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FAILURE FREQUENCY CALCULATION TECHNIQUE IN LOGICAL-PROBABILISTIC MODELS

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Armen. S. Stepanyants, Valentina S. Victorova
(Institute of Control Science, Russian Academy of Sciences, Moscow, Russia)
e-mail: lfvrk@ipu.ru, ray@ipu.ru

The technique for calculation failure frequency measure of reliability in class of logical-probabilistic-models is proposed. The technique is applicable for models of redundant repairable systems which are not limited by serial-parallel structures. In conjunction with system decomposition the techniques makes it possible to analyze high dimensional systems very efficiently.

1. INTRODUCTION

Most of the papers regarding algorithmization of reliability&safety characterization in class of logical-probabilistic-models are devoted to truth probability estimation of some logical function defined on Boolean variables (elements of the analyzed system). It should be mentioned that in logical-probabilistic-models one can calculates only so-called differential measures, that is measures of some state or transition **in given moment of time, for example, system availability (unavailability).** In this paper we examine a system of repairable elements with given failure and repair time distribution and specify its availability and failure frequency (both stationary and not stationary). Using availability and failure frequency it is possible to calculate other reliability indices of the system.

Failure frequency is important reliability measure. It characterizes system transitions in the space of states, for example transition from good state to failed state. It is necessary to calculate this index when doing effectiveness, safety, risk analysis. Failure frequency is defined as time derivative of average number of failures. Therefore average number of failures (in the general case average number of transitions) one can calculate via integration failure frequency (transitions frequency) in given time interval. Failure frequency and number of failures are the main measures when calculating cost (loss) per unit time. Mean system cost on time interval (0,t) (mean effectiveness E(0,t)) in systems with multilevel performance is defined as:

$$E(0,t) = \sum_i \int_0^t \Pr_i(t) h_i dt + \sum_{i,j} \int_0^t \omega_{i,j}(t) h_{i,j} dt \,,$$

where $\Pr_i(t)$ – system state i probability in time point t;

$\omega_{i,j}$ – frequency of transitions from the *ith* state to the *jth* state;

h_i – reward (gain or loss) per unit time associated with state i;

$h_{i,j}$ – nonrecurring gain or loss per transition from the *ith* state to the *jth* state.

The first integral presents average holding time in each system states on time interval (0,t) multiplied by reward per unit. The second integral presents average number of transitions weighted by nonrecurring reward. So, if some faults bring to the damage of adjoining equipment or processed part, then with the help of failure frequency we can estimate total loss. Known, traditional estimation of average effectiveness $E(t)$ in time moment t gives too optimistic result:

$$E(t) = \sum_i \Pr_i(t) E_i(t) \,,$$

$E_i(t)$ – effectiveness in state i (particularly, $E_i(t)$ can be equal to h_i).

It should be mentioned that using failure frequency one can calculate interval reliability index for repairable systems, which is not directly calculated in logical-probabilictic models.

Known method of failure frequency calculation, which is based on formula of joint event union, leads to considerable timetable and even to inability of obtaining accurate estimate at high dimensionality because of enumeration type of the algorithm.

In this paper we suggests less time-consuming method of calculation failure frequency for the high dimensional systems.

2. PROBLEM DESCRIPTION

Let elements of a system x_i , $i=\overline{1,n}$ and system $S(\mathbf{x})$, $\mathbf{x}=\{x_i\}$ can be in two state – good and failed

$$x_i = \begin{cases} 1, & \text{when element } i \text{ is good} \\ 0, & \text{when element } i \text{ is failed} \end{cases} \quad (1)$$

$$S(\mathbf{x}) = \begin{cases} 1, & \text{when system is good} \\ 0, & \text{when system is failed} \end{cases} \quad (2)$$

Let system state exhaustively defined by state of its elements in time point t. Denote minimal path sets of the system by $\mathbf{A}=\{A_j\}$, minimal cut sets by $\mathbf{C}=\{C_j\}$.

Then systems availability in time point t can be defined as

$$S(\mathbf{x},t) = \left\{ \bigvee_{j=1}^{r} A_j \right\} = 1, \quad (3)$$

and unavailability

$$\bar{S}(\mathbf{x},t) = \left\{ \bigvee_{j=1}^{l} C_j \right\} = 1. \quad (4)$$

Every minimal path (cut) corresponds to conjunction of some numbers of good (failure) elements $\mathbf{x}=\{x_i\}$.

Availability (unavailability) of a system is defined as:

$$P\{S(\mathbf{x},t)=1\} = P\{\bar{S}(\mathbf{x},t) = 0\} = P\{\bigvee_{j=1}^{r} A_j = 1\} = 1 - P\{\bigvee_{j=1}^{l} C_j = 1\}, \quad (5)$$

$$P\{S(\mathbf{x},t)=0\} = P\{\bar{S}(\mathbf{x},t) = 1\} = P\{\bigvee_{j=1}^{l} C_j = 1\} = 1 - P\{\bigvee_{j=1}^{r} A_j = 1\}, \quad (6)$$

where $P\{.\}$ – occurrence probability of events in brackets in time point t.

Numerous methods and algorithms were designed for calculating availability (unavailability) indexes. The main purpose of these works was to increase efficiency of transformation of logical expressions (3) and /or (4) for obtaining probability (5) and /or (6). The problem lays in exponential growth of computational complexity in the system dimension increase (number of elements, number of minimal cut or path sets). Thus, in calculating unavailability by (6), using formula of joint event union, we obtain the following expression

$$Q(t) = \sum_{i_1=1}^{l} P\{C_{i_1}\} - \sum_{i_1=1}^{l-1}\sum_{i_2>i_1}^{l} P\{C_{i_1} \wedge C_{i_2}\} + \sum_{i_1=1}^{l-2}\sum_{i_2>i_1}^{l-1}\sum_{i_3>i_2}^{l} P\{C_{i_1} \wedge C_{i_2} \wedge C_{i_3}\} - \ldots + \\ +(-1)^{l-1} P\{C_1 \wedge C_2 \wedge \ldots \wedge C_l\}. \quad (7)$$

Number of terms in right side of equation (7) will be $2^l - 1$. Besides generation software algorithm for crossing symbol subsets of paths (cuts) is complex task. For instance, famous test example of naval electrical power system, known as «I.A. Ryabinin task №35» [10], has 15 elements, 31 minimal cut sets and 92 minimal path sets ($2^{31} > 2 \cdot 10^9$). Software, implementing this method for reliability indexes calculation (e.g., Risk Spectrum), in large dimension makes only approximate calculus, which gives rough estimate for system of elements with not a high reliability. It should be mentioned that in [1-3,5,6,8,9] were suggested quite effective calculation methods for availability (unavailability) in (5), (6) interpretation.

Failure frequency is expected number of failures in given moment of time t (i.e. in (t, $t+\Delta t$) when $\Delta t \to 0$). This implies appearance of at least one cut set in time moment $t+\Delta t$. Let e_i – is occurrence event of *ith* cut set in (t, $t+\Delta t$), where $e_i(t+\Delta t)$ – conjunction of n_i variables (elements), forming C_i cut set. At ordinary failure flow assumption appearance e_i in Δt means, that in moment t ($n_i - 1$) elements of C_i were failing and then in Δt one good element failed. Denote this event as $e_i^{'}$. Using formula of total probability we can define occurrence probability of event e_i in Δt:

$$P\{e_i\} = \omega_i^*(t)\Delta t = \sum_{j_i=1}^{n_i} [\omega_{j_i}(t) \prod_{g_i \neq j_i}^{n_i} Q_{g_i}(t)]\Delta t \ , \qquad (8)$$

where $\omega_{j_i}(t)$, $Q_{g_i}(t)$ - failure frequency and unavailability of element j_i, g_i in time moment t; $\omega_i^*(t)$ – failure frequency, conditioned of appearance C_i cut set.

Well known method [7] of calculation failure frequency ω_S also is based on formula (7):

$$\omega_S \Delta t = P\{(S(\mathbf{x},t)=1) \wedge (\bigcup_{i=1}^{l} e_i)\} = P\{\bigcup_{i=1}^{l} e_i\} - P\{(S(\mathbf{x},t)=0) \wedge (\bigcup_{i=1}^{l} e_i)\} = (\omega_{S1} - \omega_{S2})\Delta t \qquad (9)$$

$$\omega_{S1}\Delta t = \sum_{i_1=1}^{l} P\{e_{i_1}\} - \sum_{i_1=1}^{l-1}\sum_{i_2>i_1}^{l} P\{e_{i_1} \cap e_{i_2}\} + \sum_{i_1=1}^{l-2}\sum_{i_2>i_1}^{l-1}\sum_{i_3>i_2}^{l} P\{e_{i_1}\cap e_{i_2} \cap e_{i_3}\} - \ldots$$
$$+ (-1)^{l-1} P\{e_1 \cap e_2 \cap \ldots \cap e_l\},$$

(10)

$$\omega_{S2}\Delta t = \sum_{i=1}^{l}\Big[\sum_{j\neq i}^{l} P\{C_j \wedge e_i\} - \sum_{\substack{j_1=1\\ j_1\neq i}}^{l-1} \sum_{j_2=j_1+1}^{l} P\{C_{j_1} \wedge C_{j_2} \wedge e_i\} + \ldots +$$
$$(-1)^{l-2} P\{C_1 \wedge \ldots \wedge C_{i-1} \wedge C_{i+1} \wedge \ldots \wedge C_l \wedge e_i\}\Big] -$$
$$- \sum_{i_1=1}^{l-1}\sum_{i_2=i_1+1}^{l}\Big[\sum_{j\neq i_1,i_2}^{l} P\{C_j \wedge (e_{i_1}\cap e_{i_2})\} - \sum_{\substack{j_1=1\\ j_1,j_2\neq i_1,i_2}}^{l-1} \sum_{j_2=j_1+1}^{l} P\{C_{j_1}\wedge C_{j_2} \wedge (e_{i_1}\cap e_{i_2})\} + \ldots + \qquad (11)$$
$$(-1)^{l-3} P\{C_1 \wedge C_2 \wedge \ldots \wedge C_{i_1-1} \wedge C_{i_1+1} \wedge \ldots \wedge C_{i_2-1} \wedge C_{i_2+1} \wedge \ldots \wedge C_l \wedge (e_{i_1}\cap e_{i_2})\}\Big] +$$
$$\ldots + \sum_{j=1}^{l} (-1)^{l-1} P\{C_j \wedge (e_1 \cap \ldots \cap e_{j-1} \cap e_{j+1} \cap \ldots \cap e_l)\}.$$

Event $\{C_{j_1} \wedge C_{j_2} \wedge (e_{i_1} \cap e_{i_2})\}$ implies, that in time point t the system was failed because of realization of two cut sets C_{j_1}, C_{j_2} and during Δt general elements for cut sets C_{i_1}, C_{i_2} have failed (i.e. in (t, $t+\Delta t$) cut sets i_1 and i_2 have occurred). If there is no such element, then probability of occurrence just two or more cut sets during Δt is equal to zero. General term is

$$P\{C_{j_1} \wedge C_{j_2} \wedge \ldots \wedge \ldots \wedge C_{j_U} \wedge (e_{i_1} \cap e_{i_2} \cap \ldots \cap e_{i_G})\} = \omega_{GU}(t)\Delta t \prod_{1\ldots U}^{1\ldots G} Q(t)\,, \qquad (12)$$

where $\omega_{GU}(t)$ - frequency of general elements group entering cut set G and not entering U from the others ($l - G$); $\prod_{1\ldots U}^{1\ldots G} Q(t)$ - product of unavailability of all elements entering G and U cut sets from other (l – G) except those elements, which are used in calculation of $\omega_{GU}(t)$ ($\omega_{GU}(t)$ are calculated similarly to (5), but with regard to group of general elements entering G cut sets). Every element in the product is included only once. Failure frequency calculation by (9) - (11) – is more laborious task (about three times), than availability (unavailability) calculation by (7).

Advantageous process of the high dimension problem solving is decomposition of structure or logical representation of the system. At structural decomposition one can appropriate:

1. singly connected decomposition, when appropriated subsystems (assemblies, modules, …) connect each other only through two nodes, and at that one node is input, the other node is output, i.e. this is series connection of the subsystems. Each of the subsystem can correspond to redundancy structure with some logical function (generally *k* out of *m*) in output node (element). In this process we avoid complicated calculation of reliability, safety, technical effectiveness indexes;
2. multiply connected decomposition, when separating subsystems can involve any numbers of inputs/outputs. Only restriction on connection acyclicity exists:
   a. all input nodes of subsystem L^k are either heading nodes or they are connected with other elements (not entering into L^k) through the input edges of L^k;
   b. all output nodes are either terminal nodes or they are connected with other elements through the outgoing edges of L^k.

   This process has disadvantages relating to complexity of the subsystem separation and indexes aggregation (e.g. failure frequency). But it is very efficien at solving high dimension problem and analyzing features of «reliability behaviour »;
3. decomposition by divisible event group of element's states
4. logical decomposition. In this process we do not make any transformation of the system structure. In this case the task of reliability modeling is simplified by dividing logical criteria of the system performance. For aggregation of indexes we can suggest method using theorem of probability of joint events sum, making easy to calculate bilateral estimation of the reliability indexes.

Decomposition methods, especially those, which have described in pt. 1 and 3, are known [7, 8, 11] and are used extensively for availability (unavailability) indexes calculation. In this paper we suggest method of failure frequency calculation based on decomposition technique by pt.3. For reduction calculation effort we also suggest decomposition by pt.1. Expression for failure frequency calculation, using decomposition by pt.1 and separation into series and parallel groups of elements and consequent convolution in one element with equivalent value of failure frequency, is the following:

– parallel schema *(1 out of m)*

$$\omega_{1\,\text{из}\,m}\{t\} = \sum_{j=1}^{m}[\omega_j(t)\prod_{g\neq j}^{m} Q_g(t)], \tag{13}$$

– series schema (*m out of m)*

$$\omega_{m\,\text{из}\,m}\{t\} = \sum_{j=1}^{m}[\omega_j(t)\prod_{g\neq j}^{m} R_g(t)], \tag{14}$$

– parallel schema *(k out of m)*

$$\omega_{k\,\text{из}\,m}\{t\} = \sum_{i_1<i_2<\ldots<i_{m-k}}^{m} Q_{i_1}Q_{i_2}\ldots Q_{i_{m-k}}\Big[\sum_{j=1,\,j\neq i_1,i_2,\ldots i_{m-k}}^{m}[\omega_j(t)\prod_{g=1,g\neq j,i_1,i_2,\ldots i_{m-k}}^{m} R_g(t)]\Big], \tag{15}$$

where $R_i(t)$, $Q_i(t) = 1 - R_i(t)$, $\omega_i(t)$ - availability, unavailability, failure frequency of element *i*.

Expression (13) – (15) can be derived from (9) – (11) or drawn directly from

$$\omega_S = P\{(S(\mathbf{x},\mathrm{t}) = 1) \wedge (\bigcup_{i=1}^{l} e_i^{'})\}\,. \tag{16}$$

Redundant structure *k out of m* very often consists of identical elements, in this case expression (15) takes on form

$$\omega_{k\ \text{из}\ m}\{t\} = C_m^{m-k} Q^{m-k}(t) k\omega(t) R^{k-1}(t)\,, \tag{17}$$

where C_m^{m-k} - number of $(m - k)$ out of m combinations of elements.

In [1] method of recursive variables increase was suggested for availability (unavailability) calculation. The kernel of problem is follows. Let

$$\begin{aligned} p^{(k)} &= P\{S(x_1,...,x_k;t)=1/x_{k+1}=1, x_{k+2}=1,...,x_n=1\} \\ r^{(k)} &= P\{S(x_1,...,x_k;t)=1/x_{k+1}=0, x_{k+2}=1,...,x_n=1\} \end{aligned} \tag{18}$$

When calculating we use the formula

$$p^{(k+1)} = R_{k+1}(t) p^{(k)} + Q_{k+1}(t) r^{(k)}\ , \tag{19}$$

where $R_{k+1}(t) = 1 - Q_{k+1}(t) = P\{x_{k+1}(t)=1\}, \quad p^{(0)} = 1$. Sequentially calculating $p^{(1)}, \dots , p^{(n)}$, on the *nth* last step of recursion we'll get system availability.

3. TECHNIQUE FOR FAILURE FREQUENCY CALCULATION

Method of recursive variables increase (18), (19) is also applicable for failure frequency calculation. Let

$$\begin{aligned} \omega^{(k)}(t)\Delta t &= P\Big\{(S(\mathbf{x},t)=1) \wedge (\bigcup_{i=1}^{l} e_i^{'}) / x_{k+1} = x_{k+2} = \ldots = x_n = 1\Big\}, \\ \text{v}^{(k)}(t)\Delta t &= P\Big\{(S(\mathbf{x},t)=1) \wedge (\bigcup_{i=1}^{l} e_i^{'}) / x_{k+1} = 0, x_{k+2} = \ldots = x_n = 1\Big\}. \end{aligned} \tag{20}$$

Lemma.

System failure frequency can be recursively calculated like that:

$$\omega^{(k+1)}(t) = R_{k+1}(t)\omega^{(k)}(t) + Q_{k+1}(t)\text{v}^{(k)}(t) + P^{x_{k+1}}_{\text{предотк}}(t)\omega_{k+1}(t),\ \ \omega^{(0)}(t) = 0,\ \ \omega_{\text{сист}}(t) = \omega^{(n)}(t)\,, \tag{21}$$

где $P^{x_{k+1}}_{\text{предотк}}(t) = (p^{(k)} - r^{(k)})$, $k = (0,1, \dots, n\text{-}1)$

Lemma Proving. On the $(k+1)$th recursion step elements $x_{k+2}, \dots, x_n$ of the system are completely reliable and we consider divisible group of disjoint events relative to the element x_{k+1}:

- element x_{k+1} is good in time point t. Probability of this event is $R_{k+1}(t)$, and failure frequency is equal to $\omega^{(k)}$ in accordance with expression (20);
- element x_{k+1} in time point t is failed. Probability of this event is $Q_{k+1}(t)$, and failure frequency is equal to $\text{v}^{(k)}$ in accordance with expression (20);
- element x_{k+1} failed on $(t,t+\Delta t)$. Probability of this event is $\omega_{k+1}(t)\Delta t$ (i.e. failure frequency is equal to $\omega_{k+1}(t)$). For the system to transfer to failed state at failure of element x_{k+1} it is necessary to be in the state that are previous to failure and the element x_{k+1} is good, but it's further failure results in failure of the system. Let us denote probability of such subsets as $P^{x_{k+1}}_{\text{предотк}}(t)$. It is proved in *appendix* that

$$P^{x_i}_{\text{предотк}}(t) = R_{\text{сист}}(t) / \{x_i = 1\} - R_{\text{сист}}(t) / \{x_i = 0\}\,, \tag{22}$$

where $R_{\text{сист}}(t) / \{A\}$ - conditional system availability subject to A.

Taking into account (22) we can come up on the $(k+1)$th step $P^{x_{k+1}}_{\text{предотк}}(t) = p^{(k)} - r^{(k)}$.

In accordance with formula of total probability we come up on (21). Note that (22) is Birnbaum reliability measure [7, 11].

Failure frequency calculation method (20), (21) (like availability calculation technique (18), (19)) can be used without system decomposition. But we advise to use system decomposition for overcoming dimensionality problem and rising performance of numerical algorithms. In case of system decomposition we suggest the following algorithm for failure frequency calculation.

– All series, parallel, *k out of m* reliability schemes are enlarged in one element with failure frequency, calculated by (13) - (15) (for availability calculation one may use well-known formulas for series-parallel schemes).

– As a result of several enlarging iterations one can get irreducible system part. In this case failure frequency calculation is implemented by (20), (21). And it is recommended to assign greater numbers to those elements, which incomes in different conjunctions several times. After that while these elements are treated as good (in accordance with expression (20)) it is possible to use simple formulas for series-parallel structures. At formalizing the step of minimal cut sets calculation one make use algorithm, proposed in [4, 5], which allows to pick out the elements, "making" reliability structure irreducible.

General way of $r^{(k)}$ and $\nu^{(k)}$ calculation includes the following. At each recursion step value x_i, stated in condition, are substituted in logical expressions (3), (4) and final expressions are transformed into probabilistic functions relative to availability index and failure frequency (in given step). At "hand-made" calculation one can figure the resulting structures, then all advantages of formulas (13) – (15), (17) together with decomposition and aggregation will be evident. Note, that in general case some steps of $r^{(k)}$ and $\nu^{(k)}$ calculation will include substeps, if resulting logical expressions will not fit series-parallel structures. In this case at given step one have to solve new task with resultant reduced logical description.

4. EXAMPLE

Let us consider irreducible bridge structure (Figure 1) and make failure frequency calculation by two stated above methods.

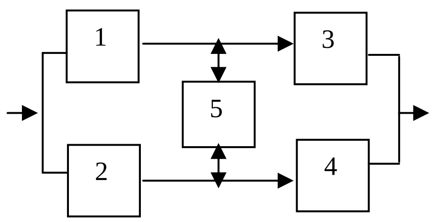

Figure 1. Bridge scheme.

$C_1 = \bar{1}\cdot\bar{2};\ C_2 = \bar{3}\cdot\bar{4};\ C_3 = \bar{1}\cdot\bar{5}\cdot\bar{4};\ C_4 = \bar{2}\cdot\bar{5}\cdot\bar{3}$ (indexes are used instead of elements x_i, conjunction symbol are substituted by product character).

1. Under (9) – (12).

$$\omega_{S1} = (\omega_1 Q_2 + \omega_2 Q_1) + (\omega_3 Q_4 + \omega_4 Q_3) + (\omega_1 Q_5 Q_4 + \omega_5 Q_1 Q_4 + \omega_4 Q_1 Q_5) + (\omega_2 Q_5 Q_3 + \omega_5 Q_2 Q_3 + \omega_3 Q_2 Q_5) - \omega_1 Q_2 Q_5 Q_4 - \omega_2 Q_1 Q_5 Q_3 - \omega_4 Q_1 Q_5 Q_3 - \omega_3 Q_2 Q_5 Q_4 - \omega_5 Q_1 Q_2 Q_3 Q_4. \quad (23)$$

All e_i and and cross product e_i, e_j were included in ω_{S1} (cross product $e_1 \cap e_2$ has not common elements, therefore for this event failure frequency is zero).

$$\omega_{S2} = [(\omega_1 Q_2 + \omega_2 Q_1)\, Q_3 Q_4 + \omega_2 Q_1 Q_5 Q_4 + \omega_1 Q_2 Q_5 Q_3]_1 - [\omega_2 Q_1 Q_3 Q_5 Q_4 + \omega_1 Q_2 Q_5 Q_3 Q_4]_2 + [(\omega_3 Q_4 + \omega_4 Q_3)\, Q_1 Q_2 + \omega_3 Q_1 Q_5 Q_4 + \omega_4 Q_2 Q_5 Q_3]_3 - [\omega_3 Q_1 Q_2 Q_5 Q_4 + \omega_4 Q_1 Q_2 Q_5 Q_3]_4 + [(\omega_4 Q_5 + \omega_5 Q_4)\, Q_1 Q_2 + (\omega_1 Q_5 + \omega_5 Q_1)\, Q_3 Q_4 + (\omega_1 Q_4 + \omega_4 Q_1)\, Q_2 Q_5 Q_3]_5 - [\omega_5 Q_1 Q_2 Q_3 Q_4 + \omega_4 Q_1 Q_2 Q_5 Q_3 + \omega_1 Q_3 Q_4 Q_2 Q_5]_6 + [(\omega_3 Q_5 + \omega_5 Q_3)\, Q_1 Q_2 + (\omega_2 Q_5 + \omega_5 Q_2)\, Q_3 Q_4 + (\omega_2 Q_3 + \omega_3 Q_2)\, Q_1 Q_5 Q_4]_7 - [\omega_5 Q_1 Q_2 Q_3 Q_4 + \omega_3 Q_1 Q_2 Q_5 Q_4 + \omega_2 Q_1 Q_3 Q_5 Q_4]_8 - [\omega_1 Q_3 Q_4 Q_2 Q_5 + \omega_1 Q_3 Q_4 Q_2 Q_5 - \omega_1 Q_3 Q_4 Q_2 Q_5]_9 - [\omega_2 Q_1 Q_3 Q_5 Q_4 + \omega_2 Q_1 Q_3 Q_5 Q_4 - \omega_2 Q_1 Q_3 Q_5 Q_4]_{10} - [\omega_4 Q_1 Q_2 Q_5 Q_3 + \omega_4 Q_1 Q_2 Q_5 Q_3 - \omega_4 Q_1 Q_2 Q_5 Q_3]_{11} - \omega_3 Q_1 Q_2 Q_5 Q_4 + \omega_3 Q_1 Q_2 Q_5 Q_4 - \quad (24)$$

$\omega_3 Q_1 Q_2 Q_5 Q_4]_{12} - [\omega_5 Q_1 Q_2 Q_3 Q_4 + \omega_5 Q_1 Q_2 Q_3 Q_4 - \omega_5 Q_1 Q_2 Q_3 Q_4]_{13}$.

In ω_{S2} were included the following events: $e_1 \Lambda C_i$ – the first square bracket (denoted as $[\dots]_1$); $e_1 \Lambda C_i \Lambda C_j$ – the second square bracket (event $C_3\ \Lambda C_4$ is the failure of all system elements there fore failure frequency of this event is zero); $e_2 \Lambda C_i$, $e_3 \Lambda C_i$, $e_4 \Lambda C_i$, - square brackets 3, 5, 7; $e_2 \Lambda C_i \Lambda C_j$, $e_3 \Lambda C_i \Lambda C_j$, $e_4 \Lambda C_i \Lambda C_j$ - square brackets 4, 6, 8; $e_1 \cap e_3 \Lambda C_2$, $e_1 \cap e_3 \Lambda C_4$, $e_1 \cap e_3 \Lambda C_2 \Lambda C_4$ – square bracket 9; $e_1 \cap e_4 \Lambda C_2$, $e_1 \cap e_4 \Lambda C_3$, $e_1 \cap e_4 \Lambda C_2 \Lambda C_3$ – square bracket 10; $e_2 \cap e_3 \Lambda C_1$, $e_2 \cap e_3 \Lambda C_4$, $e_2 \cap e_3 \Lambda C_1 \Lambda C_4$ – square bracket 11; $e_2 \cap e_4 \Lambda C_1$, $e_2 \cap e_4 \Lambda C_3$, $e_2 \cap e_4 \Lambda C_1 \Lambda C_3$ – square bracket 12; $e_3 \cap e_4 \Lambda C_1$, $e_3 \cap e_4 \Lambda C_2$, $e_3 \cap e_4 \Lambda C_1 \Lambda C_2$ – square bracket 13.

Cross products e_i of order 3 and 4 for ω_{S1} и ω_{S2} have not common elements there fore this term of failure frequency is equal to zero. But all these cross products must be done (by men or by computer).

Final expression for failure frequency in accordance with (9) will be:

$$\begin{aligned}
&\omega_{\text{сист}} = \omega_{S1} - \omega_{S2} = [\omega_1 (Q_2 + Q_4 Q_5 - Q_2 Q_4 Q_5 - Q_2 Q_3 Q_4 - Q_2 Q_3 Q_5 - \\
&Q_3 Q_4 Q_5 + 2 \cdot Q_2 Q_3 Q_4 Q_5)] + [\omega_2 (Q_1 + Q_3 Q_5 - Q_1 Q_3 Q_5 - Q_1 Q_3 Q_4 - \\
&Q_1 Q_4 Q_5 - Q_3 Q_4 Q_5 + 2 Q_1 Q_3 Q_4 Q_5)] + [\omega_1 (Q_2 + Q_4 Q_5 - Q_2 Q_4 Q_5 - \\
&Q_2 Q_3 Q_4 - Q_2 Q_3 Q_5 - Q_3 Q_4 Q_5 + 2 Q_2 Q_3 Q_4 Q_5)] + [\omega_3 (Q_4 + Q_2 Q_5 - \\
&Q_2 Q_4 Q_5 - Q_1 Q_2 Q_4 - Q_1 Q_4 Q_5 - Q_1 Q_2 Q_5 + 2 Q_1 Q_2 Q_4 Q_5)] + [\omega_4 (Q_3 + \\
&Q_1 Q_5 - Q_1 Q_3 Q_5 - Q_1 Q_2 Q_3 - Q_2 Q_3 Q_5 - Q_1 Q_2 Q_5 + 2 Q_1 Q_2 Q_3 Q_5)] + \\
&[\omega_5 (Q_1 Q_4 + Q_2 Q_3 - Q_1 Q_2 Q_4 - Q_1 Q_3 Q_4 - Q_1 Q_2 Q_3 - Q_2 Q_3 Q_4 + 2 Q_1 Q_2 Q_3 Q_4)].
\end{aligned} \qquad (25)$$

2. After calculating conditional availability by (18), (19) we can calculate failure frequency.

Let us define logical operability function throw minimal path sets

$$S(\mathbf{x}, t) = \left\{ \bigvee_{j=1}^{4} A_j \right\} = 1, \quad A_1 = 1 \cdot 3; \ A_2 = 2 \cdot 4; \ A_3 = 1 \cdot 5 \cdot 4; \ A_4 = 2 \cdot 5 \cdot 3 .$$

$$\begin{aligned}
&p^{(1)} = R_1 p^{(0)} + Q_1 r^{(0)} = R_1 \cdot 1 + Q_1 \cdot 1 = 1 \quad (r^{(0)} = P\{S(\mathbf{x}) = 1 / x_1 = 0, x_2 = \dots = x_5 = 1\} = 1), \\
&p^{(2)} = R_2 p^{(1)} + Q_2 r^{(1)} = R_2 \cdot 1 + Q_2 P\{S(\mathbf{x}) = S(x_1) = 1 / x_2 = 0, x_3 = x_4 = x_5 = 1\} = R_2 + Q_2 R_1, \\
&p^{(3)} = R_3 p^{(2)} + Q_3 r^{(2)} = R_3 (R_2 + Q_2 R_1) + Q_3 r^{(2)} = R_3 (R_2 + Q_2 R_1) + Q_3 P\{S(\mathbf{x}) = \\
&S(x_1, x_2) = 1 / x_3 = 0, x_4 = x_5 = 1\} = R_3 (R_2 + Q_2 R_1) + Q_3 (R_2 + Q_2 R_1) = R_2 + Q_2 R_1, \\
&p^{(4)} = R_4 p^{(3)} + Q_4 r^{(3)} = R_4 (R_2 + Q_2 R_1) + Q_4 P\{S(\mathbf{x}) = S(x_1, x_2, x_3) = 1 / x_4 = 0, x_5 = 1\} = \\
&R_4 (R_2 + Q_2 R_1) + Q_4 R_3 (R_2 + Q_2 R_1) = (R_2 + Q_2 R_1)(R_4 + Q_4 R_3), \\
&R_{\text{сист}} = P\{S(\mathbf{x},t)=1\} = p^{(5)} = R_5 p^{(4)} + Q_5 r^{(4)} = R_5 (R_2 + Q_2 R_1)(R_4 + Q_4 R_3) + \\
&Q_5 P\{S(\mathbf{x}) = S(x_1, x_2, x_3, x_4) = 1 / x_5 = 0\} = R_5 (R_2 + Q_2 R_1)(R_4 + Q_4 R_3) + \\
&Q_5 (1 - (1 - R_1 R_3)(1 - R_2 R_4)).
\end{aligned} \qquad (26)$$

From (20), (21).failure frequency is

$$\begin{aligned}
&\omega^{(1)} = R_1 \omega^{(0)} + Q_1 \mathrm{v}^{(0)} + \omega_1 (p^{(0)} - r^{(0)}) = R_1 \cdot 0 + Q_1 P\{(S(\mathbf{x}, t) = 1) \wedge \\
&(\bigcup_{i=1}^{4} e_i^{'}) / x_1 = 0, x_2 = \dots = x_5 = 1\} + \omega_1 \cdot (1 - 1) = 0; \\
&\omega^{(2)} = R_2 \omega^{(1)} + Q_2 \mathrm{v}^{(1)} + \omega_2 (p^{(1)} - r^{(1)}) = R_2 \cdot 0 + Q_2 P\{(S(\mathbf{x}, t) = 1) \wedge \\
&(\bigcup_{i=1}^{4} e_i^{'}) / x_2 = 0, x_3 = x_4 = x_5 = 1\} + \omega_2 (1 - R_1) = Q_2 \omega_1 + \omega_2 (1 - R_1) = Q_2 \omega_1 + \omega_2 Q_1;
\end{aligned} \qquad (27)$$

$$\omega^{(3)} = R_3\omega^{(2)} + Q_3\mathrm{v}^{(2)} + \omega_3(p^{(2)} - r^{(2)}) = R_3(Q_2\omega_1 + \omega_2 Q_1) +$$
$$Q_3 P\{(S(\mathbf{x},t)=1) \wedge (\bigcup_{i=1}^{4} e_i^{'}) / x_3 = 0, x_4 = x_5 = 1\} + \omega_3(R_2 + Q_2R_1 - R_2 - Q_2R_1) =$$
$$R_3(Q_2\omega_1 + \omega_2 Q_1) + Q_3(Q_2\omega_1 + \omega_2 Q_1) = Q_2\omega_1 + \omega_2 Q_1;$$
$$\omega^{(4)} = R_4\omega^{(3)} + Q_4\mathrm{v}^{(3)} + \omega_4(p^{(3)} - r^{(3)}) = R_4(Q_2\omega_1 + \omega_2 Q_1) +$$
$$Q_4 P\{(S(\mathbf{x},t)=1) \wedge (\bigcup_{i=1}^{4} e_i^{'}) / x_4 = 0, x_5 = 1\} + \omega_4(R_2 + Q_2R_1 - R_3(R_2 + Q_2R_1))$$
$$= R_4(Q_2\omega_1 + \omega_2 Q_1) + Q_4(\omega_3 \cdot (1 - Q_1Q_2) + R_3(\omega_1 Q_2 + \omega_2 Q_1)) + \omega_4 Q_3(R_2 + Q_2R_1);$$
$$\omega_{\text{сист}} = \omega^{(5)} = R_5\omega^{(4)} + Q_5\mathrm{v}^{(4)} + \omega_5(p^{(4)} - r^{(4)}) = R_5[R_4(Q_2\omega_1 + \omega_2 Q_1) +$$
$$Q_4[\omega_3(1 - Q_1Q_2) + R_3(\omega_1Q_2 + \omega_2Q_1)] + \omega_4 Q_3(R_2 + Q_2R_1)] +$$
$$Q_5[(\omega_3R_1 + \omega_1R_3)(1 - R_2R_4) + (\omega_2R_4 + \omega_4R_2)(1 - R_1R_3)] +$$
$$\omega_5[(R_2 + Q_2R_1)(R_4 + Q_4R_3) - (1 - (1 - R_1R_3)(1 - R_2R_4))].$$

Let us make a comments to some calculation of $r^{(k)}$ and $\mathrm{v}^{(k)}$.

$r^{(0)} = P\{S(\mathbf{x}) = 1 / x_1 = 0, x_2 = x_3 = x_4 = x_5 = 1\} = 1$ - on conditions that in time point t x_1 failed and all other elements are good, $S(\mathbf{x})$ =1 is persistent event and required probability is 1.

$\mathrm{v}^{(0)}\Delta t = P\{(S(\mathbf{x},t)=1) \wedge (\bigcup_{i=1}^{4} e_i^{'}) / x_1 = 0, x_2 = \ldots = x_5 = 1\}$ - if elements 2 – 5 are good in time point t system failure is impossible (i.e. $\bigcup_{i=1}^{4} e_i$ - null event), so $\mathrm{v}^{(0)} = 0$.

$r^{(1)} = P\{S(\mathbf{x},t) = S(x_1,t) = 1 / x_2 = 0, x_3 = x_4 = x_5 = 1\} = R_1$ - at element x_2 failure and elements $x_3 - x_5$ in good state and after substitution into $S(\mathbf{x})$ we get

$S(\mathbf{x}) = x_1$, and $r^{(1)} = R_1$.

$\mathrm{v}^{(1)}\Delta t = P\{(S(\mathbf{x},t)=1) \wedge (\bigcup_{i=1}^{4} e_i^{'}) / x_2 = 0, x_3 = x_4 = x_5 = 1\} = \omega_1\Delta t$ - substituting $x_2 = 0$, $x_3 = x_4 = x_5 = 1$ into paths $A_i(t)$ and cuts $C_i(t+\Delta t)$, we have $S(\mathbf{x}) = x_1$, $e_1(t+\Delta t) = \bar{x}_1 \cdot 1 = \bar{x}_1$, other $e_i = 0$. In order to occur failure of x_1 in $t+\Delta t$, it is necessary that x_1 was good in time point t:

$e_1^{'}(t) = x_1$. And, $\mathrm{v}^{(1)} = \omega_1$.

$r^{(2)} = P\{S(\mathbf{x}) = S(x_1, x_2) = 1 / x_3 = 0, x_4 = x_5 = 1\} = R_2 + Q_2R_1$ - substitute variables values into condition $S(\mathbf{x},t)$, expressed throw paths. As a result $S(\mathbf{x},t) = S(x_1,x_2,t) = x_1+x_2$, and it is easy to define $r^{(2)} = R_2 + Q_2R_1$ (parallel connection of x_1 and x_2).

$\mathrm{v}^{(2)}\Delta t = P\{(S(\mathbf{x},t)=1) \wedge (\bigcup_{i=1}^{4} e_i) / x_3 = 0, x_4 = x_5 = 1\} = Q_2\omega_1 + \omega_2 Q_1$ - we already got expression for $S(\mathbf{x},t)$ subject to condition, at condition substitution into cuts, which must occur in point (t, $t+\Delta t$), we have $e_1(t, t+\Delta t) = C_1(t, t+\Delta t) = \bar{x}_1 \cdot \bar{x}_2$. For this cut to occur in (t, $t+\Delta t$) it is necessary to implement event $e_1^{'}(t) = x_1 \cdot \bar{x}_2 + x_2 \cdot \bar{x}_1$ in time point t. ($S(\mathbf{x},t) = x_1+x_2$) Λ ($e_1^{'}(t) = x_1 \cdot \bar{x}_2 + x_2 \cdot \bar{x}_1$) = $x_1 \cdot \bar{x}_2 + x_2 \cdot \bar{x}_1$, so $\mathrm{v}^{(2)} = Q_2\omega_1 + \omega_2 Q_1$ (this can be write immediately in terms of $S(\mathbf{x},t)$ for parallel connection (13)).

If in $\omega_{\text{сист}}$ (27) all R_i will be replaced by (1-Q_i) we shall get (25).

<u>Resume from example</u>. When calculating by known method by (9) – (12) it was done L steps of logical expression transformation (logical expression includes 4 cut sets):

– For ω_{S1} calculation:

$L(\omega_{S1}) = (C_4^1=4)+(C_4^2=6)+(C_4^3=4)+(C_4^4=1)=15.$ (28) (28)

In (28) the first bracket is equal to the number of terms of first sum in expression for ω_{S1} (10) and associates with first four parentheses in (23). The second bracket in (28) is equal to the number of terms of second (double) sum in ω_{S1} expression (10) and associates with other five summands (with minus sign; the sixth summand is zero in (23)). The other two brackets in (28) associate with the number of terms of the third (triple) sum and last summand for ω_{S1} in (10). In (23) this terms are absent because they are equal to zero.

– For ω_{S2} calculation in accordance with (24):

$$L(\omega_{S2}) = [(C_4^1=4)\cdot(C_3^1+C_3^2+C_3^3=7)]_1+[(C_4^2=6)\cdot(C_2^1+C_2^2=3)]_2+[(C_4^3=4)\cdot(C_1^1=1)]_3 = 50. \quad (29)$$

The first parentheses in each square bracket defines number of occurrence of one, two, three cuts e_i on Δt. The second parentheses defines number of combinations of possible cuts in time point t.

As a result of fulfilment of some steps it is possible to get empty sets. Terms in expression (10, 11) for these steps are zero. It is necessary to emphasize that all these steps must be executed including those steps, which results in zero.

Thereby, total amount of steps L = 65.

In suggested method ((20), (21) taking into account (18), (19)) there are no combinatorial steps. In suggested method the calculation steps are recursive iteration of variables increase. For the example under consideration it was done 5 steps of calculation $r^{(k)}$ ($r^{(0)},\ldots, r^{(4)}$) and 5 steps of calculation $\nu^{(k)}$ ($\nu^{(0)}, \ldots, \nu^{(4)}$).

The form of final result in suggested method is well-behaved viz approximation to final result are calculated by summation of recursive iteration terms.

Expected number of failures on time interval (0,t) is

$$N_{\text{отк}}(t) = \int_0^t \omega_{\text{сист}}(t)dt\,. \quad (30)$$

Let assume exponential distribution of operating and recovery time ($\tau_{\text{восст.}}$) for all elements, and element 1 is identical to element 2 (i.e. $\lambda_1= \lambda_2= \lambda_{1,2}$, $\mu_1= \mu_2= \mu_{1,2}$), element 3 is identical to element 4 ($\lambda_3= \lambda_4= \lambda_{3,4}$, $\mu_3= \mu_4= \mu_{3,4}$). Availability and failure frequency of the elements are

$$R_i(t) = \frac{\mu_i}{\lambda_i+\mu_i} + \frac{\lambda_i}{\lambda_i+\mu_i}\exp\{-(\lambda_i+\mu_i)t\}\,,\quad \omega_i = \frac{\mu_i\cdot\lambda_i}{\lambda_i+\mu_i} + \frac{\lambda_i\cdot\lambda_i}{\lambda_i+\mu_i}\exp\{-(\lambda_i+\mu_i)t\}\,,\ i=(1\text{-}5).$$

Let $\lambda_{1,2}=1\cdot10^{-3}$, $\mu_{1,2}=1/20=0.05$, $\lambda_{3,4}=2\cdot10^{-3}$, $\mu_{3,4}=1/15=0.0667$, $\lambda_5=5\cdot10^{-3}$, $\mu_5=1/10=0.1$ (unit is hour^{-1}). Figures 2, 3 show failure frequency and expected number of failures dependence on time. One can see that interval of unstationarity is sufficiently small (~$3\tau_{\text{system recovery}}$; this is known theoretical result at $\mu_i >>\lambda_i$), that is why it is acceptable to calculate stationary measures for a systems with long operation life. In this case availability (R_i) and failure frequency (ω_i) will include only first terms, which is not dependant on time, and expected number of failures will be

$$N_{\text{стац. отк}}(t) = \omega_{\text{стац.сист}}t\,. \quad (31)$$

5. CONCLUSION

In this paper we suggest the method of calculation failure frequency measure for system with complex structure and known availability and failure frequency of system's elements (in accordance with expressions (20), (21)). The method is well algorithmizied and together with considered techniques of decomposition and aggregation ((13) - (15), (17)) makes it possible to analyze the system with considerably high dimensions. The method is more effective in comparison with other known methods.

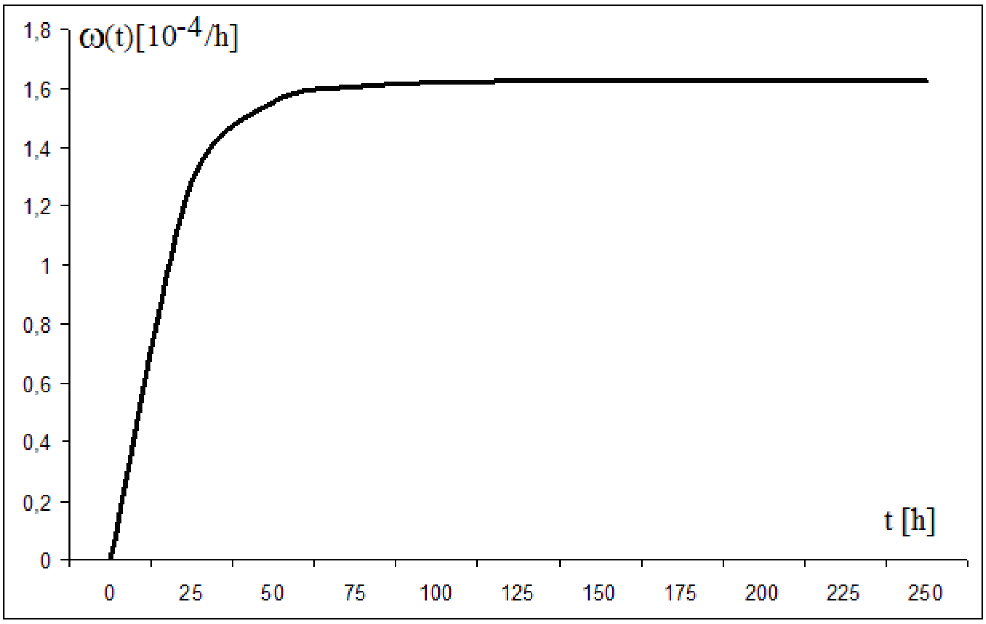

Figure 2. Failure frequency time dependence.

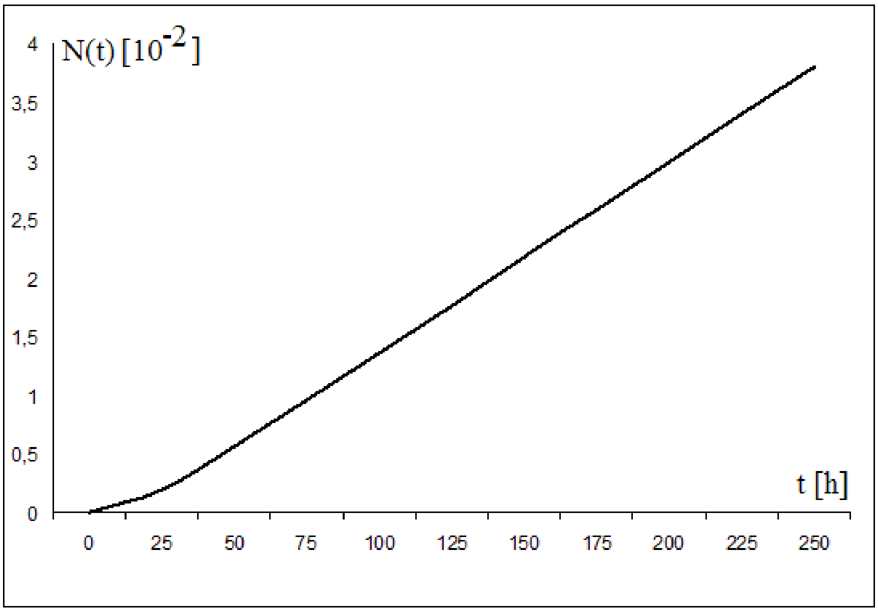

Figure 3. Expected number of failures time dependence.

6. APPENDIX

Proving the expression (22) for $P^{x_i}_{\text{prefailed}}(t)$ calculation.

Let logical function is defined through the cut sets $\{C_k\}$ in the form of (4) (we shall use product · and summation + character instead of conjunction Λ and disjunction V symbols), i.e. unoperability function is $\overline{S(\mathbf{x},t)} = C_1 + C_2 + \ldots + C_l$, and operability function is $\overline{\overline{S(\mathbf{x},t)}} = S(\mathbf{x},t) = \overline{C_1} \cdot \overline{C_2} \cdot \ldots \cdot \overline{C_l}$.

Let us divide cut set $\{C_k\}$ into two subset: $\{C_{ki}\} = \{ C_{1i}, C_{2i}, \ldots, C_{ki} \}$ and $\{C_{kj}\} = \{ C_{1j}, C_{2j}, \ldots, C_{kj} \} = \{C_k\} \setminus \{C_{ki}\}$; $\{C_{ki}\} \cup \{C_{kj}\} = \{C_k\}$, $\{C_{ki}\} \cap \{C_{kj}\} = \varnothing$; $\{C_{ki}\}$ – set of cuts, which include element *i*, $\{C_{kj}\}$ - set of cuts, which do not include element *i*. Then system operability function is $S(\mathbf{x},t)=\overline{C_{1j}}\cdot\overline{C_{2j}}\cdot\ldots\cdot\overline{C_{kj}}\cdot\overline{C_{1i}}\cdot\overline{C_{2i}}\cdot\ldots\cdot\overline{C_{ki}},\quad ki+kj=l$.

It is necessary to find probability of subset of pre failed states when the *ith* element is good. Failure of the *ith* element results in system failure. And this probability must be formed from original system description $S(\mathbf{x},t)$, but not from specially constructed logical function for this subset of pre failed states. Let define logical function of required pre failed states in the form of $y_{\text{prefailed}}(i)=y_1\cdot y_2$. Cuts $\{C_{kj}\}$, in which the *ith* element is not included, must not exist in time point *t* (as these pre failed states are states of system operability), so $y_1=\overline{C_{1j}}\cdot\overline{C_{2j}}\cdot\ldots\cdot\overline{C_{kj}}$. Let consider the cuts, which include the *ith* element. As the *ith* element failure on $(t, t+\Delta t)$ results in system failure, then at least one of events $\{C_{ki}\}$ must exist in time point *t*, provided the *ith* element is map out. So as to exist pre failed state in time *t* with the *ith* good element it is necessary to implement a function with $\{C_{ki}\}$ $x_i = 0$: $y_2=(C_{1i}/\{x_i=0\})+(C_{2i}/\{x_i=0\})+\ldots+(C_{ki}/\{x_i=0\})$.

Thereby

$$y_{\text{prefailed}}(i)=y_1\cdot y_2=\overline{C_{1j}}\cdot\overline{C_{2j}}\cdot\ldots\cdot\overline{C_{kj}}\cdot((C_{1i}/\{x_i=0\})+(C_{2i}/\{x_i=0\})+\ldots+(C_{ki}/\{x_i=0\}))=$$
$$\overline{C_{1j}}\cdot\overline{C_{2j}}\cdot\ldots\cdot\overline{C_{kj}}\cdot\overline{\overline{((C_{1i}/\{x_i=0\})+(C_{2i}/\{x_i=0\})+\ldots+(C_{ki}/\{x_i=0\}))}}=$$
$$\overline{C_{1j}}\cdot\overline{C_{2j}}\cdot\ldots\cdot\overline{C_{kj}}\cdot(1-(\overline{C_{1i}}/\{x_i=0\})\cdot(\overline{C_{2i}}/\{x_i=0\})\cdot\ldots\cdot(\overline{C_{ki}}/\{x_i=0\})).$$

Let define this probability

$$P\{\overline{C_{1j}}\cdot\overline{C_{2j}}\cdot\ldots\cdot\overline{C_{kj}}\cdot(1-(\overline{C_{1i}}/\{x_i=0\})\cdot(\overline{C_{2i}}/\{x_i=0\})\cdot\ldots\cdot(\overline{C_{ki}}/\{x_i=0\}))=1\}=P\{\overline{C_{1j}}\cdot\overline{C_{2j}}\cdot\ldots\cdot\overline{C_{kj}}=1\}-$$
$$P\{\overline{C_{1j}}\cdot\overline{C_{2j}}\cdot\ldots\cdot\overline{C_{kj}}\cdot(\overline{C_{1i}}/\{x_i=0\})\cdot(\overline{C_{2i}}/\{x_i=0\})\cdot\ldots\cdot(\overline{C_{ki}}/\{x_i=0\})=1\}.$$

Note, that

1. $P\{\overline{C_{1j}}\cdot\overline{C_{2j}}\cdot\ldots\cdot\overline{C_{kj}}=1\}=P\{(S(\mathbf{x},t)/\{x_i=1\})=1\}$ - system availability at $x_i = 1$, because $(S(\mathbf{x},t)/x_i=1)=(\overline{C_{1j}}\cdot\overline{C_{2j}}\cdot\ldots\cdot\overline{C_{kj}}\cdot(\overline{C_{1i}}\cdot\overline{C_{2i}}\cdot\ldots\cdot\overline{C_{ki}})/\{x_i=1\})=\overline{C_{1j}}\cdot\overline{C_{2j}}\cdot\ldots\cdot\overline{C_{kj}}$. Element x_i is a part of all $\{C_{ki}\}$ and his operability provides $((\overline{C_{1i}}\cdot\overline{C_{2i}}\cdot\ldots\cdot\overline{C_{ki}})/\{x_i=1\})=1$ regardless of state of other elements.
2. $P\{(\overline{C_{1j}}\cdot\overline{C_{2j}}\cdot\ldots\cdot\overline{C_{kj}})\cdot((\overline{C_{1i}}/\{x_i=0\})\cdot(\overline{C_{2i}}/\{x_i=0\})\cdot\ldots\cdot(\overline{C_{ki}}/\{x_i=0\}))=1\}=$ $P\{(S(\mathbf{x},t)/\{x_i=0\})=1\}$ - system availability at $x_i = 0$, which are obvious.

Thus, $P^{x_i}_{\text{предотк}}(t)=R_{\text{сист}}(t)\big/\{x_i=1\}-R_{\text{сист}}(t)\big/\{x_i=0\}$.

REFERENCES

1. Akulova L.G. 1983. *About stochastic complexity of reliability calculation of Boolean systems.* Iaroslavl. ISU press. (in Russian).
2. Cherniak A.A. 1991. Combinatorial graph method of reliability analysis of complex systems with monotone Boolean functions. *Automatics and Remote Control.* №4. pp. 165 – 174. (in Russian).
3. Cherniak A.A., Cherniak J.A. 1998. Logical probabilistic method of analysis of high dimensional multivalued systems. *Automatics and Remote Control.* №1. pp. 171 – 180. (in Russian).
4. Filin B.P. 1982. About analytical techniques of approximate calculation of complex system reliability. *Automatics and Remote Control.* №11. C.159 – 170. (in Russian).
5. Filin B.P. 1989. *Techniques of analysis structural reliability of communication network.* M. Radio and Connection press. (in Russian).
6. Filin B.P. 1989. About duality principle in problems of analysis of complex systems structural reliability. *Automatics and Remote Control.* №6. pp. 158 – 172. (in Russian).
7. Henley E.G., Kumamoto H. 1984. *Reliability Engineering and Risk Assessment*. M. Industry press.
8. Mojaev A.S. 1988. *General logical-probabilistic method of complex system reliability analysis.* Leningrad. "VMA" Tutorial. (in Russian).
9. Mojaev A.S., Alekseev A.O. 1994. Automated structural-logical modeling and probabilistic analysis of complex system. *Theory and informational technology of complex systems safety modeling.* St. Petersburg. Issue 2. (in Russian).
10. *Ryabinin I.A. 1971. Basis of theory and calculation of power systems. Leningrad. Shipbuilding press. (in Russian)*
11. *Ryabinin I.A., Parfenov U.M. 1978. Reliability measures estimation. Proceedings of USSR Academy of Sciences. Energetics and Transport. №6. pp. 22 – 32. (in Russian).*

PHASE TRANSITION IN RENEWAL SYSTEMS WITH COMMON RESERVE

Tsitsiashvili G. Sh.
●
Institute for Applied Mathematics, Far Eastern Branch of RAS
690041, Vladivostok, Radio str. 7,
guram@iam.dvo.ru

Introduction

Mathematical models of renewal systems with a common reserve have been introduced and analyzed detailed in the monograph [1]. In [2] a phenomenon of a phase transition in the aggregated renewal system with the unload reserve is analyzed as analytically so numerically. But the mathematical method applied in this paper is too specific to analyze the phase transition phenomenon in general renewal systems with the common reserve. This phenomenon is connected with a reform of municipal engineering systems.

In this paper a method based on a definition of a state in which a birth and death process describing this system has a maximal limit probability is suggested. This method allows to construct convenient upper bounds of the limit probability for other states and to analyze phase transition phenomenon. The obtained bounds depend on transition intensities of the birth and death processes which describe aggregations of renewal systems with unload, under load and load reserves. The suggested method allows an analyzing of a renewal system with a competition between the repair places also.

Preliminaries

At first consider a renewal system with an unload reserve. This system has a single work place with failure intensity a, a single repair place with repair intensity b, $c=b/a$ and two elements. Suppose that work and repair times are independent random variables and have exponential distributions with the parameters a,b accordingly. Take n independent copies of this system and aggregate them so that we have a renewal system with an unload reserve and with n work places, n repair places and $2n$ elements. Our initial problem is to analyze a limit by $n\to\infty$ of the stationary probability P_n that there are elements in all n work places. This probability may be interpreted as a probability that all n aggregated renewal subsystems work. It was shown in [2] that the formulas

$$\lim_{n\to\infty} P_n = 1,\ c>1,\ \lim_{n\to\infty} P_n = 0,\ c<1, \tag{1}$$

describing the phase transition in the aggregated system are true.

Consider now a renewal system with an unload reserve which has p work places, q repair places and r elements, $p\le r, q\le r$. Aggregate n independent copies of this system and obtain a renewal system with an unload reserve and with np work places, nq repair places and nr elements. Our problem is to analyze the limit by $n\to\infty$ of the stationary probability Π_n that there are elements in all np work places of the aggregated system. Remark that for a family of independent renewal systems this probability equals $Q^n \to 0$, $n \to \infty$, where $Q < 1$ is the stationary probability that the renewal system with p work places, q repair places and r elements work.

Introduce the birth and death process $x_n(t)$ which characterizes a number of elements in a work phase of the aggregated system that is in the work places and in a queue to these places. This process has the states set $U_n = \{0,...,r\}$ and the birth and death intensities

$$\lambda_n(k) = b\min(nr-k, nq), \ \mu_n(k) = a\min(k, np), \ c_n(k) = \frac{\lambda_n(k-1)}{\mu_n(k)}. \tag{2}$$

The discrete Markov process $x_n(t)$ with the finite states set U_n is ergodic and its limit distribution $\pi_n(k) = \lim_{t\to\infty} P(x_n(t) = k)$ satisfies the equalities

$$\pi_n(k-1) = \frac{\pi_n(k)}{c_n(k)}, \quad k \in U_n, \quad k > 0, \tag{3}$$

and the limit probability, that all or practically all subsystems in the aggregated system work, satisfy the equalities

$$\Pi_n = \lim_{t\to\infty} P(x_n(t) \ge np) = \sum_{k \ge np} \pi_n(k),$$
$$\Pi_{n,\varepsilon} = \lim_{t\to\infty} P(x_n(t) \ge n(p-2\varepsilon)) = \sum_{k \ge n(p-2\varepsilon)} \pi_n(k).$$

Auxiliary Statements

Introduce the monotonically no increasing and continuous functions

$$C_n(v) = \frac{c\min\left(r - v + \frac{1}{n}, q\right)}{\min(v, p)}, \ \frac{1}{n} \le v \le r; \ C_n(v) = \frac{c\min(r-v, q)}{\min(v, p)}, \ 0 \le v \le r, \tag{4}$$

with $C_n(1/n) = C(0) = +\infty$. Remark that the equality

$$C_n(k/n) = c_n(k), \ 0 \le k \le r, \tag{5}$$

and the inequalities

$$C(v) \le C_n(v) \le C\left(v - \frac{1}{n}\right), \ \frac{1}{n} \le v \le r, \tag{6}$$

are true. Define the sets

$$V_n = \left[\frac{1}{n}, r-q\right] \cap \left[p + \frac{1}{n}, r\right], \ V = [0, r-q] \cap [p, r]$$

and note that these sets contain more than a single point if and only if $r - q > p$.

Lemma 1. *1)Suppose that $cq \neq p$ or $r-q \leq p$ then there are the real numbers w_n, w which are the single roots of the equations*

$$C_n(v)=1 \ , \ \frac{1}{n} \leq v \leq r \ , \tag{7}$$

$$C_n(v)=1 \, , \ 0 \leq v \leq r \ , \tag{8}$$

respectively and satisfy the inequalities

$$w \leq w_n \leq w + \frac{1}{n} \, . \tag{9}$$

There is the integer k_n which satisfies the formulas

$$c_n(k) > 1, \ k < k_n \, ; \ c_n(k_n) \geq 1; \ c_n(k) < 1, \ k > k_n \, , \tag{10}$$

$$nw - 1 \leq k_n \leq nw + 1 \tag{11}$$

2) If the conditions $cq = p$, $r-q > p$ are true then the equations (7), (8) have the roots sets V_n, V respectively and for $k_n^- = np+1$, $k_n^+ = n(r-q)$ we have

$$c_n(k) > 1, \ k < k_n^- \, ; \ c_n(k) = 1, \ k_n^- \leq k \leq k_n^+ \, ; c_n(k) < 1, \ k > k_n^+ \, . \tag{12}$$

Proof. The functions $C_n(v), C(v)$ are monotonically no increasing. If the condition $r-q \leq p$ takes place then the sets V_n, V contain no more than a single point. So the functions $C_n(v), C(v)$ are strictly monotonically decreasing. If $cp \neq q, r-q > p$ then the sets V_n, V contain more than a single point but on these sets the functions $C_n(v) = C(v) = cp/q \neq 1$. Consequently in the condition of the statement 1) there are single roots w_n, w of the equations (7), (8) respectively.

The inequality (9) is a corollary of the formula (6). Define k_n as a minimal integer which is not larger than w_n .Then from the formula (5) we obtain (10) and from the formulas (10) and from the formulas (9), (10) - the inequalities (11).

If the condition of the statement 2) is true then the sets V_n, V are segments which contain more than a single point and $C_n(v) = C(v) = cp/q = 1$ on these sets. Consequently the segments V_n, V are the sets of the equations (7), (8) roots respectively. The formula (12) is a corollary of the formula (5).

Lemma 2. *1)Suppose that $cp \neq q$ or $r-q \leq p$ then there is a positive number ε_1 so that for any ε, $0 < \varepsilon < \varepsilon_1$, we have*

$$A_n = \sum_{0 \leq k \leq n(w-2\varepsilon)-1} \pi_n(k) \to 0 \, , \ n \to \infty \, . \tag{13}$$

$$B_n = \sum_{n(w+2\varepsilon) \leq k \leq nr} \pi_n(k) \to 0 \ , \ n \to \infty \, . \tag{14}$$

2) If the condition $cp = q,\ \ r - q > p$ *are true then there is a positive number* ε_1 *so that for any* $\varepsilon,\ \ 0 < \varepsilon < \varepsilon_1$, *the following formula is true*

$$C_n = \sum_{0 \le k \le n(p-2\varepsilon)-1} \pi_n(k) \to 0\ ,\ n \to \infty . \tag{15}$$

Proof. 1) Denote $S_n = \pi_n(k_n) < 1$, from the lemma 1 we have that the limit distribution of the process $x_n(t)$ satisfies the equalities

$$\pi_n(k) = S_n \prod_{k < i \le k_n} C_n^{-1}\left(\frac{i}{n}\right),\ k < k_n ; \tag{16}$$

$$\pi_n(k) = S_n \prod_{k_n < i \le k} C_n\left(\frac{i}{n}\right),\ k > k_n . \tag{17}$$

As $w \notin V$ so there is $\varepsilon_1 > 0$ so that for any $\varepsilon,\ \ 0 < \varepsilon < \varepsilon_1$, the following inequalities

$$\psi^{-1} = C(w-\varepsilon) > C(w) = 1 > C(w+\varepsilon) = \varphi \tag{18}$$

are true. Then for $k < n(w-2\varepsilon) < k_n$ from the formulas (4)-(6), (16), (18) we obtain

$$\pi_n(k) \le S_n \prod_{k < i \le n(w-\varepsilon)} C^{-1}\left(\frac{i}{n}\right) \le \prod_{k < i \le n(w-\varepsilon)} C^{-1}(w-\varepsilon)$$

and consequently $\pi_n(k) \le \psi^{n(w-\varepsilon)-k-1}$, $0 \le k \le n(w-\varepsilon)-1$. Summarize the last inequality by $k, 0 \le k \le n(w-2\varepsilon)-1$, and obtain the formula

$$A_n \le \frac{\psi^{n\varepsilon}}{1-\psi} \to 0,\ n \to \infty$$

so the formula (13) is proved.

For $k > n(w+\varepsilon)+1 > k_n$ from the formulas (4)-(6), (17), (18) we obtain

$$\pi_n(k) \le S_n \prod_{n(w+\varepsilon)+1 \le i \le k} C\left(\frac{i-1}{n}\right) \le \prod_{n(w+\varepsilon) \le i \le k-1} C(w+\varepsilon)$$

and consequently $\pi_n(k) \le \varphi^{k-n(w+\varepsilon)-1}$, $n(1+\varepsilon) \le k \le nr$. Summarize the last inequality by k, $n(1+2\varepsilon) \le k \le nr$, and obtain the formula

$$B_n \le \frac{\varphi^{n\varepsilon-2}}{1-\varphi} \to 0,\ n \to \infty$$

so the formula (14) is proved.

2) Denote $R_n = \pi_n\left(k_n^-\right) < 1$, from the lemma 1 we have that the limit distribution of the process $x_n(t)$ satisfies the equalities

$$\pi_n(k) = R_n \prod_{k<i\le k_n^-} C_n^-\left(\frac{i}{n}\right),\ k<k_n^- .$$

As $V=[p,r-q]$ so for any $\varepsilon,\ 0<\varepsilon<p$, we have

$$\delta^{-1} = C(p-\varepsilon) > C(p) = 1.$$

Then for $k<n(p-2\varepsilon)<k_n^-$ the formulas (4)-(6) we obtain

$$\pi_n(k) \le S_n \prod_{k<i\le n(w-\varepsilon)} C^{-1}\left(\frac{i}{n}\right) \le \prod_{k<i\le n(p-\varepsilon)} C^{-1}(p-\varepsilon).$$

And so $\pi_n(k)\le \delta^{n(p-\varepsilon)-k-1}$, $0\le k\le n(p-\varepsilon)-1$. Summarize the last inequality by k, $0\le k\le n(p-2\varepsilon)-1$, and obtain the formula

$$C_n \le \frac{\delta^{n\varepsilon}}{1-\delta}\to 0,\ n\to\infty.$$

So the formula (15) is proved.

Main results

For the aggregation of n renewal systems with the unload reserve and with np work places, nq repair places and nr elements the following statement takes place.

Theorem 1. *1) Suppose that* $cp\ne q$ or $r-q\le p$ *then we have*

$$\lim_{n\to\infty}\Pi_n = 1,\ w>p;\ \lim_{n\to\infty}\Pi_n = 0,\ w<p. \tag{19}$$

2) If the conditions $cp=q$, $r-q>p$ are *true then there is a positive number* ε_1 *so that for any* ε, $0<\varepsilon<\varepsilon_1$ *the following formula is true*

$$\Pi_{n,\varepsilon} = 1 - C_n \to 1,\ n\to\infty. \tag{20}$$

Proof. 1) Suppose that $w>p$ and choose ε from the conditions $0<\varepsilon<\varepsilon_1$, $w-2\varepsilon>p$ then we have

$$1-\Pi_n = \sum_{0\le k<np}\pi_n(k) \le \sum_{0\le k\le n(w-2\varepsilon)-1}\pi_n(k) = A_n \to 0,\ n\to\infty.$$

So the first formula from (19) is true.

Suppose that $w<p$ and choose ε from the conditions $0<\varepsilon<\varepsilon_1$, $w+2\varepsilon<p$ then we obtain

$$\Pi_n = \sum_{k\ge np}\pi_n(k) \le \sum_{n(w+2\varepsilon)\le k\le nr}\pi_n(k) = B_n \to 0,\ n\to\infty.$$

So the second formula from (19) is true also.

2) The formula (20) is a direct corollary of the formulas (15).

Consider an aggregation of n renewal systems with a load reserve which have np work places, nq repair places and nr elements. In the renewal system with the load reserve elements which are in a queue to the work places fail with the same intensity as working elements. The birth and death process $x_n(t)$ which characterizes a number of elements in the work phase is ergodic. It has the states set U_n and its birth and death intensities satisfy the equalities $\lambda_n(k) = b\min(nr-k, nq)$, $\mu_n(k) = ak$ instead of (2). For the aggregation of n renewal systems with the load reserve the following statement is true.

Theorem 2. *The formulas (19) are true with* w *which is the single root of the equation* $\min(r-v,q) = cv$ *by* v.

Proof. The theorem 2 proof practically repeats the proof of the section 1) from the theorem 1. A main difference is in a replacement of (4) by the equalities

$$C_n(v) = \frac{\min(r-v,q)}{c\left(v-\frac{1}{n}\right)},\ \frac{1}{n} \le v \le r;\ C_n(v) = \frac{\min(r-v,q)}{cv},\ 0 \le v \le r, \tag{21}$$

and in a fact that the equation $C(v) = 1$ has only the single root.

Consider now an aggregation of n renewal systems with an under load reserve. It has np work places, nq repair places and nr elements. In this system elements in a queue to work places may fail with the positive intensity $d < a$, $c' = d/b$. The birth and death process $x_n(t)$ which characterizes a number of elements in the work phase is ergodic. It has the states set U_n and its birth and death intensities satisfy the equalities $\lambda_n(k) = b\min(nr-k, nq)$, $\mu_n(k) = dk + (a-d)\min(k, np)$ instead of (2). For the aggregation of n renewal systems with the under load reserve the following statement is true.

Theorem 3. *The formulas (19) are true with* w *which is the single root of the equation* $c'v + (1-c')\min(v,p) = c\min(r-v,q)$ *by* v.

Proof. The theorem 3 proof practically repeats the theorem 2 proof. A main difference is in a replacement of (21) by the equalities

$$C_n(v) = \frac{c\min(r-v,q)}{c'\left(v-\frac{1}{n}\right) + (1-c')\min\left(v-\frac{1}{n}, p\right)},\ \frac{1}{n} \le v \le r,$$

$$C_n(v) = \frac{c\min(r-v,q)}{c'v + (1-c')\min(v,p)},\ 0 \le v \le r.$$

Remark 1. *As in the theorems 1- 3 conditions the inequality* $w > p$ *(the inequality* $w < p$*) is equivalent to the inequality* $C(p) > 1$ *(to the inequality* $C(p) > 1$*) so the formula (19) may be rewritten as follows*

$$\lim_{n\to\infty} \Pi_n = 1,\ c\min(r-p,q) > p;\ \lim_{n\to\infty} \Pi_n = 0,\ c\min(r-p,q) < p. \tag{22}$$

Consider the same aggregated systems but with a competition between the repair places. An element arrives in the repair phase and receives information about times of its repair at all places. Then the place with a minimal time of a repair is selected. During this time all other repair places do not work.

These systems are described by similar birth and death processes with the following redefinition of the birth intensity $\lambda_n(k) = nbq\min(nr-k,1)$.

Theorem 4. *For the aggregated renewal systems with the common unload (load or under load) reserve the following formulas are true*

$$\lim_{n\to\infty}\Pi_n = 1,\ cq > p;\ \lim_{n\to\infty}\Pi_n = 0,\ cq < p. \qquad (23)$$

Proof. Consider the aggregated renewal system with the common reserve. The theorem 4 proof practically repeats the theorem 1 proof. A main difference is in a replacement of (4) by the following equalities: for the unload reserve

$$C_n(v) = C(v) = \frac{cq}{\min(v,p)}$$

and for the load reserve

$$C_n(v) = C(v) = \frac{cq}{v}$$

and for the under load reserve

$$C_n(v) = C(v) = \frac{cq}{c'v + (1-c')\min(v,p)}.$$

Remark 2. *A comparison of the theorems 1-3 with the theorem 4 allows to derive that the common reserve with the competition of the repair places gives better results in terms of the work probability* Π_n.

REFERENCES

1. Gnedenko, B.V., Beliaev, Yu.K. and Soloviev, A.D. Mathematical methods in reliability theory. Science, Moscow. 1965. (In Russian).
2. Tsitsiashvili, G.Sh. and Markova, N.V. Cooperative effects in renewal systems with common reserve and competition of repair places. Communication of Dependability and Quality Management. 2005. no 3, 71—75.

THE AGEING OF SIGNALLING EQUIPMENT AND THE IMPACT ON MAINTENANCE STRATEGIES

Mr. Marc Antoni MEng,
●
FIRSE, Infrastructure SNCF, Paris, France
e-mail:

SUMMARY

Research projects of SNCF (French railway) aim at reducing the costs of infrastructure possessions and improving the operational equipment availability and safety. This permanent search for a better regularity led the SNCF to analyse the maintenance approach of signalling equipment in detail. Until now, it was commonly acknowledged that signalling equipment, which consists of many electronic devices, is not subject to ageing. In this study, a Weibull lifetime model, able to describe an ageing phenomenon, is used and it can be shown that the deterioration is statistically significant. The validity of the model is tested. We also analyse the influence of environmental covariates. We simulate different scenarios in order to investigate the impact of several maintenance strategies as well as on future maintenance costs, on the amount of components to replace based on the mean age of the network. It can be shown that in most cases a systematic replacement strategy offers the best solution.

1. INTRODUCTION

The purpose of this study is to estimate the lifetime distribution of signalling equipment. We investigate if there is an ageing phenomenon. If signalling equipment deteriorates, the current maintenance strategy, based on curative replacements, is perhaps not optimal. We developed statistical models to estimate lifetime and maintenance costs. These methods are applied to the maintenance of signalling equipment especially to electronic units (electronic fail-safe devices). Even though the mathematical models are basically used for an economic optimisation of the infrastructure maintenance, we specify for every chosen maintenance strategy the expected number of defects and failures.

In 2006, RFF (the French infrastructure owner) and SNCF analysed the consequences of a significant effort of renewal of the French network without being able to agree on its possible consequences in terms of decreasing maintenance costs and of the ageing of the network.

The present work is based on methods accepted by both partners that were developed in order to answer, at least qualitatively, the following questions:

- Is it possible to find an economic optimum between maintenance and renewal for the current network?
- What would be the consequences on the quality of the French railway network if the current expenses remain unaffected or if they change significantly?

Generally, the former studies of the SNCF are based on estimates of the national average costs of maintenance with the implicit assumption that the level of renewal avoids any ageing of the network. They are simple to use but they estimate imperfectly the variation of maintenance costs due to the network ageing.

