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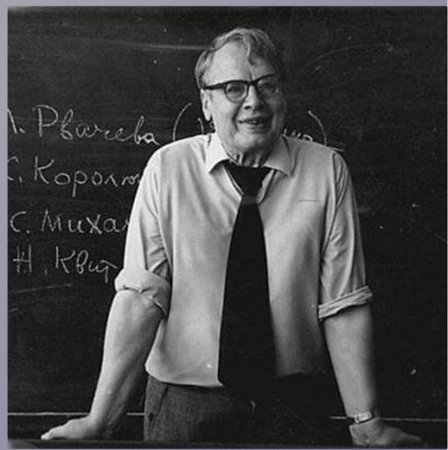

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RELIABILITY:
THEORY&APPLICATIONS

RELIABILITY
RISK ANALYSIS
MAINTENANCE
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Special Issue

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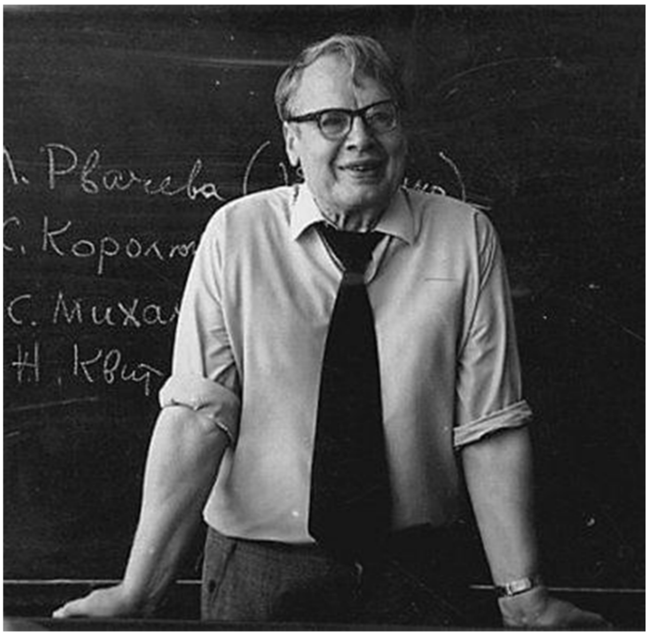
Journal of International Group on Reliability





RELIABILITY:

THEORY & APPLICATIONS

Vol.2 No.4 (23),
December, 2011

Special Issue

100th anniversary of Boris Vladimirovich
Gnedenko's birthday

San Diego
2011

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THE 100TH ANNIVERSARY OF BORIS GNEDENKO BIRTHDAY

I. Ushakov
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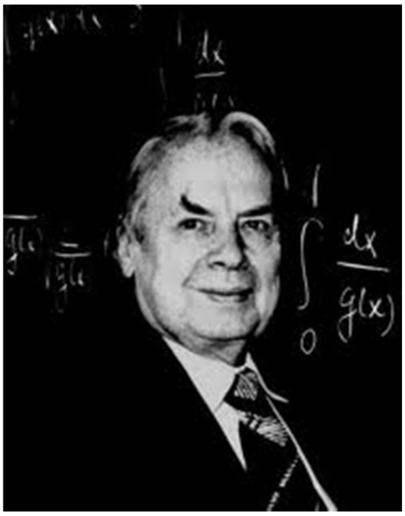
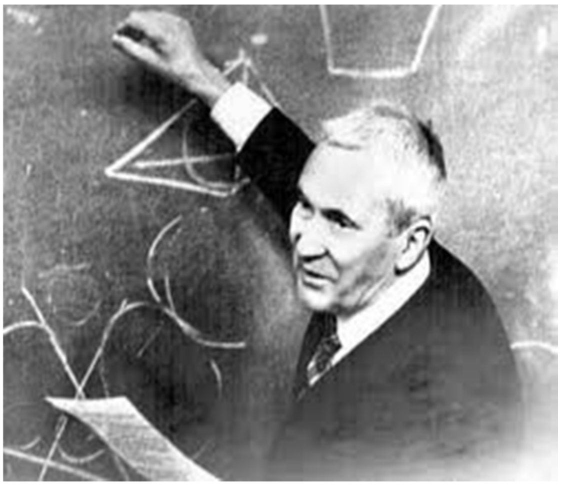
e-mail: igusha22@gmail.com



Academician of the Ukrainian Academy of Sciences Boris Vladimirovich Gnedenko by a common International opinion represents one of the most prominent mathematicians who are working in the area of Probability Theory. He combines an exceptionally delicate possession of classical mathematical methods with a deep understanding of a wide range of problems of the modern probability theory and with a permanent interest to practical applications.

A.N. Kolmogorov
"On Gnedenko's works on
the Probability Theory"
Probability Theory and Its Applications,
vol.VII, no.3, 1962.

Boris Vladimirovich Gnedenko was born on January 1st 1912 in Simbirsk, city located on the bank of the Great Russian river Volga. When Boris was only fifteen years old, he tried to enter the University of Saratov, a city on the Southern part of Volga river. However, he was turned down due to his young age. Boris was firm in his decision: he made a complaint and sent it to then Minister of Education Anatoly Lunachartsky. Soon he got a letter of permission that allowed him to enter university.



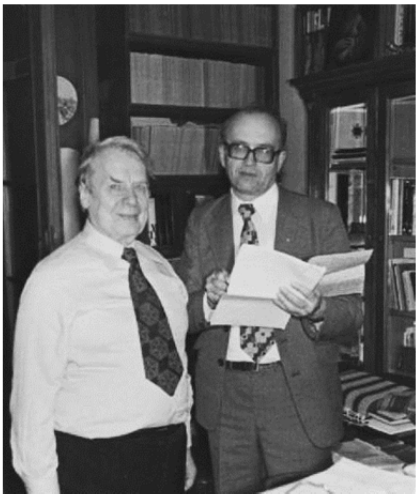
After graduation in 1930 Gnedenko taught at the Textile Institute in Ivanovo, a city, east of Moscow. In 1933 he published his first papers on the Queuing Theory concerning using machines in textile manufacture.

In 1934 Gnedenko undertook research at the Institute of Mathematics at Moscow State University. Gnedenko's supervisor was famous Russian mathematician Alexander Khinchin[1]. When in 1935 he left to spend two years at Saratov University, Andrei Kolmogorov[2] took supervision over Gnedenko's studies.

[1] **Alexander Yakovlevich Khinchin** (1894 –1959) was a Soviet mathematician and one of the most significant people in the Soviet school of probability theory.
[2] **Andrei Nikolayevich Kolmogorov** (1903 –1987) was a Russian mathematician, preeminent in the 20th century, who advanced various scientific fields, among them probability theory, topology, intuitionist logic, turbulence, classical mechanics and computational complexity.

Andrei Kolmogorov – probably, the greatest mathematician of the XX century – influenced most of all on Boris Gnedenko: he not only taught mathematics; he discussed with his pupil art, music, poetry. Such relations with the pupil led to their intimate friendship that continued entire life.

During the summer of 1937 Gnedenko went on a hiking expedition to the Caucasus along with some fellow researchers. There he met Andrei Kolmogorov and joined him to continue his vacation.



In 1937 Gnedenko presented his Candidate of Science[3] dissertation on the theory of infinitely divisible distributions. After the award of C.Sc. degree he was appointed as an assistant researcher at the Mathematics Institute of the Moscow State University.

In the beginning of November of 1937 Boris Gnedenko was drafted to the army and was sent to city of Bryansk located in the Western part of the Soviet Union. However, in few weeks he was arrested and put in local jail.

It happened that one of his companion of that Caucasus trip informed KGB (then NKVD) about "hostile Kolmogorov's attitude". KGB needed some "official confirmations", so they seized Gnedenko, who was the best Kolmogorov's friend, to get some "needed information" from him. Gnedenko was imprisoned with more than hundred other prisoners in a jail cell intended for six people. His interrogators demanded that he had to confirm that Andrei Kolmogorov was the ringleader of a group of "enemies of the people" centered in the Mathematics Department of the Moscow University. Daily interrogations during almost half a year period (sometimes several days in a row he was held under bright light of spotlight) did not broke Boris Gnedenko. Though he was promised to be released if he would "cooperate" with investigators and confirmed Kolmogorov's guilt, Boris Gnedenko refused to give such false evidence.

I had known about this in a very interesting way. In 1972 at the banquette following defense of Dr.Sc. degree by Alexander Dmitrievich Solovyev[4], Andrei Kolmogorov pronounced a toast for Boris Gnedenko who was an actual Solovyev's advisor. At the end Andrei Kolmogorov added: "However, first of all, I would like to drink this glass of wine for Boris as a Man!" Such toast, being rather unusual for Kolmogorov, made me curious. I came to the place where Kolmogorov sat, found a free place and asked him what he meant by the conclusion of his toast. He answered: "You know, Igor Alexeevich, Boris saved a life of one man…" I knew Andrei Kolmogorov in person; we met occasionally frequently enough, so I understood that he would say nothing more.

Some days later I came to Boris Gnedenko home. We had a lot of various work together: I was his deputy in journal "Reliability and Quality Control" (now Methods of Quality Management"), his deputy in Moscow Reliability Consulting Center on Reliability and some other activities. So, I visited Boris Gnedenko sometimes twice in a week. (Here, by the way, I frequently met Andrei Kolmogorov who lived in the same building). At that particular day, Boris Gnedenko stayed too long at his Department, and I had a chance to talk a bit with his wife.

She told me the story about her husband detention, and explained that "if Boris would be broken by KGB, Andrej would be severe punished". (Soviet regime used to "enemies of the people" only

[3] Candidate of Science (C.Sc.) is the first postgraduate scientific degree in Russia and the former Soviet Union. The second (and highest) postgraduate is Doctor of Science (Dr.Sc.). Difference in these degrees reminds the difference between Associate Professor and Full Professor. For information: in the former Soviet Union there were under hundred thousand C.Sc.'s, and under ten thousand Dr.Sc.'s.

[4] **Alexander Dmitrievch Solovyev** (1927-2001) was a prominent Russian mathematician, one of the founders of the Soviet School of Reliability.

"the highest level of punishment", i.e. death penalty.) She asked me not to tell about this anybody and motivated it: "Neither Boris, nor Andrei like to remember tha t time…"

I kept this secret (even from Boris Gnedenko himself) until he told this story during his interview to journal "Statistical Sciences" in 1991 (I was a translator for him).

Now let us return to the following events.

Without warning Boris Gnedenko was released after six months of imprisonment. After a real battle with Soviet bureaucracy, Andrei Kolmogorov and Alexander Khinchin reinstated Gnedenko to his post of Assistant Professor in 1938. He retained a "black mark" on his record indicating that he was not to be trusted. Because of his "disloyalty", he was not allowed to join the Soviet army in 1941 when the German forces attacked.

At the beginning of June 1941 he defended his Doctor of Science dissertation that consisted of two parts: Theory of Summation of Independent Random Variables and Theory of Maximum Term of a Variation Series.

In 1945, on the recommendation of Kolmogorov, Gnedenko was elected to the Ukrainian Academy of Sciences. He became professor at Lvov University. In 1949 Gnedenko moved to the Kiev University and later in 1955 he was appointed as Head of the Physics, Mathematics and Chemistry Section of the Ukrainian Academy of Sciences and he became Director of the Kiev Institute of Mathematics.

He also initiated works in area of computer programming and was the author of the first textbook on the subject published in the USSR.

Here he raised a group of talented pupils: Bronius Grigelionis[5] Igor Kovalenko[6], Vladimir Korolyuk[7], Tadeush Marianovich[8], Vladimir Mikhalevich[9], Manfred Shneps-Shneppe[10] , Anatoly Skorokhod[11] Ekaterina Yustchenko[12] and others who represented Soviet School of Probability Theory and Statistics.

In 1960 Boris Gnedenko returned to the Moscow University, becoming Head of the Department of Probability Theory in 1966. He held this post for thirty years until his death.

Gnedenko wrote hundreds of papers and tens of books. His books were re-published in the Soviet Union, translated into different languages. In 1949 he published a work, jointly with Kolmogorov, *Limit Distributions for Sums of Independent Random Variables* which contains a description of much of his early research.

One of Gnedenko's most famous books is *Course in the Theory of Probability* which first appeared in 1950. Written in a clear and concise manner, the book was very successful in providing a first introduction to probability and statistics. It has gone through eight Russian editions and has been

[5] **Bronius Grigelionis** (born 1935) is a Lithuanian mathematician, academician of the Lithuanian Academy of Sciences.

[6] **Igor Nikolayevich Kovalenko** (born 1935) is a Ukrainian mathematician working in reliability and queuing theories; academician of the Ukrainian Academy of Sciences.

[7] **Vladimir Semenovich Korolyuk** (born 1925) is a Ukrainian mathematician who made significant contributions to probability theory and its applications, academician of the Ukrainian Academy of Sciences.

[8] **Tadeush Pavlovich Marianovich** (born 1938) is a Ukrainian mathematician, Deputy Director of Institute of Cybernetics, correspondent member of the Ukrainian Academy of Sciences.

[9] **Mikhalevich Vladimir Sergeevich** (1930 – 1994) was a Ukrainian mathematician, Deputy Director of Institute of Cybernetics, academician of the Ukrainian Academy of Sciences.

[10] **Shneps-Shneppe Manfred Alexandrovich** (born 1935) is a Latvian mathematician, now a Professor of the Moscow University.

[11] **Skorokhod Anatoly Vladimirovich** (1930—2011) was Ukrainian mathematician, academician of the Ukrainian Academy of Sciences.

[12] **Ekaterina Logvinovna Yustchenko** (1919-2001) was Ukrainian cybernetic, author of one of the first computer languages of high level, correspondent member of the Ukrainian Academy of Sciences.

translated into English, German, Polish and Arabic. In 1966, along with Igor Kovalenko, he published "*Introduction to Queuing Theory*".

In his later work Gnedenko had been interested in probability theory applications to areas such as reliability and quality control. In 1965 he wrote an excellent text "*Mathematical methods of reliability*" in 1965 with Yuri Belyaev[13] and Alexander Solovyev. This book became "The Reliability Bible" in the Soviet Union and soon had been translated into many languages.

We should mention also about Gnedenko's interest in the history of mathematics. His "*Outline of the History of Mathematics in Russia" is* a fascinating book which looks at the history of mathematics in Russia. The work of many famous mathematicians is discussed in detail such as that of Nikolai Lobachevsky[14], Victor Bunyakovsky[15], Mikhail Ostrogradsky[16], Pafnuty Chebyshev[17], Andrei Markov[18], Alexander Lyapunov[19], and Sofia Kovalevskaya[20].

Boris Gnedenko was a brilliant lecturer. During our multiple mutual business trips, I was lucky to attend a number of his lectures for audience of various levels: from practical engineers to postgraduate students. He easily found a path to soul and brain of any audience.

His seminar at Moscow State University attracted tens of applied mathematicians and practical engineers. Then Boris Gnedenko established the Reliability Consulting Center that served for reliability and quality practical engineers. Gnedenko's authority and personal charisma help to involve tens of high level professional for consulting and lecturing on voluntary basis. Consultations took place every day (and sometimes more than one a day); twice a month there was a lecturing day of 2 lectures. People came to these lectures from all parts of the Soviet Union: from Western boarders to Far East, from Kola Peninsula to Caucasus and Central Asia.

At the end of this note I would like to share with my own experience of relations with Boris Gnedenko. As I already wrote, I met him very often in informal situations. We met at several editorial boards or at scientific councils where both are members, however mostly I visited Boris Gnedenko at his home. Every time I was invited in the host's home office. Usually a soft classical music was heard… Any discussion began with showing me something new: new collection of poetry, new album with beautiful reproductions, new musical records… Our tastes in fine art coincide: we both loved French impressionists very much. I loved them very much, however Boris Vladimirovich in addition knew them very well! He taught me that knowledge of a history is a necessary condition for real understanding in any area of interests.

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[13] **Yuri Konstantinovich Belyaev** (born 1932) is a Russian mathematician, one of founders of the Soviet School of Reliability Theory.

[14] **Nikolai Ivanovich Lobachevsky** (1792 –1856) was a Russian mathematician and geometer, renowned primarily for his pioneering works on hyperbolic geometry.

[15] **Viktor Yakovlevich Bunyakovsky** (1804–1889) was a Russian mathematician, member and later Vice president of the Petersburg Academy of Sciences.

[16] **Mikhail Vasilyevich Ostrogradsky** (1801 –1862) was a Russian and Ukrainian mathematician, mechanician and physicist. Ostrogradsky is considered to be a disciple of Leonhard Euler and one of the leading mathematicians of Imperial Russia.

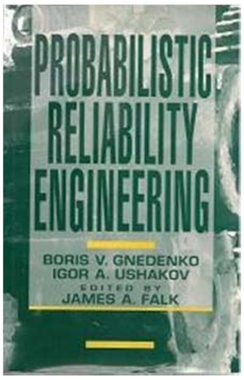
[17] **Pafnuty Lvovich Chebyshev** (1821 –1894) was a Russian mathematician.

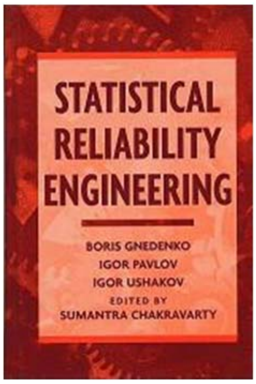
[18] **Andrei Andreievich Markov** (1856 – 1922) was a prominent Russian mathematician. He is best known for his work on theory of stochastic processes. A primary subject of his research later became known as Markov chains.

[19] **Alexander Mikhailovich Lyapunov** (1857 –1918) was a Russian mathematician, mechanician and physicist.

[20] **Sofia Vasilyevna Kovalevskaya** (1850 –1891), was the first major Russian female mathematician,

After my move to the United States, Boris Gnedenko twice visited me there. First time he visited me when I had an open heart surgery. He came with his son Dmitry just the day before my hospitalization. In few days we began long but slow walking tours in Arlington and Washington.





We talked about everything except mathematics. (At that time we already completed our two books on probabilistic and statistical reliability engineering for John Wiley & Sons.)

When we discussed poetry, Boris Gnedenko told me that his beloved poet was Alexander Pushkin[21], and I responded that mine is Vladimir Mayakovsky[22]; and Pushkin does not touch my heart. Boris Gnedenko responded me in a very much his style: "Love to Pushkin comes with age…" (Pushkin indeed became closer to me though still is not my beloved… And I will be 80 very soon☹.) Another time when I told that doesn't know Mendelssohn's[23] music except his "Violin Concerto" and, of course, "Wedding March", he softly advised me: "You should listen him more…" And indeed, Mendelssohn became one of my most beloved composers!

Such was Boris Gnedenko: always soft and polite, however firm and principal in his opinion. He was very considerate to people around him, he was very tolerant to other opinions even if they contradicts his own. He was a very educated man in many areas in sciences, music and art, however he never showed his superiority.

Boris Gnedenko possessed an unusual sense of humor. He could find unexpected words in various situations. Once we walked through the famous Arlington Military Cemetery, and I remember how he pronounced with a soft and smile: "We are here at the meeting with our future…"

I understand that I have an exceptional luckiness that I met and had such a close relations with this Great Man.

San Diego, December 2011.

[21] **Alexander Sergeievich Pushkin** (1799 – 1837) was a Russian poet and writer who is considered by many to be the greatest Russian poet and the founder of modern Russian literature.
[22] **Vladimir Vladimirovich Mayakovsky** (1893 –1930) was a Russian and Soviet poet and, among the foremost representatives of early XX century Russian Futurism.
[23] **Felix Mendelssohn** (1809 – 1847) was a German composer, pianist and conductor.

БОРИС ВЛАДИМИРОВИЧ ГНЕДЕНКО

(01.01.1912—27.12.1995)

Dmitry B. Gnedenko
•
e-mail: dmitry@gnedenko.com

Борис Владимирович Гнеденко родился 1 января (по новому стилю) 1912 года в Симбирске (ныне Ульяновск).

Его дед Василий Ксенофонтович Гнеденко и бабушка Анастасья Изотовна (оба по отцовской линии) — крестьяне Полтавской губернии, перебравшиеся в семидесятых годах XIX века в Казанскую губернию, где они получили землю в деревне Базарные Матаки. Отец – Владимир Васильевич Гнеденко – закончил землестроительное училище и работал землемером. Мама – Мария Степановна – родилась в Костроме, закончила прогимназию (семилетнее училище), в которой получила музыкальную специализацию (игра на фортепьяно), дававшую право преподавать музыку.

В 1915 году семья переехала в Казань, где одновременно с работой землемера Владимир Васильевич с осени 1916 года стал студентом физико-математического факультета университета. Весной 1918 года по ложному доносу одного из коллег Владимир Васильевич был арестован и полгода провел в концлагере под Казанью. Его здоровье было сильно подорвано, и по возвращении домой осенью 1918 года он был вынужден оставить студенческую скамью.

Этой же осенью 1918 года Борис Владимирович (Б.В.) поступил в школу. Как он сам пишет в своих воспоминаниях: «Все бы хорошо, если бы не было арифметики. Я действительно не любил арифметику, хотя складывал, вычитал, умножал и делил совсем неплохо. Я увлекался поэзией».

В связи с состоянием здоровья отца семья в 1922 году переезжает в Галич, где Владимир Васильевич работает старшим землеустроителем. К приезду семьи в Галич набор в школы был закончен, и этот год с Борисом и его братом Глебом занимается мама. «Мама узнала программу и начала заниматься с нами, чтобы мы не отстали. Достали учебник грамматики, арифметику Киселева, учебник географии Иванова. Я с особым удовольствием читал учебник географии и учил правила грамматики русского языка».

В апреле 1925 года семья переезжает в Саратов. Это было связано с тем, что родители начали беспокоиться о дальнейшем образовании своих детей, которые через два года должны были закончить школу.

В Саратове братья были зачислены в школу № 3, бывшее реальное училище. Выяснилось, что они серьезно отстали по химии и математике. На осень им были назначены переэкзаменовки по этим предметам. Это оказалось очень полезным. «Мы сумели продумать весь материал по математике и по химии, прорешать по многу десятков задач, и осенью, благодаря этому, переэкзаменовка прошла благополучно. Более того, химия и математика стали восприниматься совершенно свободно, задачи не вызывали никаких трудностей, и я начал решать задачи сразу в уме, как только узнавал условие. По математике и химии я выдвинулся в число первых учеников класса. Одноклассники стали обращаться ко мне за помощью. Математика стала мне нравиться... Мне нравилось учиться, дополнительно читать книги, решать нестандартные задачи... Я достал сборник конкурсных задач, предлагавшихся на вступительных экзаменах в Петроградский институт инженеров путей сообщения. Ни одна задача из этого сборника не вызвала у меня затруднений... Я отдавал себе отчет в том, что хочу учиться дальше и буду добиваться этого права. Я тщательно изучил правила приема в вузы страны и повсюду наталкивался на одно требование, которому я не удовлетворял, – поступающему должно исполниться 17 лет, мне же было только 15... Брат хотел стать или инженером, или физиком, а я мечтал о кораблестроении. Я даже послал в Ленинградский кораблестроительный институт письмо с просьбой допустить меня к вступительным экзаменам в мои пятнадцать лет».

Из города на Неве на это письмо Б.В. получил отказ. Тогда он посылает письмо народному комиссару просвещения А.В.Луначарскому с просьбой разрешить ему поступать в Саратовский университет. К началу вступительных экзаменов разрешение было получено.

С осени 1927 года Б.В. – студент физико-математического факультета Саратовского университета. «В мае 1930 года нам объявили, что мы будем заниматься все лето, с тем чтобы в сентябре разъехаться по местам работы. Было решено организовать ускоренный выпуск... Экзамены были сданы, и в середине августа нам были выданы документы об окончании Саратовского университета. Я не испытывал от этого ни радости, ни удовлетворения. Я понимал, что получено ущербное образование и нужно приложить много собственных усилий, чтобы исправить положение дел».

Один из университетских преподавателей Б.В. – профессор Георгий Петрович Боев – в это время был приглашен заведовать кафедрой математики в организуемый в Иваново-Вознесенске Текстильный институт и, в свою очередь, пригласил Б.В. на должность ассистента этой кафедры.

В Иваново-Вознесенске Б.В. преподавал и занимался вопросами применения математических методов в текстильном деле. Здесь им были написаны его первые работы по теории массового обслуживания, здесь Б.В. увлекся теорией вероятностей. Этот период деятельности сыграл огромную роль в его формировании как ученого и педагога.

Понимая необходимость углубления своих математических знаний, Б.В. в 1934 году поступает в аспирантуру механико-математического факультета МГУ. Его научными руководителями становятся А.Я.Хинчин и А.Н.Колмогоров.

В аспирантуре Б.В. увлекся предельными теоремами для сумм независимых случайных величин. 16 июня 1937 года он защитил кандидатскую диссертацию на тему «О некоторых результатах по теории безгранично делимых распределений», и с 1 сентября этого же года он – младший научный сотрудник Института математики МГУ.

В работах А.Я.Хинчина и Г.М.Бавли было установлено, что класс возможных предельных распределений для сумм независимых случайных величин совпадает с классом безгранично делимых распределений. Оставалось выяснить условия существования предельных распределений и условия сходимости к каждому возможному предельному распределению. Заслуга постановки и решения этих задач принадлежит Б.В.Гнеденко. Для решения возникших проблем Б.В. предложил оригинальный метод, получивший название метода сопровождающих безгранично делимых законов (идея метода появилась в октябре 1937 года и опубликована в "Докладах АН СССР" в 1938 году). Он позволил единым приемом получить все ранее найденные в этой области результаты, а также и ряд новых.

В ночь с 5-го на 6-ое декабря 1937 года Борис Владимирович был арестован. Ему предъявили надуманное обвинение в контрреволюционной деятельности и участии в контрреволюционной группе, возглавляемой профессором А.Н.Колмогоровым. Его водили на допросы, во время одного из которых ему не давали спать в течение восьми суток. Требовали подписать бумаги, подтверждающие обвинения. Борис Владимирович не подписал ничего, что могло бы быть поставлено в вину ему, А.Н.Колмогорову или кому-либо другому. В конце мая 1938 года его освободили.

С осени 1938 года Б.В. – доцент кафедры теории вероятностей механико-математического факультета МГУ, ученый секретарь Института математики МГУ. К этому периоду относятся работы Б.В.Гнеденко, в которых дано решение двух важных задач. Первая из них касалась построения асимптотических распределений максимального члена вариационного ряда, выяснения природы предельных распределений и условий сходимости к ним. Вторая задача касалась построения теории поправок к показаниям счетчиков Гейгера-Мюллера, применяемых во многих областях физики и техники.

В начале июня 1941 года Б.В. защитил докторскую диссертацию, состоящую из двух частей: теории суммирования и теории максимального члена вариационного ряда.

В годы Великой Отечественной войны Б.В. принимал активное участие в решении многочисленных задач, связанных с обороной страны.

В феврале 1945 года Борис Владимирович избирается членом-корреспондентом АН УССР и направляется Президиумом АН УССР во Львов для восстановления работы Львовского университета.

Во Львове Б.В. читает разнообразные курсы лекций: математический анализ, вариационное исчисление, теорию аналитических функций, теорию вероятностей, математическую статистику и др., в окончательной формулировке доказывает локальную предельную теорему для независимых, одинаково распределенных решетчатых слагаемых (1948 г.), начинает исследования по непараметрическим методам статистики. Во Львове им были воспитаны талантливые ученики – Е.Л.Рвачева (Ющенко), Ю.П.Студнев, И.Д.Квит и др.

Курс лекций по теории вероятностей послужил Борису Владимировичу основой для написания учебника «Курс теории вероятностей» (1949 г.). Эта книга многократно издавалась в разных странах и является одним из основных учебников по теории вероятностей и в наши дни. В эти же годы им совместно с А.Н.Колмогоровым написана монография «Предельные распределения для сумм независимых случайных величин» (1949 г.), за которую авторы были удостоены премии АН СССР им. П.Л.Чебышева (1951 г.). Совместно с А.Я.Хинчиным Б.В. пишет «Элементарное введение в теорию вероятностей» (1946 г.), которое, в свою очередь, выдержало множество изданий в СССР и за рубежом. Кроме этого Борисом Владимировичем была написана замечательная книга «Очерки по истории математики в России» (1946 г.).

В 1948 году Б.В. избирается академиком АН УССР, и в 1950 году Президиум АН УССР переводит его в Киев. Здесь он возглавляет только что созданный в Институте математики АН УССР отдел теории вероятностей и одновременно начинает заведовать кафедрой теории вероятностей и алгебры в Киевском университете. Очень скоро около него образовалась группа молодежи, заинтересовавшейся теорией вероятностей и математической статистикой. Первыми киевскими учениками Б.В. были В.С.Королюк, В.С.Михалевич и А.В.Скороход.

В это время Б.В. увлекся сам и увлек многих своих учеников и коллег задачами, связанными с проверкой однородности двух выборок. В.С.Королюк, В.С.Михалевич, Е.Л.Рвачева (Ющенко), Ю.П.Студнев и др. получили серьезные результаты в этой области.

В конце 1953 года Б.В.Гнеденко был направлен в ГДР для чтения лекций в университете им. Гумбольдта (Берлин). Он провел там весь 1954 год. За это время Б.В. сумел заинтересовать большую группу молодых немецких математиков (И.Керстан, К.Маттес, Д.Кёниг, Г.-И.Россберг, В.Рихтер и др.) задачами теории вероятностей и математической статистики. Правительство ГДР наградило Бориса Владимировича серебряным орденом «За заслуги перед Отечеством», а университет им. Гумбольдта избрал его**.** почетным доктором.

Вернувшись в конце 1954 года в Киев, Б.В. по поручению Президиума АН УССР возглавил работу по организации Вычислительного центра. Был создан коллектив, в которыйвошли сотрудники лаборатории академика С.А.Лебедева, автора первой в континентальной Европе ЭВМ, получившей название МЭСМ (малая электронная счетная машина). Лаборатория к этому времени возглавлялась её старейшими сотрудниками – Е.А.Шкабарой и Л.Н.Дашевским, т.к. сам С.А.Лебедев уже переехал в Москву, где ему была поручена организация Института точной механики и вычислительной техники. В этот коллектив вошли и математики, среди которых в первую очередь надо назвать В.С.Королюка, Е.Л.Ющенко и И.Б.Погребысского. Началась работа по проектированию универсальной машины «Киев» и специализированной машины для решения систем линейных алгебраических уравнений.

Одновременно Б.В. начал читать в университете курс программирования для ЭВМ и возглавил работу по написанию учебника по программированию. Этот курс (первая в СССР книга по программированию в открытой печати) был издан в Москве в 1961 году (авторы – Б.В.Гнеденко, В.С.Королюк, Е.Л.Ющенко). В это же время (1955 г.) Президиум АН УССР возложил на Б.В.Гнеденко обязанности директора Института математики АН УССР и председателя бюро физико-математического отделения АН УССР.

В этот период Борис Владимирович начинает разрабатывать два новых направления прикладных научных исследований – теорию массового обслуживания (ТМО) и применение математических методов в медицине.

К первому он привлек И.Н.Коваленко, Т.П.Марьяновича, Н.В.Яровицкого, С.М.Броди и др. Б.В. применил методы ТМО к расчету электрических сетей промышленных предприятий. В 1959 году были изданы «Лекции по теории массового обслуживания» (выпуск 1), прочитанные Б.В. в КВИРТУ[24] в 1956-57 годах. Затем последовали выпуски 1-2 (1960 г.), выпуски 1-3 (1963 г., совместно с И.Н.Коваленко). Эти книги послужили основой для монографии «Введение в теорию массового обслуживания» (1966 г.), написанную Б.В.Гнеденко и И.Н.Коваленко.

Второе направление связано с разработкой электронного диагноста сердечных заболеваний. Над этой проблемой работали Б.В.Гнеденко, Н.М.Амосов, Е.А.Шкабара и М.А.Куликов. В начале 1960 года была завершена сборка первого в мире диагноста.

Переехав в июле 1960 года Москву, Борис Владимирович возобновляет работу на механико-математическом факультете МГУ. Работа вновь полностью захватила его: чтение разнообразных лекционных курсов, новые ученики, новые обязанности.

В 1961 году Б.В. вместе с Я.М.Сориным, Ю.К.Беляевым, А.Д.Соловьевым, Я.Б.Шором организует семинар по надежности при Политехническом музее, который эффективно работал в течение многих лет. Вскоре появляется необходимость организации отдельного семинара специально по математическим методам теории надежности. Этот семинар начинает работать на механико-математическом факультете МГУ под руководством Б.В.Гнеденко, А.Д.Соловьева, Ю.К.Беляева и И.Н.Коваленко, который в это время работал в Москве. Семинар по математическим методам в теории надежности регулярно работал до конца восьмидесятых годов. Он помог в научном отношении встать на ноги многим своим участникам, теперь широко известным специалистам в области надежности, таким как Е.Ю.Барзилович, В.А.Каштанов, И.А.Ушаков и др. Этот семинар повлиял, в свою очередь, и на своих руководителей и подтолкнул Б.В.Гнеденко, Ю.К.Беляева и А.Д.Соловьева к написанию широко известной у нас и за рубежом монографии «Математические методы в теории надежности» (1965 г.). За цикл работ в области надежности Б.В. вместе с ближайшими сподвижниками был удостоен в 1979 году Государственной премии СССР.

