 3-level counter-terrorism protection should be extremely small.



Thus an adequate mathematical structure of such process is the so-called “branching structure” [*Gnedenko & Ushakov*, 1995]. The upper layer is presented by Federal counter-terrorism protection, the middle layer by State level protection, and the lower layer by defended objectes.



III. DEFINITION AND NOTATIONS

The notations given in Part I of the paper are repeated for the reader’s convenience:

$F_i\,(\varphi_i)$ – subjective probability that an object within the country will be protected against terrorists’ attack of type i under condition that on Federal level one spends φ_i resources. (Notice that this type of protection may not be applicable to all objects. For instance, the increasing control of purchasing chemical materials for WMD design has no relations to possible hijacking.);

$S_i^{(k)}\,(\sigma_i^{(k)})$ – subjective probability that an object within State k will be protected against terrorists’ attack of type i under condition that on this State level one spends $\sigma_i^{(k)}$ resources;

(k, j) – notation for object j within State k

$L_i^{(k,j)}\,(\lambda_i^{(k,j)})$ – subjective probability that particular object (j, k) will be protected against terrorists’ attack of type i under condition that one spends $\lambda_i^{(k,j)}$ resources;

$W^{(k,j)}$	– "weight" (or "measure of priority") of object (j, k);
$G_{k,j}$	– set of possible types of terrorists' attacks against objects (k,j).
n_k	– number of defended objects within State k;
N	– number of states.

IV. MODEL OF BRANCHING STRUCTURE

As shown in [*Gnedenko & Ushakov*, 1995], if for each object of lower layer is chosen an individual index of its effectiveness (or on the contrary, its loss), then the total effectiveness might be considered as a sum of individual indices. It follows from the following simple theorem from the Probability Theory: Mathematical expectation of the sum of random variables equals to the sum of the mathematical expectations of random variables irrespective of dependence of the variables.

Indeed, introduce the so-called indicator function of the type:

$$\delta_{(k,j)} = \begin{cases} 1, \text{ if the attack on object } (k, j) \text{ has occurred,} \\ 0, \text{ otherwise.} \end{cases}$$

Then random loss for object (k, j) is equal to $\delta_{(k,j)} W^{(k,j)}$ and total random loss of all objects is

$$\sum_{k=1}^{N} \sum_{j=1}^{n_k} \delta_{(k,j)} W^{(k,j)}$$

Mathematical expectation of this sum of random variables is defined as

$w_{\text{Total}}\ \{F_i,\ \forall i;\ S_i^{(k)},\ 1\le k \le N;\ L_i^{(k,j)},\ 1\le j \le n_k\}=$

$$E\left\{\sum_{k=1}^{N}\sum_{j=1}^{Nk}\delta_{(k,j)}W^{(k,j)}\right\}=\sum_{k=1}^{N}\sum_{j=1}^{Nk}E\left\{\delta_{(k,j)}\right\}W^{(k,j)}=\sum_{k=1}^{N}\sum_{j=1}^{Nk}\left(1-P^{(k,j)}\right)W^{(k,j)}=\sum_{k=1}^{N}\sum_{j=1}^{Nk}w^{(k,j)} \quad (1)$$

where $P^{(k,j)} = 1-(1-F^{(k,j)})\cdot(1-S^{(k,j)})\cdot(1-L^{(k,j)})$, and in turn, these values are defined as

$F^{(k,j)} = \min\ \{ F_i\ ,\ i\in G_{k,j}\}$; $S^{(k,j)} = \min\ \{ S_i\ ,\ i\in G_{k,j}\}$; $L^{(k,j)} = \min\ \{ L_i\ \}$.

In other words, formula (1) gives the total expected loss with taking into respect their "weights".

At the same time, it is easy to calculate the total expenses, C_{Total}, on all protection measures on all three layers:

$$C_{\text{Total}}\ \{\varphi_i,\ \forall i;\ \sigma_i^{(k)},\ 1\le k\le N;\ \lambda_i^{(k,j)},\ 1\le j\le n_k\}=\sum_{\forall i}\varphi_i+\sum_{\forall i}\sum_{k=1}^{N}\sigma_i^{(k)}+\sum_{\forall i}\sum_{k=1}^{N}\sum_{i=1}^{n_k}\lambda_i^{(k,j)} \quad (2)$$

Having objective functions (1) and (2), one can formulate the following optimization problems:

Direct Problem:

Optimally allocate total available resources that guarantee the minimum possible loss of defended objects against terrorists' attacks, i.e.

$$\min \{ w_{\mathrm{Total}} \mid C_{\mathrm{Total}} \}$$

Inverse Problem:

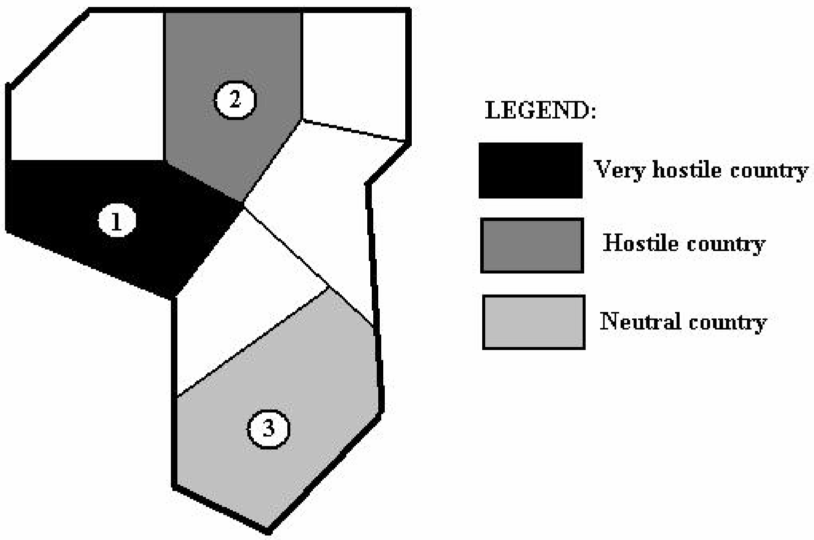
Optimally allocate resources that guarantee the acceptable expected loss of defended objects against terrorists' attacks with minimum possible expenses, i.e.

$$\min \{ C_{\mathrm{Total}} \mid w_{\mathrm{Total}} \}$$

Solution of these problems with the use of the steepest descent method is demonstrated on a simple illustrative numerical example.

V. EXAMPLE: EMBASSY PROTECTION

There are three embassies within a geographical zone. Embassies are assumed of different indices if priority ("weights") and located in the countries with different attitude to the embassies. The problem is to protect these Embassies from terrorists' attacks. Assume that there are available resources for embassy protection within given zone (financial, military, logistics, etc.). How should they be allocated in the most reasonable way?



Let the following data be given by counter-terrorism experts. Assume that we consider only 3 Embassies. The characteristics of these Embassies are as follows:

Embassy-1:
"Weight" of importance = 10; level of protection with no special measures $P_1^{(0)}$ =0.5.

Safety	0.9	0.95	0.97	0.99
Expenses	2	4	7	12

Embassy-2:
"Weight" of importance = 3; level of protection with no special measures $P_2^{(0)}$=0.8.

Safety	0.9	0.95	0.97	0.99
Expenses	1	2	4	8

Embassy-3:
"Weight" of importance = 7; level of protection with no special measures $P_3^{(0)}$ =0.9.

Safety	0.9	0.95	0.97	0.99
Expenses	0.5	1	2	5

The "weight" of importance might depend, for instance, on the size of the Embassy (number of employees) or its political significance.

Solution:

Calculate "discrete gradients" (relative increments) for each Embassy k by using the formula:

$$\gamma_k^{(s)} = W_k \frac{P_k^{(s)} - P_k^{(s-1)}}{C_k^{(s)} - C_k^{(s-1)}},$$

where W_k = "weight" of importance of the Embassy k,

$P_k^{(s)}$ = level of protection at Step s of the process of defense improving,

$C_k^{(s)}$ = expenses related to the level of protection at Step s of the process of defense improving.

Let us construct the following table that will be used (in a very simple way!) for getting an optimal allocation of money for defense all 3 Embassies.

No.	**Value of "gradient" step-by-step**		
	Embassy-1	**Embassy-2**	**Embassy-3**
1	$10 \cdot \frac{0.9-0.5}{2} = 2$	$3 \cdot \frac{0.9-0.8}{1} = 0.3$	$7 \cdot \frac{0.95-0.9}{1} = 0.35$
2	$10 \cdot \frac{0.95-0.9}{4-2} = 0.25$	$3 \cdot \frac{0.95-0.9}{2-1} = 0.15$	$7 \cdot \frac{0.97-0.95}{2-1} = 0.14$
3	$10 \cdot \frac{0.97-0.95}{7-4} = 0.067$	$3 \cdot \frac{0.97-0.95}{4-2} = 0.03$	$7 \cdot \frac{0.99-0.97}{5-3} = 0.07$
4	$10 \cdot \frac{0.99-0.97}{12-7} = 0.004$	$3 \cdot \frac{0.99-0.97}{8-4} = 0.0015$	*