The development of the economic model described in this article is based on the failure models. Unit costs for every replacement and every renewal are then associated with the failure model. Sometimes it is possible to repair components. A repaired component is less expensive than a new component but it does not have the same failure distribution.

We construct the failure model as follows: we are first interested in the lifetime distribution for an unreplaced component, we analyse the impact of environmental variables on the lifespan (ageing), and we then calculate the expected number of components to be replaced. We then describe the construction of the cost model and finally we describe the different scenarios that were created in order to analyse the impact of several maintenance strategies on the network development.

2. NOTATION

In this article, we will use the following definitions:

The term maintenance is used to indicate the replacement of a component by another component, only when it is necessary.

The term renewal is used for a massive replacement making it possible to concentrate the effort and to limit the encumbrance on the network due to work. The replacements costs are lower, but the components could still have lasted several years.

3. DATA

A database containing information of all signalling equipment on the French network is used for the analysis. This database contains the state of every component including the date of installation, of repair, of replacement, of storing and of re-employment.

4. METHODS

4.1. Lifetime Estimation

We define the survival model for unreplaced components by fixing the failure rate $\lambda(t)$. The failure rate represents the probability of observing an instantaneous failure given that the component has not failed before time t. If the components are subjected neither to wear nor to ageing, it is possible to consider a distribution with a constant failure rate. In that case the exponential distribution is obtained. The failure of the signalling equipment treated in this study is usually modelled using an exponential model. One of the aims of this analysis was to show that even equipment with electric components can deteriorate.

For mechanical or electric components, if they are a part of a large system, the standard distribution commonly used is a distribution with a power failure rate: it represents reality very well. One then obtains a class of failure distribution: the Weibull distributions. They can be described by:

- failure rate $\lambda(t) = 1/\eta\beta \cdot t\beta\text{-}1$
- density function $f(t) = \beta/\eta\beta \cdot t\beta\text{-}1 \cdot \exp(-(t/\eta)\ \beta)$
- cumulated density function $F(t) = 1 - \exp(-(t/\eta)\ \beta)$

where $\beta > 0$, $\eta > 0$ and $t > 0$. The Weibull distribution depends on two parameters (there is also a version of the

Weibull distribution depending on three parameters but this distribution is not treated here):

- The shape parameter β which determines the distribution of the failures on the time axis:
  - If it is small, the failures are distributed over a large time span,
  - If it is high, the failures appear within a short time interval,

– The scale parameter η defines the time scale.

The obtained functions (represented with respect to the reduced variable tr = t/η are presented in Figure 1 and

Figure 2. In the case of signalling components the value of the parameter β should be between 1 and 3.

For the signalling components that are still in use today, we obtain censored data. We cannot observe the date of replacement for these components. The method used for the analysis has therefore to be adapted to censored data. The treatment of censored data in parametric probability models is described in Klein & Moeschberger [4].

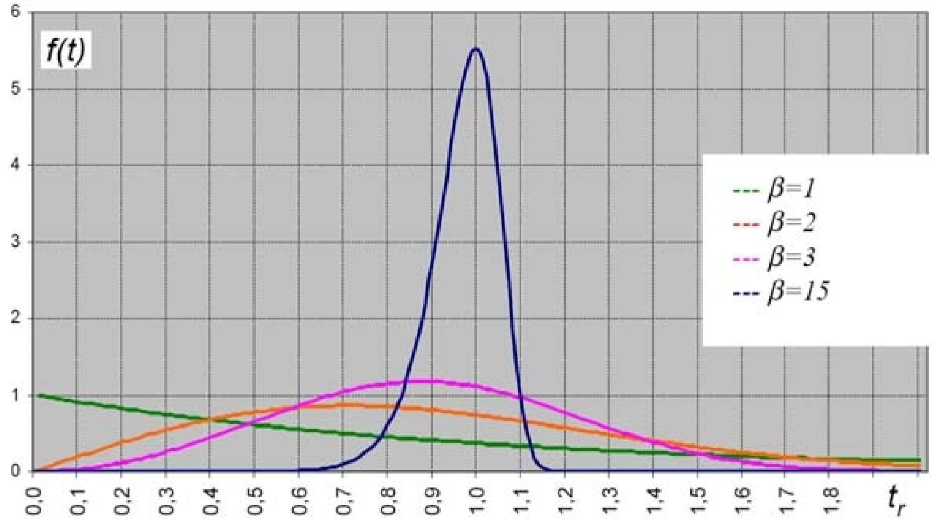

Figure 1: Density function for different Weibull distributions.

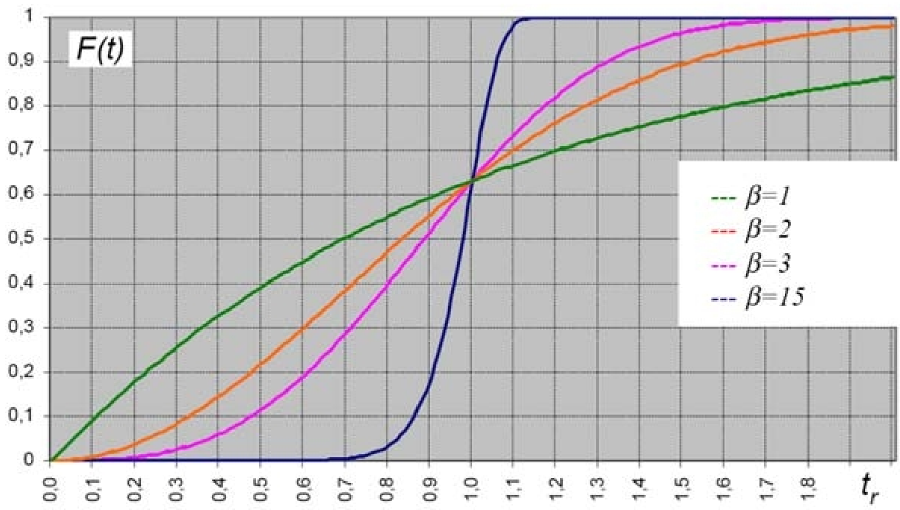

Figure 2 : Cumulated density function for different Weibull distributions.

We use the Kolmogorov-Smirnov test in order to verify if the Weibull distribution is adapted for the lifetime modelling of the specific electronic unit. In most cases, the ageing phenomenon is evident. In some cases we find ageing phenomena that seem more complex than the one modelled by the Weibull distribution. As explained in more detail in section 7.1, at the moment we are testing other distributions that remain however close to the Weibull distribution.

4.2. Replacement model

In practice, it is necessary to take into account the effect of successive replacement for the components in service. A failed component is replaced by a new one which, in turn, will be subject to wear or ageing. The renewal density $h(t)$, that gives the replacement due to a failure at time t, can be written as a sum of n- convolutions:

$$h(t) = \sum_{n=1}^{\infty} -[(1 - F(t))']^{*n} , \tag{1}$$

where * denotes the convolutions.

There is no simple analytical expression for this function, even when considering the particular case of a Weibull distribution. The term can however be calculated numerically. Figure 3 shows the influence of the parameters β and η on the amplitude of the oscillation. The system converges more or less fast towards an asymptotic value.

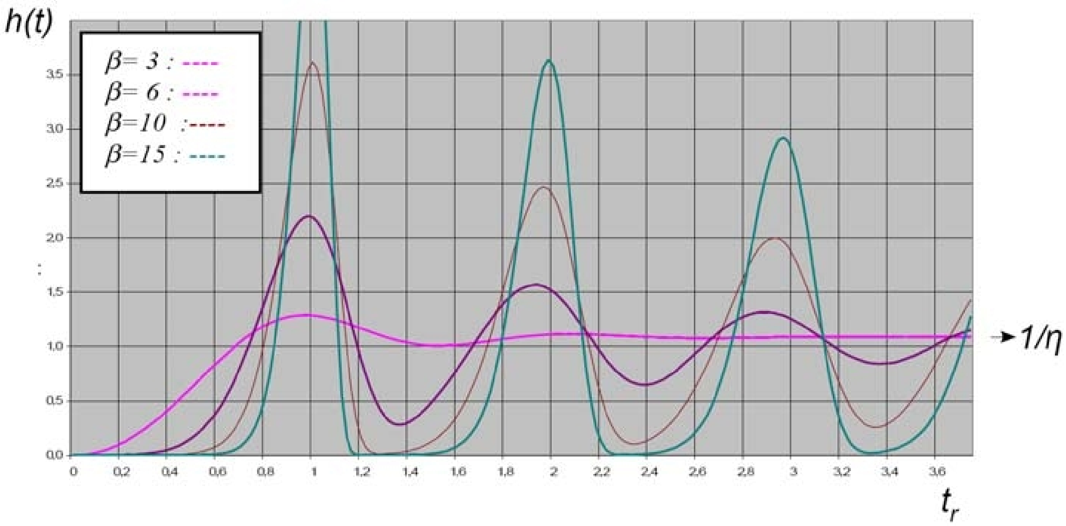

Figure 3 : The function h(t), the renewal density.

It can be seen that for a small β (this is the case for signalling units) the failures are spread over the time, and that the overlap of the successive replacements, leads to a fast attenuation of the replacement peaks. From the second cycle of replacement on, the number of failures is close to the limit value. For high β, the failures are concentrated on a time interval, the overlap between successive replacements is small and the attenuation of the replacement peaks is very slow.

We have to bear in mind that at this phase we consider implicitly that components are replaceable infinitely without reduction of their life expectancy. This hypothesis could be accepted in the case of components interacting few with other components. But it is also possible that the system, which is after many replacements very different from the initial one, influences the lifetime of the components. Every component, new or second- hand, will be stressed under conditions that are not the original ones and the lifetime will be progressively reduced in the course of time. In this article we do not treat the case of interacting components but we mention in the section 7.2 how we want to take into account this phenomenon.

4.3. Covariates

There are several covariates influencing the lifetime of the components. They belong either directly to the component or to its environment. For example, if the electronic unit is installed inside an equipment centre, it has a different lifetime distribution compared to a component next to the track without protection. The used covariates included in the study are: maximum speed on the railway track, climate, localisation but also the type of the electronic unit among all components fulfilling the same functionality and the storing time before the installation. We use two ways to treat the variables in the model: either we include them into an accelerated failure model based on a Weibull distribution or we use them as segmentation variables and estimate the lifetime for every segment separately.

For every component the most influencing covariate is determined. We find that the storing time is one of the most important covariates.

4.4. Cost model: new component

We want to optimize the global maintenance costs, including renewals and heavy maintenance (for example removal for reparation in a workshop). The real maintenance cost model is of course more complicated as compared to the following model as there are implementation constraints. However, The presented cost model considers the main expenditures.

The failure model defined above allows us to obtain in a simple way the cost model for the concerned component. The annual maintenance expense are, on the one hand, due to expenses related to failures resulting in the replacement of components (product of the unit expenses cu for every replacement and the number of replacement to be carried out) and, on the other hand, of expenses ci not directly related to the replacement of components (surveillance operations and common maintenance). In this way, the maintenance expense per year for a new installation are given by $Y(t) = c_i(t) + c_u \cdot n \cdot h(t)$ where n is the number of components of the installation and $h(t)$ is the renewal density at time t. The expected cumulated maintenance expenses at time T are then:

$$E(T) = \sum_{t=0}^{T-1} Y(t) . \quad (2)$$

The renewal expenses are chosen as constant: X = constant. The value of X is based on real costs stated over the last years. The maintenance expenses including renewal are defined as the sum of the maintenance expenses and renewal expenses over a period T. In order to calculate the expected costs during the lifetime of an installation, we define the expected annual maintenance costs that include periodical renewal by the expression:

$$C(T)/T = [X + E(T)]/T , \quad (3)$$

where X are the renewal expenses based on a complete renewal of the components. The minimum of these expected costs defines the optimal renewal period T_0. This period is a function of the parameters of the failure model (β , η) and of the ratio between the regeneration expenses by means of a systematic renewal X and by means of maintenance Y. For a component having a high parameter β the optimal value for the renewal period T_0 is not very sensitive with respect to the X/Y ratio. On the opposite, for low β as in our case, the optimal regeneration period exists only if the X/Y ratio is also low. The optimum period T_0 gives the date from which continuing the common maintenance is no longer the best economic solution.

For the calculation of the maintenance costs Y(t) it is important to consider all maintenance actions (repair, replacement, and re-employment). There are also costs of inspections and common maintenance. This study tries to compare the current maintenance strategy that is based on curative maintenance to a new one that includes systematic renewal. It has to be mentioned that even if we use a systematic replacement strategy there are still components that can fail between the renewal cycles. These costs have to be considered. As signalling equipment is essential for passenger safety we always analyse the number of expected failures in addition to the generated costs. The inspection intervals have to be adjusted in order to optimise the detection of failures for redundant systems and the detection of deviations from the regular mode for non redundant systems.

4.4.1. Unavailability costs

The failure of a component and its replacement often result in a limitation or a cessation of the traffic. This temporary closing of a line has consequences in terms of regularity. There can also be a financial impact (refund of the passengers). This is also true for renewal works. These "expenses of unavailability" are a function of the importance of the installation, of the robustness of the system functioning in disturbed situations, of the aptitude of the installation to manage disturbed situations. These costs are however difficult to evaluate as there are several possibilities to obtain them. Thus, the current model does not take this element into account.

4.5. Simulation: existing network

For the electronic signalling components there is currently no systematic renewal. The components are individually replaced after failure. In this case, it is not possible to neglect successive replacements of the same component, and the function h(t) (cf. equation (1)) will be used.

In order to quantify the impact of ageing on the maintenance strategies we used three scenarios:

- replacement after failure (corrective maintenance),
- replacement of a fixed number of components (fixed budget maintenance). Failed components are replaced and 10 % of the installed components are replaced preventively every year. Old components or repaired components are replaced first,
- replacement depending on the age of the component (time-conditioned maintenance). Failed components are replaced and at a given interval all installed components are replaced systematically and preventively.

The data mentioned above enables us to build a new database containing the equipment currently in use. The Weibull distribution gives us an estimation of the number of components likely to fail. Like for the cost calculations for new components (cf. section 4.4), we evaluate every maintenance action (repair, replacement, and re-employment) economically. It is then possible to simulate the impact of the ageing on the three proposed maintenance strategies. We calculate the expected number of failures, the expected number of components to replace and the development of the maintenance costs over time. It is then possible to find the optimal inspection interval.

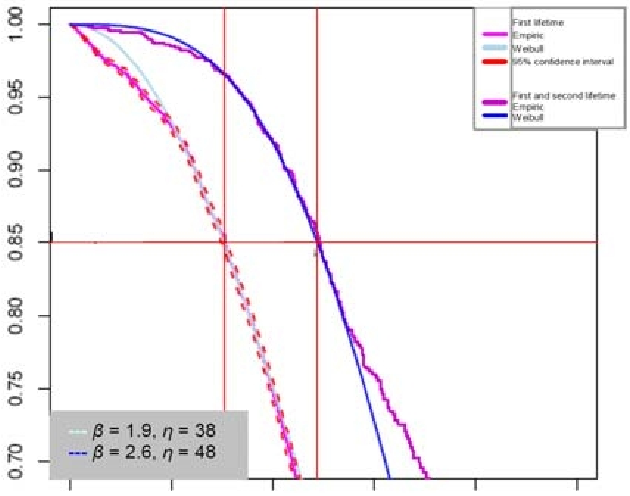

Figure 4 : Reliability estimation for signalling equipment. There are two lifetimes: the first one before reparation, the second one ignores the reparation.

5. RESULTS

Figure 4 shows the results of a lifetime estimation for track circuits transmitters. This type of unit can be repaired and there are therefore two reliability curves. The dark blue one gives the sum of the first and the second lifetime.

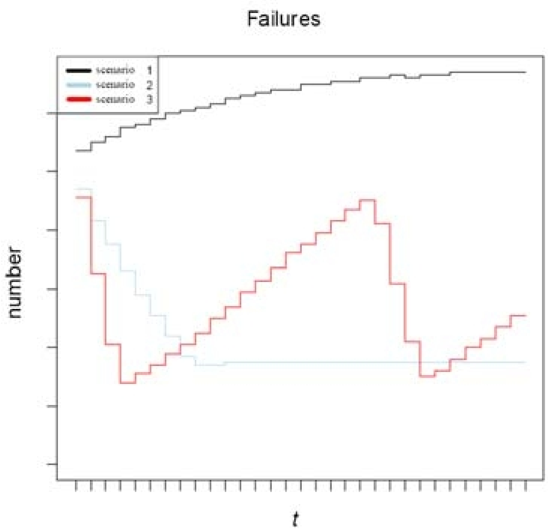

Figure 5 : Expected failures for the three scenarios.

The results of the simulations can be seen in Figure 5 and in Figure 6. As mentioned above we also consider the number of failures. From Figure 5 it can be seen that a change of the maintenance strategy does not increase the number of failures. Figure 6 gives the corresponding costs. The expenses are discounted with a rate of 4 %. It can be seen that the red scenario is more expensive at the beginning, but after a certain time, the cumulated expenses are lower than for the other strategies. It appears that a renewal strategy minimizes the global lifecycle costs.

6. CONCLUSION

The result enables SNCF engineers to adapt the current guidelines and to predict future maintenance expenditures.

6.1. Developed method

The approach is based on methods usually used in reliability. It allows a good modelling of the intuitively perceived phenomena: the more the equipment is regenerated, the lower the regular maintenance costs and conversely, regular maintenance alone does not allow a piece of equipment to last indefinitely as it ages, and finally there is often an economic optimum for the systematic renewal. The calibration of the model is based on accessible real data.

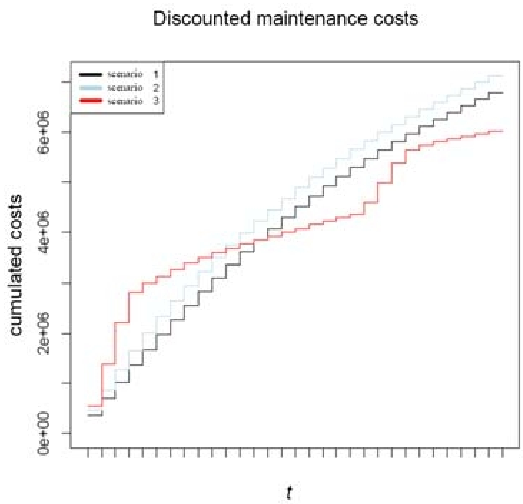

Figure 6 : Expected cumulated costs for the three scenarios.

The method is very general. The application range is very wide. All replaceable infrastructure equipments can be used for such a study. The proposed method is already used for track and overhead line components [1].

6.2. Extension to other questions

This approach can be applied to particular sub populations: high speed lines, lines with dedicated traffic. It is also possible to use this type of model on a particular line in order to predict failure trends. This knowledge allows the Maintenance Engineering Department of the SNCF to confirm in an objective way its new maintenance strategies.

7. PERSPECTIVES

7.1. Extension of the Weibull distribution

Some of the signalling components seem to follow a more complicated probability distribution. The Weibull distribution is able to model the overall behaviour but it seems that the characteristics of components at the beginning and at the end of their lifetime, change. This could be due to the different parts of the component

(electrical parts, chemical parts, mechanical parts). At the moment we are working on a more detailed modelling of the lifetime and parameter estimation. As this concerns only one sub-model of the proposed method, we can keep the cost model and the different scenarios. We only have to change the input for the calculation of the h(t) function.

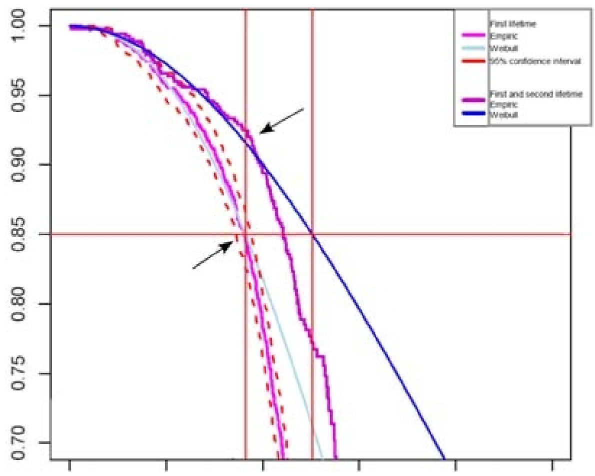

Figure 7 : Reliability estimation for signalling equipment. The Weibull model is not a very good fit. It can be seen that there is a turning point in the curve probably coming from several superposed reliability functions.

Figure 7 shows an example of a type of electronic unit that cannot be modelled by the Weibull distribution. It seems that there are two different reasons for failure: the first one is completely random. The second one should be related to wear or deterioration and increases with time. We are testing at the moment the Bertholon distribution, a combination of the exponential and Weibull distribution [2][3].

7.2. Ageing of the system

The correct functioning of an electric unit depends on the functioning of other components. For example, the transmitter of a track circuit wears more quickly if the electronic separation joint of the transmitted is not correctly electrically adjusted. Electrical fail-safe devices have characteristics that change with time: by replacing a component of an electronic board, the component will be stressed within an operating sphere different from what it would have been with a new board. The more the “neighbours” are used and were replaced, the less the system will endure. This phenomenon could be clearly detected on signalling equipment after several reparations.

In order to represent this phenomenon in an adapted way, we introduce a third parameter, the rate K. This rate represents the life expectancy reduction after each replacement of a component: $\eta_n = \eta_{n-1} \cdot K = \eta_0 \cdot K^n$, where n is the rank of replacement. This parameter K depends on the equipment and on its environment. It can be estimated from feedback data.

The introduction of a power coefficient leads to a very short life time for components that were replaced several times. This representation is satisfactory.

Figure 8 illustrates the increase of the number of replacement when the system gets older; the example uses K = 80 %.

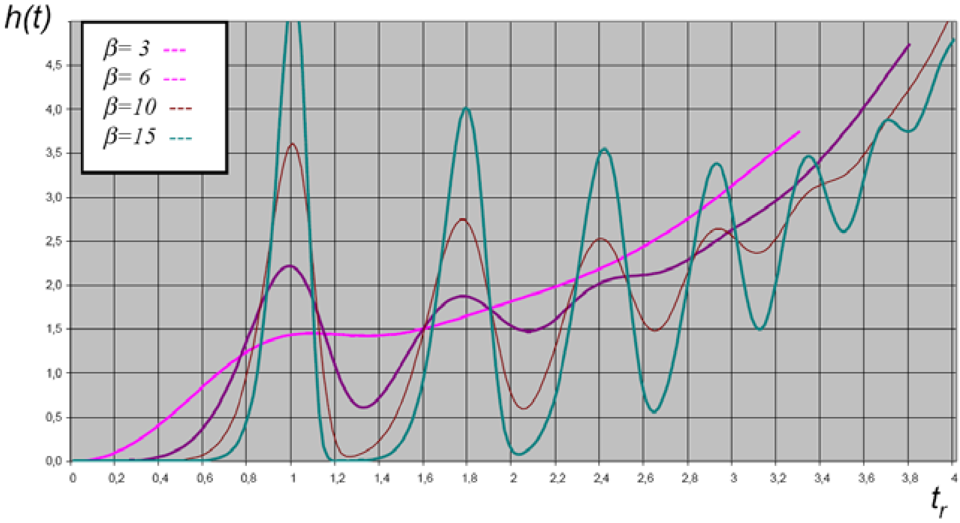

Figure 8 : Density function h(t) when including the ageing of the system.

8. REFERENCES

1. Antoni, M. & Meier-Hirmer, C. 2008. Economic correlation between maintenance and regeneration - optimisation of maintenance strategies for tracks, signalling equipment and overhead line components. In Proceedings of the WCRR.
2. Bertholon, H. 2001. Une modélisation du vieillissement (in French). Ph. D. thesis, Université Joseph Fourier, Grenoble.
3. Bertholon, H., Bousquet, N. & Celeux, G. 2004. Une modélisation de durée de vie à risques de défaillance concurrents (in French). In Proceedings of Journées de Statistique 36.
4. Klein, J. P. & Moeschberger, M. L. 1997. Survival Analysis - Techniques for Censored and Truncated

COMMUNICATION OF COST-EFFECTIVENESS OF SAFETY MEASURES BY USE OF A NEW VISUALIZING TOOL

E.B. Abrahamsen & T. Aven

University of Stavanger, Stavanger, Norway

e-mail: eirik.b.abrahamsen@uis.no, terje.aven@uis.no

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W. Røed

Proactima AS, Stavanger, Norway

e-mail: wr@proactima.com

ABSTRACT

A cost-effectiveness analysis (CEA) is often used as basis for comparisons between competing safety measures. In a CEA indices such as the expected cost per expected number of lives saved are calculated. These indices are presented to the decision-makers, and seen in relation to reference values, they form the basis for assessment of the effectiveness of the safety measures.

The appropriateness of using cost-effectiveness indices based on expected values have been thoroughly discussed in the literature. It is argued that uncertainty is not properly taken into account by the CEA, and extended frameworks for CEA are required. This paper represents a contribution to this end, by presenting a diagram that visualizes uncertainty in addition to the expected values as in the traditional CEA. The diagram is meant to be a presentation tool for semi-quantitative cost-effectiveness analyses used as a part of a screening process to identify safety measures to be assessed in a more detailed analysis. In the paper we discuss the use of the diagram in communication between analysts and other stakeholders, in particular the decision-makers. An example is presented to illustrate the applicability of the tool.

1 INTRODUCTION

Tremendous resources are spent on safety measures and the need for tools for supporting the decision-making is large. Cost-effectiveness analysis is such a tool, and it has shown to give useful support for comparisons between competing safety measures.

Different cost-effectiveness measures are used reflecting that there are many ways of expressing cost-effectiveness. We may think of a safety measure as cost-effective if it is (Petitti 2000):

- Less costly and at least as effective
- More effective and more costly, with the added benefit worth the added cost
- Less effective and less costly, with the added benefit of the alternative not worth the added cost
- Cost saving with an equal or better outcome

Quantitatively, and more precise, the cost-effectiveness can be expressed as a cost-effectiveness ratio, the ratio of change in expected costs to the change in expected effects. This type of ratio (index) usually forms the basis for communication of cost-effectiveness between analysts and other stakeholders.

A cost-effectiveness ratio produced through a cost-effectiveness analysis provides valuable insights. However, many analysts and researchers have pointed out that cost-effectiveness indices based on expected values are not appropriate for evaluation and communication of cost-effectiveness - a picture of cost-effectiveness needs to include a broader reflection of uncertainties. The main problems are that the expected values are conditional on specific background knowledge, and the expected values could produce poor predictions. Surprises may occur, and by just addressing expected values such surprises may be overlooked (Aven 2008). Taleb makes a similar conclusion using the black swan logic (Taleb 2007). The inability to predict outliers (black swans) implies the inability to predict the course of history. An outlier lies outside the realm of regular expectations, because nothing in the past can convincingly point at its occurrence. We find also similar ideas underpinning approaches such as the risk governance framework (Renn 2008) and the risk framework used by the UK Cabinet Office (Cabinet Office 2002).

To improve the communication of the cost-effectiveness of safety measures between analysts and other stakeholders, a cost-effectiveness-uncertainty-diagram is presented in this paper. The diagram visualizes uncertainty in addition to the expected values as in the traditional cost-effectiveness analysis.

The diagram should not be looked at as a tool for visualising results from detailed cost-effectiveness analyses. The diagram is meant to be a presentation tool for semi-quantitative cost-effectiveness analyses used as a part of a screening process to identify safety measures to be assessed in a more detailed analysis.

The paper is organized as follows. In Section 2 we review and discuss the use of cost-effectiveness analysis in evaluation of safety measures. In Section 3 the visualisation tool for cost-effectiveness is presented. Then in Section 4 an example is used to illustrate the applicability of the tool. Finally, in Section 5 we draw some conclusions.

2 REVIEW AND DISCUSSION OF THE COST-EFFECTIVENESS ANALYSES

In evaluation of safety measures a cost-effectiveness analysis is often adopted. The decision on whether a safety measure should be implemented or not is by using such an analysis to large extent based on the calculated cost-effectiveness ratio. The ratios can be expressed either as a cost-effectiveness ratio, or as an effectiveness-cost ratio (Boardman et.al 2006). The review and discussion of the cost-effectiveness analysis that follows, focuses on the cost-effectiveness ratio which is by far the more commonly used ratio.

The method will be illustrated by an example of two competing safety measures; safety measure 1 and safety measure 2. The following notation is used in the example:

- C_i; the investment cost associated with safety measure *i* (to simplify we assume that there is no annual cost associated with the safety measure)
- Z_i; the total effect related to loss of lives if safety measure *i* is implemented (to simplify we assume that this is the only effect of interest)
- R; the reference value. The value clarifies how much money the decision-maker is willing to pay to obtain one unit of effectiveness.

In order to compare the cost-effectiveness between the two measures, the cost-effectiveness ratio for both measures is calculated. The cost-effectiveness ratio for safety measure 1 and safety measure 2 is equal to C_1 / Z_1 and C_2 / Z_2, respectively. Safety measure 1 is more cost-effective than safety measure 2 if $C_1 / Z_1 < C_2 / Z_2$. To see whether safety measure 1 is preferred to status quo or not, the cost-effectiveness ratio has to be compared with the reference value, R. Implementation of the safety measure is preferred to status quo if the decision-maker is willing to pay more to obtain one unit of effectiveness than the cost-effectiveness index expresses, which means that safety measure 1 is preferred to status quo if $R > (C_1 / Z_1)$.

In practical situations we cannot determine the cost and the effects with certainty. There is often large uncertainty about *C* and *Z*. As a result predictions are required, and the natural choice is to use expected values.

For example, let us look at a simplified case of a safety investment. The decision-problem is to decide whether or not a safety measure should be implemented. We assume that the expected investment cost is £0.8 million, and that the expected number of fatalities is reduced from 2.7 to 1.9 if the safety measure is implemented.

The calculated cost-effectiveness index for the safety measure is (in million pounds):

$$\frac{EC}{EA} = \frac{0.8}{2.7 - 1.9} = 1$$

This value is often referred to as the implied value of a statistical life or the Implied Cost of Averting a Fatality (ICAF). We see that a cost-effectiveness analysis does not explicitly set a value to the benefit, e.g. value of a statistical life, as is required in a cost-benefit analysis, ref R2P (2001).

In many cost-effectiveness analyses we see that the decision is strongly based on the calculated cost-effectiveness index, which for this decision problem means that the decision-maker will prefer to invest in the safety measure if the decision maker's valuation of a statistical life (R) is higher than £1 million, while an investment in the safety measure will not be preferred if the valuation of a statistical life (R) is less than £1 million.

Valuable insight is provided through cost-effectiveness indices, but there is a need for a broader reflection of uncertainties, as discussed in Abrahamsen et.al 2004 and Aven (2008). The main argumentation is as mentioned in the introduction, that the expected values are conditional on specific background knowledge, and the expected values could produce poor predictions. To see this more clearly we can write the expected values in mathematical terms like E[X|K], where X is an observable quantity such as cost and K is the background knowledge. The background knowledge covers historical system performance data, system performance characteristics and knowledge about the phenomena in question. Assumptions and presuppositions are an important part of this knowledge. A result is that a true objective expectation value does not exist. Different analysts could come up with different values dependent on the assumptions and presuppositions made.

3 THE COST-EFFECTIVENESS-UNCERTAINTY-DIAGRAM

To improve the communication of the cost-effectiveness of safety measures between analysts and other stakeholders, we suggest to use a cost-effectiveness-uncertainty-diagram. This diagram better reflects the uncertainties than the cost-effectiveness indices.

The diagram reflects information about cost-effectiveness through three dimensions: 1) uncertainty, 2) expected cost and 3) the expected lives saved. The cost-effectiveness-uncertainty-diagram reflects the three dimensions by showing the expected cost on the x-axis, the expected saved lives on the y-axis and the uncertainty through different bubble sizes, see Figure 1. The cost-effectiveness of the safety measures is evaluated based on these three dimensions, and is represented by a colour (red, yellow and green). The red, yellow and green colour is in the diagram presented as black, dark grey and light grey, respectively. In the diagram below, four safety measures A, B, C and D are presented.

In the diagram attention is given to the expected number of saved lives as the expected effect, but could easily be adjusted to cover other dimensions of losses, for example related to the environment.

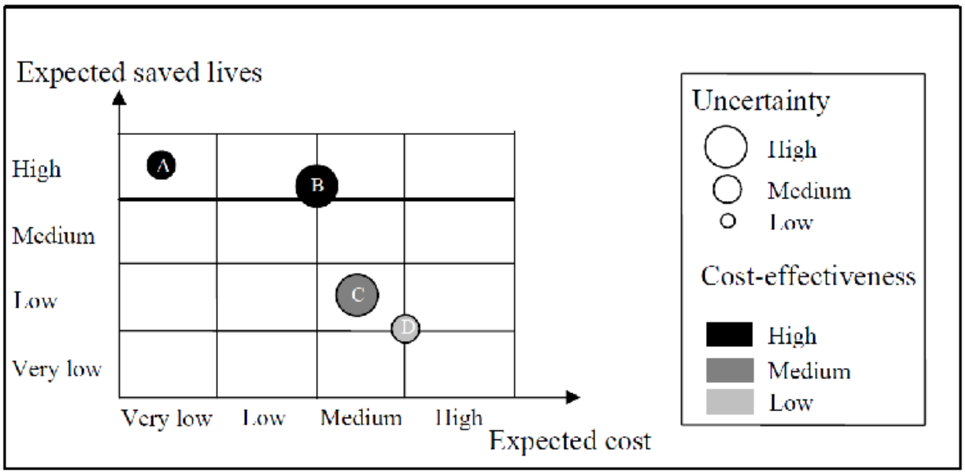

Figure 1. Graphical presentation of four safety measures in the cost-effectiveness-uncertainty-diagram.

The classification of safety measures into the cost-effectiveness-uncertainty-diagram is carried out on the basis of an understanding of the different dimensions as described in the following:

Expected Cost (EC):

The expected implementation cost of the safety measure. The expected implementation cost is considered as the centre of gravity of the probability distribution of the implementation cost.

Expected number of lives saved (EX):

The expected number of lives saved if the safety measure is implemented. The expected number of lives saved is considered as the centre of gravity of the probability distribution of the number of lives saved.

Uncertainty:

Uncertainty reflects the expected values' predictability of the real outcomes. High uncertainty in the cost-effectiveness-uncertainty-diagram may for example express that the assigned expected cost (EC) can give a poor prediction of the future cost.

In the cost-effectiveness-uncertainty-diagram four categories are used for both the cost and effectiveness dimensions, while three categories are used for the uncertainty dimension. Of course, the method may easily be adapted to more or less categories.

The categorisation process should be based on some guidelines or criteria to ensure consistency. In the following one example is given for all dimensions. The category classifications will be case-specific and subject to judgement by the analyst, but the descriptions could serve as guideline.

Expected cost:

- Very low: EC < £10.000
- Low: £10.000 ≤ EC < £100.000
- Medium: £100.000 ≤ EC < £1.000.000
- High: EC ≥ £1.000.000

Expected number of lives saved:

- Very low: EX < 0.01
- Low: 0.01 ≤ EX < 0.05
- Medium: 0.05 ≤ EX < 0.1
- High: EX ≥ 0.1

Alternatively, qualitative (non-quantified) categories may be used. This is in particular relevant in cases where a qualitative risk analysis is carried out.

Uncertainty:

Low uncertainty

All of the following conditions are met:

- The phenomena involved are well understood; the models used are known to give predictions with accuracy
- The assumptions made are seen as very reasonable
- Much reliable data are available
- There is broad agreement among experts
- Low variation in populations (low stochastic uncertainty)

High uncertainty:

One or more of the following conditions are met:

- The phenomena involved are not well understood; models are non-existent or known/believed to give poor predictions
- The assumptions made represent strong simplifications
- Data are not available, or are unreliable
- There is lack of agreement/consensus among experts
- High variation in populations (high stochastic uncertainty)

Medium uncertainty:

Conditions between those characterising high and low uncertainty, e.g.:

- The phenomena involved are well understood, but the models used are considered simple/crude
- Some reliable data are available

Note, that the degree of uncertainty must be seen in relation to the effect/influence the uncertainty has on the predicted values. For example, a high degree of uncertainty combined with high effect/influence on the predicted values will lead to that the conclusion that the uncertainty factor is high. However, if the degree of uncertainty is high but the predicted values are relatively insensitive to changes in the uncertain quantities, then the uncertainty classified in the diagram could be low or medium.

Table 1. Cost-effectiveness categories.

Category	Description of cost-effectiveness category	Expected cost (EC)	Expected number of lives saved (EX)	Uncertainty
High cost-effectiveness	Measures associated with very low or low expected costs and with medium or high expected number of lives saved. (Independent of the uncertainty)	Very low	Medium	Low, medium or high
		Low	Medium	
		Very low	High	
		Low	High	
Low cost-effectiveness	Measures associated with medium or high expected costs, with very low or low expected number of lives saved and with low uncertainty.	Medium	Very low	The cost-effectiveness category will be considered one category up (medium) if the uncertainty is considered medium or high
		High	Very low	
		Medium	Low	
		High	Low	
Medium cost-effectiveness	Measures included in categories between those characterising high and low cost-effectiveness.	High	High	The cost-effectiveness category will be considered one category up (high) if the uncertainty is considered medium or high
		High	Medium	
		Medium	High	
		Medium	Medium	
		Low	Low	
		Low	Very low	
		Very low	Low	
		Very low	Very low	

The cost-effectiveness of the safety measures have to be decided through an evaluation of the three dimensions mentioned above. The categorisation of the cost-effectiveness should again be based on some guidelines to ensure consistency. One possible way for categorisation of the cost-effectiveness of the safety measures is given in Table 1.

Incorporation of the uncertainty dimension can lead to a reclassification of the cost-effectiveness for a safety measures seen in relation to a traditional cost-effectiveness analysis. We may start the cost-effectiveness classification by first rank the safety measures according to the two standard dimensions expected cost and expected (effectiveness) number of lives saved. Then we may adjust these up or down in case the uncertainties are considered high or low. In the example discussed in the next section, the uncertainties are considered high and hence the cost-effectiveness for the safety measures should be considered reclassified.

4 AN EXAMPLE

A risk analysis has been carried out for an existing road tunnel consisting of one tube with one lane in each direction. This is a low-traffic tunnel located on the countryside. The number of cars driving through the tunnel is, in average, two cars in each direction per minute. There are however large differences in traffic density during one day. For example in the morning and in the afternoon the traffic density is considerably higher than average, while in night hours there is hardly any traffic at all.

The tunnel is located in a district where, according to the geologists, the risk of rock fall is considerably higher than in most other tunnels. One potential risk reducing measure is to install rock protection bolts. This is however an expensive safety measure. Alternative measures have been considered, related to ordinary traffic accidents. For example, it has been discovered that the illumination of the lighting is not sufficient, increasing the risk of traffic accidents particularly when entering and leaving the tunnel. Now, as a simplification for the purpose of the example, suppose two risk reducing measures only are considered:

- A: Install rock protection bolts in order to reduce the risk of rock fall
- B: Install more light fixtures and then increase the illumination to prevent traffic accidents

The alternative measures, and the associated properties in terms of expected number of lives saved, uncertainty and expected cost is presented below.

Risk reducing measure (A)
Install rock protection bolts.

Expected cost (EC):
High: > £ 1 million

Expected number of lives saved (EX):
Category medium: Even though the probability of rock fall is by geologists considered high, the probability of a car being hit is low due to the low traffic density. Based on calculations, the expected number of lives saved is in category medium (between 0.05 and 0.1)

Uncertainty:
Category high: The typical situation is that one by one car drives through the tunnel every now and then. This means that most likely zero or one car will be hit by a rock fall even though the fall zone may be substantial. However, occasionally, and in particular during ruch hours, the cars tend to pass lines of typically 5 or 10. It is possible, though not likely, that an entire line of 10 cars will be hit by one single rock fall. This means that the number of lives saved by the rock protection bolts could be in the range from zero to 20 or more persons. Based on this, the uncertainty of the effect of the bolts is considered high.

Risk reducing measure (B)
Install new light fixtures

Expected cost (EC):
Medium: Between £ 100.000 and £1 million

Expected number of lives saved (EX):
Category medium: Based on experience from other tunnels high illumination is important to prevent traffic accidents. In this tunnel, the illumination is lower than what is considered best practice. The expected number of lives saved by increasing the illumination to best practice level is in the category medium.

Uncertainty:
Category low: The traffic accidents being prevented by installing new light fixtures involves 1 or 2 cars. Then the real number of lives saved by the risk reducing measure is 0-4, depending on the number of persons in each car. The phenomena involved are well-understood, and there is broad agreement among the experts on what the result of such accidents may be. Based on this uncertainty category low is applied.

Now, which of the two safety measures is the most cost-effective one? If only the expected risk reducing effect and the expected cost were taken into consideration, the natural candidate would be measure B: This is the less expensive safety measure, and since the expected number of lives saved is equal for A and B it may be argued that B is the most cost-effective one.

Taking the uncertainty dimension into consideration, and applying the method presented in Table 1, it may be argued that safety measure A is the most cost effective one. The rationale behind this is that considering the expected number of lives saved and the expected cost only is not sufficient to evaluate cost-effectiveness: We have to take the uncertainties into consideration. In our case, it is considerable uncertainty about the number of lives saved of safety measure A; the rock protection bolts, in particular. The expected effect is low, since it is expected that the persons in zero or one car only will be saved. However, the actual number of lives saved could be much higher as described above. According to Table 1 we may change the cost-effectiveness by one category due to such considerable uncertainty.

The risk reducing measures are plotted in the cost-effectiveness-uncertainty-diagram in Figure 2 below.

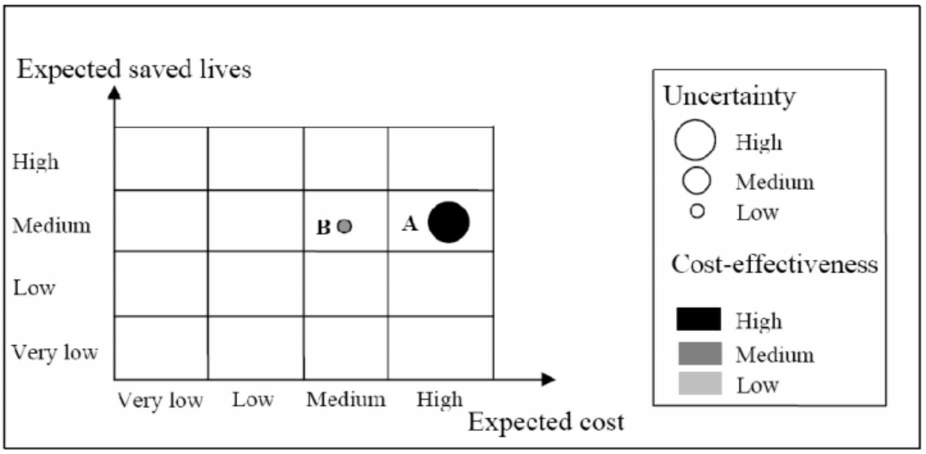

Figure 2. Graphical presentation of the risk reducing measures.

5 CONCLUSION

Communication between analysts and other stakeholders of safety measures' cost-effectiveness is usually based on cost-effectiveness indices. These indices are based on expected values. In the literature it is argued that such indices are not appropriate for evaluation and communication of cost-effectiveness. A broader reflection of uncertainties is needed.

This paper presents a cost-effectiveness-uncertainty-diagram. By extending the cost-effectiveness description to also cover uncertainties beyond the expected values, we believe that the cost-effectiveness-uncertainty-diagram would be better able to provide a broad, informative and balanced picture of cost-effectiveness.

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REFERENCES

1. Abrahamsen EB, Aven T, Vinnem JE, Wiencke HS. 2004. Safety management and the use of expected values. Risk, decision and policy 9: 347-357.

2. Aven T. 2008. Risk analysis – Assessing uncertainties beyond expected values and probabilities. Wiley. N.J.
3. Boardman EA, Greenberg DH, Vining AR, Weimer DL. 2006. Cost-benefit analysis – Concepts and practice. New Jersey: Prentice Hall.
4. Cabinet Office. 2002. Risk: improving government's capability to handle risk and uncertainty. Strategy unit report. UK.
5. Petitti DB. 2000. Meta-analysis decision analysis, and cost-effectiveness analysis. New York: Oxford University Press.
6. Renn O. 2008. Risk governance: coping with uncertainty in complex world. London: Earthscan.
7. R2P2. 2001. Health and Safety Executive, Reducing Risks, Protecting People: HSE's decision-making process, HSE Books, Sudbury.
8. Taleb NN. 2007. The black swan: the impact of the highly improbable. London: Penguin.

WEB SOFTWARE RELIABILITY ENGINEERING

G. Albeanu, A. Averian, I. Duda
•
Spiru Haret University, Bucharest, Romania

e-mail: g.albeanu.mi@spiruharet.ro

ABSTRACT

There is an increasing request for web software systems, some of them to be used very intensive. The customers ask not only for fast design and implementation, but also for a high quality product. Considering reliability as an important quality attribute, this paper describes the current state of the art in designing, implementing, and testing web software. An important attention is given to software vulnerabilities and how to deliver secure software.

1 INTRODUCTION

Recently, a special class of distributed software was born, and is used intensively by people working on their terminals, situated in office or at home. The object we are talking about is called *web application*, which "is a collection of servlets, html pages, classes, and other resources that can be bundled and run on multiple containers from multiple vendors", according to Davidson & Coward (1999). However, the term is used generic for web sites (web servers), and every software application using Internet environment.

According to Pickering (2005), "in most server architectures, the failure of any one system or service in the path between server and user will in effect cause failure of the entire application as far as the user is concerned". The term *failure* is used according to (Randell et al. 1986), i.e. "the event of a system deviating from its specified behaviour".

Another important aspect deals with software security. Even the security is managed separately; before the security hole is patched any failures of the application will have great impact on the application reliability. This makes difficult the usage of the standard software reliability growth models for insecure systems.

When speaking about web-servers, we have to take into account many technologies (hardware and software), each one having its own failure modes and sources of delay and unreliability, as proved by Pickering (2005).

According to the study of (Randell et al. 1986), the reliability of the web-based applications can be considered as a special case of distributed software running on distributed computer systems, over different kind of networks (local area networks, wide area networks etc.). If the nodes of the network are assumed to be perfect and the connections among nodes are assumed to fail in a statistically independent manner, then the network reliability can be computed as presented in (Shier 1991). However, if the nodes are imperfect, then the usage of the algorithm provided by (Lin et al. 1999) is an efficient solution.

The aim of this paper is to describe the relevant aspects of web software coding, testing and reliability analysis and to outline some best practices when thinking in terms of web software reliability engineering.

The above ideas motivate us to organize the paper as follows. The second section considers the web-based software design for reliability, and covers the state of the art in implementing and testing web applications. The management of the software vulnerabilities is described in the third section. For a secure web-server, aspects concerning the reliability growth modeling are considered in the fourth section. Two case studies are discussed. The first one refers to a Virtual Campus project (VC), while the second considers the DISTeFAX project. Finally some concluding remarks are formulated.

2 WEB-BASED SOFTWARE ENGINEERING

As a general rule, the web-based software is built in order to provide some functionality using different web services protocols and frameworks oriented to a specific application, as mentioned by Chu & Qian (2009). For instance, *E-business XML* (or ebXML) is a useful protocol when processing electronic business information over various platforms. Also, *ApacheAxis2* is a framework supporting many protocols, including SOAP (*Single Object Access Protocol*) for exchanging information in a decentralized distributed environment. We refer to SOAP, because "web services usually use SOAP over HTTP", as Maeda et al. (2003) remarked.

Some years ago, speaking about the future of software reliability engineering, Lyu (2007) said: "the traditional solution that software designers adopted – carefully elicit change requests, prioritize them, specify them, design changes, implement and test, then redeploy the software – is no longer viable." Nowadays, agile methodologies based on software components, including open source, are used to deal with rapidly software releasing, increasing reliability and diminishing the software costs (Averian et al. 2009).

According to Wasserman (2005), "the most heavily used websites are characterized by high reliability, high availability, high security, and rapid interactive response". The discussion, in (Wasserman 2005), is oriented to the following design principles: abstraction, modularity, multi layer architecture, and logging for analyzing and testing. It is important to notice that these principles are independent on the web services provided by the web application.

The *abstraction* is used in all web-application life cycle (Schneidewind 2003, Wasserman 2005): requirements' specification (use case diagrams, scenarios, work flow models, conceptual data models etc.), project design (by objects: images/audio/video, menus, buttons, text fields etc.), coding (based on templates), and testing (failure trees, root cause analysis, etc.).

Modularity promotes the component-based paradigm and the reuse principle, a smart usage of reusable components for constructing quality software by reducing the verification costs, increasing the software reliability, and reducing the development time (Albeanu et al. 2009a, Averian et al. 2009).

Recently, the application software follows a *multi-layer architecture*, being developed similar with some parts of the operating systems. Three-tier architecture is based on the following entities: client, server, and database. As Wasserman (2005) said, web applications have "well-known and widely followed n-tier site architecture" based on pattern design (configuration modes described using XML or other pattern languages), plug-ins (assuring an extensible architecture), with a modular structure using a specific user interface (based on languages and technologies like HTML, Flash, JavaScript, etc.).

For analyzing and testing web software, there are available a large collection of tools (components), ready to be embedded into the web application, or activated in order to monitor different aspects related to the website activity. These tools generate log files useful for "studying system performance, identifying errors, and determining general patterns of use", as mentioned by Wassweman (2005).

Web services are offered by different web servers for specific activities. This is the reason for Chu & Qian (2009) to say: "e-business application development has certain characteristics that make it different from traditional software development". This observation is also valid for other fields asking for high security assurance.

According to (Chu & Qian 2009), the following requirements should be taken into account for specific web applications, like those from e-business field: *service composition* (developed based on a complete system model), *formal semantics* (in order to use automated tools for service design and verification), and *systematic service design methodology* (for supporting service reuse). In this way, service reuse at different levels of granularity is also provided.

Zaupa et al. (2008), using the product line concept, proposed a web application development strategy oriented on services. In this manner, there are three stages to be followed during the development process: 1) Application domain definition; 2) Services development, and 3) Application generation.

The set of requirements has to be stable when classical software development methodologies will be used, according to Schneidewind (2003). However, in an agile framework, the requirements of a web application could be easy updated during the starting period of any iteration (when applying an iterative prototyping approach). The above three stages can be iterated when an agile methodology is used and will consider the requirements obtained in one of the following methods: classical, using UML notation, navigation-based templates, hypermedia modeling based on object thinking, and other ad-hoc, but documented models. According to (Averian et al. 2009), such an agile methodology will increase the quality of the software under development.

In order to decrease the number of faults (local or sub-network faults), the software team have to be experienced with existing vulnerabilities and security improvement mechanisms. Based on the preliminary study of Albeanu (2009), and the investigation described by (Albeanu et al. 2009a), such aspects will be detailed in the next section.

Another important aspect of web applications deals with *interoperability*. Many web applications accept as input and produce as output different objects. In order to be used/viewed/printed/listen, the object format will be a known one, and secure plug-in components will be available for clients. Here, we think about a web application in a multi server – multi client architecture. The multi server architecture is required for increasing reliability and availability by sharing connections (in a *round-robin* fashion and/or by *load balancing*). This is also the case of all software intensive systems where the *availability* is an important quality attribute, according also to (Pickering 2005, da Silva Filho 2005).

Web application *robustness* is another quality requirement: “the property of a system or a component that is totally correct in respect to a complete specification, thus its behavior is predictable for all possible operational environments”, as defined in (Calori & Stalhane 2007). In order to obtain a robust web application, software engineering plays an important role. The analysis of robustness can proceed according to some methods, like those discussed by Calori & Stalhane (2007), but for critical applications like *e-business* or *e-campus total management*, the operational environment, including the security profile, will be simulated in order to test all specified requirements.

3 SOFTWARE VULNERABILITIES

If omitting the failures generated by cyber attacks, we refer to the intrinsic reliability. In large, the software reliability covers also aspects related with security holes that permit to attackers the crashing of the web application. These security holes are generated by software vulnerabilities as defined in the following. Software vulnerability deals with insecure programming and the possible insertion, by mistake, of the following classes of bugs, as identified by Albeanu (2009) and described in (Albeanu et al. 2009a, b):

- *memory-management* (buffer/stack overflow, format string vulnerability, boundary condition checking);
- *concurrency-management* (e.g. race condition involving a security check);
- *I/O-management* (e.g. input validation mistake, SQL injection, incomplete application protocol validation and verification);
- *inconsistent integration of security technologies* (e.g. configuration errors, environmental errors, incomplete access control procedures);
- *numerical inconsistencies* (e.g. integer overflow, division by zero, XOR based encryption);

– *vulnerable entry points* (command-line parameters, the environment array of strings, default input files, default passwords, inherited file data structures, inherited attributes when working with extended classes, incorrect specification of web graph nodes).

There are possible mistakes not only during design, but also during testing and implementation phases. Environmental and administrative mistakes are common when speaking about web-servers.

The vulnerabilities are possible to be identified: (1) manually (by experts), (2) automated (by bottom up and/or top down testing) (3) by black box testing, (4) by white-box analysis, (5) using scanners, and (6) combined various methods. The software trustability will be increased by testing the software using environment perturbation (taken different actions on files, other processes, network etc.).

According to Knight & Elder (2001), when coordinated security attacks are identified, "additional protection mechanisms such as closing connections over a wide area together with longer term measures such as changing cryptographic keys" are required for such faults. Non-local fault tolerance can be implemented using a specialized cryptographic protocol implemented on a cluster of servers.

If the development is based on the component-based approach, and inadequately secure components are embedded, the wrapping technique will be used for the components accommodation. Such an approach was described early in (Randell & Dobson 1986): "design means of masking or of detecting and recovering from, the security errors which might arise", and used also for the project presented in (Albeanu et al. 2009a, b).

In order to minimize the security type vulnerabilities, the prevention of the cyber attacks is the best strategy and may use the following technologies (Maeda et al. 2003): security tokens, digital signatures, encryption, and other security tools according to the security management procedure. Taking into account the above classes of bugs and the mentioned security technologies, the following types of web application attacks will be rejected: imposture (impersonation), repudiation (refusing acknowledgment), information disclosure (without permission), information altering, denial of services, and gaining the privilege of administrators or owner applications.

According to Guo & Sampath (2008), the following classes should be taken into consideration: *data storage* class covering all possible faults related to data structures, *logic faults* generated during implementing algorithms and the application control flow (some of them being related to session/paging faults, inconsistent browser interaction parsing faults, mistakes in coding encoding/decoding and encryption/decryption algorithms), *data input* faults generated by input validation mistakes related to files and forms, *appearance* faults generated by inappropriate coding for controlling the display of the web-pages, and *linking* faults due to mistakes in controlling the transfer to different locations in the World Wide Web (URL – Uniform Resource Locator). The last class is reach for the case of web applications working with URL data bases. Comparing the two classifications mentioned above we found that the taxonomy detailed by Albeanu (2009) and described in (Albeanu et al. 2009a, b) is rich enough and contains also cookies' manipulation, communication encryption, user authentication, account management, and accessing/using resources without permission.

During web software development, a model that accurately describes the vulnerabilities is required. As mentioned by (Albeanu et al. 2009a, b), "the most used vulnerability models use VCG (Vulnerability Cause graphs), C/DFG (Control/Data Flow Graphs), and decision trees". For the web applications investigated in section 4.3, the VCG approach was used. This is similar to root cause analysis method.

A VCG structure for a specific vulnerability can be built along five steps per vertex (Byers et a. 2006): validity analysis, split requirement analysis, conversion to compound vertex analysis, predecessor identification, predecessor post-processing. The VCG is stored in the vulnerability database in order to establish a complete relation between vulnerabilities and causes. The vulnerability database considers also other supplementary information like: flaw description, example of code containing the flaw, and preventive advice, as mentioned by Albeanu(2009).

Other methods uses FMEA and soft computing techniques as those described in (Calori & Stalhane 2007).

A global analysis considers both hardware and software fault categories when studying the web application reliability (Littlewood & Strigini 2000, Lyu 2007, Pham 2003). A separate analysis can be developed in the case of software faults only.

4 WEB SOFTWARE RELIABILITY

4.1 Network reliability and performability

As Pickering (2005) already identified, an important factor influencing the web-server reliability is the network reliability and availability. As measures of availability the most important are the connectivity and the performability. When a failure occurs, the network could not be able to perform at the same parameters as when working without failure. In this way, there is a strong relation between the network failure performability and the network reliability.

Various services are provided over multiple interconnected networks with different technologies and infrastructure by different suppliers (providers).

Modelling the network as an undirected simple graph, the network reliability is studied, to assure, at least theoretically, a solution to the following problems: (1) Compute the probability that there is a path between two distinguished vertices a, and b [terminal connectivity]; (2) Compute the probability that all vertices remain connected. It is clear that both combinatorial and statistical methods are mixed in order to compute the network reliability. Analysing network reliability is more important for the case of content replications motivated by requests for decreasing the answer time to a large number of simultaneously queries.

Considering the most used types of distributed web-servers (DWS) the following architectures are possible: cluster based (with virtual IP address depending on the web service visible to the clients, and a real IP address of the cluster nodes (CN), but hidden to clients), virtual cluster (the nodes sharing the same IP, and only one node will keep a message from clients), and distributed cluster (every node having its IP, and the message being redirected by a dynamic procedure applied related to the Domain Name System). The redistribution is implemented in a switching (SW) system.

The web based system reliability can be computed as in the case of serial systems: R(DWS) = R(SW) x R(CN). It is clear that R(CN) depends on the cluster topology, but experimentally we found that the estimation of R(CN) depends also on the method of content mirroring, the best results being obtained for complete replication.

Suppose G represents the network of cluster nodes that can perform if and only if it is connected, and Gr a random subgraph of G. If every edge e of G has associated a failure probability pe, then the probability that Gr remains connected is the same as G still perform. The network reliability computation can be realized using the classical results presented by Shier (1991) and Shooman (2002).

The web applications are composed by a large number of software components, many of them used in a reusable manner. The most used protocol for inter-component communication is the client-server mechanism. In this case, a component-dependency graph is built, and the component reliability is estimated (for white-box components) or approximated (in the case of black-box components) based on its average execution time, using the methodology described by Hu (2007). Based on the architecture style (sequencing, looping, concurrency supporting, fault-tolerant style, refinement, or a mixed style), and taking into account the transition probabilities among the components (estimated during a benchmarking period) the overall system reliability can be computed using methods as described in (Davila-Nicanor & Mejia-Alvarez 2004, Suri & Bhushan 2007, Tsai et al. 2004).

For the virtual campus project it was found that tree based architecture provides a high degree of reliability, and the computing of architecture reliability was fast using the MFST method described in (Lin et al. 1999). For a cluster having five nodes the best performance was obtained in a partition of type (2, 3), being also a strong fault-tolerant architecture. Actually, the web application is distributed using an architecture of type (1, 1), the availability of service being 99%.

The DISTeFAX software was designed and implemented as a secure software system facilitating the processing of meta-faxes (multipart documents obtained from individual files generated by different applications like text editors, spread sheet processors, image processing applications, etc.)

The DISTeFAX application has two main parts according to the client-server design methodology and based on the reusing principle as an agile approach for short-time releasing of software to the customers. The server part is based on the *sqLite* open source software assessed as a very fast database machine. The activities related to sending and receiving faxes are managed by a component built on top the *eFax* module (as faxing driver). The main functionality of the client module is connected with sending facsimiles and previewing and sending those received – without using any fax-modem; the faxmodems being installed only on the fax server. For managing the visualization of the facsimiles, a wrapped component based on *libtiff* was integrated into the client module.

4.2 Web software testing

It is a general assumption that fault removal is successful in the case of many software reliability growth models, as Littlewood & Strigini (2000) already remarked. For web software, only faults generating security holes are successfully removed (it is imperative necessary).

When speak about web application testing, there are two interpretations. The first one is related to software validation as mentioned by (Davila-Nicanor & Mejia Alvarez 2004, Eaton & Memon 2004, Suri & Bhushan 2007, Guo & Sampath 2008):

– *establishing the level of usability* (offering an easier navigation; conformity with standards related to content organization, and the visibility of the navigation graph);

– *checking for browser/platform compatibility* (for assuring also the portability at operating system level, and provide wide access to the web site, including by mobile technologies);

– *assuring functionalities* (the content of pages inclusive the scripts is syntactically correct and free of bugs, and all links are active; all inputs are validated, cookies are checked for correctness and security; if a database is maintained then testing all aspects related to storage, code, protocols is required);

– *communication interface testing* (checking the network/cluster connectiveness and the correctness of data transportation, including encryption protocols and acknowledgement mechanism);

– *load testing* (in order to establish the level of performance under stress testing).

– *vulnerability scanning for security assessment* (as mentioned above and detailed by Albeanu et al. (2009a, b));

The second one addresses the *unit testing* (by assertions on some regions of code – for assuring fault prevention) and the *debugging process* connected to failures, mainly by load testing (under heavy exposure). This kind of testing is useful to estimate the software reliability, and a database of failures (type, level of severity, etc.), bugs (identifier, type, location, if possible to generate security holes, …), as a time series database useful to establish both inter-failure time and cumulative numbers of failures in order to support fault/failure forecasting, will be recorded.

Even already established a user profile, the web application is, in general, open to many users. This is why we decompose the user profile in a *public profile* and a *private profile*. It is compulsory to release a bug-free web application according to the public profile (free access services), even the testing was stopped for the private profile (controlled access services) because of some schedule constraints.

When working with components, and for some of them creating some wrappers, a regression testing is required. In general, web applications are developed in an agile methodology (mainly extreme programming, or adaptive software development), and an agile testing approach is selected. In this case, the analysis of collected data is organized in batches, every batch corresponding to software life cycle iteration. For the VC project, three builds were analysed along their life cycle.

4.3 Case studies

In the following the web software reliability is analyzed using standard software reliability models (Pham 2003), as those provided by SMERFS (Farr 2003).

The web application implementing the university virtual campus was developed during three versions/builds. For all versions, the time series corresponding to test debugging were collected (Table 1), and analyzed using SMERFS, as described in (Albeanu et al. 2009b).

Table 1. VC - Time between failure data along three builds

Build No.	Number of failure	Time between failure data [days]
1	19	1, 2, 1, 3, 5, 1, 2, 4, 8, 6, 11, 17, 19, 35, 22, 52, 28, 62, 74
2	18	1, 1, 1, 2, 1, 3, 2, 4, 3, 14, 18, 11, 33, 24, 53, 71, 59, 72
3	16	1, 1, 1, 1, 2, 2, 5, 13, 12, 10, 21, 28, 38, 58, 74, 57

During analysis, for both projects, five models were selected, namely: Model 1 - Moranda's geometric model (assuming that the software is never error-free and as debugging progresses the faults become harder to detect, with the detection rate forming a geometric progression and being constant between error occurrences), Model 2 – Quadratic Littlewood-Verrall, Model 3 – Musa's basic, Model 4 – Musa's logarithmic, and Model 5 – Nonhomogeneous Poisson Process model (for execution time).

For every model the statistics concerning accuracy, bias, noise, and trend are computed. These statistics follow specific mathematical formulas depending on the model. The noise of the model is computed based on the Braun statistics (the Braun indicator is a measure of variability giving the quantitative inference about the model's noisy).

There are computed also important estimates obtained during models' execution, like: IIF – Initial Intensity Function (initial hazard rate), CIF – Current Intensity Function (current hazard rate), PrfLvl – the "Purity" Level (the ratio of the changing in the hazard rate function from the starting point to the ending and the initial value), MTBNF – Current Mean Time Between Next Failure, and KS – the measure for Goodness-of-fit calculation.

In the case of VC project, using the Goodness-of-fit measure we obtain three well-suited models for data fitting: Quadratic Littlewood-Verall, Musa's Basic, and the Nonhomegeneous Poisson Process model. These can also be identified in the pictures giving the raw and predicted data for the third builds of the project (Figures 1-3). It can be observed that Musa's logarithmic model is most pessimistic, while the best prediction is obtained using the fifth model, for short intervals of time.

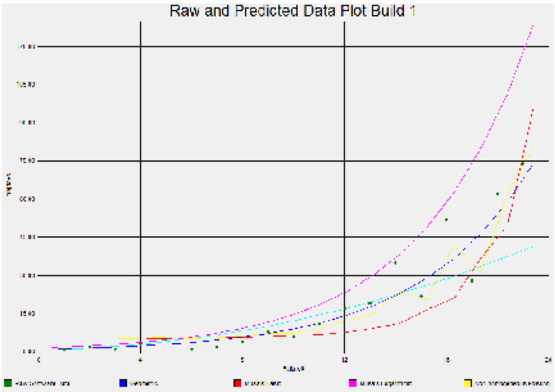

Figure 1. SRGM Analysis / VC-Build 1

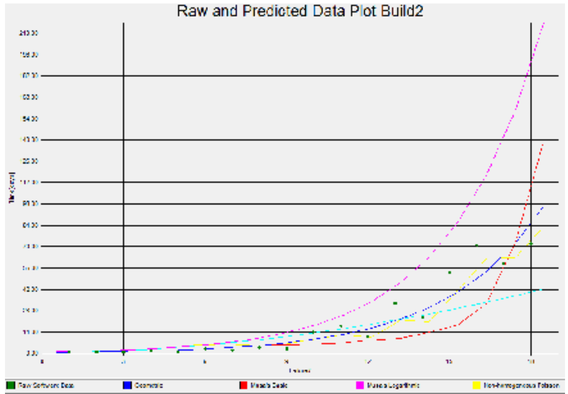

Figure 2. SRGM Analysys / VC-Build 2

The analyze using SMERF software established that for the sequence of failure data related to the DISTeFAX project (shown by value in Figure 4), the Musa's Logarithmic model was ranked first (related to accuracy, bias, noise and trend), as revealed in Figure 5, but not recommended as the best for fitting when consider the Kolmogorov distance. The SMERF software indicates as adequate for fitting only Musa's Basic model and the Non-homogeneous Poisson model.

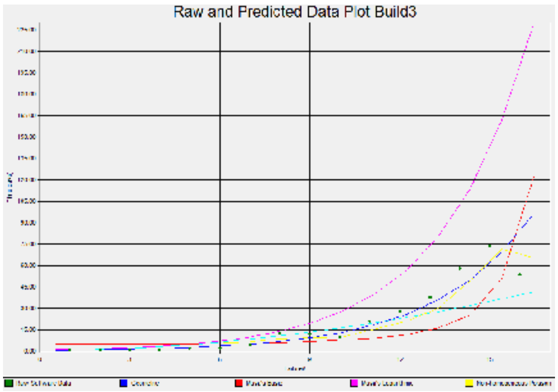

Figure 3. SRGM Analysis / VC-Build 3

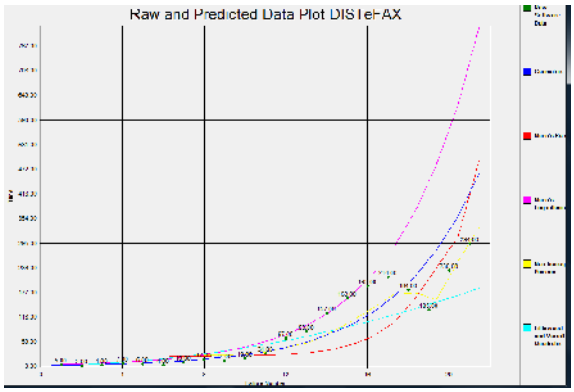

Figure 4. SRGM Analysis / DISTeFAX

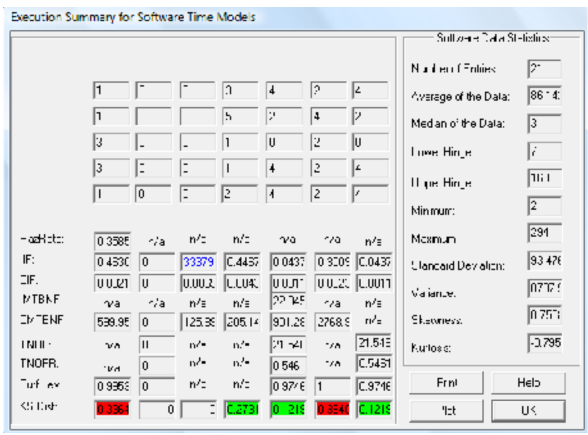
Figure 5. DISTeFAX project - Execution Summary

Combinations of models of type Hsu & Huang (2009) are also possible, when they are shared basic common assumptions, and will be considered for future investigation.

5 CONCLUDING REMARKS

This paper considers a special case of distributed software, namely the web applications, which ask not only for basic quality characteristics of software, but also have to be vulnerability-free, that means be able to prevent, detect and recover a good state after a possible cyber attack.

Starting with the development of a virtual campus for a large size university the software team had to solve important problems related to web application software engineering, time releasing constraints and to provide a high quality product. The most part of practical aspects useful for finalizing such a project were covered and outlined above.

During DISTeFAX development, the team made use of many components (some of them open source), and was responsible for assuring a high degree of security by a smart approach in vulnerability identification and removal. Such activity improves also the DISTeFAX reliability.

Finally, we appreciate that a guide of best practices for web application software reliability engineering is necessary to be developed in short time to be available for students in software engineering, practitioners, and customers.

Acknowledgement. This version extends the material given in (Albeanu et al. 2009b) with more considerations about software vulnerability and a second case study, namely the DISTeFAX project.

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REFERENCES

Albeanu, G. 2009. Increasing software quality by software vulnerability engineering. *Optimum Q* 20(2): 13-24.

Albeanu, G., Madsen, H. & Averian, A. 2009a. On the influence of software vulnerabilities on software reliability: The case of open source component based software. In R. Bris, C. Guedes Soares & S. Martorell (eds.) *Reliability, Risk and Safety: Theory and Applications, Proceedings of ESREL Conference* 2:1341-1346.

Albeanu, G., Averian A. & I. Duda. 2009b. Towards web applications reliability engineering. In Krzysztof Kolowrocki & Enrico Zio (eds.) *Proceedings of SSARS* 1:15-22.

Averian, A., Duda, G., & Albeanu, G. 2009. Quality assurance for agile component-based software development. In H Pham, T Nakagawa (eds.) *Proceedings of the 15th ISSAT International Conference on Reliability and Quality in Design*: 100-104.

Byers, D., Ardi, S., Shahmehri, N., Duma, C. 2006. Modelling Software Vulnerabilities with Vulnerability Cause Graphs, *Proceedings of the 22th IEEE International Conference on Software Maintenance* (ICSM'06).

Calori, I.C. & Stalhane, T. 2007. FMEA and BBN for robustness analysis in web-based applications, In Aven & Vinnem (eds), *Risk, Reliability and Societal Safety*, *Proceedings of ESREL*: 2341-2347.

Chu, W. & Qian, D. 2009. Design Web Services: Towards Service Reuse at the Design Level, *Journal Of Computers* 4(3): 193-200.

Davidson, J.D. & Coward, D. 1999. *Java™ Servlet Specification*, v2.2, Sun Microsystems.

Davila-Nicanor, L. & Mejia-Alvarez, P. 2004. Reliability Improvement of Web-Based Software Applications, *Proceedings of the Fourth International Conference on Quality Software*, CINVESTAV-IPN: 180- 188.

Eaton, C. & Memon, A. 2004. Evaluating Web Page Reliability across Varied Browsing Environments, *Proceedings of the 15th IEEE International Symposium on Software Reliability Engineering (ISSRE'04)*, France.

Efax. *The efax home page*: http://www.cce.com/efax/

Farr, W. 2003. *SMERFS*. http://www.slingcode.com/smerfs/compare.php

Guo, Y. & Sampath, S. 2008. Web Application Fault Classification - An Exploratory Study, *Proceedings of ESEM'08*: 3003-305, http://doi.acm.org/10.1145/1414004.1414060.

Hsu, C.J. & Huang, C.Y. 2009, Reliability Analysis Using Weighted Combinational Models for Web-based Software. *Proceedings of the 18th International World Wide Web Conference (WWW 2009)*: 1131-1132.

Hu, H. 2007. Reliability Analysis for Component-based Software System in Open Distributed Environments. *International Journal of Computer Science and Network Security* 7(5): 193-202.

Knight J.C. & Elder, M.C. 2001. Fault Tolerant Distributed Information Systems, *Proceedings of the 12th International Symposium on Software Reliability Engineering*, IEEE: 132-137.

Leffler, S. 1997. *libtiff - TIFF Library and Utilities*: http://www.libtiff.org/ (last updated: 2003)

Lin, M.S., Chen, D.J. & Horng, M.S. 1999. The Reliability Analysis of Distributed Computing Systems with Imperfect Nodes, *The Computer Journal* 42(2), 129-141.

Littlewood, B. & Strigini, L. 2000. Software Reliability and Dependability: a Roadmap. *Proceedings of the Conference on the Future of Software Engineering*: 175-188.

Lyu, M.R. 2007. Software Reliability Engineering: A Roadmap, *Proceedings of FOSE*: 153-170.

Maeda, T., Nomura, Y. & Hara, H. 2003. Security and Reliability for Web Services, *FUGITSU Sci. Tech. J*. 39(2):214-223.

Pickering, R. 2005. *Web Server Reliability*, http://www.ipcortex.co.uk/wp/app-reliability3.pdf, Ipcortex.

Pham, H. 2003. Recent Studies in Software Reliability Engineering. In Pham Hoang (ed.), *Handbook of Reliability Engineering*: 285-302, London: Springer.

Randell, B. & Dobson, J.E. 1986. Reliability and Security Issues in Distributed Computing Systems, *IEEE Proceedings of the 5th Symposium on Reliability in Distributed Software and Database Systems*: 113-118.