В связи с задачами надежности Б.В. вновь вернулся к исследованию предельных теорем для сумм независимых случайных величин, но уже в случайном числе. К этому направлению исследований Б.В. привлекает многих своих учеников. За эти работы в 1982 году ему присуждается премия им. М.В.Ломоносова первой степени, а в 1986 году – премия Минвуза СССР.

Б.В. не переставал интересоваться вопросами истории математики, подключив своих учеников и к этому направлению работ. В различных отечественных и зарубежных журналах печатались его статьи по этому направлению исследований, а его "Очерк по истории теории вероятностей" дает наиболее полное представление о его взглядах на историю этой науки.

Совместно с А.И.Маркушевичем Б.В. руководил работой семинара по вопросам преподавания в средней школе. Он тесно сотрудничал с редакциями журналов «Вестник высшей школы» и «Математика в школе». В этих и многих зарубежных журналах, в сборниках научно-методического совета Минвуза СССР им было опубликовано большое

[24] Киевское высшее инженерное радиотехническое училище.

число статей по различным аспектам преподавания. По этим же вопросам Б.В. написал в эти годы и несколько книг.

В январе 1966 года А.Н.Колмогоров передал Б.В.Гнеденко руководство кафедрой теории вероятностей механико-математического факультета МГУ, которой Б.В. заведовал до последних дней своей жизни.

Еще работая во Львове, Б.В. много времени и сил отдавал работе в обществе «Знание». С 1949 года он последовательно избирался председателем областного правления общества, возглавлял республиканскую физико-математическую секцию общества, являлся членом Президиума правления Всесоюзного общества «Знание», председателем общества «Знание» Московского университета.

Б.В. был членом редколлегий ряда отечественных и зарубежных журналов, являлся членом Королевского Статистического Общества (Великобритания), был избран почетным доктором Берлинского университета, почетным доктором Афинского университета.

В последние годы жизни, зная суровый приговор врачей, Б.В. продолжает руководить кафедрой, выдвигает и осуществляет идею создания на механико-математическом факультете экономической специализации и подготовки в ее рамках специалистов в области актуарной и финансовой математики. Кроме этого он намечает список книг, которые надо успеть написать за оставшееся время. И он пишет. Окончательно ослепнув, диктует, но выполняет намеченное.

27 декабря 1995 года Бориса Владимировича не стало. Он похоронен на Кунцевском кладбище в Москве.

Б.В.Гнеденко оставил много учеников. Среди них – академики и члены-корреспонденты различных академий, профессора и доценты. В их памяти сохраняются незабываемые дни приобщения к науке и самостоятельному творчеству под руководством большого ученого и педагога, часы непосредственного общения с Человеком большой эрудиции и высокой культуры.

Б.В. ГНЕДЕНКО: БИБЛИОГРАФИЯ[25]

1933 год

1. Методика составления эмпирических формул (совм. с Г.П.Боевым. Бюллетень ИВНИТИ, N3, 24-37; № 6, 46-65).
2. О связи коэффициента неровноты с вариационным коэффициентом (совм. с Г.П.Боевым. Бюллетень ИВНИТИ, N7, 48-53).
3. К вопросу о распределении обратных величин (Бюллетень ИВНИТИ, № 8-9, 55 - 62).
4. О статистическом распределении степенных функций (Сборник научно-исследовательских работ ИВНИТИ, 18 - 22).
5. О нормировании методом станкообходов (совм. с Г.В.Соколовым. Сборник научно-исследовательских работ ИВНИТИ, 157 - 182).

1934 год

6. К методике и технике установления потерь в производительности оборудования из-за неуспеваемости (Бюллетень ИВНИТИ, № 3-4, 114 -117).
7. Вычисление среднего перехода между станками (Бюллетень ИВНИТИ, № 1-2, 117-122).
8. О вычислении среднего перехода между станками конечных размеров (Бюллетень ИВНИТИ, № 10-12, 118-122)
9. О среднем простое станков при многостаночном обслуживании из-за неуспеваемости рабочего (Известия хлопчато-бумажн. промышл., № 11, 15 - 18).

1936 год

10. Методика составления эмпирических зависимостей и номограмм в текстильном деле (совм. с Г.П.Боевым и Ю.С.Виноградовым. Гизлегпром, 1-128).
11. О Θ в формуле Лагранжа (Математическое просвещение, № 7, 31 - 35).

1937 год

12. О единственности системы ортогональных функций, инвариантной относительно дифференцирования (ДАН СССР, т.14, N4, 159 - 161).
13. Об одном характеристическом свойстве безгранично делимых законов распределения (Бюллетень МГУ, Математика и механика, т.1, вып.5, 10-16).
14. О характеристических функциях (Бюллетень МГУ, т.1, вып. 5, 17 - 18).

[25] В библиографию включено все, что удалось найти из написанного Б.В. Гнеденко, начиная с книг и заканчивая газетными статьями. Библиография разбита по годам, и для каждого года представлено все, что публиковалось в течение этого года, включая переиздания. При этом работы для каждого года располагаются в одном определенном порядке: книги, научные статьи, статьи по различным аспектам преподавания, рецензии, статьи из журналов общего характера, газетные публикации. Если одна и та же статья публиковалась в течение года в разных местах, то она указывается под одним номером с перечислением выходных данных всех изданий. Иногда в течение года встречаются статьи с одним и тем же названием, но различным содержанием. Они перечисляются под разными номерами, по возможности рядом.

1938 год

15. О сходимости законов распределения сумм независимых слагаемых (ДАН СССР, т. 18, № 4-5, 231 - 234).

1939 год

16. О предельных теоремах для сумм независимых слагаемых (ДАН СССР, т. 22, № 2, 61 - 64).
17. К теории предельных теорем для сумм независимых случайных величин (Изв. АН СССР, серия математич. № 2, 181-232).
18. К теории предельных теорем для сумм независимых случайных величин (исправления к статье под тем же названием) (Изв. АН СССР, серия математич. № 6, 643-647).
19. Об областях притяжения устойчивых законов (ДАН СССР, т. 24, № 7, 640 - 642).
20. К теории областей притяжения устойчивых законов (Ученые записки МГУ, вып. 30, 61 - 81).
21. О предельных законах теории вероятностей (ДАН СССР, т. 23, № 9, 868 - 871).
22. О сходимости законов распределения нормированных сумм независимых случайных величин (совм. с А.В.Грошевым. Математический сборник, т. 6, (48), № 3, 521 - 541).
23. Обзор современного состояния теории предельных законов для сумм независимых слагаемых (Ученые записки Тюменского пединститута, вып. 1, 5 - 28).
24. Андрей Николаевич Колмогоров (совм. с П.С.Александровым, В.В.Степановым, И.Г.Петровским, А.Я.Хинчиным. Статья напечатана под рубрикой «Кандидаты в действительные члены Академии Наук СССР»). («Правда», 19 января).

1940 год

25. Несколько теорем о степенях функций распределения. (Ученые записки МГУ , вып. 45, 61 - 71).

1941 год

26. К теории счетчиков Гейгер-Мюллера (Журнал эксперимент. и теоретич. физики, т. 11, вып. 1, 101 - 106).
27. Предельные теоремы для максимального члена вариационного ряда (ДАН СССР, т. 32, № 1, 7-9).
28. Зона действия истребителя-перехватчика (отчет сдан заказчику).
29. Исследование влияния ветра на траекторию самолета, летящего на радиостанцию с постоянным курсовым углом и постоянной скоростью (отчет сдан заказчику).
30. О некоторых задачах теории стрельбы (отчет сдан заказчику).

1942 год

31. О локально устойчивых законах распределения (ДАН СССР, т.35, № 9, 295 -298).
32. Локально устойчивые законы распределения (Изв. АН СССР, серия математич., № 6, 291 - 308).
33. Исследование роста однородных случайных процессов с независимыми приращениями (ДАН СССР, т. 36, № 1, 3-4).

34. Средний расход снарядов в случае поражения цели одним попаданием (отчет сдан заказчику).

1943 год

35. О росте однородных случайных процессов с независимыми приращениями (Изв. АН СССР, серия математич., т.7, 89 - 110).
36. Sur la distribution limite du terme maximume d'une serie aleatoire (Annals of Mathematics v.44, № 3, 423-453).
37. О росте однородных случайных процессов с независимыми однотипными приращениями (ДАН СССР, т.11, № 3, 103-107).
38. О законе повторного логарифма для однородных случайных процессов с независимыми приращениями (ДАН СССР, т. 11, № 7, 291 - 293).
39. Решение одной задачи теории ошибок механизмов от торцевого биения (отчет сдан заказчику).

1944 год

40. Элементы теории функций распределения случайных векторов (Успехи математических наук, т. X, 230 - 244).
41. Предельные законы для сумм независимых случайных величин (Успехи математических наук, т. X, 115 - 165).
42. Теория ошибок одного аэрокартографического прибора (совм. с инженер-майором Бордюковым; отчет сдан заказчику).

1946 год

43. Элементарное введение в теорию вероятностей (совм. с А.Я. Хинчиным. ГИТТЛ, 1 - 128).
44. Очерки по истории математики в России (ГИТТЛ, 1 - 247).
45. Краткие беседы о зарождении и развитии математики (Изд. Академии педагогических наук РСФСР, 1 - 40).
46. Видатний математик (до 145-річчя з дня народження М.В. Остроградського) (За радянські кадри, 14.X., № 29-30).
47. О работе университетского издательства (Газета «Московский Университет», 23.04).

1947 год

48. Как математика изучает случайные явления (Изд. АН УССР, 1 - 75).
49. К вопросу об ошибках рычажного пантографа (совм. с М.П.Бордюковым. Вестник Военно-инжен. Академии им. Куйбышева, т. 48, вып. 6, 68 - 79).
50. Об эллипсоидах рассеивания (Ученые записки Львовского университета, т. 5, вып. 2, 116 - 120).
51. О функциях от случайных величин (Известия Львовского университета, т. 5, вып. 2, 121 - 128).
52. Стефан Банах (Ученые записки Львовского университета, т. 5, вып. 1, 5 - 9).

1948 год

53. Об одной теореме С.Н.Бернштейна (Изв. АН СССР, серия математ., т. 12, 97 - 100).

54. К теории роста однородных случайных процессов с независимыми приращениями (Труды Ин-та математики АН УССР, № 10, 60 - 82).
55. Про одну характеристичну властивість нормального закону розподілу (совм. с Е.Л.Рвачевой. ДАН УРСР, № 3, 3 - 5).
56. Об одном характеристическом свойстве нормального закона распределения (совм. с Е.Л.Рвачевой. Труды Ин-та математики АН УССР, № 11, 36 - 42).
57. О локальной предельной теореме теории вероятностей (Успехи матем. наук, т. 3, вып 8, 187 - 194).
58. Развитие теории вероятностей в России (Труды Ин-та истории естествознания, т. 2, 390 - 425).
59. Русская школа теории вероятностей (М.Л. Изд-во АН СССР. Институт истории естествознания. "Труды совещания по истории естествознания 24-26 декабря 1946 г. Тезисы", 192 - 193).
60. Теория вероятностей (совм. с А.Н.Колмогоровым. «Математика в СССР за 30 лет», Гостехиздат, 701 – 727, 739-756).
61. Математика в Московском университете в XX в (до 1940г) (совм. с П.С.Александровым и В.В.Степановым. Историко-математические исследования, 1-я серия, вып. 1, 9 - 42).
62. Михаил Васильевич Остроградский (Сборник "Люди русской науки", ГИТТЛ, т. 1, 99-104).
63. Пафнутий Львович Чебышев (Сборник "Люди русской науки", ГИТТЛ, т. 1, 111-121).
64. Андрей Андреевич Марков (Сборник "Люди русской науки", ГИТТЛ, т. 1, 179-185).
65. Рецензия на книгу Г. Крамера «Случайные величины и распределения вероятностей» (Успехи математических наук, т. 3, вып. 6, 220-221).
66. Рецензия на книгу Г. Крамера «Математические методы статистики» (Успехи математических наук, т. 3, вып. 4, 184-186).

1949 год

67. Предельные распределения для сумм независимых случайных величин (совм. с А.Н.Колмогоровым. ГИТТЛ, 1 - 264).
68. Курс теорії імовірностей («Радянська школа», Київ, Львів, 1949, 1-128).
69. О локальной предельной теореме для случая бесконечной дисперсии (Труды Ин-та математики АН УССР, № 12, 22 - 30).
70. О некоторых свойствах предельных распределений для нормированных сумм (Украин. матем. журнал, т.1, 3-8).
71. О локальной теореме для областей нормального притяжения устойчивых законов (ДАН СССР, т. 66, № 3, 325 - 326).
72. О локальной теореме для предельных устойчивых распределений (Украин. математ. журнал, № 4, 3 - 15).
73. Последовательный анализ (Труды 2-ого Всесоюзного совещания по математической статистике, Ташкент, 5 - 23).
74. О работах Н.И.Лобачевского по теории вероятностей (Историко-математические исследования, 1-я серия, вып. 2, 129 - 136).

1950 год

75. Курс теории вероятностей (ГИТТЛ, 1 - 360).
76. Курс теорії імовірностей («Радянська школа», Київ, Львів, 1 - 387).
77. Элементарное введение в теорию вероятностей (совм. с А.Я.Хинчиным. ГТТИ, 2-е издание, 1 - 144).

78. Об области притяжения нормального закона (ДАН СССР, т.71, № 3, 425 - 428).
79. Несколько замечаний к теории областей притяжения устойчивых законов (совм. с В.С.Королюком. ДАН УРСР, № 4, 275 - 278).
80. Теория вероятностей и познание реального мира (Успехи матем. наук, т. 5, вып. 1, 3 - 23).
81. Про деякі питання викладання математики в університеті (газета «За радянські кадри», 18 вересня).
82. «Арифметика» Магницького (Зірка, 8.I., № 6).

1951 год

83. Fuggetlen valosznusegi valtozok osszegeinek hatareloszlassi (es A.N.Kolmogorov. Akademiai Kiado, Budapest,1-256).
84. Limit theorems for sums of independent random variables (American Mathematical Society, 1 - 82). (См. 1944 г., № 38).
85. О максимальном расхождении двух эмпирических распределений (совм. с В.С. Королюком. ДАН СССР, т.80, № 4, 525 - 528).
86. Про імовірне відхилення (Доповіді Академії наук УРСР, 1951, № 2).
87. Несколько замечаний о локальной теореме теории вероятностей (Ученые записки Киевского ун-та, т. X, вып. 1, Математический сборник № 5, 21 - 28).
88. О работах М.В.Остроградского по теории вероятностей (Историко-математические исследования, 1-я серия, вып. 4, 99 - 123).
89. М.В.Остроградський (совм. с Е.Я.Ремез. Віст. АН УРСР, № 9, 61 - 70).
90. Михаил Васильевич Остроградский (Успехи матем. наук, т.6, вып. 5, 3 - 25).
91. Попередне повідомлення про рукописи М.В.Остроградського (совм. с Е.Я.Ремез. Віст. АН УРСР, № 8, 52 - 63).
92. М.В.Остроградский (Украинский матем. журнал, т.3, № 3, 235 - 239).
93. М.В.Остроградский (Общество по распространению полит. и научн. знаний УССР, Киев, 1 - 41).
94. Історія математики як дисципліна викладання і як предмет наукового дослідження (Метод. збірник "Математика в школі", вип.5, 14 - 31).
95. Про бесіди з історії науки на уроках математики (Журн. "Радянська школа", № 3, 44 - 49).
96. М.В.Остроградський (газета "За радянські кадри", 28 вересня).
97. Видатний учений і педагог (совм. с Е.Л. Рвачевой. Київська правда, 23.08., № 189).
98. Механіко-математичний (За радянські кадри, 11.06., № 15).
99. Выдающийся ученый и педагог (Учительская газета, 22.09., № 76).
100. Выдающийся русский ученый (Красная звезда, 23.09., № 244).

1952 год

101. Элементарное введение в теорию вероятностей (совм. с А.Я.Хинчиным. ГИТТЛ, 3-е изд., 144 стр.).
102. Elementy rachunku pravdopodobienstwa (совм. с А.Я. Хинчиным. Warsawa, Panstwowe wydawnictwo naukowe, 1 - 155).
103. М.В.Остроградский (Очерки жизни, научной и педагогической деятельности, ГИТТЛ, 1 - 331).
104. З історії математики в Россії (Радянська школа, 1-40).
105. Декілька зауважень до статей О.А.Ілляшенка і Й.І. Гіхман (ДАН УРСР, № 1, 10 - 12).

106. Об одной задаче сравнения двух эмпирических распределений (совм. с Е.Л.Рвачевой. ДАН СССР, т. 82, № 4, 513-516).
107. Некоторые результаты о максимальном расхождении между двумя эмпирическими распределениями (ДАН СССР, т. 82, № 5, 661 - 663).
108. О распределении числа выходов одной эмпирической функции распределения над другой (совм. с В.С. Михалевичем. ДАН СССР, т. 82, № 6, 841 - 843).
109. Две теоремы о поведении эмпирических функций распределения (совм. с В.С. Михалевичем. ДАН СССР, т. 85, № 1, 25 - 27).
110. Порівняння эфективності деяких методів перевірки однорідності статистичного матеріалу (совм. с Ю.П.Студневым. ДАН УРСР, № 5, 359 - 363).
111. Зависимость неровноты пряжи от длины образца (Текстильная промышленность, № 3, 27 - 31).
112. О полных ортогональных системах тригонометрических функций (Вопросы элементарной и высшей математики, вып.1, изд. Харьк. Гос. Унив., 24 - 34).
113. Выдающийся русский ученый М.В.Остроградский (Изд. общества "Знание", Москва, 1 - 24).
114. Про філософські проблеми математики в зв'язку з її викладанням (Метод. збірник "Математика в школі", вип.7, 7-23).
115. Выдающийся русский ученый (Газета "Защитник Отечества", № 121, 26.5).

1953 год

116. Introducere elementara in calculul probabilitatilor (si A.Hinchin. Bucuresti, Editure Tehnica, 1 - 116).
117. О роли максимального слагаемого при суммировании независимых случайных величин (Украинский матем. журнал, т. 5, № 3, 291 - 298).
118. О некоторых свойствах срединного уклонения (Труды института математики и механики АН УзССР, вып. 10, ч. 1, 26-35).
119. Об одной работе П.Л.Чебышева, не вошедшей в полное собрание сочинений (Историко-математ. исследов., вып. 6, 215-222).
120. Две лекции по философским вопросам математики (Киевский Гос. Университет, Научные записки, т. XII, вып. VI, Математический сборник № 7, 5 - 36).
121. 250 років "Арифметики" Магницького (совм. с И.Б.Погребысским. Вісник АН УРСР, № 7, 53 - 63).
122. Про боротьбу матеріалізму з ідеалізмом в математиці (Вісник АН УРСР, № 11, 27 - 37).
123. Pafnutij Lvovic Cebysev (Prelozeno z knihy «Prehled dejin matematiky v Rusku», str 112-125, a dodatek 3, str 232-239)(Casop. pestov. mat., 78, № 1, 89-103)
124. Aleksandr Michajlovic Ljapunov (Prelozeno z knihy «Prehled dejin matematiky v Rusku», str. 133-143)(Casop. pestov. mat., 78, № 1, 105-112)
125. Колмогоров А.Н. (БСЭ, т. 22, стр. 13).
126. Советская школа теории чисел. Советская школа теории вероятностей (Mathematik,1, 3 - 68, Russische fachtexte fur den Hochschulunterricht, heft 6, Deutscher Verlag der Wissenschaften, Berlin).

1954 год

127. Курс теории вероятностей (ГИТТЛ, 2-ое изд., 1 - 411).
128. Limit Distributions for the Sums of Independent Random Variables (with A.N. Kolmogorov. Addison-Wesley, USA, 1 - 264).

129. Limit Distributions for the Sums of Independent Random Variables (Translated and annotated by K.L. Chung. With an Appendix by J.L. Doob. Addison-Wesley Publishing Company, Inc., Cambridge, Mass. IX + 264 pp)
130. Bevezetes a valoszinugszamitasba (совм. с А.Я. Хинчиным. Budapest, Mϋvelt nép Konivkiado, 1 - 141).
131. Elementarny wstep do rachunku prawdopodobienstwa (совм. с А.Я. Хинчиным. Warszawa, PWN, 1 - 158).
132. Elementarni uvod do theorie pravdepodobnosti (совм. с А.Я. Хинчиным. Praga, statni naklad. technike liter., 1 - 115).
133. Предельные теоремы для сумм независимых слагаемых и цепей Маркова (Украинский матем. журнал, т. 6, № 1, 5 - 20).
134. Локальная предельная теорема для плотностей (ДАН СССР, т. 95, № 1, 5 - 7).
135. Проверка неизменности распределения вероятностей в двух независимых выборках (с добавлением «Kriterien fur die Unveranderlichkeit derWahrscheinlichkeitsverteilung von zwei unabhungigen Stichprobenreihen» (Mathematische Nachrichten, Band 12, Heft 1/2, 29 - 66).
136. О локальной предельной теореме для одинаково распределенных независимых слагаемых (Wissensch. Zeitschr. Humboldt Univ. zu Berlin, N.4, 287 - 293).
137. Розвиток теорії імовірностей на Україні (совм. с И.И.Гихманом. Праці Київ. ун-та, № 11, 59 - 94).
138. Wahrscheinlichkeitsrechnung und Mathematische Statistik (und L.Kalujnin. Das Hochschul-wesen, № 8-9, 50 - 54).
139. Über die Formen der Kollektiven Wissenschaftlichen Arbeit in sowietischen Hochschulwesen (Das Hochschulwesen, n. 3, 1 - 6).
140. Über den Kampf zwischen Materialismus und den Idealismus in der Mathematik (zusam. L.A.Kalujnin. Wiss. Zeitschr. Technischen Hochschul. Drezden, B. 3, No 5, 631 - 638).
141. Despre lupta dintre materialism si idealism in matematica (Anal Romino-Sovietice, matem.-fizica, № 3, 68 - 80).
142. О математической жизни в ГДР (совм. с Л.А.Калужниным. Успехи матем. наук, т. 9, вып., 4, 133 - 154).
143. Подготовка математиков в Дрезденской высшей технической школе (Вестник высшей школы, № 11, 59 - 61).
144. Рецензия на полное собрание сочинений П.Л.Чебышева (УМН, т.9, вып. 4, 263 - 266).
145. Die Wahrscheinlichkeitsrechnung und die Erkenntnis der realen Welt (Naturwissenschaftliche Reiche, 22 oktober 1954, № 38, Wissenschaftliche Beilage des Forum).
146. Wahrscheinlichkeitsrechnung und ihre praktische Anwendung (Neues Deutschland, 24.X, № 250).
147. Wahrscheinlichkeitsrechnung und Produktion (Tagliche Rundschau, 16.X, № 240).

1955 год

148. Курс теории вероятностей (Пекин, 1 - 426).
149. Предельные распределения для сумм независимых слагаемых (совм. с А.Н.Колмогоровым. Пекин, 1 - 280).
150. Elementare Einfurung in die Wahrscheinlichkeitsrechnung (mit A.Chintschin. Berlin, Deut. Verlag der Wissen., 1 - 136).
151. О популярных лекциях по математике и ее истории (Киев, общество "Знание", 1 - 44).
152. А.Я.Хинчин (к 60-летию со дня рождения) (УМН, т.X, вып.3, 197 - 212).
153. М.В.Остроградский (БСЭ, 2-ое изд., т. 31, 346 - 347).

154. Über die Ausbildung der Mathematik und Physiklehrer in der Sowietunion (Math. und Physik in der Schule, № 11, 489 - 497).
155. Walka między materialismem a idealismem w matematyce (совм.с Л.А.Калужниным. Matematika, Czasopismo dla nauczicieli, № 5-6, 1 - 18).
156. Букви замість цифр (Зірка, 08.04., № 15).

1956 год

157. Курс теории вероятностей (пер. на китайский со 2-ого издания, Пекин, 1 - 449).
158. Über die Nachprüfung statistischer Hipotesen mit Hilfe der Variationsreihe (Bericht über die Tagung Wahrscheinlichkeitsrechnung und die Mathem. Statistik, in Berlin, vom.19 bis 22 october 1954, 97 - 107, Deutscher Verlag der Wissenschaften, Berlin).
159. Непараметрические задачи статистики. (Совместно с И.И.Гихманом и Н.В.Смирновым. Труды 3-го Всесоюзного математического съезда, т. 2, 47-48).
160. The main stages in the history of the theory of probability (Actes du VIII-e Congress International d'histoire des sciences Florence, 128 - 131).
161. A.I.Hincin (Analele Romino-Sovietice, Matematica-Fizica, 10, № 3, 115 - 124).
162. Masinele electronice de calculat (Analele Romino-Sovietice, Matemaatica-Fizica, № 4, 5 - 15).
163. О развитии математики на Украине (совм. с И.Б.Погребысским. Историко-математич. исследования, 1-я серия, вып.9, 403 - 426, вып.10, 766).
164. Развитие теории вероятностей на Украине (совм. с И.И.Гихман. Историко-математич. исследования, 1-я серия, вып.9, 477 - 536).
165. О работах Гаусса по теории вероятностей ("Карл Фридрих Гаусс", сборник статей к 100-летию со дня смерти, Изд. АН СССР, 217 - 240).
166. О некоторых задачах истории математики (Труды третьего Всесоюзного математического съезда (июнь-июль 1956 г., Москва), т.II, Краткое содержание обзорных и секционных докладов, 100 - 101, Москва, изд-во АН СССР).
167. Евгений Яковлевич Ремез (совм. с И.Б. Погребысским. УМЖ, т. 8, № 2, 218-222).
168. Предисловие к сборнику «Wahrscheinlichkeitsrechnung und Mathematischestatistik» (Deutsche Verlag der Wissenschaften, Berlin).
169. Begrussungsansprache ("Ber. Tag. Wahr. und Math. Stat". Berlin, 3-5).
170. Сучасні швидкодіючі обчислювальні машини (газ."Радянська Україна", 19 декабря).
171. 20 000 обчислень на секунду (журн. "Україна", № 12).
172. Гениальный математик (Наука и життя, № 2, 29 - 30).
173. Activitate ştiinţifică legată de practică (Scinteia, 11.05., № 3592).

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174. Курс теории вероятностей (Япония).
175. Lehrbuch der Wahrscheinlichkeitsrechnung (Akademie-Verlag, Berlin, 1 - 387).
176. Rozklady graniczne sum zmiennych losowych niezaleznych (i A.Kolmogorov. PWN, Warszawa, 1 - 262).
177. Элементарное введение в теорию вероятностей (совм. с А.Я.Хинчиным. ГИТТЛ, 4-е изд., 144 стр.).
178. О некоторых задачах истории математики (Труды третьего Всесоюзного математического съезда (июнь-июль 1956 г., Москва), т. III, обзорные доклады, секция истории математики, 579 - 583, Москва, изд-во АН СССР).
179. О некоторых задачах истории математики(совм.с И.Б.Погребысским. Укр. матем. журн., т.IX, № 4, 359 - 368).

180. О некоторых советских работах по теории информации (Труды I-ой Пражской конференции по теории информации, статистическим решающим функциям и случайным процессам, Прага, 21-28).
181. О некоторых задачах теории вероятностей (УМЖ, т.9, N4, 359 - 368).
182. Електронні цифрові машини (совм. с В.М.Глушковым. Вісник АН УРСР, № 9, 3 - 10).
183. Радянська математика за сорок років (ВАН УРСР, № 11, 29 - 41).
184. П.Л.Чебышев (БСЭ, т. 47, 81 - 82).
185. Über die Arbeiten von C.F. Gauss zur Wahrscheinlichkeitsrechnung (C.F.Gauss Gedenkband, Teubner, Leipzig, 193 - 204).
186. Mathematical Education in the USSR (Amer. Math. Monthly, v. 64, № 6, 389 - 408).
187. Despre formarea notiunilor matematice (Anallele Romino Sovietice, Matematicā–Fizicā, No 1, 89 - 100).
188. Перша нарада математиків України (ВАН УРСР, № 10, 69 - 72).
189. Первое совещание математиков Украины (Успехи матем. наук, т. XII, вып.6, 215 - 220).
190. Два совещания по теории информации (УМЖ, т. IX, № 3, 345 - 347).
191. Месяц у математиков Румынской Народной Республики (УМЖ, т.9, № 1, 111 - 112).
192. Вклад українських математиків (Наука і життя, № 8, 3-5).
193. Пути улучшения научной работы (Правда Украины, 24 апреля, № 97).
194. Про математичну освіту (Радянська Освіта, 14 августа, № 137).
195. За дальший розвиток математичної науки на Україні (совместно с И.Ф.Тесленко. Радянська Освіта, 20 июля, № 29).
196. Будущее науки необозримо (Правда Украины, 18 августа, № 193).
197. Мощь и зрелость нашей науки и техники (Правда Украины, 17 октября, № 244).
198. Успіхи українських математиків (Літературна газета, 30 липня, № 59).
199. Вивчайте іноземні мови (совм. с Л.А. Калужниным. За радянські кадри, 26.01., № 7).
200. Мир потрібний усім народам (Радянська Україна, 01.05., № 103).

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201. Элементарное введение в теорию вероятностей (совм. с А.Я.Хинчиным. Пекин, 132 стр.).
202. On limit theorems of the theory of probability (АН УССР, Институт Математики, Киев, 1 - 35).
203. Про одну задачу масового обслуговування (ДАН УРСР, № 5, 477 - 479).
204. Непараметрические задачи статистики (совм. с И.И. Гихманом и Н.В.Смирновым. Труды 3-го Всесоюзного математического съезда, т. 3, 320 - 334).
205. О критерии Вилкоксона сравнения двух выборок (Бюлл. Польской Акдемии наук, серия матем., астрон. физич. наук, т.6, № 10, 611 - 614).
206. Education Cientifico-Matematica, en la Union de Republicas Socialistas Sovieticas (Nueva Education, Peru, 23-30).
207. O nekterych ukolech historie matematiky (a I.B. Pogrebysskiy. Pokroku matematiky, Fisiky a astronomie, t.3, N5, 526- 535).
208. О некоторых задачах истории математики (ИМИ, 1-я серия, вып. 11, 47 - 62).
209. Об истории математики и ее значении для математики и других наук (совм. с И.Б.Погребысским, ИМИ, 1-я серия, вып. 11, 441-460).
210. О работах Л.Эйлера по теории вероятностей, теории обработки наблюдений, демографии и страхованию (Сборник "Л.Эйлер, 250-летие со дня рождения", Изд-во АН СССР, 184 - 209).

211. Очерк жизни, научного творчества и педагогической деятельности М.В.Остроградского (совм. с И.А.Мароном, в книге «Избранные труды М.В. Остроградского», Изд. АН СССР, 380 - 457).
212. Десять лет "Историко-математических исследований" (совместно с И.Б.Погребысским. УМЖ, т.Х, № 2, 229 - 230).
213. Десять лет "Историко-математических исследований" (совместно с И.Б.Погребысским. УМН, т. 13, № 5, 229 - 233).
214. Про міжнародні зв'язки Інституту математики АН УССР (совм. с В.Т. Гаврилюк,. ВАН УРСР, № 3, 66 - 67).
215. Республиканская конференция по вопросам статистических методов анализа и контроля производства (АН СССР, УМЖ, т.Х, № 1).
216. Рецензия на книгу Л.Шметтерер «Введение в математическую статистику» (АН СССР, "Теория вероятностей и ее применение", т.III, вып. 1, 118 - 120; «Новые книги за рубежом», серия А, № 9, 13 – 15).
217. Рецензия на книгу О.Оническу, Г.Михок, Ч.Ионеску Тульча "Теория вероятностей и ее приложения" (АН СССР, "Теория вероятностей и ее применение", т.III, вып. 1, 117 - 118; «Новые книги за рубежом», серия А, № 8, 20 – 23).
218. Рецензия на книгу А.Реньи "Исчисление вероятностей" (АН СССР, "Теория вероятностей и ее применение", т.III, вып. 1, 115 - 116).
219. Рецензия на книгу У.Гренандер и М.Розенблат "Статистический анализ стационарных временных рядов" (совм. со Н.П. Слободенюком) (АН СССР, "Теория вероятностей и ее применение", т.III, вып.4, 475 - 477, «Новые книги за рубежом», серия А, № 8, 24 – 27).
220. Рецензия на книгу Saunders L., Fleming R. «Математика и статистика для фармацевтов, биологов и химиков («Новые книги за рубежом», серия А, № 10, 9 – 10).
221. Рецензия на книгу Myslivec V. «Статистические методы в сельскохозяйственных и лесоводческих исследованиях» («Новые книги за рубежом», серия А, № 10, 11).
222. Наука, яка творить чудеса (Південна правда, 14 июня, № 117).
223. Математическая статистика и производство (Известия, 18 июля, № 145).
224. К итогам Всесоюзного совещания по теории вероятностей и математической статистике (Коммунист, 30 сентября, № 230, Ереван).
225. Математическое образование (Известия, 21 декабря, № 303).
226. Посланец дружбы (Правда Украины, 17.05., № 114).