Now the number of all cells in the Table by their decreasing:

No.	Value of "gradient" step-by-step		
	Embassy-1	Embassy-2	Embassy-3
1	1	3	2
2	4	5	6
3	8	9	7
4	10	11	*

These numbers give the order of introduction to the corresponding protective measures. So, the final results are given below:

Initial expected loss is equal to

$w^{(0)} = W_1\cdot(1 - P_1^{(0)}) + W_2\cdot(1 - P_1^{(0)}) + W_3\cdot(1 - P_1^{(0)}) =\cdot 10\cdot 0.5 + 3\cdot 0.2 + 7\cdot 0.1 = 3.8.$

(1) After the 1st step the total expected loss is equal to

$w^{(1)}_{Total} = W_1\cdot(1 - P_1^{(1)}) + W_2\cdot(1 - P_1^{(0)}) + W_3\cdot(1 - P_1^{(0)}) =\cdot 10\cdot 0.1 + 3\cdot 0.2 + 7\cdot 0.1 = 1.8$

and the spent resources are equal to

$C^{(1)} = 2$

(2) After the 2nd step the total expected loss is equal to

$w^{(2)}_{Total} = W_1\cdot(1 - P_1^{(1)}) + W_2\cdot(1 - P_1^{(1)}) + W_3\cdot(1 - P_1^{(0)}) =\cdot 10\cdot 0.1 + 3\cdot 0.1 + 7\cdot 0.1 = 1.5$

and the spent resources are equal to

$C^{(2)} = 2 + 1 = 3.$

(3) After the 3rd step the total expected loss is equal to

$w^{(3)}_{Total} = W_1\cdot(1 - P_1^{(1)}) + W_2\cdot(1 - P_1^{(1)}) + W_3\cdot(1 - P_1^{(1)}) =\cdot 10\cdot 0.1 + 3\cdot 0.1 + 7\cdot 0.05 = 1.15$

and the spent resources are equal to

$C^{(3)} = 2 + 1 + 1 = 4.$

(4) After the 4th step the total expected loss is equal to

$w^{(4)}_{Total} = W_1\cdot(1 - P_1^{(2)}) + W_2\cdot(1 - P_1^{(1)}) + W_3\cdot(1 - P_1^{(1)}) =\cdot 10\cdot 0.05 + 3\cdot 0.1 + 7\cdot 0.05 = 0.9$

and the spent resources are equal to

$C^{(4)} = 2 + 1 + 1 + 2 = 6.$

(5) After the 5th step the total expected loss is equal to

$w^{(5)}_{Total} = W_1\cdot(1 - P_1^{(2)}) + W_2\cdot(1 - P_1^{(2)}) + W_3\cdot(1 - P_1^{(1)}) =\cdot 10\cdot 0.05 + 3\cdot 0.05 + 7\cdot 0.05 = 0.75$

and the spent resources are equal to

$C^{(5)} = 2 + 1 + 1 + 2 + 1 = 7.$

(6) After the 6th step the total expected loss is equal to

$w^{(6)}_{Total} = W_1\cdot(1 - P_1^{(2)}) + W_2\cdot(1 - P_1^{(2)}) + W_3\cdot(1 - P_1^{(2)}) =\cdot 10\cdot 0.05 + 3\cdot 0.05 + 7\cdot 0.03 = 0.61$

and the spent resources are equal to

$C^{(6)} = 2 + 1 + 1 + 2 + 1 + 1 = 8.$

(7) After the 7th step the total expected loss is equal to

$w^{(7)}_{Total} = W_1\cdot(1 - P_1^{(2)}) + W_2\cdot(1 - P_1^{(2)}) + W_3\cdot(1 - P_1^{(3)}) =\cdot 10\cdot 0.05 + 3\cdot 0.05 + 7\cdot 0.01 = 0.47$

and the spent resources are equal to

$C^{(7)} = 2 + 1 + 1 + 2 + 1 + 1 + 2 = 10.$

(8) After the 8th step the total expected loss is equal to

$w^{(8)}_{Total} = W_1\cdot(1 - P_1^{(3)}) + W_2\cdot(1 - P_1^{(2)}) + W_3\cdot(1 - P_1^{(3)}) =\cdot 10\cdot 0.03 + 3\cdot 0.05 + 7\cdot 0.01 = 0.37$

and the spent resources are equal to

$C^{(8)} = 2 + 1 + 1 + 2 + 1 + 1 + 2 + 3 = 13.$

(9) After the 9[th] step the total expected loss is equal to

$w^{(9)}_{\text{Total}} = W_1 \cdot (1 - P_1^{(3)}) + W_2 \cdot (1 - P_1^{(3)}) + W_3 \cdot (1 - P_1^{(3)}) = \cdot 10 \cdot 0.03 + 3 \cdot 0.03 + 7 \cdot 0.01 = 0.31$