Schneidewind N.F. 2003. Life Cycle Core Knowledge Requirements for Software Reliability Measurement, Reliability Review, *The R&M Engineering Journal* 23(2):18-29.

da Silva Filho, A. M. 2005. Toward More Available Software-Intensive Systems, *The Fifth International Requirements Engineering for High-Assurance Systems Workshop*, http://www. sei.cmu.edu/community/rhas-workshop/2005/ silvafilho.pdf.

Shier, R.D. 1991. *Network Reliability and Algebraic Structures*, Clarendon Press, Oxford.

Shooman, M.L. 2002. Reliability of Fault-Tolerant Computing Systems and Networks, *The Journal of Reliability Analysis Center* 4: 1-7&11-12.

SQLite. *SQLite home page*: http://www.sqlite.org/

Suri, P.K. & Bhushan, B. 2007. Reliability Evaluation of Web Based Software, *International Journal of Computer Science and Network Security* 7(9): 151-156.

Tsai, W.T., Zhang, D., Chen, Y., Huang, H., Paul, R. & Liao, N. 2004. A Software Reliability Model For Web Services, *8th IASTED International Conference on Software Engineering and Applications*: 144 - 149.

Unidrv. *Universal Printer Driver Components*. Microsoft: http://msdn.microsoft.com/en-us/library/ms801791.aspx

Zaupa, F., Gimenes, I.M.S., Cowan, D., Alencar, P. & Lucena, C.J.P. 2008. A Service-oriented Process to Develop Web Applications. *Journal of Universal Computer Science* 14(8): 1368-1387.

Wasserman, A.I. 2005. *Principles for the Design of Web Applications*. http:// www.se-hci.org/bridging/interact2005 /07_Wasserman.pdf.

CORROSION MECHANISMS AND THEIR CONSEQUENCES FOR NUCLEAR POWER PLANTS WITH LIGHT WATER REACTORS

H.-P. Berg

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Bundesamt für Strahlenschutz, Salzgitter, Germany

e-mail: hberg@bfs.de

ABSTRACT

It is well known that operational conditions in light water reactors strongly influence the corrosion processes. This paper gives an overview which types of corrosion are identified in operating practice based on the evaluation of events which are reported to the authorities in line with the German reporting criteria. It has been found that the main contributor is the stress corrosion cracking. Several examples of different corrosion mechanisms and their consequences are provided for PWR although a high standard of quality of structures, systems and components has been achieved. Recommendations have been given to check the plant specifications concerning the use of auxiliary materials or fluids during maintenance as well as to examine visually the outer surfaces of austenitic piping with regard to residua of adhesive or adhesive tapes within the framework of in-service inspections. However, events in the last two years show that such problems cannot be totally avoided.

1 INTRODUCTION

In light-water reactor (LWR) plants corrosion processes are strongly affected by operational measured variables such as environment medium, construction, material and/or mechanical load. The substantial material strains by the operating pressure, the mass flow, the temperature of the cooling water, the special requirements of water chemistry (conductivity) represent a special hazard regarding corrosive material changes in this range. It requires complex testing facilities and measures (recurrent in-service inspections), in order to exclude to a large extent an occurring of disturbances and/or to prevent and/or limit, if necessary, effects of an event (e.g. by loss of coolant). Thus the safety-relevant components and systems are supervised in determined periods by recurrent in-service inspections regarding aging phenomena and aspects of their behaviour. In case of irregularities this leads to repairs or in individual cases to the change of the components concerned. Events which occurred in a plant are reported to other plants, so that necessary precaution measures are performed also in these plants. The evaluation of the result of the event analyses demonstrate the current safety status of the plant. The operational experiences are efficiently recorded by the application of national and international data bases such as those implemented by the Electric Power Research Institute in the US.

2 OPERATIONAL FACTORS INFLUENCING CORROSION PROCESSES

Important influence factors which can favour corrosion processes at safety-relevant components are the operating conditions existing in LWR plants such as water chemistry, assigned materials, mechanical and thermal loads, neutron irradiation, operational state (full power or outage) and geometrical factors. In particular in the first years of nuclear energy production, corrosion damages in the nuclear power plants led to undesired consequences. However, these undesired consequences could be reduced due to further developments and realizations in the condensate or feed-water treatment as by the implementation of more highly alloyed steel (thermal treatment, material status) as well optimization of manufacturing and construction of endangered components.

2.1 Assigned materials

Beside the austenitic CrNi steel which is used primarily world-wide also nickel base alloys are used as construction materials in LWR plants. In particular the nickel base alloy such as Inconel-600 (NiCr15Fe), which have been implemented in pressurized water reactors (PWR) of American, French and Japanese design particularly for steam generator tubes have shown a larger number of cases of damage due to cracking corrosion. Corrosion-supported cracking appeared at steam generator heat tubes in the tube plate area and also at control rod execution connecting pieces of reactor pressure vessel head and - internals. In German LWR, nickel base alloys are used to a substantially smaller extent. E.g. the tubes of steam generators consist of Incoloy-800 which has a less corrodibility. In comparison to Inconel-600 this alloy has a substantially higher chrome content. However, also in Germany for closure head penetrations nickel base alloys are used of the type Inconel-600. But this material has so far shown not any corrodibility.

Due to the fact that more corrosion-resistant materials are selected for the primary circuit of a PWR plant, that the material status has been approved by thermal treatment and reduction of mechanical tensions as well as that adherence to the required quality criteria of cooling agent chemistry (pH value, electrical conductivity, concentration of damaging ions) is achieved, damage cases due to corrosion cracking are limitable regarding the state of the art of science and technology.

2.2 Water chemistry

The most important operating medium in the primary circuit of the pressurized water reactor is water. For the minimization of corrosion processes and undesired lining formation on the hot water-affected metallic material surfaces the characteristics of the operating medium are influenced by chemical-technological measures. Aimed substances are added to the operating medium, which positively affect the corrosion procedures apart from the technological processes. From the requirements of technical rules such as German utility specific (VGB) guides, ISO standards or TRD sheets (special technical rules for steam boilers) and the American Water Chemistry Guidelines for PWR and BWR (boiling water reactors), the relevance of water chemistry in the power plant operation is evident. The first VGB guides were provided since 1925 and are regularly revised with regard to the current state-of-the-art.

2.3 Mechanical loads by temperature influence

Components such as tubes and containers, which contain steam or water with changing temperature, can be destroyed because of the periodic expansion and contraction of the material (thermal periods) and a corrosive medium by corrosion fatigue. The complicated interfaces between material, mechanical loads and medium is shown in Figure 1. The load cases, which can provide information about corrosion fatigue, are usually very plant specific. From the behaviour of other identically constructed nuclear installations operating experience for the own plant cannot be concluded in a straight forward manner. These plant-specific experiences have to be determined specifically and evaluated by fatigue monitoring which is performed parallel to operation.

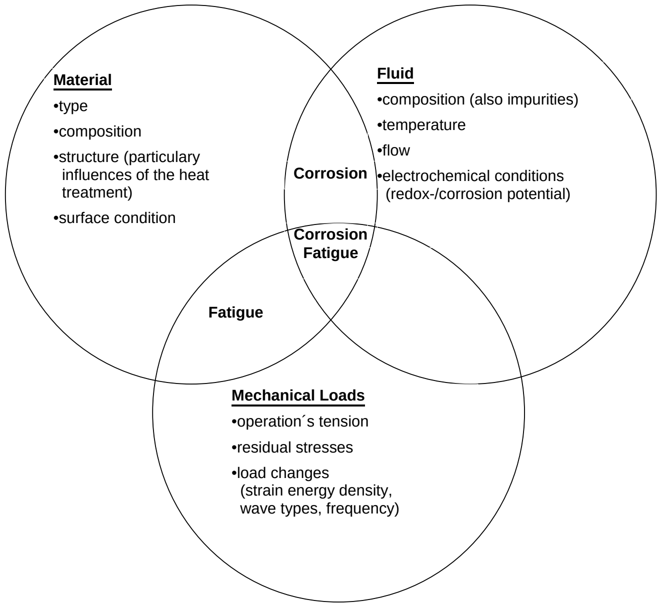

Figure 1. Factors influencing corrosion fatigue.

The investigation regarding fatigue is performed plant specific and is done in Germany following „general analysis of the mechanical behaviour" or as "component specific check" described in the relevant German Nuclear Safety Standard KTA 3201.2 (KTA 1996).

Avoiding of cracking due to corrosion fatigue is increased due to strict application of existing guidelines, monitoring of safety-relevant components by non-destructive testing methods, intensified observation of endangered components and avoidance of notches (increased local expansions and plasticizing), high sulfur content in the material, modifications in the tube diameters, water circulation with high oxygen content and low flow rate.

Mechanical and thermal loads on components can be reduced by careful plant operation.

2.4 Neutron irradiation

It is to be assumed that with increasing operation times of nuclear installations and the associated rising of neutron fluences the importance of material modifications, caused by radiation, increases. In this case also processes in the material can be effective, which appear in case of very high neutron fluences and may lead to material damages. The corrosion behaviour of metallic materials can be influenced by radiation in two different ways:

– Irradiation-induced modifications of the microstructure (radiation-induced grain boundary segregations and concentration modifications at the grain boundaries e.g. by radiation-induced chrome depletion). Radiation conditioned material modifications regarding radiation-induced stress corrosion can not easily be differentiated from the inter granular stress corrosion.

– Irradiation-induced modification of the water-chemical operating conditions. Also under reducing water-chemical conditions damage cases have been detected (cracking at austenitic screws within the core area of a PWR). These damages happen due to radiolysis and formation of oxygen and H_2O_2.

3 CORROSION TYPES

The differentiation of occurring corrosion types can be done according to the visible picture of the corrosive attack. An outline of the schematic partitioning is shown in Figure 2.

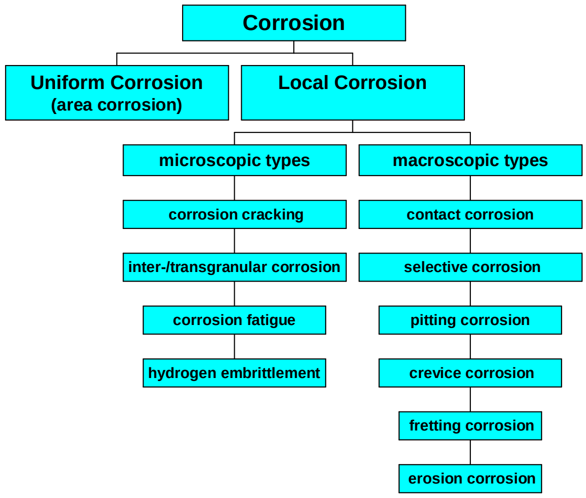

Figure 2. Outline of corrosion types.

In practice, special attention is dedicated to the local corrosion, because this type of corrosion can lead to unexpected damage (Gersinska 2003). A further distinction of the local corrosion into a macroscopic and microscopic type of attack appears appropriate (Schlicht-Szesny 2001). In contrast to the macroscopic type of attack, practically no "visible" corrosion product occurs in the microscopic type. These corrosion types are mostly caused by unexpected material failure, without any preceding considerable material losses.

4 INSPECTIONS OF CORROSION FINDINGS

Recurrent in-service inspections with suitable procedures as well as the execution of operating supervisions (e.g. leakage monitoring, oscillation monitoring etc.) represent the most important measures in order to determine material damages and thus also corrosion damages. As an additional measure for extending the level of knowledge on existing operating-conditioned damage mechanisms, supplementing tests are performed e.g. more detailed non destructive tests as well as destructive investigations at representative places of structures, systems and components, which were removed in the framework of exchange measures.

An overview of applied test kinds, and test techniques contains Table 1 (cf. (KTA 1999)). All specified testing methods only respond when separations in the material are present. Crack growth investigations on several years or fracture-mechanics crack growth computations give information over the further course of a crack.

In the following in the testing methods which are usually applied in nuclear power plants are presented briefly:

With the help of visual examinations, which are performed by application of suitable video cameras, leakages and breaks by piping systems are predominantly determined.

The ultrasonic testing (US) represents a frequently applied non-destructive inspection method in the context of the monitoring measures in nuclear installations. It enables the check of errors by the echolot principle. Short ultrasonic impulses with high pulse rate are led into the material, which are reflected at available material defects and represented at the testing set, according to the run time of the ultrasonic impulse.

In the nuclear technology the US check serves, in particular, for identification of the location of crack at surfaces of welding seams. The US testing method enables a good controllability of thick-walled components and of cracks as well as the determination of crack depths. However, practical experiences show that cracks often transmit only weak US signals. Additionally, echoes of cracks in geometrically more complicated places of clutters, as they frequently occur for example at welding seam roots, are often very difficult to differentiate from background echoes.

Table 1. Inspection types, procedures and techniques

Type of Test	Test Procedure	Test Technique
Examination with regard to cracks in the surface or in near-surface regions	Magnetic particle flaw detection	Magnetic particle examination, magnaflux examination
	Liquid penetrant examination	e.g. dye penetrant examination
	Ultrasonic examination procedure	e.g. surface waves, mode conversion, dual search units with longitudinal waves, electromagnetic ultrasonic waves
	Eddy-current examination procedure	Single frequency, multiple frequency
	Radiographic examination procedure	X-ray Radioisotope
	Selective visual examination	With or without optical means
Volumetric examination	Ultrasonic examination procedure	e.g. single probe technique with straight (ES) or angle beam scanning, tandem (angled pitch-catch) technique, mode conversion
	Radiographic examination procedure	X-ray Radioisotope
	Eddy-current examination procedure for thin walls	Single frequency Multiple frequency
Integral examination	Integral visual examination	—
	Pressure test	—
	Functional test	—

The eddy current examination is an important testing method in the nuclear technology that is based on the principle of the electromagnetic induction or eddy currents. In an electrically leading material with a field coil or a measuring probe, by which an alternating current flows, a magnetic field is produced that induces an electrical eddy current. This produces again a further magnetic field, which influenced the exciting magnetic field and thus causes a deviation of the impedance. The modification of the impedance of the receipt coil indicates cracks in the component. However, also local deviations of the electrical conductivity, the magnetic permeability and component geometry can lead to fault signals.

In case of radiographic examination procedure the material errors can be seen as shadows on a radiographic film or a picture-giving electronic system. In the framework of the recurrent in-service inspections, the radiographic examination procedure is not often used, which is justified by the relatively difficult searching of cracks. Inspection of thin-walled, austenitic welding seams show, however, that in case of good instrumentation prerequisites and using improved inspection techniques the finding of cracks leads to a satisfying inspection result.

5 SELECTED CORROSION FINDINGS AT PRESSURE CONDUCTING COMPONENTS IN PWR-SYSTEMS

The substantial material strains due to operating pressure, mass flow, and temperature of the cooling water, the special request of water chemistry (conductivity), radiation influences represent a higher hazard regarding corrosive material modifications in the primary circle of a nuclear power plant. This hazard requires complex testing facilities and measures in order to prevent or limit if necessary effects in case of occurring disturbances or of an incident (e.g. by loss of coolant). Thus the piping systems and pressure conducting components in LWR plants are of special interest in safety engineering. Apart from operating pressure and temperature, to the characteristics of the used materials as well as the sizing and constructional execution of piping systems in LWR plants, also the degree of purity of the cooling water plays an important role regarding corrosion processes.

In Figure 3 the number of reportable events whose causes were attributed to corrosion is presented for the period 1989 to 2001 for all nuclear power plants operated at present in Germany.

In this time period 136 events were reported in thirteen PWR plants and 86 events in five BWR plants.

Figure 4 lists the more recent number of reportable events whose causes were attributed to corrosion is presented for the period 2002 to 2008 for all nuclear power plants operated at present in Germany. In this time period 37 events were reported in twelve PWR plants and 41 events in five BWR plants, i.e. one PWR is out of operation in the meantime.

It is to be considered that the oldest PWR was already in operation since 1968 (the first one out of operation) and the youngest PWR since the end of 1988. The oldest BWR is since the year 1976 at the grit, the youngest since 1984.

Regarding the operational aging phenomena a rise of the events would be to be expected with increase of the operational years. However, an easy minimization of the occurrences is to be determined. Evaluation of reportable events allows to introduce preventive measures against certain corrosion phenomena in order to avoid corrosion damage. Due to the operational experience existing safety concepts could be further developed and effectively used in the context of nuclear safety research.

Root cause analyses of corrosion-afflicted safety-relevant components resulted in different corrosion types such as pitting corrosion, surface (uniform) corrosion, stress corrosion, corrosion fatigue, erosion corrosion, strain induced cracking corrosion as well as gap, contact, idle, and cavitation corrosion as well as hydrogen induced corrosion.

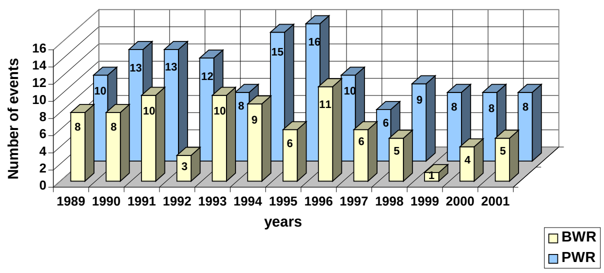

Figure 3. Reportable events regarding corrosion in German nuclear power plants (13 PWR, 5 BWR).

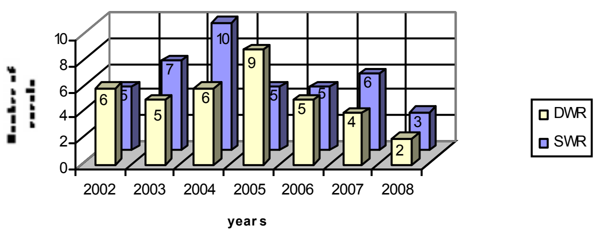

Figure 4. Reportable events regarding corrosion in German nuclear power plants (12 PWR, 5 BWR).

Figure 5 contains the distribution of the different corrosion types for all reportable events of all PWR and five BWR which are in operation in the Federal Republic of Germany in the two time schedules shown in Figures 3-4. As a result one can see that stress corrosion was most frequently identified both in PWR and in BWR plants. Pitting corrosion occurs in the PWR plants with 19% whereas corrosion fatigue in the BWR systems occurs with 17% fatigue.

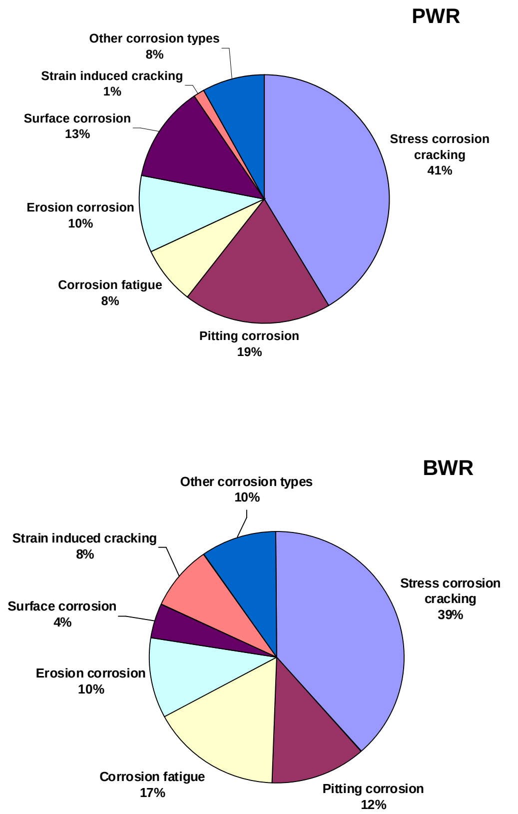

Figure 5. Distribution of corrosion types in PWR and BWR plants in Germany (Evaluation of reportable events 1968 – 2001).

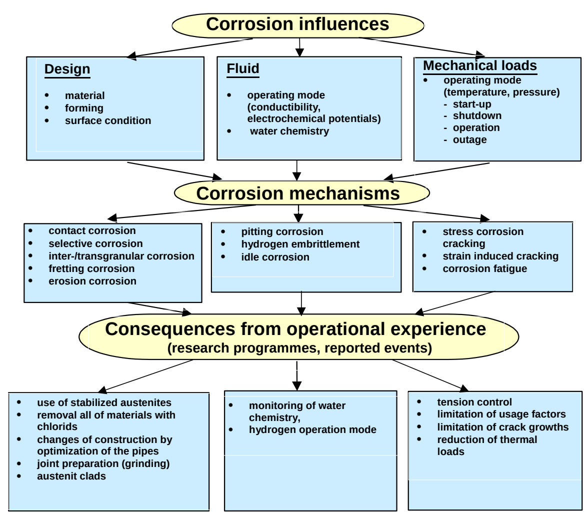

Figure 6. Root causes of different corrosion mechanisms and the operational implementation of protection goals in nuclear power plants.

Figure 6 shows an overview of the root causes of different corrosion mechanisms, the developed corrosion types and those preventive measures implemented in nuclear power plants.

In the following some practical examples of different corrosion types are explained, identified on the basis of root cause analyses by the operators of nuclear power plants with PWR.

5.1 Stress corrosion

During the spent fuel exchange, a fuel element (three service lives) was inspected which was detected by a Sipping test as a faulty element. The investigation showed reduced frictional forces of fuel rods and a cladding damage by fretting in the area of the first measuring rod. During the endoscopy of the rod meshes one found a broken and a marked out feather/spring. The findings were detected by visual examination and eddy current examination of fuel rods as well as frictional force measurement and endoscopy investigations.

As a root cause, inter granular stress corrosion of the Inconel feathers/springs was determined due to insufficient thermal treatment. The recovery of the damage took place by removing the faulty fuel rod from the fuel element. As a precautionary measure the fuel elements designated for the reapplication were enhanced by using an Inconel rod instead of zircaloy. The fuel elements designated for the new application were already designed using zircaloy.

5.2 Pitting and transgranular stress corrosion

During a compression test sample of the leakage detection line of the reactor pressure vessel (RVP) flange gasket at the pipe line section, a leakage occurred within the area of the RVP isolation.

A material investigation of the faulty tube part resulted in trans granular stress corrosion and pitting corrosion as a cause starting from a limited area with chloride deposits inside the tube. Those limited deposit and damage areas were determined by the temperature distribution (reactor pressure vessel outside temperature to ambient temperature) in the line.

For the recovery of the damage, the concerned and a comparable piece of piping was exchanged by two new tube parts with more corrosion-resistant material. A check of the comparable piece of pipe by a surface crack check of the half shells after isolating did not result in any findings.

5.3 Chloride-induced stress corrosion

In the context of the annual complete overhaul in a plant by a compression test of the leakage exhausting line, tear findings at small lines of the control valves of the pressurizer equipment station were determined. Due to these findings extensive additional examinations of the small lines of the pressure owner armature station took place. Tear findings resulted in the case of two further leakage exhausting lines as well as at the stuffing box return pipe of the pressurizer relief / isolating valve.

All other inspections of the small lines showed no findings. The tear findings did not have influence on the function behaviour of the pressure water relief valves. As a cause, a chloride-induced trans granular stress corrosion starting from the tubing inside was determined. The places corroded were in the transition from the thermally isolated to the not-isolated part of the piping within the areas where steam condenses.

A concentration of minimal chloride quantities which can not be excluded could not be avoided in this area. The damage was recovered by the exchange of the affected piping. To clarify the root cause further investigations of the concerned piping are performed. As a precautionary measure, an improved monitoring of the plugging book and leakage exhausting lines was realized by annual compression tests and non-destructive tests every four years.

Also in case of stabilized austenitic steel stress corrosion can occur under unfavourable conditions. Thus e.g. in earlier years the phenomenon of the "trans granular stress corrosion" was determined in particular within the area of austenitic piping and identified by extensive investigations as "chloride-induced stress corrosion". Due to the fact that the cooling agent does not contains chloride, no findings in respective through piping were observed. A "chloride-induced stress corrosion" can develop only if chloride containing substances together with water (sometimes only humidity) react due to concentration or deposits.

More general one can say that chloride-induced trans granular stress corrosion cracking (TGSCC) has occurred in German plants on fasteners as well as at inner and outer surfaces of piping all made of stabilised stainless steel due to contact with chloride-containing lubricants, sealing and auxiliary materials or fluids. The fasteners affected were located in the connections of core barrel/core baffle and in the reactor pressure vessel internals.

To prevent future damage, a chloride-free lubricant is to be used and changes are made in the design of the bolts to reduce notch stresses, TGSCC at inner surfaces has mainly been observed in small diameter pipes due to chloride-containing seals, which were replaced with chloride-free ones to prevent future damage. However, in some small pipes in which moistening and drying alternate for technological reasons, chloride concentrations may reach a critical level due to fortification even without any outer chloride source.

In the 90s some crack incidents from outer surface occurred even at piping of larger nominal bore. They have had no direct impact on plant safety. None of the events with leakage, which occurred at operating systems, would have led to an actuation of the safety systems, even in the case of pipe rupture. The availability of the safety systems concerned was given because of their redundancy. However, there was a loss of reliability in operating and safety systems. Recommendations have been given to check the plant specifications concerning the use of auxiliary materials or fluids during maintenance as well as to examine visually the outer surfaces of austenitic piping with regard to residua of adhesive or adhesive tapes within the framework of in-service inspections (Michel et al. 2001), (Schulz 2001).

In the revision of the year 2007 cracks were found in austenitic armatures of a nuclear power plant with BWR (cf. Fig. 7). 23 of 34 analyzed armatures are showing typical intra granular corrosion cracking.

This phenomenon typically occurs in the temperature region between 60°C und 90°C under stagnating medium conditions with a concentration of chlorine ions. Obviously it’s most important to avoid chloride concentrations as far as possible in the future.

5.4 Flow-accelerated corrosion

Flow-accelerated corrosion (FAC) is a world-wide problem in carbon or low-alloy steel piping of water/steam circuits of power plants. The experience with FAC on carbon steel piping in plants with PWR is summarised in Figure 8 (see (Schulz 2001)). To avoid FAC, in the 80s the German utilities replaced their condenser tubes made of copper alloys with new ones made of stainless steel or titanium. This replacement action creates suitable conditions for changing the water chemistry to „High“-All Volatile Treatment (HAVT, pH level >9.8).

Furthermore, the implementation of the basic safety concept led to improved flow conditions. In consequence, no damage with safety relevance has occurred in German NPPs due to FAC.

Boric acid corrosion of plants with PWRs may cause boric acid corrosion damage to low-alloy or carbon steel base material. Corresponding incidents occurred e.g. in the 80s in some US plants in areas on the reactor vessel head. In Germany, it is good practice not to operate with primary coolant leakage. In so far, boric acid corrosion is not an issue in Germany.

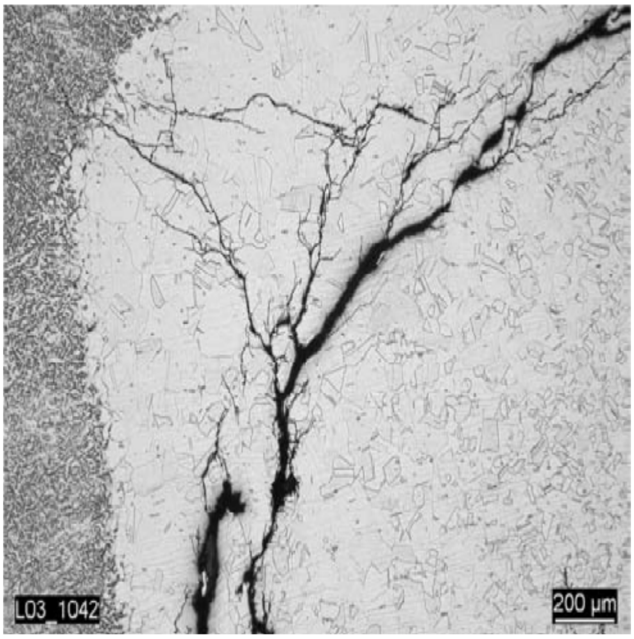

Figure 7. A typical example of a chloride-induced stress corrosion.

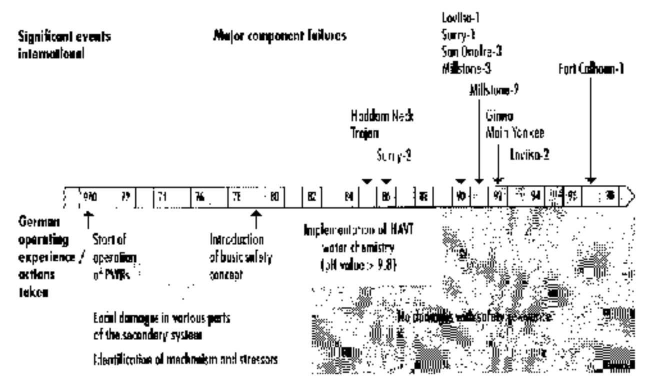

Figure 8. Thinning in PWR carbon steel piping due to flow-accelerated corrosion.

5.5 Idle corrosion

In the reactor cooling system of a PWR plant findings were determined in the area of the ground hand plating of the main cooling agent line. The findings were analyzed in the context of the recurrent in-service inspection (visual examination of the interior surface of the component) in the current revision by means of submarine. In relation to preceding checks the interior surfaces were investigated with an improved video system. First findings were identified in the area of the ground hand plating of the cold TH feeding connecting piece in one loop. After extension of the examination to all loops several displays were determined. The displays were situated all in the area of the ground hand plating. The integrity of the pressure boundaries was not impaired. As a root cause production related local and slag inclusions in ground position transitions of the hand plating were identified. The evaluation is not yet completed; however, operators and experts determined idle corrosion as a root cause. Whereas ferritic and austenitic steel is steady to a corrosion attack during the operation phase due to adjusted water chemistry, an idle corrosion is possible in case of a longer system status at lower temperature and sufficiently loosened oxygen in the medium. The following assumptions are currently under discussion:

1. During operation the corrosion potentials are alike with contacting ferritic and austenitic material areas (e.g. ferritic basic material with austenitic plating). By the use of an anti-corrosive protective layer consisting of magnetite of both materials the same corrosion potentials are assumed. Because the temperature was below the operating temperature sturdier magnetite is converted into the fewer sturdy trivalent ferric oxide/hydroxide. Breaking the ferric oxide/hydroxide layer and the medium enriched with oxygen cause a corrosion attack of the unprotected ferrite.

2. The active corrosion in the ferritic material affected via a locally different concentration of oxygen in the medium with ventilated (cathode) and not ventilated areas (anode) at the metal surface. The difference between the oxygen concentration and an oxygen depletion within the hollow area causes a potential difference and produces a current flow, which leads to the local dissolution of metal.

Results of these discussions are expected up to the end of the first quarter of 2010.

6 FINAL CONSIDERATIONS

The corrosion protection in the nuclear power plant technology is primarily the result of the operational experience over many years. During the last thirty years the high quality standard was developed by construction, manufacturing and quality assurance. It corresponds to the guidelines of the German Reactor Safety Commission and the relevant safety rules of the Nuclear Safety Standards Committee for German nuclear power plants.

By the application of optimized manufacturing processes and inspection techniques, materials with high-quality properties, in particular the tenacity, conservative limitation of the voltages, minimization of voltage peaks by optimal construction (avoidance of notches, sharp edges etc.) as well as evaluation of occurred failures the important measured influence factors of the corrosion phenomena could be reduced. The design of safety-relevant components is executed in nuclear power material -, manufacturing and test-fairly manner, also concerning the recurrent in-service inspections.

Thus following the state of the art for example in case of the reactor pressure vessel, welding seam constructions are minimized and replaced by smooth forgings. The use of qualitatively perfect product forms as well as a qualified and controlled welding seam manufacturing, by which the growth of stress corrosion is reduced as a consequence of the elimination of chromium carbides at the grain boundaries, is at present standard.

Numerous research work in the field of corrosion in nuclear installations is in close connection with the study of the boundaries, where corrosion cracking can occur. Experiences on the measured variables, which for example, determine the effectiveness of the environment medium, the height of the voltages and the corrosion resistance of the materials, allow to improve the construction and production of component components, but also the operation of nuclear power plants in an optimal manner.

Operational modifications such as aging by increase of the corrodibility are pursued by recurrence in the course of the decades with the help of further developed measuring technique and better estimation of life times. Likewise it is possible by the modern measuring technique to identify findings which could now be made visible but not in earlier years.

There are, nevertheless, still questions which are not yet sufficiently clarified in the area of the corrosion research. For a further extension of the present level of knowledge, investigations would be desirable in particular to the following topics:

– investigations of individual alloying constituents of austenitic materials and nickel based alloys, by which the radiation-induced stress corrosion can be influenced.

– investigations to radiation-influenced material modifications, radiation-induced stress corrosion and its distinction from inter granular stress corrosion.

– investigations of linings of intergranular cracking particularly in stabilized austenitic CrNi steel. Further autoclave experiments in sulfate and/or chloride-doped water.

One intergranular cracking phemomen is the irradiation assisted stress corrosion cracking which is investigated experimentally in more detail in (Fukuya et al. 2008) providing further insights.

For predictions, a mechanistic understanding of key parameters has to be developed, reliable predictive models have to formulated based on the mechanistic understanding and cost-effective mitigation technologies for stress corrosion cracking derived and are part of current comprehensive research (Dyle 2008).

In contrast to leaks and breaks of pipelines, quantitative predictions and reliability values for the different failure mechanisms are still not yet available. Therefore, large investigation programmes for different types of nuclear power plants have been performed on international level (Havel 2003) or are explicitly planned for 2009 and the following years, e. g. by the Electric Power Research Institute (Dyle 2008), (EPRI 2009 and 2010) within the primary systems corrosion research projects. Main topics of these projects are the identification of the key knowledge gaps in material degradation that could pose a threat to long-term reliable operation of light water reactors, improving the mechanistic understanding of crack initiation and early crack propagation processes that control stress corrosion cracking, development of improved predictive models and countermeasures for material corrosion in reactor internals and improved prediction and evaluation of environmentally assisted cracking in light water reactor structural materials as well as the development of reliable methods to predict and mitigate the early stages of damage and to significantly extend the life time of components

REFERENCES

Dyle, R. (2008). EPRI presentation. *NRC/DOE Workshop on Research and Development Issues*, February 2008.

Electric Power Research Institute (2009). EPRI Portfolio 2009-41.01-1, Primary Systems Corrosion Research, 2009 Nuclear Research Areas.

Electric Power Research Institute (2010). EPRI Portfolio 2010-41.01-1, Primary Systems Corrosion Research, 2010 Nuclear Research Areas.

Fukuya, K et al. (2008). Effects of dissolved hydrogen and strain rate on IASCC behavior in highly irradiated stainless steels. *Journal of Nuclear Science and Technology* 45, 452-458.

Gersinska, R. (2003). *Aktueller Kenntnisstand der Schadensbildungen durch interkristalline Spannungsrisskorrosion in austenitischen Rohrleitungen*. BfS-SK-IB-01, BfS Salzgitter, Februar 2003 (in German).

Havel, R. (2003). IAEA Extrabudgetary programme on mitigation of intergranular stress corrosion cracking in RBMK reactors.

Jiao, Z. & Was, G.S. (2008). Localized deformation and IASCC initiation in austenitic stainless steels. *Journal of Nuclear Materials* 382, 203-209.

Kerntechnischer Ausschuss. (1996). KTA 3201.2. *Components of the Reactor Coolant Pressure Boundary of Light Water Reactors;Part 2: Design and Analysis.*

Kerntechnischer Ausschuss. (1999). KTA 3210.4, *Components of the Reactor Coolant Pressure Boundary of Light Water Reactors; Part 4: In-service Inspections and Operational Monitoring.*

Michel, F., Reck, H. & Schulz, H. (2001). Experience with piping in German NPPs with respect to ageing-related aspect. *Nuclear Engineering and Design* 207, 307-316.

Schlicht-Szesny. (2001). *Korrosionsvorgänge und Korrosionsschäden in kerntechnischen Anlagen*. BfS-KT-IB-87, BfS Salzgitter, Dezember 2001 (in German).

Schulz, H. (2001). Limitations of the inspection and testing concepts for pressurised components from the viewpoint of operating experience. *EUROSAFE 2001. Nuclear Installation Safety, Seminar I*, 51-69.

IMPROVED APPROACH FOR ESTIMATING LEAK AND BREAK FREQUENCIES OF PIPING SYSTEMS IN PROBABILISTIC SAFETY ASSESSMENT

H.-P. Berg, R. Gersinska

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Bundesamt für Strahlenschutz, Salzgitter, Germany

e-mail: hberg@bfs.de

J. Sievers

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Gesellschaft für Anlagen- und Reaktorsicherheit, Köln, Germany

ABSTRACT

The estimation of leak and break frequencies in piping systems is part of the probabilistic safety assessment of technical plants. In this paper, the statistical method based on the evaluation of the German operational experience for piping systems with different diameters is described because an earlier estimation has been updated and extended introducing new methodical aspects and data. Major point is the inclusion of structure reliability models based on fracture mechanics calculation procedures. As an example of application the statistical estimation method for leak and break frequencies of piping systems with a nominal diameter of 50 mm (the volume control system of a German pressurized water reactor) was updated. Moreover, the evaluation of the operational experience was extended to 341 years with respect to cracks, leaks and breaks in the volume control system of German pressurized water reactors (PWR). Using the actual data base, new calculations of leak and break frequencies have been performed and the results have been compared with the previous values.

1 INTRODUCTION

In general, the likelihood of leaks of piping systems is of importance for the safety of process plants like chemical plants, both onshore and offshore industries and for nuclear power plants.

In case of all kind of process plants, leak and break frequencies are an input to any probabilistic safety assessment (PSA) of the process plant, usually called quantitative risk assessment (QRA) for these types of plants.

As explained in (Spuge 2005), standardized leak frequencies have been developed, based on recent data from offshore process, for different types of process equipment to ensure that consistent frequencies are available for any equipment type and hole size.

In the nuclear field, a report has been recently issued by the US regulatory body (U.S. NRC 2008) describing the development of leak frequency estimates, in particular for the loss-of-coolant accidents (LOCA), as a function of effective break size and operating time through the end of the plant license-renewal period. The estimates were based on an expert elicitation process consolidating operating experience and insights from probabilistic fracture mechanics studies with knowledge about the plant design, operation history and material performance during operation.

The elicitation required that each member of a group of international experts assessed qualitatively and quantitatively the important factors contributing to LOCA frequencies and quantify their uncertainties. Each member estimated the leak frequencies based on four reference cases.

The expert estimation for different systems and components was achieved by a factor relative to one reference case of his choice. After estimation each expert was asked in an interview for the rationale behind the given factor. A statistical evaluation of all answers was performed. Finally, the individual estimates were aggregated to obtain group estimates. Leak frequencies were provided for mean, median, 5^{th} and 95^{th} percentiles.

Compared to earlier evaluations for pressurized water reactors, the results of the elicitation are generally in good agreement, only for medium LOCA sizes (30 – 100 cm^2) the results of the elicitation process are significantly higher because of the high potential of the damage mechanism "primary water stress corrosion cracking".

In Germany, in accordance with § 19a of the national Atomic Energy Act a regulatory guideline exists for performing the probabilistic safety assessment (PSA) in the context of comprehensive safety reviews.

In addition, the Working Group „Probabilistic Safety Assessment for Nuclear Power Plants", leaded by BfS, had compiled two technical documents on methods and data for PSA (FAK PSA 2005 a), (FAK PSA 2005 b) which are usually updated after about five years. These documents also provide guidance how to deal with leak and break frequencies of pipes within a PSA.

This paper describes the statistical method, meanwhile updated, by including structure reliability models and using the recently extended database. Substantial relevant aspects were identified with reference to the determination of leak and break frequencies and proposals are provided for an update based on the current state-of-the-art.

2 DETERMINATION OF LEAK FREQUENCIES

2.1 Basic information

A leak in consequence of the failure of a piping can be caused by a wall-penetrating crack, by a break or by leaks at a solvable connection. According to experience a piping failure arises rarely at unimpaired ranges of the piping, but obviously more frequently on leak-relevant positions.

Typical examples for these positions are flanges, connections to components, elbow unions, reductions, reinforcement for pipe brackets, banks of tubes in heat exchangers and dissimilar welding seams.

Within such ranges stress enhancements exist, caused by changes in stiffness, inhomogeneous temperatures and flow within feeding ranges as well as by external additional loads, as e.g. bending moments or forces. Damages can, then, develop due to irregularities of the surface or by small flaws resulting from manufacturing in welding seams, which are in these ranges and which were not found either during the manufacture quality control process or were not evaluated as relevant findings.

Leak-relevant positions can be also in ranges which are affected by local corrosion-conditioned influences (e.g. enrichments, deposits, condensation, protective layer disturbances).

The following damage mechanisms are to be regarded at least:

- cracking due to thermal or mechanical fatigue or corrosion (e.g. stress corrosion cracking),
- material weakening by (planar) corrosion or erosion,
- overload (e.g. by internal pressure, temperature, malfunctioning of supports and shock absorbers, water hammer, condensation impact, ignition of radiolysis gas),
- assembly and maintenance error,
- external effects, e.g. from assembly and transport operations, earthquake.

Which types of damage causes are to be considered with the examined system depend particularly on the material, the dimensions, the medium and the operating conditions.

For example, mechanical oscillations can occur particularly with small nominal sizes, while thermal alternating loads, e.g. as a consequence of leaks of shutoff devices, are of higher importance with larger nominal sizes.

Influences from the commissioning phase or from longer shutdown periods can increase the frequency of certain damage causes, as e.g. from assembly and maintenance faults or due to corrosion mechanisms. In that context, also corrosion releasing aids during assembling and maintenance (e.g. chloride contamination by tapes and foils or lubricants) has to be considered.

Occurrence frequencies of leaks can be determined by a statistic evaluation of the operational experience. For the definition of the population to be included into this evaluation it is necessary to evaluate the comparability of the systems, materials, water chemistry, manufacture conditions and the quality and completeness of the experience feedback from the plants.

If possible statistics should be provided regarding the number of the leak-relevant positions of a system and/or a nominal size class. For the determination of the frequency of an event, the use of a statistic on precursor events is better than a zero-error statistic for the event (e.g. break). The correlation between the frequency of the precursor events and the event which has to be evaluated is to be estimated then by the damage mechanisms and the potential for the initial event.

In this context leaks due to wall-penetrating cracks should be considered to determine break frequencies.

No precursor events of a leak within certain systems with very high quality standard are to be expected.

This, at present, essentially applies to the main piping of the pressurized and boiling water reactors, which are laid out on the German basic safety principles. In such cases it is possible to determine extremely small leak frequencies (e.g. < 10^{-7}/a), which are clearly under the frequencies calculated from a zero-error statistics.

Such analyses were accomplished by different organisations with the help of probabilistic fracture-mechanics methods with comparable results. Usually for such piping a break preclusion is assumed.

For a break exclusion beside the concept of basis safety as laid down in documents of the German Reactor Safety Commission (RSK 1979), (RSK 1981) – an advisory board for the regulatory body –, a number of further additional measures are necessary for the qualification of piping.

These principle requirements, together with the work procedures for the quality assurance derived from it, were further developed and explained in (Beliczey & Schulz 1990), (Beliczey 1995), (Bieniussa et al. 1995) and (European Commission 2000).

2.2 Classification of the leaks

In the following leaks are defined as a comprehensive term for wall-penetrating cracks or breaks. For the determination of leak frequencies it is important in each case to define the structures the leak frequencies are to be referred.

Possible reference measures are:

– an entire system (e.g. not closable section of the emergency cooling and residual heat removal system TH, or a simply closable section),

– the unit of length of piping of a certain nominal size, or

– a welding seam.

The experience shows that the frequency of leaks depends on the regarded structure (e.g. straight tube, welding seam). Experience has shown that leaks from cracks preferably arise in the vicinity of welding seams.

The frequency of cracks is dominant at welding seams in close proximity to structural discontinuities (e.g. binding of a piping to a component).

These considerations are important for the selection of the leak-relevant design features and positions. Leaks in piping can be classified according to the following criteria:

- system (and/or function of the piping or the part of piping),
- plant condition with occurrence of the leak (e.g. operation conditions),
- design feature and/or position,
- leak size (related to the flow cross section F of the piping), and
- nominal size of the piping.

2.3 Data base

As ideal source for the estimation of the frequency of leaks the system dependent operational experience is considered. With very rare events, however, further considerations must be added. Because, if apart from the findings of the zero-error statistics no further realizations are used, a very conservative statement about the occurrence frequency results.

Independently from the fact whether on a system or on leak-relevant positions referred leak frequencies are to be determined, there is always the difficulty to obtain the knowledge of the structure of the system or of the number of the leak-relevant positions in all statistically seized plants.

For example, the determination of the leak-relevant positions can take place after studying the appropriate flow chart with the help of a plant inspection. In rare cases one will be able to determine these positions alone from the flow plans and piping isometries.

Due to the generally missing detailed knowledge of the plants considered in the statistics it is accepted that the number of the leak-relevant positions is in a certain system section of the plant which can be examined equal to the average value of this number in all plants considered for the statistics.

From this principle it can only be deviated when it is known from the plant under consideration that there is a substantially deviation concerning the number of certain positions from the average.

Thus, for example for the not-closable part of systems such as emergency cooling and residual heat removal system (TH) or volume control system two leak-relevant positions are assumed: one at the connection with the main cooling line (HKL), one at the isolation valve.

If a system section is very safety-relevant, one can be sure that the leak occurrences of all sizes were described in the usual operational experience documents.

Therefore, one will be able to consult the international operational experience (for example from USA, Japan, France) for larger parts than a nominal diameter of DN 15 mm, e.g. for breaks of not closable piping in the main cooling cycle.

As far as possible, it is reasonable to make use of common international databases such as the OPDE database (Lydell & Riznic 2008). Current results from this database show that leak frequencies dominate the whole piping failure frequency.

However, as a general principle, only those plants with similar materials should be considered. A restriction on the zero-error statistics in the not closable sections of German nuclear power plants would be too conservative.

2.4 Methodology for the determination of leak and break frequencies

In the following, the applied methodology for the determination of leak and break frequencies by means of evaluating the operating experience and the used statistic procedures is explained.

For the nominal diameter (given in mm) range 50 ≤ DN ≤ 150 the frequency of a wall-penetrating crack (leak) is given through the so-called Thomas formula (Thomas 1981):

$$\lambda_L = C \cdot \left(L_D \cdot D\right) / t_D^x \tag{1}$$

L_D Number of the leak-relevant positions,
D=DN Nominal diameter,
t_D Wall thickness of the piping with diameter D,
x Exponent with values within the range 2 to 3.

$$C = \frac{\sum_{D=50}^{D=150} N_{L,D}}{\sum_{D=50}^{D=150} \left(L_D \cdot D / t_D^x\right) \cdot T_D} \tag{2}$$

T_D Actual operation time,
$N_{L,D}$ Number of the arisen leaks (diameter D)

The constant C includes the operational experience in terms of number of leaks in piping with different nominal diameter forming a population, in relation to the operation time, to broaden the statistical basis for the leak frequency of the respective nominal diameter DN considered.

In the nominal diameter range 25 ≤ DN ≤ 250 the break frequency is estimated according to:

$$\lambda_B = \lambda_L \cdot 2{,}5 / DN \tag{3}$$

The evaluation of austenitic piping under fatigue load in (Beliczey & Schulz 1990) serves as basis. For DN < 50 as far as possible a direct statistic evaluation of leak and of break occurrences takes place. For primary cycle systems with 150 < DN < 250 the leak frequency corresponds to the same as for DN 150.

For primary cycle systems with DN ≥ 250, without the main cooling line, during basis-safety principle the following statements are valid:

The break frequency per leak relevant position λ_B is smaller than 10^{-7}/a for small systems (L_D< 10). λ_B is smaller than 10^{-8}/a for large systems. For the main cooling line, the break frequency λ_B is small (<10^{-7}/a) compared to the entire line.

In order to consider the uncertainties during the definition of certain input data, distributions are to be considered for these certain input values which determine the range for the uncertainty of the respective result.

2.5 Example of use for determination of leak and break frequencies by means of operating evaluation and statistic procedures

As an example the determination of the frequency of a break in the volume control system (TA) is described for a German pressurized water reactors (PWR).

In a first step, an adjustment of the example discussed in the existing document on PSA data (FAK PSA 2005) has been performed by a new evaluation of the operational experience in recent years.

The operating experience was extended from so far 191 years (until 1995) on to now 341 years (until 2006). With these updated data the leak and break frequencies were calculated new.

Table 1 gives an overview of the number of leak-relevant positions of the TA-system for 14 PWR divided into three groups A, B and C with structurally similar plants broken down into the operating conditions hot/cold.

Group of A covers 5 plants, B 8 plants and C is represented by only one plant. The range of the operational experience until 2006 amounts to 151 years (A), 153 years (B) and 37 years (C).

Table 2 shows the results of the new evaluation of the operational experience for the volume control system of German PWR. The reference time amounted to so far 191 years and with the new data now 341 years.

Table 1. Overview of the number of leak-relevant positions of the TA-system

DN [mm]	A cold	A hot	B cold	B hot	C cold	C hot
100	10	7	11	1	-	-
80	50	2	70	36	3	-
50	23	20	36	36	88	30
25	43	15	92	45	33	3
15	62	4	119	20	125	24

Based on the methodology described in this paper the break frequency per leak relevant position in the piping range of the nominal diameter DN 50 was calculated.

Table 3 shows the results of the new calculation of the example compared with the results given in (FAK PSA 2005).

This comparison shows that the new calculated frequencies have not changed significantly due to the evaluation of extended operational experience.

This result is exemplary and might not be typical for the behaviour of piping systems. Due to ageing effects, the influence of in-service-inspections, repairs performed and replacing of components the change in the number of leaks in relation to the operation time might lead to changes in the leak frequencies resulting from operational experience.

Table 2. Results of the new evaluation of the operational experience for the volume control system.

DN [mm]	Number of events (until 2006)	Number of leaks as break precursor in (FAK PSA 2005)	Number of leaks as break precursor (new evaluation)
15	6	2	4
25	11	6	6
50	8	2	7
80	3	1	2
100	5	-	1
Sum	33	11	20

Table 3. Comparison of the new calculation of the example with the results given in (FAK PSA 2005).

Measures for the break frequency distribution λ_B for DN 50 [mm]	**Former example in reference (FAK PSA 2005)**	**New calculation**
$\lambda_{B,5}$ (5%-quantile)	$2\cdot10^{-6}$	$4\cdot10^{-6}$
$\lambda_{B,50}$ (50%-quantile)	$7\cdot10^{-6}$	$1\cdot10^{-5}$
$\lambda_{B,95}$ (95%-quantile)	$3\cdot10^{-5}$	$4\cdot10^{-5}$
Expected value	$1\cdot10^{-5}$	$1\cdot10^{-5}$

2.6 Disadvantages of the methodology

Although the method as described is quite successful, there are some disadvantages and limits which should be mentioned:

– the specification of leak-relevant positions is very complex and not well-defined,

– the interpretation of the leaks as break precursor requires a large experience in fracture-mechanics and knowledge about the system,

– the result within a system section can be differentiated not further (e.g. regarding possible different loads for different positions).

These disadvantages are the reason for the coupling of the methodology with structural reliability models.

2.7 Structural reliability models

The described methodology which is based on statistics is not suitable for possible leak-relevant special characteristics (e.g. concerning the loading of pipes). For that purpose the use of structural reliability models would be necessary.

With the structural reliability programmes today it is possible to calculate the quantitative probabilities of leaks and breaks dependent on the position (e.g. for a certain welding seam) for certain damage mechanisms.

One proceeds as follows. For the substantial input measures (e. g. geometrical data, parameters characterising material properties, cracks, loading) distributions are identified. From this, for example, by applying Monte Carlo procedures a multiplicity of parameter combinations is randomly determined. With the help of fracture-mechanics procedures the growth of an initial crack for the respective parameter combination is determined. Altogether one receives a prognosis of the damage development of certain defect geometries under the loads which are to be expected.

Sections of a system can be differentiated regarding their failure relevance for the determination of the time and position dependent probability of leak by the employment of the structural reliability programmes. The probabilistic computation models are well suitable for the calculation of leak and break probabilities of piping and to determine trends quantitatively concerning the change of influence parameters.

Restrictions are seen in particular concerning the accuracy of absolute leak and break probabilities. The results depend to some extent strongly on the uncertainties during the definition of distributions for the relevant input parameters. In this context, parameters such as crack geometry, expected loads and those for the characterisation of the damage mechanisms play a substantial role.

A systematic comparison of different structural reliability programmes was made in the framework of (NURBIM 2004). Besides one US, English and Swedish programme, the structural reliability programme PROST developed by GRS (Grebner et al. 2004) participated in this comparison. The evaluation of the results shows that all programmes achieve the expected trends in the probability of leaks with variation of the input parameters. The probabilities of leak of the different codes agree well for the piping geometries considered.

Most of the structural reliability programmes available provide possibilities to include the effects of in-service-inspections and repair measures in the calculations on leak and break probabilities. A matter of further research might be the inclusion of time depending effects (e.g. due to ageing) in the input data of the structural reliability programmes.

3 INTEGRITY CONCEPT FOR PIPING SYSTEMS

As described above, methods based on statistics and structural reliability models are applied to get information on frequencies of possible leaks or breaks. Technically, precautionary measures are taken to exclude failures of safety relevant systems.

In Germany, a so-called integrity concept is applied (Hoffmann et al. 2007), in particular to exclude catastrophic failures of safety relevant pressure retaining components in nuclear power plants during operation.

This integrity concept is based on the requirements of assured basic safety characteristics such as design, construction, material properties and manufacturing. Complementary instruments which are implemented are the principle of multiple checking, worst-case principle, comprehensive plant monitoring, e.g. in the frame of ageing programmes, as well as the principle of verification of the actual quality status. This verification is performed on a continuous basis and, in addition, checked during the comprehensive safety review every ten years as part of the regulatory surveillance process.

Fracture mechanics safety analysis with postulated defect sizes as well as the experimental results of load behaviour to be expected are essential parts of the integrity concept. The measures determined in this way shall ensure that no major deviations from design values occur which has to be confirmed by periodic in-service inspections.

4 RISK-INFORMED IN-SERVICE INSPECTION

As explained above, the overall aim of the programme for in-service inspection of the piping at a nuclear power plant is to inspect the piping and identify areas of degradation that can be repaired before a failure occurs. The programme of inspections that is carried out has been based on a traditional deterministic approach and engineering judgement.

The use of risk informed in-service inspection methodologies in planning piping inspections in nuclear power plants is becoming mor and more common. The aim of the risk informed approach is to integrate service experiences, plant and operating conditions, other deterministic information and risk insights and to use the insights provided by the PSA to revise the programme of inspections that are carried out (in terms of the frequency of inspections, methods used, sample size, etc.), see for example (Berg 2009). As a consequence, the approach focuses on the segments of the pipe work that have the highest risk significance and reduces the inspections carried out on those with a low risk significance.

One recent approach (Männistö et al. 2009) is a fully quantitative risk informed in-service inspection methodology combining probabilistic fracture mechanics and discrete time Markov models.

Several different approaches have been developed for carrying out risk informed in-service inspection (European Commission 2005). Although the main steps are similar, the different methods and procedures differ considerably from each other in the way the evaluation and the selection of inspection sites are performed. All known risk informed in-service inspection methodologies are restricted to piping (NEA/OECD 2007).

Insights from the level 1 PSA should be used as one of the inputs in determining the piping segments to be addressed by the risk-informed in-service inspection project, the risk significance of the segments of piping being addressed, the target probabilities for the piping segments that are inspected and the changes in the risk which result from changes to the in-service inspection programme.

For piping failures leading to initiating events, the PSA should be used to determine the conditional core damage probability. For piping failures leading to the failure of standby systems or failure of systems on demand, the PSA should be used to calculate the conditional core damage frequency.

However, the piping failures that lead to the unavailability or failure on demand of safety systems are not generally included in the PSA model since the contribution to the failure probability of safety systems from failure of the pipe work is negligible in comparison to that from a failure of active components.

The rigorous way of determining the risk significance of all the segments of pipe work included in the risk-informed in-service inspection project would be to revise the PSA model to include these pipe work segments explicitly so that the core damage frequency and conditional core damage probability could be determined directly. This approach has been used in some countries.

When the revised in-service inspection programme has been defined, the PSA should be used to determine the risk insights needed for comparison against the decision criteria or guidelines used to assess the acceptability of the change in the in-service inspection programme.

This can be done by estimating what the change in the initiating event frequency or the component failure probability would be as a result of changes in the in-service inspection programme and rerunning the PSA or by carrying out sensitivity studies. In this process, the associated PSA limitations in terms of modelling details, scope, etc. should be recognized and taken into account.

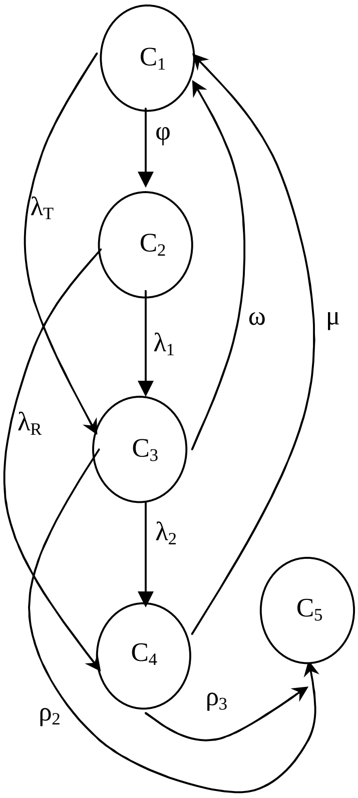
5 MARKOV MODELS FOR ESTIMATING PIPE FAILURES

As explained above, there are several different approaches to estimate pipe leak and break frequencies. One is based on statistical estimation from large databases and the other one on probabilistic fracture mechanics. In (Simola et al. 2004), the purposes of the approaches and the differences in modeling and data use are highlighted. The results of the break frequencies obtained by the two approaches are quite different, but one approach does not give systematically higher values than the other one.

It should be mentioned that the statistical analysis approach has also been developed in using a Markov model to allow an explicit modeling with respect to risk-informed in-service inspection strategies for piping systems in nuclear power plants (Fleming 2004).

The model described in (Fleming 2004) contains four pipe element states where one of it is a success state and may have the capability to model the main known pipe failure mechanisms. These failure mechanisms include damage mechanisms that operate in pipe base metal (e.g. flow accelerated corrosion), those that act on welds or in the heat affected zone near welds (e.g. thermal fatigue), combinations of mechanisms involving wall thinning and crack propagation, damage unrelated mechanisms such as those associated with severe loading such as water hammer and overpressure, and failures due to various combinations of these failure mechanisms.

However, the most general Markov model as a further development of (Fleming 2004) that has the capability to model at least all known pipe failure mechanisms is shown in Figure 1. It includes a further pipe element state compared with (Fleming 2004).



Pipe Element States

C_1 –Success, no detectable damage state
C_2 – Category 2 events, welding failures
C_3 – Category 3 events, part-cracks, full-cracks, reportable events
C_4 - Category 4 events, through-wall leaks
C_5 – Rupture or severe events

State Transition Rates

φ – category C_2 events occurrence rate
λ_1 – part-crack failure rate, given welding failures
λ_2 – leak failure rate, given part-crack failures
λ_R – leak failure rate given welding failures
λ_T - part-crack failure rate given success state
ρ_3 – rupture failure rate given leaks
ρ_2 – rupture failure rate given part-crack failures
ω – repair rate of part crack failures
μ – repair rate of leaking failures

Figure 1. Five state Markov model for all pipe failure mechanisms

The only 'success state' in the Markov model shown in Figure 1 is C_1, the others states are 'failure states' of different failure types with different severity of consequences.

When the solutions to the respective differential equations are solved, the time dependent probabilities of the piping component occupying each state can be determined.

Under the assumption that all the transition rates are constant, the Markov model equations consist of a set of coupled linear differential equations with constant coefficients. These equations can be solved analytically or numerically.

The appropriate reliability metric of the Markov model that quantifies the time dependent pipe rupture frequency is the system failure rate or hazard rate, as defined in the following.

To determine the system failure rate or hazard rate, one way is to first determine the system reliability function for the model and then to derive the hazard rate as a function of the reliability function according to the definition of the hazard rate as explained below. One approach is to focus on pipe ruptures and seek to estimate pipe rupture frequencies. Thus, instead of the definition of C_1 as the only 'success state', one can declare any state except that for rupture a 'success state'. This means that only the rupture state is a 'failure state'.

Using this concept, the reliability function for the Markov model, r{t}, is then given by

$$r\{t\} = 1 - C_5\{t\} = C_1\{t\} + C_2\{t\} + C_3\{t\} + C_4\{t\} \quad (4)$$

Under the above mentioned boundary condition, one can define from equation (4) the hazard rate for pipe ruptures (C_5), h{t}, as

$$h\{t\} = -\frac{1}{r\{t\}}\frac{dr\{t]}{dt} = \frac{1}{1 - C_5\{t\}}\frac{dC_5\{t\}}{dt} \quad (5)$$

The hazard rate, h{t}, is the time dependent frequency of pipe ruptures. The time dependent form of this rate strongly depends on the boundary conditions of the model and an asymptotic rate, which is a function of the parameters (transition rates) of the model.

6 CONCLUDING REMARKS

This paper explains the updated the method for the determination of leak and break frequencies in piping of German nuclear power plants which is proposed to be included in the revision of the documents on methods and data volume for the probabilistic safety assessment. The statistic methodology is based on the evaluation of the updated German operational experience for piping of different nominal diameters.

A direct generic statistical evaluation of the operating experience is only possible for small diameter piping (DN<50).

For larger pipings an estimation of leak frequencies needs additional assumptions like expert judgement and/or precursor evaluation, the consideration of equivalent systems or the results of trend analyses, for example performed in the frame of comprehensive safety reviews.

A detailed evaluation, e.g. of primary circuit leak frequencies, in a specific plant is time consuming and plant-specific data may be spare. In that case, it is recommended to use available generic frequency data. However, it has to be shown that the plant under consideration – a process plant or a nuclear power plant – is comparable to the plant where the generic data set results from.

This paper provided, in addition, an example for using the statistical method based on the evaluation of the German operating experience with nuclear power plants. The determination of the break frequency in the volume control system is described by expanding the operational experience from originally approx. 191 reactor years to 341 reactor years. Under these updated boundary conditions new computations of the leak and break frequencies were accomplished. The results show that the calculated break frequencies have not changed significantly due to the evaluation of extended operational experience compared with earlier results.

A further development of the methodology took place via an inclusion of structural reliability models based on fracture-mechanics computation methods.

There are several challenges in evaluating pipe failure probabilities and also in analyzing the effectiveness of inspections.

Moreover, pipe failure databases have been collected internationally, but the data is usually collected from nuclear power plants under a certain inspection programme which makes it difficult to compare different inspection programmes.

Because estimates of failure rates for nuclear power plant piping systems are important inputs to PSA and to risk-informed applications such as the approach of risk-informed in-service inspection as described above, the treatment of uncertainties is an important issue. Sources of uncertainty include failure data reporting issues, scarcity of data, inappropriate characterization of component populations as well as uncertainties about the physical characteristics of the failure mechanisms and root causes. A possible methodology for quantifying these uncertainties is provided in (Fleming & Lydell 2004).

As shortly mentioned, a combined method based on the application of probabilistic fracture mechanics and a Markov model has been developed as well as a statistical analysis approach also using a Markov model; however, it might be more likely to use semi-Markov processes for that purposes.

REFERENCES

Beliczey, S. (1995). *Grundlegende statistische Methoden.* GRS-Fachseminar zu „Ermittlung der Häufigkeiten von Lecks und Brücken in druckführenden Systemen für PSA“, Köln, 18.-20. September 1995 (published in German).

Beliczey, S. & Schulz, H. (1990). Comments on probabilities of leaks and breaks of safety-related piping in PWR plants. *Int. J. Pres. Ves. & Piping* Vol. 43, 219-227.

Berg, H.P. (2009). Overview on the different applications of probabilistic safety assessment for nuclear power plants. *Kerntechnik 74*, No 3, 106-110.

Bieniussa, K., Schulz, H., Goetsch, D., & Jalouneix, J. (1995). Integrity of the Reactor Coolant Boundary of the European Pressurized Water Reactor - Requirements for the Application of the Break Precluding Concept to the Main Coolant Lines, *Proceedings of the Seminar on Leak-Before-Break in Reactor Piping and Vessels*, Lyon. France, October 9-11, 1995. NUREG/CP-0155.

European Commission. (2000). *Final Report: Comparison of National Leak-Before-Break Procedures and Practices - Summary of Results and Potential for Greater Harmonisation.* Revision 2, EU Study Contract B7-5200/97/000782/ MAR/C2 of DG XI, April 2000.

European Commission. (2005). *Report on the Regulatory Experience of Risk-informed In-Service Inspection of Nuclear Power Plant Components and Common Views.* EUR 21320 EN, August 2004, Luxembourg, 2005.

Facharbeitskreis Probabilistische Sicherheitsanalyse für Kernkraftwerke (FAK PSA) (2005 a). *Methoden zur probabilistischen Sicherheitsanalyse für Kernkraftwerke, Stand: August 2005.* BfS-SCHR-37/05, Salzgitter, October 2005 (published in German).

Facharbeitskreis Probabilistische Sicherheitsanalyse für Kernkraftwerke (FAK PSA) (2005 b). *Daten zur probabilistischen Sicherheitsanalyse für Kernkraftwerke, Stand: August 2005.* BfS-SCHR-38/05, Salzgitter, October 2005 (published in German).

Fleming, K. (2004) Markov models for evaluating risk-informed in-service inspection strategies for nuclear power plant piping systems. *Reliability Engineering & Safety System* 83, 27-45.

Fleming, K. & Lydell, B. (2004). Database development and uncertainty treatment for estimating pipe failure rates and rupture frequencies. *Reliability Engineering & Safety System* 86, 227-246.

Grebner, H., Schimpfke, T., Peschke, J. & Sievers, J. (2004). *Weiterentwicklung der struktur-mechanischen Analysemethodik zur Bestimmung der Strukturzuverlässigkeit passiver Komponenten.* GRS-A-3236 (published in German).

Hoffmann, H. et al. (2007). Integrity concept for piping systems with corresponding leak and break postulates in German nuclear power plants. *VGB Power Tech* 7, 3-16.

Lydell, B. & Riznic, J. (2008). OPDE – the international pipe failure data exchange project. *Nuclear Engineering and Design* 238, 8, 2115-2123.

Männistö, I., Cronvall, O. & Simola, K. (2009). A Quantitative Method for RI-ISI Assessment, *Proceedings of the 20th International Conference on Structural Mechanics in Reactor Technology (SMiRT 20)*, Espoo, Finland, August 2009, Paper 1975 (on CD).

Nuclear Energy Agency/OECD. (2007). *Use and Development of Probabilistic Safety Assessment*, NEA/CSNI/R (2007) 12, November 2007.

NURBIM (2004). *Nuclear Risk-Based Inspection Methodology for Passive Components (NURBIM), 2004*. EU Contract FIKS-CT-2001-00172, Final report.

Reaktor-Sicherheitskommission (RSK) (1979). *2. Anhang zu den RSK-Leitlinien für Druckwasser-reaktoren 1979. (2. Ausgabe vom 24. Jan. 1979) Kapitel 4.2, Rahmenspezifikation Basissicherheit von druckführenden Komponenten, Stand 25.04.1979.*

Reaktor-Sicherheitskommission (RSK) (1981). *RSK-Leitlinien für Druckwasserreaktoren*. Ausgabe 10/1981.

Schulz, H. (1995). The Evolution of the Break Preclusion Concept for Nuclear Power Plants in Germany. *Proceedings of the Seminar on Leak-Before-Break in Reactor Piping and Vessels*. Lyon, France, October 9-11, 1995. NUREG/CP-0155.

Simola, K. et al. (2004). Comparison of approaches for estimating pipe rupture frequencies for risk-informed in-service inspections. *Reliability Engineering & Safety System* 84, 65-74.

Spuge, J. (2005). New generic leak frequencies for process equipment. *Process Safety Progress* 24, No 4, 249-257.

Thomas, H. M. (1981). Pipe and Vessel Failure Probability. *Rel. Engineering* 2., 83-124.

U.S. Nuclear Regulatory Commission (2008). *Estimating loss-of-coolant accident (LOCA) frequencies through the elicitation process, main report.* NUREG-1829, Vol. 1, April 2008.

RELIABILITY AND AVAILABILITY OF A GROUND SHIP-ROPE TRANSPORTER IN VARIABLE OPERATION CONDITIONS

A. Blokus-Roszkowska
K. Kołowrocki
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Gdynia Maritime University, Gdynia, Poland

e-mail: ablokus@am.gdynia.pl

ABSTRACT

In the paper the environment and infrastructure influence of the ground ship-rope transporter operating in Naval Shipyard in Gdynia on its operation processes is considered. The results are presented on the basis of a general model of technical systems operation processes related to their environment and infrastructure. The transporter operation process is described and its statistical identification is given. Next, the reliability, risk and availability evaluation of the transporter in variable operation conditions is presented. In addition, the reliability and availability basic characteristics of the system assuming its components' failure dependence are determined. Finally, the obtained results for the ground ship-rope transporter under the assumption that its components are dependent and independent are compared.

1 DESCRIPTION OF THE GROUND SHIP-ROPE TRANSPORTER IN NAVAL SHIPYARD IN GDYNIA

The ground ship-rope transporter in the Naval Shipyard in Gdynia is used to transfer ships coming to the shipyard for repairs from the platform to the repair post and back from the repair post to the platform.

Figure 1. The ship at the repair post R4.

First during ship docking the ship settled in special supporting carriages on the platform is raised to the wharf level and then the ship is transferred from the platform with the rope broaching machine on a traverse. Next the ship with the traverse, on which the ship is settled, is shifted in the repair post direction. Then after stretching the ropes from the ship to the broaching machine through some blocs, the ship is transferred from the traverser to the repair post. After some repair measures, the ship is transferred back to the traverser and then on the platform. Finally, during undocking the ship on the platform is moved down to the water.

There are nine repair posts, denoted by symbols R1-R9. The first repair post R1 can be lengthening to the post R1/B1 for long ships. There are also available two repair depots denoted by symbols B and D. Generally all kind of repairs can be carried out in any repair post. The repair posts R1 and R2 are equipped in crane. The submarines are repaired in the depot. Additionally large vessels are transferred to the repair post R1/B1. The scheme of the plan of repair post placing is given in Figure 2.

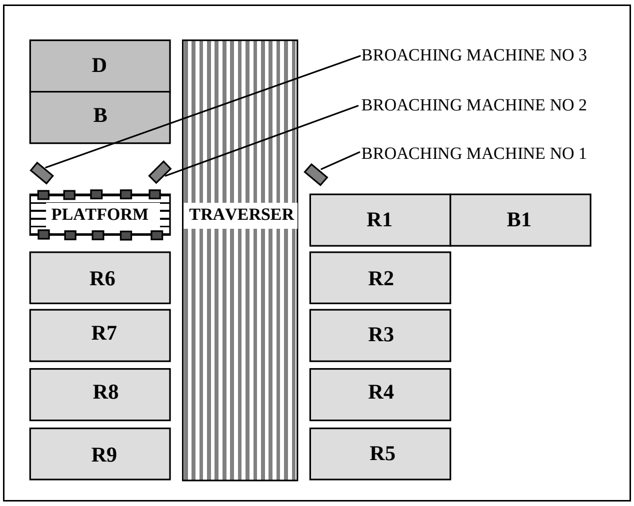

Figure 2. The scheme of the plan repair post placing.

The ground ship-rope transporter in the Naval Shipyard in Gdynia is composed of three broaching machines working independently equipped in the steel ropes "Drumet" with the diameter 30 mm. The load of steel ropes in the broaching machines is measured as a power consumption of amperage. The maximum of power consumption of broaching machines is 100 Ampere.

The ground ship-rope transporter reliability depends strongly on the tonnage of transferred ships and the place where the ship should be transferred. The broaching machines in the transportation system are numbered 1, 2, 3. There is used one or there are used two or possibly three broaching machines depending on weight and length of the ship and on which repair post the ship should be transferred. All three broaching machines are working in the extreme situation when large vessel over 1800 tonnes is transferred.

2 OPERATION PROCESS AND ITS STATISTICAL IDENTIFICATION

We analyze the ground ship-rope transporter in Naval Shipyard in Gdynia taking into account the system operation process and its varying in time reliability structures. Considering the weight and size of the vessel i.e. the system's loading and the place where the ship is transferred, that has influence on the decision which broaching machines are used we can distinguish following eight operation states:

– an operation state z_1 – the system is without loading, the time of waiting for the ship,

– an operation state z_2 – the ship with a tonnage up to 1300 tonnes is transferred from the platform to the traverser, from the traverser to the repair posts R1-R5 and from the repair posts R6-R9 to the traverser (the broaching machine no. 1 is used),

– an operation state z_3 – the ship with a tonnage up to 1300 tonnes is transferred from the traverser to the repair posts R6-R9, from the repair posts R1-R5 to the traverser and from the traverser to the platform (the broaching machine no. 3 is used),

– an operation state z_4 – the ship with a tonnage up to 1300 tonnes is transferred from the repair posts R1-R5 to the traverser and the access to the broaching machine number 3 is difficult (the broaching machine no. 2 is used),

– an operation state z_5 – the ship with a tonnage over 1300 up to 1800 tonnes is transferred from the platform to the traverser, from the traverser to the repair posts R1-R5 or from the repair posts R6-R9 to the traverser (the broaching machines 1 and 3 are used),

– an operation state z_6 – the ship with a tonnage over 1300 up to 1800 tonnes is transferred from the platform to the traverser, from the traverser to the repair posts R1-R5 or from the repair posts R6-R9 to the traverser and the access to the broaching machine number 3 is difficult (the broaching machines 1 and 2 are used),

– an operation state z_7 – the ship with a tonnage over 1300 up to 1800 tonnes is transferred from the traverser to the repair posts R6-R9, from the repair posts R1-R5 to the traverser or from the traverser to the platform (the broaching machines 2 and 3 are used),

– an operation state z_8 – the ship with a tonnage over 1800 tonnes is transferred (all broaching machines 1, 2 and 3 are used).