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227. Лекции по теории массового обслуживания (Изд. КВИРТУ, вып. I, 1 - 104).
228. Lehrbuch der Wahrscheinlichkeitstheorie (Academy Verlag, Berlin, 2 auflage).
229. Granzverteilungen von Summen unabhengiger Zufallsgrossen (und A.Kolmogorov. Academie Verlag, 1 - 279).
230. Про одне узагальнення формул Ерланга (ДАН УРСР, № 4, 347 - 360).
231. Математическая статистика (совм. с И.И.Гихманом. Математика за 40 лет, ГИФМЛ, т. I, 797 - 808).
232. О методике определения расчетных нагрузок промышленных предприятий (совм. с Б.С. Мешелем. Электричество, № 2, 13 – 15).
233. Об оценке эффективности уточнения расчетов электрических нагрузок промышленных сетей (совм. с Б.С. Мешелем. Электричество, № 11, 70 - 72).
234. Несколько замечаний к двум работам Д.Баррера (Buletinul Institutului Politehnic, din iasi, seria noua, t.5(9), fas.1-2, 111 - 118).
235. Математика (МСЭ, т. 5, 1018 - 1024).

236. O nekterych ulohach teorie pravdopodobnesti (Pokroky matematiky, fisiky a astronomie IV, № 1, 1 - 12, Československa Akademie VĚD).
237. Исследования по теории вероятностей и математической статистике в системе АН УССР (УМЖ, т.II, № 2, 123 - 136).
238. О работах А.М.Ляпунова по теории вероятностей (ИМИ, вып.12, 135 - 160).
239. Розвиток теорії ймовірностей у роботах О.М.Ляпунова (АН УРСР, IМЗ, № 1, 133 - 139).
240. Про дослідження Л.Ейлера з теорії ймовірностей, теорії обробки спостережень, демографії та страхування (АН УРСР, IМЗ, № 1, 71 - 76).
241. O historii matematyki i jej znaczeniu dla matematyki i innych nauk (i I.Pogrebyski. **Roczn.** Polsk. Towarz. Mat., ser. II, Wiadomosti matematyczne, t.3, № 1, 49 - 64).
242. О роли математических методов в биологических исследованиях (АН СССР, Вопросы философии, № 1, 85 - 97).
243. Роль математики в биологических исследованиях ("Философские проблемы современного естествознания", Труды Всесоюзного совещания по философским вопросам математики, изд-во АН СССР, 494 - 498).
244. О математическом образовании в высших военно-учебных заведениях (Вестник противовоздушной обороны, № 12, 14 - 19).
245. Sur l'enseignement superieur des mathematiques (Portugal, Gaseta de Matematica, v.XX, N. 76-77, 15-25).
246. The formation of mathematical concept (Actes IX Congr. internat. Histoire Scinces, Barcelona-Madrid, 1-7 Sept. 1959, p. 472).
247. "Материализм и эмпириокритицизм" В.И.Ленина и философские вопросы математики (Збірник «Геніальний філософський твір. В.I.Ленина», Изд. АН УРСР, 300 - 318).
248. Про об’єктивний характер математичних понять (Збірник «Теоретична зброя комунізму», Державне видавництво політичної літератури УРСР, 249- 271).
249. "Despre lupta dintre materialism si idealism in matematica (совм. с Л.А. Калужниным, Gaseta matematica si fisica, seria A, vol.XI, № 7, 1959, Бухарест).
250. Рецензия на книгу Т.В.Андерсона «Введение в многомерный статистический анализ» (АН СССР, "Теория вероятностей и ее применения", т.IV, вып.2, 247 - 248; «Новые книги за рубежом», серия А, № 4, 14 – 16).
251. Рецензия на книгу М.Фиш "Теория вероятностей и математическая статистика" (АН СССР, "Теория вероятностей и ее применения", т.IV, вып.3, 365 - 367; «Новые книги за рубежом», серия А, № 2, 16 – 20).
252. Рецензия на книгу McCarthy P.J. «Введение в статистические рассуждения» («Новые книги за рубежом», серия А, № 4, 17 – 19).
253. Рецензия на книгу Quenoille M.A. «Основа статистических рассуждений» («Новые книги за рубежом», серия А, № 5, 17 – 18).
254. Рецензия на книгу Vincze István «Статистический контроль качества» (совм. с М.Г. Фариничем, «Новые книги за рубежом», серия А, № 6, 35 –37).
255. Рецензия на книгу Ionescu H.M. «Элементы математической статистики» («Новые книги за рубежом», серия А, № 7, 8 –9).
256. Рецензия на книгу Sahleanu V. «Математические методы в медико–биологических исследованиях («Новые книги за рубежом», серия А, № 7, 9 – 11).
257. Рецензия на книгу Gumbel E.J. «Статистика крайних» («Новые книги за рубежом», серия А, № 9, 14 – 15).
258. Рецензия на книгу Dugué D. «Трактат по теоретической и прикладной статистике» («Новые книги за рубежом», серия А, № 9, 19 – 21).
259. Рецензия на книгу Mises R.V. «Вероятность, статистика и истина» (2-ое изд.) («Новые книги за рубежом», серия А, № 10, 3 – 4).

260. Рецензия на книгу Pollaczek F. «Стохастические проблемы, возникающие при изучении формирования очереди у кассы и аналогичных явлений» («Новые книги за рубежом», серия А, № 11, 8 - 10).
261. Математика у природознавстві ("Наука і життя", № 4, 17 - 19).
262. Мрія стала дійсністю ("Наука і життя", № 10, 5).
263. Кібернетика (совм. с Е.А. Шкабара. "Наука і життя", № 12, 9 - 11).
264. Этого требует сердце (совм. с Н.М.Амосовым и М.Вепринцевым. Комсомольская правда, 5 апреля).
265. Объеднати зуссиля (Вечірній Київ, 3 июня, № 129).
266. Всесоюзное совещание по теории вероятностей (Советское Закарпатье, 8 октября, № 237).
267. Вікопомна подія (Вечірній Київ, 14.08., № 217).
268. На благо народов (Правда Украины, 21.10., № 245).

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269. Introduction a la theorie des probabilites (совм. с А.Я.Хинчиным. Dunod, Paris, 1 - 157).
270. Elementare Einfuhrung in die Wahrscheinlichkeitsrechnung (mit A.Chintschin. 2 verb. Aufl. Berlin, Otsch. Verl. Wiss.)
271. Лекции по теории массового обслуживания (КВИРТУ вып.1- 2, 1 - 207).
272. О предельных теоремах теории вероятностей (Proceedings of Intern. Congress of Mathemat., 14 - 21 August, 1958; 518 – 528. Cambridge Univ. Press, New Jork).
273. Über einige Aspekte der Entwiklung der Warteschlangen (Mathematik, Technik, Wirtschaft, Heft 4, 162 - 166).
274. Об одной задаче массового обслуживания (Труды 2-ого Пражского совещания по теории информации, статистическим решающим функциям и случайным процессам, Прага, 177 - 183).
275. О некоторых задачах теории массового обслуживания (Труды Всесоюзного совещания по теории вероятностей и математич. статистике, Изд. АН Арм.ССР, Ереван, 15 - 24).
276. О некоторых аспектах развития теории массового обслуживания (Mathematic Technic Wirtschaft, b.7)
277. К проекту "руководящих указаний" по расчету электрических нагрузок промышленных предприятий (совм. с Б.С.Мешелем. Промышленная энергетика, № 3, 41 - 44).
278. О математических методах в экономических исследованиях (Экономика Советской Украины, № 4, 74 - 81).
279. О некоторых разделах теории вероятностей, имеющих непосредственное отношение к проблемам биологии и медицины (Применения математических методов в биологии, Изд. ЛГУ. 6 - 16).
280. Математическая статистика и задачи практики (Вест. АН СССР, № 2, 38 - 43; Analele Romino-Sovietice, № 3, 43 - 49).
281. The teaching of probability theory and mathematical statistics (Report of 2 conf. on mathem.educ. in South Asia, Tata inst. of Fundamental Research, Bombay, 1 - 32).
282. О работах Н.В.Смирнова (совм. с А.Н.Колмогоровым и др. Теория вероятностей и ее применения, т. V, вып. 4, 437 - 440).
283. Биографический очерк и комментарии в книге Е.Е.Слуцкий «Избранные труды» (Москва, изд-во АН СССР, 5 - 11, 283 - 286).
284. А..Я.Хинчин (совм. с А.Н.Колмогоровым. Успехи матем. наук, т.XV, вып. 4, 97 - 110).

285. А.Я.Хинчин (Теория вероятностей и ее применения, т.V, вып.1, 3 - 6).
286. Г.П.Боев (совм. с Н.Чудаковым, Известия ВУЗов, Математика, № 1(14), 245 - 246).
287. Предисловие к книге Д.Я.Стройка "Коротка історія математики" (Радянська школа, Київ, 5 - 7).
288. Рецензия на книгу "Математика в СССР за сорок лет" (совм. с И.Б. Погребысским. УМН, т.XV, вып.5, 235 - 236).
289. Рецензия на книгу Zemanek Heinz «Элементарная теория информации» («Новые книги за рубежом», серия А, № 2, 17).
290. Рецензия на книгу Borel E. «Курс теории вероятностей», т. I, вып. 1 («Новые книги за рубежом», серия А, № 3, 11 – 12).
291. Рецензия на книгу Mittenecker E. «Планирование экспериментов и статистические выводы из них», (изд. 2) («Новые книги за рубежом», серия А, № 3, 12).
292. Рецензия на книгу Lindgren B.W., McElrath G.W. «Введение в теорию вероятностей и статистику» («Новые книги за рубежом», серия А, № 5, 15 – 16).
293. Рецензия на книгу Lukacs E. «Характеристические функции» («Новые книги за рубежом», серия А, № 11, 8 – 10).
294. Рецензия на книгу Derman C., Klein M. «Теория вероятностей и статистические выводы для инженеров» («Новые книги за рубежом», серия А, № 12, 9).
295. Математика і сучасність (газета «Радянська Освіта», 18 июля).
296. Мрії стають дійсністю (Радянська Україна, 17 травня, № 113).
297. Про кибернетику (совм. с Е.А.Шкабара. журн. "Дніпро", № 4, 138 - 142).
298. Життя і математика (Літературна газета, 8 июля, № 54).
299. Об юношеских математических школах (совм. с К.И.Швецовым. газета «Радянська Освіта», 20 февраля).
300. Чудово (Вечірній Київ, 23.01., № 19).

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301. Элементы программирования (совместно с В.С.Королюком и Е.Л.Ющенко. Москва, Физматгиз).
302. Курс теории вероятностей (Физматгиз, Изд. 3-е, 1-406).
303. Элементарное введение в теорию вероятностей (совм. с А.Я.Хинчиным. ГИФМЛ, изд. 5-ое, 1 - 144).
304. An Elementary introduction to the theorie of probability (and A.Khintchin. S.Francisco and London, Freeman and Co., 1 - 139).
305. Эхтимоллар незариясидан бошлангич маьлумотлар (совм. с Хинчиным. Ташкент, 1 - 126).
306. Теоретико-вероятностные основы статистического метода расчета электрических нагрузок промышленных предприятий (Изв. ВУЗов, Электромеханика, № 1, 90 - 99).
307. О статистических методах расчета и исследования электрических нагрузок промышленных предприятий (совм. с Б.С. Мешелем. Электричество, № 2, 81 - 85).
308. О статистической оценке режимов сетей населенных пунктов (совм. с Б.С. Мешелем. Электричество, № 6, 71 - 74).
309. Теория вероятностей и некоторые ее применения (Морской сборник, № 9, 31 - 41).
310. Некоторые вопросы кибернетики и статистики (Сборник «Кибернетику - на службу коммунизму», т.I, 55 - 70).
311. Asymptotic expansions in probability theory (with W. Koroluk and A. Skorochod. Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics and Probability, v. 2, 153 – 170. Univ. California Press).

312. Математические параметры универсальной цифровой автоматической машины "Киев" (совм. с В.М.Глушковым и Е.Л.Ющенко. Сборн. трудов по вычисл. математике и техн., вып. 2, Изд. АН УССР, 7 - 14).
313. Імовірностей теорія (УРЕ, т. 5,396-397).
314. Математическая статистика - мощное орудие в работе заводской лаборатории (Заводская лаборатория, № 10, 1251 - 1253).
315. Каждому специалисту нужно знать математическую статистику (Вестник высшей школы, № 12, 29 - 30).
316. А.Я.Хинчин (Математическое просвещение, № 6, 3 - 6).
317. О статье А.Я.Хинчина "Частотная теория Р.Мизеса и современные идеи теории вероятностей" (Вопросы философии, № 1, 91 - 92).
318. A.Ia.Khinchin (Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics and Probability, v. 2, 1 - 15).
319. Михаил Васильевич Остроградский («Люди русской науки», Москва, Физматгиз, 104 - 110).
320. Пафнутий Львович Чебышев («Люди русской науки», Москва, Физматгиз, 129 - 140).
321. Андрей Андреевич Марков («Люди русской науки», Москва, Физматгиз, 193 - 199).
322. Комментарии к работам М.В. Остроградского по теории вероятностей (М.В. Остроградский «Полное собрание трудов», изд-во Академии наук Украинской ССР, Киев, 343-344, 347-348, 365, 371-372).
323. Некоторые экономические проблемы технического прогресса ("Коммунист", № 10, 23 - 34).
324. Что такое теория надежности? (Математика в школе, № 6, 8 - 14).
325. Про математичну освіту в радянській школі (Викладання математики в школі, вип.1, изд. "Радянська школа", 5 - 12).
326. Предисловие к 3-му изданию книги А.Я.Хинчина "Цепные дроби". (Москва, Физматгиз).
327. Рецензия на книгу Ю.В.Линника "Разложения вероятностных законов" (АН СССР, Теория вероятностей и ее применения, т.VI, вып. 2).

328. Рецензия на книгу Lehmann E. «Проверка статистических гипотез» («Новые книги за рубежом», серия А, № 1, 11).
329. Рецензия на книгу Avondo Bodini G., Brambilla F. «Теория очередей» (часть I. Статистика) («Новые книги за рубежом», серия А, № 2, 6 – 8).
330. Рецензия на книгу Diamond S. «Информация и ошибка» («Новые книги за рубежом», серия А, № 2, 8 – 9).
331. Рецензия на книгу Scheffй H. «Дисперсионный анализ» («Новые книги за рубежом», серия А, № 3, 22).
332. Это нужно внедрять (О применении математических методов при решении прозводственно-экономических проблем) (Наука и жизнь, № 9, 26-29).

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333. The theory of probability (USA, Chelsea publ. Co., 1 - 459).
334. Курс теории вероятностей («Giao trinh ly thuyet xac suat») (Hanoi, 1 - 389).
335. Lehrbuch der Wahrscheinlichkeitsrechnung (Academie-Verlag, Berlin, 3 erweiterte Auflage, 1 - 393).
336. Limit Distributions for the Sums of Independent Random Variables (with A.N. Kolmogorov. Translated and annotated by K.L. Chung. With an appendix by G.L. Doob. Addison-Wesley Publishing Company, Inc.,Cambridge, Mass. IX + 264 pp. 2-nd print)

337. An elementary introduction to the theory of probability (and A.Khinchin. New York, Dover Publications, Inc., 1 - 130).
338. О понятии надежности (совм. с Ю.К.Беляевым. Вопросы радиоэлектроники, серия 12, вып. 13, 3 - 11).
339. Основные направления исследований в теории массового обслуживания (совм. с Ю.К.Беляевым и И.Н.Коваленко. Труды 6-го Всесоюзн. совещ. по теории вероятн. и матем. статистике, Вильнюс, 341 - 355).
340. Математика(УРЕ, т.8, 536-537).
341. Математична статистика(УРЕ, т.8, 537-538).
342. Математичні машини та прилади (УРЕ, т.8, 538-539).
343. Языком математики (Изд. «Знание», IX серия, Физика и химия, № 7, 1 - 30).
344. Einige Fragen der Kybernetik und Statistik (Sowietwissenschaft, Verlag Kultur und Fortschritt, 10, 1071 - 1090).
345. Применение математических методов при обработке результатов биологических наблюдений (совм. с С.В.Фоминым и Я.И.Хургиным. Сборник "Биологические аспекты кибернетики", Изд. АН СССР, 103 - 111).
346. Роль математики в развитии техники и производства (Математика в школе, № 1, 25 - 35).
347. Замечания к статье С.И.Петухова "Решение одной задачи теории массового обслуживания" (Морской сборник, № 2, 44 - 47).
348. А.И.Фетисов (совм. с А..Я.Маргулисом. Математика в школе, № 1, 76 - 77).
349. Прекрасный памятник выдающемуся ученому и педагогу (Математика в школе, № 3, 87).
350. Рецензия на книгу «Труды второй Пражской конференции по теории информации, статистическим решающим функциям и теории случайных процессов» («Новые книги за рубежом», серия А, № 1, 10 – 12).
351. Рецензия на книгу «Вероятность и статистика» (том, посвященный Г. Крамеру) («Новые книги за рубежом», серия А, № 4, 9 – 11).
352. Рецензия на книгу Mosteller F., Rourke R.E.K., Thomas G.B. «Теория вероятности с приложениями к статистике» («Новые книги за рубежом», серия А, № 5, 8 – 10).
353. Рецензия на книгу Plackett R.L. «Принципы регрессионного анализа» («Новые книги за рубежом», серия А, № 7, 12 – 13).
354. Рецензия на книгу Rashevsky N. «Математические принципы в биологии и их применения» («Новые книги за рубежом», серия А, № 11, 18 – 20).
355. Современная математика и строительство коммунизма (Народное образование, № 5, 24 - 28).
356. Математика вокруг нас (Наука и человечество, т.I, 106 - 117).
357. Математика вокруг нас (Наука и жизнь, № 8, 18 - 23).
358. Слово теории вероятностей (Знание - сила, № 5, 14 - 15).
359. Математика и надежность (Московская правда, 13 июня, № 137).
360. На уровне XIX века (Учительская газета, 21 июня, № 73).
361. Математика вторгается в жизнь (Московский комсомолец, 18 октября, № 229).
362. Патриарх русской математики (Восточно-сибирская Правда, 04.01., № 3).
363. Кафедра намечает планы (Московский университет, 13.02.).
364. Книжные полки науки (совм. с Баландиным, Реутовым и др. Неделя, 25.11-1.12.).

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365. The theory of probability (New York, Chelsea publ. Co., 1 - 471).
366. Елементарно въведение в теорията на вероятностите (совм. с А.Я.Хинчиным. Малка математическа библиотека, София, изд-во "Техника", 1 - 128).

367. Elementarny wstȇp do rachunku prawdopodobienstwa (совм. с А.Я. Хинчиным. Warszawa, PWN, 1 - 199).
368. Teoria de las probabilidades (совм. с А.Я. Хинчиным. Buenos - Aires, 1 - 153).
369. Лекции по теории массового обслуживания (совм. с И.Н.Коваленко. вып. 1 - 3, КВИРТУ, 1 - 316).
370. Элементы программирования (совм. с В.С.Королюком и Е.Л.Ющенко. 2-ое изд., Физматгиз, 1 - 348).
371. М.В.Остроградский. Жизнь и работа. Научное и педагогическое наследство (совм. с И.Б. Погребысским. Москва, изд-во АН СССР, 5-270).
372. Niektore zagadnienia cybernetyki i statystyki (Roczniki Polskiego Towarzystwa Matematycznego, seria II: Wiadomosci Matematyczne VII, 65 - 85).
373. Надежность (совм. с Я.Б.Шором. справочник "Автоматизация производства и промышленная электроника", Изд. БСЭ, т. 2, 348-353).
374. О проблемах истории математики в России и СССР и о работах в этой области за 1956 - 1961 гг. (совм. с И.Б. Погребысским, И.З. Штокало и А.П. Юшкевичем. ИМИ, 1-я серия, вып. 15, 11 - 36).
375. Проблемы истории математики нового времени (совм. с К.А.Рыбниковым и Н.И.Симоновым. ИМИ, 1-я серия, вып. 15, 73 - 96).
376. О работах А.Н.Колмогорова по теории вероятностей (УМН, т. 18, вып. 5, 5 - 11).
377. О работах А.Н.Колмогорова по теории вероятностей (совм. с Н.В.Смирновым. АН СССР, Теор. вероятн. и ее применения, т. VIII, № 2, 167 - 174).
378. А.Н.Колмогоров как педагог (совм. с П.С.Александровым. УМН, т. 18, вып. 5, 115 - 120).
379. А.Н.Колмогоров (Математика в школе, № 2, 67 - 68).
380. Первые шаги в развитии счета (Математика в школе, № 4, 5 - 10).
381. Лекции по истории математики. Вводная лекция (Математика в школе, № 1, 3 - 13).
382. Математика древних народов Двуречья (Математика в школе, № 6, 3 - 12).
383. Математика в биологии и медицине (Биология в школе, № 5, 73-79).
384. О программированном обучении (Морской сборник, № 9, 13 - 20).
385. Предисловие редактора и заключительная статья в книге А.Я.Хинчина «Педагогические статьи» (Изд-во Акад. пед. наук РСФСР, 3 - 12, 180 - 203).
386. Предисловие редактора и заключительная статья «О некоторых постановках задач и результатах теории массового обслуживания» в книге А.Я.Хинчина "Работы по теории массового обслуживания" (Физматгиз, 4 - 7, 221 - 235).
387. За борбата между материализма и идеализма в математиката (совм. с Л.А. Калужниным, Математика и физика, № 1, 1 - 9, № 2, 1 - 5, София).
388. Предисловие к книге В.А.Вышенского, М.И.Ядренко "Збiрник задач для учасникiв математичних олiмпiад" (Киев, изд-во "Радянська школа").
389. Предисловие к книге А.М.Кондратова "Числа вместо интуиции" (изд-во "Знание", IX-ая серия "Физика и химия", № 8).
390. Предисловие к книге Р.Х.Зарипова "Кибернетика и музыка" (изд-во "Знание", IX-ая серия "Физика и химия", № 18).
391. Рецензия на книгу Cox D.R. «Теория восстановления» («Новые книги за рубежом», серия А, № 4, 14 – 17).
392. Рецензия на книгу Saaty Th.L. «Элементы теории очередей» («Новые книги за рубежом», серия А, № 7, 15 – 18).
393. Рецензия на книгу Parratt L.G. «Вероятность и экспериментальные ошибки в естествознании» («Новые книги за рубежом», серия А, № 8, 25 – 27).
394. Рецензия на книгу Beneљ J. «Статистическая динамика теории регулирования» («Новые книги за рубежом», серия А, № 10, 15 – 17).

395. Рецензия на книгу Bailey N.T. «Введение в математическую теорию генетического сцепления» («Новые книги за рубежом», серия В, № 10, 18 – 19).
396. Рецензия на книгу Rosenblutt M. «Случайные процессы» («Новые книги за рубежом», серия А, № 11, 27 – 30).
397. Надежность - ключ автоматизации (Известия, 24 апреля, № 157).
398. Ключевая проблема современной техники (Учительская газета, 1 октября, № 116).
399. Выдающийся математик современности (Известия, 24.04., № 98).
400. Рисовать картины будущего ("Искусство кино", № 6, 108-116).

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401. Элементарное введение в теорию вероятностей (совм. с А.Я.Хинчиным. изд-во «Наука», изд. 6-е, 1 - 146).
402. Elementen der programmirung (mit Koroluk und Justenko, Teubner, Leipzig, 1 - 327).
403. Bevezetes a programozasba (Koroluk und Juscenko, Budapest, v.1 und v.2, 1 - 228, 1 - 204).
404. О критерии знаков (Доклады Болгарской АН, т.17, № 9, 793 - 796).
405. О ненагруженном дублировании (Известия АН СССР, Техническая кибернетика, № 4, 3 - 12).
406. О дублировании с восстановлением (Известия АН СССР, Техническая кибернетика, № 5, 111 - 118).
407. Об определении оптимального числа причалов (совм. с М.Н. Зубковым. «Морской сборник», № 6, 30 - 39).
408. Статистические методы в теории надежности (Стандартизация, № 6, 16 - 23; изд-во «Знание», 1-25).
409. Роль математики в развитии современного естествознания (Сборн. "Диалектика в науках о неживой природе", Изд. «Мысль», 45 - 85).
410. Статистически методи в теория на сигурността (Физико-математическое списание, т.7, вып. 2, 120 - 134).
411. Шо е теория на масовото обслужване (Физико-математическое списание, т.7, вып.3, 200 - 211).
412. За подготовка на учителя по математика (Математика и физика, София, № 4, 4 - 11, № 5, 1 - 9).
413. Математически страни на теорията на сигурността (Новости в автоматиката и телемеханиката, кн. 4, София, 5 - 34).
414. Из история на математиката (Математика, № 2, 6 - 9, София).
415. Mathematik in Biologie und Medizin (Biologie in der Schule, B.13, H.3, 107 - 111).
416. Возможности и нужды молодой науки (журн. "Научно-техн. общества СССР", № 12, 2 - 5).
417. О теории массового обслуживания (Математика в школе, № 3, 10 - 20).
418. Наука о случайном (Детская энциклопедия, 2-ое изд., 452 - 461).
419. О математических методах теории надежности (Детская энциклопедия, 2-ое изд., 461 - 465).
420. О воспитании учителя математики (Математика в школе, № 6, 8 - 20).
421. Предисловие к книге Эмиля Бореля "Вероятность и достоверность" (изд. 2-ое, М. "Наука").
422. Рецензия на книгу M.Rosenblutt "Random Processes" (Annals of Mathematical Statistics, v.35, no.4, 1832 - 1833).
423. Рецензия на книгу Walsh J.E. «Справочник по непараметрическим статистикам» («Новые книги за рубежом», серия А, № 1, 18 – 20).

424. Рецензия на книгу «Математические проблемы в биологических науках» (под редакцией R.E. Bellman) (совм. с Загорской И.Б.. «Новые книги за рубежом», серия А, № 3, 24 – 28).
425. Рецензия на книгу Takбcs L. «Введение в теорию очередей» («Новые книги за рубежом», серия А, № 4, 15 – 18).
426. Рецензия на книгу Freeman H. «Введение в теорию статистических выводов» («Новые книги за рубежом», серия А, № 10, 14 – 17).
427. Рецензия на книгу Bodion G. «Диалектическая теория вероятностей, включающая ее классическое и квантовое исчисление» (совм. с Г.А.Зайцевым. «Новые книги за рубежом», серия А, № 12, 9 – 11).
428. Рецензия на книгу «Исследования по порядковым статистикам» (под редакцией A.E. Sarhan и B.G. Greenberg) («Новые книги за рубежом», серия А, № 12, 11 – 13).
429. Стандарт высокого качества (совм. с А.И.Бергом, Я.М.Сориным и Я.Б.Шором. Известия, 9 января, № 8).
430. Великий рыцарь науки (Красная звезда, 13 февраля, № 39).
431. Надежность и математика (Неделя, 10 - 16 мая 1964, № 20).
432. Качеством можно управлять (совм. с Я.М.Сориным. Известия, 20 июля, № 172).
433. По някой в проси на училищното образование (Учительско дело, София, 23 октября, № 84).
434. Сколько стоит нагнетатель? (Неделя, 5-11.07).
435. Возможности и нужды молодой науки (Журнал «Научно-технические общества СССР», № 12, 2-5).

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436. Курс теории вероятностей (Наука, изд. 4-ое, 1 - 400).
437. Математические методы в теории надежности (совм. с Ю.К. Беляевым и А.Д. Соловьевым. изд. Наука, 1 - 524).
438. Lehrbuch der Wahrscheinlichkeitsrechnung (Academie-Verlag, Berlin, 4 Auflage).
439. Об одной задаче теории дублирования (Морской сборник, № 1, 14 - 23).
440. Об одном аспекте проблемы оператор-машина (Вопросы радиоэлектроники, вып. 25, серия 12, 3 - 11).
441. Математика (УРЕ, т. 17, 465-468).
442. Обеспечение качества, надежности и долговечности массовой продукции и статистические методы исследования (Стандартизация, № 5, 4 - 6).
443. К вопросу надежности сельскохозяйственной техники (совм. с В.П.Поповым. Тракторы и сельхоз. машины, № 6, 21 - 24).
444. O teorii obslugi («Matematika», czasopismo dla nauczycieli, Warszawa, № 1(85), 1 - 9, № 2(86), 50 - 54).
445. Предисловие к книге Т.Саати "Элементы теории массового обслуживания и ее приложения" (Изд-во "Сов. Радио", 5 - 14).
446. Über Bednungstheorie (Math. in der Schule, № 5, 325 - 340).
447. С езика на математиката (Народна просвета, София, 1 - 47).
448. Математиката на древните народи от Месопотомия (Математика, БНР, № 2, 1 - 4, № 3, 6-10).
449. Математические модели и программированное обучение (совм. с Ю.И.Берилко. Советская педагогика, № 10, 140 - 142).
450. Символ прогрессивных идей и методов в педагогике (Вестник Высшей Школы, № 5, 13 - 20).
451. О перспективах математического образования (Математика в школе, № 6, 2- 11).

452. Perspektiven der mathematischen Ausbildung (Band "Problem des Mathematikunterrichts", Berlin, 28 - 59).
453. Meine Idealvorstellung vom Mathematiklehrer (Band "Problem des Mathematikunterrichts", Berlin, 88 - 104).
454. Mathematical education in the USSR (The Australian Mathematics Teacher, v.21, № 3, 49 - 59).
455. Итоги открытого конкурса на учебники по математике (совм. с И.С.Петраковым. Математика в школе, № 2, 4 - 9).
456. К столетию Московского математического общества (Математика в школе, № 2, 95 - 96).
457. Предисловие к книге Э. Гумбель «Статистика экстремальных значений» (Москва, Мир, 5-7).
458. Рецензия на книгу Христов Вл. «Основы теории вероятностей и математической статистики с приложениями в технике и экономике» («Новые книги за рубежом», серия А, № 2, 15 – 18).
459. Рецензия на книгу Bergstrqm H. «Предельные теоремы для сверток» («Новые книги за рубежом», серия А, № 8, 20 – 22).
460. Рецензия на книгу Spitzer F. «Принципы случайного блуждания» («Новые книги за рубежом», серия А, № 9, 32 – 33).
461. Рецензия на книгу «Концепция информации в современных науках» («Новые книги за рубежом», серия А, № 11, 5 – 7).
462. Теория массового обслуживания (Экономическая газета, 3 февраля, № 5).
463. Слагаемые надежности машин (совм. с А.Башкирцевым и В.Поповым. Правда, 25 февраля, № 56).
464. Статистические методы в промышлености (совм. Я.Б.Шором. Известия, 14 апреля, № 88).
465. Проблемы повышения надежноости (Труд, 2 июля 1965, № 153).
466. О некоторых вопросах кибернетики (Советская Аджария, 1 сентября, № 171).
467. Человек. Время. Надежность. (Московская правда, 21 января).
468. Брак в солидных обложках (совм. с А.Н. Колмогоровым, А.А. Дородницыным, А. Вайнштейном. Известия, 23.01., № 19).

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469. The theory of probability (Chelsea publ., New York, 1 - 479).
470. Введение в теорию массового обслуживания (совм. с И.Н.Коваленко. Наука, 431 стр.).
471. Математическая теория надежности и ее применения в машиностроении (Труды конференции "Надежность и долговечность машин и приборов", вып. 1, 73 - 80).
472. Математические вопросы теории надежности (совм. с Ю.К.Беляевым и И.Н.Коваленко. Итоги науки, ВИНИТИ, 7 - 53).
473. Об эффективности восстановления резервных устройств (Сборник "Предельные теоремы и стат. выводы", АН УзССР, Институт математики им. В.И. Романовского. Изд-во «Фан», Ташкент, 44 - 59).
474. О некоторых предельных теоремах в теории резервирования с восстановлением (Вопросы радиоэлектроники, сер.12, вып.13, 3 - 15).
475. Роль истории физико-математических наук в развитии современной науки ("История и методология естественных наук", Изд. МГУ, вып.5, 5 - 14).
476. Об алгоритмическом подходе к обучению (совм. с Б.В.Бирюковым. предисловие к книге Л.Н.Ланда "Алгоритмизация в обучении", изд. Просвещение, 11 - 33).

477. О методах комбинаторики в теории вероятностей и математической статистике (Математика в школе, № 5, 18 - 21).
478. О математических моделях в педагогике (Вестник высш. школы, № 9, 25 - 31).
479. Mathematik rund um uns (Wissenschaft und Menschheit, 78 - 89).
480. Т.А.Сарымсаков (УМН, т.XXI, вып.3, 248 - 253).
481. А.П.Юшкевич (совм. с И.Г.Башмаковой, Математика в школе, № 4, 84 - 85).
482. О вступительных экзаменах в МГУ в 1966 г. (совм. с М.К.Потаповым. Математика в школе, № 6, 37 - 49).
483. Предисловие к книге Ж. Мот «Статистические предвидения и решения на предприятии» (Москва, Прогресс, 5-9).
484. Предисловие к книге Б.А. Козлова и И.А. Ушакова «Краткий справочник по теории надежности радиоэлектронной аппаратуры» (Москва, Советское Радио, 3-4).
485. Рецензия на «Историко-математические исследования», вып. 16 (УМН, т.XXI, вып.3, 262 - 264).
486. Рецензия на книгу Drooyan J., Hadel W. «Программированное введение в числовые системы» («Новые книги за рубежом», серия А, № 2, 6).
487. Рецензия на книгу Barlow R.E., Proschan F. «Математическая теория надежности» («Новые книги за рубежом», серия А, № 8, 24 – 26).
488. Шостаковичу – 60 лет (Советская музыка, № 9, 15).
489. Алгебра прогресса (Московская правда, 20 апреля, № 92).
490. Генеральная задача (Горьковский рабочий, 9 июня, № 133).
491. Математика и современность (Красная звезда, 16 августа).
492. Молоді – математичний журнал (совм. с М.Чайковским, М.Ядренко, А.Конфоровичем, Г.Саковичем. Радянська освіта, 15 октября, № 83).