and the spent resources are equal to

$C^{(9)} = 2 + 1 + 1 + 2 + 1 + 1 + 2 + 3 + 2 = 15.$

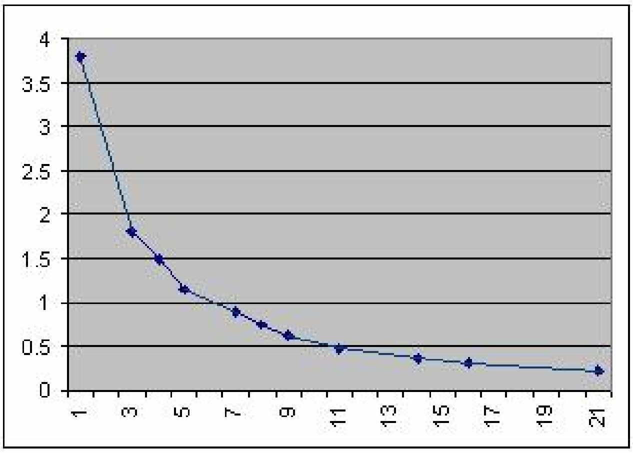
(10)After the 10[th] step the total expected loss is equal to

$w^{(10)}_{\text{Total}} = W_1 \cdot (1 - P_1^{(3)}) + W_2 \cdot (1 - P_1^{(3)}) + W_3 \cdot (1 - P_1^{(3)}) = \cdot 10 \cdot 0.01 + 3 \cdot 0.03 + 7 \cdot 0.01 = 0.21$

and the spent resources are equal to

$C^{(10)} = 2 + 1 + 1 + 2 + 1 + 1 + 2 + 3 + 2 + 5 = 20.$

The process of constructing cost-effectiveness curve can be continued. Graphical presentation of the steepest descent solution is presented below.



Conclusions

This theoretical approach can be used for the assessing, planning, modeling, and managing of cost-effective counter-terrorism measures. Some ideas of this were applied to modeling survivability of National Energy System or the former USSR [*Rudenko & Ushakov*, 1979; *Kozlov et al*., 1986; *Rudenko & Ushakov*, 1989]. Further development of proposed theoretical approach and its implementation for various possible scenarios can significantly boost the analytic resources and predictive capabilities of fighting against terrorism. The approach is powerful enough for the solution of complex and highly unstructured problems. Based on this approach, one can formulate much more complex and realistic problems to include various "what-if" scenarios and additional information: known gaps in security system, counter-terrorism intelligence, impact of preemptive strike against terrorist groups, fuzzy information about terrorist plans and capabilities, etc. Also, the proposed approach can be used to identify the most appropriate security measures and develop optimal strategy aimed at providing maximum possible protection against terrorist threat. Finally, it may be useful in exploring the impact of budget cuts and resource reallocation scenarios on safety issues.

References

I. Ushakov. "Counter-terrorism: Protection Resources Allocation. Part I. Minimax Criterion". "Reliability: Theory and Practice" (vol.1, No.2), 2006.

I. Ushakov. "Cost-effective approach to counter-terrorism". Int'l Journal *Communication in Dependability and Quality Management* (vol.8, No.3),2005.

B. Gnedenko, I. Ushakov "Probabilistic Reliability Engineering". *John Wiley & Sons, Inc., New York*, 1995.

I. Ushakov. "Handbook of Reliability Engineering". (Section 8.6 "Vulnerability of Complex Systems"). *John Wiley & Sons, Inc., New York*, 1994.

Yu. Rudenko, I. Ushakov "Reliability Energy Systems, 2nd edition" (in Russian). Original Russian title: "Надежность систем энергетики". *"Nauka", Novosibirsk, Siberian Branch of the Academy of Sciences of the USSR*, 1989.

M. Kozlov, Yu. Malashenko, V. Rogozhin, I. Ushakov (team lead), **T. Ushakova**. "Computer model of Energy Systems Survivability: Methodology, Model, Implementation" (in Russian). Original Russian title: "Моделирование живучести систем энергетики: методология, модель, реализация". *The Computer Center of the Academy of Sciences of the USSR, Moscow*, 1986.

Yu. Rudenko, I. Ushakov "On evaluation of survivability of complex energy systems". Original title: "К вопросу оценки живучести сложных систем энергетики". *Journal of the Academy of Sciences of the USSR, issue "Energy and Transportation", №1*, 1979.