On the basis of the statistical data coming from experts using the ground ship-rope transporter in Naval Shipyard in Gdynia (Blokus-Roszkowska et al. 2009) the transition probabilities p_{bl} from the operation state z_b into the operation state z_l, $b, l = 1,...,8$, $b \neq l$, were evaluated. Their approximate evaluations are given in the matrix below.

$$[p_{bl}] = \begin{bmatrix} 0 & 0.3529 & 0.3529 & 0 & 0.0441 & 0 & 0.1618 & 0.0883 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

On the basis of statistical data coming from experiment (Blokus-Roszkowska et al. 2009) it is possible to evaluate approximately the conditional mean values $M_{bl} = E[\theta_{bl}]$, $b, l = 1,...,8$, $b \neq l$, of the lifetimes in the particular operation states.

$$M_{12} = 3613.33,\ M_{13} = 2620.21,\ M_{14} = 0,\ M_{15} = 3405.00,\ M_{16} = 0,\ M_{17} = 2001.36,\ M_{18} = 9229.17,$$
$$M_{21} = 65.25,\ M_{31} = 65.61,\ M_{41} = 0,\ M_{51} = 73.00,\ M_{61} = 0,\ M_{71} = 92.72,\ M_{81} = 120.00.$$

Hence, by (Blokus-Roszkowska et al. 2008b, Kołowrocki & Soszyńska 2008, Soszyńska 2006) the unconditional mean sojourn times in the particular operation states are determined from the formula

$$M_b = E[\theta_b] = \sum_{l=1}^{8} p_{bl} M_{bl},\ b = 1,...,8,$$

and takes values:

$$M_1 \cong 3494.92,\ M_2 \cong 65.25,\ M_3 \cong 65.61,\ M_4 = 0,\ M_5 \cong 73.00,\ M_6 = 0,\ M_7 \cong 92.72,\ M_8 \cong 120.00.$$

The limit values of the transient probabilities $p_b(t)$ at the operational states z_b, according to results given in (Blokus-Roszkowska et al. 2008b, Grabski 2002, Kołowrocki & Soszyńska 2008) are equal to:

$$p_1 = 0.9790,\ p_2 = 0.0064,\ p_3 = 0.0065,\ p_4 = 0,\ p_5 = 0.0009,\ p_6 = 0,\ p_7 = 0.0042,\ p_8 = 0.0030. \quad (1)$$

3 RELIABILITY OF THE GROUND SHIP-ROPE TRANSPORTER

According to rope reliability data given in their technical certificates and experts' opinions based on the nature of wire failures the following reliability states have been distinguished:

- a reliability state 3 – a wire is new, without any defects,
- a reliability state 2 – the corrosion of wire is greater than 0% and less than 25%,
- a reliability state 1 – the corrosion of wire is greater than or equal to 25% and less than 50%,
- a reliability state 0 – otherwise (a wire is failed).

The system consists of three broaching machines – subsystems $S_1,\ S_2,\ S_3$ linked in series. Further assuming that the ground ship-rope transporter is in the reliability state subset $\{1,2,3\},\{2,3\},\{3\},$ when all its subsystems are in this subset of reliability states, we conclude that the ground ship-rope transporter is a series system of subsystems $S_1,\ S_2,\ S_3$. In our further analysis considering broaching machines we will discuss the reliability of the rope system only, so we say that the broaching machine is in the reliability state subset $\{1,2,3\},\{2,3\},\{3\},$ if the rope in this broaching machine is in this state subset.

We assume that the reliability function of the subsystem $S_i,\ i=1,2,3,$ is given by the vector

$$\boldsymbol{R}_i(t,\cdot) = [\boldsymbol{R}_i(t,0), \boldsymbol{R}_i(t,1), \boldsymbol{R}_i(t,2), \boldsymbol{R}_i(t,3)],\ t \in <0,\infty),$$

with the co-ordinates

$$\boldsymbol{R}_i(t,u) = P(S_i(t) \ge u \mid S_i(0) = 3) = P(T_i(u) > t),\ t \in <0,\infty),\ u = 0,1,2,3,\ i = 1,2,3,\ \boldsymbol{R}_i(t,0) = 1.$$

$T_i(u)$, i = 1,2,3 are independent random variables representing the lifetimes of subsystems S_i in the reliability state subset $\{u, u + 1, ..., 3\}$, while they were at the reliability state 3 at the moment $t = 0$ and $S_i(t)$ are the subsystems S_i reliability states at the moment t, $t \in <0,\infty)$.

Then as the system is composed of three broaching machines – subsystems $S_1,\ S_2,\ S_3$ linked in series, according to results given in (Blokus-Roszkowska et al. 2008a), the reliability of the ground ship-rope transporter is defined by the vector

$$\overline{\boldsymbol{R}}(t,\cdot) = [1, \overline{\boldsymbol{R}}(t,1), \overline{\boldsymbol{R}}(t,2), \overline{\boldsymbol{R}}(t,3)],\ t \in <0,\infty),$$

where

$$\overline{\boldsymbol{R}}(t,u) = \prod_{i=1}^{3} \boldsymbol{R}_i(t,u),\ t \in <0,\infty),\ u = 1,2,3. \quad (2)$$

Each broaching machine $S_1,\ S_2,\ S_3$ is equipped with one rope that is composed of 6 identical strands. Each strand consists of 36 wires with a webbing core. We consider the wires as basic components of the system. The rope is in the reliability state subset $\{1,2,3\},\{2,3\},\{3\},$ if all 6 strand are in this subset, so it is a series system. After some consultations with experts we assume that the strand does not satisfy the technical conditions after breaking 6 of its 36 wires. With this assumption we conclude that the rope is in the reliability state subset $\{1,2,3\},\{2,3\},\{3\},$ when all six strands of the rope are in this state subset and each of the strand is in the reliability state subset $\{1,2,3\},\{2,3\},\{3\},$ if at least 30 out of its 36 wires are in this state subset. Thus, we obtain that the rope is a regular 4-states "30 out of 36"-series system composed of k_n = 6 series-linked strands with l_n = 36 parallel-linked components (wires). As each broaching machine has only one rope we can say that the broaching machines i.e. subsystems $S_1,\ S_2,\ S_3$, are also regular 4-state "30 out of 36"-series systems.

Moreover we assume that the ground ship-rope transporter subsystems $S_i,\ i=1,2,3,$ are composed of identical 4-state components (wires), having the multi-state reliability functions

$$R^{(b)}(t,\cdot) = [1, R^{(b)}(t,1), R^{(b)}(t,2), R^{(b)}(t,3)],$$

with exponential co-ordinates $R^{(b)}(t,1)$, $R^{(b)}(t,2)$ and $R^{(b)}(t,3)$ different in various operation states z_b, $b = 1,2,...,8$.

As all three subsystems S_i, $i = 1,2,3$, are identical "30 out of 36"-series systems in our further analysis we denote their reliability functions by $\overline{\boldsymbol{R}}^{(6)}_{6,36}(t,\cdot)$.

At the system operational state z_1 the system is composed of subsystems S_1, S_2 and S_3 linked in series. Thus, according to (2), the system reliability function is a vector:

$$\overline{\boldsymbol{R}}(t,\cdot) = [1, \overline{\boldsymbol{R}}(t,1), \overline{\boldsymbol{R}}(t,2), \overline{\boldsymbol{R}}(t,3)], \quad t \in <0,\infty),$$

where

$$\overline{\boldsymbol{R}}(t,u) = \left[\overline{\boldsymbol{R}}^{(6)}_{6,36}(t,u)\right]^3, \ t \in <0,\infty), \ u = 1,2,3. \tag{3}$$

At the system operational state z_1 components of subsystems S_1, S_2 and S_3 (wires in the ropes) have identical following conditional reliability functions co-ordinates:

$$R^{(1)}(t,1) = \exp[-0.0097t], \quad R^{(1)}(t,2) = \exp[-0.0147t], \quad R^{(1)}(t,3) = \exp[-0.0278t], \quad t \geq 0.$$

Thus, considering (3) and from (Blokus-Roszkowska et al. 2008b), the conditional multi-state reliability function of the ground ship-rope transporter at the operational state z_1 is given by:

$$[\overline{\boldsymbol{R}}(t,\cdot)]^{(1)} = [1, [\overline{\boldsymbol{R}}(t,1)]^{(1)}, [\overline{\boldsymbol{R}}(t,2)]^{(1)}, [\overline{\boldsymbol{R}}(t,3)]^{(1)}],$$

where

$$[\overline{\boldsymbol{R}}(t,1)]^{(1)} = \left[[\overline{\boldsymbol{R}}^{(6)}_{6,36}(t,1)]^{(1)}\right]^3 = [\sum_{i=0}^{6}\binom{36}{i}[1-\exp[-0.0097t]]^i \exp[-(36-i)0.0097t]]^{18}, \tag{4}$$

$$[\overline{\boldsymbol{R}}(t,2)]^{(1)} = \left[[\overline{\boldsymbol{R}}^{(6)}_{6,36}(t,2)]^{(1)}\right]^3 = [\sum_{i=0}^{6}\binom{36}{i}[1-\exp[-0.0147t]]^i \exp[-(36-i)0.0147t]]^{18}, \tag{5}$$

$$[\overline{\boldsymbol{R}}(t,3)]^{(1)} = \left[[\overline{\boldsymbol{R}}^{(6)}_{6,36}(t,3)]^{(1)}\right]^3 = [\sum_{i=0}^{6}\binom{36}{i}[1-\exp[-0.0278t]]^i \exp[-(36-i)0.0278t]]^{18}, \tag{6}$$

for $t \geq 0$.

The expected values and standard deviations of the ground ship-rope transporter conditional lifetimes in the reliability state subsets calculated from the above result given by (4)-(6), according to results given in (Blokus-Roszkowska et al. 2008b, Kołowrocki 2004) at the operation state z_1 are respectively given in years by:

$$\mu_1(1) \cong 9.4539, \ \mu_1(2) \cong 6.3866, \ \mu_1(3) \cong 3.3772, \tag{7}$$

$$\sigma_1(1) \cong 2.0576, \ \sigma_1(2) \cong 1.5939, \ \sigma_1(3) \cong 0.8422, \tag{8}$$

and further, using (7), from (Kołowrocki 2004) it follows that the conditional lifetimes in the particular reliability states at the operation state z_1 in years are:

$$\overline{\mu}_1(1) \cong 3.0673, \ \overline{\mu}_1(2) \cong 3.0094, \ \overline{\mu}_1(3) \cong 3.3772.$$

At the operational state z_2 the ship is transferred using the broaching machine number 1, so the system is composed of subsystem S_1. The scheme of the ground ship-rope transporter at the operational state z_2 is showed in Figure 3.

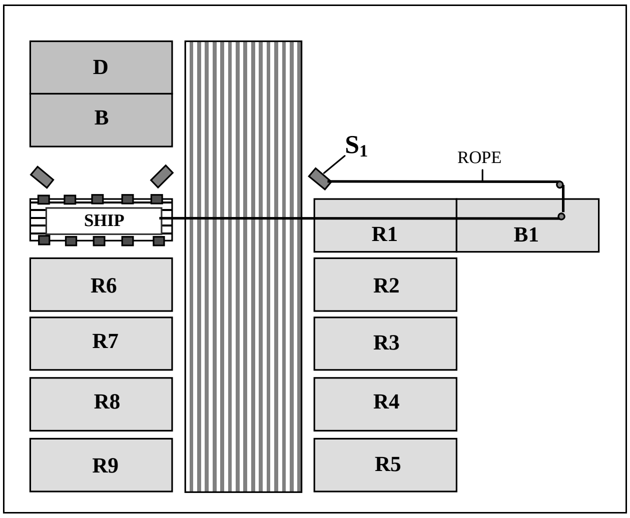

Figure 3. The scheme of the ground ship-rope transporter at the operational state z_2.

We assume that at the operational state z_2 wires in the ropes have following exponential conditional reliability functions co-ordinates:

$$R^{(2)}(t,1) = \exp[-0.0158t],\ R^{(2)}(t,2) = \exp[-0.0235t],\ R^{(2)}(t,3) = \exp[-0.0388t],\ \ t \ge 0.$$

As the system is composed only of subsystem S_1 the conditional multi-state reliability function of the ground ship-rope transporter at the operational state z_2 is given by:

$$[\overline{\boldsymbol{R}}(t,\cdot)]^{(2)} = [1, [\overline{\boldsymbol{R}}(t,1)]^{(2)},\ [\overline{\boldsymbol{R}}(t,2)]^{(2)}, [\overline{\boldsymbol{R}}(t,3)]^{(2)}],$$

where

$$[\overline{\boldsymbol{R}}(t,1)]^{(2)} = [\overline{\boldsymbol{R}}^{(6)}_{6,36}(t,1)]^{(2)} = [\sum_{i=0}^{6}\binom{36}{i}[1-\exp[-0.0158t]]^i \exp[-(36-i)0.0158t]]^6, \tag{9}$$

$$[\overline{\boldsymbol{R}}(t,2)]^{(2)} = [\overline{\boldsymbol{R}}^{(6)}_{6,36}(t,2)]^{(2)} = [\sum_{i=0}^{6}\binom{36}{i}[1-\exp[-0.0235t]]^i \exp[-(36-i)0.0235t]]^6, \tag{10}$$

$$[\overline{\boldsymbol{R}}(t,3)]^{(2)} = [\overline{\boldsymbol{R}}^{(6)}_{6,36}(t,3)]^{(2)} = [\sum_{i=0}^{6}\binom{36}{i}[1-\exp[-0.0388t]]^i \exp[-(36-i)0.0388t]]^6, \tag{11}$$

for $t \ge 0$.

The expected values and standard deviations of the ground ship-rope transporter conditional lifetimes in the reliability state subsets calculated from the above result given by (9)-(11), and from (Kołowrocki 2004) at the operation state z_2 given in years are:

$$\mu_2(1) \cong 7.7309,\ \mu_2(2) \cong 5.2210,\ \mu_2(3) \cong 3.1622, \tag{12}$$

$$\sigma_2(1) \cong 2.1062,\ \sigma_2(2) \cong 1.4722,\ \sigma_2(3) \cong 0.8912, \tag{13}$$

and further, using (12), from (Kołowrocki 2004) it follows that the conditional lifetimes in the particular reliability states at the operation state z_2 in years are:

$$\bar{\mu}_2(1) \cong 2.5099,\ \bar{\mu}_2(2) \cong 2.0588,\ \bar{\mu}_2(3) \cong 3.1622.$$

At the system operational state z_3 the system is composed of subsystem S_3. The ship is transferred using the broaching machine number 3 and the scheme is showed in Figure 4.

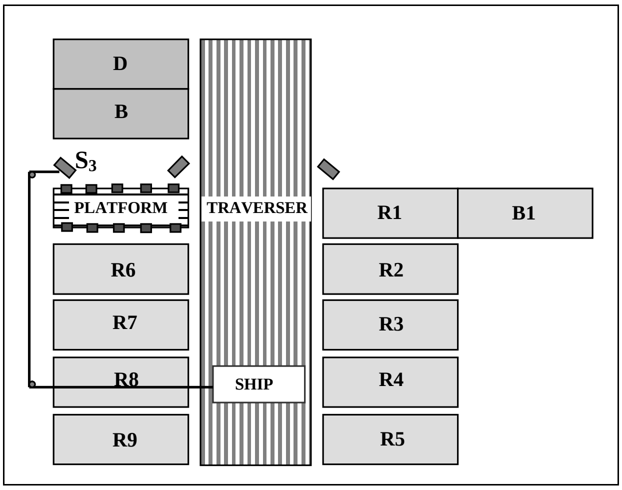

Figure 4. The scheme of the ground ship-rope transporter at the operational state z_3.

At the operational state z_4 the ship is transferred using the broaching machine number 2, so the system is composed of subsystem S_2.

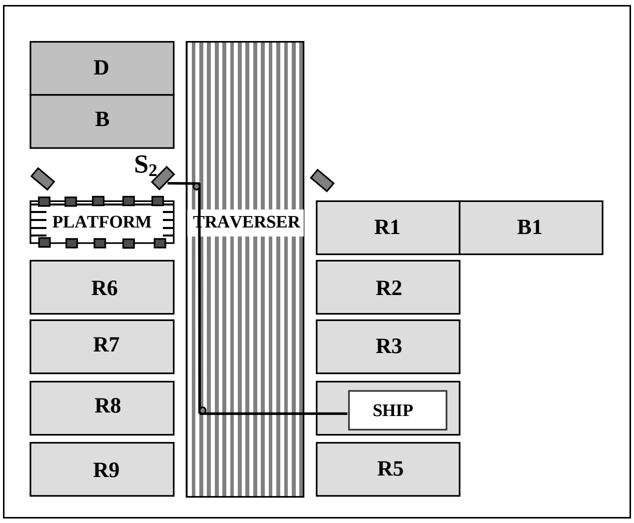

Figure 5. The scheme of the ground ship-rope transporter at the operational state z_4.

At the operation states z_3 and z_4 the system similarly as at the operation state z_2 is composed of one rope. As all ropes are composed of identical wires the conditional reliability function of the ground ship-rope transporter at the operation states z_3 and z_4 are the same as at the operation state z_2.

At the system operational state z_5 the system is composed of subsystems S_1 and S_3 linked in series. At the operational state z_5 the ship is transferred using the broaching machines number 1 and 3 and the scheme of the ground ship-rope transporter at the operational state z_5 is showed in Figure 6. Thus the system is a series system composed of identical two subsystems S_i, $i = 1,3$, and its reliability function is a vector:

$$\overline{\boldsymbol{R}}(t,\cdot) = [1, \overline{\boldsymbol{R}}(t,1), \overline{\boldsymbol{R}}(t,2), \overline{\boldsymbol{R}}(t,3)], \quad t \in < 0, \infty),$$

where

$$\overline{\boldsymbol{R}}(t,u) = \left[\overline{\boldsymbol{R}}^{(6)}_{6,36}(t,u)\right]^2, \; t \in < 0, \infty), \; u = 1,2,3. \tag{14}$$

The subsystems S_1 and S_3 are 4-state "30 out of 36"-series systems, in which components (wires in the ropes) have identical following conditional reliability functions co-ordinates:

$$R^{(5)}(t,1) = \exp[-0.0175t],\ R^{(5)}(t,2) = \exp[-0.0361t],\ R^{(5)}(t,3) = \exp[-0.0551t],\ t \geq 0.$$

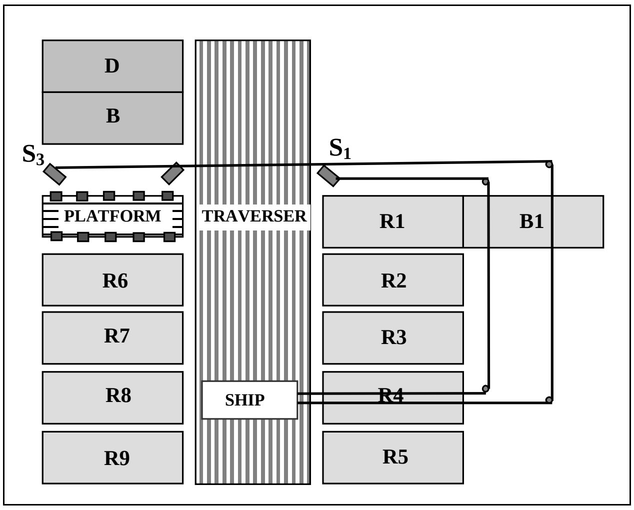

Figure 6. The scheme of the ground ship-rope transporter at the operational state z_5.

Thus, considering (14) and from (Blokus-Roszkowska et al. 2008b), the conditional multi-state reliability function of the ground ship-rope transporter at the operational state z_5 is given by:

$$[\overline{\boldsymbol{R}}(t,\cdot)]^{(5)} = [1, [\overline{\boldsymbol{R}}(t,1)]^{(5)}, [\overline{\boldsymbol{R}}(t,2)]^{(5)}, [\overline{\boldsymbol{R}}(t,3)]^{(5)}],$$

where

$$[\overline{\boldsymbol{R}}(t,1)]^{(5)} = \left[[\overline{\boldsymbol{R}}^{(6)}_{6,36}(t,1)]^{(5)}\right]^2 = [\sum_{i=0}^{6}\binom{36}{i}[1-\exp[-0.0175t]]^i \exp[-(36-i)0.0175t]]^{12}, \quad (15)$$

$$[\overline{\boldsymbol{R}}(t,2)]^{(5)} = \left[[\overline{\boldsymbol{R}}^{(6)}_{6,36}(t,2)]^{(5)}\right]^2 = [\sum_{i=0}^{6}\binom{36}{i}[1-\exp[-0.0361t]]^i \exp[-(36-i)0.0.361t]]^{12}, \quad (16)$$

$$[\overline{\boldsymbol{R}}(t,3)]^{(5)} = \left[[\overline{\boldsymbol{R}}^{(6)}_{6,36}(t,3)]^{(5)}\right]^2 = [\sum_{i=0}^{6}\binom{36}{i}[1-\exp[-0.0551t]]^i \exp[-(36-i)0.0551t]]^{12}, \quad (17)$$

for $t \geq 0$.

The expected values and standard deviations, from results in (Kołowrocki 2004), of the ground ship-rope transporter conditional lifetimes in the reliability state subsets at the operation state z_5 counted in years are:

$$\mu_5(1) \cong 5.8962,\ \mu_5(2) \cong 2.8583,\ \mu_5(3) \cong 1.8727, \quad (18)$$

$$\sigma_5(1) \cong 1.5326,\ \sigma_5(2) \cong 0.7421,\ \sigma_5(3) \cong 0.4852. \quad (19)$$

Hence the conditional lifetimes in the particular reliability states at the operation state z_5 in years are:

$$\bar{\mu}_5(1) \cong 3.0379,\ \bar{\mu}_5(2) \cong 0.9856,\ \bar{\mu}_5(3) \cong 1.8727.$$

At the operation states z_6 and z_7 the system similarly as at the operation state z_5 is composed of two ropes, thus the conditional reliability function of the ground ship-rope transporter at the operation states z_6 and z_7 are the same as at the operation state z_5.

At the system operational state z_6 the system is composed of subsystems S_1 and S_2 linked in series. The ship is transferred using the broaching machines number 1 and 2 and the scheme of the ground ship-rope transporter at the operational state z_6 is presented in Figure 7.

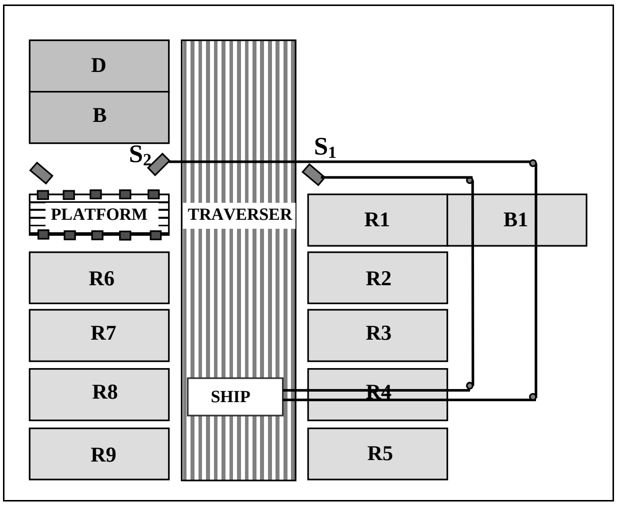

Figure 7. The scheme of the ground ship-rope transporter at the operational state z_6.

Whereas at the system operational state z_7 the system is composed of subsystems S_2 and S_3 linked in series. Then the ship is transferred using the broaching machines number 2 and 3 and the scheme of this situation is showed in Figure 8.

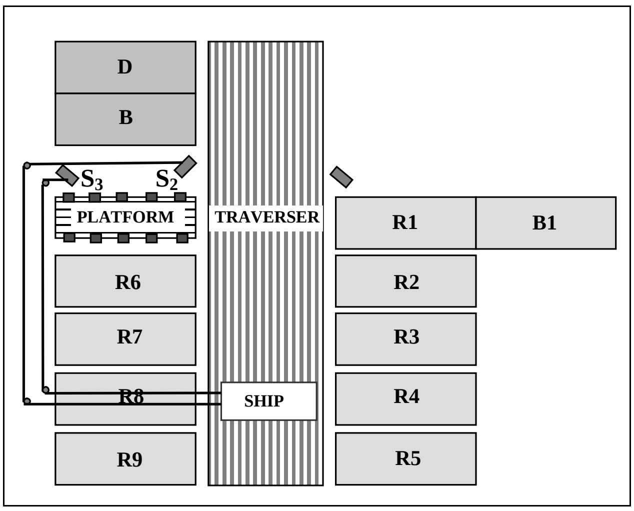

Figure 8. The scheme of the ground ship-rope transporter at the operational state z_7.

At the operational state z_8 the system is composed of subsystems S_1, S_2 and S_3 linked in series. At the operational state z_8 the ship is transferred using all three broaching machines 1,2 and 3 (Figure 9). Thus the system is a series system composed of three identical subsystems S_i, $i = 1,2,3,$ and its reliability function, according to (3), is a vector:

$$\overline{\boldsymbol{R}}(t,\cdot) = [1, \overline{\boldsymbol{R}}(t,1), \overline{\boldsymbol{R}}(t,2), \ \overline{\boldsymbol{R}}(t,3)], \quad t \in <0,\infty),$$

where

$$\overline{\boldsymbol{R}}(t,u) = \left[\overline{\boldsymbol{R}}^{(6)}_{6,36}(t,u)\right]^3, \ t \in <0,\infty), \ u = 1,2,3. \tag{20}$$

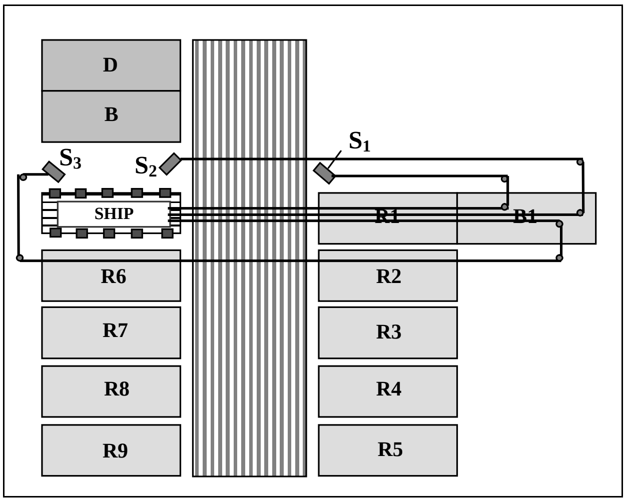

Figure 9. The scheme of the ground ship-rope transporter at the operational state z_8.

The subsystems S_1, S_2 and S_3 are 4-state "30 out of 36"-series systems, in which components (wires in the ropes) have identical following conditional reliability functions co-ordinates:

$$R^{(8)}(t,1) = \exp[-0.0215t],\ R^{(8)}(t,2) = \exp[-0.0394t],\ R^{(8)}(t,3) = \exp[-0.0607t],\ t \geq 0.$$

Thus, considering (20) and from (Blokus-Roszkowska et al. 2008b), the conditional multi-state reliability function of the ground ship-rope transporter at the operational state z_8 is given by:

$$[\overline{\boldsymbol{R}}(t,\cdot)]^{(8)} = [1, [\overline{\boldsymbol{R}}(t,1)]^{(8)}, [\overline{\boldsymbol{R}}(t,2)]^{(8)}, [\overline{\boldsymbol{R}}(t,3)]^{(8)}],$$

where

$$[\overline{\boldsymbol{R}}(t,1)]^{(8)} = \left[[\overline{\boldsymbol{R}}^{(6)}_{6,36}(t,1)]^{(8)}\right]^3 = [\sum_{i=0}^{6}\binom{36}{i}[1-\exp[-0.0215t]]^i \exp[-(36-i)0.0215t]]^{18}, \quad (21)$$

$$[\overline{\boldsymbol{R}}(t,2)]^{(8)} = \left[[\overline{\boldsymbol{R}}^{(6)}_{6,36}(t,2)]^{(8)}\right]^3 = [\sum_{i=0}^{6}\binom{36}{i}[1-\exp[-0.0394t]]^i \exp[-(36-i)0.0.394t]]^{18}, \quad (22)$$

$$[\overline{\boldsymbol{R}}(t,3)]^{(8)} = \left[[\overline{\boldsymbol{R}}^{(6)}_{6,36}(t,3)]^{(8)}\right]^3 = [\sum_{i=0}^{6}\binom{36}{i}[1-\exp[-0.0607t]]^i \exp[-(36-i)0.0607t]]^{18}, \quad (23)$$

for $t \geq 0$.

The expected values and standard deviations of the ground ship-rope transporter conditional lifetimes in the reliability state subsets calculated from the above result, according to results given in (Kołowrocki 2004) at the operation state z_8, in years, are respectively given by:

$$\mu_8(1) \cong 4.3668,\ \mu_8(2) \cong 2.3829,\ \mu_8(3) \cong 1.5467, \quad (24)$$

$$\sigma_8(1) \cong 0.8427,\ \sigma_8(2) \cong 0.5935,\ \sigma_8(3) \cong 0.3841. \quad (25)$$

and further, using (24) and from (Kołowrocki 2004), the conditional lifetimes in the particular reliability states at the operation state z_8 in years are:

$$\bar{\mu}_8(1) \cong 1.9839,\ \bar{\mu}_8(2) \cong 0.8362,\ \bar{\mu}_8(3) \cong 1.5467.$$

In the case when the operation time is large enough its unconditional multi-state reliability function of the ground ship-rope transporter is given by the vector

$$\overline{\boldsymbol{R}}(t,\cdot) = [1, \overline{\boldsymbol{R}}(t,1), \overline{\boldsymbol{R}}(t,2), \overline{\boldsymbol{R}}(t,3)],\ \ t \geq 0,$$

where according to (Blokus-Roszkowska et al. 2008b, Soszyńska 2006), the vector co-ordinates are given respectively by:

$$\overline{\boldsymbol{R}}(t,u) = \sum_{i=1}^{8} p_i [\overline{\boldsymbol{R}}(t,u)]^{(i)},\ t \geq 0,\ u = 1,2,3, \quad (26)$$

where $[\overline{\boldsymbol{R}}(t,u)]^{(i)}$, $i = 1,\ldots,8$, are given by (4)-(6), (9)-(11), (15)-(17), (21)-(23).

The mean values and the standard deviations of the ground ship-rope transporter unconditional lifetimes in the reliability state subsets, according to (Blokus-Roszkowska et al. 2008b) and after considering (7)-(8), (12)-(13), (18)-(19), (24)-(25) and (1), respectively are:

$$\mu(1) = \sum_{i=1}^{8} p_i \, \mu_i \; (1) \cong 9.3996, \; \sigma(1) \cong 2.0901, \tag{27}$$

$$\mu(2) = \sum_{i=1}^{8} p_i \, \mu_i \; (2) \cong 6.3424, \; \sigma(2) \cong 1.6234, \tag{28}$$

$$\mu(3) = \sum_{i=1}^{8} p_i \, \mu_i \; (3) \cong 3.3613, \; \sigma(3) \cong 0.8532. \tag{29}$$

Next, the unconditional mean values of the ground ship-rope transporter lifetimes in the particular reliability states, by (Kołowrocki 2004) and considering (27)-(29), in years are:

$$\bar{\mu}(1) = \mu(1) - \mu(2) = 3.0572, \; \bar{\mu}(2) = \mu(2) - \mu(3) = 2.9811, \; \bar{\mu}(3) = \mu(3) = 3.3613.$$

If the critical reliability state is r = 2, then according to (Blokus-Roszkowska et al. 2008b, Kołowrocki 2004), the system risk function takes the form

$$\boldsymbol{r}(t) = 1 - \overline{\boldsymbol{R}}(t,2) = 1 - \sum_{i=1}^{8} p_i [\overline{\boldsymbol{R}}(t,2)]^{(i)}, \; t \geq 0.$$

where $\overline{\boldsymbol{R}}(t,2)$ is the unconditional reliability function of the ground ship-rope transporter at the critical state.

Hence, the moment when the system risk function exceeds a permitted level, for instance δ = 0.05, from (Blokus-Roszkowska et al. 2008a, Kołowrocki 2004), is

$$\tau = \boldsymbol{r}^{-1}(\delta) \cong 3.685 \text{ years} \cong 3 \text{ years } 250 \text{ days}.$$

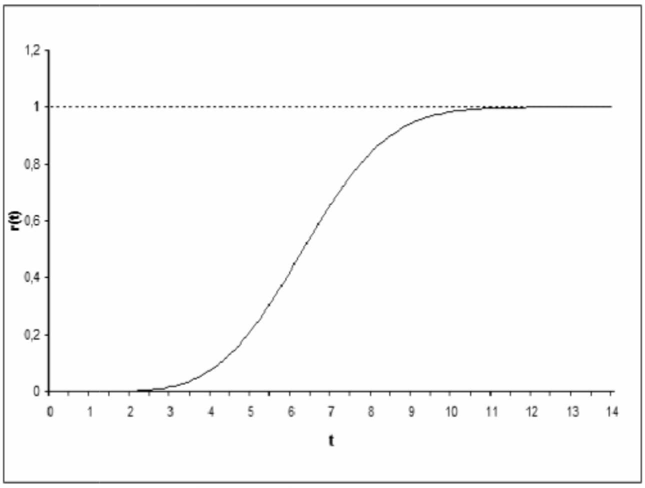

Figure 10. The graph of the ground ship-rope transporter risk function $r(t)$.

4 AVAILABILITY OF THE GROUND SHIP-ROPE TRANSPORTER

In this point the asymptotic evaluation of the basic reliability and availability characteristics of renewal systems with non-ignored time of renovation are determined in an example of the ground ship-rope transporter.

Assuming that the ground ship-rope transporter is repaired after its failure and that the time of the system renovation is not ignored and it has the mean value $\mu_0(2) = 0.0014 \cong 12$ hours and the standard deviation $\sigma_0(2) = 0.0002 \cong 2$ hours, applying theoretical results presented in (Blokus-Roszkowska et al. 2008a), we obtain the following results:

– the distribution function of the time $\bar{\bar{S}}_N(2)$ until the *Nth* system's renovation, for sufficiently large *N*, has approximately normal distribution $N(6.3438N, 1.6234\sqrt{N})$, i.e.,

$$\bar{\bar{F}}^{(N)}(t,2) = P(\bar{\bar{S}}_N(2) < t) \cong F_{N(0,1)}(\frac{t - 6.3438N}{1.6234\sqrt{N}}),\ t \in (-\infty, \infty),\ N = 1,2,...,$$

– the expected value and the variance of the time $\bar{\bar{S}}_N(2)$ until the *Nth* system's renovation take respectively forms

$$E[\bar{\bar{S}}_N(2)] \cong 6.3438N,\ D[\bar{\bar{S}}_N(2)] \cong 2.6354N,$$

– the distribution function of the time $\bar{S}_N(2)$ until the *Nth* exceeding the reliability critical state 2 of this system takes form

$$\bar{F}^{(N)}(t,2) = P(\bar{S}_N(2) < t) = F_{N(0,1)}(\frac{t - 6.3438N + 0.0014}{1.6234\sqrt{N}}),\ t \in (-\infty, \infty),\ N = 1,2,...,$$

– the expected value and the variance of the time $\bar{S}_N(2)$ until the *Nth* exceeding the reliability critical state 2 of this system take respectively forms

$$E[\bar{S}_N(2)] \cong 6.3424N + 0.0014(N-1),\ D[\bar{S}_N(2)] \cong 2.6354N,$$

– the distribution of the number $\bar{\bar{N}}(t,2)$ of system's renovations up to the moment $t,\ t \geq 0,$ is of the form

$$P(\bar{\bar{N}}(t,2) = N) \cong F_{N(0,1)}(\frac{6.3438 - t}{0.6445\sqrt{t}}) - F_{N(0,1)}(\frac{6.3438(N+1) - t}{0.6445\sqrt{t}}),\ N = 1,2,...,$$

– the expected value and the variance of the number $\bar{\bar{N}}(t,2)$ of system's renovations up to the moment $t,\ t \geq 0,$ take respectively forms

$$\bar{\bar{H}}(t,2) \cong 0.1576t,\ \ \bar{\bar{D}}(t,2) \cong 0.0103t,$$

– the distribution of the number $\bar{N}(t,2)$ of exceeding the reliability critical state 2 of this system up to the moment $t,\ t \geq 0,$ is of the form

$$P(\bar{N}(t,2) = N) \cong F_{N(0,1)}(\frac{6.3438N - t - 0.0014}{0.6445\sqrt{t + 0.0014}}) - F_{N(0,1)}(\frac{6.3438(N+1) - t - 00014}{0.6445\sqrt{t + 0.0014}}),\ N = 1,2,...,$$

– the expected value and the variance of the number $\bar{N}(t,2)$ of exceeding the reliability critical state 2 of this system up to the moment $t,\ t \geq 0,$ are respectively given by

$$\bar{H}(t,2) \cong 0.1576(t + 0.0014),\ \ \bar{D}(t,1) \cong 0.0103(t + 0.0014),$$

– the availability coefficient of the system at the moment *t* is given by the formula

$$K(t,2) \cong 0.9998,\ t \geq 0,$$

– the availability coefficient of the system in the time interval $< t, t+\tau), \tau > 0,$ is given by the formula

$$K(t,\tau,2) \cong 0.1576 \int_{\tau}^{\infty} \bar{\boldsymbol{R}}(t,2)dt,\ t \geq 0,\ \tau > 0,$$

where the reliability function of a system at the critical state $\bar{\boldsymbol{R}}(t,2)$ is given by the formula (26).

5 THE GROUND SHIP-ROPE TRANSPORTER WITH DEPENDENT FAILURES OF COMPONENTS

From practical point of view it seems reasonable to consider the ground ship-rope transporter assuming component failures' dependence (Blokus-Roszkowska & Kołowrocki 2009). Indeed, failures of some wires in ropes have influence on the remaining wires and may cause their reliability characteristics worsening. Thus, the assumption about dependence of wires seems to be natural and justified.

The increased load caused by one or several components' failures may cause the increase of the failure rates of the rest components. We consider an equal load sharing model that is widely described in (Blokus-Roszkowska 2007a, b).

A multi-state "*m* out of *n*"-series system with dependent components is considered as a system of linked independently in series multi-state "*m* out of *n*" subsystems composed of components with failure dependency. In each of these subsystems we assume the following model of failure dependency. After getting out *v* components in a subsystem, of the reliability state subset $\{u, u+1, \dots, z\}$, $u = 1,2,\dots,z$, the increased load is shared equally among others. The number of components *v*, that are getting out of the reliability state subset can be equal to $v = 0,1,2,\dots,l_i - 1$, where l_i, $i = 1,2,\dots,k$, is number of components in the *i*-th subsystem.

We denote by $T_{ij}(u)$, $i = 1,2,\dots,k$, $j = 1,2,\dots,l_i$, $u = 1,2,\dots,z$, the random variables representing the lifetimes of components E_{ij} in the state subset $\{u, u+1,\dots, z\}$, and $T(u)$, $u = 1,2,\dots,z$, is a random variable representing the lifetime of a system in this reliability state subset. Then the reliability of remaining not failed components is getting worse so that the mean values of the *i*-th, $i = 1,2,\dots,k$, subsystem component lifetimes in the state subset $\{u, u+1, \dots, z\}$, are of the form

$$E[T_{ij}'(u)] = E[T_{ij}(u)] - \frac{v}{l_i} E[T_{ij}(u)] = \frac{l_i - v}{l_i} E[T_{ij}(u)], \ j = 1,2,\dots,l_i, \ \ v = 0,1,2,\dots,l_i - 1, \ \ i = 1,2,\dots,k, \ u = 1,2,\dots,z.$$

The ground ship-rope transporter as a system with dependent failures of components is described in (Blokus-Roszkowska & Kołowrocki 2009). In this paper there are quoted only some final values of reliability characteristics to compare them with results obtained in the previous point. Additionally the availability analysis of the ground ship-rope transporter in Naval Shipyard in Gdynia assuming the wires' failure dependence is presented.

The mean values and the standard deviations of the ground ship-rope transporter unconditional lifetimes in the reliability state subsets, according to results given in (Blokus-Roszkowska et al. 2008b, Soszyńska 2006), counted in years respectively are:

$$\mu(1) = \sum_{i=1}^{8} p_i \mu_i(1) \cong 8.7940, \ \sigma(1) \cong 2.2355, \tag{30}$$

$$\mu(2) = \sum_{i=1}^{8} p_i \mu_i(2) \cong 5.7981, \ \sigma(2) \cong 1.4867, \tag{31}$$

$$\mu(3) = \sum_{i=1}^{8} p_i \mu_i(3) \cong 3.0731, \ \sigma(3) \cong 0.7797. \tag{32}$$

Next, the unconditional mean values of the ground ship-rope transporter lifetimes in the particular reliability states, by (Kołowrocki 2004) and considering (30)-(32), in years are:

$$\bar{\mu}(1) = \mu(1) - \mu(2) = 2.9959, \ \bar{\mu}(2) = \mu(2) - \mu(3) = 2.725, \ \bar{\mu}(3) = \mu(3) = 3.0731.$$

Next, assuming that the ground ship-rope transporter is repaired after its failure and that the time of the system renovation is not ignored and it has the mean value $\mu_0(2) = 0.0014 \cong 12$ hours and the standard deviation $\sigma_0(2) = 0.0002 \cong 2$ hours, applying results given in (Blokus-Roszkowska et al. 2008a), we obtain the following results:

– the distribution function of the time $\bar{\bar{S}}_N(2)$ until the *Nth* system's renovation, for sufficiently large *N*, has approximately normal distribution $N(5.7995N, 1.4867\sqrt{N})$, i.e.,

$$\bar{\bar{F}}^{(N)}(t,2) = P(\bar{\bar{S}}_N(2) < t) \cong F_{N(0,1)}\left(\frac{t - 5.7995N}{1.4867\sqrt{N}}\right), \ t \in (-\infty, \infty), \ N = 1,2,\dots,$$

– the expected value and the variance of the time $\bar{\bar{S}}_N(2)$ until the *Nth* system's renovation take respectively forms

$$E[\bar{\bar{S}}_N(2)] \cong 5.7995N, \ D[\bar{\bar{S}}_N(2)] \cong 2.2103N,$$

– the distribution function of the time $\bar{S}_N(2)$ until the *Nth* exceeding the reliability critical state 2 of this system takes form

$$\bar{F}^{(N)}(t,2) = P(\bar{S}_N(2) < t) = F_{N(0,1)}(\frac{t - 5.7995N + 0.0014}{1.4867\sqrt{N}}),\ t \in (-\infty,\infty),\ N = 1,2,\ldots,$$

– the expected value and the variance of the time $\bar{S}_N(2)$ until the *Nth* exceeding the reliability critical state 2 of this system take respectively forms

$$E[\bar{S}_N(2)] \cong 5.7981N + 0.0014(N-1),\ D[\bar{S}_N(2)] \cong 2.2103N$$

– the distribution of the number $\bar{\bar{N}}(t,2)$ of system's renovations up to the moment $t,\ t \geq 0,$ is of the form

$$P(\bar{\bar{N}}(t,2) = N) \cong F_{N(0,1)}(\frac{5.7995 - t}{0.6173\sqrt{t}}) - F_{N(0,1)}(\frac{5.7995(N+1) - t}{0.6173\sqrt{t}}),\ N = 1,2,\ldots,$$

– the expected value and the variance of the number $\bar{\bar{N}}(t,2)$ of system's renovations up to the moment $t,\ t \geq 0,$ take respectively forms

$$\bar{\bar{H}}(t,2) \cong 0.1724t,\ \ \bar{\bar{D}}(t,2) \cong 0.0113t,$$

– the distribution of the number $\bar{N}(t,2)$ of exceeding the reliability critical state 2 of this system up to the moment $t,\ t \geq 0,$ is of the form

$$P(\bar{N}(t,2) = N) \cong F_{N(0,1)}(\frac{5.7995N - t - 0.0014}{0.6173\sqrt{t + 0.0014}}) - F_{N(0,1)}(\frac{5.7995(N+1) - t - 00014}{0.6173\sqrt{t + 0.0014}}),\ N = 1,2,\ldots,$$

– the expected value and the variance of the number $\bar{N}(t,2)$ of exceeding the reliability critical state 2 of this system up to the moment $t,\ t \geq 0,$ are respectively given by

$$\bar{H}(t,2) \cong 0.1724(t + 0.0014),\ \ \bar{D}(t,1) \cong 0.0113(t + 0.0014),$$

– the availability coefficient of the system at the moment *t* is given by the formula

$$K(t,2) \cong 0.9998,\ t \geq 0,$$

– the availability coefficient of the system in the time interval $< t, t+\tau), \tau > 0,$ is given by the formula

$$K(t,\tau,2) \cong 0.1724\int_{\tau}^{\infty}\bar{\boldsymbol{R}}(t,2)dt,\ t \geq 0,\ \tau > 0,$$

where the reliability function of a system at the critical state $\bar{\boldsymbol{R}}(t,2)$ is given by the formula

$$\bar{\boldsymbol{R}}(t,2) = \sum_{i=1}^{8} p_i[\bar{\boldsymbol{R}}(t,2)]^{(i)},\ \ t \geq 0,$$

where

$$[\bar{\boldsymbol{R}}(t,2)]^{(1)} = [\sum_{j=0}^{6}\frac{(0.5292t)^j}{j!}\exp[-0.5292t]]^{18}$$

$$[\bar{\boldsymbol{R}}(t,2)]^{(i)} = [\sum_{j=0}^{6}\frac{(0.846t)^j}{j!}\exp[-0.846t]]^{6},\ \ i = 2,3,4,$$

$$[\bar{\boldsymbol{R}}(t,2)]^{(i)} = [\sum_{j=0}^{6}\frac{(1.2996t)^j}{j!}\exp[-1.2996t]]^{12},\ \ i = 5,6,7,$$

$$[\bar{\boldsymbol{R}}(t,2)]^{(8)} = [\sum_{j=0}^{6}\frac{(1.4184t)^j}{j!}\exp[-1.4184t]]^{18},\ \ t \geq 0.$$

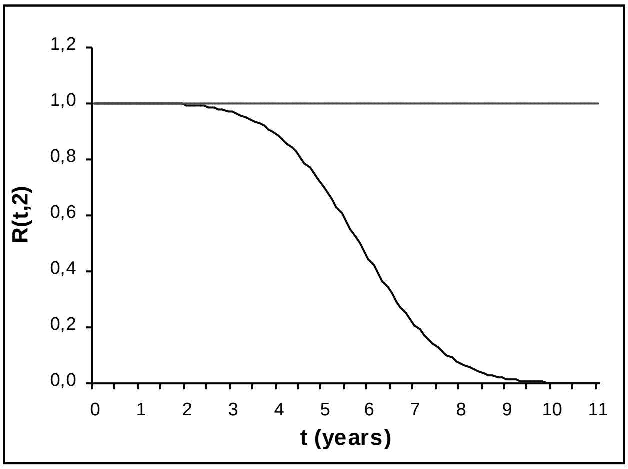

Figure 11. The graph of the unconditional reliability function of the ground ship-rope transporter with dependent failures of components.

Now we can compare the expected values of the ground ship-rope transporter unconditional lifetimes in the reliability state subsets in the case when wires failure in dependent and independent way. We can notice that these values under the assumption that wires failure in dependent way in the reliability state subset {1,2,3} are shorten for about 6.4% and in the reliability state subsets {2,3}, {3} are shorten for about 8.6% than in the case when wires are independent. Comparing also the expected values of the time until the *Nth* system's renovation we also conclude that there are lower for about 8.6% in the case the wires failure in dependent way than independently.

The obtained results illustrate that the increased load of remaining un-failed components causes shortening the lifetime of these components. That fact can be interpreted as a decrease of their reliability faster than for the systems with independent components.

6 CONCLUSIONS

In the paper a practical application of the theoretical results of reliability, risk and availability evaluation of industrial systems in variable operation conditions is presented. The ground ship-rope transporter in Naval Shipyard in Gdynia is considered in varying in time operation conditions with its different reliability structure and its components' reliability functions in different operation states. The results presented in the paper can suggest that it seems reasonable to continue the investigations focusing on the methods of reliability, risk and availability analysis of complex multi-state systems and the methods of safety evaluation related to the multi-state systems in variable operation processes and their applications to the ground ship-rope transporters used in shipyards.

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REFERENCES

Blokus, A., Kołowrocki, K. & Soszyńska, J. 2005. Reliability of port transportation systems related to their operation processes. *Proceedings 12-th International Congress of the International Maritime Association of the Mediterranean - IMAM 2005. Maritime Transportation and Exploitation of Ocean and Coastal Resources*: 1487-1495.

Blokus-Roszkowska, A. 2007a. *Reliability analysis of homogenous large systems with component dependent failures* (*in Polish*). Ph.D. Thesis. Maritime University, Gdynia – Warsaw: System Research Institute.

Blokus-Roszkowska, A. 2007b. On component failures' dependency influence on system's lifetime. *International Journal of Reliability, Quality and Safety Engineering. Special Issue: System Reliability and Safety* 14(6): 1-19.

Blokus-Roszkowska, A, Guze, S., Kołowrocki, K., Kwiatuszewska-Sarnecka, B., Soszyńska, J. 2008a. General methods for safety, reliability and availability evaluation of technical systems. WP 3 – Task 3.1 – English – 31.05.2008. *Poland-Singapore Joint Project. MSHE Decision No. 63/N-Singapore/2007/0.*

Blokus-Roszkowska, A., Guze, S., Kołowrocki, K., Kwiatuszewska-Sarnecka, B., Soszyńska, J. 2008b. Models of safety, reliability and availability evaluation of complex technical systems related to their operation processes. WP 4 – Task 4.1 – English – 31.05.2008. *Poland-Singapore Joint Project. MSHE Decision No. 63/N-Singapore/2007/0.*

Blokus-Roszkowska, A., Kołowrocki, K., Namysłowski, M. 2009. Data mining for identification and prediction of safety and reliability characteristics of complex industrial systems and processes. WP 6 – Task 6.2 – English – 31.01.2009. Sub-Task 6.2.4 – Appendix 4A. Preliminary statistical data collection of a shipyard ground ship-rope transporter operation process and its preliminary statistical identification. *Poland-Singapore Joint Project. MSHE Decision No. 63/N-Singapore/2007/0.*

Blokus-Roszkowska, A., Kołowrocki, K. 2009. Reliability and risk analysis of a ground ship-rope transporter in variable operation conditions. *Proceedings of the European Safety and Reliability Conference – ESREL 2009*: 1643-1648.

Grabski, F. 2002. *Semi-Markov Models of Systems Reliability and Operations*. Warsaw: Systems Research Institute, Polish Academy of Sciences.

Kołowrocki, K. 2004. *Reliability of Large Systems*. Amsterdam - Boston - Heidelberg - London - New York - Oxford - Paris - San Diego - San Francisco - Singapore - Sydney – Tokyo: Elsevier.

Kołowrocki, K. & Soszyńska, J. 2006. Reliability and availability of complex systems. *Quality and Reliability Engineering International*. 22(1): 79-99.

Kołowrocki, K. & Soszyńska, J. 2008. A general model of technical systems operation processes related to their environment and infrastructure. WP 2 – Task 2.1 – English – 31.05.2008. *Poland-Singapore Joint Project. MSHE Decision No. 63/N-Singapore/2007/0.*

Smith, R. L. 1982. The asymptotic distribution of the strength of a series-parallel system with equal load-sharing. *Annals of Probability* 10: 137-171.

Soszyńska, J. 2006. Reliability evaluation of a port oil transportation system in variable operation conditions. *International Journal of Pressure Vessels and Piping* 83(4): 304-310.

UNAVAILABILITY CALCULATIONS WITHIN THE LIMITS OF COMPUTER ACCURACY

R. Briš
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Technical University of Ostrava, Faculty of Electrical Engineering and Computer Science, Ostrava, Czech Republic

e-mail: radim.bris@vsb.cz

ABSTRACT

The paper presents a new analytical algorithm which is able to carry out direct and exact reliability quantification of highly reliable systems with maintenance (both preventive and corrective). A directed acyclic graph is used as a system representation. The algorithm is based on a special new procedure which permits only summarization between two or more non-negative numbers that can be very different. If the summarization of very small positive numbers transformed into the machine code is performed effectively no error is committed at the operation. Reliability quantification is demonstrated on a real system from practice.

1 INTRODUCTION

It is a simulation method which is used for the quantification of reliability when accurate analytic or numerical procedures do not lead to satisfactory computations of system reliability. A direct simulation technique has been improved by the application of a parallel algorithm [1] to such extent that it can be used for real complex systems which can be modelled and quantitatively estimated from the point of view of the reliability without unreal simplified conditions which analytic methods usually require. However, if it is necessary to work and quantitatively estimate highly reliable systems, when unreliability indicators (i.e. system non-functions) move in the order 10^{-5} and higher (i.e.10^{-6} etc.), the simulation technique, whatever improved, can meet the problems of prolonged and inaccurate computations.

Highly reliable systems appear more often and in research they are closely connected with a penetrative increase of progress. We can observe the systems for example in space applications where complex device is often designed for a few tens of years without a possibility of help of human hand. Safety systems of nuclear power stations represent other example of highly reliable systems. They have to be reliable enough to comply with still increasing internationally agreed safety criteria and moreover they are mostly so called sleeping systems which start and operate only in the case of big accidents. Their hypothetical failures are then not apparent and thus reparable only at optimally selected inspective times. We can add some more examples. The question is how to model the behaviour of these systems and how to find their efficient procedure for estimation and computation of reliability indicators.

2 A PROBLEM FORMULATION AND COMPONENT MODELS

Let us have a system assigned with a help of a directed acyclic graph (AG) [1]. Terminal nodes of the AG that represent functionality of input system components are established by the definition of deterministic or stochastic process, to which they are subordinate. From them we can compute a time course of the availability coefficient, possibly unavailability of individual terminal nodes, using methodology of basic renewal theory, as for example in [2]. The aim is then to find a correspondent time course of the unavailability coefficient for the highest SS node which represents reliability behaviour of the whole system.

2.1 Models of components – terminal nodes

In the first phase of the research, an exponential distribution for the time to a failure will be supposed, possibly for the time to a restoration. Under this condition, all frequently used models with both preventive and corrective maintenance may be described by three of the following models:

- Model with elements (terminal nodes in AG) that can not be repaired
- Model with repairable elements (CM – Corrective Maintenance) for apparent failures, i.e. a model when a possible failure is identified at the occurrence and immediately afterwards it starts a process leading to its restoration.
- Model with repairable elements with hidden failures, i.e. a model when a failure is identified only at special deterministically assigned times, appearing with a given period (moments of periodical inspections). In the case of its occurrence at these times an analogical restoration process starts, as in the previous case.

An analytical accurate computation of time dependence of the (un)availability coefficient was for the first two situations explained enough and derived in [2]. Let us remind that in the first case of the element that can not be repaired a final course of unavailability coefficient *P(t)* is presented by a distribution function of time to failure of the element:

$$P(t) = 1 - e^{-\lambda t}, \tag{1}$$

where λ is the failure rate.

In the second case we can derive a relation on the basis of Laplace´s transformation for a similar coefficient

$$\begin{aligned} P(t) &= 1 - \left[\frac{\mu}{\lambda+\mu} + \frac{\lambda}{\lambda+\mu} e^{-(\lambda+\mu)t} \right] \\ &= \frac{\lambda}{\lambda+\mu}\left[1 - e^{-(\lambda+\mu)t}\right], \quad t > 0 \end{aligned} \tag{2}$$

where μ is the repair rate.

The third model is rather more general than the earlier stated in the model with periodical exchanges [2] where we assumed a deterministic time to repair. If we hereby presume that a time to the end of repair is exponential random variable, it is necessary to derive an analytical computation of time course of the function of unavailability coefficient.

2.2 Unavailability coefficient for a model with repairable elements and hidden failures

With the same indication of failure and restoration intensities as given above we can describe the unavailability coefficient with the following function:

$$P(\tau) = (1 - P_C).(1 - e^{-\lambda\tau}) + P_C\left[1 + \frac{\mu}{\mu - \lambda}(e^{-\mu\tau} - e^{-\lambda\tau})\right], \quad \tau > 0 \tag{3}$$

where τ is a time which has passed since the last planned inspection, Pc is the probability of a non-functional state of an element at the moment of inspection at the beginning of the interval to the next inspection.

Proof of the relationship (3) brings ref. [3].

Note:

1. For the purposes of an effective computer calculation the expression in the brackets can be converted into the formation:

$$\left[1 + \frac{\mu}{\mu - \lambda}(e^{-\mu\tau} - e^{-\lambda\tau})\right] = 1 - \frac{\mu}{\mu - \lambda}e^{-\lambda\tau}\left[1 - e^{-(\mu - \lambda)\tau}\right], \quad \tau > 0 \tag{4}$$

2. In other hypotheses we will need this expression to be always positive, what is also easy to proof.

3 THE NEW ALGORITHM

3.1 Probabilities of functional and non-functional state

It is evident that probabilities of a functional p and non-functional state q comply with a relation

$q + p = 1.$

Taking into consideration the final accuracy of real numbers in a computer it is important which one from p or q we count. If we want to fully use the machine accuracy, we have to compute the smaller one from both probabilities.

Example:

We take into account the following sum:

0.000 002 7816 + 0.999 997 2184

If we counted hypothetically on a computer with three-digit decimal numbers, then for the value of q = 0.00000278, we would instead of a correct value p = 0.9999972184 have only p = 1**.**

In return for q = 1- p, we would get: q = 1- p = 0, keeping at disposal p = 1.

It is apparent that it gets to a great loss of accuracy if we firstly counted p instead of q. Our result will be maximally precise saving accuracy of q.

Seeing that probabilities of a non-function state of a highly reliable system is very small, we have to concentrate on numerical expression of these probabilities. For these purposes it is necessary to reorganize the computer calculation and set certain rules which do not have the influence on accuracy of the computation at the numeration process.

3.2 Probability calculation of non-functional states of terminal nodes

The probability calculation of non-functioning state (unavailability coefficient) of the simplest possible not repaired element (or terminal node) can be done by the use of relation (1).

Similarly, for other models of system elements the computation of an expression

$$1 - e^{-x}, \tag{5}$$

for $x \geq 0$ is a crucial moment at the probability numerical expression of a non-function state (unavailability coefficient).

For values x<<1, i.e. near 0, direct numerical expression written by the formula would lead to great errors! At subtraction of two near numbers it gets to a considerable loss of accuracy. On personal computer the smallest number ε, for which it is numerically evident that

$$1 + \varepsilon \neq 1,$$

is approximately 10^{-18}.

If

$$x \approx 10^{-25},$$

the real value of the expression (5) will be near 10^{-25}. A direct numerical calculation of the expression gets a zero!

As the algorithm was created in a programming environment Matlab, for the need of this paper was used the Matlab function "exmp1" which enables exact calculation of the expression (5) based on Taylor's decomposition.

3.3 The numeration substance of probability of a non-functional state of a node

The probability of a non-functional state of a node of an AG, for which the individual input edges are independent, is in fact given by going over all possible combinations of probabilities of the input edges (such combinations that case failure of the node). For 20 input edges we have regularly a million combinations.

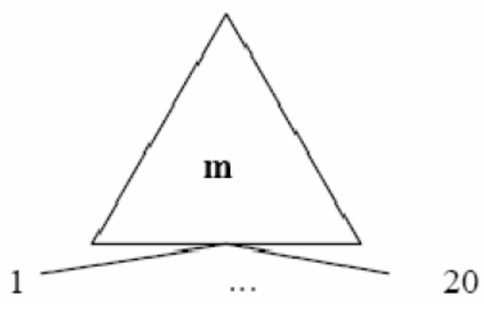


Figure 1. One node of the acyclic graph with 20 edges.

One partial contribution to the probability of a non-functional state of the node in Figure1 has a form:

$$q_1 . q_2 ... q_{i-1} p_i . q_{i+1} ... q_{j-1} . p_j . q_{j+1} ... q_{20},$$

where a number of occurring probabilities *p* (here the number equals to 2) can not reach "m". This fact is in the context of definition of the internal node of AG [1], which is correctly functioning just in the case when at least **m** inferior nodes (either terminal or non-terminal) are correctly functioning.

The probability of a non-functional state of the node is generally given by a sum of a big quantity of very small numbers. These numbers are generally very different!

If the sum will be carried out in the way that the addition runs in the order from the biggest one to the smallest ones, certainly a lost stems from rounding off, more than the addition runs in the order from the smallest ones to the biggest values. And even in this second case there is not possible to determine effectively how much accuracy "has been lost".

Note: In the case of dependence of the input edges (terminal nodes) we cannot express the behaviour of an individual node numerically. There is necessary to work with the whole relevant sub-graph. Combinatorial character for the quantification will stay nevertheless unchanged.

3.4 The error-free sum of different non-negative numbers

The first step to the solution of this problem is to find a method for the "accurate" sum of many non-negative numbers.

The arithmetic unit of a computer (PC) works in a binary scale. A positive real number of today's PC contains 53 valid binary numbers, see Figure 2. A possible order ranges from approximately -1000 to 1000.

Figure 2. A positive real number in binary scale.

The line indicated as "order" means an order of a binary number.

The algorithm for the "accurate" quantification of sums of many non-negative numbers consists from a few steps:

1. The whole possible machine range of binary positions (bites) is partitioned into segments of 32 positions for orders, according to the following scheme in Figure 3:
2.

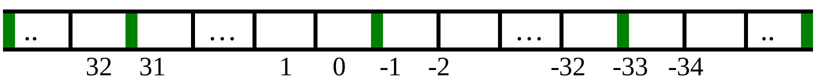


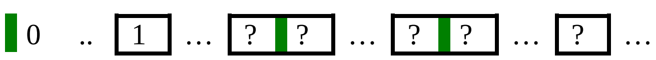
Figure 3. Segments composed from 32 binary positions.

The number of these segments will be approx.:

$$\frac{2000}{32} \cong 63$$

3. Total sum is memorized as one real number, which is composed from 32 bite segments. Each from these segments has additional 21 bites used as transmission.

4. At first a given non-zero number of the sum that must be added is decomposed according to before assigned firm borders (step 1) mostly into three parts containing 32 binary numbers of the number at most, according to the scheme in Figure 4. The individual segments are indexed by numbers 1-63.



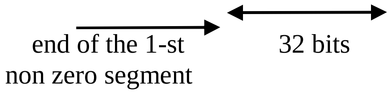


Figure 4. Decomposition of a given non-zero number

5. Then the individual parts of this decomposed number are added to the corresponding members of the sum number, as in Figure 5.

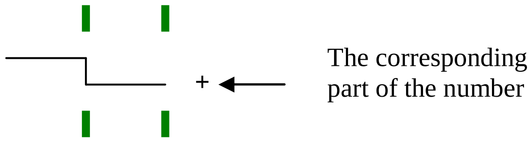


Figure 5. Adding a number to the sum number

6. Always after the processing of 2^{21} numbers (the limit is chosen so that it could not lead to overflowing of the sum number at any circumstances) a modification of the sum number is carried out which is indicated as the “clearance” process. Upwards a transmission is separated (in the following Figure 6 it is identified by a symbol β) which is added to the upper sum.

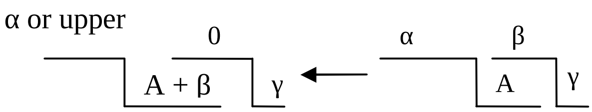


Figure 6. Clearance process.

7. If a final sum is required, at first the clearance process has to be carried out. Then the group of three sums is elaborated, from which the upper is the highest non-zero one (identified by a symbol α in Figure 7). We make a sum of these three sums as usual in a binary scale, when *p* in the following expression is given by an index of the highest non-zero segment:

$$sum = \alpha .2^{p} + \beta .2^{p-32} + \gamma .2^{p-64}$$

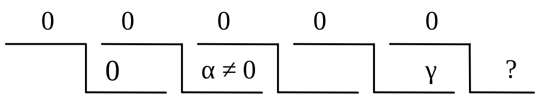


Figure 7. Demonstration of the final summarization.

So numbers in their full machine accuracy (53 binary numbers beginning with 1) are the input for this process of adding. The output is the only number in the same machine accuracy (53 binary numbers beginning with 1). The number is mechanically the nearest number to the real accurate error-free sum which contains in principle up to 2000 binary numbers.

3.5 Permissible context of the usage not leading to the loss of accuracy

The probability of a non-functional state of a repairable component (repairable component with hidden failures) is given by the formula (3), which can be simplified as

$$P(\tau) = (1 - P_C).\alpha(\tau) + P_C.\beta(\tau),$$

where P_C is the probability of a non-functional state of an element at the moment of the inspection at the beginning of the interval till the next inspection; α, β are non-negative mechanically accurate and numerically expressed functions.

One contribution to the computation of a non-functional state of a node generally has a form

$$q_1.q_2...(1 - q_k)....$$

In both cases occurs (*1- q*). It has already been explained that when we use $p \equiv 1- q$, it can come to the catastrophic loss of accuracy. A basic question then comes out: Do we have to modify further the stated patterns for the purpose of removing the subtraction? Fortunately not. In the introduced context of the product $(1- q).\alpha$, where α is expressed numerically in a full machine accuracy there is no loss in machine accuracy! Thanks to rounding off the final product to 53 binary numbers, lower orders of the expression *(1-q)*, i.e. a binary numbers on 54^{th} place and other places behind the first valid number, can not practically influence the result.

3.6 Determination of system probability behaviour according to a graph structure

Let all elements appearing in the system are independent. The probability of a non-functional state of a system, assigned by the help of AG, is thus simply gained on the basis of estimation of all nodes upwards. For instance for the AG in Figure 8 the following steps have to be made:

- numerical expression of the probability of a non-functional state of terminal nodes, i.e. elements 8,9,10 and 5,6,7
- numerical expression of the probability of a non-functional state of an internal node 4 which
  is given by the following sum:

$$\begin{aligned}&q_8.q_9.q_{10} + (1-q_8).q_9.q_{10}\\ &+q_8.(1-q_9).q_{10} + q_8.q_9.(1-q_{10})\\ &+q_8.(1-q_9).(1-q_{10})\\ &+(1-q_8).q_9.(1-q_{10})\\ &+(1-q_8).(1-q_9).q_{10}\end{aligned}$$

- numerical expression of the probability of a non-functional state of an internal node 3 which
  is given by the only item

$$q_5.q_6.q_7$$

- numerical expression of the probability of a non-functional state of a terminal node 2
- numerical expression of the probability of a non-functional state of the highest SS node 1 which is given:

$$\begin{aligned}&q_2.q_3.q_4 + (1-q_2).q_3.q_4\\ &+q_2.(1-q_3).q_4 + q_2.q_3.(1-q_4)\end{aligned}$$

In the case of AG with dependent elements, where every multiple used node causes dependence, the situation is much more complex. We have to decompose a set of nodes to a disjunctive system of mutually independent subsets. The process has been also implemented to the new algorithm.

Note: Internal numeration of nodes is such that the node with a less number can not be inferior to the node with greater number. Nodes are numbered in the decreasing order of numbers.

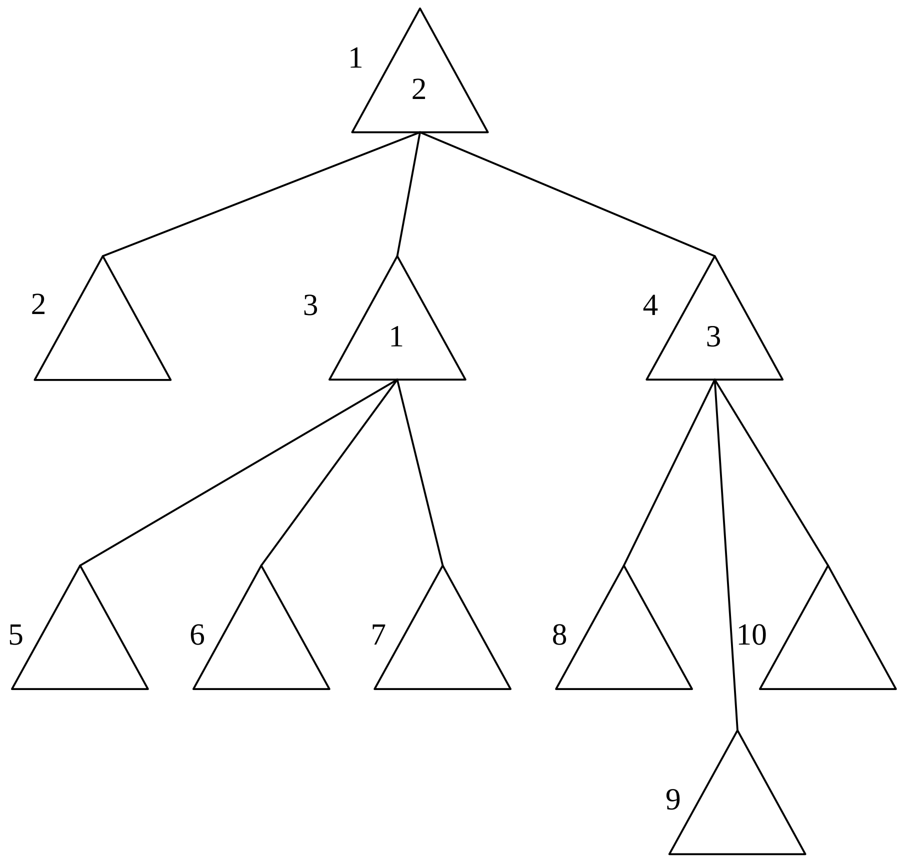


Figure 8: The system assigned by the help of a structure AG

4 EXAMPLE OF COMPUTATIONS WITH HIGHLY RELIABLE SYSTEMS

Let us consider a very reliable electronic system from practice – driving light for a rolling-stock, for example let us consider an electric locomotive. The light of the locomotive is composed from 15 parallel branches of LED diodes. We define the critical failure of the light as follows: the critical failure of the system occurs just in case when tree branches are in failure simultaneously. Other words the critical failure occurs if three out of fifteen branches fail. One branch of LED diodes is repairable component with exponential time to failure with mean 5.10^4. Repair time is also exponentially distributed with mean time to repair 6 hours.

Figure 9 demonstrates the time dependent unavailability coefficient within first 200 hours. Exact computation results show that the unavailability coefficient given by equation (1) was stabilized after 30 hours of operation on a value around 8.10^{-10}.

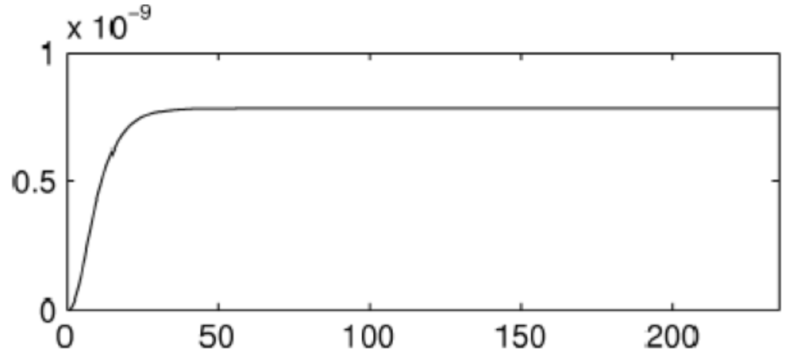


Figure 9: The time dependent unavailability coefficient within first 200 hours.

5 CONCLUSIONS

Maintaining the full machine accuracy requires mainly not to carry out subtraction of near values. All required outputs are therefore necessary to express in the form of the sum of numbers with consistent sign (in our case non-negative).

A problem of a sum of many non-negative numbers can be solved by decomposing a sum into more partial sums which can be carried out without a loss! The process has been numerically realized within a programming environment Matlab.

Numerical expression of probabilities of a non-functional state of one node of an AG has a combinatorial character. We have to go over all combinations of input edges behaviour leading to a non-functional state of the node. The astronomic increase of combinations with the increasing number of elements causes that the program will be usable only up to a certain size of a system. Already at moderate exceeding the critical size of the system it comes to enormous increase of machine time. The computation above run below 1s, on Pentium (R) 4 CPU 3.40GHz, 2.00 GB RAM.

The algorithm enables to carry out exact unavailability analysis of real maintained systems with both preventive and corrective maintenance. The future research will continue with the aim to use the algorithm for maintenance optimization, i.e. to find such a maintenance strategy to minimize the maintenance cost at a prescribed maximal unavailability level.

6 ACKNOWLEDGEMENT

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7 REFERENCES

1. Briš R. (2008). Parallel simulation algorithm for maintenance optimization based on directed Acyclic Graph. Reliab Eng Syst Saf 2008;93:852-62.
2. Briš R. and Drábek V. (2007). Mathematical Modeling of both Monitored and Dormant Failures, Proceedings of the 17th Advances in Risk and Reliability Technology Symposium AR2TS, Edited by Lisa Bartlett, Published by Loughborough University, Loughborough, Leicestershire, LE11 3TU, pg. 376-393(2007), ISBN 0 904947 62 9.