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493. The Theory of Probability (Chelsea Publishing Company, New York, 1 - 529).
494. On some stochastic problems of reliability theory (together Yu.Beliaev and A.Soloviev. Proc. of the 5-th Berkeley symp. on math. statistics and probabil., v.3, 259 - 270).
495. Some theorems on standbys (Proc. of the 5-th Berkeley symp. on math. statistics and probabil., v. 3, 285 - 291).
496. О связи теории суммирования независимых случайных величин с задачами теории массового обслуживания и теории надежности (Revue Roumaine de Math. pures et appliquees, t.XII, № 9, 1243 - 1253).
497. О некоторых задачах теории массового обслуживания (совм. с И.Н.Коваленко. Изв. АН СССР, Техническая кибернетика, № 5, 88 - 100).
498. Асимптотические методы в вопросах исследования операций (Материалы симпозиума "Исследование операций и анализ развития науки", Изд. АН СССР, ч.2, 23 - 37).
499. О надежности оператора в системе человек - машина (Материалы конф. "Научн. и практ. проблемы больших систем", сб. 2, 76 - 80).
500. Применение теории массового обслуживания к задачам больших систем (совм. с И.Н.Коваленко. Научные и практ. проблемы больших систем, 54 - 55).
501. Теория вероятностей и задачи стандартизации (Стандарты и качество, № 1, 75 - 77; № 2, 72 - 75; № 4, 76 - 78,;№ 5, 77 - 79; № 7, 31 - 34).
502. Стандартизация и математика (Стандарты и качество, № 6, 86 - 92).
503. Развитие теории вероятностей в Московском университете (Вестник МГУ, Серия «Математика», № 6, 30 - 51).
504. Математика в СССР за 50 лет (Математика в школе, № 6, 5 - 13).

505. О проблемах школьного математического образования (Физико - математическо списание, т. X, кв. 4, 311 - 324).
506. О книге Б.Е.Бердичевского "Оценка надежности аппаратуры автоматики" (Стандарты и качество, № 8, 59).
507. Предисловие к книге Э.Николау "Введение в кибернетику" (изд-во "Мир").
508. Предисловие к книге Л.Феликса "Элементарная математика в современном изложении" (изд-во "Просвещение").
509. Предисловие к книге Дж. Букан, Э. Кенигсберг «Научное управление запасами» (Москва, Наука, 6-7).
510. Рецензия на книгу Walsh J.E. «Справочник по непараметрическим статистикам» (том II) («Новые книги за рубежом», серия А, № 5, 29 – 31).
511. Рецензия на книгу Atkinson R.C., Bower G.H., Grothers E.J. «Введение в математическую теорию обучения» («Новые книги за рубежом», серия А, № 6, 34 – 36).
512. Рецензия на книгу Lamperti J. «Probability» («Новые книги за рубежом», серия А, № 7, 12 - 14).
513. Рецензия на книгу Schmetterer L. «Введение в математическую статистику» (изд. 2) («Новые книги за рубежом», серия А, № 8, 34 – 36).
514. Рецензия на книгу Richter H. «Теория вероятностей» (изд. 2) («Новые книги за рубежом», серия А, № 10, 20 - 22).
515. Проблемы больших систем (совм. с Л. Дудкиным и Б. Коробочкиным. Экономическая газета, № 48).
516. На т'рсещите п'тя к'м знанието (Студентска трибуна, бр. 2 (542), год.XIX, София, 3 октября).
517. Математика. Кибернетика (Книжное обозрение, 10.06.).
518. А двоечник спокоен (совм. с Н. Виленкиным, Н. Розовым, А. Халамайзером. Литературная газета, № 33).
519. Теория становится жизнью (газета «Московский университет». 21 февраля, № 11).

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520. Limit Distributions for the Sums of Independent Random Variables (with A.N.Kolmogorov. 2-nd rev. ed. — Reading, MA: Addison-Wesley, USA, ix + 293 p.)
521. Lehrbuch der Wahrscheinlichkeitsrechnung (Academy Verlag, Berlin, 5 erweiterte Auflage, 1 - 399).
522. Osnovi teorije vjerojatnosti (совм.с А.Я. Хинчиным. Zagreb, Technicka kniga, 1 - 114).
523. Introduccion a la teoria de las probabilidades (совм. с А.Я. Хинчиным. Barcelona, Montaner y Simon, S.A., 1 - 182).
524. The theory of probability (Chelsea Publishing Company, New York).
525. Metode matematice in teoria sigurantei (совм. с Ю.К.Беляевым, А.Д.Соловьевым. Bucuresti, 1 - 560).
526. Metody matematiczne w teorii nezavodnosci (совм. с Ю.К.Беляевым, А.Д.Соловьевым. Warszawa, 1 - 484).
527. Introduction to Queuing theorie (with I.N.Kovalenko. Ierusalem, 1 - 281).
528. Mathematische Methoden der Zuverlässigkeitstheorie (mit Beljaev und Solovjev. Berlin, Academie-Verlag, B.1, 1 - 222, B.2, 1 - 262).
529. Беседы о математической статистике (Знание, Математика и кибернетика, № 6, 1-48).
530. Применение вероятностных методов (совм. с Е.Ю.Барзиловичем и Е.В.Чепуриным. Изв. АН СССР, Техническая кибернетика, № 6, 27 - 40).
531. Асимптотические методы в теории надежности (Стандарты и качество, № 2, 63 - 64).

532. О ненагруженном резервировании с восстановлением (совм. с Ю.Насром. Автоматика и телемеханика, № 7, 105 - 111).
533. Наука, практика, надежность (Научная мысль, Вестник АПН, 28 - 37).
534. О математических методах в здравоохранении и медицинском приборостроении (Новости приборостроения, Москва, ВНИИ мед. приборостроения, 5 - 7).
535. Вопросы математизации современного естествознания (Сборник "Диалект. материал. и современное естествознание", изд. Наука, 171 - 206).
536. Die Mathematik in der UdSSR wahrend der letzen 50 Jhare (Math. in der Schule, J.6, N.7, 483 - 495).
537. Современная математика и будущий инженер (Вестник Высшей школы, № 1, 45 - 53).
538. Задачи стандартизации и математика (Сборник «Актуальные проблемы стандартизации», изд. "Стандарты", 16 - 31).
539. O matematickom obrazovanju (Prilog broju, 68, 1 - 4, Югославия).
540. Статистическое мышление и школьное математическое образование (Математика в школе, № 1, 8 - 15).
541. Теория вероятностей и комбинаторика (совм. с И.Е.Журбенко. Математика в школе, № 2, 72 - 84, № 3, 30 - 49).
542. О месте риторики в преподавании математики (Математика в школе, № 3, 89).
543. Предисловие и заключительная статья к книге A.Ya.Khinchin "The Teaching of Mathematics" (The English Universities Press LTD, London, p. IX – XX, 102 – 107, 113, 115 – 116).
544. Предисловие к книге "Программированное обучение за рубежом" (совм. с И.И. Тихоновым. изд-во «Высшая школа», 5 - 19).
545. Рецензия на книгу Chakravarti I.V., Laha R.G., Roy J. «Справочник по методам прикладной статистики» (в 2-х томах) («Новые книги за рубежом», серия А, № 5, 29-31).
546. Рецензия на книгу «Избранные работы Эгона Пирсона» («Новые книги за рубежом», серия А, № 6, 21-22).
547. Рецензия на книгу Moran P.A.P. «Введение в теорию вероятностей» («Новые книги за рубежом», серия А, № 8, 25-27).

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548. Курс теории вероятностей (Изд. Наука, изд. 5-ое, 1 - 400).
549. The theory of probability (MIR publishers, 1 - 405).
550. Элементарное введение в теорию вероятностей (совм. с А.Я.Хинчиным. Пер. на японский, Токио, 6 - 161).
551. Элементарное введение в теорию вероятностей (совм. с А.Я.Хинчиным. Пер. на арабский, изд-во «Мир», 1 - 202).
552. Mathematical methods of reliability theory (with Beljaev and Solovyev. Academic Press, New York, 1 - 506).
553. Elements de programmation sur ordinateurs (avec Koroliouk et Iouchtchenko. Dunod, Paris, 1 - 362).
554. Об одной теореме переноса (совм. с Х.Фахимом. ДАН СССР, т. 187, № 1, 15 - 17).
555. Резервирование с восстановлением и суммирование случайного числа слагаемых (Венгрия, Труды совещания по теории надежности в Тихани, 16 – 19 сентября 1969 г., 1-9)
556. О связи задачи суммирования случайных величин с теорией резервирования (Сборник научных статей, вып. № 4, РВВИАУ им. Алксниса, Рига, 23 - 34).
557. О статистических методах в теории надежности (Сборник "Основные вопросы надежности и долговечности машин, МАТИ, 22 - 42).

558. Об основных направлениях математических исследований в теории надежности (Труды совещания по матем. теории надежности, Ташкент, изд. АН УзССР, 3 - 18).
559. О статистических задачах теории надежности (совм. с Ю.К.Беляевым. Труды совещания по матем. теории надежности, Ташкент, изд. АН УзССР, 19 - 25).
560. О применении нормального распределения при обработке опытных данных (Вестник машиностроения, № 2, 12 - 13).
561. О роли и месте теории надежности в процессе создания сложных систем (совм. с И.А.Ушаковым, Б.А.Козловым. Сборник «Теория надежности и массовое обслуживание», изд. «Наука», 14 - 32, Предисловие, 7 - 13).
562. Всесторонне развивать науку о надежности (совм. с Я.М. Сориным. Изд-во Стандартов, сборник «Надежность и контроль качества», № 1, 3-9).
563. Математические методы–основа контроля качества и надежности промышленной продукции (Надежность и контроль качества, № 2, 3 - 13).
564. Random processes and their application to demography and Insurance (8-th ASTIN Colloquium, Sopot, Sept. 1969).
565. Some remarks concerning the reports presented by P. Thyrion, H. Bühlmann and R. Buzzi (8-th ASTIN Colloquium, Sopot, Sept. 1969).
566. Несколько замечаний к одной теореме И.Н.Коваленко (совм. с Б.Фрайером. Литов. матем. сборник, т. IX, № 3, 463 - 470).
567. Вопросы теории испытаний изделий на качество и надежность (Стандарты и качество, № 5).
568. О статье Я.М.Сорина "Задачи служб надежности на современном этапе" (Стандарты и качество, № 9, 52 - 53).
569. Об одной математической модели в задачах инженерной психологии (Zastosowania matematyki, юбилейный том, посвященный Хуго Штейнхаусу, v. X).
570. Математические методы в стандартизации (совм. с Я.Б. Шором. Стандарты и качество, № 1, 8 – 13).
571. Methody matematiczne w normalizacji (Normalizacja, № 4, 181 - 184).
572. Wahrscheinlichkeitsrechnung und Kombinatorik (zus. Schurbenko. Mathem. in der Schule, № 3, 170 - 210, № 4, 284 - 295).
573. О преподавании биологии и математической статистики (Биология в школе, № 5, 42 - 43).
574. Об образовании математических понятий («Математика в современном мире», Изд. «Знание», серия «Математика и кибернетика», № 9, 3 - 10).
575. О пропаганде математических знаний (Слово лектора, № 1, 92 - 95).
576. О формирану наставника математике (Nastava matematike i fizike, serija B, XVII - XVIII).
577. О математике во ВТУЗе (Сборник научн. статей, № 4, РВВИАУ им. Алксниса, Рига, 5 - 16).
578. Ленинская теория познания и вопросы математизации современного знания (Вестник АН СССР, № 5, 53 - 60).
579. Леонтий Магницкий и его «Арифметика» (совм. с И.Б.Погребысским. Математика в школе, № 6, 78 - 82).
580. Сергей Натанович Бернштейн (совм. с А.Н.Колмогоровым. Теория вероятностей и ее применения, т.12, вып.3, 532-535).
581. Виктор Иосифович Левин (совм. с А.Я.Маргулисом. Математика в школе, №6, 65).
582. Предисловие к сборнику «Теория надежности и массовое обслуживание» (Москва, Наука, 7 – 13).
583. Предисловие к книге Барлоу, Прошан "Матем. теория надежности" (Москва, Изд. «Советское радио»).

584. Предисловие и примечания редактора к книге Э. Бореля «Вероятность и достоверность» (Москва, Наука, 5-6, 105-110).
585. Предисловие к книге А.Реньи "Диалоги о математике" (изд-во «Мир», 5 - 19).
586. Предисловие к книге С.С. Демидова «Проблемы Гильберта» (Москва, изд-во «Знание»).
587. Рецензия на книгу А.П.Юшкевича «История математики в России» (совм. с И.Г. Башмаковой. Математика в школе, № 4, 86 –87).
588. Рецензия на книгу Pal Revesz «The laws of large numbers» (Успехи математических наук, т. XXIV, вып. 2, 260 –261).
589. Технический прогресс и математическое образование (Социалистическая индустрия, 1 августа, № 28).

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590. Элементарное введение в теорию вероятностей (совм. с А.Я.Хинчиным. изд-во «Наука», изд. 7-е).
591. Lehrbuch der Wahrscheinlichkeitsrechnung (Academie-Verlag, Berlin, 6 Auflage).
592. A megbizhatōsāgelmelet matematikai modszerei (совм. с Ю.К.Беляевым и А.Д.Соловьевым. Müszaki könyvkiadó, Budapest).
593. О некоторых нерешенных задачах теории массового обслуживания (Труды VI Международного конгресса по телетраффику, Мюнхен, 9 - 15 сентября 1970, 227/1 - 227/17).
594. Статистические методы в теории надежности (ВНИИПТМАШ, Труды. Надежность подъемно-транспортных машин, вып. 1(96), 6 - 12).
595. О математических методах в теории надежности (Венгрия, Труды совещания по теории надежности в Тихани, 14 - 19 сентября 1970).
596. Проблемы надежности (Техника и вооружение, № 4, 38 - 39).
597. Развитие прикладных методов теории вероятностей (История отечественной математики, Киев, т.4, книга 2, 7 - 13, 52 - 62).
598. Итоги дискуссии по поводу статьи П.С. Суханова «Об одном противоречии системы предпочтительных чисел» (совм. с С.В. Крейтером. Стандарты и качество, № 8, 25 - 27).
599. Проблемы математизации современного естествознания (Сборник «Диалектика и современное естествознание», изд. «Наука», 82 - 102).
600. Научно-технический прогресс и математика (изд. «Знание», серия «Математика и кибернетика», № 10, 3 - 17).
601. О будущем прикладной математики (Сборник "Будущее науки", вып. 3, изд. «Знание», 82 - 102), (Наука и жизнь, № 1, 42 –47, 71).
602. Nauczanie a efektiwność badań naukowych (Zagadnenia naukoznawstwa, 3 (23), 70 - 78).
603. Предисловие к брошюре А.Реньи "Письма о вероятности" (Изд. «Мир», 5 - 15).
604. В.И.Ленин и развитие математики в Советском Союзе (Математика в школе, № 1, 4 - 12).
605. В.И.Ленин и методологические проблемы математики (изд. «Знание», серия «Математи-ка и кибернетика», № 1, 1 - 32).
606. В.И.Ленин и методологические вопросы математики (УМН, т. 25, вып. 2, 5 - 14).
607. Ленинская теория познания и математическое образование (Вестник высшей школы, № 4, 77 - 81).
608. Lenin a matematyka w Zwianzku Radzieckim (Zagadnenia naukoznawstwa, 2 (22), 3 - 31).
609. Фридрих Енгелс за философските проблеми на математиката (Българска Академия на науките, Физико математическо списание, том 13(46), кн. 4, 296 – 306).

610. Рецензия на книгу Beard R.T., Pentikdinen T., Pesonen E. «Теория риска» («Новые книги за рубежом», серия А, № 4, 31 – 33).
611. Рецензия на книгу Szaby A. «Начала греческой математики» (совм. с И.Б. Погребысским, «Новые книги за рубежом», серия А, №5 ,5 –6).
612. Рецензия на книгу Onicescu O. «Теория вероятностей и ее применения» («Новые книги за рубежом», серия А, № 6, 17 – 19).
613. Рецензия на книгу Weinberg F. «Основы теории вероятностей и статистики и их применение к исследованию операций» («Новые книги за рубежом», серия А, № 6, 19 – 21).
614. Рецензия на книгу Hбjek J. «Курс по непараметрической статистике» («Новые книги за рубежом», серия А, № 7, 16 – 17).
615. Практичность абстракции (Неделя, 18 - 24 мая, № 21).
616. Счет время бережет (Известия, 13 июля 1970).

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617. Einfuhrung in die Bedienungstheorie (zus. I.N.Kovalenko. Berlin, Academie-Verlag, 1 - 450).
618. Математические методы в теории надежности (совм. с Ю.К.Беляевым и А.Д.Соловьёвым. т. 1, Япония).
619. Wstep do teorii obslugi masowej (совм. с И.Н. Коваленко. Warszawa, Panstwowe Wydawnictwo Naukowe).
620. Курс теории вероятностей (Япония, т.I).
621. Лекции по теории суммирования случайного числа независимых величин (Варшава, изд-во университета, 1 - 42).
622. Применение теории массового обслуживания к задачам больших систем (совм. с И.Н.Коваленко. «Большие системы, теория, методология, моделирование»; изд. «Наука», 7 - 9, 105 - 122).
623. Theorie und Praxis der Productionssicherheit (Ideen des exakten Wissens, № 6, 411 – 414).
624. Свойства решений задачи с потерями в случае периодичности интенсивностей (совм. с И.П. Макаровым. Дифференциальные уравнения, т. VII, № 9, 1696 – 1698, изд. «Наука и техника», Минск).
625. Беседы за математическата статистика (Малка математическа библиотека, изд. «Техника», София, 5-61).
626. За бъдещето на приложната математика (София, Математика, година X, книжка 5, 5 – 9).
627. Сообщение на заседании НТС (Тезисы докладов, конференций и совещаний, Москва, ЦНИИ информации и технико-экономических исследований рыбного хозяйства, вып. 3, 28 – 30).
628. Об источниках нового в математике (Белград, «Dijalektika», broj. 3, godina VI).
629. Предисловие и послесловие составителя сборника «Проблемы современной математики» (Москва, Знание, серия «Математика и кибернетика», № 10, 3, 45 – 48).
630. О роли математики в ускорении темпов научно-технического прогресса (Математика в школе, № 5, 4 – 11).
631. Mathematik und Leben (Berlin, Wissenschaft und fortschritt, № 6, 256 – 259).
632. Ф.Энгельс о философских проблемах математики (Вестник МГУ (философия), № 2, 20 - 27; Математика в школе, № 1, 4 – 11).
633. Гордость отечественной науки. К 150-летию со дня рождения П.Л.Чебышева. (Вестник высшей школы, № 5, 76 - 80).
634. Пафнутий Львович Чебышев (Знание–сила, № 10, 22 – 23).
635. Иосиф Бенедиктович Погребысский (Математика в школе, № 6, 91 – 92).

636. Вступительные экзамены на естественные факультеты МГУ (Математика в школе, № 1, 50 - 55).
637. Рецензия на книгу «История математики» (т. I) (Вестник АН СССР № 10, 123 – 124).
638. Рецензия на «Французско–русский математический словарь» (Успехи математических наук, т. XXVI, вып. 3, 249 – 251).
639. Рецензия на книгу Broad C.D. «Индукция, вероятность и причинность» (совм. с И.Б. Погребысским, «Новые книги за рубежом», серия А, №3 ,5 –6).
640. Рецензия на книгу Stųrmer H. «Полумарковские процессы с конечным множеством состояний» («Новые книги за рубежом», серия А, № 4, 24 – 25).
641. Математика и технический прогресс (Приокская правда, 29 сентября, № 231).

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642. Математические методы в теории надежности (совм. с Ю.К. Беляевым и А.Д. Соловьёвым. Япония, т.II).
643. Курс теории вероятностей (Япония, т.II).
644. Methodes Mathematiques en Theorie de la Fiabilite (совм. с Ю.К. Беляевым и А.Д. Соловьёвым. М. Изд-во «Мир»)
645. О задачах теории массового обслуживания (МГУ, Сборник трудов II Всесоюзного совещания-школы по теории массового обслуживания, Дилижан, 41 - 51).
646. Limit theorems for sums of a random number of positive independent random variables (Proceedings of the Sixth Berkeley Symposium on Mathematical Statistics and Probability, University of California Press, v. II, 537 - 549).
647. Асимптотические методы в вопросах исследования операций (Сборник "Исследование операций. Методологические аспекты", Москва, Наука, 29 - 42).
648. О статистических методах в социальных науках (АН СССР, Математизация научного знания, вып.V, 50 - 60).
649. О некоторых вопросах надежности как предмета исследования и преподавания (Надежность и долговечность машин и оборудования, 62 - 71).
650. Беседи върху теория на вероятностите и комбинаторика (София, «Математика», вып. 4, 2 - 8; вып. 5, 1 - 6).
651. Математика многонациональной советской страны и научно-технический прогресс (Изд-во "Знание", сборник "Математика и научно-технический прогресс", серия "Математика, кибернетика", № 11, 29 - 58).
652. Математика - наука древняя и молодая («Архитектура математики», изд-во «Знание», серия «Математика, кибернетика» № 1, 19 - 32).
653. Иосиф Бенедиктович Погребысский (УМН, т. XXVII, вып.1, 227 - 235).
654. Георгий Федорович Рыбкин (совм. с П.С.Александровым, А.Н.Колмогоровым, А.И.Маркушевичем и др. УМН, т.27, вып.5, 223-225).
655. XIII Международный конгресс по истории науки (Математика в школе, № 1, 94 - 96, совм. с С.С. Демидовым).
656. О математике в СССР за 50 лет его существования (Математика в школе, № 6, 5 – 12).
657. Математика и научно-технический прогресс (Изд-во "Просвещение", сборник «Школьникам о XXIV съезде КПСС», 110 - 119).
658. Технический прогресс и математическое образование (Изд-во "Высшая школа", сборник научно-методических статей по математике, вып. 2, 22 - 27).
659. Математизация науки и математическое образование (Вестник высшей школы, № 1, 40 – 45).
660. Статистическое мышление и школьный курс математики (Изд-во "Знание", сборник «Новое в школьной математике», 165 - 180).

661. Обзор статей, посвященных факультативному курсу теории вероятностей (Математика в школе, № 2, 47 – 48).
662. Статистическо образование в училищата и университетите (Българска Академия на науките, Физико математическо списание, т. 15, кн. 4, 321 - 327).
663. Наше всеобщее достояние (к 25-летию Всесоюзного общества "Знание") ("Знание-сила", № 6, 1-2).
664. В единстве к свету (газета «Московский университет», 7 марта).
665. Сучасна школа. Здібності й підготовка до самостійной праці (Радянська Україна, 16 января, № 13).

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666. The theory of probability (Mir publishers, Moscow, Second Printing).
667. Беседы о теории массового обслуживания (Изд-во «Знание», серия "Математика, кибернетика", № 9, 3 - 63).
668. Приоритетные системы обслуживания (совм. с Э.А. Даниеляном, Б.Н. Димитровым и др. Изд-во Московского университета, 3 - 447).
669. Elementare Einfuhrung in die Wahrscheinlichkeitsrechnung (und A.J. Chintschin. Berlin, 1 - 174).
670. О работах по приоритетным системам обслуживания (совм. с Э.А.Даниеляном. Дополнение к книге Н.Джейсуол "Очереди с приоритетами", Изд-во "Мир",255 - 271,).
671. Asymptotic Problems in Queueing Theory (Proceedings of the Prague symposium on asymptotic statistics, 107 - 125).
672. Über einige neue Probleme der Bedienungstheorie (Urania, Leipzig, h. 4, 72 – 75; h. 5, 72 - 75).
673. Об условиях существования финальных вероятностей у Марковского процесса (совм. с А.Д.Соловьевым. Math. Operationsforsch. u. Statist., 4, h.5, 379 - 390).
674. Statistical Problems in Teletraffic Theory (with M.A. Schneps-Schneppe. Stockholm, Seventh International Teletraffic Congress, VI, p. 141/1 – 141/6).
675. Математико-статистические методы на службу стандартизации и контроля качества (Минск, Тезисы докладов конференции «Проблемы подготовки и повышения квалификации специалистов в области стандартизации», 32 - 35).
676. Математика и современное естествознание (Синтез современного научного знания, Москва, Наука, 143 - 158).
677. Полвека советской математической науки (Слово лектора, Знание, № 1, 38 – 44; № 2, 32 - 38).
678. Математиката в СССР за 50 години от неговото съществуване (София, Физико математическо списание, т. 16, кн. 1, 3 - 14).
679. Mathematik und Ausbildung von Ingenieuren (Wissenschaftliche Zeitschrift der Technischen Universitat Dresden, 22, h.5).
680. Некоторые проблемы преподавания теории телетраффика и статистического моделирования (совм. с Г.П.Башариным. "Электросвязь", № 9, 73 - 78).
681. Беседи въерху теория на вероятностите и комбинаторика (София, «Математика», кн. 1, 6 - 14).
682. Методологические предпосылки применения количественных методов в педагогических исследованиях (Тезисы докладов к семинару «Объективные характеристики, критерии, оценки и измерения педагогических явлений и процессов», 3 - 4).
683. Колмогоров А.Н. (БСЭ, т. 12, стр. 437).
684. Андрей Николаевич Колмогоров. К 70-летию со дня рождения (УМН, т.XXVIII, вып. 5, 5 -15).

685. Ученый и педагог. К 70-летию А.Н. Колмогорова (журнал "Математика в школе", № 2, 88 - 89).
686. Андрей Николаевич Колмогоров (София, Физико математическо списание, т. 16, кн. 3, 226 - 228).
687. Математик (о творческом пути А.Н.Колмогорова) ("Московский комсомолец", 6.05.1973).
688. Математика и современность (газета «Московский университет», 27 апреля).
689. Науки о случайном (газета «Московский университет», 13 февраля).

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690. О работах по статистическим методам теории надежности и теории массового обслуживания в АН СССР (совм. с Ю.К.Беляевым. Известия АН СССР, Техническая кибернетика, № 3, 19 -23).
691. Об исследованиях по теории информации в системе АН СССР (Известия АН СССР, Техническая кибернетика, № 3, 24-26).
692. О математической теории надежности (Сборник "Математика в нашей жизни", Москва, изд-во «Знание», серия «Математика, кибернетика», № 10, 43 - 62).
693. Новите задачи на теорията на масовото обслужване (София, Проблеми на съвременната математика, т. 2, 179 - 185).
694. Беседи върху теория на вероятностите и комбинаторика (София, Математика, № 1, 5 – 13; № 2, 4 – 11; № 3, 10 – 19; № 4, 6 - 11).
695. О дефиницији математике (Beograd, Nastava Matematike, № 1, 81 - 84).
696. Об исследованиях по истории математики, проводящихся в Советском Союзе (Ванкувер, Труды Международного Конгресса математиков, 549 - 560).
697. Role of practice in development of the theory of probability (XIV-th International Congress of the History of Science, Tokyo&Kyoto, Japan, 19 - 27 august 1974, abstracts of Papers, 14, Science Council of Japan).
698. Вплив П.Л.Чебишова на розвиток теорії ймовірностей (Київ, Нариси з історіі природознавства і техніки, вип. XVIII, 13 - 23)
699. Академия наук и прогресс математики (Квант, № 4, 3 – 11; № 5, 18 - 25).
700. Академия наук и развитие математики (Математика в школе, № 1, 4 - 11).
701. Академия наук и развитие математического просвещения в СССР (Математика в школе, № 2, 7 - 14).
702. Прикладные аспекты преподавания математики (Сборник "Математическое образование сегодня", № 6, 30 - 52).
703. Заведующий кафедрой (Вестник высшей школы, № 3, 51 - 59).
704. Нужны специализированные группы (Вестник высшей школы, № 8, 57 -58).
705. Политехнические аспекты преподавания математики в средней школе (Математика в школе, № 6, 18 - 24).
706. Приобщение к мышлению (Сборник "Этюды о лекторах", изд-во «Знание», 204 - 211).
707. Научно-технический прогресс и математика (Стенограмма кинозаписи лекции, изд - во «Знание», 3 - 18).
708. Международный конгресс математиков (Вестник высшей школы, № 12, 45 - 48).
709. Лев Аркадьевич Калужнин (УМН, т. XXIX, вып.4, 193 - 197).
710. Александр Яковлевич Маргулис (Математика в школе, № 1, 84).
711. Предисловие (Сборник "Математическое образование сегодня", Москва, изд-во «Знание», серия «Математика, кибернетика», № 6, 3 - 4).
712. Предисловие (Сборник "Математика в нашей жизни", Москва, изд-во «Знание», серия «Математика, кибернетика», № 10, 3 - 4).

713. Послесловие к статье Пичурина Л.Ф. "Школьная математика и вузовское преподавание" (Вестник высшей школы, № 7, 25-27).

1975 год

714. О надежности дублированной системы с восстановлением и профилактическим обслуживанием (совм. с М.Динич, Ю.Насром. Известия АН СССР, Техническая кибернетика, № 1, 66 – 71).
715. Приближенная модель одной физической задачи (Саранск, сборник "Управление, надежность, навигация", вып. 3, 125 - 127).
716. О некоторых вопросах управления научными исследованиями (Тезисы докладов к семинару "Вопросы целевого управления", 9-10 декабря, 5 - 7).
717. Об источниках нового в математике («Современная культура и математика», изд - во «Знание», серия "Математика, кибернетика", № 8, 35 - 51), (София, Поредица "Математика, физика, химия", № 1, 32 - 46).
718. Проблемы современной математики («Материалы в помощь лектору, выступающему по проблемам физики и математики», изд-во «Знание», 5 - 10).
719. О математизации научного знания ("Коммунист", № 5, 73 - 80).
720. Научно-технический прогресс и математика («Материалы в помощь лектору, выступающему по проблемам физики и математики», изд-во «Знание», 11 - 16).
721. Die Wahrscheinlichkeitsrechnung und der wissenschaftlich-technische Fortschritt (Berlin, Alpha, № 1, 1 – 2, 24).
722. Научно-технический прогресс и математика (Слово лектора, № 7, 57 - 64).
723. Чтобы лучше готовить математиков в университетах (Вестник высшей школы, № 9, 54 - 57).
724. Полезная форма повышения квалификации математиков (Вестник высшей школы, N7, 84 - 87).
725. О некоторых вопросах преподавания математики в средних специальных учебных заведениях (Методические рекомендации по математике, "Высшая школа", вып.1, 5 - 12), (София, сб. "Осъвременяване на обучението по математика", ч.1, 151 - 162).
726. Об исследованиях по истории математики в Советском Союзе (Математика в школе, № 6, 8 - 16).
727. Теория отражения и математика (Математика в школе, № 4, 4 - 12).
728. Алексей Дмитриевич Семушин (совм. с А.Я.Маргулисом, Г.Г.Масловой. «Математика в школе», № 1, 89).
729. Предисловие к книге "Статистические задачи обработки систем и таблицы для числовых расчетов показателей надежности" (Москва, "Высшая школа").
730. Предисловие к книге Б. Козлова и И.А. Ушакова «Справочник по расчету надежности» (Москва, «Советское радио»)
731. Gruβschreiben von Prof. Dr. B.Gnedenko (Leipzig, Tagung der Konferenz der Mathematikmethodiker, 25 - 26 September, 5 - 6).

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732. The Theory of Probability (Mir Publishers, Moscow, third Printing).
733. Элементарное введение в теорию вероятностей (совм. с А.Я. Хинчиным. "Наука", 8-ое изд., 5 - 167).
734. О длительности безотказной работы дублированной системы с восстановлением и профилактикой (совм. с И.М.Махмудом. Известия АН СССР, Техническая кибернетика, № 3, 86 - 91).