3. Bris R. (2008). Exact reliability quantification of highly reliable systems with maintenance. In Safety, Reliability and Risk Analysis: Theory, Methods and Applications – Martorell et al. (eds), 2008 Taylor & Francis Group, pg. 489-496, ISBN 978-0-415-48513-5.

ANALYSIS AND OPTIMIZATION OF POWER TRANSMISSION GRIDS BY GENETIC ALGORITHMS

F. Cadini, E. Zio, L.R. Golea, C.A. Petrescu

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Politecnico di Milano - Dipartimento di Energia, Via Ponzio 34/3, 20133 Milano, Italy

francesco.cadini@polimi.it

ABSTRACT

Two applications of multi-objective genetic algorithms (MOGAs) are reported with regards to the analysis and optimization of electrical transmission networks. In a first case study, an analysis of the topological structure of a network system is carried out to identify the most important groups of elements of different sizes in the network. In the second case study, an optimization method is devised to improve the reliability of power transmission by adding lines to an existing electrical network.

1 INTRODUCTION

In this paper, two applications of multi-objective genetic algorithms (MOGAs) are reported with regards to the analysis and optimization of electrical transmission networks.

In the first case study, Genetic Algorithms (GAs) are used within a multiobjective formulation of the search problem, in which the decision variables identify groups of components and the objectives are to maximize the importance of the groups while minimizing their dimension.

In the second case study, a GA method is developed for identifying strategies of expansion of an electrical network in terms of new lines of connection to add for improving the reliability of its transmission service, while maintaining limited the investment cost. To realistically restrict the search space to small numbers of new connections, the so-called guided multi-objective genetic algorithm (G-MOGA) has been applied. In this approach, the search is based on the guided domination principle which allows to change the shape of the dominance region specifying maximal and minimal trade-offs between the different objectives so as to efficiently guide the MOGA towards Pareto-optimal solutions within these boundaries (Zio et al. 2009).

The paper is organized as follows. Section 2 presents the group closeness centrality measure which can be used to quantify the importance of groups of nodes. The concept of network global reliability efficiency is also presented. In Section 3 and Section 4, the case studies regarding the IEEE 14 BUS network system (Christie 1993) and IEEE RTS 96 (Billinton & Li 1994) are presented and solved by MOGA. Conclusions on the outcomes of the analysis are eventually drawn in Section 5.

2 TOPOLOGICAL GROUP CLOSENESS CENTRALITY AND GLOBAL RELIABILITY EFFICIENCY

Mathematically, the topological structure of a network can be represented as a graph $G(N,K)$ with N nodes connected by K edges. The connections are defined in an $N \times N$ adjacency matrix $\{a_{ij}\}$ whose entries are 1 if there is an edge joining node i to node j and 0 otherwise.

The group closeness centrality (Everett & Borgatti 1999), $C^C(g)$, is based on the idea that a node can quickly interact with all other nodes if it is easy accessible (close to) all others. If d_{ij} is the topological shortest path length between nodes i and j (i.e., the minimum number of arcs on a path connecting them), the closeness of a group g of N_g nodes is the sum of the distances from the members of the group to all vertices outside the group:

$$C^C(g) = \frac{N - N_g}{\sum_{i \in g, j \in G} d_{ij}} \tag{1}$$

This measure is normalized by dividing the distance score into the number of non-group members, with the result that larger numbers indicate greater centrality.

When the group consists of a single node, the group closeness centrality is the same as the individual node closeness centrality (Freeman 1979, Sabidussi 1966, Wasserman & Faust 1994).

To capture the failure behavior of the network, the reliability of its connecting edges is included in the framework of analysis by means of the formalism of weighted networks, the weight w_{ij} associated to the edge between the pair of nodes i and j being its reliability:

$$p_{ij} = e^{-\lambda_{ij} \cdot T} \tag{2}$$

where λ_{ij} is the failure rate of edge ij linking nodes i and j and T is a reference time ($T = 1$ year, in this work).

On the basis of the adjacency and reliability matrices $\{a_{ij}\}$ and $\{p_{ij}\}$, the matrix of the most reliable path lengths $\{rd_{ij}\}$ can be computed (Zio 2007). The group reliability closeness centrality can then be computed as in equation 1, with rd_{ij} replacing d_{ij}.

The global reliability efficiency $RE[G]$ of the graph G can also be defined as (Zio 2007):

$$RE[G] = \frac{1}{N(N-1)} \sum_{i, j \in G, i \neq j} \left(1 / rd_{ij}\right) \tag{3}$$

3 CASE STUDY 1: IEEE 14 BUS ELECTRICAL TRANSMISSION NETWORK

The topological structure of the electrical transmission network system of the IEEE (Institute of Electrical and Electronic Engineers) 14 BUS) is considered for the analysis of the importance of groups of components, measured in terms of reliability closeness centrality. The system considered represents a portion of the American Electric Power System and consists of 14 bus locations connected by 20 lines and transformers. The topology of the system can be represented by the graph $G(14,20)$ of Figure 1.

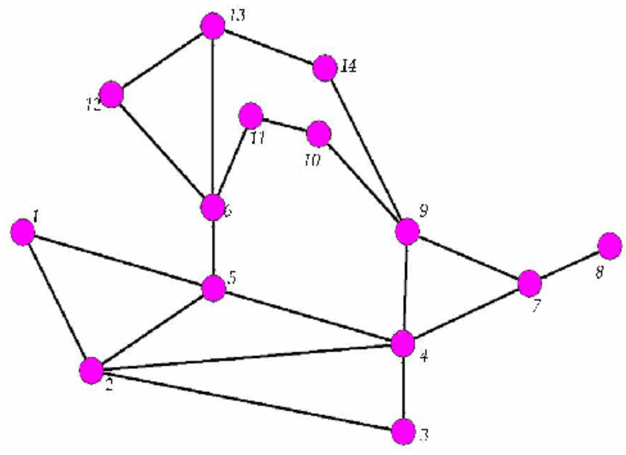

Figure 1. Graph representation of the IEEE 14 BUS transmission network

A MOGA has been implemented to identify the most reliability-central groups of nodes of different sizes in the network of Figure 1, considering as objective functions the group reliability closeness centrality measure and the dimension of the group.

Figure 2 shows the results obtained on the importance of the group in terms of reliability closeness centrality. In the Figure, the values of the objective functions in correspondence of all the nondominated groups of nodes contained in the MOGA archive at convergence are shown to identify the two-dimensional Pareto-optimal surface (circles). The results are compared for validation with those obtained by exhaustive computation of all groups of nodes (i.e., the computation of the group reliability closeness centrality measure for all the possible combinations of n out of N nodes; due to the fact that the number of groups obtained is 2^N, its implementation is feasible here thanks to the small size of the network but would require impractical computational resources for large networks).

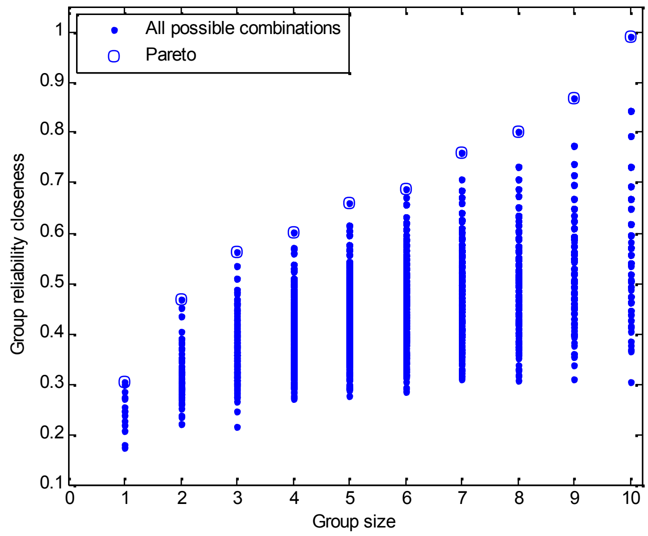

Figure 2. Results of the multi-objective search of the most central groups of nodes in terms of reliability closeness centrality

Actually, different groups of equal size can have the same centrality measure value: Table 1 reports all the nondominated solutions contained in the archive, identified by the MOGA.

In the present case, the smallest group with maximal reliability closeness is of size 10 and there are 2 of these. The group {1, 2, 3, 5, 7, 10, 11, 12, 13, 14} is particularly interesting because it does not contain the highly central node {4} and contains the node {1} that have the smallest individual reliability closeness centrality measure, as it can be seen in Table 2.

Table 1. Pareto optimal results of the multi-objective search for reliability closeness centrality groups

Group reliability closeness centrality	Group Size	Components
0.303	1	4
0.47	2	(4, 6), (6, 9)
0.562	3	(2, 6, 9)
0.602	4	(1, 2, 6, 9), (1, 3, 6, 9), (2, 3, 6, 9)
0.659	5	(1, 2, 3, 6, 9)
0.688	6	(1, 2, 3, 6, 7, 9), (1, 2, 3, 6, 8, 9)
0.761	7	(1, 2, 3, 5, 7, 10, 13), (1, 2, 3, 5, 7, 11, 13), (1, 2, 3, 6, 7, 10, 13), (1, 2, 3, 6, 7, 10, 14), (1, 2, 3, 6, 7, 11, 13), (1, 2, 3, 6, 7, 11, 14)
0.802	8	(1, 2, 3, 5, 7, 10, 11, 13), (1, 2, 3, 5, 7, 10, 12, 13), (1, 2, 3, 5, 7, 11, 12, 14), (1, 2, 3, 6, 7, 10, 11, 13), (1, 2, 3, 6, 7, 10, 12, 13), (1, 2, 3, 6, 7, 11, 12, 14) …
0.868	9	(1, 2, 3, 5, 7, 10, 11, 12, 13), (1, 2, 3, 5, 7, 10, 11, 13, 14), (1, 2, 3, 6, 7, 11, 12, 13, 14) …
0.99	10	(1, 2, 3, 5, 7, 10, 11, 12, 13, 14), (1, 2, 3, 6, 7, 10, 11, 12, 13, 14)

Table 2. Individual reliability closeness centrality

Node	Reliability closeness centrality
4	0.3031
9	0.2998
5	0.2835
7	0.2742
6	0.2716
14	0.253
10	0.2448
13	0.2448
11	0.2371
2	0.2272
8	0.2184
12	0.2081
3	0.1793
1	0.1723

4 CASE STUDY 2: IEEE RTS 96 ELECTRICAL TRANSMISSION NETWORK

The transmission network system IEEE RTS 96 (Figure 3a) (Billinton 1994) consists of 24 bus locations (numbered in bold in the Figure) connected by 34 lines and transformers. The transmission lines operate at two different voltage levels, 138 kV and 230 kV. The 230 kV system is the top part of Figure 3a, with 230/138 kV tie stations at Buses 11, 12 and 24.

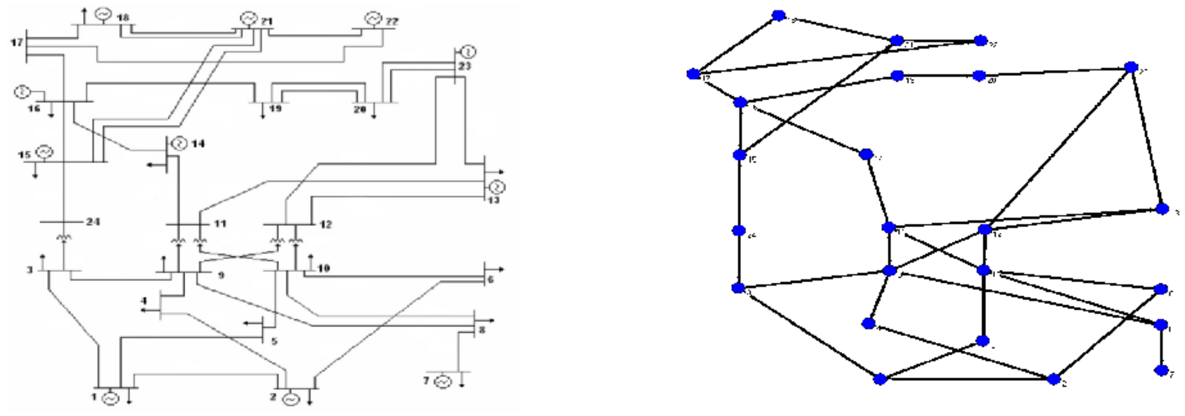

Figure 3. a) IEEE RTS 96 transmission network; b) IEEE RTS 96 graph representation

Figure 3b gives the representation of the graph $G(24,34)$ of the transmission network; the corresponding 24×24 adjacency matrix $\{a_{ij}\}$ has entry equal to 1 if there is a line or transformer between bus locations i and j and 0 otherwise.

A MOGA has been constructed for identifying the best improvements in the connection of the network, aimed at increasing its global reliability efficiency in transmission at acceptable costs. The improvements are obtained by addition of new lines between nodes with no direct connection in the original network. Given the lack of geographical information on the nodes locations, for simplicity and with no loss of generality, three typologies of lines have been arbitrarily chosen as the minimum, the mean and the maximum values of the failure rates of the transmission lines taken from (Billinton 1994):

$$\lambda_1 = 0.2267 \text{ outages/yr}$$
$$\lambda_2 = 0.3740 \text{ outages/yr}$$
$$\lambda_3 = 0.5400 \text{ outages/yr}$$

The addition of a new line requires an investment cost assumed inversely proportional to the failure rate. The network cost can be then defined as:

$$C[G] = \sum_{i,j\in N, i\neq j} \left(1/\lambda_{ij}\right) \tag{4}$$

The reliability cost of the original IEEE RTS 96 is $C\left[G\right] = 332.0120$ in arbitrary monetary units and the reliability efficiency is $RE\left[G\right] = 0.2992$, which is a relatively high value representative of a globally reliable network.

From the algorithmic point of view, a proposal of improvement amounts to changing from 0 to 1 the values of the elements in the adjacency matrix corresponding to the added connections. The only physical restriction for adding direct new connections is that the connected nodes must be at the same voltage level (138 or 230 kV), otherwise the addition of a transformer would also be needed. From the genetic algorithm point of view, the generation of proposals of network improvements can be achieved by manipulating a population of chromosomes, each one with a number of bits equal to 214 which is double the number of zeros (i.e., the number of missing direct connections *ij*) in the upper triangular half of the symmetric adjacency matrix $\{a_{ij}\}$. The bits are dedicated to each missing direct connection *ij* so as to code the three different available types of lines with failure rates λ_1, λ_2 and λ_3: in other words, the bit-string (00) is used to code the absence of connection, (01) connection line with a λ_1-type line, (10) connection with a λ_2-type line and (11) connection with a λ_3-type line. The initial population of 200 individuals is created by uniformly sampling the binary bit values.

During the genetic search, each time a new chromosome is created, the corresponding matrices $\{a_{ij}\}$ and $\{p_{ij}\}$ are constructed to compute the values of the two objective functions, network global reliability efficiency and cost of the associated improved network.

Figure 4 shows the Pareto dominance front (squares) obtained by the MOGA at convergence after 10^3 generations; the circle represents the original network with $RE[G] = 0.2992$ and $C[G] = 332.0120$, while the star represents the network fully connected by the most reliable transmission lines $\lambda_1 = 0.2267$ occ/yr, for which $RE[G] = 0.57$ and $C[G] = 804.1072$.

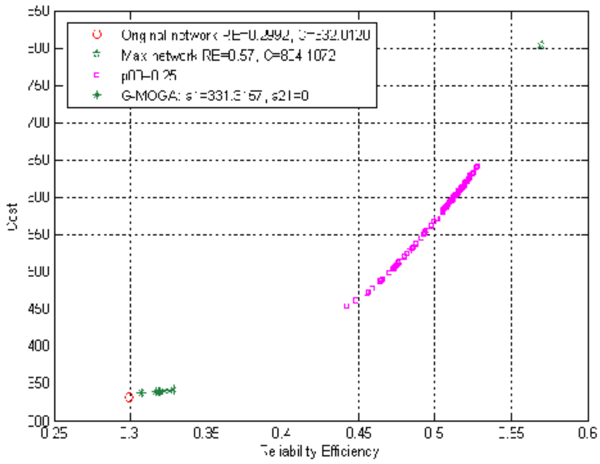

Figure 4. Pareto front reached by the MOGA

The optimality search is biased from the beginning (from the initial population) towards highly connected network solutions, because the string (00) has a probability of 0.25 whereas the probability of adding a connection of any one of the three available types (i.e., the probability of the strings 01, 10, 11) is 0.75; this drives the population evolution to highly connected networks in the Pareto front (squares in Figure 4), all with values $RE[G] \geq 0.4417$, $C[G] \geq 454.4738$ and numbers of added connections exceeding 60.

In practical applications only a limited number of lines can be added, due to the large investment costs and other physical constraints. To drive the genetic search towards low cost solutions (i.e., low number of added lines) maximal and minimal trade-offs to the two objectives of the optimization (network global reliability efficiency and cost) can be defined within a Guided Multi-Objective Genetic Algorithm (G-MOGA) scheme, (Zio 2007). The preferential optimization has been performed by using G-MOGA, with the same population size, evolution procedures and parameters of the previous search. In this approach, the search is guided by defining the maximal and minimal trade-offs that allow to identify a precise section of the Pareto front. The values of the trade-off parameters have been set by trial-and-error to $a_{12} = 331.3157$ and $a_{21} = 0$; the search converges to a small number of solutions in a Pareto front which is more concentrated on low cost networks, characterized by a limited number of added connections (asterisks in Figure 4).

Table 3 lists the five solutions of lowest cost identified by the G-MOGA search: the added connections improve the network global reliability efficiency and they do so with relatively small costs.

Table 3. The five solutions on the Pareto front obtained by the G-MOGA

G-MOGA	
Reliability Efficiency	Cost
0.3072	337.6
0.3168	339.4
0.3186	339.4
0.3187	339.4
0.3193	339.4

5 CONCLUSIONS

In this paper, the electrical transmission network system of the IEEE 14 BUS has been taken as case study for the MOGA analysis of the importance of groups of components, measured in terms of their centrality in the structure of interconnection paths. The results obtained using the group reliability closeness centrality measure as importance indicator have shown that the groups classified as most central indeed contain the nodes of individual highest centrality but may also include nodes with a relatively low centrality.

Also, a MOGA for improving an electrical transmission network (IEEE RTS 96) has been implemented with the objective of identifying the lines to be added for maximizing the network transmission reliability efficiency, while maintaining the investment costs limited. A preferential procedure of optimization has been implemented for individuating realistic network expansion solutions made of few new transmission lines.

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REFERENCES

Billinton, R., Li, W. (1994). *Reliability Assessment of Electric Power Systems Using Monte Carlo Methods*, pp.229-308.

Christie, R. (1993). The IEEE 14 BUS data can be found on: http://www.ee.washington.edu/research/pstca/, University of Washington.

Everett, M.G., Borgatti, S.P. (1999). The centrality of groups and classes, *Journal of Mathematical Sociology*, Vol. 23, N. 3, pp. 181-201.

Freeman, L.C. (1979). Centrality in Social Networks: Conceptual Clarification, *Social Networks* 1, pp. 215-239.

Sabidussi G. (1966). The Centrality Index of a Graph, *Psychometrika*, n.31.

Wasserman S., Faust K. (1994). Social Networks Analysis, Cambridge U.P., Cambridge, UK.

Zio, E. (2007). From Complexity Science to Reliability Efficiency: A New Way of Looking at Complex Network Systems and Critical Infrastructures, *Int. J. Critical Infrastructures*, Vol. 3, Nos. 3/4, pp. 488-508.

Zio E., Baraldi P., Pedroni N. (2009). Optimal power system generation scheduling by multi-objective genetic algorithms with preferences, *Reliab. Eng. Sys. Safety*, Vol. 94, Issue 2, pp. 432-444.

POISSON PROCESSES WITH FUZZY RATE

R. Guo
University of Cape Town, Cape Town, South Africa
Renkuan.Guo@uct.ac.za
•
D. Guo
South African National Biodiversity Institute, Cape Town, South Africa
Guo@sanbi.org
•
C. Thiart
University of Cape Town, Cape Town, South Africa
Christien.Thiart@uct.ac.za

ABSTRACT

Poisson processes, particularly the time-dependent extension, play important roles in reliability and risk analysis. It should be fully aware that the Poisson modeling in the current reliability engineering and risk analysis literature is merely an ideology under which the random uncertainty governs the phenomena. In other words, current Poisson Models generate meaningful results if randomness assumptions hold. However, the real world phenomena are often facing the co-existence reality and thus the probabilistic Poisson modeling practices may be very doubtful. In this paper, we define the random fuzzy Poisson process, explore the related average chance distributions, and propose a scheme for the parameter estimation and a simulation scheme as well. It is expecting that a foundational work can be established for Poisson random fuzzy reliability and risk analysis.

1 INTRODUCTION

It should be fully aware that vagueness is an intrinsic feature in today's diversified business environments, just as Carvalho and Machado (2006) commented, "In a global market, companies must deal with a high rate of changes in business environment. … The parameters, variables and restrictions of the production system are inherently vagueness." Therefore the co-existence of random uncertainty and fuzzy uncertainty is inevitable reality of safety and reliability analysis and modelling.

It is a well-established fact that Poisson processes and particularly the non-stationary Poisson processes play important roles in safety and reliability modeling. Many researchers contributed to the probabilistic developments, see Crow (1974), Guo and Love (1992, 1994, 2004), Guo et al. (2007), Love and Guo (1991).

Logically, it is obvious that probabilistic modeling is only a good approximation to real world problem when random uncertainty governs the phenomenon. If fuzziness and randomness both appear then probabilistic modeling may be questionable. Therefore, developing the appropriate models for modeling fuzziness and randomness co-existence is necessary.

In this paper, we are trying to offer a systematic treatment for the random fuzzy Poisson processes not only in the mathematical sense (building models based on postulates and definitions) but also in the statistical sense (estimation and hypothesis testing based on sample data).

2 FOUNDATION OF RANDOM FUZZY VARIABLE

Without a solid understanding of the intrinsic feature of random fuzzy variable, there is no base for exploring the modelling of random fuzzy processes. Therefore, it is necessary to briefly review Liu's hybrid variable theory established on the axiomatic credibility measure and probability measure foundations.

First let us review the credibilistic fuzzy variable theory. Let Θ be a nonempty set, and $\mathcal{P}(\Theta)$ the power set on Θ. Each element, let us say, $A\subset\Theta$, $A\in\mathcal{P}(\Theta)$ is called an fuzzy event. A number denoted as $\mathrm{Cr}\{A\}$, $0\le\mathrm{Cr}\{A\}\le 1$, is assigned to event $A\in\mathcal{P}(\Theta)$, which indicates the credibility grade with which event $A\in\mathcal{P}(\Theta)$ occurs. $\mathrm{Cr}\{A\}$ satisfies following axioms given by Liu (2004, 2007):

Axiom 1: $\mathrm{Cr}\{\Theta\}=1$.

Axiom 2: $\mathrm{Cr}\{\cdot\}$ is non-decreasing, i.e., whenever $A\subset B$, $\mathrm{Cr}\{A\}\le\mathrm{Cr}\{B\}$.

Axiom 3: $\mathrm{Cr}\{\cdot\}$ is self-dual, i.e., for any $A\in 2^{\Theta}$, $\mathrm{Cr}\{A\}+\mathrm{Cr}\{A^c\}=1$.

Axiom 4: $\mathrm{Cr}\{\cup_i A_i\}=\sup_i\left[\mathrm{Cr}\{A_i\}\right]$ for any $\{A_i\}$ with $\mathrm{Cr}\{A_i\}\le 0.5$.

Definition 1: (Liu (2004, 2007)) Any set function $\mathrm{Cr}:\mathcal{P}(\Theta)\to[0,1]$ satisfies *Axioms 1-4* is called a credibility measure. The triple $(\Theta,\mathcal{P}(\Theta),\mathrm{Cr})$ is called the credibility measure space.

It should be fully aware that credibility measure only follows sub-σ-additive property, but probability measure does enjoy the σ-additive property. This character of credibility measure relaxes the assumptions of the set mapping so that it might cover a wider category of real world uncertain problems, but brings new difficulties in its mathematical treatments.

Definition 2: A fuzzy variable ξ is a measurable mapping, i.e., $\xi:(\Theta,\mathcal{P}(\Theta))\to(\mathrm{R},\mathcal{B}(\mathrm{R}))$. The measurable mapping is characterized by the membership of the pre-image of event $B=(-\infty,r]$ under fuzzy variable ξ to the power set $\mathcal{P}(\Theta)$. In other words,

$$\forall B\in\mathcal{B}(\mathrm{R}),\{\theta\in\Theta:\xi\in B\}\in\mathcal{P}(\Theta)\tag{1}$$

The measurability of fuzzy variable ξ definitely induces a measure on the measurable space $(\mathrm{R},\mathcal{B}(\mathrm{R}))$. Let us denote the induced measure as μ^c. For $\forall B\in\mathcal{B}(\mathrm{R})$, the induced measure is

$$\mu^c\{B\}=\mathrm{Cr}\{\theta\in\Theta:\xi\in B\}=\mathrm{Cr}\{\theta\in\Theta:\xi(\omega)\le r\}\tag{2}$$

Therefore, further denote $\mu^c=\mathrm{Cr}\circ\xi^{-1}$ and specifically, the distribution is defined by the induced measure

$$\Lambda(x)=\mu^c\{(-\infty,r]\}=\mathrm{Cr}\{\theta\in\Theta:\xi(\theta)\le r\}\tag{3}$$

The induced distribution by fuzzy variable ξ is just the credibility distribution which characterizes the measurement of vague (or fuzzy) uncertainty associated with every event with fuzzy variable ξ.

Definition 3: (Liu (2004, 2007)) The credibility distribution $\Lambda:\mathrm{R}\to[0,1]$ of a fuzzy variable ξ on $(\Theta,\mathcal{P}(\Theta),\mathrm{Cr})$ is

$$\Lambda(x)=\mathrm{Cr}\left\{\theta\in\Theta\,\middle|\,\xi(\theta)\le x\right\}\tag{4}$$

Credibility measure, as an axiomatic measure development, the set class, power set $\mathcal{P}(\Theta)$ plays the critical roles in defining set function credibility measure Cr as well as the measurability of fuzzy variable. However, it is necessary to keep in mind that power set $\mathcal{P}(\Theta)$ is the largest σ-algebra of space Θ. The establishment of set function on power set inevitably brings different feature from that establishing probability measure on the smallest σ-algebra $\mathcal{A}(\Omega)$ of a space Ω.

Therefore, a fuzzy variable is not a fuzzy set in the sense of Zadeh's fuzzy theory (1965, 1978), in which a fuzzy set is defined by a membership function.

Liu (2004, 2007) defined a random fuzzy variable as a mapping from the credibility space $(Q,2^Q,\mathrm{Cr})$ to a set of random variables. Again, we should be aware that a random fuzzy variable here takes real numbers as its values, which behaves very similar to a random variable. We would like to present an intuitive definition similar to that of stochastic process in probability theory and expect readers who are familiar with the basic concept of stochastic processes can understand the comparative definition.

Definition 4: ((Guo et al, 2007)) A random fuzzy variable, denoted as $\xi=\left\{X_{\beta(\theta)},\theta\in\Theta\right\}$, is a collection of random variables X_β defined on the common probability space $(\Omega,\mathcal{A},\Pr)$ and indexed by a fuzzy variable $\beta(\theta)$ defined on the credibility space $(Q,2^Q,\mathrm{Cr})$.

Similar to the interpretation of a stochastic process, $X=\left\{X_t,t\in\mathbf{R}^+\right\}$, a random fuzzy variable is a bivariate mapping from $\left(\Omega\times\Theta,\mathcal{A}\times 2^\Theta\right)$ to the space $\left(\mathrm{R},\mathcal{B}(\mathrm{R})\right)$, where $\mathcal{B}(\mathrm{R})$ denotes Borel σ-algebra on real number set $\mathrm{R}\equiv(-\infty,\infty)$. As to the index, in stochastic process theory, index used is referred to as *time* typically, which is a positive (scalar variable), while in the random fuzzy variable theory, the "index" is a fuzzy number (i.e., variable), say, β. Using uncertain parameter as index is not starting in random fuzzy variable definition. In stochastic process theory we already know that the stochastic process $X=\left\{X_{\tau(\omega)},\omega\in\Omega\right\}$ uses stopping time $t(w)$, $w\,\mathrm{O}\,\mathrm{W}$, in which an (uncertain) random variable is used as its index.

In random fuzzy variable theory, there are different types of chances measures proposed for characterizing a random fuzzy variable. What we are going to use is the average chance measure, denoted as ch, which will plays a similar role to a probability measure, denoted as $\Pr$, in probability theory.

Definition 5: (Liu (2004, 2007)) Let x be a random fuzzy variable, then the average chance measure denoted by $\mathrm{ch}\{\mathrm{g}\}$, of a random fuzzy event $\{\mathrm{x}\,\mathrm{J}\ x\}$, is

$$\mathrm{ch}\{\mathrm{x}\,\mathrm{J}\ x\}=\mathop{\mathrm{T}}_0^1 \mathrm{Cr}\left\{\mathrm{q}\,\mathrm{O}\,\mathrm{Q}|\Pr\{\mathrm{x}(\mathrm{q})\mathrm{J}\ x\}\,\mathrm{i}\ \mathrm{a}\right\}d\mathrm{a} \tag{5}$$

Then function $\mathrm{Y}(\mathrm{g})$ is called as average chance distribution if and only if

$$\mathrm{Y}(x)=\mathrm{ch}\{\mathrm{x}\,\mathrm{J}\ x\} \tag{6}$$

It is quite important to emphasize here that the definition of random fuzzy variable is constructive. The mapping order is essential. The following theorem is actually a summary of Liu's definition and examples in his book. For example, if a random variable η has zero mean and a fuzzy variable ζ, then the sum of the two, $\eta+\zeta$, results in a random fuzzy variable ξ, Liu (2004, 2007). Accordingly, let $\eta \sim N(0,\sigma^2)$, i.e., a normal random variable with zero mean and variance σ^2, and let ζ be a triangular fuzzy number (i.e., variable), then $\xi=\eta+\zeta$ is a normal random fuzzy variable, denoted as $\mathrm{x}\overset{d}{:}N(\mathrm{z},\mathrm{s}^2)$. Liu (2004, 2007) also mentioned an example in which an exponential density, $\mathrm{b}e^{-\mathrm{b}t}$ having a fuzzy parameter b. We state Liu's ideas formally as a theorem.

Theorem 1: Let ζ be a fuzzy variable defined on the credibility space $(\Theta,\mathcal{P}(\Theta),\mathrm{Cr})$ and τ be a random variable defined on the probability space $(\Omega,\mathcal{A}(\Omega),P)$, then

(1) Let $\oplus$ be an arithmetic operator, which can be "$+$", "$-$", "$\times$" or "$\div$" operation, such that $\zeta\oplus\tau$ maps from $(\Theta,\mathcal{P}(\Theta),\mathrm{Cr})$ to a collection of random variables on $(\Omega,\mathcal{A}(\Omega),P)$, denoted by

ξ. Then ξ is a random fuzzy variable defined on hybrid product space $(\Theta, \mathcal{P}(\Theta), \mathrm{Cr}) \times (\Omega, \mathcal{A}(\Omega), P)$.

(2) Let $f: \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ be a continuous, such that $f(\zeta, \tau)$ maps from $(\Theta, \mathcal{P}(\Theta), \mathrm{Cr})$ to a collection of random variables on $(\Omega, \mathcal{A}(\Omega), P)$, denoted by ξ. Then $\xi = f(\zeta, \tau)$ is a random fuzzy variable defined on hybrid product space $(\Theta, \mathcal{P}(\Theta), \mathrm{Cr}) \times (\Omega, \mathcal{A}(\Omega), P)$.

(3) Let $F(x;\theta)$ be the probability distribution of random variable τ with parameter θ (possible vector-valued), then $F(x;\zeta)$ defines a random fuzzy variable ξ on the hybrid product space $(\Theta, \mathcal{P}(\Theta), \mathrm{Cr}) \times (\Omega, \mathcal{A}(\Omega), P)$.

Note that the Theorem 1 is merely specifying three subfamilies of random fuzzy variables. Particularly, the Item (1) and (2) are strictly stating Liu's definition of random fuzzy variable, Liu (2004, 2007), for avoiding the possible confusion with general hybrid variable, particularly, fuzzy random variable. A straightforward example for Item (2) are linear function $f(\zeta, \tau) = \alpha_0 + \alpha_1 \zeta + \alpha_2 \tau,\ \alpha_0, \alpha_1, \alpha_2 \in \mathbb{R}$. Another example for Item (2) is $f(\zeta, \tau) = (\tau/\varsigma)^{\alpha},\ \alpha > 0$. As the Item (3), it is a direct extension to Liu's exponential distribution with fuzzy rate parameter, Liu (2004, 2007).

3 RANDOM FUZZY POISSON PROCESSES

It is well-known fact that a Poisson random variable N takes nonnegative integer-value with probability:

$$\Pr\{N = k\} = \frac{\lambda^k}{k!} e^{-\lambda}, k = 0, 1, 2, \cdots \tag{7}$$

where $\lambda > 0$ is the parameter representing the rate of event occurrences.

3.1 Probabilistic Poisson processes

In stochastic process theory, Poisson process may be defined in different ways although revealed the same intrinsic features. Grimmett and Stirzaker (1992) stated a formal definition as following:

Definition 6: A Poisson process with intensity λ is a process $N = \{N(t), t \ge 0\}$ taking values in $S = \{0, 1, 2, \cdots\}$ such that:

(a) $N(0) = 0$; if $s < t$, then $N(s) \le N(t)$;

(b) $\Pr\{N(t+h) = n + m \mid N(t) = n\}$

$$= \begin{cases} \lambda h + o(h) & \text{if } m = 1 \\ o(h) & \text{if } m > 1 \\ 1 - \lambda h + o(h) & \text{if } m = 0 \end{cases}$$

(c) if $s < t$ then number $N(t) - N(s)$ of an emission in the interval $(s, t]$ is independent of the times of emissions during $[0, s)$.

It is fairly straightforward that at any time t, $N(t)$ is a Poisson random variable with rate λt with probability:

$$\Pr\{N(t) = k\} = \frac{(\lambda t)^k}{k!} e^{-\lambda t}, k = 0, 1, 2, \cdots \tag{8}$$

Associated with probabilistic Poisson processes, the critical fact is the distributions of the inter-arrival times which have many applications to reliability and risk analysis.

Theorem 2: The successive inter-arrival (sojourn) times in a Poisson process $N = \{N(t), t \geq 0\}$ with intensity λ are *i.i.d.* variables with common probability density function $\lambda e^{-\lambda t}$.

Theorem 3: The waiting time to the n^{th} event, W_n, in a Poisson process $N = \{N(t), t \geq 0\}$ with intensity λ has a gamma distribution with probability density function $f_{W_n}(t) = \frac{(\lambda t)^{n-1}}{(n-1)!}\lambda e^{-\lambda t}$.

The proof of theorem 2 is an application of Poisson process definition, while the proof of theorem 3 is completed by noticing the fact that the distribution of waiting times is merely that of *n* *i.i.d.* exponential variables.

3.2 Random fuzzy stationary Poisson processes

According to Theorem 1, an intuitive formation of a random fuzzy Poisson process is to assume the intensity λ to be a credibilistic fuzzy variable defined on credibility space $(\Theta, \mathcal{P}(\Theta), \mathrm{Cr})$ with credibility distribution function Λ.

Definition 7: A random fuzzy Poisson process with credibilistic fuzzy intensity λ on credibility space $(\Theta, \mathcal{P}(\Theta), \mathrm{Cr})$ is a process $N = \{N(t), t \geq 0\}$ taking values in $S = \{0,1,2,\cdots\}$ such that:

(a) $N(0) = 0$; if $s < t$, then $N(s) \leq N(t)$;

(b) $\Pr\{N(t+h) = n + m \mid N(t) = n\}$

$$= \begin{cases} \lambda h + o(h) & \text{if } m = 1 \\ o(h) & \text{if } m > 1 \\ 1 - \lambda h + o(h) & \text{if } m = 0 \end{cases}$$

(c) if $s < t$ then number $N(t) - N(s)$ of an emission in the interval $(s, t]$ is independent of the times of emissions during $[0, s)$.

It is obvious that in Definition 7 the intensity parameter λ is a credibilistic fuzzy parameter (credibilistic fuzzy variable, indeed). For a given value of parameter $\lambda = \lambda_0$, $N = \{N(t), t \geq 0\}$ is just a probabilistic Poisson process. However, if λ is a fuzzy parameter, then for any given time t, the count $N(t)$ is a random fuzzy variable according to Theorem 1. Therefore, Definition 7 defines a stationary random fuzzy Poisson process.

Theorem 4: The successive inter-arrival (sojourn) times in a random fuzzy Poisson process $N = \{N(t), t \geq 0\}$ with credibilistic fuzzy intensity λ having a piecewise linear credibility distribution

$$\Lambda(x)=\begin{cases} 0 & x\le a \\ \dfrac{x-a}{2(b-a)} & a<x\le b \\ \dfrac{1}{2} & b<x\le c \\ \dfrac{x+d-2c}{2(d-c)} & c<x\le d \\ 1 & x>d \end{cases} \tag{9}$$

are *i.i.d.* random fuzzy variables with common average chance density:

$$\psi(t)=\frac{e^{-at}-e^{-bt}}{2(b-a)t^2}+\frac{be^{-bt}-ae^{-at}}{2(b-a)t} +\frac{e^{-ct}-e^{-dt}}{2(d-c)t^2}+\frac{ce^{-ct}-de^{-dt}}{2(d-c)t} \tag{10}$$

Proof: Note that

$$\Pr\{T(\lambda)\le t\}=1-e^{\lambda t} \tag{11}$$

Therefore event $\{\theta:\Pr\{T(\lambda(\theta))\le t\}\ge \alpha\}$ is a fuzzy event and is equivalent to the fuzzy event $\{\theta:\lambda(\theta)\ge -\ln(1-\alpha)/t\}$. As a critical toward the derivation of the average chance distribution, it is necessary to calculate the credibility measure for fuzzy event $\{\theta:\lambda(\theta)\ge -\ln(1-\alpha)/t\}$, i.e., obtain the expression for

$$\text{Cr}\{\theta:\lambda(\theta)\ge -\ln(1-\alpha)/t\} \tag{12}$$

Recall that for the credibilistic fuzzy variable, λ , the credibility measure takes the form

$$\text{Cr}\{\theta:\lambda(\theta)\le x\}=\begin{cases} 0 & x\le a \\ \dfrac{x-a}{2(b-a)} & a<x\le b \\ \dfrac{1}{2} & b<x\le c \\ \dfrac{x+d-2c}{2(d-c)} & c<x\le d \\ 1 & x>d \end{cases} \tag{13}$$

Accordingly, the range for integration with α can be determined as shown in Table 1. Recall that the expression of $x=-\ln(1-\alpha)/t$ appears in Equations (12) and (13), which facilitates the link between intermediate variable α and average chance measure.

Table 1. Range analysis for α

x	α and credibility measure expression	
$-\infty<x\le a$	Range for α	$0\le\alpha\le 1-e^{-at}$
	$\text{Cr}\{\lambda(\theta)\ge -\ln(1-\alpha)/t\}$	1

$a < x \le b$	Range for α	$1-e^{-at} < \alpha \le 1-e^{-bt}$
	$\mathrm{Cr}\{\lambda(\theta) \ge -\ln(1-\alpha)/t\}$	$1-(x-a)/(2(b-a))$
$b < x \le c$	Range for α	$1-e^{-bt} < \alpha \le 1-e^{-ct}$
	$\mathrm{Cr}\{\lambda(\theta) \ge -\ln(1-\alpha)/t\}$	0.5
$c < x \le d$	Range for α	$1-e^{-ct} < \alpha \le 1-e^{-dt}$
	$\mathrm{Cr}\{\lambda(\theta) \ge -\ln(1-\alpha)/t\}$	$(d-x)/(2(d-c))$
$d < x < +\infty$	Range for α	$1-e^{-dt} < \alpha \le 1$
	$\mathrm{Cr}\{\lambda(\theta) \ge -\ln(1-\alpha)/t\}$	0

The average chance distribution for the exponentially distributed random fuzzy lifetime is then derived by splitting the integration into five terms according to the range of α and the corresponding mathematical expression for the credibility measure $\mathrm{Cr}\{\theta:\lambda(\theta) \ge -\ln(1-\alpha)/t\}$, which is detailed in Table 3. Then the exponential random fuzzy lifetime has an average chance distribution function:

$$\begin{aligned}\Upsilon(t) &= \int_0^1 \mathrm{Cr}\{\theta:\lambda(\theta) \ge -\ln(1-\alpha)/t\}\mathrm{d}\alpha \\ &= 1+\frac{e^{-bt}-e^{-at}}{2(b-a)t}+\frac{e^{-dt}-e^{-ct}}{2(d-c)t}\end{aligned} \tag{14}$$

and the average chance density is

$$\begin{aligned}\upsilon(t) &= \frac{e^{-at}-e^{-bt}}{2(b-a)t^2}+\frac{be^{-bt}-ae^{-at}}{2(b-a)t} \\ &\quad +\frac{e^{-ct}-e^{-dt}}{2(d-c)t^2}+\frac{ce^{-ct}-de^{-dt}}{2(d-c)t}\end{aligned} \tag{15}$$

This concludes the proof.

Similar to the probabilistic reliability theory, we define a reliability function or survival function for a random fuzzy lifetime and accordingly name it as the average chance reliability function, which is defined accordingly as

$$\bar{\Upsilon}(t) = 1-\Upsilon(t) \tag{16}$$

Then, for exponential random fuzzy lifetime, its average chance reliability function is

$$\bar{\Upsilon}(t) = \frac{e^{-at}-e^{-bt}}{2(b-a)t}+\frac{e^{-ct}-e^{-dt}}{2(d-c)t} \tag{17}$$

Theorem 5: The waiting time to the n^{th} event, W_n, in a random fuzzy Poisson process $N=\{N(t), t \ge 0\}$ with fuzzy intensity λ having a credibility distribution Λ has a average chance distribution function:

$$\Upsilon(t) = \int_0^1 \left[1-\Lambda\left(\frac{\chi^2_{2n,1-\alpha}}{2t}\right)\right]d\alpha \tag{18}$$

Proof: Note that the waiting time for a fixed λ_0 follows a gamma density $f_{W_n}(t)=\frac{(\lambda t)^{n-1}}{(n-1)!}\lambda e^{-\lambda t}$.

Further note that $2\lambda_0 W_n \sim \chi^2_{2n}$, therefore event

$$
\begin{aligned}
&\left\{\theta : \Pr\left\{W_n(\theta) \le t\right\} \ge \alpha\right\} \\
&= \left\{\theta : \Pr\left\{2\lambda W_n(\theta) \le 2\lambda t\right\} \ge \alpha\right\} \\
&= \left\{\theta : \Pr\left\{\chi^2_{2n}(\theta) \le 2\lambda t\right\} \ge \alpha\right\} \\
&= \left\{\theta : \lambda(\theta) \ge \frac{\chi^2_{2n,1-\alpha}}{2t}\right\}
\end{aligned}
\tag{19}
$$

Hence the average chance distribution

$$
\begin{aligned}
&\Psi_{W_n}(t) \\
&= \int_0^1 \mathrm{Cr}\left(\theta : \Pr\left\{W_n(\theta) \le t\right\} \ge \alpha\right) d\alpha \\
&= \int_0^1 \left(1 - \Lambda\left(\frac{\chi^2_{2n,1-\alpha}}{2t}\right)\right) d\alpha
\end{aligned}
\tag{20}
$$

If we specify the form of the credibility distribution of Λ, then specific form of average chance distribution should be obtained.

3.3 Time-dependent random fuzzy Poisson processes

In reliability engineering and risk analysis, the non-stationary Poisson processes enjoy wide applications because the intensity function is time-dependent. It is expected that the mathematical treatments may be much more complicated since the fuzzy functional nature of intensity when the parameters are credibilistic fuzzy variables. For a concrete discussion purpose, we narrow our attention to a linear intensity function:

$$
\lambda(t) = \beta_0 + \beta_1 t, \ \beta_0 > 0, \ \beta_1 > 0 \tag{21}
$$

Further, we assume that β_0 and β_1 both have piecewise linear credibility distribution:

$$
\Lambda_i(x) = \begin{cases} 0 & x < a_i \\ \dfrac{x - a_i}{2(b_i - a_i)} & a_i \le x < b_i \\ \dfrac{x + c_i - 2b_i}{2(c_i - b_i)} & b_i \le x < c_i \\ 1 & x \ge c_i \end{cases}, \ i = 0,1 \tag{22}
$$

Then the integrated the intensity function (mean measure):

$$
m(t) = \beta_0 t + \beta_1 t^2 \tag{23}
$$

will have a credibility distribution:

$$\Lambda_{m(t)}(y)=\begin{cases} 0 & y<a \\ \dfrac{y-a}{2(b-a)} & a\le y<b \\ \dfrac{y+c-2b}{2(c-b)} & b\le y<c \\ 1 & y\ge c \end{cases} \tag{24}$$

where

$$\begin{aligned} a &= a_0 t + a_1 t^2 \\ b &= b_0 t + b_1 t^2 \\ c &= c_0 t + c_1 t^2 \end{aligned} \tag{25}$$

In general, the credibility distribution of the integrated intensity function $m(t)$, it is necessary to apply Zadeh's (1978) extension principle, denoted as $\Lambda_{m(t)}$, but for the piecewise linear credibility distribution case, the mathematical arguments are simpler.

Now let us derive the average chance distribution for the inter-arrival times.

$$\Psi_T(t)=\int_0^1 \mathrm{Cr}\left(\theta : \Pr\{T(\theta)\le t\}\ge\alpha\right)d\alpha \tag{26}$$

Note that for the first arrival time,

$$\begin{aligned} &\{\theta : \Pr\{T_1(\theta)\le t\}\ge\alpha\} \\ &=\left\{\theta : 1-\exp\left(-\int_0^t(\lambda_0+\lambda_1 u)\,du\right)\ge\alpha\right\} \\ &=\left\{\theta : 1-e^{-m(t)}\ge\alpha\right\} \\ &=\{\theta : m(t)\ge-\ln(1-\alpha)\} \end{aligned} \tag{27}$$

Therefore, the average chance distribution for T_1, the first inter-arrival time, is

$$\begin{aligned} &\Psi_{T_1}(t) \\ &=\int_0^1 \mathrm{Cr}\left(\theta : \Pr\{T(\theta)\le t\}\ge\alpha\right)d\alpha \\ &=\int_0^1 \mathrm{Cr}\left(\theta : m(t)\ge-\ln(1-\alpha)\right)d\alpha \end{aligned} \tag{28}$$

It is noticed that $y=-\ln(1-\alpha)$, therefore,

$$\mathrm{Cr}\{m(t)>y\}=\begin{cases} 1 & y<a \\ \dfrac{2b-2-y}{2(b-a)} & a\le y<b \\ \dfrac{c-y}{2(c-b)} & b\le y<c \\ 0 & y\ge c \end{cases} \tag{29}$$

Table 2. Range analysis for α

y	α and credibility measure expression	
$-\infty < y \le a$	Range for α	$0 \le \alpha \le 1-e^{-m(t)}$
	$\mathrm{Cr}\{m(\theta) \ge -\ln(1-\alpha)\}$	1
$a < y \le b$	Range for α	$1-e^{-at} < \alpha \le 1-e^{-bt}$
	$\mathrm{Cr}\{m(\theta) \ge -\ln(1-\alpha)\}$	$(2b-a-y)/(2(b-a))$
$b < y \le c$	Range for α	$1-e^{-bt} < \alpha \le 1-e^{-ct}$
	$\mathrm{Cr}\{m(\theta) \ge -\ln(1-\alpha)\}$	$(c-y)/(2(c-b))$
$c < y < +\infty$	Range for α	$1-e^{-dt} < \alpha \le 1$
	$\mathrm{Cr}\{m(\theta) \ge -\ln(1-\alpha)\}$	0

Hence,

$$\begin{aligned}
\Psi_{T_1}(t) &= \int_0^1 \mathrm{Cr}\left(\theta : m(t) \ge -\ln(1-\alpha)\right) d\alpha \\
&= \int_0^{1-e^{-m(a)}} 1 \cdot d\alpha + \int_{1-e^{-m(a)}}^{1-e^{-m(b)}} \frac{2b-a+\ln(1-\alpha)}{2(b-a)} d\alpha \\
&+ \int_{1-e^{-m(b)}}^{1-e^{-m(c)}} \frac{c+\ln(1-\alpha)}{2(b-a)} d\alpha + \int_{1-e^{-m(a)}}^{1} 0 \cdot d\alpha \\
&= 1-e^{-m(a)} + \frac{2b-a}{2(b-a)}\left(e^{-m(a)} - e^{-m(b)}\right) \\
&+ \frac{1}{2(b-a)} \int_{1-e^{-m(a)}}^{1-e^{-m(b)}} \ln(1-\alpha) d\alpha \\
&+ \frac{c}{2(c-b)}\left(e^{-m(b)} - e^{-m(c)}\right) \\
&+ \frac{1}{2(c-b)} \int_{1-e^{-m(b)}}^{1-e^{-m(c)}} \ln(1-\alpha) d\alpha
\end{aligned} \tag{30}$$

Note that

$$\int \ln(1-\alpha) d\alpha = (1-\alpha) - (1-\alpha)\ln(1-\alpha) \tag{31}$$

Hence

$$\begin{aligned}
\int_{1-e^{-m(a)}}^{1-e^{-m(b)}} \ln(1-\alpha) d\alpha &= \left(e^{-m(b)} - e^{-m(a)}\right) \\
&- m(b)e^{-m(b)} + m(a)e^{-m(a)}
\end{aligned} \tag{32}$$

and

$$\int_{1-e^{-m(b)}}^{1-e^{-m(c)}} \ln(1-\alpha)d\alpha = \left(e^{-m(c)} - e^{-m(b)}\right) \\ - m(c)e^{-m(b)} + m(b)e^{-m(b)} \tag{33}$$

Combine above arguments, it is established that:

$$\begin{aligned} &\Psi_{T_1}(t) = \\ &1 - e^{-m(a)} + \frac{2b-a-1}{2(b-a)}\left(e^{-m(a)} - e^{-m(b)}\right) \\ &+ \frac{1}{2(b-a)}\left(-m(b)e^{-m(b)} + m(a)e^{-m(a)}\right) \\ &+ \frac{c-1}{2(c-b)}\left(e^{-m(b)} - e^{-m(c)}\right) \\ &+ \frac{1}{2(c-b)}\left(-m(c)e^{-m(c)} + m(b)e^{-m(b)}\right) \end{aligned} \tag{34}$$

Next let us derive the i^{th} inter-arrival time. Recall that conditioning on the $(i-1)^{st}$ occurrence time w_{i-1}, the mean measure is

$$m(t \mid w_{i-1}) = \beta_0 (t - w_{i-1}) + \beta_1 \left(t^2 - w_{i-1}^2\right) \tag{35}$$

Accordingly, the credibility distribution for $m(t \mid w_{i-1})$ is

$$\Lambda_{m(t|w_{i-1})}(y) = \begin{cases} 0 & y < a \\ \dfrac{y-a}{2(b-a)} & a \le y < b \\ \dfrac{y+c-2b}{2(c-b)} & b \le y < c \\ 1 & y \ge c \end{cases} \tag{36}$$

where

$$\begin{aligned} a &= a_0 (t - w_{i-1}) + a_1 \left(t^2 - w_{i-1}^2\right) \\ b &= b_0 (t - w_{i-1}) + b_1 \left(t^2 - w_{i-1}^2\right) \\ c &= c_0 (t - w_{i-1}) + c_1 \left(t^2 - w_{i-1}^2\right) \end{aligned} \tag{37}$$

Thus, the average chance distribution for the i^{th} inter-arrival time is:

$$
\begin{aligned}
&\Upsilon_{T_i}(t)= \\
&1-e^{-m(a|w_{i-1})}+\frac{2b-a-1}{2(b-a)}\left(e^{-m(a|w_{i-1})}-e^{-m(b|w_{i-1})}\right) \\
&+\frac{1}{2(b-a)}\left(-m(b\,|\,w_{i-1})e^{-m(b|w_{i-1})}+m(a\,|\,w_{i-1})e^{-m(a|w_{i-1})}\right) \\
&+\frac{c-1}{2(c-b)}\left(e^{-m(b|w_{i-1})}-e^{-m(c|w_{i-1})}\right) \\
&+\frac{1}{2(c-b)}\left(-m(c\,|\,w_{i-1})e^{-m(c|w_{i-1})}+m(b\,|\,w_{i-1})e^{-m(b|w_{i-1})}\right)
\end{aligned}
\tag{38}
$$

It is necessary to emphasize that in the expression of the average chance distribution of the inter-arrivals (either Equation (34) for the first arrival, or Equation (38) for the i^{th} arrival) the time t factor is containing in the parameters (a,b,c) as shown in Equation (25) for the first arrival and Equation (37) for the i^{th} arrival. Also, the functions for parameters (a,b,c) are changed for the successive arrivals as indicated in Equation (37).

4 A PARAMETER ESTIMATION SCHEME

The parameter estimation is in nature an estimation problem of credibility distribution from fuzzy observations. Guo and Guo (2009) recently proposed a maximally compatible random variable to a credibilistic fuzzy variable and thus the fuzzy estimation problem is converted into estimating the distribution function of the maximally compatible random variable. The following scheme is for estimating the piecewise linear credibility distribution.

Definition 8: Let X be a random variable defined in $(\mathbb{R},\mathcal{B}(\mathbb{R}))$ such that

$$\mu^c=\mathrm{Cr}\circ\xi^{-1}=\mu=P\circ X^{-1} \tag{39}$$

Then X is called a maximally compatible to fuzzy variable ξ.

In other words, random variable X can take all the possible real-values the fuzzy variable ξ may take with and the distribution of X , $F_X(r)$ equals the credibility distribution of ξ, $\Lambda_\xi(r)$ for all $r\in\mathbb{R}$.

It is aware that the induced measure $\mu^c=\mathrm{Cr}\circ\xi^{-1}$ and measure $\mu=P\circ X^{-1}$ are defined on the same measurable space $(\mathbb{R},\mathcal{B}(\mathbb{R}))$. Furthermore, we notice that the pre-image $\xi^{-1}(B)\in\mathcal{P}(\Theta)$, but, the pre-image $X^{-1}(B)\in\mathcal{A}(\Theta)\subset\mathcal{P}(\Theta)$, which implies that for the same Borel set $B\in\mathcal{B}(\mathbb{R})$, the pre-images under fuzzy variable ξ and random variable are not the same. It is expected that

$$\{\theta\in\Theta:X(\theta)\le r\}\ne\{\theta\in\Theta:\xi(\theta)\le r\} \tag{40}$$

but

$$
\begin{aligned}
&\Pr\{\theta\in\Theta:X(\theta)\le r\} \\
&=\mathrm{Cr}\{\theta\in\Theta:\xi(\theta)\le r\}
\end{aligned}
\tag{41}
$$

The statistical estimation scheme for parameters (a,b,c) of the credibility distribution based on fuzzy observations $\{x_1,x_2,\cdots,x_n\}$ can be stated as:

Estimation Scheme 1:

Step 1: Rank fuzzy observations $\{x_1,x_2,\cdots,x_n\}$ to obtain "order" statistics $\{x_{(1)},x_{(2)},\cdots,x_{(n)}\}$ in ascending order;

Step 2: Set $\hat{a} = x_{(1)}$ and $\hat{c} = x_{(n)}$;

Step 3: Set a tentative estimator for b,

$$\hat{b}_e = \frac{4\overline{x}_n - x_{(1)} - x_{(n)}}{2} \tag{42}$$

where

$$\overline{x}_n = \frac{1}{n}\sum_{i=1}^{n} x_i \tag{43}$$

Step 4: Identify $x_{(i_0)}$ from $\left\{x_{(1)}, x_{(2)}, \cdots, x_{(n)}\right\}$ such that $x_{(i_0)} \le \hat{b}_e < x_{(i_1)}$ and $1 < i_0 < i_1$, then we may see $\left\{x_{(1)}, x_{(2)}, \cdots, x_{(i_0)}\right\}$ as a set of order statistics from uniform [*a*,*b*]. Hence the "sufficient" statistic for parameter b is $x_{(i_0)}$.

Then $\left(\hat{a}, \hat{b}, \hat{c}\right) = \left(x_{(1)}, x_{(i_0)}, x_{(n)}\right)$ is the parameter estimator for the piecewise linear credibility distribution.

$$\hat{\Lambda}(x) = \begin{cases} 0 & x < \hat{a} \\ \dfrac{x - \hat{a}}{2\left(\hat{b} - \hat{a}\right)} & \hat{a} \le x < \hat{b} \\ \dfrac{x + \hat{c} - 2\hat{b}}{2\left(\hat{c} - \hat{b}\right)} & \hat{b} \le x < \hat{c} \\ 1 & x \ge \hat{c} \end{cases} \tag{44}$$

The next issue is how to extract the information on intensity rate λ in stationary random fuzzy Poisson process.

It is noticed that for probabilistic Poisson process case, the interpretation of intensity λ is the occurrence rate in unit time. Based on such an observation, therefore, for any individual value λ_0 the fuzzy intensity may take, it results in a probabilistic Poisson process. Sample this Poisson process until n_0 events and record the total waiting time w_{n_0}, then $\hat{\lambda}_0 = n_0 / w_{n_0}$ is an estimate of intensity λ_0. Repeat the sampling procedure from the random fuzzy Poisson process as many times as possible, say, m times, then the intensity "observation" sequence is

$$\left\{\hat{\lambda}_1, \hat{\lambda}_2, \cdots, \hat{\lambda}_m\right\} = \left\{\frac{n_0^1}{w_{n_0}^1}, \frac{n_0^2}{w_{n_0}^2}, \cdots, \frac{n_0^m}{w_{n_0}^m}\right\} \tag{45}$$

Apply the Estimation Scheme 1 to the estimated rate observations $\left\{\hat{\lambda}_1, \hat{\lambda}_2, \cdots, \hat{\lambda}_m\right\}$ the piecewise linear credibility distribution shown in Equation (44).

For the non-stationary random fuzzy Poisson process, the mean measure involves two linear piecewise credibility distributions for fuzzy parameters β_0 and β_1 respectively.

The scheme can state as follows:

Step 1: Sampling procedure from the random fuzzy Poisson process $N = \{N_t, t \ge 0\}$ m times. Let the i^{th} n_0 events the waiting time are $\left\{w_1^i, w_2^i, \cdots, w_{n_0}^i\right\}$.

Step 2: For the i^{th} sample, perform the maximum likelihood estimation and obtain the parameter pair $\left(\hat{\beta}_0^i, \hat{\beta}_1^i\right)$ which is regarded as the fuzzy parameters taking values. Repeat the estimation process until all m MLE pairs $\left\{\left(\hat{\beta}_0^i, \hat{\beta}_1^i\right), i = 1, 2, \cdots, m\right\}$ are obtained.

Step 3: Applying the Estimation Scheme 1 to fuzzy sequences $\{\hat{\beta}_0^1, \hat{\beta}_0^2, \cdots, \hat{\beta}_0^m\}$ and $\{\hat{\beta}_1^1, \hat{\beta}_1^2, \cdots, \hat{\beta}_1^m\}$ respectively, the parameters $(\hat{a}_0, \hat{b}_0, \hat{c}_0)$ and $(\hat{a}_1, \hat{b}_1, \hat{c}_1)$ define the two piecewise linear credibility distributions for β_0 and β_1 respectively.

5 A SIMULATION SCHEME

Simulation of a random fuzzy Poisson process is intrinsically two-stage procedure: a fuzzy parameter simulation for generating realizations $\{\lambda_1, \lambda_2, \cdots, \lambda_n\}$ from a piecewise linear credibility distribution function Λ and a waiting times sequence:

$$W_n = \sum_{i=1}^{n} T_i \tag{46}$$

where $T_1, T_2, \cdots, T_n$ are i.i.d. exponential with common probability density function $\lambda e^{-\lambda t}$.

As to the fuzzy parameter simulation, we utilize the maximally compatible random variable to a fuzzy variable concept and the inverse transformation of the probability distribution function approach for generating fuzzy variable realizations. An algorithm is stated as follows:

Fuzzy simulation scheme 1:

Step 1: Simulating uniform random variable uniform[0,1], and denote the simple random sample as $\{u_1, u_2, \cdots, u_n\}$;

Step 2: Set $\Lambda(x_i) = u_i$, $(i = 1, 2, \cdots, n)$;

Step 3: Set x_i, $(i = 1, 2, \cdots, n)$:

$$x_i = \begin{cases} a + 2(b - a)u_i & \text{if } 0 \le u_i \le 0.5 \\ 2b - c + 2(c - b)u_i & \text{if } 0.5 \le u_i \le 1 \end{cases} \tag{47}$$

Then $\{x_1, x_2, \cdots, x_n\}$ is a sample from the fuzzy variable ξ with a piecewise linear credibility distribution Λ.

Next we state a random fuzzy Poisson process simulation scheme.

Simulation scheme 2: Simulating a stationary random fuzzy Poisson process.

Step 1: Simulate a sequence of uniform (0,1), denoted as $\{u_1, u_2, \cdots, u_n\}$,

Step 2: $\{\tau_1, \tau_2, \cdots, \tau_n\}$

$$\tau_i = -\frac{1}{\beta}\ln(1 - u_i),\ i = 1, 2, \cdots, n \tag{48}$$

are the exponentially distributed random lifetime.

Step 3: in term of Fuzzy Simulation Scheme 1, Fuzzy variable sample $\{x_1, x_2, \cdots, x_n\}$ is obtained.

Step 4: $\{T_1, T_2, \cdots, T_n\}$

$$T_i = -\frac{1}{z_i}\ln(1 - u_i),\ i = 1, 2, \cdots, n \tag{49}$$

which construct a stationary random fuzzy Poisson process $N = \{N_t, t \ge 0\}$.

As to the time-dependent random fuzzy Poisson process, we state an algorithm to illustrate the idea. For example, we simulate a random fuzzy Poisson process with power law intensity function having a fuzzy scale parameter.

Simulation scheme 3: Simulating a time-depenedent random fuzzy Poisson waiting times $\{W_i, i = 1, 2, \cdots, n\}$, which forms a power law process. Note that the conditional Weibull distribution

$$F(t\,|\,x) = 1 - \exp\left(-\left(\left(\frac{t+x}{\eta}\right)^{\beta} - \left(\frac{x}{\eta}\right)^{\beta}\right)\right) \quad (50)$$

Then, **Step 1:** For given sample from Uniform(0,1), $\{u_1, u_2, \cdots, u_n\}$,

$$W_i = \eta\left(\ln\left(e^{\left(\frac{W_{n-1}}{\eta}\right)}\right) - \ln(1-u_i)\right)^{1/\beta}, \quad i = 1, 2, \cdots, n \quad (51)$$

are waiting times in the random fuzzy Poisson process with power law.

Step 2: As to the fuzzy scale parameter, η, in term of *Fuzzy Simulation Scheme 1*, $\{x_1, x_2, \cdots, x_n\}$ is the sample from the fuzzy scale parameter Finally, let

$$W_i = x_i\left(\ln\left(e^{\left(\frac{W_{n-1}}{x_i}\right)}\right) - \ln(1-u_i)\right)^{1/\beta}, \quad i = 1, 2, \cdots, n \quad (52)$$

will generate random fuzzy waiting times $\{W_1, W_2, \cdots, W_n\}$ with fuzzy scale parameter η, which construct a random fuzzy Poisson process with power law intensity having a fuzzy scale parameter η.

6 CONCLUSION

In this paper, we give a systematic treatment of random fuzzy Poisson processes not only from the stationary one and then non-stationary one, but also a parameter estimation scheme as well as a simulation scheme is proposed. In this way, the foundation for the random fuzzy Poisson processes is formed although in its infant stage, particularly, the time-dependent random fuzzy Poisson process. The applications to reliability engineering fields and the risk analysis now can extend from random uncertainty only cases to randomness and fuzziness co-existence cases. It is expecting that this development will help the reliability and risk analysis researchers as well as reliability analysts and engineers.

7 REFERENCES

Carvalho, H. and Machado, V.C. 2006. Fuzzy set theory to establish resilient production systems. *Proceedings of the IIE Annual Conference and Exhibition*.

Crow, L.H. 1974. Reliability analysis for complex, repairable systems. *Reliability and Biometry*, Proschan, F. and Serfling, R. J. (Eds.) Philadelphia, Pennsylvania, SIAM, pp. 379-410.

Grimmett, G.R. and Stirzaker, D.R. 1992. *Probability and Random Processes*. 2nd Edition. London: Clarendon Press, Oxford.

Guo, R. and Love, C.E. 1992. Statistical Analysis of An Age Model for Imperfectly Repaired System. *Quality and Reliability Engineering International*, 8: 133-146.

Guo, R. and Love, C.E. 1994. Simulating Nonhomogeneous Poisson Processes with Proportional Intensities. *Naval Research Logistics*, 41: 507-522.

Guo, R., and Love, C.E. 2004. Fuzzy Covariate Modelling an Imperfectly Repaired System, *Quality Assurance*, 10(37), 7-15, 2004.

Guo, R., Zhao, R.Q., Guo, D. and Dunne, T. 2007. Random Fuzzy Variable Modeling on Repairable System. *Journal of Uncertain Systems*, 1(3): 222-234, August, 2007.

Guo, R. and Guo, D. 2009. Statistical Simulating Fuzzy Variable, Proceedings of the Eighth International Conference on Information and Management Sciences, Kunming & Banna, China, July 20-28, 2009, pp. 775-778. Series of Information & Management Sciences, Series Editor: Eldon Y. Li, Cal Poly State University.

Love, C.E. and Guo, R. 1991. Using Proportional Hazard Modelling in Plant Maintenance. *Quality and Reliability Engineering International*, 7: 7-17.

Love, C.E. and Guo, R. 1991. Application of Weibull Proportional Hazards Modelling to Bad-As-Old Failure Data. *Quality and Reliability Engineering International*, 7: 149-157.

Liu, B.D. 2004. *Uncertainty Theory: An Introduction to Its Axiomatic Foundations*. Berlin: Springer-Verlag Heidelberg.

Liu, B.D. 2007. *Uncertainty Theory: An Introduction to Its Axiomatic Foundations*. 2nd Edition. Berlin: Springer-Verlag Heidelberg.

Zadeh, L. A. 1965. Fuzzy sets, *Information and Control*, 8: 338-353.

Zadeh, L. A. 1978. Fuzzy sets as a basis for a theory of possibility, *Fuzzy Sets and Systems*, 1: 3-28.

MODELLING THE SHIP SAFETY ON WATERWAY ACCORDING TO NAVIGATIONAL SIGNS RELIABILITY

S. Guze
L. Smolarek
•
Maritime University, Gdynia, Poland

e-mail: sambor@am.gdynia.pl
leszsmol@am.gdynia.pl

ABSTRACT

An approach to safety analysis connected with consecutive "m out of n" systems is presented. Further, the consecutive "*m* out of *n*: G" system is defined and the recurrent formulae for its reliability function evaluation are proposed. Next the IALA buoys and leading lights system are introduced. Moreover, the safety states model for ship navigation are defined. Further, analysis of safety during manoeuvre in restricted area with curved draws is illustrated.

1 INTRODUCTION

The safety of passengers and cargo involved in the process of transport is one of the most important criteria for the evaluation of the process. In the maritime transport the most important factors making up the security include: the technical efficiency of the ship, the qualifications of the people in charge of the ship and the conditions under which the transport process takes a place. There are many hazard situations, in maritime transport, particularly in restricted waterways. In such situations it is useful to have methods to assess the safety of traffic. They allow the evaluation of the activities what lead to settle the hazard situation and allow the evaluation of quality control and assessment in terms of traffic safety (Pietrzykowski 2003, Purcz. 1998, Smolarek 2009). This assessment can help to develop the best control or the best manoeuvre for given hazard situation (Fuji 1977, Gucma 1998, Pietrzykowski 2003, Purcz. 1998, Smolarek 2009).

In the case of shipping on the restricted waters important aspects of safety are the technical characteristics of vessel, the type of waterway and its navigational infrastructure (Fuji 1977, IALA NAVGUIDE 2006, Kopacz et. al. 2001, Kopacz et. al. 2003).

In the case of shipping on the restricted waters the technical characteristics of vessel, the type of waterway and its navigational infrastructure are important aspects of its safety (Fuji 1977, Kopacz et. al. 2001, Kopacz et. al. 2003).

Navigational infrastructure is a set of basic navigation, stable and distributed objects and systems necessary to ensure adequate level of maritime safety (Kopacz et. al. 2003).

The paper is devoted to the combining the results on reliability of the two-state consecutive "*m* out of *n*: F" and consecutive "*m* out of *n*: G" systems (Antonopoulou et. al. 1987, Barlow et. al 1975, Guze 2007a,b, Hwang 1982, Kołowrocki 2004, Malinowski 2005) into the safety analysis of the ship on restricted waterway (Kopacz et. al. 2003).

2 TWO-STATE CONSECUTIVE "*M* OUT OF *N: F*" SYSTEMS

In the case of two-state reliability analysis of consecutive "*m* out of *n*" systems we assume that (Guze 2007a, Malinowski 2005):

- n is the number of system components,
- E_i, $i = 1,2,...,n,$ are components of a system,
- T_i are independent random variables representing the lifetimes of components E_i, $i = 1,2,...,n,$
- $R_i(t) = P(T_i > t), t \in <0,\infty),$ *is* a reliability function of a component E_i, $i = 1,2,...,n,$
- $F_i(t) = 1 - R_i(t) = P(T_i \le t), t \in <0,\infty),$ is the distribution function of the component E_i lifetime T_i, $i = 1,2,...,n,$ also called an unreliability function of a component E_i, $i = 1,2,...,n.$

Definition 1. A two-state system is called a two-state consecutive "*m* out of *n*: F" system if it is failed if and only if at least its *m* neighbouring components out of n its components arranged in a sequence of *E*$_1$, *E*$_2$, …, *E*$_n$, are failed.

After assumption that:

- T is a random variable representing the lifetime of the consecutive "*m* out of *n*: F" system,
- $\boldsymbol{CR}_n^{(m)}(t) = P(T > t), t \in <0,\infty),$ is the reliability function of a non-homogeneous consecutive "*m* out of *n*: F" system,
- $\boldsymbol{CF}_n^{(m)}(t) = 1 - \boldsymbol{CR}_n^{(m)}(t) = P(T \le t), t \in <0,\infty),$ is the distribution function of a consecutive "*m* out of *n*: F" system lifetime T ,

we can formulate the following auxiliary theorem [5].

Lemma 1. The reliability function of the two-state consecutive "*m* out of *n*: F" system is given by the following recurrent formula

$$\boldsymbol{CR}_n^{(m)}(t) = \begin{cases} 1 & \text{for } n < m, \\ 1 - \prod_{i=1}^{n}\left(1 - R_i(t)\right) & \text{for } n = m, \\ R_n(t)\boldsymbol{CR}_{n-1}^{(m)}(t) & \\ + \sum_{j=1}^{m-1} R_{n-j}(t)\boldsymbol{CR}_{n\text{-}j\text{-}1}^{(m)}(t) & \\ \cdot \prod_{i=n-j+1}^{n}\left(1 - R_i(t)\right) & \text{for } n > m, \end{cases} \tag{1}$$

for $t \in <0,\infty)$.

Definition 2. The consecutive "*m* out of *n*: F" system is called homogeneous if its components lifetimes T_i have an identical distribution function

$F(t) = P(T_i \le t),\ i = 1,2,\dots,n,\ t \in <0,\infty),$

i.e. if its components E_i have the same reliability function

$R(t) = 1 - F(t),\ t \in <0,\infty).$

Lemma 1 simplified form for homogeneous systems takes the following form.

Lemma 2. The reliability function of the homogeneous two-state consecutive "*m* out of *n*: F" system is given by the following recurrent formula

$$\boldsymbol{CR}_n^{(m)}(t) = \begin{cases} 1 & \text{for } n < m, \\ 1 - (1 - R(t))^n & \text{for } n = m, \\ R(t)\boldsymbol{CR}_{n-1}^{(m)}(t) & \\ + R(t)\sum_{j=1}^{m-1}(1 - R^j(t)) & \\ \cdot \boldsymbol{CR}_{n-j-1}^{(m)}(t) & \text{for } n > m, \end{cases} \tag{2}$$

for $t \in <0,\infty)$.

3 TWO-STATE CONSECUTIVE "*M* OUT OF *N*: G" SYSTEMS

Definition 3. A two-state system is called a two-state consecutive "*m* out of *n*: G" system if it is good if and only if at least its *m* neighbouring components out of n its components arranged in a sequence of $E_1, E_2, \dots, E_n$, are good.

In further analysis we assume, that:

- T is a random variable representing the lifetime of the consecutive "*m* out of *n*: G" system,
- $\boldsymbol{CRG}_n^{(m)}(t) = P(T > t), t \in <0,\infty),$ is the reliability function of a non-homogeneous consecutive "*m* out of *n*: G" system,
- $\boldsymbol{CFG}_n^{(m)}(t) = 1 - \boldsymbol{CRG}_n^{(m)}(t) = P(T \le t), t \in <0,\infty),$ is the distribution function of a consecutive "*m* out of *n*: G" system lifetime T.

Thus, we can formulate the following auxiliary theorem (Malinowski 2005).

Lemma 3. The reliability function of the two-state consecutive "*m* out of *n*: G" system is given by the following recurrent formula

$$\boldsymbol{CRG}_n^{(m)}(t) = \begin{cases} 0 & \text{for } n < m, \\ \prod_{i=1}^{n} R_i(t) & \text{for } n = m, \\ (1 - R_n(t))\boldsymbol{CRG}_{n-1}^{(m)}(t) & \\ + \sum_{j=1}^{m-1} (1 - R_{n-j}(t))(1 - \boldsymbol{CRG}_{n\text{-}j\text{-}1}^{(m)}(t)) & \\ \cdot \prod_{i=n-j+1}^{n} R_i(t) & \text{for } n > m, \end{cases} \tag{3}$$

for $t \in < 0, \infty)$.

From the above theorem, as a particular case for the homogeneous system, i.e. system composed of components with identical reliability, we immediately get the following corollary.

Corollary 4. The reliability function of the homogeneous two-state consecutive “*m* out of *n*: G” system is given by the following recurrent formula

$$\boldsymbol{CRG}_n^{(m)}(t) = \begin{cases} 0 & \text{for n} < m, \\ [R(t)]^n & \text{for } n = m, \\ (1 - R(t))\boldsymbol{CRG}_{n\text{-}1}^{(m)}(t) & \\ + (1 - R(t)) \sum_{j=1}^{m-1} R^j(t) & \\ \cdot (1 - \boldsymbol{CRG}_{n\text{-}j\text{-}1}^{(m)}(t)) & \text{for n} > m, \end{cases} \tag{4}$$

for $t \in < 0, \infty)$.

4 THE MAIN KIND OF NAVIGATION INFRASTRUCTURE IN WATERWAYS DESIGN

The classification of navigation infrastructure is as follows (Kopacz et. al. 2001, 2003):

- signalling – warning and visual positioning infrastructure;
- radio-navigation positioning infrastructure;
- vessel traffic monitoring, information and navigation support infrastructure.

Every kind of the infrastructure has components in the form of an object or a system of navigation infrastructure.
An object is a simple element, for example a buoy or lighting tower. The objects create system of navigation infrastructure.
For safe navigation in restricted or limited areas IALA introduced the system of buoys and leading lights. It can be helpful to define a clearing line for the limits of safe navigation (IALA NAVIGUIDE 2006).
There are major parameters which are important for the optimum number and arrangement of buoys and leading lights. These parameters depend on the average channel width, the channel length, whether the section is straight or curved.
In the other hand the optimum separation distance between buoys and the numbers of buoys and leading lights are important. The distance is depended on the average width of the section concerned and its curvature. It is obvious that in the sections of waterway which have the greatest risk of groundings or collisions, the numbers of buoys and leading lights should be highest (IALA NAVIGUIDE 2006).

5 SAFETY ANALYSIS OF SHIP ON WATERWAY

Definition 4. The system is in safety state if the ship operator has full navigational information.

Definition 5. The system is in dangerous state if the ship operator has insufficient navigational information.

Under above definitions we define the set of safety states as

$$S = \{S_S, S_D\},$$

where:

S_S – state of safety,
S_D – state of dangerous.

Thus, after assumption that:

n_S – limit number for safety state;
n_D – limit number for dangerous state.

and considering formulae (1)-(4), we can define probabilities of states as follows:

$$- P(S_S) = \boldsymbol{CRG}_n^{(n_s)}(t), \text{ for } t \in <0, \infty).$$
$$- P(S_D) = 1\text{-}\boldsymbol{CR}_n^{(n_D)}(t), \text{ for } t \in <0, \infty).$$

It means that

- probability that the system is in safety state is equal to probability that at least n_S neighbouring components are good;

- probability that the system is in dangerous state is equal to probability that at least n_D neighbouring components are failed.

6 APPLICATION

Let us consider the vessel waterway given in Fig 1.

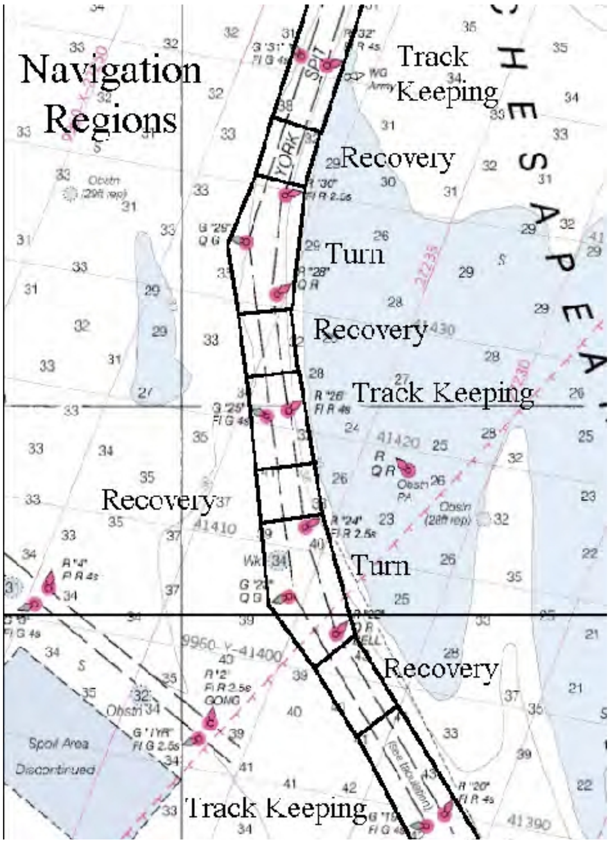

Figure 1. The vessel manoeuvring phases (IALA NAVIGUIDE 2006).

In particular case we have on the track 12 components of buoys system. We assume that for phase of track keeping ship operator need at least two navigational signs fo safety manoeuvring and in the phases of turn recovery the same operator need at least three signs. Thus, the number limits for safety states are give as

$n_S = 3,\ n_D = 2.$

Because the probabilities of buoys' visibility are the same, the probabilites of respective states are given as

$\boldsymbol{P(S_S)} = \boldsymbol{CRG}_{12}^{(3)}(t)$, where

- for $n < 3$

$$\boldsymbol{CRG}_1^{(3)}(t) = \boldsymbol{CRG}_2^{(3)}(t) = 0, \text{ for } t \in <0, \infty). \tag{5}$$

- for $n = 3$

$$\boldsymbol{CRG}_3^{(3)}(t) = [R(t)]^3, \text{ for } t \in <0, \infty). \tag{6}$$

- for $n > 3$

$$\boldsymbol{CRG}_n^{(3)}(t) = (1 - R(t))\Big[\boldsymbol{CRG}_{n\text{-}1}^{(3)}(t)$$

$$- R(t)\boldsymbol{CRG}_{n\text{-}2}^{(m)}(t) - R^2(t)\boldsymbol{CRG}_{n\text{-}3}^{(m)}(t)$$

$$+ R(t) + R^2(t)\Big] \text{ for } t \in <0, \infty). \tag{7}$$

And for

$$\boldsymbol{P(S_D)} = 1 - \boldsymbol{CR}_{12}^{(3)}(t), \text{ where}$$

- for $n < 2$

$$\boldsymbol{CR}_1^{(2)}(t) = 0, \text{ for } t \in <0, \infty). \tag{8}$$

- for $n = 2$

$$\boldsymbol{CR}_2^{(2)}(t) = 1 - 2R(t) + R^2(t), \text{ for } t \in <0, \infty). \tag{9}$$

- for $n > 3$

$$\boldsymbol{CR}_n^{(2)}(t) = 1 - R(t)\Big[\boldsymbol{CR}_{n\text{-}1}^{(2)}(t) + (1 - R(t))\boldsymbol{CR}_{n\text{-}2}^{(2)}(t)\Big], \tag{10}$$

for $t \in <0, \infty)$.

In particular case when the lifetimes of buoys have exponential distribution function of the form

$$F(t) = 1 - e^{-0.01t}, \text{ for } t \in <0, \infty),$$

i.e. if the reliability function of the particular buoys are given by

$$R(t) = e^{-0.01t}, \text{ for } t \in <0, \infty).$$

Considering (5)–(10), we get the following reccurent formula for the probabilities of safety states

a) safety state S_S

- for $n < 3$

$$\boldsymbol{CRG}_1^{(3)}(t) = \boldsymbol{CRG}_2^{(3)}(t) = 0, \text{ for } t \in <0, \infty). \tag{11}$$

- for $n = 3$

$$\boldsymbol{CRG}_3^{(3)}(t) = e^{-0.03t}, \text{ for } t \in <0, \infty). \qquad (12)$$

- for $n > 3$

$$\boldsymbol{CRG}_n^{(3)}(t) = \left(1 - e^{-0.01t}\right)\left[\boldsymbol{CRG}_{n\text{-}1}^{(3)}(t) - e^{-0.01t}\boldsymbol{CRG}_{n\text{-}2}^{(m)}(t) - e^{-0.02t}\boldsymbol{CRG}_{n\text{-}3}^{(m)}(t) + e^{-0.01t} + e^{-0.02t}\right], \text{ for } t \in <0, \infty). \qquad (13)$$

b) in the dangerous state S_D

- for $n < 2$

$$\boldsymbol{CR}_1^{(2)}(t) = 0, \text{ for } t \in <0, \infty). \qquad (14)$$

- for $n = 2$

$$\boldsymbol{CR}_2^{(2)}(t) = 1 - 2e^{-0.01t} + e^{-0.02t}, \text{ for } t \in <0, \infty). \qquad (15)$$

- for $n > 3$

$$\boldsymbol{CR}_n^{(2)}(t) = 1 - e^{-0.01t}\left[\boldsymbol{CR}_{n\text{-}1}^{(2)}(t) + \left(1 - e^{-0.01t}\right)\boldsymbol{CR}_{n\text{-}2}^{(2)}(t)\right], \qquad (16)$$

for $t \in <0, \infty)$.

Then the values of the particular probabilities of the safety states, calculated by the computer program based on the formulae (11)-(16), are presented in the Tables 1-2 and illustrated in Figure 2.

Table 1. The values of probabilities of the dangerous state of navigational signs

t	$P(S_D) = 1 - \boldsymbol{CR}_{12}^{(2)}(t)$
0.0	0.0000
5.0	0.0248
10.0	0.0885
15.0	0.1762
20.0	0.2753
25.0	0.3766
30.0	0.4737
35.0	0.5626
40.0	0.6415
45.0	0.7095
50.0	0.7671
55.0	0.8149
60.0	0.8541
65.0	0.8857
70.0	0.9111
75.0	0.9312
80.0	0.9470
85.0	0.9594
90.0	0.9690

95.0	0.9764
100.0	0.9821
105.0	0.9865
110.0	0.9898
115.0	0.9923
120.0	0.9942
125.0	0.9957
130.0	0.9968
135.0	0.9976
140.0	0.9982
145.0	0.9987
150.0	0.9990
155.0	0.9993
160.0	0.9995

Table 2. The values of probabilities of the safety state of navigational signs

t	$\boldsymbol{P(S_s)} = \boldsymbol{CRG}_{12}^{(3)}(t)$
0.0	0.0000
50.0	0.3990
100.0	0.4637
150.0	0.4871
200.0	0.4833
250.0	0.4156
300.0	0.3151
350.0	0.2194
400.0	0.1447
450.0	0.0924
500.0	0.0578
550.0	0.0357
600.0	0.0219
650.0	0.0134
700.0	0.0082
750.0	0.0050
800.0	0.0030
850.0	0.0018
900.0	0.0011
950.0	0.0007
1000.0	0.0004

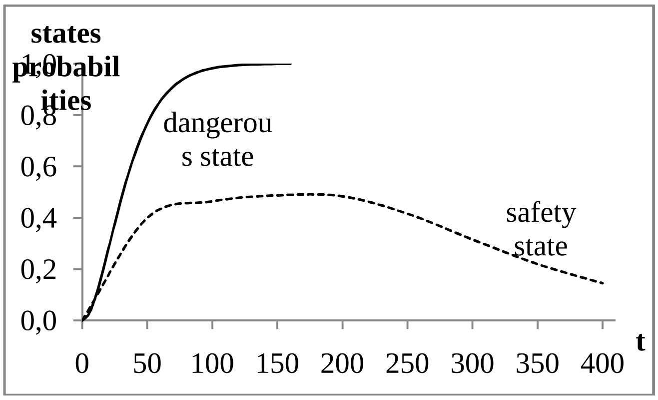

Figure 2. The graphs of particular states probabilities

7 CONCLUSIONS

The paper is devoted to an approach to safety analysis of ship in restricted waterways because of navigational infrastructure. The recurrent formulae for two-state reliability functions, a general one for non-homogeneous and its simplified form for homogeneous two-state consecutive "*m* out of *k*: G" systems have been proposed. The formulae for a homogeneous two-state consecutive "*m* out of *k*: F" and a homogeneous two-state consecutive "*m* out of *k*: G" has been applied to evaluation of ship safety in limited waterway.

Further, the safety model was used to the safety of ship on exemplary limited area with 12 navigational signs. The probabilities of respective states was evaluated and illustrated.

The transition probabilities between states depend of navigational signs technical reliability and waterway shape.

The calculated examples show us the possibilities of practical model usage.

REFERENCES

Antonopoulou J. M., Papstavridis S., 1987. Fast recursive algorithm to evaluate the reliability of a circular consecutive-k-out-of-n: F system. *IEEE Transactions on Reliability*, Tom R-36, Nr 1, 83 – 84.

Barlow R. E., Proschan F., 1975. *Statistical Theory of Reliability and Life Testing. Probability Models*. Holt Rinehart and Winston, Inc., New York.

Fuji Y., 1977. The behaviour of ships in limited waters. *Proc. Of the 24th International PIANC Congress*, Leningrad.

Gucma L., 1998. Kryterium bezpieczeństwa manewru na torze wodnym. *Materiały na Konferencję Explo-Ship*, WSM, Szczecin.

Guze S., 2007. Numerical approach to reliability evaluation of two-state consecutive „k out of n: F" systems. *Proc.1st Summer Safety and Reliability Seminars, SSARS 2007*, Sopot, 167-172.

Guze S., 2007. Numerical approach to reliability evaluation of non-homogeneous two-state consecutive „k out of n: F" systems. *Proc. Risk, Quality and Reliability, RQR 2007*, Ostrava, 69-74.

Hwang F. K., 1982. Fast Solutions for Consecutive-k-out-of-n: F System. *IEEE Transactions on Reliability*, Vol. R-31, No. 5, pp 447-448.

IALA NAVGUIDE. 2006. Aids for Navigations. *IALA-AISM*, France.

Kołowrocki K., 2004. *Reliability of Large Systems*, Elsevier.

Kopacz Z., Morgaś W., Urbański J., 2001. The maritime Safety system. Its components and elements. *The Journal of Navigation*, No 2.

Kopacz Z., Morgaś W., Urbański J., 2003. Navigational infrastructure. Its state. Its establishing and its changes. *Annual of Navigation*, No 6.

Malinowski J., 2005, Algorithms for reliability evaluation of different type network systems, *WIT*, Warsaw.

Pietrzykowski Z., 2003. Procedury decyzyjne w sterowaniu statkiem morskim. *Zeszyty Naukowe WSM Szczecin*, Nr 72, Szczecin.

Purcz, 1998. Ship collision aspect unique to inland waterways. Ship Collision Analysis.*Gluver H. And Olsen D. (edts.), Balkema*, Rotterdam.

Smolarek L., 2009. Finite Discrete Markov Model of Ship Safety. TransNav'2009.Symposium Proceedings, Gdynia.

RELIABILITY, RISK AND AVAILABILITY BASED OPTIMIZATION OF COMPLEX TECHNICAL SYSTEMS OPERATION PROCESSES PART 1 THEORETICAL BACKGROUNDS

K. Kolowrocki, J. Soszynska.

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Gdynia Maritime University, Gdynia, Poland

e-mail: katmatkk@am.gdynia.pl, joannas@am.gdynia.pl

ABSTRACT

A convenient new tool for solving the problem of reliability and availability evaluation and optimization of complex technical systems is presented. Linking a semi-markov modeling of the system operation processes with a multi-state approach to system reliability and availability analysis is proposed to construct the joint general model of reliability and availability of complex technical systems in variable operation conditions. This joint model and a linear programming is proposed to complex technical systems reliability and availability evaluation and optimization respectively.

1 INTRODUCTION

Most real technical systems are very complex and it is difficult to analyze their reliability and availability. Large numbers of components and subsystems and their operating complexity cause that the evaluation and optimization of their reliability and availability is complicated. The complexity of the systems' operation processes and their influence on changing in time the systems' structures and their components' reliability characteristics is often very difficult to fix and to analyze. Usually the system environment and infrastructure have either an explicit or an implicit strong influence on the system operation process. As a rule some of the initiating environment events and infrastructure conditions define a set of different operation states of the technical system. A convenient tool for solving this problem is a semi-markov modeling of the system operation processes linked with a multi-state approach for the system reliability and availability analysis and a linear programming for the system reliability and availability optimization.

2 MODELLING SYSTEM OPERATION PROCESS

We assume that the system during its operation process is taking $v, v \in N,$ different operation states. Further, we define the system operation process $Z(t)$, $t \in < 0, +\infty >,$ with discrete operation states from the set of states $Z = \{z_1, z_2, \ldots, z_v\}$. Moreover, we assume that the system operation process $Z(t)$ is semi-markov (Grabski 2002) with the conditional sojourn times θ_{bl} at the operation states z_b when its next operation state is z_l, $b, l = 1,2,...,v,$ $b \neq l.$ Under these assumptions, the system operation process may be described by [1], [2], [6] the vector of probabilities of the system operation process $Z(t)$ initial operation states $[p_b(0)]_{1 \times v}$, the matrix of probabilities of the system operation process $Z(t)$ transitions between the operation states $[p_{bl}]_{v\times v}$ and the matrix of conditional distribution functions of the system operation process $Z(t)$ conditional sojourn times θ_{bl} in the operation states $[H_{bl}(t)]_{v\times v}$ or equivalently by the matrix of corresponding conditional density functions $[h_{bl}(t)]_{v\times v}$.

From the formula for total probability it follows that the unconditional distribution functions of the sojourn times θ_b, $b = 1,2,...,v,$ of the system operation process $Z(t)$ at the operation states z_b, $b = 1,2,...,v,$ are given by (Kolowrocki, Soszynska 2008)

$$H_b(t) = \sum_{l=1}^{v} p_{bl} H_{bl}(t), \;\; b = 1,2,...,v.$$

(1)

Hence, the mean values $E[\theta_b]$ of the unconditional sojourn times θ_b, $b = 1,2,...,v,$ are given by

$$M_b = E[\theta_b] = \sum_{l=1}^{v} p_{bl} M_{bl}, \; b = 1,2,...,v, \tag{2}$$

where M_{bl} are defined by the formula

$$M_{bl} = E[\theta_{bl}] = \int_0^\infty t dH_{bl}(t) = \int^\infty t h_{bl}(t)dt, \quad b,l = 1,2,...,v, \; b \neq l. \tag{3}$$

The limit values of the transient probabilities $p_b(t)$ at the particular operation states are given by (Grabski 2002, Kolowrocki, Soszynska 2008)

$$p_b = \frac{\pi_b M_b}{\sum_{l=1}^{v} \pi_l M_l}, \;\; b = 1,2,...,v, \tag{4}$$

where M_b, $b = 1,2,...,v,$ are given by (2), while the stationary probabilities π_b of the vector $[\pi_b]_{1xv}$ satisfy the system of equations

$$\begin{cases} [\pi_b] = [\pi_b][p_{bl}] \\ \sum_{l=1}^{v} \pi_l = 1. \end{cases} \tag{5}$$

3 RELIABILITY, RISK AND AVAILABILITY OF MULTI-STATE SYSTEMS IN VARIABLE OPERAION CONDITIONS

In the multi-state reliability analysis to define systems with degrading (ageing) components we assume that:

– n is the number of system components,
– E_i, i = 1,2,...,n, are components of a system,
– all components and a system under consideration have the state set {0,1,...,z}, $z \geq 1$,
– the state indexes are ordered, the state 0 is the worst and the state z is the best,

– $T_i(u)$, $i = 1,2,...,n$, are independent random variables representing the lifetimes of components E_i in the state subset $\{u,u+1,...,z\}$, while they were in the state z at the moment $t = 0$,
– $T(u)$ is a random variable representing the lifetime of a system in the state subset $\{u,u+1,...,z\}$ while it was in the state z at the moment $t = 0$,
– the system state degrades with time t without repair,
– $e_i(t)$ is a component E_i state at the moment t, $t \in <0,\infty)$, given that it was in the state z at the moment $t = 0$,
– $s(t)$ is a system state at the moment t, $t \in <0,\infty)$, given that it was in the state z at the moment $t = 0$.

The above assumptions mean that the states of the system with degrading components may be changed in time only from better to worse (Kolowrocki 2004, Kolowrock 2007).
Under these assumptions, a vector

$$R_i(t,\cdot) = [R_i(t,0),R_i(t,1),...,R_i(t,z)],\ t \in (-\infty,\infty),\ i = 1,2,...,n,$$

where

$$R_i(t,u) = P(e_i(t) \geq u \mid e_i(0) = z) = P(T_i(u) > t),\ t \in (-\infty,\infty),\ u = 0,1,...,z,$$

is the probability that the component E_i is in the state subset $\{u,u+1,...,z\}$ at the moment t, $t \in <0,\infty)$, while it was in the state z at the moment $t = 0$, is called the multi-state reliability function of a component E_i.
Similarly, a vector

$$\boldsymbol{R}_n(t,\cdot) = [\boldsymbol{R}_n(t,0), \boldsymbol{R}_n(t,1),..., \boldsymbol{R}_n(t,z)],\ t \in (-\infty,\infty),$$

where

$$\boldsymbol{R}_n(t,u) = P(s(t) \geq u \mid s(0) = z) = P(T(u) > t),\ t \in (-\infty,\infty),\ u = 0,1,...,z, \quad (6)$$

is the probability that the system is in the state subset $\{u,u+1,...,z\}$ at the moment t, $t \in <0,\infty)$, while it was in the state z at the moment $t = 0$, is called the multi-state reliability function of a system.

A probability

$$\boldsymbol{r}(t) = P(s(t) < r \mid s(0) = z) = P(T(r) \leq t),\ t \in (-\infty,\infty),$$

that the system is in the subset of states worse than the critical state r, $r \in \{1,...,z\}$ while it was in the state z at the moment $t = 0$ is called a risk function of the multi-state system or, in short, a risk (Kołowrocki 2004).

Under this definition, from (6), we have

$$\boldsymbol{r}(t) = 1 - P(s(t) \geq r|s(0) = z) = 1 - \boldsymbol{R}_n(t,r),\ \ t \in (-\infty,\infty). \quad (7)$$

and if τ is the moment when the risk exceeds a permitted level δ, then

$$\tau = \boldsymbol{r}^{-1}(\delta), \tag{8}$$

where $\boldsymbol{r}^{-1}(t)$, if it exists, is the inverse function of the risk function $\boldsymbol{r}(t)$.

Further, we assume that the changes of the process $Z(t)$ states have an influence on the system multi-state components E_i reliability and the system reliability structure as well. Thus, we denote the conditional reliability function of the system multi-state component E_i while the system is at the operational state z_b, $b = 1,2,...,v$, by (Kołowrocki et all 2008, Limnios, Oprisan 2001, Ross 2007, Soszynska 2006a,b)

$$[R_i(t, \cdot)]^{(b)} = [1, [R_i(t,1)]^{(b)}, ..., [R_i(t,z)]^{(b)}], \ i = 1,2,...,n,$$

where for $t \in <0,\infty)$, $u = 1,2,...,z$, $b = 1,2,...,v$,

$$[R_i(t,u)]^{(b)} = P(T_i^{(b)}(u) > t | Z(t) = z_b)$$

and the conditional reliability function of the system while the system is at the operational state z_b, $b = 1,2,...,v$, by

$$[\boldsymbol{R_n}(t,\cdot)]^{(b)} = [1, [\boldsymbol{R_n}(t,1)]^{(b)}, ..., [\boldsymbol{R_n}(t,z)]^{(b)}],$$

where for $t \in <0,\infty)$, $u = 1,2,...,z$, $b = 1,2,...,v$.

$$[\boldsymbol{R_n}(t,u)]^{(b)} = P(T^{(b)}(u) > t | Z(t) = z_b),$$

and $T^{(b)}(u)$ is the system conditional lifetime at the operational state z_b, dependent on the components conditional lifetimes at the operational state z_b.

The reliability function $[R_i(t,u)]^{(b)}$ is the conditional probability that the component E_i lifetime $T_i^{(b)}(u)$ in the state subset $\{u, u+1,..., z\}$ is greater than t, while the process $Z(t)$ is at the operation state z_b. Similarly, the reliability function $[\boldsymbol{R_n}(t,u)]^{(b)}$ is the conditional probability that the system lifetime $T^{(b)}(u)$ in the state subset $\{u, u+1,..., z\}$ is greater than t, while the process $Z(t)$ is at the operation state z_b. In the case when the system operation time θ is large enough, the unconditional reliability function of the system

$$\boldsymbol{R_n}(t,\cdot) = [1, \boldsymbol{R_n}(t,1), ..., \boldsymbol{R_n}(t,z)], \tag{9}$$

where

$$\boldsymbol{R_n}(t,u) = P(T(u) > t), \text{ for } t \in <0,\infty), u = 1,2,...,z,$$

and $T(u)$ is the unconditional lifetime of the system in the system reliability state subsets is given by

$$\boldsymbol{R}_n(t,u) \cong \sum_{b=1}^{v} p_b [\boldsymbol{R}_n(t,u)]^{(b)} \text{ for } t \geq 0,\ u = 1,2,...,z, \tag{10}$$

and the mean value of the system unconditional lifetime in the system reliability state subsets is

$$\mu(u) \cong \sum_{b=1}^{v} p_b \mu_b(u),\ \ u = 1,2,...,z, \tag{11}$$

where

$$\mu_b(u) = \int_0^\infty [\boldsymbol{R}_n(t,u)]^{(b)} dt,\ \ u = 1,2,...,z, \tag{12}$$

and p_b are given by (4) and the variance of the system unconditional lifetime in the system reliability state subsets is

$$\sigma^2(u) = 2\int_0^\infty t\ \boldsymbol{R}_n(t,u)dt - [\mu(u)]^2,\ \ u = 1,2,...,z. \tag{13}$$

Additionally, according to (3.19) (Blokus et all 2008), we get the following formulae for mean values of the unconditional lifetime of the system in particular reliability states

$$\bar{\mu}(u) = \mu(u) - \mu(u+1),\ \ u = 0,1,...,z-1,\ \ \bar{\mu}(z) = \mu(z). \tag{14}$$

The main characteristics of multi-state renewal system with ignored time of renovation related to their operation process can be approximately determined by using results of the research report (Blokus et all 2008) formulated in the form of the following theorem.

Theorem 3.1

If components of the multi-state renewal system with ignored time of renovation at the operational states have exponential reliability functions and the system reliability critical state is r, $r \in \{1,2,...,z\}$, then:

i) the distribution of the time $S_N(r)$ until the *Nth* exceeding of reliability critical state *r* of this system, for sufficiently large *N*, has approximately normal distribution $N(N\mu(r), \sqrt{N}\sigma(r))$, i.e.,

$$F^{(N)}(t,r) = P(S_N(r) < t) \cong F_{N(0,1)}\left(\frac{t - N\mu(r)}{\sqrt{N}\sigma(r)}\right),\ t \in (-\infty,\infty),\ \ r \in \{1,2,...,z\},$$

ii) the expected value and the variance of the time $S_N(r)$ until the *Nth* exceeding the reliability critical state *r* of this system take respectively forms

$$E[S_N(r)] = N\mu(r),\ \ D[S_N(r)] = N[\sigma(r)]^2,\ r \in \{1,2,...,z\},$$

iii) the distribution of the number $N(t,r)$ of exceeding the reliability critical state *r* of this system up to the moment $t,\ t \geq 0$, for sufficiently large *t*, is approximately of the form

$$P(N(t,r)=N) \cong F_{N(0,1)}\left(\frac{N\mu(r)-t}{\sigma(r)\sqrt{\frac{t}{\mu(r)}}}\right) - F_{N(0,1)}\left(\frac{(N+1)\mu(r)-t}{\sigma(r)\sqrt{\frac{t}{\mu(r)}}}\right),\ N=0,1,2,\ldots,\ r\in\{1,2,\ldots,z\},$$

iv) the expected value and the variance of the number $N(t,r)$ of exceeding the reliability critical state *r* of this system at the moment $t,\ t\geq 0,$ for sufficiently large *t*, approximately take respectively forms

$$H(t,r)=\frac{t}{\mu(r)},\ \ D(t,r)=\frac{t}{[\mu(r)]^3}[\sigma(r)]^2,\ r\in\{1,2,\ldots,z\},$$

where and $\mu(r)$ and $\sigma(r)$ are given by (11)-(13) for $u=r$.

The main characteristics of multi-state renewal system with non-ignored time of renovation related to their operation process can be approximately determined by using results of the research report [1] formulated in the form of the following theorem.

Theorem 3.2

If components of the multi-state renewal system with non-ignored time of renovation at the operational states have exponential reliability functions, the system reliability critical state is r, $r\in\{1,2,\ldots,z\},$ and the successive times of system's renovations are independent and have an identical distribution function with the expected value $\mu_o(r)$ and the variance $\sigma_o^2(r),$ then:

i) the distribution function of the time $\bar{\bar{S}}_N(r)$ until the *Nth* system's renovation, for sufficiently large *N*, has approximately normal distribution

$$N(N(\mu(r)+\mu_o(r)),\sqrt{N(\sigma^2(r)+\sigma_o^2(r))}),\text{ i.e.,}$$

$$\bar{\bar{F}}^{(N)}(t,r)=P(\bar{\bar{S}}_N(r)<t)\cong F_{N(0,1)}\left(\frac{t-N(\mu(r)+\mu_o(r))}{\sqrt{N(\sigma^2(r)+\sigma_o^2(r))}}\right),\ t\in(-\infty,\infty),\ N=1,2,\ldots,\ r\in\{1,2,\ldots,z\},$$

ii) the expected value and the variance of the time $\bar{\bar{S}}_N(r)$ until the *Nth* system's renovation take respectively forms

$$E[\bar{\bar{S}}_N(r)]\cong N(\mu(r)+\mu_o(r)),\ \ D[\bar{\bar{S}}_N(r)]\cong N(\sigma^2(r)+\sigma_o^2(r)),\ r\in\{1,2,\ldots,z\},$$

iii) the distribution function of the time $\bar{S}_N(r)$ until the *Nth* exceeding the reliability critical state *r* of this system takes form

$$\bar{F}^{(N)}(t,r)=P(\bar{S}_N(r)<t)\cong F_{N(0,1)}\left(\frac{t-N(\mu(r)+\mu_o(r))+\mu_o(r)}{\sqrt{N(\sigma^2(r)+\sigma_o^2(r))-\sigma_o^2(r)}}\right),\ t\in(-\infty,\infty),\ \ N=1,2,\ldots,$$

$r\in\{1,2,\ldots,z\},$

iv) the expected value and the variance of the time $\bar{S}_N(r)$ until the *Nth* exceeding the reliability critical state *r* of this system take respectively forms

$$E[\bar{S}_N(r)] \cong N\mu(r) + (N-1)\mu_o(r),\ D[\bar{S}_N(r)] \cong N\sigma^2(r) + (N-1)\sigma_o^2(r),\ r \in \{1,2,...,z\},$$

v) the distribution of the number $\bar{\bar{N}}(t,r)$ of system's renovations up to the moment $t, t \geq 0,$ is of the form

$$P(\bar{\bar{N}}(t,r) = N) \cong F_{N(0,1)}\left(\frac{N(\mu(r)+\mu_o(r)) - t}{\sqrt{\frac{t}{\mu(r)+\mu_0(r)}(\sigma^2(r)+\sigma_o^2(r))}}\right) - F_{N(0,1)}\left(\frac{(N+1)(\mu(r)+\mu_o(r)) - t}{\sqrt{\frac{t}{\mu(r)+\mu_0(r)}(\sigma^2(r)+\sigma_o^2(r))}}\right),$$

$N = 1,2,...,\ r \in \{1,2,...,z\},$

vi) the expected value and the variance of the number $\bar{\bar{N}}(t,r)$ of system's renovations up to the moment $t, t \geq 0,$ take respectively forms

$$\bar{\bar{H}}(t,r) \cong \frac{t}{\mu(r)+\mu_o(r)},\ \bar{\bar{D}}(t,r) \cong \frac{t}{(\mu(r)+\mu_o(r))^3}(\sigma^2(r)+\sigma_o{}^2(r)),\ r \in \{1,2,...,z\},$$

vii) the distribution of the number $\bar{N}(t,r)$ of exceeding the reliability critical state r of this system up to the moment $t, t \geq 0,$ is of the form

$$P(\bar{N}(t,r) = N) \cong F_{N(0,1)}\left(\frac{N(\mu(r)+\mu_o(r)) - t - \mu_0(r)}{\sqrt{\frac{t+\mu_0(r)}{\mu(r)+\mu_0(r)}(\sigma^2(r)+\sigma_o^2(r))}}\right) - F_{N(0,1)}\left(\frac{(N+1)(\mu(r)+\mu_o(r)) - t - \mu_0(r)}{\sqrt{\frac{t+\mu_0(r)}{\mu(r)+\mu_0(r)}(\sigma^2(r)+\sigma_o^2(r))}}\right),$$

$N = 1,2,...,\ r \in \{1,2,...,z\},$

viii) the expected value and the variance of the number $\bar{N}(t,r)$ of exceeding the reliability critical state r of this system up to the moment $t, t \geq 0,$ for sufficiently large t, are approximately respectively given by

$$\bar{H}(t,r) \cong \frac{t+\mu_0(r)}{\mu(r)+\mu_o(r)},\ \bar{D}(t,r) \cong \frac{t+\mu_0(r)}{(\mu(r)+\mu_o(r))^3}(\sigma^2(r)+\sigma_o{}^2(r)),\ r \in \{1,2,...,z\},$$

ix) the availability coefficient of the system at the moment t is given by the formula

$$A(t,r) \cong \frac{\mu(r)}{\mu(r)+\mu_o(r)},\ \ t \geq 0,\ r \in \{1,2,...,z\},$$

x) the availability coefficient of the system in the time interval $< t, t+\tau), \tau > 0,$ is given by the formula

$$A(t,\tau,r) \cong \frac{1}{\mu(r)+\mu_o(r)}\int_\tau^\infty \boldsymbol{R}_n(t,r)dt,\ \ t \geq 0,\ \ \tau > 0,\ r \in \{1,2,...,z\},$$

where $\boldsymbol{R}_n(t,r)$ is given by the formula (10) and $\mu(r)$ and $\sigma(r)$ are given by (11)-(13) for $u = r$.

4 OPTIMIZATION OF A SYSTEM OPERATIONPROCESS

4.1 Optimal transient probabilities maximizing system lifetime

Considering the equation (10), it is natural to assume that the system operation process has a significant influence on the system reliability. This influence is also clearly expressed in the equation (11) for the mean values of the system unconditional lifetimes in the reliability state subsets.

From linear equation (11), we can see that the mean value of the system unconditional lifetime $\mu(u)$, $u = 1,2,...,z,$ is determined by the limit transient probabilities p_b, $b = 1,2,...,\nu,$ of the system operation states given by (4) and the mean values $\mu_b(u)$, $b = 1,2,...,\nu,$ $u = 1,2,...,z,$ of the system conditional lifetimes in the reliability state subsets $\{u, u+1,..., z\}$, $u = 1,2,..., z,$ given by (3.6). Therefore, the system lifetime optimization approach based on the linear programming can be proposed. Namely, we may look for the corresponding optimal values $\dot{p}_b$, $b = 1,2,...,\nu,$ of the transient probabilities p_b, $b = 1,2,...,\nu,$ in the system operation states to maximize the mean value $\mu(u)$ of the unconditional system lifetimes in the reliability state subsets $\{u, u+1,..., z\}$, $u = 1,2,..., z,$ under the assumption that the mean values $\mu_b(u)$, $b = 1,2,...,\nu,$ $u = 1,2,...,z,$ of the system conditional lifetimes in the reliability state subsets are fixed. As a special case of the above formulate system lifetime optimization problem, if r, $r = 1,2,...,z,$ is a system critical reliability state, then we want to find the optimal values $\dot{p}_b$, $b = 1,2,...,\nu,$ of the transient probabilities p_b, $b = 1,2,...,\nu,$ in the system operation states to maximize the mean value $\mu(r)$ of the unconditional system lifetimes in the reliability state subset $\{r, r+1, ,..., z\}$, $r = 1,2,..., z,$ under the assumption that the mean values $\mu_b(r)$, $b = 1,2,...,\nu,$ $r = 1,2,...,z,$ of the system conditional lifetimes in this reliability state subset are fixed. More exactly, we formulate the optimization problem as a linear programming model with the objective function of the following linear form

$$\mu(r) = \sum_{b=1}^{\nu} p_b \mu_b(r) \tag{15}$$

for a fixed $r \in \{1,2,..., z\}$ and with the following bound constraints

$$\sum_{b=1}^{\nu} p_b = 1, \tag{16}$$

$$\breve{p}_b \le p_b \le \hat{p}_b, \; b = 1,2,...,\nu, \tag{17}$$

where

$$\mu_b(r), \; \mu_b(r) \ge 0, \; b = 1,2,...,\nu,$$

are fixed mean values of the system conditional lifetimes in the reliability state subset $\{r, r+1,..., z\}$ and

$\breve{p}_b$, $0 \le \breve{p}_b \le 1$ and $\hat{p}_b$, $0 \le \hat{p}_b \le 1,$ $\breve{p}_b \le \hat{p}_b$, $b = 1,2,...,\nu,$
(18)

are lower and upper bounds of the unknown transient probabilities p_b, $b = 1,2,...,\nu$, respectively.
Now, we can obtain the optimal solution of the formulated by (15)-(18) the linear programming problem, i.e. we can find the optimal values $\dot{p}_b$ of the limit transient probabilities p_b, $b = 1,2,...,\nu$, that maximize the objective function (15). First, we arrange the system conditional lifetime mean values $m^{(b)}(r)$, $b = 1,2,...,\nu$, in non-increasing order

$$\mu_{b_1}(r) \geq \mu_{b_2}(r) \geq \ldots \geq \mu_{b_\nu}(r), \text{ where } b_i \in \{1,2,...,\nu\} \text{ for } i = 1,2,...,\nu.$$

Next, we substitute

$$x_i = p_{b_i}, \ \breve{x}_i = \breve{p}_{b_i}, \ \hat{x}_i = \hat{p}_{b_i} \text{ for } i = 1,2,...,\nu \quad (19)$$

and we maximize with respect to x_i, $i = 1,2,...,\nu$, the linear form (15) that after this transformation takes the form

$$\mu(r) = \sum_{i=1}^{\nu} x_i \mu_{b_i}(r) \quad (20)$$

for a fixed $r \in \{1,2,...,z\}$ with the following bound constraints

$$\sum_{i=1}^{\nu} x_i = 1, \quad (21)$$

$$\breve{x}_i \leq x_i \leq \hat{x}_i, \ i = 1,2,...,\nu, \quad (22)$$

where

$$\mu_{b_i}(r), \ \ \mu_{b_i}(r) \geq 0, \ \ i = 1,2,...,\nu,$$

are fixed mean values of the system conditional lifetimes in the reliability state subset $\{r, r+1,..., z\}$ arranged in non-increasing order and

$$\breve{x}_i, \ 0 \leq \breve{x}_i \leq 1 \text{ and } \hat{x}_i, \ 0 \leq \hat{x}_i \leq 1, \ \breve{x}_i \leq \hat{x}_i, \ i = 1,2,...,\nu, \quad (23)$$

are lower and upper bounds of the unknown limit transient probabilities x_i, $i = 1,2,...,\nu$, respectively.
We define

$$\breve{x} = \sum_{i=1}^{\nu} \breve{x}_i, \ \ \hat{y} = 1 - \breve{x} \quad (24)$$

and

$$\breve{x}^0 = 0, \ \hat{x}^0 = 0 \text{ and } \breve{x}^I = \sum_{i=1}^{I} \breve{x}_i, \ \hat{x}^I = \sum_{i=1}^{I} \hat{x}_i \quad \text{for } I = 1,2,...,\nu. \quad (25)$$

Next, we find the largest value $I \in \{0,1,...,\nu\}$ such that

$$\hat{x}^I - \breve{x}^I < \hat{y} \tag{26}$$

and we fix the optimal solution that maximize (20) in the following way:

i) if $I = 0$, the optimal solution is

$$\dot{x}_1 = \hat{y} + \breve{x}_1 \text{ and } \dot{x}_i = \breve{x}_i \text{ for } i = 2,3,...,\nu; \tag{27}$$

ii) if $0 < I < \nu$, the optimal solution is

$$\dot{x}_i = \hat{x}_i \text{ for } i = 1,2,...,I,\ \dot{x}_{I+1} = \hat{y} - \hat{x}^I + \breve{x}^I + \breve{x}_{I+1}$$

and

$$\dot{x}_i = \breve{x}_i \text{ for } i = I+2, I+3,...,\nu; \tag{28}$$

iii) if $I = \nu$, the optimal solution is

$$\dot{x}_i = \hat{x}_i \text{ for } i = 1,2,...,\nu. \tag{29}$$

Finally, after making the inverse to (19) substitution, we get the optimal limit transient probabilities

$$\dot{p}_{b_i} = \dot{x}_i \text{ for } i = 1,2,...,\nu, \tag{30}$$

that maximize the system mean lifetime given by the linear form (15) giving its optimal value in the following form

$$\dot{\mu}(r) = \sum_{b=1}^{\nu} \dot{p}_b \mu_b(r) \tag{31}$$

for a fixed $r \in \{1,2,...,z\}$.

From the above, replacing r by u, $u = 1,2,...,z$, we obtain the corresponding optimal solutions for the mean values of the system unconditional lifetimes in the reliability state subsets $\{u, u+1,...,z\}$ of the form

$$\dot{\mu}(u) = \sum_{b=1}^{\nu} \dot{p}_b \mu_b(u) \text{ for } u = 1,2,...,z,$$

(32)

and by (13) the corresponding values of the variances of the system unconditional lifetimes in the system reliability state subsets is

$$\dot{\sigma}^2(u) = 2\int_0^\infty t\ \dot{\boldsymbol{R}}_n(t,u)dt - [\dot{\mu}(u)]^2,\ u = 1,2,...,z, \tag{33}$$

where $\dot{\mu}(u)$ is given by (32) and $\dot{\boldsymbol{R}}_n(t,u)$, according to (9)-(10), is the coordinate of the corresponding optimal unconditional multistate reliability function of the system

$$\dot{\boldsymbol{R}}_{\boldsymbol{n}}(t,\cdot) = [1, \dot{\boldsymbol{R}}_{\boldsymbol{n}}(t,1), ..., \dot{\boldsymbol{R}}_{\boldsymbol{n}}(t,z)], \quad (34)$$

given by

$$\dot{\boldsymbol{R}}_n(t,u) \cong \sum_{b=1}^{v} \dot{p}_b[\boldsymbol{R}_n(t,u)]^{(b)} \text{ for } t \geq 0, \quad u = 1,2,..., z, \quad (35)$$

and by (14) the optimal solutions for the mean values of the system unconditional lifetimes in the particular reliability states are of the form

$$\dot{\bar{\mu}}(u) = \dot{\mu}(u) - \dot{\mu}(u+1), \; u = 0,1,..., z-1, \; \dot{\bar{\mu}}(z) = \dot{\mu}(z). \quad (36)$$

Moreover, considering (7) and (8), the corresponding optimal system risk function and the moment when the risk exceeds a permitted level δ, respectively are given by

$$\dot{\boldsymbol{r}}(t) = 1 - \dot{\boldsymbol{R}}_n(t,r), \; t \in (-\infty,\infty), \quad (37)$$

and

$$\dot{\tau} = \dot{\boldsymbol{r}}^{-1}(\delta), \quad (38)$$

where $\dot{\boldsymbol{r}}^{-1}(t)$, if it exists, is the inverse function of the risk function $\dot{\boldsymbol{r}}(t)$.
Finally, replacing $\mu(r)$ by $\dot{\mu}(r)$ and $\sigma(r)$ by $\dot{\sigma}(r)$ in the expressions for the renewal systems characteristics pointed in Theorem 1 and Theorem 2, we get their corresponding optimal values.

4.2. Optimal sojourn times in operation states maximizing system lifetime

Replacing in (4) limit transient probabilities p_b in operational states by their optimal values $\dot{p}_b$ found in the previous section and the mean values M_b of the unconditional sojourn times in operational states by their corresponding unknown optimal values $\dot{M}_b$ we get the system of equations

$$\dot{p}_b = \frac{\pi_b \dot{M}_b}{\sum_{l=1}^{v} \pi_l \dot{M}_l}, \; b = 1,2,..., v. \quad (39)$$

After simple transformations the above system takes the form

$$\begin{aligned}
&(\dot{p}_1 - 1)\pi_1\dot{M}_1 + \dot{p}_1\pi_2\dot{M}_2 + ... + \dot{p}_1\pi_v\dot{M}_v = 0 \\
&\dot{p}_2\pi_1\dot{M}_1 + (\dot{p}_2 - 1)\pi_2\dot{M}_2 + ... + \dot{p}_2\pi_v\dot{M}_v = 0 \\
&. \\
&. \\
&. \\
&\dot{p}_v\pi_1\dot{M}_1 + \dot{p}_v\pi_2\dot{M}_2 + ... + (\dot{p}_v - 1)\pi_v\dot{M}_v = 0,
\end{aligned} \quad (40)$$

where $\dot{M}_b$ are unknown variables we want to find, $\dot{p}_b$ are optimal limit transient probabilities determined by (30) and π_b are probabilities determined by (2).

Since the above system is homogeneous then it has nonzero solutions when the determinant of the system equations main matrix is equal to zero, i.e. if its rank is less than ν. Moreover, in this case the solutions are ambiguous. Anyway, if we fix the optimal values $\dot{M}_b$ of the mean values M_b of the unconditional sojourn times in operational states, for instance by arbitrary fixing one or a few of them, then it is also possible to look for the optimal values $\dot{M}_{bl}$ of the mean values M_{bl} of the conditional sojourn times in operational states using the following system of equations

$$\sum_{l=1}^{\nu} p_{bl} \dot{M}_{bl} = \dot{M}_b, \quad b = 1,2,...,\nu, \tag{41}$$

obtained from (2) by replacing M_b by $\dot{M}_b$ and M_{bl} by $\dot{M}_{bl}$, were p_{bl} are known probabilities of the system operation process transitions between the operation states.

5 REFERENCES

Blokus-Roszkowska, A., Guze, S., Kołowrocki K., Kwiatuszewska-Sarnecka, B., Soszyńska J. 2008. *Models of safety, reliability and availability evaluation of complex technical systems related to their operation processes*. WP 4 – Task 4.1 – English – 31.05.2008. Poland-Singapore Joint Project. MSHE Decision No. 63/N-Singapore/2007/0.

Grabski, F. 2002. *Semi-Markov Models of Systems Reliability and Operations*. Warsaw: Systems Research Institute, Polish Academy of Sciences.

Kołowrocki, K. 2004. *Reliability of Large Systems*, Elsevier.

Kołowrocki, K. 2007a. Reliability modelling of complex systems – Part 1. *International Journal of Gnedenko e-Forum "Reliability: Theory & Application"*, Vol. 2, No 3-4, 116-127.

Kołowrocki, K. 2007b. Reliability modelling of complex systems – Part 2. *International Journal of Gnedenko e-Forum "Reliability: Theory & Application"*, Vol. 2, No 3-4, 128-139.

Kołowrocki, K. & Soszyńska, J. 2008. *A general model of technical systems operation processes related to their environment and infrastructure*. WP 2 – Task 2.1 – English – 31.05.2008. Poland-Singapore Joint Project. MSHE Decision No. 63/N-Singapore/2007/0.

Kołowrocki, K., Soszyńska, J., Baranowski, Z., Golik, P. 2008. *Preliminary modeling, statistical identification and evaluation of reliability, risk, availability and safety of port, shipyard and ship technical systems in constant and variable operation conditions*. WP4 - Task 4.2. Preliminary reliability, risk and availability analysis and evaluation of a port oil transportation system in constant and variable operation conditions. WP4 - Sub-Task 4.2.1 – English – 30.11.2008. Poland-Singapore Joint Project. MSHE Decision No. 63/N-Singapore/2007/0.

Limnios, N., Oprisan, G. 2001. *Semi-Markov Processes and Reliability.* Birkhauser, Boston.

Ross, S. M. 2007. *Introduction to Probability Models*. Elsevier, San Diego, 2007.

Soszyńska, J. 2006a. Reliability of large series-parallel system in variable operation conditions. *International Journal of Automation and Computing*. Vol. 3, No 2: 199-206.

Soszyńska, J. 2006b. Reliability evaluation of a port oil transportation system in variable operation conditions. *International Journal of Pressure Vessels and Piping*. Vol. 83, Issue 4: 304-310.

RELIABILITY, RISK AND AVAILABILITY BASED OPTIMIZATION OF COMPLEX TECHNICAL SYSTEMS OPERATION PROCESSES PART 2 APPLICATION IN PORT TRANSPORTATION

K. Kolowrocki, J. Soszynska.

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Gdynia Maritime University, Gdynia, Poland

e-mail: katmatkk@am.gdynia.pl, joannas@am.gdynia.pl

ABSTRACT

The joint general model of reliability and availability of complex technical systems in variable operation conditions linking a semi-markov modeling of the system operation processes with a multi-state approach to system reliability and availability analysis and linear programming considered in the paper Part 1 are applied in maritime industry to reliability, risk and availability optimization of a port piping oil transportation system.

6 RELIABILITY, RISK AND AVAILABILITY EVALUATION OF A PORT OIL PIPING TRANSPORTATION SYSTEM

The oil terminal in Dębogórze is designated for the reception from ships, the storage and sending by carriages or cars the oil products. It is also designated for receiving from carriages or cars, the storage and loading the tankers with oil products such like petrol and oil.

The considered system is composed of three terminal parts *A*, *B* and *C*, linked by the piping transportation systems. The scheme of this system is presented in *Figure 1* (Kołowrocki et all. 2008).

The unloading of tankers is performed at the piers placed in the Port of Gdynia. The piers is connected with terminal part *A* through the transportation subsystem S_1 built of two piping lines composed of steel pipe segments with diameter of 600 mm. In the part *A* there is a supporting station fortifying tankers

pumps and making possible further transport of oil by the subsystem S_2 to the terminal part *B*. The subsystem S_2 is built of two piping lines composed of steel pipe segments of the diameter 600 mm. The terminal part *B* is connected with the terminal part *C* by the subsystem S_3. The subsystem S_3 is built of one piping line composed of steel pipe segments of the diameter 500 mm and two piping lines composed of steel pipe segments of diameter 350 mm. The terminal part *C* is designated for the loading the rail cisterns with oil products and for the wagon sending to the railway station of the Port of Gdynia and further to the interior of the country.

The oil pipeline system consists three subsystems:

- the subsystem S_1 composed of two identical pipelines, each composed of 178 pipe segments of length 12m and two valves,
- the subsystem S_2 composed of two identical pipelines, each composed of 717 pipe segments of length 12m and to valves,
- the subsystem S_3 composed of three different pipelines, each composed of 360 pipe segments of either 10 m or 7,5 m length and two valves.

The subsystems S_1, S_2, S_3 are forming a general port oil pipeline system reliability series structure. However, the pipeline system reliability structure and the subsystems reliability depend on its changing in time operation states (Kołowrocki et all. 2008).

Taking into account the varying in time operation process of the considered system we distinguish the following as its eight operation states:

- an operation state z_1 – transport of one kind of medium from the terminal part B to part C using two out of three pipelines in subsystem S_3,
- an operation state z_2 – transport of one kind of medium from the terminal part C (from carriages) to part B using one out of three pipelines in subsystem S_3,
- an operation state z_3 – transport of one kind of medium from the terminal part B through part A to piers using one out of two pipelines in subsystem S_2 and one out of two pipelines in subsystem S_1,
- an operation state z_4 – transport of two kinds of medium from the piers through parts A and B to part C using one out of two pipelines in subsystem S_1, one out of two pipelines in subsystem S_2 and two out of three pipelines in subsystem S_3,
- an operation state z_5 – transport of one kind of medium from the piers through part A to B using one out of two pipelines in subsystem S_1 and one out of two pipelines in subsystem S_2,
- an operation state z_6 – transport of one kind of medium from the terminal part B to C using two out of three pipelines in subsystem S_3, and simultaneously transport one kind of medium from the piers through part A to B using one out of two pipelines in parts S_1 and one out of two pipelines in subsystem S_2,
- an operation state z_7 – lack of medium transport (system is not working)
- an operation state z_8 – transport of one kind of medium from the terminal part B to C using one out of three pipelines in part S_3, and simultaneously transport second kind of medium from the terminal part C to B using one out of three pipelines in part S_3.

At the moment because of the luck of sufficient statistical data about the oil terminal operation process it is not possible to estimate its all operational characteristics. However, on the basis the still limited data, given in (Kołowrocki et all. 2008), the transient probabilities p_{bl} from the operation state z_b into the operations state z_l for $b,l = 1,2,\ldots,8,\ b \neq l,$ were preliminary evaluated. Their approximate values are included in the matrix below

$$[p_{bl}] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0.06 & 0.06 & 0.86 & 0.02 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.125 & 0 & 0 & 0 & 0 & 0.125 & 0.687 & 0.063 \\ 0.4 & 0 & 0 & 0 & 0.6 & 0 & 0 & 0 \\ 0.82 & 0 & 0 & 0 & 0.16 & 0 & 0 & 0.02 \\ 0.67 & 0 & 0 & 0 & 0 & 0 & 0.33 & 0 \end{bmatrix} \qquad (42)$$

Unfortunately, it was not possible yet to determine the matrix of the conditional distribution functions $[H_{bl}(t)]_{8x8}$ of the sojourn times θ_{bl} for $b,l = 1,2,\ldots,8,\ b \neq l,$ [1], [6] and further consequently, according to (2.1), it was also not possible to determine the vector $[H_b(t)]_{1x8}$ of the unconditional distribution functions of the sojourn times θ_b of this operation process at the operation states z_b, $b = 1,2,\ldots,8.$ However, on the basis of the preliminary statistical data coming from experiment it was possible to evaluate approximately the conditional mean values $M_{bl} = E[\theta_{bl}],\ b,l = 1,2,...,8,\ \ b \neq l,$ of sojourn times in the particular operation states defined by (3). On the basis of the statistical data given in Tables 1-10 in (Kołowrocki et all. 2008) (Appendix 1A) their approximate evolutions are as follows:

$$M_{15} = 720,\ M_{16} = 420,\ \ M_{17} = 698.95,\ \ M_{18} = 480,$$

$$M_{51} = 750,\ \ M_{56} = 564,\ \ M_{57} = 748.7,\ \ M_{58} = 540,$$

$$M_{61} = 360,\ M_{65} = 360,\ \ M_{71} = 975.3,\ \ M_{75} = 872.4,$$

$$M_{78} = 600,\ \ M_{81} = 900,\ \ M_{87} = 420. \quad (43)$$

Hence, by (2), the unconditional mean lifetimes in the operation states are

$$M_1 = E[\theta_1] = p_{15}M_{15} + p_{16}M_{16} + p_{17}M_{17} + p_{18}M_{18}$$

$$= 0.06 \cdot 720 + 0.06 \cdot 420 + 0.86 \cdot 698.95 + 0.02 \cdot 480 \cong 679.1,$$

$$M_5 = E[\theta_5] = p_{51}M_{51} + p_{56}M_{56} + p_{57}M_{57} + p_{58}M_{58}$$

$$= 0.125 \cdot 750 + 0.125 \cdot 564 + 0.687 \cdot 748.7 + 0.063 \cdot 540 \cong 712.63,$$

$$M_6 = E[\theta_6] = p_{61}M_{61} + p_{65}M_{65} = 0.4 \cdot 360 + 0.6 \cdot 360 = 360,$$

$$M_7 = E[\theta_7] = p_{71}M_{71} + p_{75}M_{75} + p_{78}M_{78} = 0.82 \cdot 975.3 + 0.16 \cdot 872.4 + 0.02 \cdot 600 \cong 951.33,$$

$$M_8 = E[\theta_8] = p_{81}M_{81} + p_{87}M_{87} = 0.67 \cdot 900 + 0.33 \cdot 420 \cong 741.6. \quad (44)$$

Since from the system of equations (5) given here in the form

$$\begin{cases} [\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7, \pi_8] \\ = [\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7, \pi_8][p_{bl}] \\ \pi_1 + \pi_2 + \pi_3 + \pi_4 + \pi_5 + \pi_6 + \pi_7 + \pi_8 = 1, \end{cases}$$

we get

$$\pi_1 = 0.396,\ \ \pi_2 = 0,\ \ \pi_3 = 0,\ \ \pi_4 = 0,\ \ \pi_5 = 0.116,\ \pi_6 = 0.038,\ \ \pi_7 = 0.435,\ \ \pi_8 = 0.015, \quad (45)$$

then the limit values of the transient probabilities $p_b(t)$ at the operational states z_b, according to (4), are given by

$$p_1 = 0.34,\ \ p_2 = 0,\ \ p_3 = 0,\ \ p_4 = 0,\ \ p_5 = 0.1,\ \ p_6 = 0.02,\ \ p_7 = 0.53,\ \ p_8 = 0.01. \quad (46)$$

From the above result, according to (34)-(35), the unconditional multistate (three-state) reliability function of the system is of the form

$$\overline{\boldsymbol{R}}_3(t,\cdot) = [1,\ \overline{\boldsymbol{R}}_3(t,1),\ \overline{\boldsymbol{R}}_3(t,2)], \tag{47}$$

with the coordinates given by

$$\overline{\boldsymbol{R}}_3(t,1) = 0.34\cdot[\overline{\boldsymbol{R}}(t,1)]^{(1)} + 0\cdot[\overline{\boldsymbol{R}}(t,1)]^{(2)} + 0\cdot[\overline{\boldsymbol{R}}(t,1)]^{(3)} + 0\cdot[\overline{\boldsymbol{R}}(t,1)]^{(4)}$$

$$+ 0.01\cdot[\overline{\boldsymbol{R}}(t,1)]^{(5)} + 0.02\cdot[\overline{\boldsymbol{R}}(t,1)]^{(6)} + 0.53\cdot[\overline{\boldsymbol{R}}(t,1)]^{(7)} + 0.01\cdot[\overline{\boldsymbol{R}}(t,1)]^{(8)}\,] \quad \text{for } t \ge 0, \tag{48}$$

$$\overline{\boldsymbol{R}}_3(t,2) = 0.34\cdot[\overline{\boldsymbol{R}}(t,2)]^{(1)} + 0\cdot[\overline{\boldsymbol{R}}(t,2)]^{(2)} + 0\cdot[\overline{\boldsymbol{R}}(t,2)]^{(3)} + 0\cdot[\overline{\boldsymbol{R}}(t,2)]^{(4)}$$

$$+ 0.1\cdot[\overline{\boldsymbol{R}}(t,2)]^{(5)} + 0.02\cdot[\overline{\boldsymbol{R}}(t,2)]^{(6)} + 0.53\cdot[\overline{\boldsymbol{R}}(t,2)]^{(7)} + 0.01\cdot[\overline{\boldsymbol{R}}(t,2)]^{(8)}\,] \text{ for } t \ge 0, \tag{49}$$

where $[\overline{\boldsymbol{R}}(t,1)]^{(b)}$, $[\overline{\boldsymbol{R}}(t,2)]^{(b)}$, $b = 1,2,...,8,$ are fixed in (Kołowrocki et all. 2008).
In [7] (Appendix 1B), it is also fixed that the mean values of the system unconditional lifetimes in the particular reliability state subsets $\{1,2\}$ and $\{2\}$ are:

$$\mu_1(1) = 0.364,\ \mu_1(2) = 0.304,$$

$$\mu_2(1) = 0.807,\ \mu_2(2) = 0.666,$$

$$\mu_3(1) = 0.307,\ \mu_3(2) = 0.218,$$

$$\mu_4(1) = 0.079,\ \mu_4(2) = 0.058,$$

$$\mu_5(1) = 0.307,\ \mu_5(2) = 0.218,$$

$$\mu_6(1) = 0.079,\ \mu_6(2) = 0.058,$$

$$\mu_7(1) = 0.110,\ \mu_7(2) = 0.085,$$

$$\mu_8(1) = 0.364,\ \mu_8(2) = 0.079. \tag{50}$$

After considering (46)-(50) and applying (11), the mean values of the system unconditional lifetimes in the reliability state subsets $\{1,2\}$ and $\{2\}$, before the optimization, respectively are:

$$\mu(1) = p_1\mu_1(1) + p_2\mu_2(1) + p_3\mu_3(1) + p_4\mu_4(1) + p_5\mu_5(1) + p_6\mu_6(1) + p_7\mu_7(1) + p_8\mu_8(1)$$

$$= 0.34\cdot 0.364 + 0.00\cdot 0.807 + 0.00\cdot 0.307 + 0.00\cdot 0.079 + 0.10\cdot 0.307 + 0.02\cdot 0.079$$

$$+ 0.53\cdot 0.110 + 0.01\cdot 0.364 \qquad \cong 0.218,$$

(51)

$$\mu(2) = p_1\mu_1(2) + p_2\mu_2(2) + p_3\mu_3(2) + p_4\mu_4(2) + p_5\mu_5(2) + p_6\mu_6(2) + p_7\mu_7(2) + p_8\mu_8(2)$$

$$= 0.34\cdot 0.304 + 0.00\cdot 0.666 + 0.00\cdot 0.218 + 0.00\cdot 0.058 + 0.10\cdot 0.218 + 0.02\cdot 0.058$$

$$+0.53\cdot 0.083+0.01\cdot 0.304 \cong 0.173, \quad (52)$$

and according to (14), the mean values of the system lifetimes in the particular reliability states $u=1$ and $u=2$, before the optimization, respectively are

$$\bar{\mu}(1)=\mu(1)-\mu(2)=0.045,\ \bar{\mu}(2)=\mu(2)=0.173. \quad (53)$$

Further, according to (13), the variances and standard deviations of the system unconditional lifetimes in the system reliability state subsets are

$$\sigma^2(1)=2\int_0^\infty t\ \overline{\boldsymbol{R}}_3(t,1)dt-[\mu(1)]^2 \cong 0.0520,\ \ \sigma(1)\cong 0.228, \quad (54)$$

$$\sigma^2(2)=2\int_0^\infty t\ \overline{\boldsymbol{R}}_3(t,2)dt-[\mu(2)]^2 \cong 0.0342,\ \ \sigma(2)\cong 0.185, \quad (55)$$

where $\overline{\boldsymbol{R}}_3(t,1),\ \overline{\boldsymbol{R}}_3(t,2)$ are given by (48)-(49) and $\mu(1)$, $\mu(2)$ are given by (51)-(52).
If the critical safety state is r =1, then the system risk function, according to (7), is given by

$$\boldsymbol{r}(t) = 1-\overline{\boldsymbol{R}}_3(t,1) \text{ for } t \ge 0, \quad (56)$$

where $\boldsymbol{R}_3(t,1)$ is given by (48).
Hence, the moment when the system risk function exceeds a permitted level, for instance δ = 0.05, from (8), is

$$\tau = \boldsymbol{r}^{-1}(\delta) \cong 0.011 \text{ years}. \quad (57)$$

Further, assuming that the oil pipeline system is repaired after its failure and that the time of the system renovation is ignored, applying *Theorem 3.1*, we obtain the following results:

i) the distribution of the time $S_N(1)$ until the *Nth* exceeding of reliability critical state 1 of this system, for sufficiently large *N*, has approximately normal distribution $N(0.218N, 0.228\sqrt{N})$, i.e.,

$$F^{(N)}(t,1)=P(S_N(1)<t)\cong F_{N(0,1)}\left(\frac{t-0.218N}{0.228\sqrt{N}}\right),\ t\in(-\infty,\infty),$$

ii) the expected value and the variance of the time $S_N(1)$ until the *Nth* exceeding the reliability critical state 1 of this system take respectively forms

$$E[S_N(1)]=0.218N,\ \ D[S_N(1)]=0.0519N,$$

iii) the distribution of the number $N(t,1)$ of exceeding the reliability critical state 1 of this system up to the moment t, $t\ge 0$, for sufficiently large t, is approximately of the form

$$P(N(t,1)=N)\cong F_{N(0,1)}\left(\frac{0.218N-t}{0.4883\sqrt{t}}\right)-F_{N(0,1)}\left(\frac{0.218(N+1)-t}{0.4884\sqrt{t}}\right),\ \ N=0,1,2,\ldots,$$

iv) the expected value and the variance of the number $N(t,1)$ of exceeding the reliability critical state 1 of this system at the moment $t, t \geq 0,$ for sufficiently large *t*, approximately take respectively forms

$$H(t,1) = 4.587t, \quad D(t,1) = 5.0095t.$$

Further, assuming that the oil pipeline system is repaired after its failure and that the time of the system renovation is not ignored and it has the mean value $\mu_0(1) = 0.005$ and the standard deviation $\sigma_0(1) = 0.005,$ applying *Theorem 3.2*, we obtain the following results:

i) the distribution function of the time $\bar{\bar{S}}_N(1)$ until the *Nth* system's renovation, for sufficiently large *N*, has approximately normal distribution $N(0.223N, 0.2279\sqrt{N})$, i.e.,

$$\bar{\bar{F}}^{(N)}(t,1) = P(\bar{\bar{S}}_N(1) < t) \cong F_{N(0,1)}\left(\frac{t - 0.223N)}{0.2279\sqrt{N}}\right), \ t \in (-\infty, \infty), \ N = 1,2,...,$$

ii) the expected value and the variance of the time $\bar{\bar{S}}_N(1)$ until the *Nth* system's renovation take respectively forms

$$E[\bar{\bar{S}}_N(1)] \cong 0.223N, \ D[\bar{\bar{S}}_N(1)] \cong 0.0519N,$$

iii) the distribution function of the time $\bar{S}_N(1)$ until the *Nth* exceeding the reliability critical state 1 of this system takes form

$$\bar{F}^{(N)}(t,1) = P(\bar{S}_N(1) < t) = F_{N(0,1)}\left(\frac{t - 0.223N + 0.005}{\sqrt{0.0519N - 0.000025}}\right), \ t \in (-\infty, \infty), \ N = 1,2,...,$$

iv) the expected value and the variance of the time $\bar{S}_N(1)$ until the *Nth* exceeding the reliability critical state 1 of this system take respectively forms

$$E[\bar{S}_N(1)] \cong 0.218N + 0.005(N-1), \ D[\bar{S}_N(1)] \cong 0.0519N + 0.000025(N-1),$$

v) the distribution of the number $\bar{\bar{N}}(t,1)$ of system's renovations up to the moment $t, t \geq 0,$ is of the form

$$P(\bar{\bar{N}}(t,1) = N) \cong F_{N(0,1)}\left(\frac{0.223N - t}{0.482\sqrt{t}}\right) - F_{N(0,1)}\left(\frac{0.223(N+1) - t}{0.482\sqrt{t}}\right) \ N = 1,2,...,$$

vi) the expected value and the variance of the number $\bar{\bar{N}}(t,1)$ of system's renovations up to the moment $t, t \geq 0,$ take respectively forms

$$\bar{\bar{H}}(t,1) \cong 4.484t, \quad \bar{\bar{D}}(t,1) \cong 4.68t,$$

vii) the distribution of the number $\overline{N}(t,1)$ of exceeding the reliability critical state 1 of this system up to the moment $t, t \geq 0,$ is of the form

$$P(\bar{N}(t,1) = N) \cong F_{N(0,1)}(\frac{0.223N - t - 0.005}{0.482\sqrt{t + 0.005}}) - F_{N(0,1)}(\frac{0.223(N+1) - t - 0005}{0.482\sqrt{t - 0.005}}),\ N = 1,2,... ,$$

viii) the expected value and the variance of the number $\overline{N}(t,1)$ of exceeding the reliability critical state 1 of this system up to the moment $t, t \geq 0,$ are respectively given by

$$\bar{H}(t,1) \cong \frac{t + 0.005}{0.223},\ \bar{D}(t,1) \cong 4.68(t + 0.005),$$

ix) the availability coefficient of the system at the moment *t* is given by the formula

$$A(t,1) \cong 0.9776,\quad t \geq 0,$$

x) the availability coefficient of the system in the time interval $< t, t + \tau), \tau > 0,$ is given by the formula

$$A(t,\tau,1) \cong 4.484 \int_{\tau}^{\infty} \overline{\boldsymbol{R}}_3\ (t,1)dt,\ \ t \geq 0,\ \ \tau > 0.$$

7 RELIABILITY, RISK AND AVAILABILITY OPTIMIZATION OF A PORT OIL PIPING TRANSPORTATION SYSTEM

The objective function (15), in this case as the critical state is $r = 1$, takes the form

$$\mu(1) = p_1 \cdot 0.364 + p_2 \cdot 0.807 + p_3 \cdot 0.307 + p_4 \cdot 0.079 + p_5 \cdot 0.307$$

$$+ p_6 \cdot 0.079 + p_7 \cdot 0.110 + p_8 \cdot 0.364. \tag{58}$$

The lower $\breve{p}_b$ and upper $\hat{p}_b$ bounds of the unknown limit transient probabilities p_b, $b = 1,2,...,8,$ coming from experts are respectively:

$\breve{p}_1 = 0.25$, $\breve{p}_2 = 0.01$, $\breve{p}_3 = 0.01$. $\breve{p}_4 = 0.01$, $\breve{p}_5 = 0.08$, $\breve{p}_6 = 0.01$, $\breve{p}_7 = 0.40$, $\breve{p}_1 = 0.01$;

$\hat{p}_1 = 0.50$, $\hat{p}_2 = 0.05$, $\hat{p}_3 = 0.05$, $\hat{p}_4 = 0.05$, $\hat{p}_5 = 0.20$, $\hat{p}_6 = 0.05$, $\hat{p}_1 = 0.75$, $\hat{p}_1 = 0.05$.

Therefore, according to (16)-(18), we assume the following bound constraints

$$\sum_{b=1}^{8} p_b = 1, \tag{59}$$

$$0.25 \leq p_1 \leq 0.50,\ 0.01 \leq p_2 \leq 0.05,\ 0.01 \leq p_3 \leq 0.05,\ 0.01 \leq p_4 \leq 0.05,$$

$$0.08 \leq p_5 \leq 0.20,\ 0.01 \leq p_6 \leq 0.05,\ 0.40 \leq p_7 \leq 0.75,\ 0.01 \leq p_8 \leq 0.05. \tag{60}$$

Now, before we find optimal values $\dot{p}_b$ of the limit transient probabilities p_b, $b = 1,2,...,\nu$, that maximize the objective function (58), we arrange the system conditional lifetime mean values $\mu_b(1)$, $b = 1,2,...,8$, in non-increasing order

$$\mu_2(1) \geq \mu_1(1) \geq \mu_8(1) \geq \mu_3(1) \geq \mu_5(1) \geq \mu_7(1) \geq \mu_4(1) \geq \mu_6(1).$$

Next, according to (19), we substitute

$$x_1 = p_2 = 0.00,\ x_2 = p_1 = 0.34,\ x_3 = p_8 = 0.01,\ x_4 = p_3 = 0.00,\ x_5 = p_5 = 0.10,$$

$$x_6 = p_7 = 0.53,\ x_7 = p_4 = 0.00,\ x_8 = p_6 = 0.02 \tag{61}$$

and

$$\breve{x}_i = 0.01,\ \hat{x}_i = 0.95 \text{ for } i = 1,2,...,\nu \tag{62}$$

and we maximize with respect to x_i, $i = 1,2,...,8$, the linear form (52) that according to (20) takes the form

$$\mu(1) = x_1 \cdot 0.807 + x_2 \cdot 0.364 + x_3 \cdot 0.364 + x_4 \cdot 0.307 + x_5 \cdot 0.307 + x_6 \cdot 0.110$$

$$+ x_7 \cdot 0.079 + x_8 \cdot 0.079 \tag{63}$$

with the following bound constraints

$$\sum_{i=1}^{8} x_i = 1, \tag{64}$$

$$0.01 \leq x_1 \leq 0.05,\ 0.25 \leq x_2 \leq 0.50,\ 0.01 \leq x_3 \leq 0.05,\ 0.01 \leq x_4 \leq 0.05,$$

$$0.08 \leq x_5 \leq 0.20,\ 0.40 \leq x_6 \leq 0.75,\ 0.01 \leq x_7 \leq 0.05,\ 0.01 \leq x_8 \leq 0.05. \tag{65}$$

where

$$\breve{x}_1 = 0.01,\ \breve{x}_2 = 0.25,\ \breve{x}_3 = 0.01,\ \breve{x}_4 = 0.01,\ \breve{x}_5 = 0.08,\ \breve{x}_6 = 0.40,\ \breve{x}_7 = 0.01,\ \breve{x}_8 = 0.01;$$

$$\hat{x}_1 = 0.05,\ \hat{x}_2 = 0.50,\ \hat{x}_3 = 0.05,\ \hat{x}_4 = 0.05,\ \hat{x}_5 = 0.20,\ \hat{x}_6 = 0.75,\ \hat{x}_7 = 0.05,\ \hat{x}_8 = 0.05. \tag{66}$$

are lower and upper bounds of the unknown limit transient probabilities x_i, $i = 1,2,...,8$, respectively.