735. Приближенная модель одной физической задачи (Саранск. Межвузовский сборник научных работ «Управление, надежность и навигация», вып. 3, 125-127).
736. Предсловие к книге Перроте А.И. "Вопросы надежности радиоэлектронной аппаратуры (М. "Советское Радио").
737. Ташмухамед Алиевич Сарымсаков (к шестидесятилетию со дня рождения) (совм. с П.С.Александровым, А.Н.Кологоровым, Ю.В.Прохоровым. УМН, т. XXXI, вып. 2, 241-246).
738. Всесоюзное совещание-семинар заведующих математическими кафедрами университетов (совм. с Б.Р.Вайнбергом и др. УМН, т. XXXI, вып 2, 247 – 253).
739. О математическом образовании в итальянской школе (совм. с М.Клерико. Математика в школе, № 5, 90 - 93).
740. О некоторых вопросах преподавания математики в средних специальных учебных заведениях (Министерство высшего и среднего специального образования СССР. «Методические рекомендации по математике». Выпуск № 1, стр. 5 – 12)
741. О развитии мышления и речи на уроках математики (Математика в школе, № 3, 8 - 13).
742. Важные аспекты проблемы качества обучения (Математика в школе, № 1, 6 - 10).
743. Что делать с «неспособными»? (газета «Советская культура», 14 сентября).

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744. О развитии теории массового обслуживания и теории надежности в СССР (совм. с Ю.К. Беляевым, И.А. Ушаковым. Известия АН СССР, Техническая кибернетика, № 5, 69 - 87).
745. Беседы о теории массового обслуживания (Япония, Гэндай - сугаку, № 10, 11 – 16; № 11, 72 – 77; № 12, 55 - 58, на японском языке).
746. Научно-технический прогресс и математизация знаний (Москва, изд - во «Знание», 3 - 61)
747. Математика - народному хозяйству (Москва, изд - во «Знание», 3 - 63. Переведена на датский язык и опубликована в Дании в 1978 году).
748. За советом в природу (Заметки о надежности в технике и живом мире) (совм. с Я.М. Сориным, М.Б. Славиным. Москва, изд - во «Знание», 3 - 128).
749. Главное направление научно-технического прогресса (Слово лектора, № 7, 31 - 39).
750. Математика: мода или необходимость? («Просто о сложном», Материалы Всесоюзной научно-методической конференции, Москва, Знание, 80 - 83).
751. Высшее математическое образование в СССР на современном этапе (Киев, сборник "Проблемы высшей школы", изд-во «Вища школа», вып. 28, 8 - 9).
752. О развитии математики в нашей стране за 60 лет Советской власти (Математика в школе, № 5, 12 - 19), (сокращенный вариант, Квант, № 11, 18 – 26).
753. Высшее математическое образование за 60 лет Советской власти (Математика в школе, № 3, 8 - 16).
754. О математике Страны Советов ("Квант", № 11, 19-26).
755. Естественные факультеты Московского университета (Математика в школе, № 1, 47 - 51).
756. Current Studies in the history of mathematics in the Soviet Union (Amer. Math. Soc. Transl, v. 109, 119 - 129).
757. Исследования по истории математики в Советском Союзе ("Нариси історії природознавства і техніки", вып. 23, 3-13).
758. Пьер Симон Лаплас (Българска Академия на науките, Физико математическо списание, т. 20, кн. 3, 252 - 259).
759. Abbildtheorie und Mathematik (Berlin, Mathematik in der Schule, № 9, 449 - 456).

760. О воспитании научного мировоззрения на уроках математики (Математика в школе, № 4, 13 - 19).
761. За развитието на мислинето и речта при уроците по математика (София, "Обучинието по математика", № 5, 6 - 12).
762. Рецензия на книгу "Хрестоматия по истории математики" под ред. А.П. Юшкевича (совм. с С.С. Петровой. УМН, т. XXXII, вып. 1, 249 - 251).
763. Нужен эксперимент (газета «Московский университет», 18 марта).

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764. The Theory of Probability (Mir Publishers, Moscow, fourth printing).
765. Lehrbuch der Wahrscheinlichkeitsrechnung (Academy Verlag, Berlin, 7 Auflage, 3 - 399).
766. Teoria della probabilita (Roma, traduzione dal ruso, 5 - 391).
767. Математика и контроль качества продукции (Изд-во «Знание», серия "Математика, кибернетика", № 11, 3 - 64).
768. Matematikkens forhold til samfundsokonomien (Tekst nr 7, Tekster fra IMFUFA, 1 - 77).
769. О методах теории массового обслуживаия («Кибернетика и диалектика», Наука, 116 - 140).
770. О математических методах кибернетики. Теория массового обслуживания (Сборник "Кибернетику - на службу коммунизму", Москва, «Энергия», т. 9, 11 - 27).
771. On some problems in queueing theory (Hungary, Colloguia mathematica societatis janos bolyai, 85 - 92).
772. К вопросу о профилактике технических систем (Саранск, сборник "Управление, надежность, навигация", вып. 4, 97 - 100).
773. Беседы о теории массового обслуживания (Япония, Гэндай - сугаку, № 2, 74 - 76, на японском языке).
774. La mathematisation de la sciece (Alap-Paris, Novosti Moscou, «La Science au 20-e siecle», t. 5, 99 - 127).
775. Математика і науково-технічний прогрес (Київ, Знання, 3 - 48, совм. с В.С. Сологубом).
776. Научно-технический прогресс и математика (Минск, «Вышейшая школа», Хрестоматия по лекторскому мастерству, 122 - 131).
777. Теория вероятностей (совм. с О.Б. Шейниным. "Математика XIX века", Москва, "Наука", 184 - 240).
778. О Всесоюзном совещании-семинаре заведующих математическими кафедрами механико-математических и физико-математических факультетов университетов (Сборник научно-методических статей по математике, 120-123).
779. Математизация знаний и особенности ее пропаганды (Слово лектора, № 11, 41 - 46).
780. Математика и оборона страны (Математика в школе, N2, 56 - 61).
781. О математическом образовании в вузах в период научно-технического прогресса (Сборник научно-методических статей по математике, вып. 7, 3-9).
782. Научно-технический прогресс и математическое образование во втузах (Сборник научно-методических статей по математике, вып. 8, 6 - 11), (Москва, Высшая школа).
783. Wybrane problemy nauczania matematyki w szkolach wyzszych (Warszawa, Zycie szkoly wyzszei, 27 - 42).
784. Совершенствовать мастерство преподавателя (Вестник высшей школы, № 3, 57 -61).
785. Статистическое мышление и школьное математическое образование (Сборник «На путях обновления школьного курса математики» Москва, Просвещение, 56 - 68).
786. Политехнические аспекты преподавания математики в средней школе (Сборник «На путях обновления школьного курса математики», Москва, Просвещение, 121 - 132).

787. Мнение кафедры теории вероятностей МГУ им. М.В. Ломоносова об учебниках для средней школы по математике (Математика в школе, № 5, 26 - 27).
788. Предисловие к четвертому изданию книги А.Я. Хинчина "Цепные дроби" (Москва, Наука, 3 - 4).
789. Über einige grundsatzliche Fragen zur Entwicklung der Mathematik im Zusammenhang mit der Erziehung zu einer wissenschaftlichen Weltanschauung (Berlin, Mathematik in der Schule, No 9, 451 - 455).
790. Ученый, педагог, реформатор (Математика в школе, № 2, 93-94).
791. Комсомол и развитие советской математики (Математика в школе, № 5, 22 - 24).
792. Памяти Рафаила Самойловича Гутера (совм. с И.В. Чувило и др. Сборник научно-методических статей по математике, вып. 8, 112 – 113).
793. Математика на каждый день (газета «Правда», 4 января. Переведена на датский язык и опубликована в Дании в 1978 году).

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794. Elementare Einfuhrung in die Wahrscheinlichkeitsrechnung (zum A.J. Chintschin. Berlin, Veb Deutscher Verlag der Wissenschaften, 3 - 174).
795. Теория на вероятностите (совм. с А.А. Гешевым. Пловдив, 3 - 219).
796. Вероятностей теория (УСЭ, т. 2, 191).
797. Zum sechsten Hilbertschen Problem (Leipzig, Ostwalds Klassiker der exakten Wissenschaften, b. 252, 145 - 150).
798. Mathematics in Sceintific Research and Education (в книге «Computers in the life sceinces», printed in Great Britain, by Biddles Ltd.Guildford, Surrey, Croom Helm London, 23 – 25).
799. Popularisation of Mathematics, Mathematical Ideas and Results in the USSR (Denmark, Tekster fra IMFUFA, nr. 18, 60 - 62).
800. О математическом образовании математика (Вестник высшей школы, № 10, 21 - 24).
801. The Mathematical Education of Engineers (совм. с Z. Khalil. Educational Studies in Mathematics, 10 (1979), 71 - 83, D. Reidel Publishing Co., Dordrecht, Holland, and Boston, USA).
802. Педагог, коллектив и воспитание творческих начал (Вестник высшей школы, № 4, 38 - 41).
803. Как подготовить творческого специалиста? (Материалы "круглого стола", Вестник высшей школы, № 3, 11).
804. О математическом творчестве (Математика в школе, № 6, 16 - 22).
805. Школьный курс математики и воспитание мировоззрения (Математика в школе, № 3, 3 - 6).
806. Предисловие к книге Х.Крамера "Полвека с теорией вероятностей: наброски воспоминаний" (Изд-во «Знание», серия "Математика, кибернетика", № 2, 3 - 4).
807. Предисловие к книге И.Г. Башмаковой "Становление алгебры" (Изд-во «Знание», серия «Математика, кибернетика», № 9, 3 - 7).
808. Предисловие к книге А.Я. Маргулиса "Серия "Математика, кибернетика" за 12 лет" (Изд - во «Знание», серия "Математика, кибернетика", № 10, 3 - 8).
809. Алексей Иванович Маркушевич (совм. с А.Н. Колмогоровым и др. Математика в школе, № 5, 77 – 78).
810. Петр Сергеевич Моденов (совм. с А.Г. Свешниковым. Математика в школе, № 1, 79 - 80).
811. Рецензия на книгу "Хрестоматия по истории математики" под ред. А.П. Юшкевича (совм. с С.С. Петровой. УМН, т. 34, вып. 1, 262 - 264).
812. Радость творчества («Учительская газета», 10 марта).

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813. Математические методы управления качеством продукции (Изд-во «Знание», 4 - 32).
814. Математика в современном мире (Москва, Просвещение, 3 - 128).
815. Теоретическая и прикладная математика (изд-во «Знание», серия "Математика, кибернетика", № 10, 50 - 62).
816. Математика в Московском Университете за первые 100 лет ("Математическая наука в МГУ", изд-во «Знание», серия "Математика, кибернетика", № 4, 5 - 20).
817. Математика в Московском Государственном университете (Квант, № 2, 2 - 9).
818. Московский университет и математическое просвещение (Математика в школе, № 2, 14 - 19).
819. О московской школе теории вероятностей (изд-во «Знание», серия "Математика, кибернетика", № 4, 30 - 44).
820. Развитие математики и математического образования (Математика в школе, № 6, 3 - 8).
821. НТП и математическое образование ("Вестник высшей школы", № 9, 52 - 54).
822. Кафедра и подготовка творческой смены ("Вестник высшей школы", № 3, 43 - 47).
823. Възпитаване на научен мироглед в уроците по математика (София, изд. «Народна просвета», сборник статей «За някои въпроси на обучинието по математика», 5-18).
824. Върху развитието на мислинето и речта в уроците по математика (София, изд. «Народна просвета», сборник статей «За някои въпроси на обучинието по математика», 18 - 28).
825. Елементи от историята на науката в уроците по математика (София, изд. «Народна просвета», сборник статей «За някои въпроси на обучинието по математика», 28 - 41).
826. Ленин и математика (Математика в школе, № 1, 3 - 8).
827. Предисловие к сборнику «Математика как профессия» (изд-во «Знание», Серия "Математика, кибернетика", № 6, 3 - 23).
828. Предисловие к книге А. Реньи "Трилогия о математике" (Москва, Мир, 5 - 16).
829. Предисловие к книге Н.А. Плохинского, "Алгоритмы биометрии" (Изд-во Московского университета, 3 - 4).
830. Предисловие к книге Н.Б. Вассоевича и др. "Коэффициент ранговой корреляции Спирмена" (изд-во Московского университета, 3).
831. Наум Яковлевич Виленкин. К 60-летию со дня рождения (совм. с С.И. Шварцбурдом, А.Г. Мордковичем. Математика в школе, № 6, 63 - 64).
832. О серии брошюр "Математика, кибернетика" (Математика в школе, № 5, 76 - 77).
833. О книге "Биографический словарь деятелей в области математики" (Математика в школе, № 4, 64 - 65).
834. Математик (К 1000-летию со дня рождения Абу Али Ибн Сины) («Комсомольская правда», 21 августа).

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835. Из истории науки о случайном (Изд-во «Знание», серия "Математика, кибернетика", № 6, 3 - 64).
836. Математическое образование в вузах (Москва, Высшая школа, 3 - 173).
837. Предельные теоремы для сумм случайного числа случайных слагаемых (совм. с Д.Б. Гнеденко. Ивановский ГУ, межвузовский сборник "Алгебраические системы", 78 - 88).
838. О формулах Эрланга для систем с потерями (совм. с О. Аннаоразовым. Изв. АН Туркменской ССР, серия физико-технических, химических и геологических наук, № 6, 99-100).

839. Математика в Московском университете (Сборник научно-методических статей по математике, вып. 9, 124 - 136, Москва, Высшая школа).
840. О месте лекции в математическом образовании (Сборник научно-методических статей по математике, вып. 9, 25 - 37, Москва, Высшая школа).
841. О призвании учителя (Математика в школе, № 5, 5 - 11).
842. Роль математики в формировании у учащихся научного мировозрения (Сурган Хумуужуулэгч, № 1, 35 - 45).
843. О воспитании научного мировоззрения на занятиях по математике (Ивановский ГУ, межвузовский сборник "Алгебраические системы", 10 - 18).
844. Слово, зажигающее сердца (Сборник "Живое слово науки", изд-во «Знание», 184 - 189).
845. Константин Петрович Сикорский. К 85-летию со дня рождения (совм. с Р.С. Черкасовым, Н.А. Курдюмовым. Математика в школе, № 5, 66).
846. Симеон Дени Пуассон (Математика в школе, № 3, 64 - 67; Болгария, "Физико-математическое списание", 23, кн.3).
847. Предисловие к книге А.Н. Колмогорова и др. "Физико-математическая школа при МГУ" (Изд-во «Знание», серия "Математика, кибернетика", № 5, 3 - 7).
848. Предисловие к книге Л.Н. Дашевского, Е.А. Шкабара "Как это начиналось" (Изд-во «Знание», серия "Математика, кибернетика", № 1, 3 -6).
849. Введение к сборнику "Труды Всесоюзной школы-семинара. Теория массового обслуживания. Баку. 1978" (Москва, ВНИИСИ, 3).
850. Историко-математические исследования (к выходу XXV тома) (Успехи математических наук, т. 36, вып. 4, 242-243).
851. Изабелла Григорьевна Башмакова. К 60-летию со дня рождения (совм. с УМН, № 36, 5(221), 211-214).
852. Изабелла Григорьевна Башмакова (совм. с П.С.Александровым, А.Н.Колмогоровым и др. "Математика в школе", № 1, 73-74).
853. Алексей Иванович Маркушевич (совм. с П.С.Александровым, А.Н.Колмогоровым и др. Болгария, "Физико-математическое списание", 23(56), №2, 150-152).
854. Сагды Хасанович Сираждинов (совм. с А.Н.Колмогоровым и др. УМН, т. XXXVI, № 1, 73-74).
855. Мордухай Моисеевич Вайнберг (Математика в щколе, № 1, 80).

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856. The Theory of Probability (Translated from the Russian, Mir, fifth printing).
857. Элементарное введение в теорию вероятностей (совм. с А.Я. Хинчиным. Изд-во «Наука», 9-ое изд., 3 - 156).
858. Математика и теория надежности (совм. с А.Д. Соловьевым. Изд-во «Знание», серия "Математика, кибернетика", № 10, 3 - 63,).
859. Формирование мировоззрения учащихся в процессе обучения математике (Москва, Просвещение, 3 - 145).
860. Математические модели старения полимерных изоляционных материалов (совм. с Р.П. Брагинским, С.А. Молчановым и др. ДАН СССР, т. 268, № 2, 281 - 284,
861. Об одном свойстве предельных распределений для максимального и минимального членов вариационного ряда (совм. с Л. Сенуси-Берекси. ДАН СССР, т. 267, № 5, 1039 - 1040).
862. Об одном свойстве логистического распределения (совм. с Л. Сенуси-Берекси. ДАН СССР, т. 267, № 6, 1293 - 1295).
863. О распределениях Лапласа и логистическом как предельных в теории вероятностей (совм. с Д.Б. Гнеденко. Сердика, Българско математическо списание, т. 8, 229 - 234).

864. Теория надежности (совм. с Ю.К. Беляевым. Математическая энциклопедия, т. III, 854 - 858).
865. Статистически методи за контрол на качеството на масовата промишлена продукция (София, "Математика", № 7, 2 - 9).
866. Математическое образование и математика в СССР за 60 лет (Математика в школе, № 6, 6 - 10).
867. Статья В.И. Ленина "О значении воинствующего материализма" и математическое образование (Математика в школе, № 4, 5 - 8).
868. Московский государственный университет (Математика в школе, № 2, 57 - 59).
869. Какъв трябва да бъде учебникът по математика за ученици (София, "Обучението по математика", № 1, 10 - 18).
870. Математика в современном мире (Вечерняя средняя школа, № 1, 30 - 33).
871. О математических способностях и их развитии (Математика в школе, № 1, 31 - 34).
872. Математика в СССР (Квант, № 11, 2 - 4).
873. Михаил Васильевич Остроградский (Квант, № 10, 5 - 10).
874. Александр Яковлевич Маргулис (Математика в школе, № 1, 80).
875. Предисловие к книге М.А. Ястребенецкий Надежность технических средств в АСУ технологическими процессами. - Москва, Энергоиздат, 1982

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876. Предисловие (совм. с Д. Кёнигом) и глава I в "Handbuch der Bedienungstheorie I" (Academie - Verlag, Berlin, 7-9, 19-38).
877. Elementare Einfuhrung in die Wahrscheinlichkeitsrechnung (und A.J. Chintschin. VEB Deutcher Verlag der Wissenschften, Berlin, 3 - 174).
878. On limit theorems for a random number of random variables (Proceedings of the Fourth USSR-Japan Symposium "Probability Theory and Mathematical Statistics", august 23-29, 1982, Springer-Verlag, Berlin, 167 - 176).
879. On some stability theorems (Proceedings of the 6th International Seminar "Stability Problems for Stochastic Models", april 1982, Springer-Verlag, Berlin, 24 - 31).
880. О свойстве продолжимости предельных распределений для максимального члена последовательности (совм. с Л. Сенуси-Берекси. Вестник Московского университета, серия 1, "Математика. Механика", № 3, 11 - 20).
881. Предельные теоремы для крайних членов вариационного ряда (совм. с А. Шерифом. ДАН СССР, т. 270, № 3, 523 - 525).
882. A characteristic property of one class of limit distributions (and S. Janjic. Math. Nachr. Bd., 113, 145 - 149).
883. Теоремы устойчивости для предельных распределений членов вариационного ряда (ТВиП, т. 28, вып. 4, 809-810).
884. Математические модели старения полимерных изоляционных материалов (совм. с Р.П. Брагинским и др. ДАН СССР, т. 268, № 2, 281 - 284).
885. О математических задачах теории массового обслуживания и надежности (совм. с Ю.К. Беляевым, И.А. Ушаковым. Известия АН СССР, Техническая кибернетика, № 6, 3 - 12).
886. Учет периодичности при оценке коэффициента загрузки диспетчера (совм. с Л.Г. Афанасьевой и Н.А. Дроздовым. II Всесоюзная конференция по управлению воздушным движением. Тезисы докладов. АН СССР. М. 51 – 53)
887. Теория вероятностей и математическая статистика (в сб. "Очерки развития математики в СССР". Киев, "Наукова думка", 500-513).
888. Математика и научное познание (Изд-во «Знание», серия "Математика, кибернетика", № 7, 3 - 64).

889. О преподавании предметов теоретико-материалистического мировоззрения в процессе преподавания математики в вузе (Сборник научно-методических статей по математике, вып. 10, 187-189).
890. О математических способностях (Сборник научно-методических статей по математике, вып. 10, 154-163).
891. Колос и машина ("Изобретатель и рационализатор", № 6, 6 - 7).
892. О продовольственной программе и математике (Математика в школе, № 2, 4 - 9).
893. Математика и производство (Квант, № 1, 3 - 6, 11).
894. Карл Маркс и математиката (София, Физико-математическо списание, т. 25, вып. 4, 267 - 276).
895. Андрей Николаевич Колмогоров. К 80-летию со дня рождения. (совм. с Н.Н. Боголюбовым, С.Л. Соболевым. УМН, т. 38, вып. 4, 11 - 23).
896. Андрей Николаевич Колмогоров (Математика в школе, № 2, 76 – 78).
897. Павел Сергеевич Александров (совм. с А.Н. Колмогоровым. Математика в школе, № 1, 47 - 48).
898. Иван Федорович Тесленко (совм. с М.И. Бурда, Р.С. Черкасовым. Математика в школе, № 2, 78 – 79).
899. Предисловие к книге Е.Ю. Барзиловича и др. "Вопросы математической теории надежности" (Москва, Радио и связь, 3 - 8).
900. Предисловие к книге «Геометрия гильбертова пространства и три принципа функционального анализа» (Изд-во «Знание», серия "Математика, кибернетика", № 6, 3).
901. О книге А.Я. Халамайзера "Математика гарантирует выигрыш" (Математика в школе, № 6, 69).
902. Дорогу осилит идущий (газета «Московский университет», 14 апреля).

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903. Теория на вероятностите и математическа статистика (совм. с А.А. Гешевым. София, изд. «Наука и изкуство», 3 - 229).
904. Matematika siuolaikiniame pasaulyje (Математика в современном мире) (Kaunas, Sviesa, 4 - 102).
905. Service systems with the time-dependent input and service intensities (Moscow,Proceedings of the Third International Seminar on Teletraffic Theory, june 20-26, 142 - 146).
906. Mathematical problems in queueing and reliability theory (with Y.K.Belyaev and I.A.Ushakov. "Engrg. Cybernetics", № 6, 62-69).
907. Принципы аналитического моделирования диспетчера в секторе управления воздушным движением (совм. с Л.Г. Афанасьевой. Пермь, Тезисы докладов Всесоюзной научно-технической конференции "Применение статистических методов в производстве и управлении", 31 мая - 2 июня, 101 - 102).
908. Особенности анализа эффективности криогенных систем и установок с резервными режимами работы (совм. с М.В. Козловым и др. Ленинград, Межвузовский сборник научных трудов "Криогенная техника и кондиционирование", 3 – 8).
909. О распределении медианы (совм. с С. Стоматовичем, А. Шукри. Вестник Московского университета, серия "Математика. Механика", № 2, 59 - 63).
910. За управлението на качеството на промишлената продукция (София, «Социално управление», № 1, 3 - 10).
911. К истории понятия вероятности случайного события (совм. с М.Т. Перес. АН СССР, Вопросы истории естествознания и техники, № 1, 71 - 75).

912. Международный математический конгресс в Варшаве (Математика в школе, № 4, 67 - 69).
913. Математическое творчество и общественный прогресс (Квант, № 2, 2 - 5).
914. Воспитание моральных принципов и математика (Математика в школе, № 5, 6 - 10).
915. Математические рукописи К.Маркса и вопросы математического образования (Математика в школе, № 2, 7 - 12).
916. Слово, зажигающее сердца (Изд-во «Знание», серия "Математика, кибернетика", № 10, 52 - 57).
917. Михаил Васильевич Остроградский (Изд-во «Знание», серия "Математика, кибернетика", № 5, 3 - 63).
918. Евгений Евгеньевич Слуцкий (Киев, "У світі математики", вып. 15, стр. 40).
919. Предисловие к книге «Handbuch der Bedienungstheorie II» (und D. König. Academie-Verlag, Berlin, 7 - 8).
920. Какими быть X-XI классам? (коллективное письмо девяти академиков с предложением к проекту реформ школы) (Известия, 26 января).
921. Вторая грамотность XX века (Известия, 18 декабря).
922. Математика и жизнь (Учительская газета, 5 июля).
923. Что может статистика (Звезда, 23 июня) (Пермь).
924. О ценности знаний (Правда, 27 февраля).

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925. Математика и контроль качества продукции (на монгольском языке, Улан-Батор, Улсын Хэвлэпийн Газар, Улаанбаатор, 3-69).
926. Математика и математическое образование в современном мире (Изд-во «Просвещение», 3-191).
927. О продолжимости предельных совместных распределений для членов вариационного ряда (совм. с Баркат Х., Хемида С. Докл. АН СССР, т. 284, 789 – 790)
928. Вероятностно-статистическое моделирование управления воздушным движением (совм. с Л.Г. Афанасьевой. Тарту, Тезисы докладов III Всесоюзной научно-технической конференции "Применение многомерного статистического анализа в экономике и оценке качества продукции", 17-18.09, 134 - 144).
929. О некоторых актуальных проблемах надежности (совм. с Е.Ю. Барзиловичем. Сборник "Проблемы надежности летательных аппаратов", Изд-во «Машиностроение», 4 – 9).
930. И не только в биологии (Вестник высшей школы, № 10, 31 - 32).
931. Предисловие, статьи: «Вероятность», «Вероятностей теория», «Математическая статистика», «Математика» (Энциклопедический словарь юного математика) (Москва, Педагогика, 5 – 7, 36 – 40, 172 – 180, 183 – 184).
932. Программа педагогических вузов по истории математики (составлена совм. с. Математика в школе, № 3, 57-60).
933. О двух совещаниях в Болгарии по вопросам школьного образования ("Информатика и вычислительная техника", 68).
934. Математика и математики в Великой Отечественной войне (Квант, № 5, 9 -15).
935. О нашем товарище (газета «Московсий университет», 1 апреля).

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936. Формиране на мироглед у учениците при обучинието по математика (София, 2 - 156).
937. Предельные теоремы для членов вариационного ряда (Первый Всемирный конгресс Общества математической статистики и теории вероятностей им. Бернулли. Тезисы. М. "Наука", т. 1. 194).

938. Математические основы исследования (Надежность и эффективность в технике, Справочник т.1, Москва, Машиностроение, 54 - 58).
939. Место математической статистики в научно-техническом прогрессе ("Заводская лаборатория", № 12, 1 - 2).
940. Вторая грамотность века (Изд-во «Знание», Слово лектора, № 3, 14 - 18).
941. К истории основных понятий теории вероятностей (История и методология естественных наук, вып. XXXII, 81 - 88).
942. Из истории начального периода истории теории вероятностей (Первый Всемирный конгресс Общества математической статистики и теории вероятностей им. Бернулли. Тезисы. М. "Наука", т. 2, 939).
943. Теория вероятностей и Я. Бернулли (совм. с С.Х. Сираждиновым. Сборник «Бернулли ученые и общество Бернулли», производственно-издательский комбинат ВИНИТИ, 24 - 37).
944. Математической подготовке - прикладную направленность (Вестник высшей школы, № 9, 49 - 52).
945. Математизация знания и вопросы математического образования (Сборник трудов "Математизация современной науки: предпосылки, проблемы, перспективы", 23 - 32).
946. Математиката и изучаването на заобикалящата ни действителност (София, "Проблеми на ученическото техническо творчество", № 3, 25 - 29).
947. Вечният стремеж към открития (София, "Наука и техника за младежта", № 3, 14 - 18).
948. О двух совещаниях в Болгарии по вопросам школьного образования (Математика в школе, № 1, 68 - 69).
949. Об упражнениях по математике (совм. с М.В. Потоцким. Сб. научно-методических статей по математике. М. "Высшая школа", вып. 13, 6-15).
950. Об исследовании по истории школьного математического образования в нашей стране, проводимом в Японии (совм. с Р.С. Черкасовым. Математика в школе, № 4, 75 - 76).
951. Из воспоминаний о В.В. Голубеве ("Голубев Владимир Васильевич. К 100-летию со дня рождения". М. Изд-во ВВИА им. Н.Е. Жуковского, 60-61).
952. Великий русский ученый и просветитель Михаил Васильевич Ломоносов (совм. с Н.П. Жидковым. Математика в школе, № 5, 49 - 54).
953. Адольф Павлович Юшкевич (с Башмаковой И.Г. и др. "Математика в школе", № 4, 72-74).

1987 год

954. Введение в теорию массового обслуживания (совм. с И.Н. Коваленко. «Наука», 2-е изд., перераб. и дополн., 336 стр.).
955. Lehrbuch der Wahrscheinlichkeitsrechnung (eighth edition. Academie-Verlag, Berlin).
956. Современные задачи теории и практики надежности (Надежность и контроль качества, № 11, 3 - 10).
957. Работы академика А.А.Маркова по теории вероятностей (С.Я.Гродзенский "Андрей Андреевич Марков", Москва, "Наука", стр. 223-237).
958. В единстве теории и практики (совм. с Д.Б. Гнеденко. Вестник высшей школы, № 4, 48 – 50).
959. Университеты и научно-технический прогресс (Изд-во «Высшая школа», сборник научно-методических статей по математике, вып. 14, 3 - 11).
960. Посев научный - жатве народной (Сборник статей "Октябрь, наука, прогресс", 108 - 117, изд-во «Советская Россия»).
961. Мечта и НТР (Слово лектора, № 9, 63 - 64).
962. О математике Страны Советов (Квант, № 11, 3 - 8).

963. Развитие школьного математического образования в Советском Союзе за 70 лет (совм. с Г.Г. Масловой, Р.С. Черкасовым. Математика в школе, № 6, 6 - 14).
964. Математика и математическое образование в Стране Советов (Математика в школе, № 4, 6 - 12; № 5, 3 - 7).
965. К вопросу о содержании факультатива по теории вероятностей (Математика в школе, № 3, 24 -25).
966. О педагогической деятельности кафедры теории вероятностей Московского университета (Вестник Московского университета, сер. 1, Математика. Механика; № 2, 91 - 94).
967. По поводу первого Всемирного конгресса Общества им. Я. Бернулли (совм. с М.А. Мирзахмедовым, Х.П. Ариповым. Математика в школе, № 1, 75 - 76).
968. Итоги работы приложения «Надежность и контроль качества» за 1987 год (Надежность и контроль качества, № 12, 3 - 6).
969. Предисловие редактора к книге С.Я.Гродзенского "Андрей Андреевич Марков" (Москва, "Наука", стр. 5-10).
970. Рецензия на книгу «Справочник по надежности технических систем» (Надежность и контроль качества, № 3, 57 – 58).
971. Рецензия на книгу «Надежность систем энергетики» (Надежность и контроль качества, № 9, 60).
972. Рецензия на журнал "Вопросы истории естествознания и техники" (Математика в школе, № 3, 73 - 74).
973. О двух сборниках трудов по философским вопросам математики (совм. с В.Н. Пономаревым, А.А. Григоряном. Математика в школе, № 5, 74 - 75).
974. Математика и экономика (совм. с Е.В. Морозовым. Газета «Ленинская правда», Петрозаводск, 22 сентяря).
975. Воспитание творчеством (Учительская газета, 8 августа).

1988 год

976. Курс теории вероятностей (Изд-во «Наука», изд. 6-е, перераб. и дополн.).
977. The theory of probability (Moscow, Mir Publishers, sixth printing).
978. Особенности перколяционной модели старения полимеров (совм. с Р.П. Брагинским, В.В. Малуновым и др. ДАН СССР, т. 303, № 3, 535 – 537).
979. Теоретическое и статистическое исследование дефектного множества в эмаль-лаковых электроизоляционных покрытиях (совм. с Р.П. Брагинским и др. ДАН СССР, т. 303, № 2, 270-274).
980. Нормирование надежности и "перестройка" взглядов (совм. с И.А. Ушаковым. Стандарты и качество, № 7, 35 - 38).
981. Совершенствование математического образования в университете (Саранск, сборник "Совершенствование содержания математического образования в школе и ВУЗе", 13 - 19).
982. О специальных курсах и семинарах естественнонаучного и прикладного характера (Изд-во «Высшая школа», сборник научно-методических статей по математике, вып. 15, 4 - 9).
983. О некоторых вопросах перестройки математического образования в университетах (совм. с Д.Б. Гнеденко. Современная высшая школа, № 3, 81 - 90).
984. Роль математических методов исследования в кардинальном ускорении научно-технического прогресса (совм. с А.И. Орловым. Заводская лаборатория, № 1).
985. О курсе математики в школах Японии (совм. с Р.С. Черкасовым. Математика в школе, № 5, 72 – 76).

986. Отстаивая исследовательский поиск (Изд-во «Наука», сборник «Путь в большую науку: академик Аксель Берг», 147 - 150).

1989 год

987. The Theory of Probability (Chelsea Publishing Company, New York).
988. Курс теории вероятностей (Египет, на арабском языке, совм. с изд-ом «Мир»).
989. Introduction to Queueing Theory (and I.N. Kovalenko. Birkhäuser, Boston, 2nd edition, revised and supplemented, 315 p.).
990. Об оценке неизвестных параметров распределения при случайном числе независимых наблюдений (Академия наук ГССР, Труды Тбилисского математического института, т. 92, 146-150).
991. О работах кафедры теории вероятностей по математической теории надежности (совм. с Ю.К. Беляевым, А.Д. Соловьевым. Теория вероятностей и ее применения, т. XXXIV, вып. 1, 191 – 196).
992. Кафедра теории вероятностей Московского университета (Теория вероятностей и ее применения, т. XXXIV, вып. 1, 119 - 127).
993. Предисловие и статьи: «Вероятность», «Вероятностей теория», «Математическая статистика», «Математика» (Энциклопедический словарь юного математика, 2-ое изд., испр. и дополн.) (Москва, Педагогика, 5 - 7, 35 - 40, 172 - 178, 183 - 184).
994. Об образовании преподавателя математики средней школы (Математика в школе, № 3, 19 - 22).
995. Математика как орудие педагогического исследования (Свердловск, сборник "Применение математических методов и ЭВМ в педагогических исследованиях", 6 - 22).
996. О роли математики в формировании у учащихся научного мировоззрения и нравственных принципов (Математика в школе, № 5, 19 - 26).