According to (24), we find

$$\breve{x} = \sum_{i=1}^{8} \breve{x}_i = 0.78,\ \hat{y} = 1 - \breve{x} = 1 - 0.78 = 0.22 \tag{67}$$

and according to (25), we find

$$\breve{x}^0 = 0,\ \hat{x}^0 = 0,\ \hat{x}^0 - \breve{x}^0 = 0,$$

$$\breve{x}^1 = 0.01 \quad \hat{x}^1 = 0.05, \quad \hat{x}^1 - \breve{x}^1 = 0.04$$
$$\breve{x}^2 = 0.26 \quad \hat{x}^2 = 0.55, \quad \hat{x}^2 - \breve{x}^2 = 0.29,$$
$$\ldots$$
$$\breve{x}^8 = 0.78 \quad \hat{x}^8 = 1.70, \quad \hat{x}^8 - \breve{x}^8 = 0.92. \qquad (68)$$

From the above, as according to (67), the inequality (26) takes the form

$$\hat{x}^I - \breve{x}^I < 0.22, \qquad (69)$$

then it follows that the largest value $I \in \{0,1,...,8\}$ such that this inequality holds is $I = 1$.
Therefore, we fix the optimal solution that maximize (63) according to the rule (28). Namely, we get

$$\dot{x}_1 = \hat{x}_1 = 0.05,$$

$$\dot{x}_2 = \hat{y} - \hat{x}^1 + \breve{x}^1 + \breve{x}_2 = 0.22 - 0.05 + 0.01 + 0.25 = 0.43, \qquad (70)$$

$$\dot{x}_3 = \breve{x}_3 = 0.01, \; \dot{x}_4 = \breve{x}_4 = 0.01, \; \dot{x}_5 = \breve{x}_5 = 0.08,$$

$$\dot{x}_6 = \breve{x}_6 = 0.40, \; \dot{x}_7 = \breve{x}_7 = 0.01, \; \dot{x}_8 = \breve{x}_8 = 0.01.$$

(71)

Finally, after making the inverse to (61) substitution, we get the optimal limit transient probabilities

$$\dot{p}_2 = \dot{x}_1 = 0.05, \quad \dot{p}_1 = \dot{x}_2 = 0.43, \quad \dot{p}_8 = \dot{x}_3 = 0.01, \; \dot{p}_3 = \dot{x}_4 = 0.01, \quad \dot{p}_5 = \dot{x}_5 = 0.08,$$

$$\dot{p}_7 = \dot{x}_6 = 0.40, \quad \dot{p}_4 = \dot{x}_7 = 0.01, \quad \dot{p}_6 = \dot{x}_8 = 0.01 \qquad (72)$$

that maximize the system mean lifetime in the reliability state subset $\{1,2\}$ expressed by the linear form (58) giving, according to (31) and (72), its optimal value

$$\dot{\mu}(1) = \dot{p}_1 \cdot 0.364 + \dot{p}_2 \cdot 0.807 + \dot{p}_3 \cdot 0.307 + \dot{p}_4 \cdot 0.079 + \dot{p}_5 \cdot 0.307 + \dot{p}_6 \cdot 0.079$$

$$+ \dot{p}_7 \cdot 0.110 + \dot{p}_8 \cdot 0.364 = 0.43 \cdot 0.364 + 0.05 \cdot 0.807 + 0.01 \cdot 0.307$$

$$+ 0.01 \cdot 0.079 + 0.08 \cdot 0.307 + 0.01 \cdot 0.079 + 0.40 \cdot 0.110 + 0.01 \cdot 0.364 = 0.274. \qquad (73)$$

Further, according to (32), substituting the optimal solution (72) in (52), we obtain the optimal solutions for the mean values of the system unconditional lifetimes in the reliability state subset $\{2\}$

$$\dot{\mu}(2) = \dot{p}_1 \cdot 0.304 + \dot{p}_2 \cdot 0.666 + \dot{p}_3 \cdot 0.218 + \dot{p}_4 \cdot 0.058 + \dot{p}_5 \cdot 0.218 + \dot{p}_6 \cdot 0.058$$

$$+ \dot{p}_7 \cdot 0.085 + \dot{p}_8 \cdot 0.304 = 0.43 \cdot 0.304 + 0.05 \cdot 0.666 + 0.01 \cdot 0.218$$

$$+ 0.01 \cdot 0.058 + 0.08 \cdot 0.218 + 0.01 \cdot 0.058 + 0.40 \cdot 0.085 + 0.01 \cdot 0.079 = 0.220. \qquad (74)$$

and according to (36), the optimal solutions for the mean values of the system unconditional lifetimes in the particular reliability states

$$\dot{\bar{\mu}}(1) = \dot{\mu}(1) - \dot{\mu}(2) = 0.054,\ \dot{\bar{\mu}}(2) = \dot{\mu}(2) = 0.220. \quad (75)$$

Moreover, according to (34)-(35) and (47)-(49), the corresponding optimal unconditional multistate reliability function of the system is of the form

$$\dot{\bar{\boldsymbol{R}}}_3(t,\cdot) = [1, \dot{\bar{\boldsymbol{R}}}_3(t,1), \dot{\bar{\boldsymbol{R}}}_3(t,2)], \quad (76)$$

with the coordinates given by

$$\dot{\bar{\boldsymbol{R}}}_3(t,1) = 0.43\cdot[\bar{\boldsymbol{R}}(t,1)]^{(1)} + 0.05\cdot[\bar{\boldsymbol{R}}(t,1)]^{(2)} + 0.01\cdot[\bar{\boldsymbol{R}}(t,1)]^{(3)} + 0.01\cdot[\bar{\boldsymbol{R}}(t,1)]^{(4)}$$

$$+ 0.08\cdot[\bar{\boldsymbol{R}}(t,1)]^{(5)} + 0.01\cdot[\bar{\boldsymbol{R}}(t,1)]^{(6)} + 0.40\cdot[\bar{\boldsymbol{R}}(t,1)]^{(7)} + 0.01\cdot[\bar{\boldsymbol{R}}(t,1)]^{(8)}], \quad (77)$$

$$\dot{\bar{\boldsymbol{R}}}_3(t,2) = 0.43\cdot[\bar{\boldsymbol{R}}(t,2)]^{(1)} + 0.05\cdot[\bar{\boldsymbol{R}}(t,2)]^{(2)} + 0.01\cdot[\bar{\boldsymbol{R}}(t,2)]^{(3)} + 0.01\cdot[\bar{\boldsymbol{R}}(t,2)]^{(4)}$$

$$+ 0.08\cdot[\bar{\boldsymbol{R}}(t,2)]^{(5)} + 0.01\cdot[\bar{\boldsymbol{R}}(t,2)]^{(6)} + 0.40\cdot[\bar{\boldsymbol{R}}(t,2)]^{(7)} + 0.01\cdot[\bar{\boldsymbol{R}}(t,2)]^{(8)}] \quad (78)$$

for $t \geq 0$, where $[\bar{\boldsymbol{R}}(t,1)]^{(b)}$, $[\bar{\boldsymbol{R}}(t,2)]^{(b)}$, $b = 1,2,...,8$, are fixed in [7].
Further, according to (13) and (32)-(33), the corresponding optimal variances and standard deviations of the system unconditional lifetime in the system reliability state subsets are

$$\dot{\sigma}^2(1) = 2\int_0^\infty t\ \dot{\bar{\boldsymbol{R}}}_3(t,1)dt - [\dot{\mu}(1)]^2 \cong 0.084,\ \dot{\sigma}(1) \cong 0.289, \quad (79)$$

$$\dot{\sigma}^2(2) = 2\int_0^\infty t\ \dot{\bar{\boldsymbol{R}}}_3(t,2)dt - [\dot{\mu}(2)]^2 \cong 0.056,\ \dot{\sigma}(2) \cong 0.237, \quad (80)$$

where $\dot{\bar{\boldsymbol{R}}}_3(t,1)$, $\dot{\bar{\boldsymbol{R}}}_3(t,2)$ are given by (77)-(78) and $\dot{\mu}(1)$, $\dot{\mu}(2)$ are given by (73)-(74).
If the critical safety state is r =1, then the optimal system risk function, according to (7) and (37), is given by

$$\dot{\boldsymbol{r}}(t) = 1 - \dot{\bar{\boldsymbol{R}}}_3(t,1) \text{ for } t \geq 0, \quad (81)$$

where $\dot{\bar{\boldsymbol{R}}}_3(t,1)$ is given by (77).
Hence and considering (38), the moment when the optimal system risk function exceeds a permitted level, for instance δ = 0.05, is

$$\dot{\tau} = \dot{\boldsymbol{r}}^{-1}(\delta) \cong 0.19 \text{ years.} \quad (82)$$

Replacing $\mu(r)$ by $\dot{\mu}(1)$ given by (73) and $\sigma(r)$ by $\dot{\sigma}(1)$ given by (79) in the expressions for the renewal systems characteristics pointed in *Theorem 1* and *Theorem 2*, we get their corresponding optimal values pointed below.
Under the assumption that the oil pipeline system is repaired after its failure and that the time of the system renovation is ignored, we obtain the following optimal results:

i) the distribution of the time $S_N(1)$ until the *Nth* exceeding of reliability critical state 1 of this system, for sufficiently large *N*, has approximately normal distribution $N(0.274N, 0.289\sqrt{N})$, i.e.,

$$F^{(N)}(t,1) = P(S_N(1) < t) \cong F_{N(0,1)}(\frac{t - 0.274N}{0289\sqrt{N}}),\ t \in (-\infty, \infty),$$

ii) the expected value and the variance of the time $S_N(1)$ until the *Nth* exceeding the reliability critical state 1 of this system take respectively forms

$$E[S_N(1)] = 0.274N,\ \ D[S_N(1)] = 0.084N\ ,$$

iii) the distribution of the number $N(t,1)$ of exceeding the reliability critical state 1 of this system up to the moment $t, t \geq 0,$ for sufficiently large *t*, is approximately of the form

$$P(N(t,1) = N) \cong F_{N(0,1)}(\frac{0.274N - t}{0.552\sqrt{t}}) - F_{N(0,1)}(\frac{0.274(N+1) - t}{0.552\sqrt{t}}),\ \ N = 0,1,2,...,$$

iv) the expected value and the variance of the number $N(t,1)$ of exceeding the reliability critical state 1 of this system at the moment $t, t \geq 0,$ for sufficiently large *t*, approximately take respectively forms

$$H(t,1) = 3.649t,\ \ D(t,1) = 4.06t.$$

Under the assumption that the oil pipeline system is repaired after its failure and that the time of the system renovation is not ignored and it has the mean value $\mu_0(1) = 0.005$ and the standard deviation $\sigma_0(1) = 0.005,$ we obtain the following optimal results:
i) the distribution function of the time $\bar{\bar{S}}_N(1)$ until the *Nth* system's renovation, for sufficiently large *N*, has approximately normal distribution $N(0.279N, 0.289\sqrt{N})$, i.e.,

$$\bar{\bar{F}}^{(N)}(t,1) = P(\bar{\bar{S}}_N(1) < t) \cong F_{N(0,1)}(\frac{t - 0.279N)}{0.289\sqrt{N}}),\ \ t \in (-\infty, \infty),\ N = 1,2,...,$$

ii) the expected value and the variance of the time $\bar{\bar{S}}_N(1)$ until the *Nth* system's renovation take respectively forms

$$E[\bar{\bar{S}}_N(1)] \cong 0.279N\ ,\ \ D[\bar{\bar{S}}_N(1)] \cong 0.084N\ ,$$

iii) the distribution function of the time $\bar{S}_N(1)$ until the *Nth* exceeding the reliability critical state 1 of this system takes form

$$\bar{F}^{(N)}(t,1) = P(\bar{S}_N(1) < t) = F_{N(0,1)}(\frac{t - 0.279N + 0.005}{\sqrt{0.084N - 0.000025}}),\ t \in (-\infty, \infty),\ N = 1,2,...,$$

iv) the expected value and the variance of the time $\bar{S}_N(1)$ until the *Nth* exceeding the reliability critical state 1 of this system take respectively forms

$E[\bar{S}_N(1)] \cong 0.274N + 0.005(N-1)$, $D[\bar{S}_N(1)] \cong 0.08352N + 0.000025(N-1)$,

v) the distribution of the number $\bar{\bar{N}}(t,1)$ of system's renovations up to the moment $t, t \geq 0,$ is of the form

$$P(\bar{\bar{N}}(t,1) = N) \cong F_{N(0,1)}(\frac{0.279N - t}{0.549\sqrt{t}} - F_{N(0,1)}(\frac{0.279(N+1) - t}{0.549\sqrt{t}},\ N = 1,2,...,$$

vi) the expected value and the variance of the number $\bar{\bar{N}}(t,1)$ of system's renovations up to the moment $t, t \geq 0,$ take respectively forms

$$\bar{\bar{H}}(t,1) \cong 3.584t,\ \ \bar{\bar{D}}(t,1) \cong 3.868t,$$

vii) the distribution of the number $\bar{N}(t,1)$ of exceeding the reliability critical state 1 of this system up to the moment $t, t \geq 0,$ is of the form

$$P(\bar{N}(t,1) = N) \cong F_{N(0,1)}(\frac{0.279N - t - 0.005}{0.549\sqrt{t+0.005}}) - F_{N(0,1)}(\frac{0.279(N+1) - t - 0005}{0.49\sqrt{t+0.005}}),\ N = 1,2,...,$$

viii) the expected value and the variance of the number $\bar{N}(t,1)$ of exceeding the reliability critical state 1 of this system up to the moment $t, t \geq 0,$ are respectively given by

$$\bar{H}(t,1) \cong \frac{t + 0.005}{0.279},\ \ \bar{D}(t,1) \cong 3.868(t + 0.005),$$

ix) the availability coefficient of the system at the moment *t* is given by the formula

$$A(t,1) \cong 0.982,\ \ t \geq 0,$$

x) the availability coefficient of the system in the time interval $< t, t+\tau), \tau > 0,$ is given by the formula

$$A(t,\tau,1) \cong 3.584\int_{\tau}^{\infty} \overline{\boldsymbol{R}}_3\ (t,1)dt,\ \ t \geq 0,\ \ \tau > 0.$$

To obtain the optimal mean sojourn times in the particular operation states maximizing the mean lifetime of the port oil piping transportation system we substitute the optimal limit transient probabilities $\dot{p}_b$ determined by (72) and probabilities π_b determined by (45) into the system of equation (40) and we get its following form

$$-0.22572\dot{M}_1 + 0.04988\dot{M}_5 + 0.01634\dot{M}_6 + 0.18705\dot{M}_7 + 0.00645\dot{M}_8 = 0$$
$$0.0198\dot{M}_1 + 0.0058\dot{M}_5 + 0.0019\dot{M}_6 + 0.02175\dot{M}_7 + 0.00075\dot{M}_8 = 0$$
$$0.00396\dot{M}_1 + 0.00116\dot{M}_5 + 0.00038\dot{M}_6 + 0.00435\dot{M}_7 + 0.00015\dot{M}_8 = 0$$
$$0.00396\dot{M}_1 + 0.00116\dot{M}_5 + 0.00038\dot{M}_6 + 0.00435\dot{M}_7 + 0.00015\dot{M}_8 = 0$$
$$0.03168\dot{M}_1 - 0.10672\dot{M}_5 + 0.00304\dot{M}_6 + 0.0348\dot{M}_7 + 0.0012\dot{M}_8 = 0$$

$$0.00396\dot{M}_1 + 0.00116\dot{M}_5 - 0.03762\dot{M}_6 + 0.00435\dot{M}_7 + 0.00015\dot{M}_8 = 0$$
$$0.1584\dot{M}_1 + 0.0464\dot{M}_5 + 0.0152\dot{M}_6 - 0.261\dot{M}_7 + 0.006\dot{M}_8 = 0$$
$$0.00396\dot{M}_1 + 0.00116\dot{M}_5 + 0.00038\dot{M}_6 + 0.00435\dot{M}_7 - 0.01485\dot{M}_8 = 0.$$
(83)

Since the above system is homogeneous then it has nonzero solutions when the determinant of the system equations main matrix is equal to zero, i.e. if its rank is less than 8. Moreover, in this case the solutions are ambiguous.
Since the second equation multiplied by five gives the third equation and the third and fourth equations are identical, then after omitting two of them (the second and the third ones), we have

$$-0.22572\dot{M}_1 + 0.04988\dot{M}_5 + 0.01634\dot{M}_6 + 0.18705\dot{M}_7 + 0.00645\dot{M}_8 = 0$$
$$0.00396\dot{M}_1 + 0.00116\dot{M}_5 + 0.00038\dot{M}_6 + 0.00435\dot{M}_7 + 0.00015\dot{M}_8 = 0$$
$$0.03168\dot{M}_1 - 0.10672\dot{M}_5 + 0.00304\dot{M}_6 + 0.0348\dot{M}_7 + 0.0012\dot{M}_8 = 0$$
$$0.00396\dot{M}_1 + 0.00116\dot{M}_5 - 0.03762\dot{M}_6 + 0.00435\dot{M}_7 + 0.00015\dot{M}_8 = 0$$
$$0.1584\dot{M}_1 + 0.0464\dot{M}_5 + 0.0152\dot{M}_6 - 0.261\dot{M}_7 + 0.006\dot{M}_8 = 0$$
$$0.00396\dot{M}_1 + 0.00116\dot{M}_5 + 0.00038\dot{M}_6 + 0.00435\dot{M}_7 - 0.01485\dot{M}_8 = 0. \quad (84)$$

As we are looking for nonzero solutions, we omit the second equation and we get

$$-0.22572\dot{M}_1 + 0.04988\dot{M}_5 + 0.01634\dot{M}_6 + 0.18705\dot{M}_7 + 0.00645\dot{M}_8 = 0$$
$$0.03168\dot{M}_1 - 0.10672\dot{M}_5 + 0.00304\dot{M}_6 + 0.0348\dot{M}_7 + 0.0012\dot{M}_8 = 0$$
$$0.00396\dot{M}_1 + 0.00116\dot{M}_5 - 0.03762\dot{M}_6 + 0.00435\dot{M}_7 + 0.00015\dot{M}_8 = 0$$
$$0.1584\dot{M}_1 + 0.0464\dot{M}_5 + 0.0152\dot{M}_6 - 0.261\dot{M}_7 + 0.006\dot{M}_8 = 0$$
$$0.00396\dot{M}_1 + 0.00116\dot{M}_5 + 0.00038\dot{M}_6 + 0.00435\dot{M}_7 - 0.01485\dot{M}_8 = 0.$$
(85)

From the above we get nonzero solutions in case when the rank of the main matrix is not greater than 4. In our case, since the above system of equations is satisfied by any values of $\dot{M}_2$, $\dot{M}_3$ and $\dot{M}_4$, than after considering expert opinions, it is sensible to assume

$$\dot{M}_2 \cong 480, \ \dot{M}_3 \cong 1440, \ \dot{M}_4 \cong 480, \quad (86)$$

and in order to get 4 nonzero solutions of the system of equations (85) to fix one of the remaining unknown variables for instance, according to (44), assuming

$$\dot{M}_6 \cong 360. \quad (87)$$

After this the system of equations (85) takes the form

$$-0.22572\dot{M}_1 + 0.04988\dot{M}_5 + 0.18705\dot{M}_7 + 0.00645\dot{M}_8 = -5.8824$$
$$0.03168\dot{M}_1 - 0.10672\dot{M}_5 + 0.0348\dot{M}_7 + 0.0012\dot{M}_8 = -1.0944$$
$$0.00396\dot{M}_1 + 0.00116\dot{M}_5 + 0.00435\dot{M}_7 + 0.00015\dot{M}_8 = 13.5432$$
$$0.1584\dot{M}_1 + 0.0464\dot{M}_5 - 0.261\dot{M}_7 + 0.006\dot{M}_8 = -5.472$$

$$0.00396\dot{M}_1 + 0.00116\dot{M}_5 + 0.00435\dot{M}_7 - 0.01485\dot{M}_8 = -0.1368. \tag{88}$$

Next, after subtracting the third equation from the fifth equation, we get

$$\begin{aligned} -0.22572\dot{M}_1 + 0.04988\dot{M}_5 + 0.18705\dot{M}_7 + 0.00645\dot{M}_8 &= -5.8824 \\ 0.03168\dot{M}_1 - 0.10672\dot{M}_5 + 0.0348\dot{M}_7 + 0.0012\dot{M}_8 &= -1.0944 \\ 0.1584\dot{M}_1 + 0.0464\dot{M}_5 - 0.261\dot{M}_7 + 0.006\dot{M}_8 &= -5.472 \\ -0.015\dot{M}_8 &= -13.68. \end{aligned} \tag{89}$$

The solutions of the above system of equations are

$$\dot{M}_1 \cong 330,\ \dot{M}_5 \cong 210,\ \dot{M}_7 \cong 280,\ \dot{M}_8 = 912. \tag{90}$$

Hence and considering (86) and (87), we get the following final solution of the equation (83)

$$\dot{M}_1 \cong 330,\ \dot{M}_2 \cong 480,\ \dot{M}_3 \cong 1440,\ \dot{M}_4 \cong 480,\ \dot{M}_5 \cong 210,\ \dot{M}_6 = 360,\ \dot{M}_7 \cong 280,\ \dot{M}_8 = 912. \tag{91}$$

Now, substituting in (41) the above mean values $\dot{M}_b$ of the system unconditional sojourn times in the particular operation states and the known probabilities p_{bl} of the system operation process transitions between the operation states given in the matrix (42), we may look for the optimal values $\dot{M}_{bl}$ of the mean values of the system conditional sojourn times in the particular operation states that maximizing the mean lifetime of the port oil piping transportation system in the reliability states subset $\{1,2\}$. The optimal values $\dot{M}_{bl},\ b,l = 1,2,...,8,\ b \neq l,$ should to satisfy the following obtained this way system of equations

$$\begin{aligned} 0.06\dot{M}_{15} + 0.06\dot{M}_{16} + 0.86\dot{M}_{17} + 0.02\dot{M}_{18} &= 330 \\ p_{21}M_{21} + p_{22}M_{22} + p_{23}M_{23} + p_{24}M_{24} + p_{25}M_{25} + p_{26}M_{26} + p_{27}M_{27} + p_{28}M_{28} &= 480 \\ p_{31}M_{31} + p_{32}M_{32} + p_{33}M_{33} + p_{34}M_{34} + p_{35}M_{35} + p_{36}M_{36} + p_{37}M_{37} + p_{38}M_{38} &= 1440 \\ p_{41}M_{41} + p_{42}M_{42} + p_{43}M_{43} + p_{44}M_{44} + p_{45}M_{45} + p_{46}M_{46} + p_{47}M_{47} + p_{48}M_{48} &= 480 \\ 0.125\dot{M}_{51} + 0.125\dot{M}_{56} + 0.687\dot{M}_{57} + 0.063\dot{M}_{58} &= 210 \\ 0.4\dot{M}_{61} + 0.6\dot{M}_{65} &= 360 \\ 0.82\dot{M}_{71} - 0.16\dot{M}_{75} + 0.02\dot{M}_{78} &= 280 \\ 0.67\dot{M}_{81} + 0.33\dot{M}_{87} &= 912. \end{aligned}$$

Unfortunately, the solution of the above system of equations are ambiguous

8 CONCLUSION

The joint general model of reliability and availability of complex technical systems in variable operation conditions linking a semi-markov modeling of the system operation processes with a multi-state approach to system reliability and availability analysis constructed in the paper Part 1 was applied to reliability evaluation of the port oil piping transportation system. The main reliability and availability characteristics were evaluated and maximized after its operation process optimization.

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9 REFERENCES

Blokus-Roszkowska, A., Guze, S., Kołowrocki K., Kwiatuszewska-Sarnecka, B., Soszyńska J. 2008. *Models of safety, reliability and availability evaluation of complex technical systems related to their operation processes*. WP 4 – Task 4.1 – English – 31.05.2008. Poland-Singapore Joint Project. MSHE Decision No. 63/N-Singapore/2007/0.

Grabski, F. 2002. *Semi-Markov Models of Systems Reliability and Operations*. Warsaw: Systems Research Institute, Polish Academy of Sciences.

Kołowrocki, K. 2004. *Reliability of Large Systems*, Elsevier.

Kołowrocki, K. 2007a. Reliability modelling of complex systems – Part 1. *International Journal of Gnedenko e-Forum "Reliability: Theory & Application"*, Vol. 2, No 3-4, 116-127.

Kołowrocki, K. 2007b. Reliability modelling of complex systems – Part 2. *International Journal of Gnedenko e-Forum "Reliability: Theory & Application"*, Vol. 2, No 3-4, 128-139.

Kołowrocki, K. & Soszyńska, J. 2008. *A general model of technical systems operation processes related to their environment and infrastructure*. WP 2 – Task 2.1 – English – 31.05.2008. Poland-Singapore Joint Project. MSHE Decision No. 63/N-Singapore/2007/0.

Kołowrocki, K., Soszyńska, J., Baranowski, Z., Golik, P. 2008. *Preliminary modeling, statistical identification and evaluation of reliability, risk, availability and safety of port, shipyard and ship technical systems in constant and variable operation conditions*. WP4 - Task 4.2. Preliminary reliability, risk and availability analysis and evaluation of a port oil transportation system in constant and variable operation conditions. WP4 - Sub-Task 4.2.1 – English – 30.11.2008. Poland-Singapore Joint Project. MSHE Decision No. 63/N-Singapore/2007/0.

Limnios, N., Oprisan, G. 2001. *Semi-Markov Processes and Reliability*. Birkhauser, Boston.

Ross, S. M. 2007. *Introduction to Probability Models*. Elsevier, San Diego, 2007.

Soszyńska, J. 2006a. Reliability of large series-parallel system in variable operation conditions. *International Journal of Automation and Computing*. Vol. 3, No 2: 199-206.

Soszyńska, J. 2006b. Reliability evaluation of a port oil transportation system in variable operation conditions. *International Journal of Pressure Vessels and Piping*. Vol. 83, Issue 4: 304-310.

SAFETY AND RISK EVALUATION OF STENA BALTICA FERRY IN VARIABLE OPERATION CONDITIONS

Krzysztof Kolowrocki, Joanna Soszynska.

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Gdynia Maritime University, Gdynia, Poland

e-mail: katmatkk@am.gdynia.pl, joannas@am.gdynia.pl

ABSTRACT

Basic safety structures of multi-state systems of components with degrading safety states related to their variable operation conditions are defined. For these systems the conditional and unconditional multi-state safety functions are determined. A semi-markov process for the considered systems operation modelling is applied. Further, the paper offers an approach to the solution of a practically important problem of linking the multi-state systems safety models and the systems operation processes models.

Theoretical definitions and results are illustrated by the example of their application in the safety and risk evaluation of the Stena Baltica ferry operating at the Baltic Sea. The ferry transportation system has been considered in varying in time operation conditions. The system safety structure and its components safety functions were changing in variable operation conditions.

1 INTRODUCTION

Taking into account the importance of the safety and operating process effectiveness of technical systems it seems reasonable to expand the two-state approach to multi-state approach in their safety analysis. The assumption that the systems are composed of multi-state components with safety states degrading in time gives the possibility for more precise analysis and diagnosis of their safety and operational processes' effectiveness. This assumption allows us to distinguish a system safety critical state to exceed which is either dangerous for the environment or does not assure the necessary level of its operational process effectiveness. Then, an important system safety characteristic is the time to the moment of exceeding the system safety critical state and its distribution, which is called the system risk function. This distribution is strictly related to the system multi-state safety function that is a basic characteristic of the multi-state system. Determining the multi-state safety function and the risk function of systems on the base of their components' safety functions is then the main research problem. Modelling of complicated systems operations' processes is difficult mainly because of large number of operations states and impossibility of precise describing of changes between these states. One of the useful approaches in modelling of these complicated processes is applying the semi-markov model (Grabski 2002). Modelling of multi-state systems' safety and linking it with semi-markov model of these systems' operation processes is the main and practically important problem of this paper. This new approach to system safety investigation is based on the multi-state system reliability analysis considered for instance in (Aven 1985, Kolowrocki 2004) and on semi-markov processes modeling discussed for instance in (Soszynska 2006, Soszynska 2007). This paper using the results of the report (Soszynska et all. 2007), is devoted to optimizing the multi-state safety function, the risk function of the ship technical system on the base of its components' safety functions and its variable in time operation process.

2 SYSTEM SAFETY IN VARIABLE OPERATION CONDITIONS

We assume that the system during its operation process has v different operation states. Thus we can define the system operation process $Z(t)$, $t \in <0,+\infty>$, as the process with discrete operation states from the set

$$Z = \{z_1, z_2, \ldots, z_v\}.$$

In practice a convenient assumption is that $Z(t)$ is a semi-markov process (Grabski 2002) with its conditional lifetimes θ_{bl} at the operation state z_b when its next operation state is z_l, $b, l = 1,2,\ldots,v$, $b \neq l.$ In this case the process $Z(t)$ may be described by:

- the vector of probabilities of the process initial operation states $[p_b(0)]_{1xv}$,
- the matrix of the probabilities of the process transitions between the operation states $[p_{bl}]_{vxv}$, where $p_{bb}(t) = 0$ for $b = 1,2,\ldots,v.$
- the matrix of the conditional distribution functions $[H_{bl}(t)]_{vxv}$ of the process lifetimes θ_{bl}, $b \neq l,$ in the operation state z_b when the next operation state is z_l, where $H_{bl}(t) = P(\theta_{bl} < t)$ for $b, l = 1,2,\ldots,v,$ $b \neq l,$ and $H_{bb}(t) = 0$ for $b = 1,2,\ldots,v.$

Under these assumptions, the lifetimes θ_{bl} mean values are given by

$$M_{bl} = E[\theta_{bl}] = \int_0^\infty t dH_{bl}(t), \ \ b, l = 1,2,\ldots,v, \ b \neq l. \tag{1}$$

The unconditional distribution functions of the lifetimes θ_b of the process $Z(t)$ at the operation states z_b, $b = 1,2,\ldots,v,$ are given by

$$H_b(t) = \sum_{l=1}^{v} p_{bl} H_{bl}(t), \ \ b = 1,2,\ldots,v.$$

The mean values $E[\theta_b]$ of the unconditional lifetimes θ_b are given by

$$M_b = E[\theta_b] = \sum_{l=1}^{v} p_{bl} M_{bl}, \ \ b = 1,2,\ldots,v, \tag{2}$$

where M_{bl} are defined by (1).

Limit values of the transient probabilities at the operation states

$$p_b(t) = P(Z(t) = z_b), \ \ t \in <0,+\infty), \ \ b = 1,2,\ldots,v,$$

are given by

$$p_b = \lim_{t \to \infty} p_b(t) = \frac{\pi_b M_b}{\sum_{l=1}^{v} \pi_l M_l}, \ \ b = 1,2,\ldots,v, \tag{3}$$

where the probabilities π_b of the vector $[\pi_b]_{1xv}$ satisfy the system of equations

$$\begin{cases} [\pi_b] = [\pi_b][p_{bl}] \\ \sum_{l=1}^{v} \pi_l = 1. \end{cases} \quad (4)$$

We assume that the system is composed of n components E_i, $i = 1,2,...,n,$ and that the changes of the operation process $Z(t)$ states have an influence on the system components E_i safety and on the system safety structure as well.

Consequently, we denote the component E_i lifetime by $T_i^{(b)}$ and by

$$s_i^{(b)}(t,\cdot) = [1,\ s_i^{(b)}(t,1),\ s_i^{(b)}(t,2),\ ...,\ s_i^{(b)}(t,z)\,],$$

where for $t \in <0,\infty)$, $b = 1,2,...,v$, $u = 1,2,...,z$,

$$s_i^{(b)}(t,u) = P(T_i^{(b)}(u) > t \,|\, Z(t) = z_b),$$

its conditional safety function while the system is at the operational state z_b, $b = 1,2,...,v$.

Similarly, we denote the system lifetime by $T^{(b)}(u)$ and by

$$\boldsymbol{s}_{n_b}^{(b)}(t,\ \cdot) = [1,\ \boldsymbol{s}_{n_b}^{(b)}(t,1),\ \boldsymbol{s}_{n_b}^{(b)}(t,2),\ ...,\ \boldsymbol{s}_{n_b}^{(b)}(t,z)\,]$$

for $n_b \in \{1,2,...,n\},$ where n_b are numbers of components in the operation states z_b and for $t \in <0,\infty)$, $n_b \in \{1,2,...,n\}$, $b = 1,2,...,v$, $u = 1,2,...,z$,

$$\boldsymbol{s}_{n_b}^{(b)}(t,u), = P(T^{(b)}(u) > t \,|\, Z(t) = z_b).$$

the conditional safety function of the system while the system is at the operational state z_b, $b = 1,2,...,v$.

Thus, the safety function $s_i^{(b)}(t,u)$ is the conditional probability that the component E_i lifetime $T_i^{(b)}(u)$ in the state subset $\{u,u+1,...,z\}$ is not less than t, while the process $Z(t)$ is at the operation state z_b. Similarly, the safety function $\boldsymbol{s}_{n_b}^{(b)}(t,u)$ is the conditional probability that the system lifetime $T^{(b)}(u)$ in the state subset $\{u,u+1,...,z\}$ is not less than t, while the process $Z(t)$ is at the operation state z_b.

In the case when the system operation time is large enough, the unconditional safety function of the system is given by

$$\boldsymbol{s}_n(t,\ \cdot) = [1,\ \boldsymbol{s}_n(t,1),\ \boldsymbol{s}_n(t,2),\ ...,\ \boldsymbol{s}_n(t,z)\,],\ t \ge 0,$$

where

$$\boldsymbol{s}_n(t,u) = P(T(u) > t) \cong \sum_{b=1}^{v} p_b \boldsymbol{s}_{n_b}^{(b)}(t,u) \quad (5)$$

for $t \ge 0$, $n_b \in \{1,2,...,n\}$, $u = 1,2,...,z,$ and $T(u)$ is the unconditional lifetime of the system in the safety state subset $\{u,u+1,...,z\}$.

The mean values of the system lifetimes in the safety state subset $\{u,u+1,...,z\}$ are

$$\mu(u) = E[T(u)] \cong \sum_{b=1}^{v} p_b \mu_b(u), \; u = 1,2,..., z, \tag{6}$$

where (Lisnianski, 2003, Soszynska 2006)

$$\mu_b(u) = \int_0^\infty \boldsymbol{s}_{\boldsymbol{n_b}}^{(b)}(t,u)dt, \;\; n_b \in \{1,2,..., n\}, \; u = 1,2,..., z.$$

(7)

The mean values of the system lifetimes in the particular safety states u, are (Kolowrocki 2004)

$$\overline{\mu}(u) = \mu(u) - \mu(u+1), \; u = 1,2,..., z-1, \; \overline{\mu}(z) = \mu(z). \tag{8}$$

3 THE STENA BALTICA FERRY DESCRIPTION

The m/v Stena Baltica is a passenger Ro-Ro ship operating in Baltic Sea between Gdynia and Karlskrona ports on regular everyday line. Her owner is Stena Line Scandinavia AB. She was build in Gdańsk Shipyard in 2005.

Figure 1. Stena Baltica Ferry

She is characterized by the following parameters: the length of 164.41m, the breadth moulded of 27.60 m, the summer load draft of 6.313 m, DWT of 4456, the displacement of 16618 tons, the cargo capacity of 466 cars, the total numbers of passengers and crew capacity of 1200 + 96 = 1296. The number of cabins is 379 with the number of beds 949 and total number of seats on a board is 981. The main engines are 4 of the kind MAN 4840 kW, the propellers are 2 of the kind Ka Me Wa with diameter 4800 mm, the BOW thrusters are 2 of the kind 1275 kW and 735 kW and the aft thruster is 1 of the kind 735 kW. The navigation and communication equipments are according to SOLAS Convention. The ferry speed is 19.5 knots (calm water) (RPM – 178). The service restriction are: maximum of 350 NM from land and wave height of 3.1 m, according to the Stockholm Agreement.

4 STENA BALTICA FERRY IN VARIABLE OPERATION CONDITIONS

We preliminarily assume that the Stena Baltica ferry is composed of five subsystems S_1, S_2, S_3, S_4, S_5 having an essential influence on her safety (Soszynska et all 2007). These subsystems are:

S_1 - a navigational subsystem,

S_2 - a propulsion and controlling subsystem,
S_3 - a loading and unloading subsystem,
S_4 - a hull subsystem,
S_5 - an anchoring and mooring subsystem,
S_6 - a protection and rescue subsystem,
S_7 - a social subsystem.

In our further ship safety analysis we will omit the protection and rescue subsystem S_6 and the social subsystem S_7 and we will consider its strictly technical subsystems S_1, S_2, S_3, S_4 and S_5 only.

Further, assuming that the ship is in the safety state subset $\{u,u+1,...,4\}$ if all its subsystems are in this subset of safety states and considering *Definition 3.4* (Kolowrocki 2004), we conclude that the ship is a series system of subsystems S_1, S_2, S_3, S_4, S_5 with a general scheme and detailed scheme presented respectively in *Figure2*.

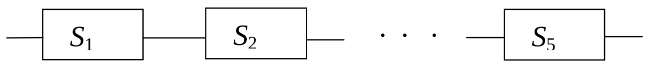

Figure 2. General scheme of ship safety structure

Taking into account the operation process of the considered ferry we distinguish the following as its eighteen operation states:

- an operation state z_1 – loading at Gdynia Port,
- an operation state z_2 – unmooring operations at Gdynia Port,
- an operation state z_3 – leaving Gdynia Port and navigation to “GD” buoy,
- an operation state z_4 – navigation at restricted waters from “GD” buoy to the end of Traffic Separation Scheme,
- an operation state z_5 – navigation at open waters from the end of Traffic Separation Scheme to “Angoring” buoy,
- an operation state z_6 – navigation at restricted waters from “Angoring” buoy to “Verko” Berth at Karlskrona,
- an operation state z_7 – mooring operations at Karlskrona Port,
- an operation state z_8 – unloading at Karlskrona Port,
- an operation state z_9 – loading at Karlskrona Port,
- an operation state z_{10} – unmooring operations at Karlskrona Port,
- an operation state z_{11} – ship turning at Karlskrona Port,
- an operation state z_{12} – leaving Karlskrona Port and navigation at restricted waters to “Angoring” buoy,
- an operation state z_{13} – navigation at open waters from “Angoring” buoy to the entering Traffic Separation Scheme,
- an operation state z_{14} – navigation at restricted waters from the entering Traffic Separation Scheme to “GD” buoy,
- an operation state z_{15} – navigation from “GD” buoy to turning area,
- an operation state z_{16} – ship turning at Gdynia Port,
- an operation state z_{17} – mooring operations at Gdynia Port,
- an operation state z_{18} – unloading at Gdynia Port.

On the basis of data coming from experts, the probabilities of transitions between the operation states are approximately given by

$$[p_{bl}] = \begin{bmatrix} 0\,1\,0 \ldots 0\,0 \\ 0\,0\,1 \ldots 0\,0 \\ \ldots \\ 0\,0\,0 \ldots 0\,1 \\ 1\,0\,0 \ldots 0\,0 \end{bmatrix},$$

and the conditional mean values of lifetimes in the operation states are:

$$M_{12} = 54.33,\ M_{23} = 2.57,\ M_{34} = 36.57,\ M_{45} = 52.5,\ M_{56} = 525.95,\ M_{67} = 37.16,$$

$$M_{78} = 7.02,\ M_{89} = 21.43,\ M_{910} = 53.69,\ M_{1011} = 2.93,\ M_{1112} = 4.38,\ M_{1213} = 23.86,$$

$$M_{1314} = 509.69,\ M_{1415} = 50.14,\ M_{1516} = 34.28,\ M_{1617} = 4.52,\ M_{1718} = 5.62,\ M_{181} = 18.74.$$

Hence, by (2), the unconditional mean lifetimes in the operation states are:

$$M_1 = 54.33,\ M_2 = 2.57,\ M_3 = 36.57,\ M_4 = 52.5,\ M_5 = 525.95,\ M_6 = 37.16,$$

$$M_7 = 7.02,\ M_8 = 21.43,\ M_9 = 53.69,\ M_{10} = 2.93,\ M_{11} = 4.38,\ M_{12} = 23.86,$$

$$M_{13} = 509.69,\ M_{14} = 50.14,\ M_{15} = 34.28,\ M_{16} = 4.52,\ M_{17} = 5.62,\ M_{18} = 18.74.$$

Since from the system of equations (4) we get

$$\pi_1 = \pi_2 = \pi_3 = \pi_4 = \pi_5 = \pi_6 = \pi_7 = \pi_8 = \pi_9 = \pi_{10} = \pi_{11} = \pi_{12} = \pi_{13} = \pi_{14} =$$

$$\pi_{15} = \pi_{16} = \pi_{17} = \pi_{18} = 0.056,$$

then the limit values of the transient probabilities $p_b(t)$ at the operational states z_b, according to (3), are given by

$$p_1 = 0.037,\ p_2 = 0.002,\ p_3 = 0.025,\ p_4 = 0.036,\ p_5 = 0.364,\ p_6 = 0.025,\ p_7 = 0.005,\ p_8 = 0.014,$$

$$p_9 = 0.037,\ p_{10} = 0.002,\ p_{11} = 0.003,\ p_{12} = 0.017,\ p_{13} = 0.354,\ p_{14} = 0.035,\ p_{15} = 0.024,\ p_{16} = 0.003,$$

$$p_{17} = 0.004,\ p_{18} = 0.013. \quad (9)$$

We assume as earlier that the ship is composed of $n = 5$ subsystems S_i, $i = 1,2,...,5$, and that the changes of the process of ship operation states have an influence on the system subsystems S_i safety and on the ship safety structure as well. The subsystems S_i, $i = 1,2,3,4,5$ are composed of five-state components, i.e. $z = 4$, with the multi-state safety functions

$$s_i^{(b)}(t,\cdot) = [1,\ s_i^{(b)}(t,1),\ s_i^{(b)}(t,2),\ s_i^{(b)}(t,3),\ s_i^{(b)}(t,4)\ ],\ t \in <0,\infty),\ b = 1,2,...,18,\ u = 1,2,3,4,$$

with exponential co-ordinates different in various operation states z_b, $b = 1,2,...,18$.
In (Soszynska et all 2007), on the basis of expert opinions concerned with the safety of the ship components the ship safety function in different operation conditions are determined.
At the operation state z_1, i.e. at the loading state the ship is built of $n_1 = 2$ subsystems S_3 and S_4 forming a series structure (Kolowrocki 2004) shown in *Figure 3*.

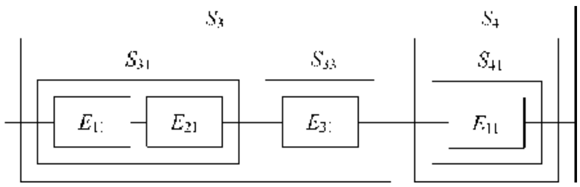

Figure 3. The scheme of the ship structure at the operation state z_1

Considering that the ship is in the safety state subsets $\{u, u+1,...,4\}$, $u = 1,2,3,4,$ if all its subsystems are in this safety state subset, according to *Definition 3.4* (Kolowrocki 2004), the considered system is a five-state series system and the conditional safety function of the ship while the ship is at the operational state z_1 is given by

$$\bar{\boldsymbol{s}}_2^{(1)}(t,\cdot) = [1, \bar{\boldsymbol{s}}_2^{(1)}(t,1), \bar{\boldsymbol{s}}_2^{(1)}(t,2), \bar{\boldsymbol{s}}_2^{(1)}(t,3), \bar{\boldsymbol{s}}_2^{(1)}(t,4)\ ], \quad (10)$$

where

$$\bar{\boldsymbol{s}}_2^{(1)}(t,u) = \boldsymbol{s}_{3;1,1,1}^{(1)}(t,u)\ \boldsymbol{s}_{1,1}^{(1)}(t,u) \ \text{ for } t \in <0,\infty),\ u = 1,2,3,4, \quad (11)$$

i.e.

$$\bar{\boldsymbol{s}}_2^{(1)}(t,1) = \exp[-0.433t]\exp[-0.05t] = \exp[-0.483t], \quad (12)$$

$$\bar{\boldsymbol{s}}_2^{(1)}(t,2) = \exp[-0.59t]\exp[-0.06t] = \exp[-0.65t] \quad (13)$$

$$\bar{\boldsymbol{s}}_2^{(1)}(t,3) = \exp[-0.695t]\exp[-0.065t] = \exp[-0.76t], \quad (14)$$

$$\bar{\boldsymbol{s}}_2^{(1)}(t,4) = \exp[-0.85t]\exp[-0.07t] = \exp[-0.92t]. \quad (15)$$

The expected values and standard deviations of the ship conditional lifetimes in the safety state subsets calculated from the above result given by (10)-(15), according to (7), at the operational state z_1 are:

$$\mu_1(1) \cong 2.07,\ \mu_1(2) \cong 1.54,\ \mu_1(3) \cong 1.32,\ \mu_1(4) \cong 1.09 \text{ years}, \quad (16)$$

$$\sigma_1(1) \cong 2.07,\ \sigma_1(2) \cong 1.54,\ \sigma_1(3) \cong 1.32,\ \sigma_1(4) \cong 1.09 \text{ years}, \quad (17)$$

and further, using (8), it follows that the ship conditional lifetimes in the particular safety states at the operational state z_1 are:

$$\bar{\mu}_1(1) \cong 0.53,\ \bar{\mu}_1(2) \cong 0.22,\ \bar{\mu}_1(3) \cong 0.23,\ \bar{\mu}_1(4) \cong 1.09 \text{ years.} \quad (18)$$

At the operation states z_2, i.e. at the cargo loading and un-loading state the ship is built of $n_2 = 3$ subsystems S_1, S_2 and S_5 forming a series structure (Kolowrocki 2004) shown in Figure 4.

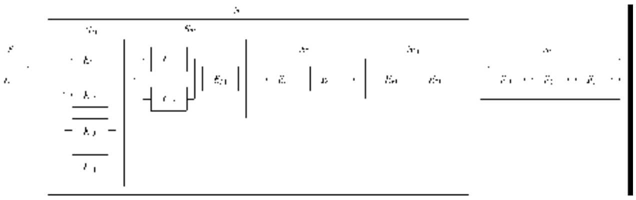

Figure 4. The scheme of the ship structure at the operation state z_2

Considering that the ship is in the safety state subsets $\{u, u+1,...,4\}$, $u = 1,2,3,4$, if all its subsystems are in this safety state subset, according to *Definition 3.4* (Kolowrocki 2004), the considered system is a five-state series system and the conditional safety function of the ship while the ship is at the operational state z_2 is given by

$$\bar{\boldsymbol{s}}_3^{(2)}(t,\ \cdot) = [1, \bar{\boldsymbol{s}}_3^{(2)}(t,\ 1), \bar{\boldsymbol{s}}_3^{(2)}(t,\ 2), \bar{\boldsymbol{s}}_3^{(2)}(t,\ 3), \bar{\boldsymbol{s}}_3^{(2)}(t,\ 4)], \quad (19)$$

where

$$\bar{\boldsymbol{s}}_3^{(2)}(t,u) = \boldsymbol{s}_{1,1}^{(2)}(t,\ u)\ \boldsymbol{s}_{7;4,2,1,1,1,1,1}^{(2)}(t,\ u)\ \boldsymbol{s}_{3;1,1,1}^{(2)}(t,\ u) \text{ for } t \in <0,\infty),\ u = 1,2,3,4, \quad (20)$$

i.e.

$$\bar{\boldsymbol{s}}_3^{(2)}(t,\ 1) = \exp[-0.033t][\ 12\exp[-0.33t] + 8\exp[-0.429t] - 16\exp[-0.363t]$$
$$- 3\exp[-0.462t]]\exp[-0.099t] = 12\exp[-0.462t] + 8\exp[-0.561t]$$
$$-16\exp[-0.495t] - 3\exp[-0.594t] \quad (21)$$

$$\bar{\boldsymbol{s}}_3^{(2)}(t,\ 2) = \exp[-0.04t][\ 12\exp[-0.38t] + 8\exp[-0.49t] + 6\exp[-0.46t]$$
$$- 16\exp[-0.42t] - 6\exp[-0.45t] - 3\exp[-0.53t]]\exp[-0.12t] = 12\exp[-0.54t] + 8\exp[-0.65t]$$
$$+ 6\exp[-0.62t] - 16\exp[-0.58t] - 6\exp[-0.61t] - 3\exp[-0.69t], \quad (22)$$

$$\bar{\boldsymbol{s}}_3^{(2)}(t,\ 3) = \exp[-0.045t][\ 12\exp[-0.43t] + 8\exp[-0.555t] + 6\exp[-0.53t]$$
$$- 16\exp[-0.48t] - 6\exp[-0.505t] - 3\exp[-0.605t]]\exp[-0.145t] = 12\exp[-0.62t] + 8\exp[-0.745t]$$
$$+ 6\exp[-0.72t] - 16\exp[-0.67t] - 6\exp[-0.695t] - 3\exp[-0.795t], \quad (23)$$

$$\bar{\boldsymbol{s}}_3^{(2)}(t,\ 4) = \exp[-0.05t][\ 12\exp[-0.47t] + 8\exp[-0.605t] + 6\exp[-0.58t]$$
$$- 16\exp[-0.525t] - 6\exp[-0.55t] - 3\exp[-0.66t]]\exp[-0.165t] = 12\exp[-0.685t] + 8\exp[-0.82t]$$

$$+ 6 \exp[-0.795t] - 16 \exp[-0.74t] - 6 \exp[-0.765t] - 3 \exp[-0.875t]. \quad (24)$$

The expected values and standard deviations of the ship conditional lifetimes in the safety state subsets calculated from the above result given by (19)-(24), according to (7), at the operational state z_2 are:

$$\mu_2(1) \cong 2.86,\ \ \mu_2(2) \cong 0.43,\ \mu_2(3) \cong 2.14,\ \ \mu_2(4) \cong 1.93 \text{ years}, \quad (25)$$

$$\sigma_2(2) \cong 2.74,\ \sigma_2(2) \cong 2.35,\ \sigma_2(3) \cong 2.05,\ \sigma_2(4) \cong 1.85 \text{ years}, \quad (26)$$

and further, using (8), it follows that the ship conditional lifetimes in the particular safety states at the operational state z_2 are:

$$\bar{\mu}_2(1) \cong 0.43,\ \bar{\mu}_2(2) \cong 0.29,\ \bar{\mu}_2(3) \cong 0.21,\ \bar{\mu}_2(4) \cong 1.93 \text{ years}. \quad (27)$$

At the remaining operation states z_3, z_4, z_5, z_6, z_7, z_8, z_9, z_{10}, z_{11}, z_{12}, z_{13}, z_{14}, z_{15}, z_{16}, z_{17} and z_{18} we proceed in an analogous way. We determined the system conditional safety functions in particular operation states and the expected values and standard deviations of the ship conditional lifetimes.

In the case when the system operation time is large enough, the unconditional safety function of the ship is given by the vector

$$\boldsymbol{s}_5(t,\ \cdot) = [1,\ \boldsymbol{s}_5(t,1),\ \boldsymbol{s}_5(t,2),\ \boldsymbol{s}_5(t,3),\ \boldsymbol{s}_5(t,4)],\ t \geq 0, \quad (28)$$

where, according to (5) and after considering the values of p_b, $b = 1,2,...,18$, given by (9), its co-ordinates are as follows:

$$\boldsymbol{s}_5(t,1) = 0.037\cdot\bar{\boldsymbol{s}}_2^{(1)}(t,1) + 0.002\cdot\bar{\boldsymbol{s}}_3^{(2)}(t,1) + 0.025\cdot\bar{\boldsymbol{s}}_2^{(3)}(t,1) + 0.036\cdot\bar{\boldsymbol{s}}_3^{(4)}(t,1) + 0.364\cdot\bar{\boldsymbol{s}}_3^{(5)}(t,1)$$
$$+ 0.025\cdot\bar{\boldsymbol{s}}_3^{(6)}(t,1) + 0.005\cdot\bar{\boldsymbol{s}}_3^{(7)}(t,1) + 0.014\cdot\bar{\boldsymbol{s}}_2^{(8)}(t,1) + 0.037\cdot\bar{\boldsymbol{s}}_2^{(9)}(t,1) + 0.002\cdot\bar{\boldsymbol{s}}_3^{(10)}(t,1)$$
$$+ 0.003\cdot\bar{\boldsymbol{s}}_2^{(11)}(t,1) + 0.017\cdot\bar{\boldsymbol{s}}_3^{(12)}(t,1) + 0.354\cdot\bar{\boldsymbol{s}}_3^{(13)}(t,1) + 0.035\cdot\bar{\boldsymbol{s}}_3^{(14)}(t,1)$$
$$+ 0.024\cdot\bar{\boldsymbol{s}}_2^{(15)}(t,1) + 0.003\cdot\bar{\boldsymbol{s}}_2^{(16)}(t,1) + 0.004\cdot\bar{\boldsymbol{s}}_3^{(17)}(t,1) + 0.013\cdot\bar{\boldsymbol{s}}_2^{(18)}(t,1), \quad (29)$$

$$\boldsymbol{s}_5(t,2) = 0.037\cdot\bar{\boldsymbol{s}}_2^{(1)}(t,2) + 0.002\cdot\bar{\boldsymbol{s}}_3^{(2)}(t,2) + 0.025\cdot\bar{\boldsymbol{s}}_2^{(3)}(t,2) + 0.036\cdot\bar{\boldsymbol{s}}_3^{(4)}(t,2)$$
$$+ 0.364\cdot\bar{\boldsymbol{s}}_3^{(5)}(t,2) + 0.025\cdot\bar{\boldsymbol{s}}_3^{(6)}(t,2) + 0.005\cdot\bar{\boldsymbol{s}}_3^{(7)}(t,2) + 0.014\cdot\bar{\boldsymbol{s}}_2^{(8)}(t,2) + 0.037\cdot\bar{\boldsymbol{s}}_2^{(9)}(t,2)$$
$$+ 0.002\cdot\bar{\boldsymbol{s}}_3^{(10)}(t,2) + 0.003\cdot\bar{\boldsymbol{s}}_2^{(11)}(t,2) + 0.017\cdot\bar{\boldsymbol{s}}_3^{(12)}(t,2) + 0.354\cdot\bar{\boldsymbol{s}}_3^{(13)}(t,2)$$
$$+ 0.035\cdot\bar{\boldsymbol{s}}_3^{(14)}(t,2) + 0.024\cdot\bar{\boldsymbol{s}}_2^{(15)}(t,2) + 0.003\cdot\bar{\boldsymbol{s}}_2^{(16)}(t,2) + 0.004\cdot\bar{\boldsymbol{s}}_3^{(17)}(t,2) + 0.013\cdot\bar{\boldsymbol{s}}_2^{(18)}(t,2), \quad (30)$$

$$\boldsymbol{s}_5(t,3) = 0.037\cdot\bar{\boldsymbol{s}}_2^{(1)}(t,3) + 0.002\cdot\bar{\boldsymbol{s}}_3^{(2)}(t,3) + 0.025\cdot\bar{\boldsymbol{s}}_2^{(3)}(t,3) + 0.364\cdot\bar{\boldsymbol{s}}_3^{(5)}(t,3) + 0.025\cdot\bar{\boldsymbol{s}}_3^{(6)}(t,3)$$

$$+0.005\cdot\bar{s}_3^{(7)}(t,3)+0.014\cdot\bar{s}_2^{(8)}(t,3)+0.037\cdot\bar{s}_2^{(9)}(t,3)+0.002\cdot\bar{s}_3^{(10)}(t,3)$$

$$+0.003\cdot\bar{s}_2^{(11)}(t,3)+0.017\cdot\bar{s}_3^{(12)}(t,3)+0.354\cdot\bar{s}_3^{(13)}(t,3)+0.035\cdot\bar{s}_3^{(14)}(t,3)$$

$$+0.024\cdot\bar{s}_2^{(15)}(t,3)+0.003\cdot\bar{s}_2^{(16)}(t,3) \qquad +0.004\cdot\bar{s}_3^{(17)}(t,3)+0.013\cdot\bar{s}_2^{(18)}(t,3), \quad (31)$$

$$\boldsymbol{s}_5(t,4)=0.037\cdot\bar{s}_2^{(1)}(t,4)+0.002\cdot\bar{s}_3^{(2)}(t,4)+0.025\cdot\bar{s}_2^{(3)}(t,4)+0.036\cdot\bar{s}_3^{(4)}(t,4)+0.364\cdot\bar{s}_3^{(5)}(t,4)$$

$$+0.025\cdot\bar{s}_3^{(6)}(t,4)+0.005\cdot\bar{s}_3^{(7)}(t,4)+0.014\cdot\bar{s}_2^{(8)}(t,4)+0.037\cdot\bar{s}_2^{(9)}(t,4)+0.002\cdot\bar{s}_3^{(10)}(t,4)$$

$$+0.003\cdot\bar{s}_2^{(11)}(t,4)+0.017\cdot\bar{s}_3^{(12)}(t,4)+0.354\cdot\bar{s}_3^{(13)}(t,4)+0.035\cdot\bar{s}_3^{(14)}(t,4)+0.024\cdot\bar{s}_2^{(15)}(t,4)$$

$$+0.003\cdot\bar{s}_2^{(16)}(t,4)+0.004\cdot\bar{s}_3^{(17)}(t,4)+0.013\cdot\bar{s}_2^{(18)}(t,4) \text{ for } t\ge 0, \quad (32)$$

where

$\bar{s}_{n_b}^{(b)}(t,u)$, $u=1,2,3,4$, $b=1,2,\ldots,18$, are given in (Soszynska et all 2007)

The mean values and standard deviations of the system unconditional lifetimes in the safety state subsets, according to (6)-(7) respectively are:

$$\mu(1)\cong 0.037\cdot 2.07+0.002\cdot 2.86+0.025\cdot 4.94+0.036\cdot 4.2+0.364\cdot 4.2+0.025\cdot 4.01$$

$$+0.005\cdot 2.86+0.014\cdot 3.53+0.037\cdot 3.53+0.002\cdot 2.86+0.003\cdot 3.91+0.017\cdot 4.2$$

$$+0.354\cdot 4.2+0.035\cdot 4.2+0.024\cdot 4.94+0.003\cdot 3.91+0.004\cdot 2.86+0.013\cdot 2.07$$

$$\cong 4.07, \quad (33)$$

$$\sigma(1)\cong 4.1,$$

$$\mu(2)\cong 0.037\cdot 1.54+0.002\cdot 2.43+0.025\cdot 3.9+0.036\cdot 3.80+0.364\cdot 3.80+0.025\cdot 3.24$$

$$+0.005\cdot 2.43+0.014\cdot 2.50+0.037\cdot 2.50+0.002\cdot 2.43+0.003\cdot 3.37+0.017\cdot 3.80$$

$$+0.354\cdot 3.80+0.035\cdot 3.80+0.024\cdot 3.90+0.003\cdot 3.37+0.004\cdot 2.43$$

$$\cong 3.59, \quad (34)$$

$$\sigma(2)\cong 3.34,$$

$$\mu(3)\cong 0.037\cdot 1.32+0.002\cdot 2.14+0.025\cdot 3.44+0.036\cdot 3.38+0.364\cdot 3.38+0.025\cdot 2.88$$

$$+0.005\cdot 2.14+0.014\cdot 2.17+0.037\cdot 2.17+0.002\cdot 2.14+0.003\cdot 3.07+0.017\cdot 3.38$$

$$+0.354\cdot 3.38+0.035\cdot 3.38+0.024\cdot 3.44+0.003\cdot 3.07+0.004\cdot 2.14+0.013\cdot 1.32$$

$$\cong 3.19, \tag{35}$$

$$\sigma(3) \cong 3.65,$$

$$\mu(4) \cong 0.037 \cdot 1.09 + 0.002 \cdot 1.93 + 0.025 \cdot 3.1 + 0.036 \cdot 3.05 + 0.364 \cdot 3.05 + 0.025 \cdot 2.61$$

$$+ 0.005 \cdot 1.93 + 0.014 \cdot 1.92 + 0.037 \cdot 1.92 + 0.002 \cdot 1.93 + 0.003 \cdot 2.76 + 0.017 \cdot 3.05$$

$$+ 0.354 \cdot 3.05 + 0.035 \cdot 3.05 + 0.024 \cdot 3.10 + 0.003 \cdot 2.76 + 0.004 \cdot 1.93 + 0.013 \cdot 1.09$$

$$\cong 2.87, \tag{36}$$

$$\sigma(4) \cong 2.75.$$

The mean values of the system lifetimes in the particular safety states, by (8), are

$$\bar{\mu}(1) = \mu(1) - \mu(2) = 0.48, \ \bar{\mu}(2) = \mu(2) - \mu(3) = 0.4,$$

$$\bar{\mu}(3) = \mu(3) - \mu(4) = 0.32, \ \ \bar{\mu}(4) = \mu(4) = 2.87. \tag{37}$$

If the critical safety state is r = 2, then the system risk function, according to (12) (Kolowrocki, Soszynska 2008) , is given by

$$\boldsymbol{r}(t) = 1 - \boldsymbol{s}_5(t,2) = 1 - [0.037 \cdot \bar{\boldsymbol{s}}_2^{(1)}(t,2) + 0.002 \cdot \bar{\boldsymbol{s}}_3^{(2)}(t,2) + 0.025 \cdot \bar{\boldsymbol{s}}_2^{(3)}(t,2) + 0.036 \cdot \bar{\boldsymbol{s}}_3^{(4)}(t,2)$$

$$+ 0.364 \cdot \bar{\boldsymbol{s}}_3^{(5)}(t,2) + 0.025 \cdot \bar{\boldsymbol{s}}_3^{(6)}(t,2) + 0.005 \cdot \bar{\boldsymbol{s}}_3^{(7)}(t,2) + 0.014 \cdot \bar{\boldsymbol{s}}_2^{(8)}(t,2)$$

$$+ 0.037 \cdot \bar{\boldsymbol{s}}_2^{(9)}(t,2) + 0.002 \cdot \bar{\boldsymbol{s}}_3^{(10)}(t,2) + 0.003 \cdot \bar{\boldsymbol{s}}_2^{(11)}(t,2) + 0.017 \cdot \bar{\boldsymbol{s}}_3^{(12)}(t,2)$$

$$+ 0.354 \cdot \bar{\boldsymbol{s}}_3^{(13)}(t,2) + 0.035 \cdot \bar{\boldsymbol{s}}_3^{(14)}(t,2) + 0.024 \cdot \bar{\boldsymbol{s}}_2^{(15)}(t,2) + 0.003 \cdot \bar{\boldsymbol{s}}_2^{(16)}(t,2)$$

$$+ 0.004 \cdot \bar{\boldsymbol{s}}_3^{(17)}(t,2) + 0.013 \cdot \bar{\boldsymbol{s}}_2^{(18)}(t,2)] \ \text{ for } t \geq 0. \tag{38}$$

Hence, the moment when the system risk function exceeds a permitted level, for instance δ = 0.05, from (13) (Kolowrocki, Soszynska 2008), is

$$\tau = \boldsymbol{r}^{-1}(\delta) \cong 0.19 \text{ years}. \tag{39}$$

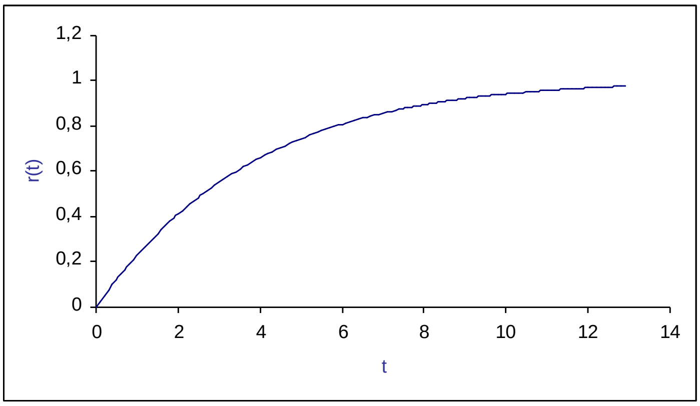

Figure 5. A graph of a risk function $r(t)$ of the ship

5 CONCLUSION

In the paper the multi-state approach (Kolowrocki 2004) to the safety analysis and evaluation of systems related to their variable operation processes has been considered. The ship safety structure and its safety subsystems characteristics are changing in different states what makes the analysis more complicated. A semi-markov model (Grabski 2002) of this ferry operation process is applied and its parameters statistical identification is performed. The Stena Baltca ferry operation process is analyzed and its operation states are defined. Preliminary collected statistical data is applied to the ferry operation process identification. Basic safety structures of multi-state systems of components with degrading safety states related to their variable operation conditions are applied to the considered ferry safety determination. For the ferry technical subsystems the conditional and unconditional multi-state safety functions are determined. The proposed approach to the solution of a practically important problem of linking the multi-state systems safety models and the systems operation processes models is applied to the preliminary evaluation of the safety function, the risk function and other safety characteristics of the Stena Baltica ferry operating with varying in time her structure and safety characteristics of the subsystems and components it is composed. The system safety structures are fixed generally with not high accuracy in details concerned with the subsystems structures because of their complexity and concerned with the components safety characteristics because of the luck of statistical data necessary for their estimation. Whereas, the input characteristics of the ferry operation process are of high quality because of the very good statistical data necessary for their estimation.

The results presented in the paper suggest that it seems reasonable to continue the investigations focusing on the methods of safety analysis for other more complex multi-state systems and the methods of safety evaluation related to the multi-state systems in variable operation processes and their applications to the ship transportation systems (Soszynska et all 2007).

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6 REFERENCES

Aven, T. 1985. Reliability evaluation of multi-state systems with multi-state components. *IEEE Transactions on reliability* 34, pp 473-479.

Dziula, P., Jurdzinski, M., Kolowrocki, K. & Soszynska, J. 2007. On multi-state approach to ship systems safety analysis. *Proc. 12th International Congress of the International Maritime Association of the Mediterranean, IMAM 2007.* A. A. Balkema Publishers: Leiden - London - New York - Philadelphia - Singapore.

Grabski, F. 2002. *Semi-Markov Models of Systems Reliability and Operations*. Warsaw: Systems Research Institute, Polish Academy of Science.

Kolowrocki, K. 2004. *Reliability of Large Systems*. Elsevier, Amsterdam - Boston - Heidelberg - London - New York - Oxford - Paris - San Diego - San Francisco - Singapore - Sydney - Tokyo.

Kolowrocki, K., Soszyńska, J. 2006. Reliability and availability of complex systems. *Quality and Reliability Engineering International* Vol. 22, Issue 1, J. Wiley & Sons Ltd. 79-99.

Kolowrocki, K., Soszynska, J. 2008. A general model of ship technical systems safety. *Proc. International Conference on Industrial Engineering and Engineering Management, IEEM 2008*, Singapore, 1346-1350.

Lisnianski, A., Levitin, G. 2003. *Multi-state System Reliability. Assessment, Optimisation and Applications.* World Scientific Publishing Co., New Jersey, London, Singapore , Hong Kong.

Meng, F. 1993. Component- relevancy and characterisation in multi-state systems. *IEEE Transactions on reliability* 42, pp 478-483.

Soszynska, J. 2006. Reliability evaluation of a port oil transportation system in variable operation conditions. *International Journal of Pressure Vessels and Piping* Vol. 83, Issue 4, 304-310.

Soszynska, J. 2007. Systems reliability analysis in variable operation conditions. *International Journal of Reliability, Quality and Safety Engineering. System Reliability and Safety* Vol. 14, No 6, 1-19.

Soszyńska, J., Kołowrocki, K., Kamiński, P., Judziński, M., Guze, S., Milczek, B., Golik P. 2007. *Preliminary modelling, statistical identification and evaluation of reliability, risk, availability and safety of port, shipyard and ship technical systems in constant and variable operation conditions.* WP4 - Task 4.2. Preliminary safety and risk analysis and evaluation of Stena Baltica ferry in constant and variable operation conditions. WP4 - Sub-Task 4.2.5 – English – 30.11.2008. Poland-Singapore Joint Project. MSHE Decision No. 63/N-Singapore/2007/0.

A COMPUTATIONAL TOOL FOR A GENERAL MODEL OF OPERATION PROCESSES IN INDUSTRIAL SYSTEMS

Mohamed Salahuddin Habibullah, Ernest Lumanpauw

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Institute Of High Performance Computing, Singapore
e-mail: *mohdsh@ihpc.a-star.edu.sg*, *lumanpauwe@ihpc.a-star.edu.sg*

Kołowrocki Krzysztof, Joanna Soszyńska

•

Gdynia Maritime University, Poland
e-mail: *katmatkk@am.gdynia.pl*, *joannas@am.gdynia.pl*

Ng Kien Ming

•

National University of Singapore, Singapore
e-mail: *kienming@nus.edu.sg*

ABSTRACT

The complexities of real industrial systems operation processes require computational methods that can analyze the large data and evaluate the behaviours of these systems. The use of methods such as Bayesian Network, Formal Safety Assessment and Statistical-Model based method were discussed as possibilities. Of which, a computational tool, based on the Semi-Markov model, was developed. This tool was then applied to analyze the behaviour of the operation processes of the oil transportation system in Dębogórze, Poland. The analyses showed that the computational solutions generated compared favourably well with some well-established analytical formulae, enabling possible extensions of the tool to include reliability and optimization evaluations to be explored.

1 INTRODUCTION

Many real industrial systems belong to the class of complex systems, resulting from the large number of components and interconnected parts, which are collectively assembled to define the operations and properties of the systems. Due to the complexity of such systems, it often causes the evaluation of the system reliability, availability and safety to become difficult. These complexities are multiplied in the case of large complex systems, where the determination of the exact reliability, availability and risk functions of the systems, leads to very complicated formulae, often useless for reliability practitioners to use. In real maritime systems, the focus area of this paper, some examples of such systems are in the piping transportation of water, gas and oil (Guze S. et al 2008) as well as in shipyard transportation using belt, rope conveyers and elevators (Blokus-R. et al 2008).

The difficulties associated with these large complex systems are further compounded when reliability optimization of these systems needs to be evaluated, with respect to their safety and costs. Due to the mathematical complexities of the current methods, such evaluations are often complicated and not possible to be performed by practitioners. In addition, the need to analyze these systems in their variable operation conditions as well as considering their changes in time reliability structures and observing the components reliability characteristics, further complicates the issue. Furthermore, in handling such systems, the large datasets generated out of these systems, which needs to be analysed and processed, often necessitates the use of data mining tools and extensive compute-power, to speed up the computational processes. Thus, the availability of a computational tool that can model such large and complex maritime industrial systems operation processes, would indeed be valuable to practitioners and users.

In developing such computational tool, various approaches have been proposed in understanding the behaviour of maritime industrial systems operation processes. The 3 most commonly researched methods are namely the Bayesian Network (BN), the Formal Safety Assessment (FSA) and the Statistical Model-based method. Here, an overview of these 3 methods and its applicability in evaluating the behaviour of maritime industrial systems operation processes is discussed.

Bayesian Network (BN) (Pearl J. 1985) is a probabilistic model, whose nodes are used to represent variables and edges, in describing variables dependence with each another. It is a popular research approach to the maritime industry and it is often considered when analyzing the probability relationship between the different types of ships and accident rates. It is also a model that can be used to clearly represent the inter-relationship between subsystems and components of a system. In modelling maritime transportation systems, some variances of the BN have emerged. The Bayesian Belief Network (BBN) (Trucco P. 2008) was primarily developed for modelling Maritime Transport System (MTS), by taking into consideration ship owners, shipyards, port authorities, regulators and their mutual influences. By considering the case for the design of High Speed Craft (HSC), the study looked into the risk analysis associated with the quantification of Human and Organizational Factors (HFO). Integrated with the Fault-Tree analysis, the BBN managed to identify the probabilistic correlations between the basic events of collision and the model under HFO conditions, for collision in the open seas. Norrington (Norrington L. et al 2008) had also used the BBN model to conduct the reliability analysis of Search-And-Rescue (SAR) operations with the UK coastal guard coordination centres. Another variant is the Fuzzy-Bayesian Network (FBN) (Eleye-D 2008), applied in marine safety assessment, by integrating human elements into quantitative analysis. For this analysis, mass assignment theory was used as the bridge to connect the human factor and the probabilistic calculation.

Formal Safety Assessment (FSA) is a structured and systematic methodology, aimed at enhancing maritime safety by using risk analysis and cost benefit assessment. It is achieved by providing justifications for the proposed regulatory measures and allowing comparisons of the different measures to be made. This is in line with the basic philosophy of the FSA in that it is a tool that can be used to facilitate transparent decision-making process. FSA is also used to help in evaluating new regulations for maritime safety and protection of marine environment, with the aim of achieving a balance between technical and operational issues, which includes human, maritime safety, protection of marine environment and costs (IMO 2002). In its application to maritime systems operation processes, Ruud (Rudd S. et al 2008) has applied the FSA in developing risk-based rules and functional requirements, for systems and components in an offshore crane system. In studying the watertight integrity of hatchways of bulk carriers, Lee (Lee J. et al 2001) has also applied the FSA, which resulted in 18 hazards to be identified. This enabled 32 risk control measures to be devised in reducing the associated risks. Wang (Wang J. 2002) also explored the use of the FSA in maritime design. By considering both offshore and marine safety, the current practices as well as recent developments in safety assessment were illustrated. This was then applied to several maritime case studies, resulting in the relationship between offshore safety and formal ship safety assessment to be described. Others have also tried incorporating new models within the FSA. One such effort is by Hu (Hu S. et al 2007) who proposed a Model based on Relative Risk Assessment (MRRA) and used it to discuss the frequency and severity criteria affecting in ship navigation.