1990 г.

997. Теория вероятностей (Киев, «Вища школа», 3-328, совм. с И.Н. Коваленко).
998. Предисловие к книге В.М.Круглова и В.Ю.Королёва "Предельные теоремы для случайных сумм" (Москва, изд-во МГУ).
999. Вначале было слово (Вестник высшей школы, № 1, 23 - 27).
1000. Ученый, учитель, гражданин (Математика в школе, № 5, 56 - 59).
1001. Воспоминания о Вячеславе Васильевиче Степанове (К 100-летию со дня рождения) (УМН, т.45, вып. 6, 165 - 169).
1002. Советская школа и В.И. Ленин (Математика в школе, № 3, 3 - 8).

1991 год

1003. Einfuhrung in die Wahrscheinlichkeitstheorie (Academy Verlag, Berlin, 9 Auflage, 475 p.).
1004. Введение в специальность математика (Москва, Наука, 3 - 237).
1005. Павел Сергеевич Александров (совм. с А.Н.Колмогоровым. Сборник "Математика в её историческом развитии", изд-во "Наука", 125-130).
1006. Математика в современном мире и математическое образование (Математика в школе, № 1, 2 - 4).
1007. Развитие мышления и речи при изучении математики (Математика в школе, № 4, 3 - 9).

1992 год

1008. Probability Theory (совм. с О.Б. Шейниным. «Mathematics in the 19th Century», Birkhäuser, Boston).
1009. Введение (Сборник "Применение вероятностных методов решения задач технического обеспечения агропромышленного производства", 4 - 10, совм. с В.И. Анискиным) (НИИ механизации сельского хозяйства).
1010. Математика и проблемы надежности и безопасности современной техники (Математика в школе, № 1, 3 - 7).
1011. Математика в Московском университете (1755 - 1933) (совм. с О.Б. Лупановым, К.А. Рыбниковым. Изд-во Московского университета, сборник "Математика в Московском университете", 3 - 19).
1012. Кафедра теории вероятностей (Изд-во Московского университета, сборник "Математика в Московском университете", 217 - 237).
1013. Ростислав Семенович Черкасов. К 80-летию со дня рождения. (совм. с Л.С. Атанасяном, И.Г. Башмаковой и др. Математика в школе, № 4 - 5, 42 – 43).

1993 год

1014. Елементарни увод у теорију вероватноħе (совм. с А.Я. Хинчиным. Београд, 7-168).
1015. О прошлом и будущем (Теорія імовірностей та математична статистика, 49, 3-26, Київ).
1016. Учитель и друг (Сборник "Колмогоров в воспоминаниях", Издательская фирма «Физико-математическая литература» ВО «Наука», 173 - 208).
1017. Педагогические взгляды Н.И. Лобачевского. К 200-летию со дня рождения (Математика в школе, № 1, 2 - 5).
1018. Знание истории науки - преподавателю школы (Математика в школе, № 3, 30 - 32).

1994 год

1019. Стандарт образования – взгляд в будущее (совм. с Д.Б. Гнеденко. Математика в школе, № 3, 2 - 3).
1020. Абак, десятичная позиционная система счисления и десятичные дроби (Математика в школе, № 1, 75 – 77).
1021. Одна русская народная задача (Математика в школе, № 2, 65).
1022. Александр Яковлевич Хинчин (Математика в школе, № 4, 70 - 73).
1023. Александр Яковлевич Хинчин (Квант, № 6, 2-6).
1024. Лобачевский как педагог и просветитель (Вестник Московского университета, сер. 1, Математика. Механика, № 2, 15 - 23).
1025. Preface to American edition («Handbook of Reliability Engineering», John Wiley, New York, XIX - XXI).
1026. Послесловие к публикации «Евгений Неглинкин» (журнал «Новое литературное обозрение», № 6, 182).

1995 год

1027. Probabilistic Reliability Engineering (and I. Ushakov. John Wiley, New York, 518 p.).
1028. О случайных величинах, обусловленных суммами независимых случайных величин (совм. с Э.М. Кудлаевым. Вестник Московского Гос. Университета, № 5, 23 – 31,
1029. Предисловие и две статьи в книге А.Я. Хинчина «Избранные труды по теории вероятностей» (Москва, Научное изд-во ТВП, VII - IX, XI - XIV, XXI - XXXVIII).

1030. Рассказ–воспоминание в книге Б.Н. Рудакова «Много лет пронеслось…» (Изд-во Московского университета, 132 - 136).

1996 год

1031. Random summation: limit theorems and applications (and V.Y. Korolev. New York, CRC Press Boca Raton, 1 – 267).
1032. Theoria de las Probabilidades (Madrid, Rubinos – 1860, Moscu, Euro – Omega).
1033. Павел Сергеевич Александров. К 100-летию со дня рождения (совм. с А.Н. Колмогоровым. Математика в школе, № 2, 2 - 4).
1034. О преподавании математики в предстоящем тысячелетии (совм. с Р.С. Черкасовым. Математика в школе, № 1, 52 - 54).

1997 год

1035. Lehrbuch der Wahrscheinlichkeitstheorie (Verlag Harri Deutsch, 10 korrigierte Auflage, 469p.).
1036. Развитие теории вероятностей («Очерки по истории математики», изд-во МГУ, 247 - 338).
1037. Мої університетські роки (У світі математики, т. 3, вип. 2, 73 - 82).
1038. Викладання і творчість (У світі математики, т. 3, вип. 2, 95 - 100).

1998 год

1039. Theory of Probability (Gordon and Breach Science Publishers, New York, sixth ed., 497 p.).
1040. Эйлер иУкраина ("У світі математики", т.4, в. 4, 32).

1999 год

1041. The Theory of Probability and the Elements of Statistics (Chelsea Publishing Company, New York).
1042. Statistical Reliability Engineering (совм. с И.В. Павловым и И.А. Ушаковым. John Wiley, New York, 3-499).
1043. Статистическое мышление и школьное математическое образование (Математика в школе, № 6, 2 – 6).
1044. Учитель в математике, учитель в жизни («Явление чрезвычайное. Книга о Колмогорове» Москва, изд-ва «Фазис» и «Мирос», 40 – 48).

2000 год

1045. Математика и жизнь (В книге «О математике», Москва, изд-во «Едиториал УРСС», 8 - 85).
1046. Об обучении математике в университетах и педвузах на рубеже двух тысячелетий (совм. с Д.Б. Гнеденко. В книге «О математике», Москва, изд-во «Едиториал УРСС», 88 – 207).

2001 год

1047. Курс теории вероятностей (Москва, изд-во «Едиториал УРСС», 7-е изд.).
1048. Очерк по истории теории вероятностей (Москва, изд-во «Едиториал УРСС»).

2002 год

1049. Математика и жизнь (В книге «О математике», Москва, изд-во «Едиториал УРСС», изд. 2-е, 8-85).
1050. Об обучении математике в университетах и педвузах на рубеже двух тысячелетий (совм. с Д.Б. Гнеденко. В книге «О математике», Москва, изд-во «Едиториал УРСС», изд. 2-е, 88 – 207).

2003 год

1051. Элементарное введение в теорию вероятностей (совм. с А.Я. Хинчиным. Москва, изд-во «Едиториал УРСС», 10-ое изд.).
1052. Беседы о математике, математиках и Механико-математическом факультете (Москва, Издательство Центра прикладных исследований при Механико-математическом факультете МГУ, 3-149).

2004 год

1053. Курс теории вероятностей (Серия "Классический университетский учебник". Москва, изд-во "Едиториал УРСС", 8-е изд.).
1054. Андрей Николаевич Колмогоров (в книге "Математики и механики -- ректоры Московского университета и деканы механико-математического факультета МГУ") (Изд-во Центра прикладных исследований при Мех-мат фак-те МГУ, 101-102).
1055. Предисловие редактора и заключительная статья "О некоторых постановках задач и результатах теории массового обслуживания" в книге А.Я. Хинчина "Работы по математической теории массового обслуживания" (Москва, изд-во "Едиториал УРСС", 2-е изд.).
1056. Предисловие к книге А. Реньи "Диалоги о математике" (Москва, изд-во "Едиториал УРСС", 2-е изд., 5-19).
1057. Предисловие к книге А.Я. Хинчина "Цепные дроби" (Москва, изд-во "Едиториал УРСС", 5-е изд.).

2005 год

1058. Введение в теорию массового обслуживания (совм. с И.Н. Коваленко. Москва, изд-во КомКнига, 3-е изд., исправленное и дополненное).
1059. Очерки по истории математики в России (Москва, изд-во КомКнига, 2-е изд., исправленное и дополненное).

2006 год

1060. Математика и жизнь (Москва, изд-во КомКнига, 3-е изд.).
1061. Об обучении математике в университетах и педвузах на рубеже двух тысячелетий (совм. с Д.Б. Гнеденко. Москва, изд-во КомКнига, 3-е изд.).
1062. Александр Яковлевич Хинчин. (В книге А.Я. Хинчина "Избранные труды по теории чисел", стр. VII-XX. Москва, изд-во МЦНМО) (В книге А.Я. Хинчина "Педагогические статьи", стр.180-196. Москва, изд-во КомКнига, 2-е изд.).

2007 год

1063. Курс теории вероятностей (Москва, изд-во ЛКИ, 9-е изд.).
1064. Введение в теорию массового обслуживания (совм. с И.Н. Коваленко. Москва, изд-во ЛКИ, 4-е изд., исправ.).
1065. Очерки по истории математики в России (Москва, изд-во ЛКИ, 3-е изд., исправ.).
1066. Математика и контроль качества продукции (Москва, изд-во ЛКИ, 2-е изд.).

2009 год

1067. Беседы о математической статистике (Москва. Книжный дом «ЛИБРОКОМ». 2-е изд., исправ.)
1068. Беседы о теории массового обслуживания (Москва. Книжный дом «ЛИБРОКОМ». 2-е изд., исправ.)
1069. Очерки по истории математики в России (Москва. Книжный дом «ЛИБРОКОМ». 4-е изд.).
1070. Беседы о математике, математиках и Механико-математическом факультете МГУ (Москва. Книжный дом «ЛИБРОКОМ». 2-ое изд.)

2010 год

1071. Введение в теорию массового обслуживания (совм. с И.Н. Коваленко. Москва, изд-во ЛКИ, 5-е изд., исправ.).
1072. Курс теории вероятностей (Москва. Книжный дом «Либроком», юбилейное 10-е изд., исправ., дополненное).
1073. Предисловие к книге Т.Л.Саати «Элементы теории массового обслуживания и её приложения» (Москва. Книжный дом «ЛИБРОКОМ». 2-е изд.).
1074. Курс теорії ймовірностей (Киев. Изд-во Киевского университета).

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Balametov A.B., Halilov, E.D.

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e-mail: balametov.azniie@gmail.com

Introduction. When calculating the energy losses in distribution networks using performance load profiles. Known methods of calculating energy losses in electric networks [1] are based on normal operating conditions and functioning of the electrical network, uninterrupted electricity supply to consumers.

Methods of calculating the losses of electricity use charts on duration. Earlier load profiles have stable characteristics and allow you to calculate the energy loss to adapt to these conditions, the simplified formulas. Currently, load profiles feeders 6-10 kV are many different forms, there were changes in the structure of energy consumption. Transition economies characterized by: non-uniformity in the daily load profiles; disconnection associated with non-payment for electricity; limitations associated with the overload of network elements, etc.

Most informative are the load profiles of individual groups of consumers for whom the known types of graphics. Load profiles of feeder formed of different consumer groups. The combination of load profiles generated total schedule feeder. Information about the probabilistic characteristics of load profiles is generally little known.

Calculation of energy losses by the method of medium loads is the use of expressions with the form factor. A calculation of the form factor reduces to obtaining the expressions having a clear and simple for hand calculation of the form. This is largely possible in a simple way to explain the patterns of influence on the profile, load losses of electricity. However, the pace of development of computer technology with great potential and their application in all spheres of government allow the use of complex computational algorithms, more flexible and accurate simulation. At the same time with this to some extent this may lose the simplicity and clarity of representation formulas for calculating the load losses.

Deterministic methods of calculation do not take into account the inaccuracy of initial data for plotting load. To overcome these deficiencies have developed methods for calculating the energy loss, based on the probabilistic representation of the graphs of electrical loads. These methods can be divided into: methods of submission of the load as a random variable and regression methods for calculating the losses.

Staging. Energy loss in the elements of an electric network is a function of the characteristics of load profiles. For the calculation of load losses in distribution networks using the method of average loads

$$\Delta W_L = \Delta P_{av} k_f^2 T \ ,$$

and the method of the number of hours the greatest losses

$$\Delta W_L = \Delta P_{max} \tau \ ,$$

where ΔP_{av} - loss of power in the network at an average load of nodes (or networks in general) for the time T; k^2_f - a square form factor graphics power or current; ΔP_{max} - loss of power in the network at maximum loads of nodes; τ - the number of hours the maximum losses.

Key indicators of load profiles in the calculation of loss are: the number of hours of peak load T_{max}, the fill factor loading schedule. Another important characteristic of load is the ratio of minimum load to maximum $k_{min}=P_{min}/P_{max}$. In the calculations of energy losses characterize the shape of load profiles parameters: the number of hours of the greatest losses τ and form factor graphics power k^2_f. The most accurate values of τ and k^2_f can be identified by well-known load profiles. Research aimed at obtaining a more accurate dependency on the parameters characterizing k^2_f load profiles led to a set of design formulas. With unknown load schedule k^2_f value can be determined using various empirical formulas [1].

The most commonly used in technical literature are the following calculation formula k^2_f depending on two parameters, k_Z and k_{min} [3]:

$$k_f^2 = 1 + \frac{(1-k_з)^2(k_з - k_{min})}{(2-k_{fill}-k_{min})k_{fill}^2} \quad \text{when } \lambda < 1;$$

$$k_f^2 = 1 + \frac{(1-k_{fill})(k_{fill} - k_{min})^2}{(1+k_{fill}-2k_{min})k_{fill}^2} \quad \text{when } \lambda \geq 1 , \tag{1}$$

$$\text{where } \lambda = \frac{k_з - k_{min}}{1-k_{fill}} .$$

In [1] based on approximations performed alternative calculations for all possible configurations of the load duration profile in increments of 0.1 in both axes (t and k_{fill}), a formula for the average expected value

$$k_f^2 = \frac{1+2k_{fill}}{3k_{fill}} \tag{2}$$

The error in the formula (1) using two parameters (k_{fill} and k_{min}) is estimated [1] is about 10.8%, and the error formula (2) uses only one parameter, k_{fill} is about 13%.

Assuming that the specification obtained by the two small parameters, in [1] proposed to use the formula (2). In this case, also noted that k_{min} has less credibility.

In [4] the error of the known empirical expression of certain k^2_f, as well as the formula (1). Accuracy in determining k^2f can increase the input except used two parameters k_{fill} and k_{min}, additional parameters characterizing the load profiles. As an additional parameter that can be taken during the duration of the maximum and minimum load graphics, etc. For example, the duration of the off-peak schedule is one of the parameters characterizing the performance load profiles. Normally, advance information on the duration of the minimum load can be obtained. Since the total load profile is formed from the sum of standard load profiles, we can evaluate the length of the minimum load. As an additional parameter taken during the duration of treatment with minimal impact - T_{min} or in relative units k_{tmin}.

It is known that more significantly affect the parameters are taken into account in determining k^2_f, the greater accuracy of simulation can be achieved. Thus, the load profile is proposed to characterize the three parameters k_{fill}, k_{min} and k_{tmin}.

In connection with the foregoing, the article examines the issues of error estimation k2f taking into account the dependence on two parameters, k_{fill} and k_{min} and three k_{fill}, k_{min} and k_{tmin}

To assess the calculation errors k^2_f in this article are considered: the use of simulation modeling of discrete possible configurations load duration profile and simulation load profiles analytical dependences in time.

Load profiles on duration are smoothly varying. In this regard, following the technique of estimating the loss of electricity distribution networks performance load duration profile as a continuously decreasing function of time. Calculations of energy losses are usually made on the PC. Therefore, excessive efforts to simplify the formulas for calculating k2f in modern conditions of development of computer technology due to the loss of accuracy unreasonable.

Simulation discrete simulation of the possible configurations of the load duration profile. Modeling of possible configurations of the load duration profile is reduced to the equivalent problem of simulation, a combination of levels of the columns of n * n and solving the following integral equations with inequalities.

Formed equation for specified values k_{fill} and k_{min} in the form of equation

$$I_1 + I_2 + I_3 \cdots I_n = n \cdot k_3. \quad (3)$$

where I_n- the current value or the power corresponding n-th stage of load profile.
Sets values of the first and the n-th column

$$I_1 = n, \quad I_n = n \cdot k_{min}. \quad (4)$$

Formed by inequalities of the form

$$I_2 \leq I_1, \; I_3 \leq I_2, \cdots I_n \leq I_{n-1}. \quad (5)$$

Solution of equations (3), (4) and (5) are integer variables $I_2 \div I_{n-1}$. Search all the options for given values of k_{fill} and k_{min} is produced by changing the values in columns (a decrease by one unit) from the load profile corresponding to the maximum k^2f toward its reduction. With respect to the discrete change of stress level $\Delta_{\Delta I}=1/n$ and the time duration of the $\Delta_{\Delta T}=1/n$ discrete step changes in the level k_{fill} has a value of $\Delta_{\Delta Kfill}=1/n^2$.

In accordance with the algorithm (3), (4) and (5) developed a mathematical model and simulation program for the possible configurations of the load duration profile. For example, Fig. 1 shows a histogram k^2_f when n=10, Δ=0.01 and 340 possible choice of load profiles when k_{fill}=0.4, k_{min}=0.1 and its comparison with the normal distribution profile.

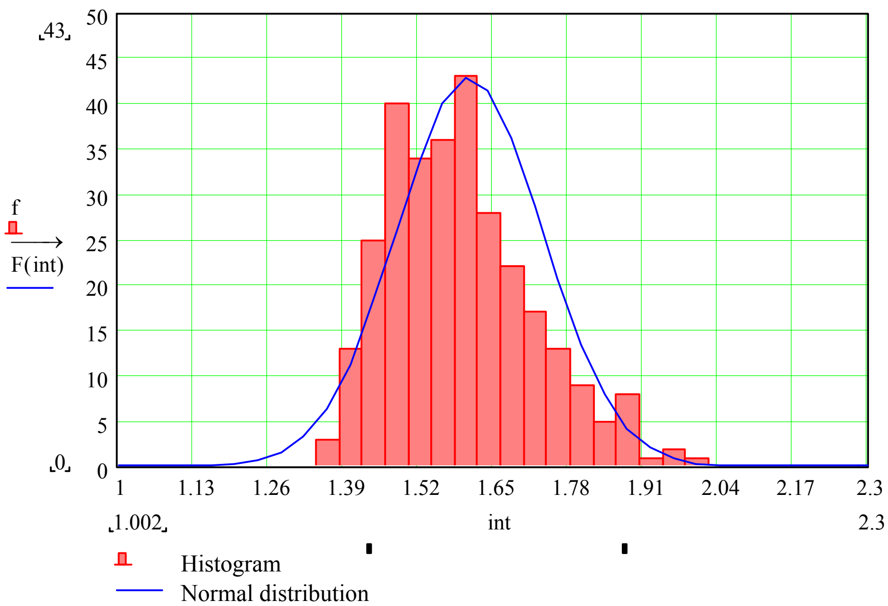

Fig. 1. Histogram k^2_f for load profiles k_{fill}=0.4, k_{min}=0.1.

The results of processing histograms k^2_f at different steps of discreteness show that the choice of the step discontinuity, having a sufficient number of possible load profiles, which allows to reliably determine characteristics of the distribution k^2_f have parameters close to the normal law. As shown in Fig.1 case, a normal law with mean 1,609 and standard deviation of 0.127.

Table 1 shows the average k^2_f for possible configurations of load profiles for the duration in increments of 0.1 according to the k_{fill} and k_{min}.

Analysis of the results shows that the use of formula (1) and (2) is associated with large systematic errors. Analysis of the results of the discrete simulation of characteristics of load profiles for various k_{fill}, depending on k_{min} shows the possibility of increasing the accuracy by obtaining appropriate and adequate dependency formulas.

Equation (2), obtained by averaging the form factor k^2_f all possible load profiles, can not completely eliminate the error. For example, for values of k_{min} = 0.1 (1) and (2) have a negative error k^2_f. For values of k_{min} = 0.2 and k_{min} = 0.3, (1) is negative, and (2) positive systematic errors. For k_{fill} = 0.4, k_{min} = 0.1, (1) has a systematic error reaches up to -35.2%, and (2) to - 19.82%.

Using discrete simulation with $k_{min}\geq 0.1$ with a limited step $\Delta_{\Delta I}$=0.1 leads to a systematic error modeling. For a preliminary comparative evaluation of the error histograms of the distribution k^2_f. For example, if k_3=0.19 and k_{min}=0.1 with a step of discreteness $\Delta_{\Delta I}$=0.1 and the time duration of the $\Delta_{\Delta T}$=0.1 there is a possibility k^2_f =3.02, then k_{min}=0.1 and a discrete step c $\Delta_{\Delta I}$=0.05, $\Delta_{\Delta T}$=0.05, n=400, a step change in the level of discreteness k_3 $\Delta_{\Delta Kfill}$=0.0025, we have over 300 options with an average k^2_f =2.22. Thus, the average value, as determined in step discrete 0.1 in this case has an error of 36%.

Table 1. Results comparing the values k^2_f by (1) and (2), depending on the k_{fill} at k_{min} = 0.1.

№	Results of the load profiles simulation			Results of the calculation according to the formulas			
	The fill factor, k_{fill}	Number of variants	Form factor, k^2_f	By formule (1)	By formule (2)	Error of formula %	
						(1)	(2)
1	0.4	340	1.58	1.45	1.50		
2	0.39	298	1.60	1.47	1.52	-8.36	-5.14
3	0.38	253	1.63	1.49	1.54	-8.59	-5.31
4	0.37	218	1.66	1.51	1.57	-8.81	-5.44
5	0.36	186	1.68	1.53	1.59	-8.8	-5.29
6	0.35	155	1.71	1.56	1.62	-8.94	-5.27
7	0.34	127	1.75	1.58	1.65	-9.53	-5.68
8	0.33	104	1.78	1.60	1.68	-9.9	-5.80
9	0.32	82	1.82	1.63	1.71	-10.47	-6.1
10	0.31	66	1.86	1.65	1.74	-11.02	-6.31
11	0.3	50	1.91	1.68	1.78	-11.8	-6.69
12	0.29	39	1.95	1.71	1.82	-12.54	-6.97
13	0.28	29	2.02	1.74	1.86	-13.96	-7.89

To ensure the adequacy of the discrete simulation problem arises of selecting a rational step, discrete, depending on the values of k_{fill} and k_{min}. For example to load profiles with k_Z <0.3 0.1 acceptance of discreteness step is coarse, in terms of number of charting options in terms of compliance with the normal distribution law.

Reduction of discrete steps increases the accuracy of the simulation. However, the sharply rising number of possible loads profiles. In this regard, along with a complete discrete simulation of characteristics of production load profiles according to the algorithm (3), (4) and (5) with a selectable discrete steps suggested below, the proposed use of a simplified simulation algorithm, which is based on the assumption that the distribution of k^2_f possible load profiles for the normal law.

Discrete simulation graphs of electrical loads for the duration of the choice of discrete steps, depending on the k_{fill} and k_{min} and receive library approximated by improving the accuracy of modeling technical energy losses in distribution networks.

Load profiles on duration are smoothly varying. Using the full discrete model leads to a systematic error and the relatively time-consuming to model. Therefore, further consider the use of simulation load profiles analytic functions, which has certain advantages dimension of the task, speed and visibility, and can explain the causes of systematic error in formula (1) and their elimination.

In this regard, following the technique of estimating the loss of electricity distribution networks performance load profiles on duration as a continuously decreasing function and obtaining the necessary characteristics of the graph by direct integration.

Method of determining k^2_f simulation load profiles analytical dependences in time. **Energy losses in the elements of an electric network are a function of the characteristics of load**

profiles. Load profiles **on duration can be expressed in different functions: parabolic when k_{fill}≥0.7; linear at k_{fill}=0.5÷0.7; exponential with k_{fill} =0.25÷0.5**; hyperbolic linear at **k_{fill} ≤ 0.25 etc. [2].**

Equation (1) obtained an approximation of load profiles for the duration of the following analytical dependences in time:

$$I = I_{max} - (I_{max} - I_{min})\left(\frac{t}{T}\right)^{\lambda} \quad \text{при}\ \lambda > 1 \tag{6}$$

$$I = I_{min} + (I_{max} - I_{min})\left(1 - \frac{t}{T}\right)^{\frac{1}{\lambda}} \quad \text{при}\ \lambda \le 1 \tag{7}$$

where - I_{max}, I_{min} values of maximum and minimum currents for the settlement period of time T.

Auxiliary factor λ determined as follows:

$$\lambda = \frac{I_{av} - I_{min}}{I_{max} - I_{av}}$$

In deriving (1) the following assumptions [3]: load profiles the load as a random variable has a beta - distribution; load profiles on duration represented by analytical dependences in time form (6) and (7). Given the fact that the analytical dependence (6) and (7) are inferable, the parameters for the beta - the distribution and, accordingly, (1).

Approximation load profiles analytical dependences of the form (6) and (7), although much more accurate simulation of energy loss, but does not completely eliminate the systematic errors [1]. In this regard, in [4, 5], attempts were made to obtain empirical relationships that eliminate these shortcomings. In [5], the choice of approximating functions load profiles different analytical dependences.

Next, we consider obtaining empirical approximation for k^2_fload profiles exponential dependence of the form

$$I = I_{min} + (I_{max} - I_{min}) \cdot e^{-(\alpha_2 t)^{\rho}} \tag{8}$$

Here, α and ρ - zoom options, determined by approximation.

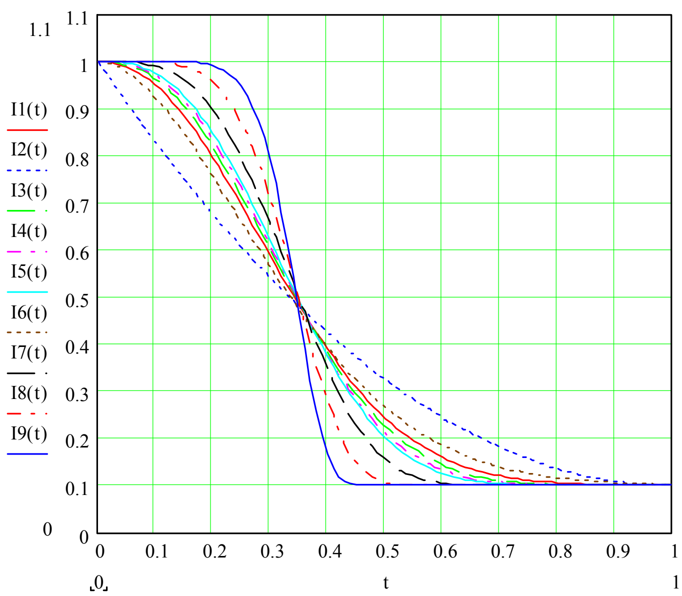

Fig. 2. Family load profiles with k_{fill}=0.4 , I_{min}=0.1 and λ=0.5 represented by a power function of the form (8) and the exponential form (7) for different ρ and α.

Improving the accuracy of modeling the energy loss and reduction of systematic errors can be achieved by selecting the type of approximating dependence (ρ and α) production load profiles. Modeling load profiles exponential form (8) by choosing ρ and α gives a family of graphs and kf are close to the real (Fig. 2).

Formulation of the problem of modeling the characteristics of production load profiles for the duration. Define the parameters α and ρ approximation load profiles dependence (8), which are also an implicit function of the parameters load profiles $\mathbf{k_{fill}}$, k_{min} and k_{tmin} .

$$k_{fill} = \int_0^1 (I_{min} + (I_{max} - I_{min}) \cdot e^{-(\alpha t)^{\rho}})dt \qquad (9)$$

Dispersion for the given load profiles is determinates by expression

$$D_i = \int_0^1 \left(I_{min} + (I_{max} - I_{min}) \cdot e^{-(\alpha t)^{\rho}} \right)^2 dt - k_{fill}^2 \qquad (10)$$

To calculate the definite integral (9) and (10) used numerical integration methods, in particular, Simpson method by selecting the corresponding α and ρ. In general, the load profiles $\mathbf{k_{fill}}$ = const have many solutions α_i and ρ_i and different dispersions.

Selection coefficients α_i and ρ_i, providing multiple solutions is a solution of the problem of minimizing the function

$$\left[\int_0^1 I_{min} + (I_{max} - I_{min}) \cdot e^{-(\alpha t)^{\rho}} dt - k_{fill} \right]^2 = 0 \qquad (11)$$

subject to the restrictions on k_{min} and k_{tmin} as

$$0 \le k_{min} < 1,$$
$$0 \le k_{t\,min} < 1.$$

Direct integration of expression (9, 10) is not possible and therefore requires the use of numerical methods.

When setting ρ is the minimization of (11) by selecting the value α, which provides a given value of k_{fill} (9)? In the case where for a given ρ can not provide appropriate value of k_{fill}, the change in ρ (increase) seek the solution set ρi and αi.

Ranges of α and ρ depend on the shape of load profiles k_{fill} and k_{min}. Modeling the load profile corresponding to given values of parameters k_{fill} and k_{min} is not always possible to provide. For example, setting the parameter α <2 often does not provide specified k_Z, k_{min} and k_{tmin} by choosing α.

As a preliminary criterion for complete selection of options appropriate approximation of (9) proposed to use the condition

$$(I_{max} - I_{min}) \cdot e^{-(\alpha t)^{\rho_i}} \le \varepsilon_{кз} \qquad (12)$$

Here is invited $\varepsilon_{кз}$ =0.0005.

If condition (12) holds, then we can assume that (8) provides an approximation having an acceptable error.

The start time of minimum load graphics in the program is determined by the condition

$$(I_{max} - I_{min}) \cdot e^{-(\alpha t)^{\rho_i}} - k_{t\,min} \le \varepsilon_{kt\,min} \qquad (13)$$

The condition of precision search k_{tmin} invited to take within $\varepsilon_{кtmin}$ =0.001 ÷ 0.01

Simulation modeling of load profiles family to determine the ranges of shape factor.

Simulation marginal production schedules by the condition of obtaining the lowest and highest values for k^2_f. Technique for modeling load schedule (8) with k_{fill} (9) reduces to the problem of finding the parameters α and ρ from $(k_p - k_{fill})^2 \rightarrow$ min. For this purpose, using quadratic interpolation functions (11) for values of α for a given ρ in three different locations

$$f(\alpha) = a + b\alpha + \alpha^2 \tag{14}$$

Algorithm for simulation of load profiles based on an iterative coordinate descent method and the method of quadratic interpolation. To find the minimum of the method of quadratic interpolation.

Programming. A program for simulation of load profiles (Fig. 3). The initial data inputs are: the filling factor of load profile k_{fill}, the ratio of minimum load to maximum kmin, the relative duration of the off-peak schedule - k_{tmin}, defined as the ratio of the length of the minimum load for the duration of the billing period. Simulation modeling of a family of load profiles by specifying parameters $\mathbf{k_{fill}}$, k_{min}, k_{tmin}. In this case, determined by α and k_{tmin}. Provides lower and upper limits of change ρ and pitch changes Δρ. Usually for 10-40 iterations can be obtained practically acceptable approximation for a load profile using a quadratic interpolation.

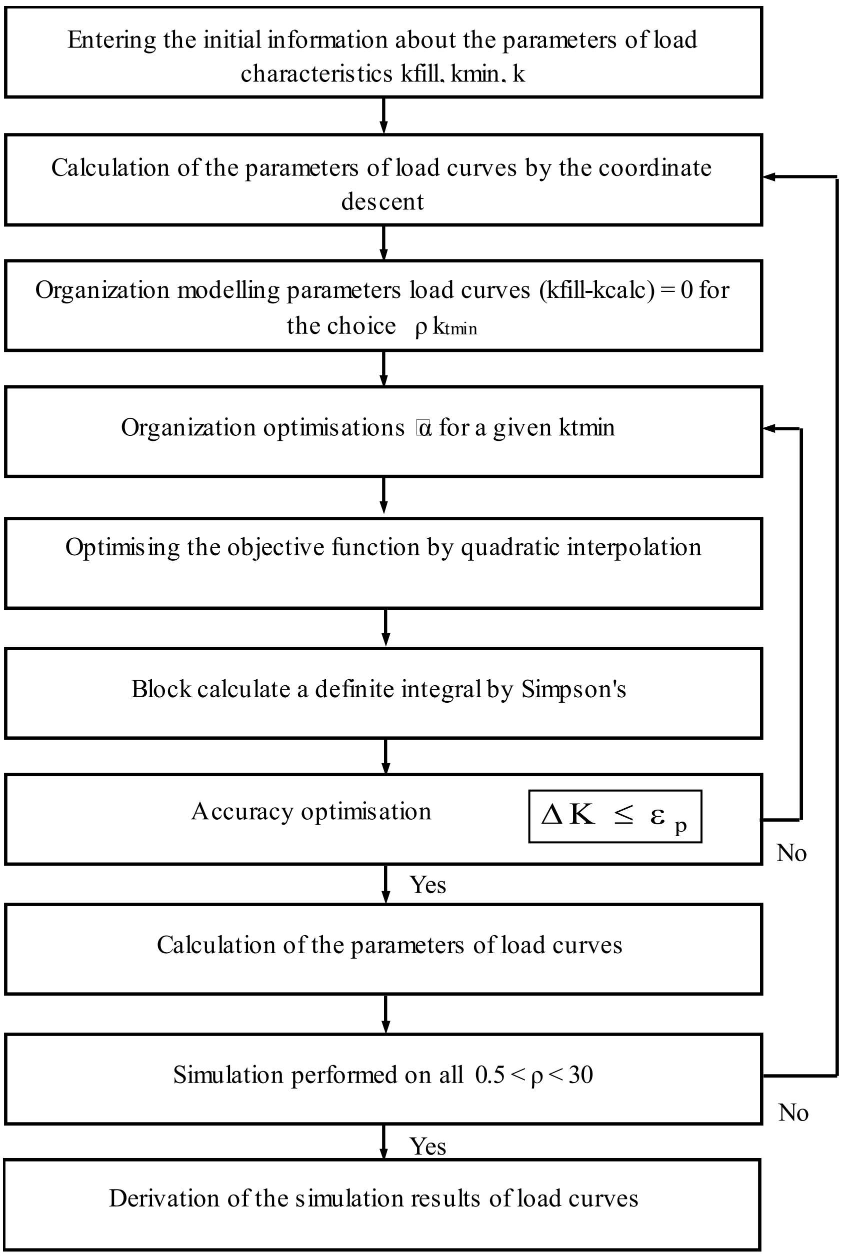

Fig. 3. Flow chart of load profiles simulation.

Recommended values of α and ρ depending on the load shape. Below are the recommended values of α and ρ to approximateload profiles function of the form (8) in duration, depending on the k_Z. For a given k_{fill} and k_{min} and relative growth ρ increases the dispersion and the value of α. When α = const and increasing ρ, k_{fill} decreases. An increase in α and ρ variance increases. To obtain relatively large k_Z must specify a relatively large value of ρ. To obtain profiles with longer duration load minimum k_{tmin} necessary to increase the value of α. To obtain profiles with longer duration of maximum load is also necessary to increase the value of α. Ranges of the dispersion of production schedules with the specified k_{fill} defined by setting ρ of $0.5 \le \rho \le 30$ and a step change in Δρ $0.1 \le \Delta\rho \le 1$, the choice of α corresponding to the set parameters and k_{fill} and k_{min}.

Numerical experiment. Simulation results for the load profile with k_{fill} = 0.4, k_{min} = 0.1 are shown in Table 2.

Table 2. Parameters of the simulation load profile.

Number	Coefficient of formula (8)		Results of simulation (9-13)		
	α_i	ρ_i	k^2_f	k_{tmin}	k_{tmax}
1	2.666	1.8	1.587		
2	2.661	1.9	1.61	0.17	0.08
3	2.658	2	1.631	0.2	0.08
4	2.679	3	1.777	0.38	0.14
5	2.719	4	1.857	0.46	0.17
6	2.755	5	1.907	0.5	0.2
7	2.783	6	1.941	0.53	0.22
8	2.806	7	1.966	0.55	0.23
9	2.825	8	1.985	0.57	0.24
10	2.841	9	2.0	0.58	0.25
11	2.854	10	2.012	0.59	0.26
12	2.865	11	2.022	0.59	0.26
13	2.875	12	2.03	0.6	0.27
14	2.883	13	2.037	0.61	0.27
15	2.891	14	2.043	0.61	0.28
16	2.897	15	2.049	0.61	0.28
17	2.903	16	2.053	0.62	0.28
18	2.908	17	2.058	0.62	0.29
19	2.906	18	2.059	0.62	0.29

Dependence of the squared form factor, the resulting simulation of load profiles for the values of k_{fill} = 0.4, k_{min} = 0.1 shows that for the same values of the k_{fill} value k^2_f varies within 1.587 ÷ 2.059. The average value is set to k^2_{fsr} =1.823. Limits k^2_f deviations from the average amount ±13%. The relative duration of minimum load varies in k_{tmin} = 0.13 ÷ 0.62.

Simulation modeling of load profiles as we obtain the limit profiles for k^2_f appropriate minimum, taking $k_{min} = 0.25 * k_{fill}$, and maximize, taking $k_{min} = 0.6 * k_{fill}$ algorithm (6-14), whose results are shown in Figure 4.

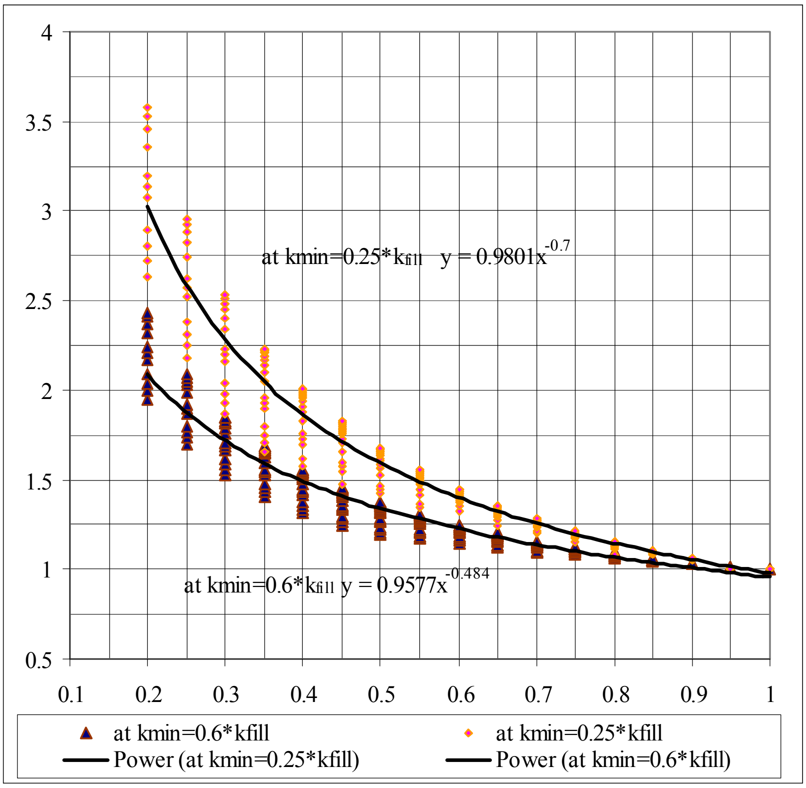

Fig. 4. Profiles of variation squared form factor of the k_Z to the possible load profiles.

Dependence of the squared form factor, the resulting simulation schedules (Table 2) and (Fig. 4) shows that for the same values of the k_Z and k_{min} time duration of treatment with minimal impact k_{tmin} and the corresponding values k^2_f vary widely. In the presence of advance information about ktmin available to assess k2f depending on three parameters: k_{fill}, k_{min} and k_{tmin}, allowing more accurate simulation.

Comparison of calculation results of simulation program schedules with the most commonly used empirical formulas.

Produced by comparing the results of the calculation k^2_f based on simulation graphs of electrical loads for the duration of the algorithms (3) - (5) and (6-14) (Table 3).

Table 3. The results of comparison k^2_f for k_{min} = 0.1 and ρ = 2 by simulation load profiles by function of the form (11) and (3) - (5)

Filling coefficient, k_3	Results of simulation (10)		Results of discrete simulation	Error of simulation (5)-(7), %
	α	$k^2_{fп}$	k^2_{fd}	$(k^2_{fд}- k^2_{fп})*100\backslash k^2_{fп}$
0.250	5.317	2.167	2.198	1.43
0.300	3.988	1.970	1.905	-3.30
0.350	3.190	1.789	1.709	-4.47
0.400	2.658	1.631	1.582	-3.00

Produced by comparing the results of the calculation for the average k^2f formula (1) and simulation plots of electrical loads for the duration of the algorithm (6-14) for load profiles, depending on the k_Z and k_{min}, taking k_{min} = ν * $\mathbf{k_{fill}}$. ν = 0.666, 0.5, 0.25, 0.125. The calculation results k^2_f shown in Table 4.

Table 4. Calculation errors k^2_f formula (1) for small values of k_{fill}

№	Filling coefficient , k_{fill}	Minimum of graph, k_{min}	Model estimated value k^2_f by		Error of k^2_f by formula (1)
			formula (1)	algorithm (8-13)	
1	0.5	0.3333	1.143	1.236	-7.52
2		0.25	1.2	1.366	-12.15
3		0.125	1.273	1.592	-20.04
4		0.0625	1.304	1.684	-22.57
5	0.4	0.2666	1.225	1.355	-9.59
6		0.2	1.321	1.585	-16.66
7		0.1	1.45	1.823	-20.46
8		0.05	1.508	1.971	-23.49
9	0.3	0.2	1.363	1.522	-10.45
10		0.15	1.527	1.792	-14.79
11		0.075	1.754	2.213	-20.74
12		0.0375	1.86	2.457	-24.30
13	0.2	0.1333	1.64	1.909	-14.09
14		0.1	1.941	2.344	-17.19
15		0.05	2.371	2.991	-20.73
16		0.025	2.577	3.371	-23.55

Empirical formula (1) has negative systematic inaccuracy, and (3) has a positive systematic inaccuracy in k^2_f compared with the average values obtained by the technique (6-14). The values of the systematic inaccuracy k^2_f vary in the range (7 ÷ 45%) depending on the k_{fill}.

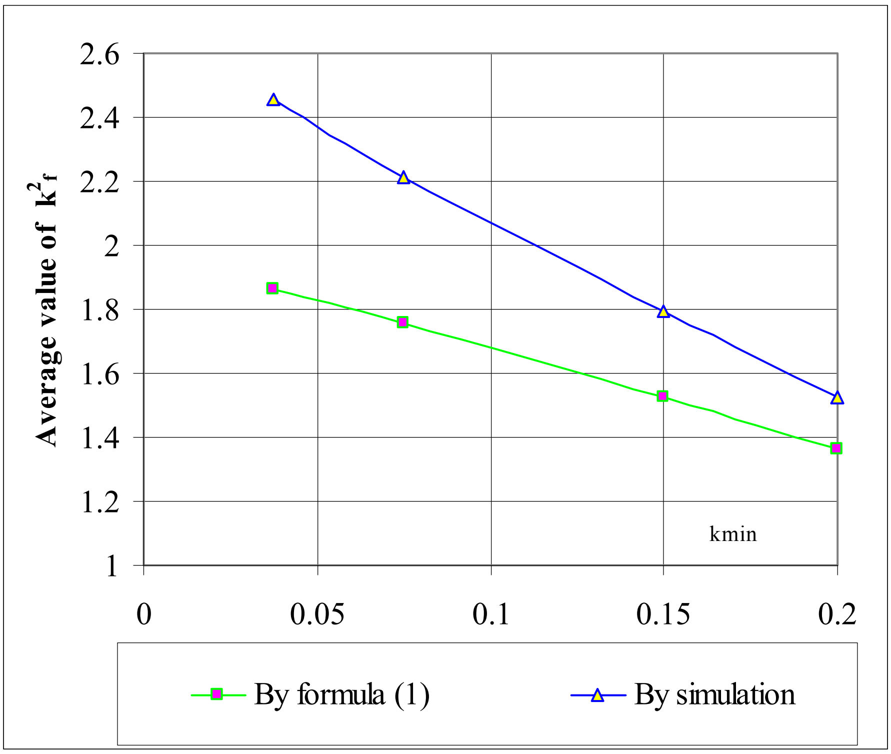

Fig. 5. Depending on the results k^2_f simulation load profiles (6-14) and by (1) $k_{fill} = 0.3$ from k_{min}.

The errors increase with decreasing k_{fill} from 0.5 in the direction of small values.

k_{fill} = 0.5 for the values of the errors in $k^2{}_f$ vary in the range (7 ÷ 15%), for k_{fill} = 0.4 in the range (10 ÷ 17%), for k_{fill} = 0.3 in the range (16 ÷ 24%) and k_{fill} = 0.2, range (22 ÷ 45%).

Depending on the results $k^2{}_f$ simulation load profiles (6-14) and by (1) k_{fill} = 0.3 from k_{min} shown in Fig. 5.

Dependence of the error in $k^2{}_f$ by (1) k_{fill} = 0.3 from k_{min} is shown in Fig. 6.

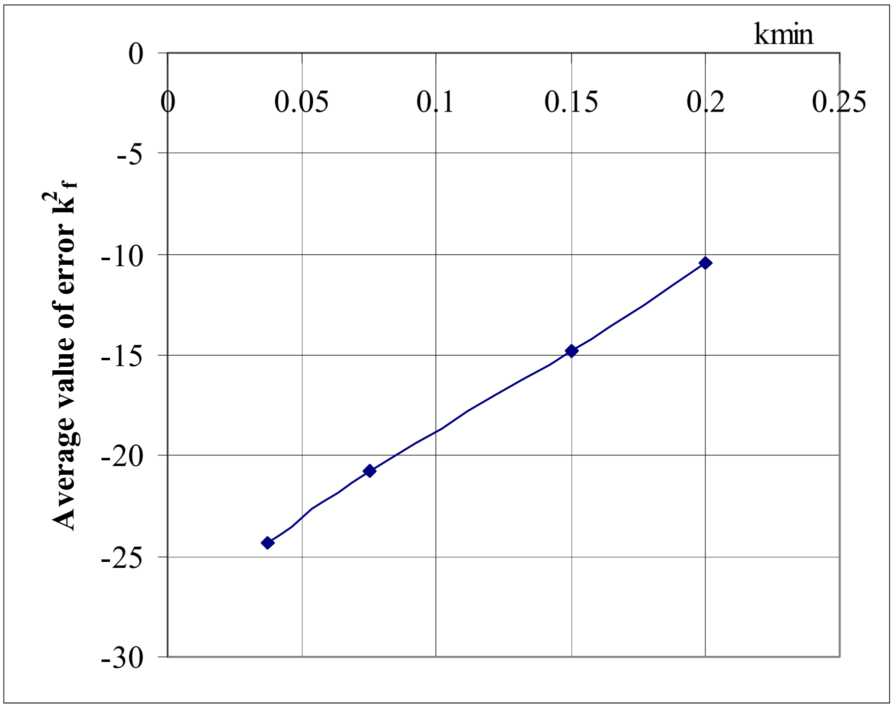

Fig. 6. Dependence of the error in $k^2{}_f$ by (1) k_Z = 0.3 from k_{min} .

A comparison of shape factor family of load profiles on duration as a power function (7) and expression (8) show that, depending on the value of t_{max} approximation coefficients α and ρload profiles take different values. $k^2{}_f$ value varies in the range 1.631 ÷ 1.856.

Approximation load profiles dependence (7) compared with (8) and algorithm (9) - (13) has a negative error of the estimated 7-30% depending on the k_{fill} and k_{min}.

The reasons for the growth of systematic errors (1) for small values of the fill factor due to the use for the approximation of production schedules depending on the form (7), which in this case, the forms chart from almost zero to a maximum load.

Using simulation diagrams of electrical loads according to the algorithm (6) - (14) allows more flexible modelling $k^2{}_f$ and meets the additional desired parameters: the duration of the minimum and maximum loads.

Thus, the use of simulation load profiles exponential dependence of the form (8) differs from the known fact that is based on close to real load profiles and, accordingly, improves the accuracy of simulation $k^2{}_f$.

Conclusions

1. The technique of simulation load profiles of possible schedules for the duration of the electrical loads in the form of a continuous function approximation schedules exponentially.

2. Produced by comparison of the calculated form factor k^2_f with the results used in practice, empirical formulas, and establishes the presence of significant systematic errors at small values of the fill factor to 30%.

3. The proposed technique for modelling the characteristics of load profiles duration as a continuous function of improving the accuracy and flexibility of modelling k^2_f and losses in distribution networks.

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RISK ASSESSMENT OF OFFENCES AT STATE BORDER USING FUZZY HIERARCHICAL INFERENCE

Alexander S. Androshchuk
•
National Academy of State Border Guard Service of Ukraine
named after Bogdan Khmelnitsky, Khmelnytskyi City, Ukraine

e-mail: asaasa@mail.ru

A model of fuzzy inference to risk assessment of offenses in control. The useage of models provides an opportunity: the use of quality indicators, taking into account inaccurate, approximate information, using knowledge of experts, which are given in the form of fuzzy rules. The complexity of constructing fuzzy model conclusion is solved by a hierarchical system of opinion and knowledge bases.

Topicality. One of the improvements in management of departments and agencies of the State Border (SB) is to increase the speed and quality [1]. Range of tasks in such an environment significantly increases, most of them are weakly-formalized and unformalized nature, the conditions of their solution is continuously deteriorating. The need for new methods of management bodies of control SB are evident during profiling (assessment) risk [2]. The most critical issue is risk assessment of offenses relative in control on SB.

Formulation of the problem. Performances of the mathematical problem of risk assessment as a feasible nonlinear object with the set of input variables $X=\{x_i\}$ and one output variable y:

$$y=f_y(x_1, x_2,\ldots, x_n)\ . \qquad (1)$$

How to choose the signs of input variables (risk indicators) offense in control. Output variable y is an indicator of the degree of possible violations of security control on SB.

To automate the risk assessment at the State Border Guard Service of Ukraine (SBGSU) is established subsystem "RISK" [3]. Analysis of its application shows that worked through the issue of automation of storage and delivery on request relevant information. Question analysis (processing) of this information remains the prerogative of the SBGSU personnel on the basis of his experience, intuition, subjective perceptions. Schengen information system (Schengen Information System - SIS) is designed to provide common external borders and integration into a single system, necessary for the Schengen Convention [4]. SIS contains data on individuals, yaks crossed the common border of the European Union (EU), illegal immigrants, lost and false travel documents, persons missing more. Risk assessment of involvement with offenders is not made.

That is, the existing information and telecommunication systems SBGSU and the EU is unable to make sufficient timely and quality support decisions on risk and need further improvement.

Analysis of recent research and publications. For risk assessment should be applied not only qualitative judgments about these risks, but also various methods of quantitative analysis.

In [5] presented research aimed at automating the distribution of capabilities in the protection of control on SB based on optimization methods. However, the tasks of risk assessment is given insufficient attention. In [6] proposed a model that is based on a binary interpretation of signs (risk indicators) describing the site. Analysis of this model showed that this interpretation is not suitable for assessing the majority of risk indicators that are qualitative in nature.

In other subject areas for risk assessment is most often used device of probability theory and mathematical statistics [7, 8]. However, given that decisions on the basis of risk assessment areas, occurs in an environment where: events do not occur with enough frequency, most signs are good quality and presented in natural-language descriptions, and their rating is based on vague opinions and estimates of experts, information the main parameters are incomplete and unclear, etc. - use probabilistic methods is impossible.

One of the promising areas of modern high-tech simulation is unclear, due to increasing trend of complexity and formal mathematical models of real systems and management processes associated with the desire to improve their adequacy and to consider a set of different factors that influence decision-making processes.

The question of risk of crossing the state border with the use of fuzzy inference models considered in [9], based on 3-5 features describing the object of research. The study of more complex models of risk assessment of offenses with a lot of signs according to the method [10] (see Table 1) is carried out.

Table 1. The tasks of analysis and risk assessment of offenses in SB

№ i/o	Name of task	Signs of description (entrances variables)		Risk estimation	Leading actions
1	Estimation of risk of offence on SB	x_1 - is **a volume:** the volume of display of offence, which causes a disturbance, is estimated		Degree of possibility of realization of offence	Adjustment problems of border dresses, Border Guard officers and border control
		How the offense with other factors – y_1	x_2 - **implementation:** the level of reacting of organization is estimated on offence		
			x_3 - is **tendencies:** it is estimated, degree of disturbance by the tendency of change of display of offence		
		The level of concern about the offense – y_2	x_4 - is **a seriousness:** the level of disturbance of organization is estimated in relation to the display of offence		
			x_5 - is **priority:** the level of disturbance of government is estimated in relation to the display of offence		
			x_6 - is **the disturbance of society:** the level of disturbance of public is estimated in relation to the display of offence		
		How the offense with other factors – y_3	x_7 - is **a generator:** the level of disturbance of influencing of offence is estimated on other offences		
			x_7 - - **STEPPYUEH:** the level of disturbance of external factors which influence on activity of service is estimated.		

The purpose of the article - a methodological study using the apparatus of fuzzy logic and presentation of the analysis automation tools for incoming data on violations of control.

The content. The authors suggest the use of fuzzy inference model for evaluation of crime control, according to the mathematical formulation of problem (1). Fuzzy logical conclusion - this approximation of "inputs-outputs" depends on the basis of linguistic expression <IF- THEN>. For example, drug smuggling, has a high volume, trends and have a high level, the degree of possibility of its occurrence in SB is high.

Structure of the fuzzy conclusion includes the following modules [12]:

fuzzy factor that converts a fixed vector of input variables (factors affecting) (*X*) in the vector of fuzzy sets that are needed for fuzzy inference;

fuzzy knowledge base, consisting of: base rules, which contains information about the dependence $Y=f(X)$ in the form of linguistic rules <IF-THEN> and intended for formal presentation of empirical knowledge, or knowledge of experts in a subject area, a database that contains parameters of membership functions and the coefficients of the importance of rules;

membership function that is used to represent linguistic terms as fuzzy sets;

fuzzy inference machine, which is rules-based knowledge base determines the value of output variable in a set $\widetilde{Y}$ of corresponding fuzzy values of input variables ($\widetilde{X}$);

defazzyfikator that converts output fuzzy sets in a clear number Y.

Content interpretation of fuzzy model involves the selection and specification of input and output variables of the fuzzy system output. Each sign, see. Table. Formalized as a level of compliance with offenses. For example, the volume: measured by the fact whether the offense generates concern - x_1: Obviously, the higher the score, the more can be carried out illegal activity.

In our case, 8 input variables. To clarify the model in the future may apply additional indicators.

Output variable is the level of the degree of possibility of offense - D: transportation of contraband, weapons, drugs, etc. on a specific area (checkpoint control on SB).

In the fuzzy model of risk of illegal activity all variables are presented as linguistic, universal set of which $U=\{u_1, u_2,\dots, u_n\}$ is measured in points in the interval of real numbers from 1 to 5 staff based on their knowledge and experience.

As a term-set of variables we will use the set L_1 = {"low", "medium", "high" level}.

Building a membership function terms "low", "medium", "high" that are used for evaluation of linguistic variables may be achieved through the method of statistical processing of expert information that is presented in [12].

The next step - building a fuzzy knowledge base. Description of the problem areas of risk control requires a large number of rules (see Table 1). When a large number of parameters of the construction of expression "input-output" becomes quite complicated. This is because in the memory of man also held up to 7 ± 2 concepts of tag [11]. In complex systems, a condition which is characterized by many features, there is a problem of completeness of knowledge base. It is known that the number of rules needed to describe the n input variables whose values are determined using m terms, equal to $R=m^n$ [12]. For example, in our case, n = 8, m = 3, then the number of rules R = 83 = 512.

Thus, the number of rules increases exponentially with increasing number of input variables, resulting in increased time and complexity of the output logical decision. Knowledge base containing a large number of rules are complex in perception, editing and use. But decision-making process requires, however, full understanding of the particular situation. This contradiction is solved by constructing a hierarchical knowledge base [13]. This approach corresponds to the hierarchical structure of the SBGSU process in special situations. In this regard, to hold the hierarchical classification of the parameters on it and build a tree report that will determine the system of nested into each other expression-knowledge of lower dimension. An example of such a tree to 8 input variables (see Table 1) Shown in Fig. 1. From the example shows that knowledge of the type $D=f_5(x_1,x_2,\dots,x_8)$ the relationship of inputs x1-x8 with the release of D, replaced by a sequence of relationships: $D=f_5(x_1,y_1,z)$, $y_1 = f_1(x_2,x_3)$, $y_2 = f_2(x_4,x_5,x_6)$, $y_3 = f_3(x_7,x_8)$, $z = f_4(y_2,y_3)$, where y_1,y_2,y_3,z - intermediate linguistic variables.

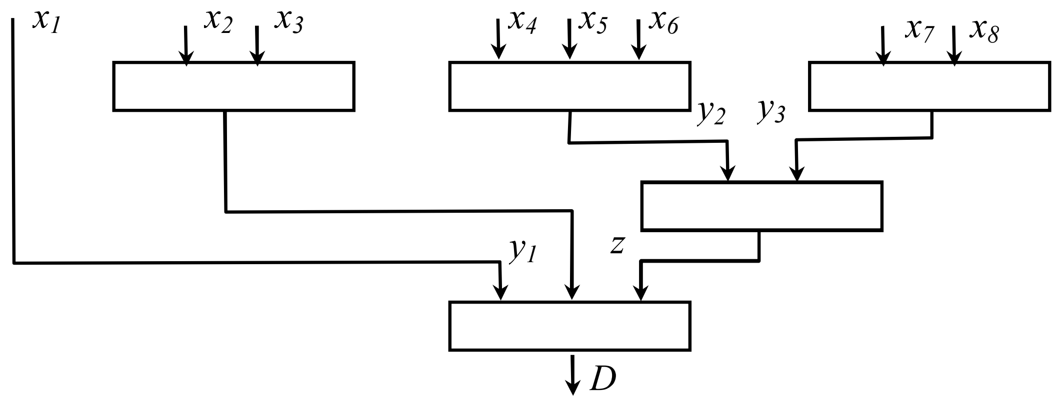


Figure 1. Hierarchical tree report

Thus the number of rules R = 33 +32 +33 +32 +32 = 27 +9 +27 +9 +9 = 81, significantly less compared with the usual conclusion.

Due to the principle of hierarchy can be considered a significant number of parameters that affect the overall assessment. Feasibility porivnevoho representation of expert knowledge is due not only to the natural hierarchy of objects of evaluation, but also the need to take account of additional parameters as the accumulation of knowledge about the object.

Based on the expert survey constructed fuzzy knowledge base of fuzzy systems $D=f_5(x_1,y_1,z)$, $y_1 = f_1(x_2,x_3)$, $y_2 =f_2(x_4,x_5,x_6)$, $y_3 = f_3(x_7,x_8)$, $z = f_4(y_2,y_3)$, which are given in Table. 2.

The method of activation will be *min*. Since all the rules as a logical connection to pidumov only applies fuzzy conjunction (the operation "AND"). As a method of aggregation will use the *min*-conjunction operation. For accumulation endings rules will use the *max*-disjunction. As defazzyfikatsiyi method will use the center of gravity method.

Feature of fuzzy inference in the hierarchical knowledge base is the lack of procedures and defazzyfikatsiyi fazzyfikatsiyi for intermediate variables (y_1, y_2, y_3 and z in Fig. 1). The result of inference in a fuzzy set directly transmitted to the machine fuzzy logical deduction the next level of hierarchy. Therefore, to describe the intermediate variables in the hierarchical fuzzy knowledge bases of a set only term-set, no definition of membership functions.

Table 2. Fuzzy Knowledge Bases

Fuzzy knowledge base on $y_1 = f_2(x_2,x_3)$			Fuzzy knowledge base on $y_2 =f_3(x_4,x_5,x_6)$			
Low	Low	Low	Low	Low	Low	Low
Middle	Low	Low	Low	Middle	Middle	Middle
Middle	High	Middle	Middle	High	Middle	Middle
High	Middle	Middle	Middle	Middle	High	Middle
High	High	High	High	High	High	High

Fuzzy knowledge base on $y_3 = f_4(x_7,x_8)$			Fuzzy knowledge base on $z = f_5(y_2,y_3)$		
Middle	Low	Low	Low	Middle	Low
Low	Middle	Middle	Low	Low	Low
Middle	Low	Middle	Middle	Low	Low

Middle	Middle	Middle	Middle	Middle	Middle
Middle	High	High	High	Middle	Middle
High	High	High	High	High	High

Fuzzy knowledge base on $D=f_1(x_1,y_1,z)$			
Low	Middle	Low	Low
Low	Low	Middle	Low
Middle	Middle	Low	Middle
Middle	Middle	Middle	Middle
High	Middle	Middle	Middle
High	Middle	High	High

The implementation model of fuzzy inference by using the package fuzzyTech [14]. It found the original value of variable D, for given input variables:

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	D
5	4	4	3	4	3	1	2	4,2
1	1	2	1	4	3	2	3	3,3

Figure 2. shows the hierarchical scheme of fuzzy inference constructed in the environment fuzzyTech 5.8.

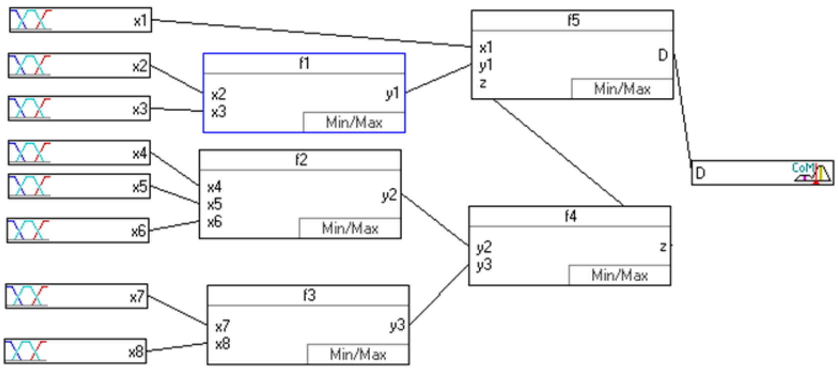


Figure 2. Hierarchical scheme of fuzzy inference

The calculated results coincide with those obtained experimentally.

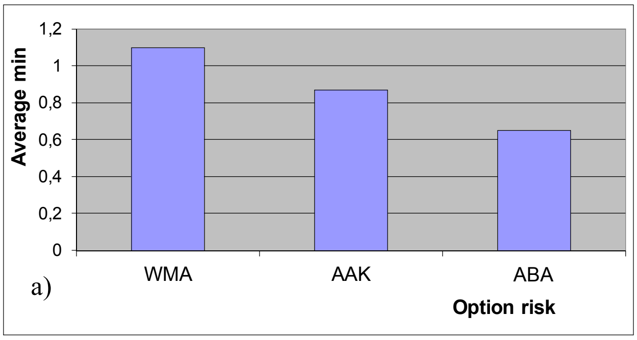
Operationalize the model implemented as a software module "Risk assessment offense", which proposed to include subsystem "RISK".

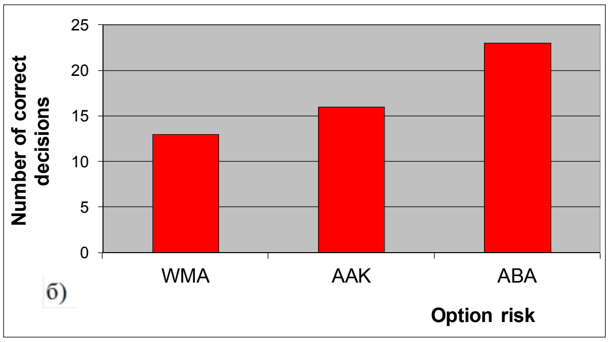
Check the adequacy of the developed model of fuzzy inference made by the experiment. The experiment was conducted at the department of integrated border management of the National Academy of SBGSU. For the experiment, were selected data on offenses that were used for illegal activity through the control on SB at different times in different parts of the border service.

The experiment involved three study groups: two control and one experimental (20 students in each). During the experiment, each listener provided information about 30 descriptions of possible situations of control, and 15 of each category (high risk and low risk). The task of the inspectors first control group consisted of assess the situation and take it to one of two categories (high and low degree of involvement in illegal activities) without the use of automation. The task of students second control group consisted of assess the situation and take it to one of two categories of application software module based on the model [6].

The task of inspectors experimental group was to assess situations and refer them to one of two categories with the use of automation (software module developed by the author of "Risk assessment offense"). After entering the last parameter is risk assessment in points. If it is within 5.4 points, the situation is evaluated highly the possibility of offense.

The researchers evaluated the following parameters: time spent on assessment of the situation, the quality of decision - evaluation coincides with the existing offenses (correct decision), the assessment does not match (wrong decision). Experimental results, which are shown in Fig. 3.a - on time for assessment of offenses in Figure 3.b - the quality of the decisions indicate that application of the developed software module based on the model of hierarchical fuzzy inference "Risk assessment offense" allows you to: reduce the time to assess the offense, 1,7 times compared with the risk assessment without automation, increase the number of correct solutions in 1,8 times compared with the risk assessment without automation and 1,2 times compared with the known approach.