The third approach is the Statistical Model-based method, defined as a set of mathematical equations which describes the object of interest in terms of random variables and its associated probability distribution. From the statistical analysis point of view, in applying such methods, availability of real data is a necessity as these data are then used to extract features in relation to maritime safety, reliability, availability and risks. One such real data is from the Marine Accident Investigation Branch (MAIB) in UK. Using data from 1992 to 1999, Wang (Wang J. et al 2005) conducted a comprehensive statistical analysis of accidents involving fishing vessels, with the results presented in tables and graphs, showing the frequency and trends of the various accidents. Jin (Jin D. 2005) also conducted similar fishing vessels accidents analyses using data from northeastern USA but investigated further by modelling the accidents using logistics probability distribution. By considering factors such as weather and vessel size into the probability function, the paper provided a complete analysis of all causal factors of the fishing accidents and how much do they affect the ship safety, leading to the conclusion that the accidents probability is affected by the weather, vessel location, time of year, and vessel characteristics. Thus, it can be seen that the primary aim of these statistical models is to simulate, and then predict and optimize the system. One particular branch of statistical methods that has such capabilities is the Markov and Semi-Markov (Heyman D. et al 2003) methods. These methods has been used to estimate the corrosion rate as well as finding the failure probability of piping system (Vinod G. et al 2003), to study the patient population profile of a clinical trial, for arbitrary many patient classes, trial sites and start-times (Felli J. C. et al 2007) and to model in conjunction with Weibull distribution holding times to the actual power-plant operating data (Perman M. et al 1997).

In this paper, the Statistical Model-based method was chosen to provide a mean of understanding the behaviour of maritime industrial systems operation processes. This is due to the fact that the Statistical Model-based method is capable of handling mathematical complexities associated with maritime systems. The ability of this method to handle large datasets together with the use of data mining tools and extensive compute-power is also a plus. Also, recent breakthroughs in the use of Semi-Markov models on maritime transportation by Kołowrocki (Guze S, et al 2008, Blokus-R et al 2008, Kołowrocki K. et al 2008) have opened many possibilities. In his papers, he has analytically modelled the maritime transportation operations as a Semi-Markov process, with the stationary limiting probability that the operation will stay in each state, are computed based on the model. He then applied this to several real problems such as shipyard rope transportation system port oil transportation and ship operational process. Although generalized BN and FSA can possibly perform similar studies, the focus here is more towards leveraging the work done by Kołowrocki (Guze S, et al 2008, Blokus-R et al 2008, Kołowrocki K. et al 2008). Comparisons of the 3 methods with actual data, on its treatment of uncertainties, will be explored and published elsewhere.

Although the Semi-Markov model can be evaluated analytically, its solution is straightforward only for simple problems, with limited number of states considered. For large and complex systems such as in real maritime transportation, the continued use of analytical computation is tedious and cumbersome. Thus, having a Semi-Markov computational tool that can model industrial systems operation processes would indeed be advantageous, as the reusability of the codes would enable users to continuously improve and optimize the operation processes. In addition, the use of such computational tool would also ease the handling of the compute-intensive large data sets associated with maritime transportation and its use together with data mining techniques.

The rest of this paper is organized as follows. Section 2 describes the proposed general Semi-Markov model of industrial systems operation processes. This is then followed by Section 3, where the developed Semi-Markov computational tool, is discussed. In Section 4, the application of the computational tool for the case of port oil transportation system process is analyzed and described. Finally, in Section 5, the paper is concluded with outlines of future possible works that could be explored.

2 GENERAL MODEL OF INUSTRIAL SYSTEMS OPERATION PROCESSES

The general model of industrial systems operation processes is formulated as a Semi-Markov process (Grabski F. 2002). The systems, during its operation processes, are taking $v, v \in N$ different operation states. Furthermore, $Z(t)$, $t \in <0,+\infty>$ is defined as the process with discrete operation states from the set of states, $Z = \{z_1, z_2, ...z_v\}$, and conditional sojourn times, θ_{bl}, at the operation states, z_b, when its next operation state is z_l, $b.l = 1,2,....,v$, $b \neq l$. Based on the above assumptions, the general system operation process may be described by:

- The initial operation state probability vector:

$$[p_b(0)]_{1xv} = [p_1(0), p_2(0),....., p_v(0)] \tag{1}$$

where $p_b(0) = P(Z(0) = z_b)$ for $b = 1,2,....,v$

- The transition probability matrix:

$$[p_{bl}]_{vcv} = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1v} \\ p_{21} & p_{22} & \cdots & p_{2v} \\ \cdots & \cdots & \cdots & \cdots \\ p_{v1} & p_{v2} & \cdots & p_{vv} \end{bmatrix} \tag{2}$$

where $p_{bb} = 0$ for $b = 1,2,....,v$

- The conditional sojourn time distribution matrix:

$$H_{bl}(t)_{1xv} = \begin{bmatrix} H_{11}(t) & H_{12}(t) & ... & H_{1v}(t) \\ H_{21}(t) & H_{22}(t) & ... & H_{2v}(t) \\ ... & ... & ... & ... \\ H_{v1}(t) & H_{v2}(t) & ... & H_{vv}(t) \end{bmatrix} \tag{3}$$

where $H_{bl}(t) = P(\theta_{bl} < t)$ for $b = 1,2,....,v$, $b \neq l$ and $H_{bb}(t) = 0$ for $b = 1,2,....,v$

- The mean value of the conditional sojourn time:

$$M_{bl} = E[\theta_{bl}] = \int_0^{\infty} t h_{bl}(t) dt \tag{4}$$

where $h_{bl}(t) = \frac{d}{dt}[H_{bl}(t)]$ for $b = 1,2,....,v$, $b \neq l$

The use of the Semi-Markov model is mainly due to the scarcity of available data. In such scenarios, the Semi-Markov model is perhaps the best available option to analyze the problems, since it is built upon theories of probability and stochastic process, and hence justifying the use of the density function to get the mean, as in equation (4). It is also common practice that as more data becomes available, data mining methods such as regression, prediction, etc as well as simulation tools can be used to build a more robust analytical model of the problem. Thus, from the law of total probability,

- The distribution function of the unconditional sojourn time θ_b is given by:

$$H_b(t) = \sum_{l=1}^{v} p_{bl} H_{bl}(t), \quad b = 1,2,....,v$$

(5)

- The mean value of the unconditional sojourn times is given by:

$$M_b = E[\theta_b] = \sum_{l=1}^{v} P_{bl} M_{bl}, \quad b = 1,2,....,v$$

(6)

The limiting probability is one of the important characteristics of the process, it is given by:

$$P_b = \lim_{t \to \infty} p_b(t) = \frac{\pi_b M_b}{\sum_{l=1}^{v} \pi_l M_l}, \quad b = 1,2,....,v$$

(7)

where the stationary distribution vector, $[\pi_b]_{1xv}$ is a vector which satisfies the following system of equations:

$$[\pi_b] = [\pi_b][p_{bl}], \quad \sum_{l=1}^{v} \pi_1 = 1$$

(8)

Thus, having obtained the limiting probability values, the expected time spent in a particular operation state for sufficiently large operation time, θ, can be approximated by:

$$E[\hat{\theta}_b] = p_b \theta, \quad b = 1,2,....,v$$

(9)

3 COMPUTATIONAL TOOL FOR MODELLING INDUSTRIAL SYSTEMS OPERATIONS PROCESSES

Here, the computational tool for the general probabilistic model of industrial systems operation processes, described in the previous section, is presented. The basis on the development of the computational model is based on the number of states, v, that the problem needs to handle. When the number of states, v, is small, it is still possible to perform all the calculations analytically by hand, as undertaken by Kołowrocki (Guze S, et al 2008, Blokus-R et al 2008, Kołowrocki K. et al 2008). However, when the value of v is large, these calculations will be overly tedious and difficult, in particular the calculations associated with equation (8). Therefore, having a computational tool software to automate the tasks in Section 2 will undoubtedly assist in analyzing and solving the model, and hence the problem.

Due to the convenience in handling large matrices, the model is coded using a matrix-based numerical programming language. In performing this, various tools currently exists and have been explored namely Matlab, GNU Octave and R. In our implementation, the GNU Octave tool is employed since it is an open-source and free programming language. Furthermore, the code developed is mostly compatible with Matlab, a widely-used commercial software.

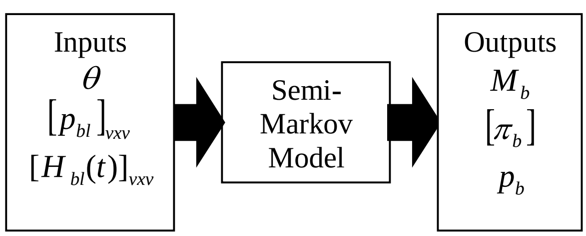

Figure 1. Block diagram of the general model of industrial systems operation processes.

Typically, the structure of a computer program is best described by its inputs, outputs and computation procedures or algorithms. In the case for general model of industrial systems operation processes considered, the inputs, outputs and model relationship can be explained by the block diagram illustrated in Figure 1. For brevity, since the model is based on the Semi-Markov process, the model is denoted as the Semi-Markov model.

From Figure 1, it can be seen that to generate the outputs, *i.e.* M_b, $[\pi_b]$, p_b, the adopted computer tool will take in the values of the conditional sojourn time, θ, the transition probability matrix, $[p_{bl}]_{vxv}$ and the conditional sojourn time distribution matrix, $[H_{bl}(t)]_{vxv}$. If the Semi-Markov model is considered as a black box, then the former 3 values will act as inputs in generating the 3 outputs.

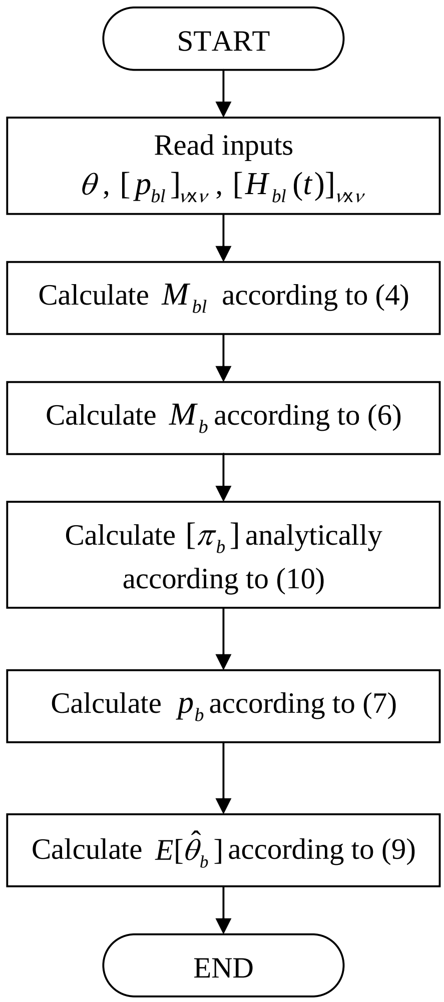

Figure 2. Flowchart of the Semi-Markov model computational procedures.

Figure 2 describes the details of the computational Semi-Markov model adopted in this paper. As shown in the flowchart, upon reading in the 3 inputs, the value of the integral, M_{bl}, needs to be evaluated. This evaluation is often difficult, especially if the density function of the conditional sojourn time distribution, $h_{bl}(t)$, is complex. Furthermore, some numerical programming languages such as Octave do not have the capabilities to handle such complex integrations. An alternative approach is to numerically approximate the mean value of M_{bl} by generating sufficiently large random samples from the corresponding distribution density function and then averaging their values. In our implementation, 100,000 samples were generated. Once this is done, the value of M_b was then evaluated.

The next stage is the most compute-intensive aspect of the calculations, which involves the evaluation of the stationary distribution vector, $[\pi_b]$. It can be seen from equation (8) that the evaluation of $[\pi_b]$ involves solving a series of linear equations, which is simple if the number of states is small. However, when the number of states considered in the problem is large, the solution process is rather complex and compute-intensive in nature. To overcome this, we propose the adoption of the following machine oriented approach. Our proposition involves letting $[P_{bl}]$ be a vxv irreducible transition probability matrix. Let us suppose I is a vxv identity matrix and ONE is a vxv matrix whose entries are all 1. This would lead to:

$$[\pi_b] = (1,...,1)(I - [p_{bl}] + ONE)^{-1} \tag{10}$$

The proof of the above proposition can be derived from equation (8), where upon simple re-arrangement leads to:

$$[\pi_b](I - [p_{bl}]) = 0 \tag{11}$$

Furthermore, from the system of equations hypotheses, it is known that $[\pi_b]$ must sum to 1. Thus, from equation (10), it leads to:

$$[\pi_b](I - [p_{bl}] + ONE) = [\pi_b]ONE = (1,...,1) \tag{12}$$

Furthermore, if we now assume that $(I - [p_{bl}] + ONE)$ has an inverse, thus solving for $[\pi_b]$ will yield equation (10) as desired. The adoption of this machine learning approach will assist in overcoming the compute-intensive nature of the calculations. Once the value of $[\pi_b]$ is obtained, as shown in the flow chart, the value of p_b can then be evaluated using equation (7). This will then lead to the value of $E[\hat{\theta}_b]$ from equation (9) to be computed.

4 APPLICATION ON A PORT OIL TRANSPORTATION SYSTEM PROCESS

The proposed computational tool described, in the previous section, is now applied on the case of analyzing the operation processes of the port oil transportation system in Dębogórze, Poland (Guze S. 2008). As shown in Figure 3, the process involves the transportation of liquid cargo to Dębogórze terminal from the pier in the Port of Gdynia. In our analysis, the considered system is composed of 3 stages, namely, the pier, the 3 terminal parts A, B and C and the 3 linked piping subsystems, S_1, S_2 and S_3. The breakdown of the subsystems is as follows:

- Subsystem S_1: Consists of 2 identical pipelines, each composed of 178 elements. In each pipeline, there are 176 pipe segments and 2 valves.
- Subsystem S_2: Consists of 2 identical pipelines, each composed of 719 elements. In each pipeline, there are 717 pipe segments and 2 valves.
- Subsystem S_3: Consists of 2 types 1 pipeline and 1 type 2 pipelines, each composed of 362 elements. In each of the type 1 pipeline, there are 360 pipe segments and 2 valves. In each of the type 2 pipeline, there are 360 pipe segments and 2 valves.

At the pier, the unloading of tankers is performed in the Port of Gdynia. The pier is connected with the terminal Part A through the transportation subsystem S_1. In terminal Part A, there is a supporting station to fortify the tankers' pumps, enabling the further transportation of the oil cargo to the terminal Part B through the subsystem S_2. The terminal Part B is further connected to the terminal Part *C* via the subsystem S_3. Finally, in the terminal Part *C*, the rail cisterns are unloaded with oil cargo products to be distributed to the rest of Poland.

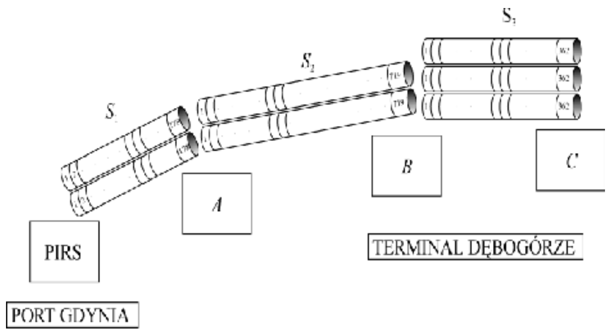

Figure 3. Schematic view of the Port Oil Transportation System.

Based on the operation processes taking place at the abovementioned system, the process, $Z(t)$, where $t \in <0,+\infty>$, is defined, with the operation states as follows:

- z_1: Transportation of 2 different medium types from terminal Part B through Part C using two out of three pipelines in subsystem S_3.
- z_2: Transportation of 1 medium type from terminal Part C through Part B using one out of three pipelines in subsystem S_3.
- z_3: Transportation of 1 medium type from terminal Part B through Part A to the pier using 1 out of 2 pipelines in subsystem S_2 and 1 out of 2 pipelines in subsystem S_1.
- z_4: Transportation of 2 medium types from the pier through Parts A and B to Part C using both pipelines in subsystem S_1, both pipelines in subsystem S_2 and 2 out of 3 pipelines in subsystem S_3.
- z_5: Transportation of 1 medium type from the pier through Part A and B to Part C using 1 out of 2 pipelines in subsystem S_1 and S_2 and 1 out of 3 pipelines in subsystem S_3.

Thus, using the 5 operation states described above, the computational procedures presented in Figure 2, can now be applied. In our analysis, due to the scarcity of historical data, the values of the transition probability matrix, $[P_{bl}]$ and the conditional sojourn time distribution matrix, $[H_{bl}(t)]$, were constructed by eliciting opinions from domain experts. This leads to the followings:

$$[H_{bl}(t)] = \begin{bmatrix} 0 & 0 & 0 & 1-e^{-37117.4t^2} & 0 \\ 0 & 0 & 1-e^{-191749t^2} & 0 & 0 \\ 1-e^{-1484695t^2} & 1-e^{-107737.1t^2} & 0 & 0 & 0 \\ 0 & 1-e^{9696341t^2} & 0 & 0 & 1-e^{-9696341t^2} \\ 0 & 0 & 0 & 1-e^{-29.1t^2} & 0 \end{bmatrix}$$

$$[P_{bl}] = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0.11 & 0.89 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

(13)

With the inputs given in equation (13), the values of the integral, M_{bl}, was then evaluated leading to the values of the unconditional sojourn time distribution function, $H_b(t)$, and its mean value M_b for $b = 1,2,....5$ to be given by:

Table 1. Values evaluated from the computational procedures.

$H_1(t) = 1 - \exp[-37117.4t^2]$	$M_1 = 0.005$
$H_2(t) = 1 - \exp[-19174.9t^2]$	$M_2 = 0.006$
$H_3(t) = 1 - 0.11 \cdot \exp[-148469.5t$ $- 0.89 \cdot \exp[-107737.1t^2]$	$M_3 = 0.003$
$H_4(t) = 1 - 0.5 \cdot \exp[-969634.1$ $- 0.5 \cdot \exp[-969634.1t^2]$	$M_4 = 0.001$
$H_5(t) = 1 - \exp[-29.1t^2]$	$M_5 = 0.164$

Here, the most compute-intensive aspect of the calculations, which involves evaluating the stationary distribution vector, $[\pi_b]$, is undertaken. Thus, using the computational procedures described in equation (10), leads to the values of the stationary distribution vector of the process to be given by:

$$[\pi_b] = \left[\frac{1}{22}, \frac{9}{22}, \frac{9}{22}, \frac{2}{22}, \frac{1}{22}\right]$$

(14)

This enables the limiting probability, P_b for $b = 1,2,....5$ to be evaluated, yielding,

$$[P_1 \quad P_2 \quad P_3 \quad P_4 \quad P_5] = [0.018 \quad 0.228 \quad 0.095 \quad 0.007 \quad 0.652]$$

(15)

Thus, from the above computations, the expected time spent in a particular operation state, $E[\hat{\theta}_b]$ for $b = 1,2,....5$, given $\theta = 365$ days, were then evaluated.

$$\lfloor E[\hat{\theta}_1] \quad E[\hat{\theta}_2] \quad E[\hat{\theta}_3] \quad E[\hat{\theta}_4] \quad E[\hat{\theta}_5] \rfloor = [6.6 \quad 83.2 \quad 34.7 \quad 2.6 \quad 238] \text{days} \qquad (16)$$

The computed values of $E[\hat{\theta}_b]$ were then compared with the analytical solutions evaluated by Kołowrocki (Guze S. 2008) showing very favourable comparisons. This provides a preliminary indication on the ability of the computational Semi-Markov model tool developed as well as its continued use to other maritime problems and possible extension to include the evaluation of reliability, availability and risk of the systems.

5 CONCLUSIONS

The paper has introduced a computational tool, based on the Semi-Markov model that can be used to analyze the general stochastic model of industrial systems operation processes. The application of the developed computational tool was then illustrated for the port oil transportation system. The results showed that the computational solutions matched well with the analytical calculations. This preliminary result from the developed computational tool showed the potential of the tools' practical usefulness in other operation process evaluations, especially under changing structures and characteristics. In the long term, the aim is for this computational tool to be extended to incorporate reliability and availability calculations as well as optimization modules, using large-scale data from the maritime industrial systems operations processes. The block diagram of the proposed process workflow is shown in Figure 3 below. The implementation and results of the extended computational tool will be published elsewhere.

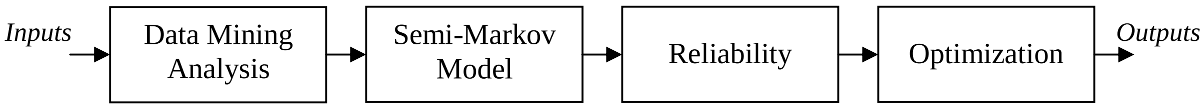

Figure 3. Block diagram of the proposed extended computational tool.

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REFERENCES

1. Guze S., Kołowrocki K., Soszyńska J. (2008), *Modelling environment and infrastructure influence on reliability and operation processes of port oil transportation system*, Proc. 2nd Summer Safety and Reliability Seminars (SSARS), Vol. 1, 179-186.
2. Blokus-Roszkowska A., Kołowrocki K. (2008), *Modelling environment and infrastructure of shipyard transportation systems and processes*, Proc. 2nd Summer Safety and Reliability Seminars (SSARS), Vol. 1, 77-84.
3. Pearl J. (1985), *Bayesian networks: A model of self-activated memory for evidential reasoning*, Proc. 7th Conference of the Cognitive Science Society, 329-334.
4. Trucco P., Cagno E., Ruggeri F., Grande O. (2008), *A bayesian belief network modeling of organizational factors in risk analysis: A case study in maritime transportation*, Reliability Engineering & System Safety, Vol 93, 845-856.
5. Norrington L., Quigley J., Russell A. and Van der Meer R. (2008), *Modelling the reliability of search and rescue operations with Bayesian Belief Network*, Reliability Engineering & System Safety, Vol 93, 940 – 949.
6. Eleye-Datubo A. G., Wall A. and Wang J. (2008), *Marine and Offshore Safety Assessment by Incorporative Risk Modeling in a Fuzzy-Bayesian Network of an Induced Mass Assignment Paradigm*, Risk Analysis, Vol 28, 95 – 112.
7. International Maritime Organization (IMO) (2002), *Guidelines for Formal Safety Assessment (FSA)*, IMO MSC/Circ 1023.
8. Ruud S., Age M. (2008), *Risk-based rules for crane safety systems*, Reliability Engineering & System Safety, Vol 93, 1369-1376.
9. Lee J., Yeo I., Yang Y. (2001), *A trial application of FSA methodology to the hatchway watertight integrity of bulk carriers*, Marine Structures, Vol 14, 651-667.

10. Wang J. (2002), *Offshore safety case approach and formal safety assessment of ships*, Journal of Safety Research, Vol 33, 81-115.
11. Hu S., Fang Q., Xia H., Xi Y. (2007), *Formal safety assessment based on relative risks model in ship navigation*, Reliability Engineering & System Safety, Vol 92, 369-377.
12. Wang J., Pillay A., Kwon Y. S., Wall A. D., Loughran C. G. (2005), *An analysis of fishing vessel accidents*, Accident Analysis & Prevention, Vol 37, 1019-1024.
13. Jin D., Thunberg E. (2005), *An analysis of fishing vessel accidents in fishing areas off the northeastern United States*, Safety Science, Vol 43, 523-540.
14. Heyman D. P., Sobel M. J. (2003), *Stochastic models in operations research*, Vol I: Stochastic processes and operating characteristics, Dover Publications.
15. Vinod G., Bidhar S. K., Kushwaha H. S., Verma A. K., Srividya A. (2003), *A comprehensive framework for evaluation of piping reliability due to erosion – corrosion for risk-informed in service inspection*, Reliability Engineering and System Safety, Vol 82, 187-193.
16. Felli J. C., Anderson W. H., Kremida J. P., Ruberg S. J. (2007), *A semi-Markov model for patient progression through clinical trials*, European Journal of Operation Research 176, 542-549.
17. Perman M., Senegacnik A., Tuma M. (1997), *Semi-Markov Models with an application to Power-Plant Reliability Analysis*, IEEE Transactions on Reliability, Vol 46, No 4.
18. Kołowrocki K., Soszyńska J. (2008), *A general model of industrial systems operation processes related to their environment and infrastructure*, Proc. 2nd Summer Safety and Reliability Seminars (SSARS), Vol. 2, 223-226.
19. Grabski F. (2002), *Semi-Markov Models of Systems Reliability and Operations*, Warsaw: Systems Research Institute, Polish Academy of Sciences.

SELECTED APPROACHES FOR RELIABILITY COMPARISON OF HIGHLY RELIABLE ITEMS

D. Valis, Z. Vintr

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University of Defence, Brno, Czech Republic

e-mail: david.valis@unob.cz, zdenek.vintr@unob.cz

M. Koucky

•

Technical University of Liberec, Liberec, Czech Republic

e-mail: miroslav.koucky@tul.cz

ABSTRACT

The application of electronic elements introduces a number of advantages as well as disadvantages. The paper deals with advanced method of dependability - reliability analysis procedure of a highly reliable item. The data on manufacturing and operating of a few hundred thousands pieces of the highly reliable devices are available and from the statistical point of view they are very important collection/set. However, concerning some pieces of the items the manufacturing procedure of them was not made, controlled and checked accurately. The procedure described in the paper is based on the thorough data analysis aiming at the operation and manufacturing of these electronic elements. As the data sets collected are statistically non-coherent the objective of the paper is to make a statistical assessment and evaluation of the results. Failure rates calculations and their relation comparability regarding the both sets are presented in the paper.

1 INTRODUCTION

As we know from previous publications the item is initialised by start power. We have also discovered from the previous publications that the reliability assessment of the items may highlight some mathematical non-coherence. The data sets which are available have different digits number therefore their comparability might be problematic. That is why the measures – failure rates calculated must be tested before claiming their comparability in terms of the functional description – characteristic of the item. All the terms used are in accordance with the IEC 60050/191.

The whole calculation has been made from the reason that unfortunately non-intentional causes resulted in non-compliance with the manufacturing process during development and manufacturing a new item. While manufacturing the item a relatively minor shortening of program protocol took place, thereby shortening the initialisation time. This situation resulted in the production of many of incorrectly manufactured items where the initialisation time was shortened by the program. The non-compliance with the manufacturing process was detected only by accident and that was after some time. However, most of the items manufactured this way have been mounted in systems and they have been in operation. In the paper we are going to address reliability assessment of a highly reliable electronic item.

In this paper the evaluated application of reliability data analysis techniques - procedure for comparison of two constant failure rates is perceived of an item produced for systems´ specific

use/utilization. Item is implemented in a system in order to control one of the step functions of the system.
Based on the assumptions and the calculation which have been made before, the reliability measure values for correctly and incorrectly programmed items were found. These values were calculated at the required confidence level. By comparing these values we were able to determine whether the error affects the item reliability during a manufacturing process.
However, concerning the field data we face a theoretical problem. The data set is apparently different concerning a digit place in terms of the operation time of the item sets. It means that correctly manufactured items obviously operate for a shorter time than the ones manufactured incorrectly. This situation can affect a calculation procedure as well as a comparison of the results. Taking into account this situation it is necessary to test the field data using the statistical test which is supposed to prove their comparability. The results of the test are mentioned in the second paper named "Statistical comparing of reliability of two sets of highly reliable items". For more details see e.g. Holub (1992) or Finn (1998).

2 APPLICATION OF A COMPARISON TECHNIQUE

In this case when taking into account two sets of objects we have to consider reliability measures where there is a presumption that the sets can be different. Time to failure is for both sets independent and fulfils the presumption of exponential distribution. For more details see IEC 60605-4, IEC 61 650 or Lipson & Sheth (1973).

It is necessary to introduce other important relations which are essential for next steps. Because it is a case of non-repaired items, we can assume that:

- accumulated operation/test time is calculated as a sum of times to failure;
- all the objects belong to the same original set.

In order to use the comparison procedures the following data are required:

- an observed number of valid failures r_1 a r_2 in two observing periods – it is fulfilled;
- accumulated valid test times T_i^* in these two periods – it is fulfilled;
- the confidence level should be stated/chosen if required;

All the information is available and it is possible to continue working with it.

Following the IEC 61 650 we choose the accurate calculation of two constant failure rates comparison using F – distribution. We calculate f using the equation (1).

$$f = \frac{r_2}{r_1 + 1} \times \frac{T_1^*}{T_2^*} \tag{1}$$

For the chosen confidence level we get the f_c (either for $1 - \alpha_0 = 0{,}90$ or for $1 - \alpha_0 = 0{,}95$) from the tables of F – distribution stated in the appendix A of the document EN 60812:2006.

$$f_c = F_{1-\alpha_0}(\nu_1, \nu_2) \tag{2}$$

where $\nu_1 = 2(r_1 + 1)$; $\nu_2 = 2r_2$.
Next we use the decision criteria given in the table 2 of the IEC 61650 stating that if $f > f_c$, then $w_1 < w_2$, or if $f < f_c$, then $w_1 = w_2$). Generally the recommended confidence level for calculation is α_0 = 5% or 10% which corresponds with $(1 - \alpha_0)$ – fractiles, that is 0,95 – fractiles or 0,90 – fractiles of F- distribution.

The calculation:

The calculation has been intentionally modified due to the industrial secret and due to not providing the sensitive data about the product. The confidence level was stated at 95%.

The mean time to failure is calculated according to the (1)

a) for incorrectly manufactured items:

$$m_{l\ F/C} = \frac{2.T^{*F/C}}{\chi^2_{\alpha,\nu}} = \frac{2.230\,995\,532\ h}{68{,}648}$$

$$m_{l\ F/C} = \frac{461\,991\,064\ h}{68{,}648} \cong 6{,}73\ .\ 10^6\ \text{h}$$

where accumulated operation time of all wrongly manufactured items according to the assumption given in IEC 61650, chapter 4, article 4, and according to the formula is $T^{*F} = \sum_{t=i}^{n} t_i^{\ F} =$ 230 995 532 h; a number of the degrees of freedom according to the formula (2) is ν = $2r^F + 1$= 2.25 + 1 = 51; the chi-square for 51 degrees of freedom and the confidence level α = 95% is 68,648.

b) for correctly manufactured items

$$m_{l\ F/C} = \frac{2.T^{*F/C}}{\chi^2_{\alpha,\nu}} = \frac{2.56\,864\,717\ h}{7{,}8}$$

$$m_{l\ F/C} = \frac{113\,729\,434\ h}{7{,}8} \cong 1{,}46\ .\ 10^7\ \text{h}$$

where accumulated operation time of all wrongly manufactured items according to the assumption given in IEC 6150, chapter 4, article 4, and according to the formula is $T^{*C} = \sum_{t=i}^{n} t_i^{\ C}$ = 56 864 717 h; a number of the degrees of freedom according to the formulae (2) is ν = $2r^C + 1$ = 2.1 + 1 = 3; the chi-square for 3 degrees of freedom and the confidence level α = 95% is 7,8.

The calculation of the *f* according to the formula (1)

$$f = \frac{r_2}{r_1+1} \times \frac{T_1^*}{T_2^*} = \frac{1}{25+1} \times \frac{230\,995\,532}{56\,864\,717}$$

$$f = 0{,}156$$

Next, the calculation of the f_c according to the formula (2)

$$f_c = F_{1-\alpha_0}(\nu_1, \nu_2) = 19{,}476$$

where $\nu_1 = 2(r_1 + 1) = 2(25+1) = 52$; $\nu_2 = 2r_2 = 2$

As the calculation introduced above and using this approach shows that $f < f_c$. Based onto the assumption of the *F*-distribution approach we can state that the failure rates of the basic sets $w_1 = w_2$, so they are constant.

3 WEIBULL REGRESS ANALYSIS APPROACH UTILISATION

Following approach is based onto Weibull regress model where the scale parameter is modelled using both two-parametric function and covariate.

We have to consider a random sample $(X_i, d_i, z_i), i = 1,....,n$, where X_i is time to failure, or time of censoring; d_i censoring indicator ($d_i = 1$, if X_i is time to failure or $d_i = 0$ if X_i is time of censoring); z_i is variable (so called covariate) having values:

z_i = 0, if X_i is time for item of *F*_type
z_i = 1, if X_i is time for item of *C*_type
Objective of the analysis is to state whether the difference in the technical life of the both types of items is significant from the statistical point of view.
The answer might be based onto the Weibull regress model where the scale parameter is modelled using both two-parametric function $\lambda(z,\beta) = \; = \exp(\beta_0 + \beta_1 z)$ and covariate *z*.
Let's assume that the *f* is the Weibull probability density function

$$f(t) = \begin{cases} \lambda \alpha t^{\alpha-1} \exp(-\lambda t^{\alpha}) & for\ t \ge 0 \\ 0 & everywhere \quad else \end{cases} \tag{3}$$

where $\lambda = \dfrac{1}{\theta^{\alpha}} > 0, \alpha > 0$ are Weibull distribution parameters (θ . scale parameter, α . shape parameter).
The Weibull reliability function is defined as follows:

$$R(t) = \begin{cases} \exp(-\lambda t^{\alpha}) & for\ t \ge 0 \\ 1 \quad everywhere \quad else \end{cases} \tag{4}$$

We assume that λ parameter is function of time for our application $\lambda(z,\beta) = \exp(\beta_0 + \beta_1 z)$, where $\beta = (\beta_0, \beta_1)$ is a vector of unknown parameters and *z* is variable (so called covariate) reaching two values in our case:
z = 0 for first type of sample (items of the F_type);
z = 1 for second type of sample (items of the C_type)
The probability density function might be stated in the following form than:

$$f(t;\beta,z) = \begin{cases} \exp(\beta_0 + \beta_1 z)\alpha t^{\alpha-1} \exp(\beta_0 + \beta_1 z) t^{\alpha}\ for\ t \ge 0 \\ \quad 0 \qquad everywhere \quad else \end{cases} \tag{5}$$

The reliability function can be expressed in following way:

$$R(t;\beta,z) = \begin{cases} \exp(-\exp(\beta_0 + \beta_1 z) t^{\alpha}\ for\ t \ge 0 \\ 1 \qquad everywhere \quad else \end{cases} \tag{6}$$

We use the method of maximal plausibility for unknown parameters estimation α, β in this regression model
Plausibility function is defined in the following form than:

$$L(y;\alpha,\beta) = \prod_{i=1}^{n} (f(y_i;\beta,z))^{d_i} (R(y_i;\beta,z))^{1-d_i}$$

The function will be expressed in following way after taking the logarithm of the function and regarding to the expressions of (5) and (6):

$$l(y;\alpha,\beta) = d \ln \alpha + \sum_{i=1}^{n} d_i((\alpha-1)y_i + \beta_0 + \beta_1 z_i) - \sum_{i=1}^{n} \exp(\alpha y_i + \beta_0 + \beta_1 z_i),$$

where $y_i = \ln t_i$ and $d = \sum_{i=1}^{n} d_i$

To find the maximally plausible estimation of the parameters α, β we have to create a system of partial differential equations. The system has presumably following form:

$$\frac{\partial l(y;\alpha,\beta)}{\partial \alpha} = \frac{d}{\alpha} + \sum_{i=1}^{n} d_i y_i - \sum_{i=1}^{n} y_i \exp(\alpha y_i + \beta_0 + \beta_1 z_i) = 0$$

$$\frac{\partial l(y;\alpha,\beta)}{\partial \beta_0} = d - \sum_{i=1}^{n} \exp(\alpha y_i + \beta_0 + \beta_1 z_i) = 0$$

$$\frac{\partial l(y;\alpha,\beta)}{\partial \beta_1} = \sum_{i=1}^{n} d_i z_i - \sum_{i=1}^{n} z_i \exp(\alpha y_i + \beta_0 + \beta_1 z_i) = 0$$

We get following estimations of the parameters applying the numerical calculation of above mentioned equations:

$$\hat{\alpha} = 1{,}2801; \hat{\beta}_0 = -17{,}8800; \hat{\beta}_1 = -1{,}9785 \quad (7)$$

Consequently we get the so called "information matrix" while conducting the second partial derivation of the plausible function logarithm:

$$\frac{\partial^2 l(y;\alpha,\beta)}{\partial\alpha^2} = -\frac{d}{\alpha^2} - \sum_{i=1}^{n} y_i^2 \exp(\alpha y_i + \beta_0 + \beta_1 z_i)$$

$$\frac{\partial^2 l(y;\alpha,\beta)}{\partial\beta_0^2} = -\sum_{i=1}^{n} \exp(\alpha y_i + \beta_0 + \beta_1 z_i)$$

$$\frac{\partial^2 l(y;\alpha,\beta)}{\partial\beta_1^2} = --\sum_{i=1}^{n} z_i^2 \exp(\alpha y_i + \beta_0 + \beta_1 z_i)$$

$$\frac{\partial^2 l(y;\alpha,\beta)}{\partial\beta_0 \partial\beta_1} = -\sum_{i=1}^{n} z_i \exp(\alpha y_i + \beta_0 + \beta_1 z_i)$$

$$\frac{\partial^2 l(y;\alpha,\beta)}{\partial\alpha \partial\beta_0} = -\sum_{i=1}^{n} y_i \exp(\alpha y_i + \beta_0 + \beta_1 z_i)$$

$$\frac{\partial^2 l(y;\alpha,\beta)}{\partial\alpha \partial\beta_1} = -\sum_{i=1}^{n} z_i y_i \exp(\alpha y_i + \beta_0 + \beta_1 z_i)$$

And finally we can also determine the standard deviations of our estimations for (7). We get at the end

$$\sigma(\hat{\alpha}) = 0{,}2369; \sigma(\hat{\beta}_0) = 1{,}5710; \sigma(\hat{\beta}_1) = 1{,}0075 \quad (8)$$

Now it is remarkable that the claim of un-existence of life time difference in terms of the both items types reliability (item_F, item_C) might be converted to hypothesis test:

$H_0 : \beta_1 = 0 \times H_1 : \beta_1 \neq 0$

(zero hypothesis equals to the goodness of fit).

The test statistic has the value of 3,856 if we use a statistical test based onto Neymann-Pearson lemma (Wald's test might be also used as an alternative but it goes to the same results). Such test is a classical statistical test based onto plausibility ratio – likelihood ratio test. Its test statistics has the following form $LR(\beta_1) = 2\left[l(y;\hat{\beta}_1) - l(y;0)\right]$ and the χ_1^2 is asymptotically distributed. The value $l(y;....)$ is value of logarithm plausibility function. The Wald's test is in our case based onto following test statistics $\frac{(\hat{\beta}_1)^2}{\mathrm{var}(\hat{\beta}_1)}$ which has also asymptotic distribution of the χ_1^2.

The result got by the Neymann-Pearson lemma calculated on the confidence level 0,05 leads to rejection of the H_0 hypothesis. Therefore we can claim that the statistical difference between both item types is significant.

4 RISK ANALYSIS RESULTING FROM THE FAILURE OCCURRENCE – FUZZY APPROACH

The description of the item behaviour presented above indicates some possible situations. Such item behaviour might cause a failure occurrence with all possible consequences. We need to assess both the potential of such situation occurrence and the consequences. Risk assessment is on of suitable tools which might be used for this purpose.

In this phase of observing and assessing the objects we are talking about possibilities of risk characteristics assessment. Since we know the item failure probability can be stated using the approaches above. Than we need to assess the consequences of the failure occurrence which is next fragment of risk (as stated in the usual form). The detection possibility is also about to be stated but is recommended to use the approaches mentioned in standards (e.g. IEC 60605-4).
Total risk number might be calculated either by common approaches or by another, non-traditional - soft, method. One of such method might be also fuzzy logic.
Let us assume that any technical object in any instant of time can occur in any operational state (operational condition, failure state or partially failure state – functionality is limited, but not lost). A transfer between these states is subject to stochastic laws. As suitable means to depict transfers between individual operational states is use a theory of Markov processes. However, we shall not deal with a description of transfers between individual operational states. A greater attention will be paid to mathematical modelling of effects related to a transfer between individual states.
As transfers between states are connected with a number of effects, it is very important to deal with them in more detail. The most important and from the respect of the function of the object also the most critical is a transfer from an operational state into a fault (using hardware approach). This transfer can result in the worst effects. However, it will depend what is the mechanism of a transfer. If a transfer is caused by a scheduled downtime of the object because of the preventive maintenance, it is unpleasant matter, but better than if, for example, a transfer caused by an unexpected failure with devastating results.
To evaluate severity of effects of failures of technical objects, we decided to use fuzzy set theory 0**.** Since this theory uses vague terms that already appear in classification of severity of failure effects, then a decision on acceptability of failure and determination on the importance of the object on which the failure appeared. Simultaneously, it is possible using this theory to assign numerical value to the studied circumstance and thus we consider it suitable. Through this theory it is also possible to include severities of failure effects D of single objects into a fuzzy set**.** Here, we shall assume that single fuzzy sub-sets consist of coefficients of failure effect severity. Based on the seriousness of these effects it will be later determined to what level are the given groups indispensable. To classify the failure effect criticality in relation to the inherent availability of technical object we have selected the following three criteria of influence on:
Function - D_1,
Safety - D_2,
Recovery-related costs - D_3.
For every of these criteria we created an ascending scale of coefficients to enable to assess a seriousness of possible effects of failure related to the individual criteria. The scale is determined by a set I with four elements $I \in \{1;2;3;4\}$, while a value of coefficient of individual effect of failure in relation to selected criteria is denoted D_i, where $i \in <1,2,3>$. The principle is that with an increasing value of coefficient increases also a severity of effect. These values serve as the basis to express a severity of failure effect D. Scales of severity criteria are in the tables 1 – 3. For more details see for example Valis & Vintr (2006), Novák (1999) or Driankov & Hellendorn & Reifrank (1993).

Tab. 1 – Categorization of failures from the viewpoint of effects on the system functionality

Definition	Coefficient of significance D_1
Even after a failure, a system is capable to fulfil all required functions.	1
A failure partially limits an ability of the system to perform a required function, but the crew can cope with the impacts.	2

A failure significantly limits an ability of the system to perform some of required functions and the crew is not capable cope with the impacts of failure with its own force.	3
A failure prevents a system to fulfil the required functions.	4

Tab. 2 – Categorization of failures from the viewpoint of safety of the system

Definition	Coefficient of significance D_2
A failure has no effect on a safety of the system, crew and environment.	1
A failure results in a lowering of safety of the system, crew and environment.	2
A failure causes a situation when the system is dangerous for the system, crew and environment.	3
A failure results in a direct threat of health and lives of people or great losses of property.	4

Tab. 3 – Categorization of failures from the viewpoint of repair cista

Definition	Coefficient of significance D_3
Removal of failure effects does not require costs higher than 0.1 % of the system purchase costs.	1
Removal of failure effects does not require costs higher than 1 % of the system purchase costs.	2
Removal of failure effects does not require costs higher than 10 % of the system purchase costs.	3
Removal of failure effects requires costs higher than 10 % of the system purchase costs.	4

The resulting coefficient D is at the same time a coefficient of seriousness of a given object and a relation expresses it:

$$D = D_1 \,.\, D_2 \,.\, D_3; \qquad D_{min} = 1,\ D_{max} = 64. \qquad (9)$$

To construct a fuzzy sub-set, a “fuzzification of values” is used. Actual observed values of physical values are bounded and are expressed by means of real numbers. Therefore as a universum of fuzzy numbers that represent vague concepts related with a classification of failure effects, a suitable closed interval for every of them will be sufficient. We will reach single classes of failure effects (seriousness) by dividing the resulting coefficient D into suitable sub-intervals (see above). For practical use and graphical representation a trapezoidal fuzzy number is suitable, see Figure 1, where μ expresses a function of applicability and x obtained fuzzy number.

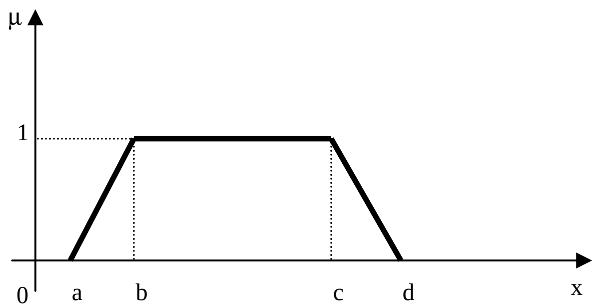

Figure 1. Example of fuzzy trapezoidal number

To determine the actual functions of applicability for fuzzified value of selected value it is enough to identify in what interval this value usually occurs. This interval is then a core of found fuzzy number and we denote it ⟨b,c⟩. For a demonstrated example, this core is always expressed by limit values of individual coefficients of significance of failures. Further, it is determined what values a variable certainly does not assume. A set of these values we assume to be expressed as (-∞;a) ∪ (d;∞), while a < b ≤ c < d. Then an interval ⟨a;d⟩ is a support-set „A" of found fuzzy number.
A function of applicability of found fuzzy number into a set „A" we express as follows:

$$\mu_A(x) = max\left(min\left(\frac{x-a}{b-a}, \frac{x-d}{c-d}, 1 \right), 0 \right). \tag{10}$$

For another procedure, it is necessary to determine individual fuzzy sets and based on them perform final categorization of failure effects. For this purpose, a four-level categorization of the failure effects recommended in many international standards, is used:

Minor: assigned fuzzy set ⟨1;4⟩;
Major: assigned fuzzy set ⟨6;16⟩;
Critical: assigned fuzzy set ⟨18;36⟩;
Catastrophic: assigned fuzzy set ⟨48;64⟩.

Figure 2 graphically represents an applicability of severity of effects of individual failures into fuzzy sets.

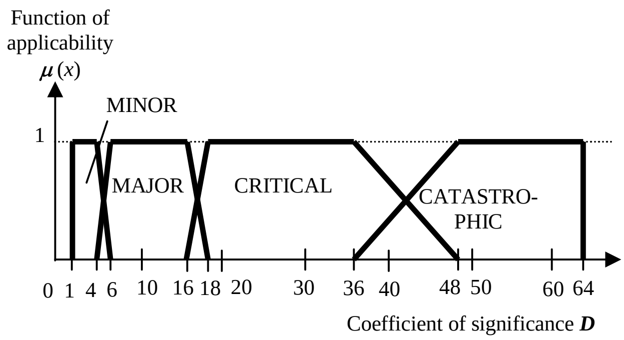

Figure 2. Graphical model of fuzzy sets for evaluation of severity of failure effects

A failure occurrence might have various consequences. Speaking about the consequences impacts in our case of the highly reliable electronic item implemented inside a complex system. Therefore the precise and adequate failure profile has to be determined in the risk analysis. The procedures described above might serve to express the total risk number consisting from the well known form:

$$R = P \times C \tag{11}$$

where P – is the probability value; and C – is the value of consequences.
The additional index of the detection might be also applied but we would recommend to follow standards like IEC 60605-4 to handle with this characteristic for risk assessment procedures.

5 CONCLUSION

The procedure as described above was used to calculate and compare the reliability measures – failure rates in this case of the single sets which served as correctly and incorrectly made electronic items. Following the obtained results a possible effect of a manufacturing error upon the items reliability was estimated. As we can see from the results although the data sets are different – they have different size of the information which they contain – we need to compare them.

Consequently we need to state if the results in the form of the failure rate are comparable and statistically same. These claims can prove the dependability of the product and finally safe the good name of the company producing a valuable goods. This fact should be referred to when carrying out statistical data evaluation using the introduced tools.
The above-mentioned ad-hoc procedure was designed as a tool to provide assessment of the effects/failure occurrence of the use of vetronics elements on the total system's dependability. This method assumes that an assessment of the effects of failures of individual subsystems will be done in a described way, at first without the vetronics components and then with the vetronics components. Based on a comparison of results of both analyses it can be assessed whether, and in what extent, vetronics can influence a dependability of the system. Finally, this method also enables to assess and to state the importance level of the individual components and subsystems from the viewpoint of capability of the system to perform required functions. It also provides to involve other criteria of evaluation such as for example security robustness or corrective maintenance costs.

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REFERENCES

IEC 60050/191 1990. International Electrotechnical Vocabulary - Part 191: Quality and dependability of services.
IEC 60605-4 Equipment reliability testing - Part 4: Statistical procedures for exponential distribution - Point estimates, confidence intervals, prediction intervals and tolerance intervals.
EN 60812:2006 Analysis techniques for system reliability - Procedure for failure mode and effects analysis (FMEA).
IEC 61650 Reliability data analysis techniques - Procedures for comparison of two constant failure rates and two constant failure (event) intensities.
Lipson, Ch., Sheth, N. J. 1973. *Statistical Design and Analysis of Engineering Experiments*; New York: Mc Graw Hill.
Holub, R. 1992. *Dependability tests (stochastic methods)*. Brno: Military Academy.
Novák, V. 1999. *Mathematical principles of fuzzy logic*, Boston: Kluwer.
Finn, J. 1998. *Electronic component reliability*, Chichester [UK]: John Wiley & Sons Ltd.
Driankov, D., Hellendoorn, H., Reifrank, M. 1993. *An introduction to fuzzy control*, Berlin: Springer-Verlag.
Vališ, D., Vintr, Z. 2006. *Dependability of Mechatronics Systems in Military Vehicle Design*. In: *Proceedings of the European Safety and Reliability Conference "ESREL 2006"*, London/Leiden/New York/Philadelphia/Singapore: Taylor&Francis Group, pp. 1703 – 1707.

MEAN TIME TO FAILURE FOR PERIODIC FAILURE RATE

Christian Tanguy
•
Orange Labs, CORE/TPN, Issy-les-Moulineaux, France

e-mail: christian.tanguy@orange-ftgroup.com

ABSTRACT

The paper is concerned with the determination of the Mean Time To Failure (MTTF) in configurations where the failure rate is periodical. After solving two configurations exactly, we show that when the period of the failure rate oscillations is small with respect to the average failure rate, the MTTF is essentially given by the inverse of the average failure rate, give or take corrections that can be expressed analytically. This could be helpful in the description of systems the environment of which is subject to changes.

1 INTRODUCTION

The occurrence of failures in systems is often described by using well-known distributions (see for instance (Kuo and Zuo 2003, Pham 2006, Rausand and Høyland 2004)) such as exponential, Weibull, etc. However, most of these distributions have associated failure rates which are constant or monotonous. This cannot realistically describe many real-life situations. To quite but a few examples, the probability of hurricanes is highest during the ‘‘right” seasons, computing systems exhibit different level activities during the day (there is also a weekly dependence because of week-ends). The efficiency of cooling units for electronic equipments in telecommunication networks depends on the ambient temperature; problems may arise in summer.

Quite naturally, the possibility of the periodicity of the failure rate has been raised. Castillo and Sieworek (1981) have considered the reliability of computing systems, and presented several data, clearly showing that hard disk failures seem to follow the workload. The influence of this workload can be taken into account quite satisfactorily by the addition of a (periodical) failure rate. A few fundamental, mathematical studies have also been devoted to the issue of periodic random environment (Dimitrov, Chukova, and Green 1997, Prakasa Rao 1997), the emphasis being laid on time distributions, nonstationary Poisson processes and other probability properties such as the ‘‘almost lack of memory”. Semi-Markov processes have also been used to model failure rates; a beautiful analytic expression has been found for the reliability in the case of a Furry-Yule process (Grabski 2002). More practical consideration emerge again, as witnessed by recent work on high-performance computing systems such as grids (Kang and Grimshaw 2007, Schroeder and Gibson 2006). To quote Kang and Grimshaw (2007)

Accurate failure prediction in Grids is critical for reasoning about QoS guarantees such as job completion time and availability

Another recent practical paper (Andrews 2005) considers a problem that could (somehow) ring a bell to all of us: what is the life expectancy of our mobile phones? In these electronic devices, the temperature of specific part of the circuits may substantially increase during operations such as finding the next antenna, working in conditions of huge traffic. It has been recognized for many decades that some processes ultimately responsible for hardware failures in electronic components have a temperature dependence which obeys the Arrhenius law (Baker 1972), used in many acceleration life test procedures. While the universality of this law is to be considered very carefully, there is no doubt that even a small increase in temperature may lead to surges in the failure rate. Should we consider the worst-case (meaning: temperature) scenario, or the most-of-the-time situation, knowing that these two hypotheses lead to mean times to failure (MTTF) differing by orders of magnitude? A review of the potential problems linked to temperature can be found in (Parry, Rantala, and Lasance 2002).

For this reason, we have tried to answer the following question: is there some way to perform a quick and not so dirty evaluation of the MTTF of a System subject to periodic failure? What are the important parameters?

Our paper is organized as follows: in section 2, we recall the well-known general expressions for the reliability and the MTTF, and compute the latter in two exactly solvable cases: in the first one, the failure rate takes two possible (constant) values; in the second one, we add an sinusoidal contribution to an otherwise constant failure rate. We show that when the oscillation period *T* of the added failure process is small compared with the otherwise expected lifetime, what really matters is merely the averaged failure rate $\overline{\lambda}$ over one period *T* (see equation (3) below). We confirm in section 3 this assertion in the general case, and provide the corrections to this asymptotic result in equation (4); a visual interpretation of the result is also provided. We conclude by a brief discussion of possible extensions of this work.

2 TWO EXACTLY SOLVABLE CASES

2.1 Link between MTTF and reliability

The reliability may be written quite generally as

$$R(t) = e^{-\int_0^t \lambda(\tau)\, d\tau}, \tag{1}$$

and the MTTF is given by

$$MTTF = \int_0^\infty t\,(-R'(t))\, dt = \int_0^\infty R(t)\, dt \quad . \tag{2}$$

Let us now turn to two cases where the MTTF may be exactly computed.

2.2 Bimodal failure rate

We assume that the failure rate λ takes two values: λ_+ if $0 < t < \alpha T$, and λ_- if $\alpha T < t < T$ (see *Figure 1*). After considering the successive intervals [n T, (n+α) T] and [(n+α) T, (n+1) T], and summing the easy to integrate exponentials, we eventually get

$$MTTF = \frac{1}{\lambda_-} + \left(\frac{1}{\lambda_+} - \frac{1}{\lambda_-} \right) \frac{1 - e^{-\alpha \lambda_+ T}}{1 - e^{-(\alpha \lambda_+ + (1-\alpha)\lambda_-)T}}$$

This relatively cumbersome expression of λ_+, λ_-, and T is actually very simple when considered in the $T \to 0$ limit, that is when the period of the oscillation is small compared with λ_+ and λ_-. We obtain

$$MTTF \to \frac{1}{\lambda_-} + \left(\frac{1}{\lambda_+} - \frac{1}{\lambda_-}\right)\frac{\alpha\,\lambda_+}{(\alpha\,\lambda_+ + (1-\alpha)\,\lambda_-)} = \frac{1}{\alpha\,\lambda_+ + (1-\alpha)\,\lambda_-}$$

which is nothing but the inverse of the average of the failure rate in the time period [0,T].

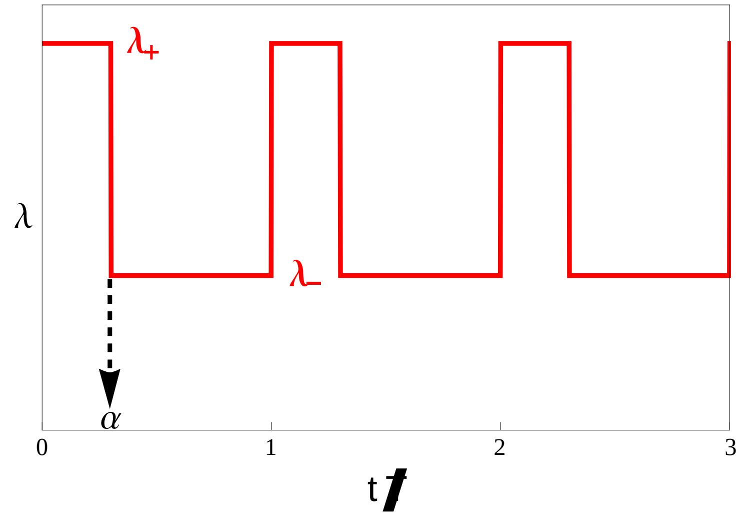

Figure 1. Simple variation of the failure rate.

Following a suggestion of Prof. O. Hryniewicz, we have also considered the case in which the system still spends a time $\alpha\ T$ in the "λ_+" state and $(1 - \alpha)\ T$ in the "λ_-" state, but when the occurrence of the "λ_-" state appears randomly at times $t_i + i\ T$ during the ith period $[i\ T, (i+1)\ T]$ $(0 < t_i < T$ for all i's). The calculations are straightforward and give

$$MTTF = \frac{1}{\lambda_+} + \left(\frac{1}{\lambda_-} - \frac{1}{\lambda_+}\right)\frac{1-e^{-(1-\alpha)\,\lambda_-\,T}}{1-e^{-(\alpha\,\lambda_+ + (1-\alpha)\,\lambda_-)T}}\ \left\langle e^{-\lambda_+\,t_i}\right\rangle,$$

where $\left\langle e^{-\lambda_+\,t_i}\right\rangle$ denotes the statistical average on the t_i's and closely looks like the moment generating function of their distribution. When t_i is always equal to $\alpha\ T$, we recover the result mentioned previously. $\left\langle e^{-\lambda_+\,t_i}\right\rangle$ may of course be larger than $e^{-\lambda_+\,\alpha\,T}$, and the MTTF is correspondingly modified. However, the asymptotic $T \to 0$ limit remains the same.

2.3 Sinusoidal failure rate

We now assume that the failure rate is given by $\lambda(t) = \lambda_0 + \lambda_1 \cos\omega t$ (see *Figure 2*), so that

$$R(t) = \exp\left(-\lambda_0\,t - \frac{\lambda_1}{\omega}\sin\omega t\right)$$

($T = \frac{2\pi}{\omega}$ is the period of the failure rate oscillations). The gist of the MTTF calculation is to expand the factor $\exp\left(-\frac{\lambda_1}{\omega}\sin\omega t\right)$ in the integral as a power series in λ_1. Each contribution is then (somewhat tediously) assessed. After some work, it is possible to show that even powers of λ_1 contribute to an hypergeometric function ${}_1F_2$ defined by

$$ {}_1F_2(\alpha;\beta,\gamma;z) = \sum_{n=0}^{\infty} \frac{\Gamma(n+\alpha)\Gamma(\beta)\Gamma(\gamma)}{\Gamma(\alpha)\Gamma(n+\beta)\Gamma(n+\gamma)} \frac{z^n}{n!} $$

where $\Gamma(z)$ is the Euler gamma function. A similar conclusion is reached for the odd powers of λ_1.

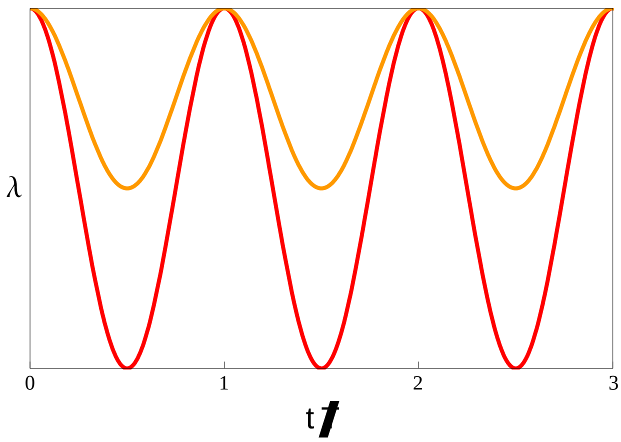

Figure 2. Sinusoidal variations of the failure rate: $\lambda_1 = \lambda_0$ (red) and $\lambda_1 = \lambda_0 / 3$ (orange).

Finally, we obtain (i is such that i^2 = -1)

$$ MTTF = \frac{1}{\lambda_0}\left({}_1F_2\left(1;1-\frac{i\lambda_0}{2\omega},1+\frac{i\lambda_0}{2\omega},\frac{\lambda_1^2}{4\omega^2}\right) - \frac{\lambda_0\lambda_1}{\lambda_0^2+\omega^2}\; {}_1F_2\left(1;\frac{3}{2}-\frac{i\lambda_0}{2\omega},\frac{3}{2}+\frac{i\lambda_0}{2\omega},\frac{\lambda_1^2}{4\omega^2}\right)\right) $$

The prefactor $1/\lambda_0$ indicates that the MTTF will be linked to the inverse of the ‘‘average” failure rate. It is indeed the exact result when $\lambda_1 = 0$, as expected. However, when $\lambda_1 > 0$, there are corrections to the simple result $1/\lambda_0$. If we assume that the two failure rates λ_0 and λ_1 are small compared with respect to ω, keeping the first two orders of the expansion, we find, expanding the hypergeometric functions ${}_1F_2$

$$ MTTF \approx \frac{1}{\lambda_0}\left(1+\frac{\lambda_1^2}{\lambda_0^2+4\omega^2}+\ldots-\frac{\lambda_0\lambda_1}{\lambda_0^2+\omega^2}-\ldots\right) $$

or

$$ MTTF \approx \frac{1}{\lambda_0}\left(1-\frac{\lambda_0\lambda_1}{\omega^2}+\frac{\lambda_1^2}{4\omega^2}\right) $$

We have displayed in Figure 3 the value of the MTTF as a function of ω, for two different values of (λ_0, λ_1), using λ_0 as a scaling parameter. We see that both curves are monotonous and that the asymptotic limits are quickly reached after initial, steep increases.

When the period of the oscillations is large, we would expect the MTTF to be the inverse of the ‘‘initial” failure rate, i.e., $\frac{1}{\lambda_0 + \lambda_1}$. This is indeed observed in Figure 3 when $\omega \to 0$. Similar curves could be drawn for higher moments of the failure time distribution. It should be noted, however, that the variation of the average of t^2 is not necessarily monotonous anymore, as shown in Figure 4.

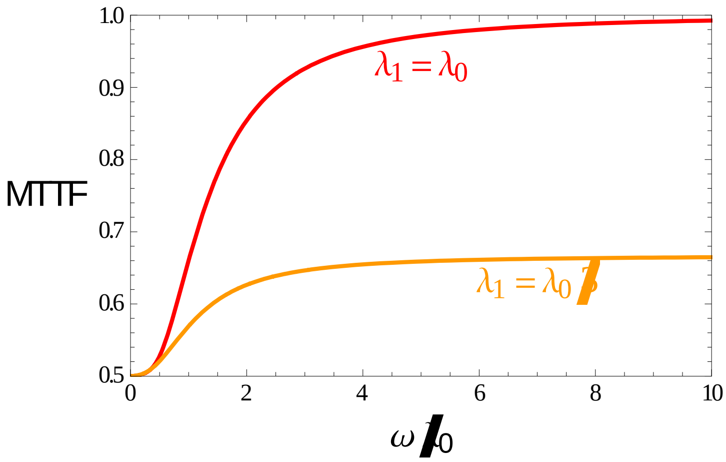

Figure 3. MTTF as a function of $\omega = 2\pi / T$ for the configuration of Figure 2 ($\lambda_0 + \lambda_1 = 2$).

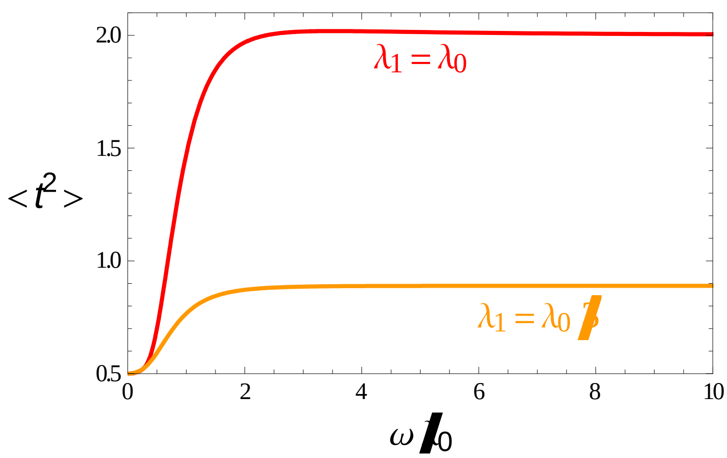

Figure 4. Same as Figure 2 for the average value of t².

3 GENERAL CASE WHEN THE OSCILLATION PERIOD IS SMALL

It may be satisfying to obtain an analytical solution to a few configurations, but this, unfortunately, is not true in general. The question is now to establish whether the MTTF may be evaluated by averaging a few quantities, and if so, the result is not too inaccurate. Recall that in many real situations, the period of the oscillations may be one day or one week — one year or more in the context of climatologic studies — and therefore much shorter than expected failure times.

3.1 Calculation

Let us now consider the general case when the oscillation period is *T*. We can define an average failure rate

$$\overline{\lambda} = \frac{1}{T} \int_0^T \lambda(t)\, dt \ . \tag{3}$$

Going back to the expression of the MTTF, we see that

$$\int_0^t \lambda(\tau)\, d\tau = \overline{\lambda}\, T \left\lfloor \frac{t}{T} \right\rfloor + \int_{T\left\lfloor \frac{t}{T} \right\rfloor}^t \lambda(\tau)\, d\tau = \overline{\lambda}\, t + \int_{T\left\lfloor \frac{t}{T} \right\rfloor}^t (\lambda(\tau) - \overline{\lambda})\, d\tau \ ,$$

where $\lfloor x \rfloor$ is the integer part of *x*. Turning now to the MTTF expression (see (1) and (2)), we have

$$\begin{aligned} MTTF &= \sum_{n=0}^{\infty} \int_{nT}^{(n+1)T} \exp\left(-\overline{\lambda}\, nT - \int_{nT}^t \lambda(\tau)\, d\tau \right) dt = \sum_{n=0}^{\infty} \exp\left(-n \overline{\lambda}\, T \right) \int_0^T \exp\left(-\int_0^t \lambda(\tau)\, d\tau \right) dt \\ &= \frac{\int_0^T \exp\left(-\int_0^t \lambda(\tau)\, d\tau \right) dt}{1 - \exp\left(-\overline{\lambda}\, T \right)} = \frac{N}{D} \end{aligned}$$

When the oscillation period is small ($T \to 0$, or the failure rate is assumed to be too small to matter during *T*), the expressions of the numerator *N* and denominator *D* may be expanded. Up to second order, we get

$$N = \int_0^T \left(1 - \int_0^t \lambda(\tau)\, d\tau + \frac{1}{2} \left(\int_0^t \lambda(\tau)\, d\tau \right)^2 - \cdots \right) dt = T \left(1 - \frac{1}{T} \int_0^T \int_0^t \lambda(\tau)\, d\tau\, dt + \frac{1}{2T} \int_0^T \left(\int_0^t \lambda(\tau) d\tau \right)^2 dt + \cdots \right),$$

while

$$D = \overline{\lambda}\, T \left(1 - \frac{1}{2} \overline{\lambda}\, T + \frac{1}{6} \left(\overline{\lambda}\, T \right)^2 + \cdots \right)$$

From the expressions of *N* and *D*, the leading order for the MTTF's expansion gives $MTTF \approx \frac{1}{\overline{\lambda}}$.

Using integrations by parts and $\tilde{\lambda}(t) = \lambda(t) - \overline{\lambda}$, we can easily obtain the corrections to the $T \to 0$ limit. After simplification, they give the main result of this paper

$$MTTF \approx \frac{1}{\overline{\lambda}} \left(1 + \frac{1}{T} \int_0^T t \tilde{\lambda}(t)\, dt + \frac{1}{2T} \int_0^T \left(\int_0^t \tilde{\lambda}(\tau) d\tau \right)^2 dt - \frac{\overline{\lambda}}{2T} \int_0^T \tilde{\lambda}(t) \left(t - \frac{T}{2} \right)^2 dt + \cdots \right) \tag{4}$$

Depending on the actual form of $\tilde{\lambda}(t)$, the first-order correction $\int_0^T t \tilde{\lambda}(t)\, dt$ may cancel (this is actually the case in Example 2.3, where the corrections to 1 are 0, $\frac{\lambda_1^2}{4\omega^2}$, and $-\frac{\lambda_0\, \lambda_1}{\omega^2}$, respectively) or be finite (in Example 2.2., it is $-\frac{\alpha(1-\alpha)}{2} (\lambda_+ - \lambda_-) T$, in agreement with the expansion of the exact result).

3.2 Visual interpretation

It is possible to understand visually why $MTTF \approx \frac{1}{\bar{\lambda}}$. We can indeed plot the ‘‘effective” reliability corresponding to the exponential distribution characterized by the average failure rate $\bar{\lambda}$, which is displayed on the left of Figure 5. As a reminder of the result of equation (2), we have shaded the area under the curve, which is nothing but the MTTF. Because of an added, periodic contribution, the true reliability $R(t)$ oscillates around $\exp\left(-\bar{\lambda}\, t\right)$ (see the right of Figure 5: the green-shaded areas correspond to a MTTF increase, while the red ones correspond to a MTTF decrease). If the oscillation period is short with respect to $1/\bar{\lambda}$, we expect the plus or minus contributions to compensate. It might nonetheless be very difficult to use such a graphical construction to provide upper or lower bounds for the MTTF in the general case.

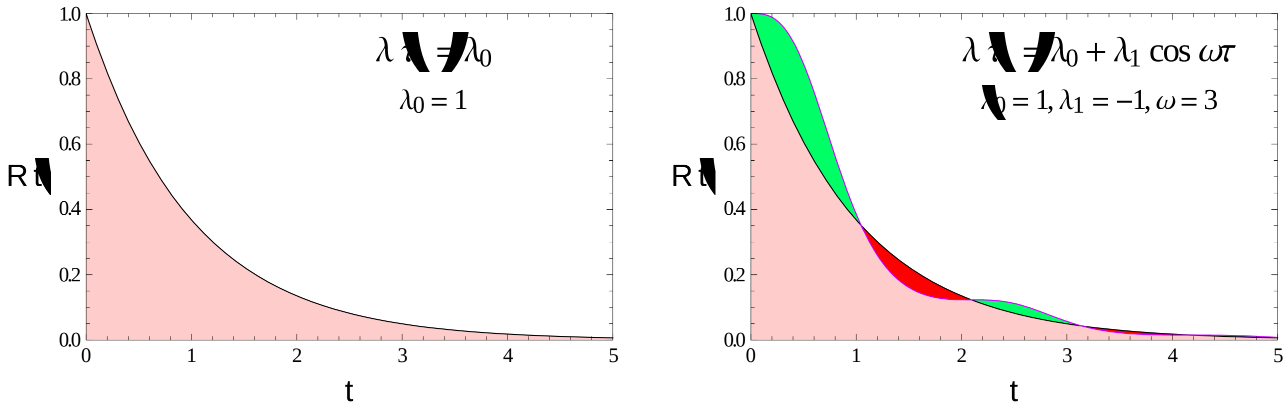

Figure 5. Comparison between the MTTFs for two different failure rates: their difference is provided by the sum of all green areas minus the sum of red areas.

4 WHAT ABOUT TEMPERATURE EFFECTS?

We mentioned in the Introduction that temperature is an important issue in the reliability of electronic components. Some data on the failure rates may be found in hardware catalogs, in operating condition (at a given temperature, mainly 20 or 25 °C). In some cases, estimates of the failure rate at *higher* temperatures are also given. It might therefore be more suitable to express the instantaneous failure rate as $\lambda(\mathrm{T}(t))$. Assuming that the Arrhenius law is valid *for a given physical process* we would have

$$\lambda(T) \;\propto\; e^{-\frac{E_a}{k\mathrm{T}}},$$

where E_a is the activation energy of the process, k the Boltzmann constant and T the temperature. The calculations of the preceding sections would have to take this further cause of variation into account. We would expect the MTTF to be weighted by the times spent in the higher temperature regimes.

5 CONCLUSION AND OUTLOOK

We have provided simple analytical results for the MTTF with a periodical failure rate, which may prove helpful when evaluating the lifetime of various kinds of components operating in environments for which the workload may induce failures to occur in a periodic manner.

Generalizations of the present results would of course include the assessment of the variation of the MTTF, when the initial distribution is not exponential, but a more realistic one.

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REFERENCES

Andrews, C. 2005 The next generation reliability standard for PAs. *Wireless Design & Development*, 1 November 2005.
Baker, E. 1972 Some effects of temperature on material properties and device reliability. *IEEE Trans. on Parts, Hybrids, and Packaging* PHP-8 (4), 4–14.
Castillo, X. & Siewiorek, D. P. 1981. Workload, performance, and reliability of digital computing systems. *Proc. of FTCS 11*, 84–89.
Dimitrov, B., Chukova, S. & Green D., Jr. 1997 Probability distributions in periodic random environment and their applications. *SIAM J. Appl. Math.* 57 (2), 501–517, and references therein.
Grabski, F. 2002 Semi-Markov models of systems reliability and operations. Systems Research Institute, Polish Academy of Sciences, Warsaw.
Kang, W. & Grimshaw, A. 2007 Failure prediction in computational grids. *Proc. Of the 40th Annual Simulation Symposium (ANSS'07)* 6, 93–116.
Kuo, W. & Zuo, M. J., 2003 *Optimal Reliability Modeling*. Hoboken: John Wiley & Sons, Inc.
Parry, John D., Rantala, J. & Lasance, C. J. M. 2002 Enhanced Electronic System Reliability—Challenges for Temperature Prediction *IEEE Trans. on Components and Packaging Technologies* 25 (4), 533–538.
Pham, H. 2006 Basic Statistical Concepts, in *Handbook of Engineering Statistics*, Pham, H. ed. Springer, New York.
Prakasa Rao, B. L. S. 1997 On distributions with periodic failure rate and related inference problems, in *Advances in Statistical Decision Theory and Applications*, Panchapakesan, S., Balakrishnan, N. & Gupta, S. S., eds, Birkhaüser, Boston, chapter 22, and references therein
Rausand, M. & Høyland, A. 2004 *System Reliability Theory*, 2nd edition. Hoboken: John Wiley & Sons, Inc.
Schroeder, B. & Gibson, G. A. 2006 A large-scale study of failures in high-performance computing systems. *Proc. of the International Conference on Dependable Systems and Networks (DSN2006)*, Philadelphia, USA, June 25-28, 2006.

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