WMA- without automation; AAK - automation (known approach);
ABA - automation (developed approach)

Figure 3. Results of experimental verification

Conclusions

Thus, the paper presents a model of fuzzy inference of risk offense to control illegal activity and by its experimental verification. Application of this model, unlike the existing offers an opportunity: the use of quality indicators, taking into account inaccurate, approximate information about important features, the usage of knowledge of experts from the Border Guard Service - experts who served in the form of fuzzy rules of inference, a more qualitative assessment of the object under research during profiling risks. The complexity of constructing fuzzy model conclusion is solved by a hierarchical system of opinion and knowledge bases. Implementation of this model within the software and algorithmic provision of information and telecommunications systems will enable SBGSU to reduce time for risk profiling and improve the quality of decisions made.

The proposed approach requires the development of formalization of knowledge and experience, which were accumulated by experts (staff officers, heads of departments (agencies, departments, administration, supervisors), teachers of educational institutions, developers of an integrated information system, "Hart", which is the prospect for further research in this direction.

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TESTING FOR SERIAL CORRELATION BY MEANS OF EXTREME VALUES

Ishay Weissman

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Faculty of Industrial Engineering and Management
Technion - Israel Institute of Technology

e-mail: ieriw01@ie.technion.ac.il

Dedicated to the hundredth birthday of Boris Gnedenko

ABSTRACT

The *largest spacing* of a sample is suggested as a possible test-statistic to detect serial dependence (correlation) among the data. A possible application is in testing the quality of random number generators, which are so important in the study of systems reliability. We compare its performance to the *Kolmogorov-Smirnov* test because of their similar nature – one is based on extreme distance between order statistics, the second on extreme discrepancy between the empirical distribution function and the theoretical one. The tests are applied to several models with serial dependence. Special attention is given to an *autoregressive model*. Based on Monte Carlo simulations, the largest spacing is more powerful for moderately large sample size, over 50, say. A surprising connection to extreme values is discovered, namely, that the *likelihood-ratio* test, which is most powerful under the autoregressive alternative, is based on lower extremes.

Keywords: Autoregressive model, Binomial model, Kolmogorov-Smirnov test, Largest spacing, Likelihood-ratio test, Monte Carlo methods, Moving-max model.

1 INTRODUCTION AND MOTIVATION

Suppose we have a sample $X_1, X_2, \ldots, X_n$ from some continuous distribution function $F0$. Suppose further that $F0$ is the uniform distribution over the unit interval [0, 1] (if not, we replace the Xi by $F_0(X_i)$). Under ideal conditions we expect that the sample is an iid sample, but we suspect that the data at hand exhibit some serial correlation among consecutive observations and we want to put it under a statistical test.

So, let $H0$ denote the null hypothesis of "iid-uniform". If the alternative is "not iid-uniform", then, as J.E. Gentle says, "*this alternative is uncountably composite and there cannot be a most powerful test. We need a suite of statistical tests. Even so, of course, not all alternatives can be addressed*" (Gentle (2003), page 71). In the context of random number generation L'Ecuyer & Simard (2007) offer a comprehensive battery of tests called **TestU01**. The authors state on page 4 that "*the number of different tests that can be defined is infinite and these tests detect different problems.*" Our aim in this paper is to attack a narrower problem, namely the existence of serial correlation. There are many possible models which possess serial correlation. To be specific, we start with the *autoregressive model*

$$X_i = \rho X_{i-1} + (1-\rho)U_i \quad (1 \le i \le n,\ 0 \le \rho < 1)\,, \tag{1}$$

where $\{U_i : i \ge 0\}$ is a $U[0,1]$-iid sequence and $X_0 = U_0$. In this setting, the problem is to test

$$H_0 : \rho = 0 \ \text{ vs. } \ H_1 : \rho > 0\,. \tag{2}$$

In Section 6 we deal with the *binomial model* and the *moving-max model*.

Back to Equation (1), a plot of consecutive pairs, *Xi*+1 vs. *Xi*, is very useful here. For instance, on the left plot of Figure 1, we have n = 2000 pairs with $\rho = 0$ and on the right, the same with $\rho = 0.1$. In the latter, the slopes below and above the data are quite visible. But as ρ decreases to 0, detection of existence of a serial correlation by the human eye becomes more and more difficult.

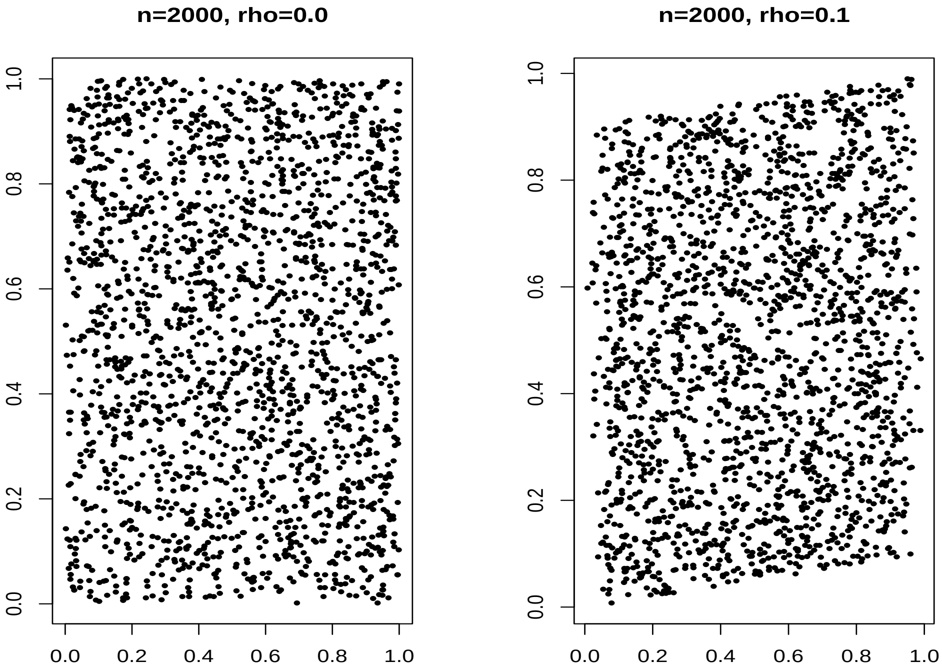

Figure 1. Pairs of Successive Numbers, X_{i+1} vs. X_i

This paper evolved as a result of the author's empirical observation that the largest spacing (LS) of a sample (see a formal definition in Section 2) is quite effective in detecting serial correlation in the sample (see Onn and Weissman (2011)). In order to study its performance with respect to other test statistics, we compare it to the Kolmogorov-Smirnov (KS) test. The KS test is chosen not only because it is so widely used as a goodness of fit test, but also because of its similar nature to LS. That is, while KS distance is the extreme vertical distance between the empirical distribution and the uniform distribution, LS is the extreme horizontal distance between consecutive order statistics. We also compare the two tests to the performance of the sample serial correlation (SSC), the least squares estimator of ρ. Under normality the latter is the natural one to use and it would be interesting to compare the performance of the first two tests to the performance of SSC under the present set-up. The comparisons in terms of power are presented in Section 3.

We then ask what is a most powerful test for the null hypothesis (2). It is somehow surprising that the answer is again an extreme statistics. In Section 4 we define the transformed data $\{T_{1i} : 1 \leq i \leq n\}$, which are iid, and show that the likelihood-ratio test (LRT) (which is most powerful) is based on $\min_{1 \leq i \leq n} T_{1i}$. In Section 5 we discuss similar issues under the *autoregressive model of order k.* Two more models which exhibit serial dependence are introduced in Section 6 and the performance of the LS and KS tests applied to these models are discussed.

Remark 1. The $\{X_i\}$ as defined by (1) are marginally not U [0, 1]-distributed if $\rho > 0$. But if the $\{U_i\}$ are discrete uniform (rather than continuous uniform), the $\{X_i\}$ can be uniform too: Suppose Y_1 and Y_2 are both U [0, 1]-distributed and the error term ε is a random variable, independent of Y_1. Suppose further that for some $\rho > 0$ one has

$$Y_2 = \rho Y_1 + (1 - \rho)\varepsilon.$$

What are the conditions on ρ and ε for this to hold? Lawrance (1992) proves that $\rho = k^{-1}$ for some integer $k \geq 2$ and ε must be uniform over the set $\{0, 1, \dots, k-1\}/(k-1)$. Hence, for very large k, ε is also (approximately) U [0, 1]-distributed. In fact, computer generated "random numbers" are of ε -type.

2 ASYMPTOTIC BACKGROUND

Largest spacing. Let $Y_1 \leq Y_2 \leq \dots \leq Y_n$ be the order statistics of $\{X_1, X_2, \dots, X_n\}$ and Y_0, Y_{n+1} be given by 0, 1 respectively. Define the *sample spacings* $V_i = Y_i - Y_{i-1}$ (i =1, 2, … , n + 1). Darling (1953) studies properties of the sample spacings (under H_0) in the context of a random partition of an interval. In particular he gives the exact joint distribution of (V_{min}, V_{max}),

$$P\{V_{\min} \geq x,\ V_{\max} \leq y\} = \sum_{j=0}^{n+1} (-1)^j \binom{n+1}{j} \{(1-(n+1-j)x - jy)_+\}^n.$$

Putting $x = 0$ we obtain the Whitworth (1897) result

$$P\{V_{\max} \leq y\} = \sum_{j=0}^{n+1} (-1)^j \binom{n+1}{j} \{(1-jy)_+\}^n. \tag{3}$$

and putting $y = 1$, we get

$$P\{V_{\min} \geq x\} = \{1 - (n+1)x\}^n \qquad (0 < x < 1/(n+1)).$$

Darling gives also the asymptotic joint distribution

$$\lim_{n\to\infty} P\left\{\frac{x}{(n+1)^2} \leq V_{\min}\ ,\ V_{\max} \leq \frac{y + \log(n+1)}{n+1}\right\} = \exp(-x - e^{-y}) \quad (x > 0,\ -\infty < y < \infty). \tag{4}$$

We learn from this equation that $Vmax$ is asymptotically Gumbel distributed and $Vmin$ is asymptotically exponential. In the notation of Gnedenko (1943), the asymptotic distribution functions of V_{max} and V_{min} are, respectively, $\Lambda(x)$ and $1-\Psi_\alpha(x)$ with $\alpha = 1$. These results can be explained by the well known fact, that if $E_1, E_2, \dots, E_{n+1}$ are independent unit-exponential and T_{n+1} is their sum, then

$$(V_1,\ V_2,\ \dots,\ V_{n+1}) \stackrel{D}{=} \frac{(E_1,\ E_2,\ \dots,\ E_{n+1})}{T_{n+1}} = \frac{n+1}{T_{n+1}} \cdot \frac{(E_1,\ E_2,\ \dots,\ E_{n+1})}{n+1},$$

independent of T_{n+1}. Since $T_{n+1}/(n+1) \to 1$ a.s., the limit in (4) follows.

Weiss (1959, 1960) writes that the statistics $R_n = V_{max} - V_{min}$ and $S_n = V_{max}{=}V_{min}$ have been proposed to test H_0 when the alternative is that the $\{X_i\}$ are iid from some df F ($\neq F_0$). He then shows that (asymptotically) the test based on S_n is equivalent to the test based on V_{min} alone, and the test based on R_n is equivalent to the one based on V_{max} alone (small values of V_{min} are critical, large values of V_{max} are critical). He farther shows that the test based on V_{min} is not consistent while the one based on V_{max} is admissible and consistent under any fixed alternative. This is a good reason to check how well V_{max} performs at the present set-up in comparison to other possible tests.

In all the tests that follow, the significance level is α = .05. In particular, the LST rejects H_0 when $V_{max} > v_n(.95)$, where $v_n(p)$ is the p-quantile of V_{max}, determined by (3). Since our study includes small values of n, we do not rely on the asymptotic distribution.

Kolmogorov-Smirnov distance. One of the most popular tests of the null hypothesis is the Kolmogorov-Smirnov test (KST). Given a sample, we define the empirical cumulative distribution function F_n by

$$F_n(x) = \frac{1}{n}\sum_{1}^{n} I\{X_i \le x\} \quad (0 \le x \le 1),$$

where $I\{A\}$ is the indicator of the event A. The Kolmogorov-Smirnov distance is defined by

$$D_n = \sup_{0\le x\le 1} |F_n(x) - x|.$$

The KST rejects the null hypothesis with significance level α = .05 if $D_n > d_n(.95)$, where $d_n(p)$ is the p-quantile of D_n. The exact distribution function of D_n is too complicated to write in a closed form. The asymptotic distribution is given by

$$\lim_{n\to\infty} P\{\sqrt{n}D_n \ge x\} = 2\sum_{i=1}^{\infty}(-1)^{i-1}\exp(-2i^2x^2).$$

The quantiles $d_n(.95)$ are well tabulated for $n \le 50$. For larger n, it is suggested to use the approximation $d_n(.95) \approx 1.36/\sqrt{n}$. However, we prefer to use estimators of $d_n(.95)$ based on Monte Carlo simulations of 10^6 replications of samples of size n. They give more accurate

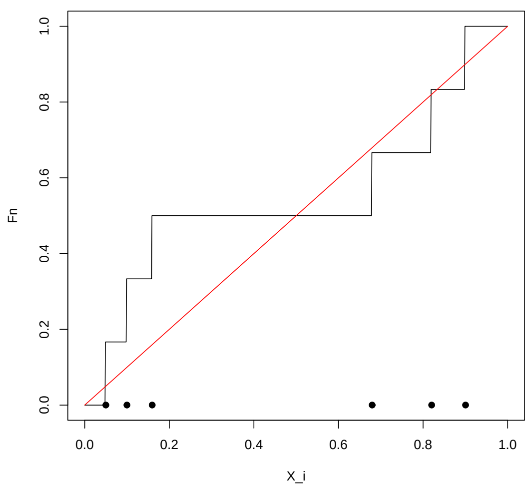

Figure 2. Example of F_n, $n = 6$

results, in the sense that the empirical powers under H_0 are closer to the nominal level of 5%.

Sample serial correlation. As mentioned above, the SSC is the least squares estimator of ρ. Under normality it is also the maximum likelihood estimator (MLE) of ρ (the MLE in our set-up, with uniform errors, is given in Section 4). The exact distribution is not known but by White (1961), SSC is asymptotically normally distributed with mean $(1 - 2n^{-1} + 4n^{-2})\rho$ and variance equal to $n^{-1}(1 - \rho^2) + n^{-2}(10\rho^2 - 1)$, (ignoring terms of order n^{-3}). Based on SSC, H_0 is rejected when SSC> $s_n(.95)$, the .95-quantile. As in the case of KS, the quantiles were estimated by $N = 10^6$ replications of samples of size n. It turns out that the asymptotic approximations are quite good (relative error ≤ 1%) when $n \geq 5000$.

3 POWER COMPARISONS

In this section we present the Monte Carlo results (using the software R) concerning the power of each test. For each selected n and ρ, we generated 10^5 samples of size n according to the autoregressive model (1). The empirical power reported is the proportion of samples for which H_0 H_0 is rejected. At an early stage of our study we thought that LS and KS distance might be highly correlated, because if LS is large, so must be at least one of the adjacent vertical distances, either at the right or at the left of the largest spacing as seen in Figure 2.

Table 1 shows the results for n = 100 and 1000. We also included the relative frequency of simultaneous rejection by LST and KST (under *Both*) and their correlation (under $r(V,D)$). It appears that they are indeed quite positively dependent. For n = 100 the LST is superior to the KST, inferior to the SSC test. For n = 1000, LST is by far superior to the other two tests. To strengthen the impression that as n increases, the superiority of LST becomes more and more apparent, we ran similar simulations for several sample sizes (again 10^5 replications for each pair (n, ρ)). The results are given in tables which appear in the Appendix. Based on these tables, we produced Figure 3. The blue graphs correspond to the likelihood ratio test, which is obviously most powerful (discussed in detail in Section 4). The other three are *ad hoc* tests applied to a particular model. The trend is clear – for n = 10, 20 the order (in terms of power) is SSC, KST, LST. For n = 50, 100 the order is SSC, LST, KST. For $n \geq 200$ the order is LST, SSC, KST. Moreover, the LST power function, for large n, tends to be very steep and approaches 1 very fast.

Table 1. Empirical Powers, $\alpha = .05$, n = 100, 1000

n = 100						n = 1000					
ρ	*SSC*	V_{max}	D_n	*Both*	$r(V,D)$	ρ	*SSC*	V_{max}	D_n	*Both*	$r(V,D)$
0	.050	.050	.050	.006	.221	0	.050	.050	.050	.003	.055
.05	.123	.044	.059	.006	.246	.01	.091	.061	.052	.003	.058
.10	.250	.145	.092	.020	.308	.02	.153	.233	.061	.013	.066
.15	.436	.302	.160	.072	.356	.03	.244	.471	.078	.035	.082
.20	.625	.592	.354	.221	.412	.04	.347	.676	.111	.071	.104
.25	.802	.749	.612	.500	.424	.05	.474	.816	.171	.133	.129
.30	.912	.914	.878	.777	.446	.06	.596	.904	.288	.248	.147
.35	.962	.968	.972	.928	.464	.07	.712	.955	.497	.455	.153
.40	.989	.985	.994	.982	.484	.08	.807	.978	.768	.728	.167
						.09	.884	.993	.940	.921	.187
						.10	.934	.997	.992	.986	.180

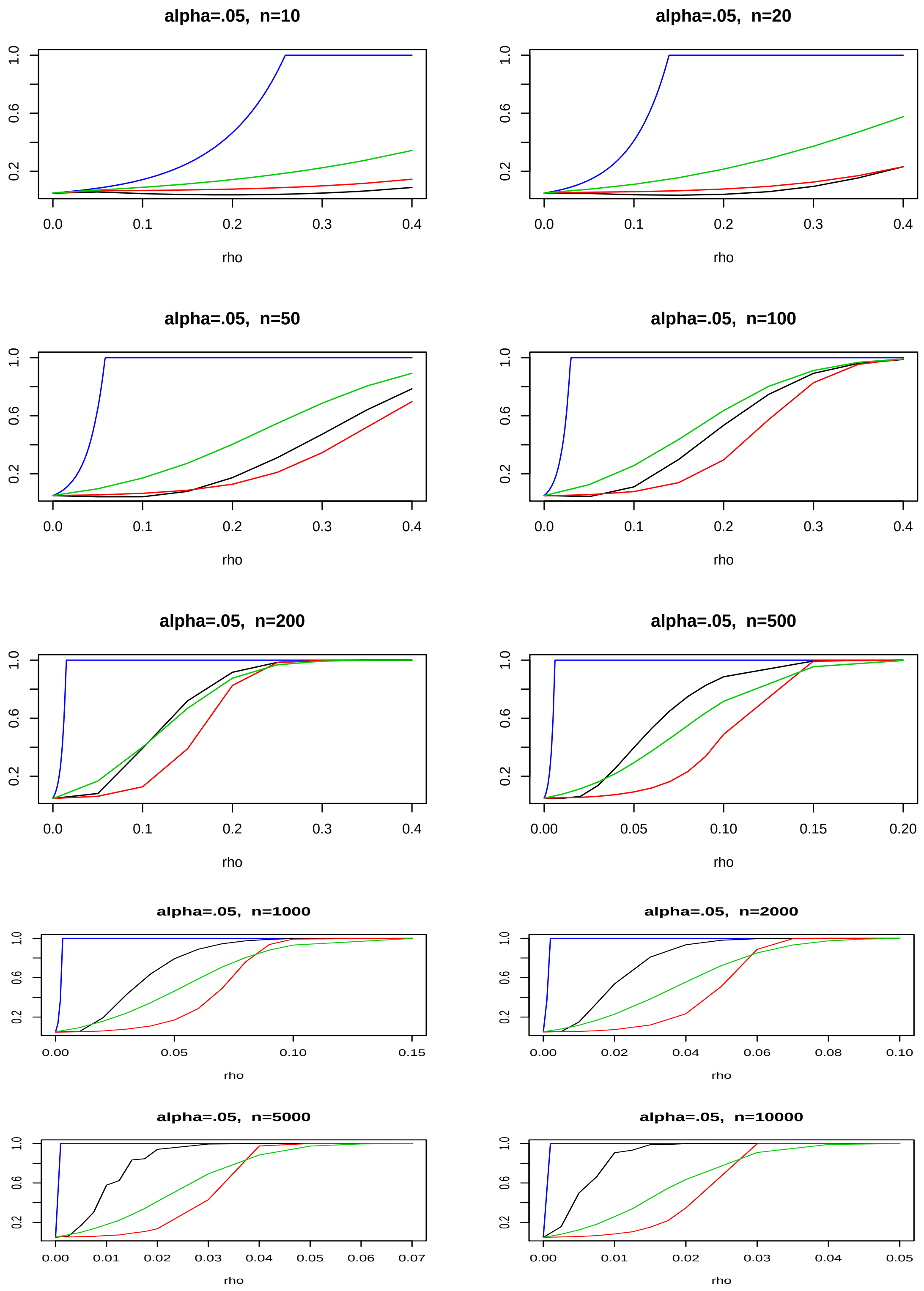

Figure 3. Power Functions, Autoregressive Model, LRT (blue), LST (black), KST (red), SSC (green)

We can confidently infer that the LST is superior to the KST in the presence of serial correlation. However, under a departure from uniformity, preserving the independence, the KST might be superior to the LST. For example, we took samples of iid $\beta(\gamma, 1)$, namely $X_i = U_i^{1/\gamma}$. Again, for each pair (n, γ), we generated 10^5 samples of size n with the proper γ. The empirical powers are shown in Figure 4. The superiority of the KST over the LST is evident. The KST is quite close to the most powerful test (the blue graph). The latter is the LRT for this model, namely, reject "$\gamma = 1$" when $-2\ \Sigma \log X_i < \chi^2_{2n}(.05)$ if the alternative is $\gamma > 1$, or when $-2\ \Sigma \log X_i > \chi^2_{2n}(.95)$ if the alternative is "$\gamma < 1$". The power of the LRT is computed directly from the χ^2-distribution, no need for simulations.

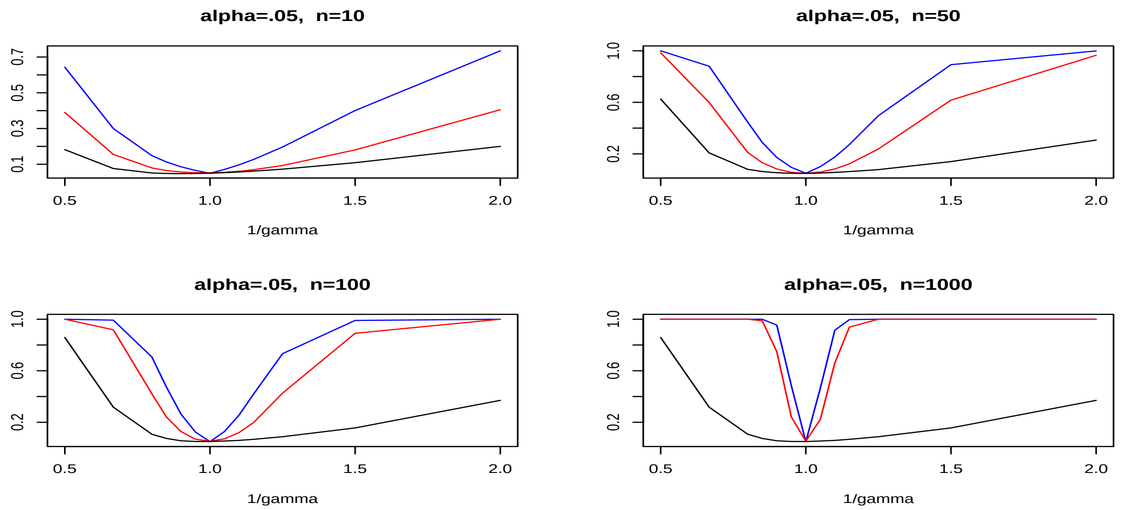

Figure 4. Power Functions, Beta Model, LRT (blue), LST (black), KST (red)

4 LIKELIHOOD-RATIO TEST

In the previous section we presented some evidence in favor of the largest spacing when the alternative is the autoregressive model. However, what is the most powerful test for this alternative? By the Neyman-Pearson theory, the answer is the likelihood-ratio test (LRT).

Let $\mathbf{X} = (X_1, X_2, \ldots, X_n)$ be defined by Equation (1). In order to compute the joint density function of $\mathbf{X}$, conditioned on $U_0 = X_0 = x_0$, we express $\mathbf{U} = (U_1, U_2, \ldots, U_n)$ in terms of $\mathbf{X}$, namely

$$U_i = (X_i - \rho X_{i-1})\,(1-\rho)^{-1}\ (1 \le i \le n).$$

The Jacobian of this transformation is $(1-\rho)^{-n}$. Since $0 \le U_i \le 1$ for all i, one has for $\mathbf{x}$ in $[0, 1]^n$

$$f_X(x) = (1-\rho)^{-n} \Pi_{i=1}^{n} \quad I\{\rho x_{i-1} \le x_i \le \rho x_{i-1} + 1 - \rho\}$$
$$= (1-\rho)^{-n} I\left\{\rho \le \min_{1\le i\le n} \quad \min\left\{\frac{x_i}{x_{i-1}},\ \frac{1-x_i}{1-x_{i-1}}\right\}\right\}. \tag{5}$$

Let

$$T_{1i} = \min\left\{\frac{X_i}{X_{i-1}},\ \frac{1-X_i}{1-X_{i-1}}\right\} \quad \left(1 \le i \le n,\ T_{1\min} = \min_{1\le i\le n} T_{1i}\right),$$

then the following facts follow from Equation (5):

Fact 1. *The $\{T_{1i}\}$ are iid uniform on $[\rho, 1]$.*

Fact 2. *The likelihood function is given by*

$$L(\rho) = (1-\rho)^{-n} I\{\rho \le T_{1min}\} \quad (0 \le \rho \le 1). \tag{6}$$

Fact 3. *The statistic T_{1min} is sufficient with respect to ρ and it is the maximum likelihood estimator (MLE) of ρ.*

For testing

$$H_0 : \rho = 0 \text{ vs. } H_1 : \rho > 0\,,$$

the likelihood function is also the likelihood-ratio. A most powerful α-level test rejects $H0$ when

$$T_{1min} > c_\alpha = 1 - \alpha^{1/n} \tag{7}$$

and the power is given by

$$\pi_\alpha(\rho) = \begin{cases} \dfrac{\alpha}{(1-\rho)^n} & if\ \rho \le c_\alpha, \\ 1 & if\ \rho \le c_\alpha. \end{cases}$$

So, in fact, the LRT is based on the minimum of $2n$ ratios. The blue graphs in Figure 3 represent the power of the LRT. No doubt the LRT is superior to all other tests, but we note its close proximity to the LST graph for large n.

If one desires $\pi_\alpha(\rho) = 1$ for $\rho \ge \rho_0$, one needs

$$n \ge \frac{\log \alpha}{\log(1-\rho_0)}. \tag{8}$$

Table 2 presents the (smallest) required sample size for $\alpha = .05$.

Table 2. Sample Size Required for Power 1, $\alpha = .05$

ρ_0	$n \geq$
.1	29
.01	299
.001	2995
10^{-m}	3×10^{m}

Since $nc_\alpha = -\log \alpha + O(n^{-1})$, for large n, $\pi_\alpha(\rho) = 1$ for $\rho \geq (-\log \alpha)=n$.

5 AUTOREGRESSIVE MODEL OF ORDER *k*

Suppose we want to protect ourselves against serial correlation of higher order. For this purpose we assume that the data have been generated by an autoregressive model of order k, namely

$$X_i = \rho_1 X_{i-1} + \rho_2 X_{i-2} + \dots + \rho_k X_{i-k} + \eta U_i \ (1 \leq i \leq n), \tag{9}$$

where $\{U_i : i \geq -k + 1\}$ is a $U(0, 1)$-iid sequence and

$$Xi = U_i \quad \text{for } i \leq 0\,, \ \ \rho j \geq 0, \ \ 0 < \eta = 1 - \sum_{j=1}^{k} p_j \leq 1.$$

The goal is to test whether $\rho j = 0$ for all j vs. the alternative that $\rho j > 0$ for at least one j, namely,

$$H_0 : \max_j \rho_j = 0 \ \text{ vs. } \ H_1 : \max_j \rho_j > 0\,.$$

The joint density function of $\mathbf{X} = (X_1, X_2, \dots, X_n)$ at $\mathbf{x} \in [0, 1]^n$, conditioned on $\{X_i = x_i : -k+1 \leq i \leq 0\}$ is given by

$$f_X(x) = \eta^{-n} \prod_{i=1}^{n} I\left\{\Sigma_{j=1}^{k} \rho_j x_{i-j} \leq x_i \leq \Sigma_{j=1}^{k} \rho_j x_{i-j} + \eta\right\} \ . \tag{10}$$

Unfortunately, a sufficient statistic of low dimension, as in the case $k = 1$, does not exist. However, if we define

$$T_{ji} = \min\left\{\frac{X_i}{X_{i-j}}, \frac{1-X_i}{1-X_{i-j}}\right\} \ (1 \leq i \leq n), \ \ T_{j\min} = \min_{1\leq i\leq n} T_{ji} \quad ,$$

Then, with probability 1, $\rho_j \leq T_{j\min}$ for $1 \leq j \leq k$.

Hence, it is reasonable to use $T^* = \max_{1\leq j\leq k} T_{jmin}$ as a test statistic, i.e., to reject $H0$ when $T^* > C_\alpha = t^*_{kn}(1-\alpha)$ - quantile of T^* under H_0. We note that under H_0, within each vector $\boldsymbol{T}_j = (T_{j1}, \dots, T_{jn})$, the T_{ji} are iid $U(0, 1)$ random variables. Moreover, T_{ji} and $T_{j'i'}$ are independent, except in the case where $i = i'$, implying that the T_{jmin} are dependent. This is why the derivation of c_α in a closed form is too complicated and we must resort to Monte Carlo methods. However, once we determine c_α for given k and n, we can guarantee power 1 for all alternatives with max $\rho_j \geq c_\alpha$. In order to compute $c_{.05}$ we generated 10^6 samples for each combination of n = 100, 1000, 10000 and k = 1, 2, . . . , 10. Here we report, in Table 3, the asymptotic values of $nc_{.05}$ with 3 significant digits.

Table 3. Asymptotic Values of $nc_{.05}$

k	$nc_{.05}$	k	$nc_{.05}$
1	3.00	6	4.40
2	3.54	7	4.51
3	3.85	8	4.62
4	4.08	9	4.72
5	4.24	10	4.79

For instance, suppose we plan to run a Monte-Carlo method based on random numbers generated by a specific software. Suppose further that we want to be sure that there is no positive serial correlation up to lag k = 8 and that we can tolerate correlations below $\rho_0 = 10^{-4}$. Then, we have to generate a sample of size $n \geq 4.62/\rho_0 = 46200$ in order that $\max_{1\leq j\leq 8} \rho_j > 10^{-4}$ will be detected with probability 1.

*Remark 2***.** The resources required to carry out this kind of a test are (almost) free. For instance, in S-PLUS or R, one command does the job: "x=runif(46200)" and it takes a fraction of a second.

*Remark 3***.** A special case of our model is when it is known that $\rho_j = 0$ for $1 \leq j \leq k-1$ for some k and we want to test whether $\rho_k = 0$. Then a test based on T_{kmin} has all the properties as the test based on T_{1min} discussed in Section 3.

Although it is hard to compete with T^*, we still wish to explore the performance of the tests considered in Section 2. Here, instead of the sample serial correlation SSC, we use the sum of squares for regression (SSREG) as the test statistic. Namely, $\mathbf{X} = (X_1,..., X_n)$ is the dependent variable and $\boldsymbol{X}_j = (X_{1-j}, ..., X_{n-j})$, $1 \leq j \leq k$, are the regressors. In Table 4 we present some Monte Carlo results for k = 2, based on 10^5 replications of samples of size n = 100.

The conclusion is similar to the one of the case k = 1. Beside the clear dominance of T^*, we note that the performance of SSREG is very poor. The reason could be that the deviates from the model are uniform rather than normal. The largest spacing test performs better than the other two.

Table 4. Empirical Powers, $k = 2$, $\alpha = .05$, $n = 100$

ρ_1	ρ_2	*SSREG*	V_{max}	D_n	T^*
0	0	.050	.050	.050	.050
.025	0	.049	.042	.055	.468
0	.025	.049	.042	.056	.459
.025	.025	.047	.045	.063	.990
.050	.050	.056	.171	.090	1.000
.075	0	.061	.072	.070	1.000
0	.075	.070	.072	.070	1.000
.075	.075	.076	.565	.171	1.000
.100	0	.079	.146	.090	1.000
0	.100	.090	.150	.091	1.000
.100	.100	.107	.848	.379	1.000
.125	0	.111	.245	.121	1.000
0	.125	.113	.250	.121	1.000
.125	.125	.146	.960	.738	1.000
.150	.150	.197	.993	.953	1.000

6 TWO MORE MODELS

We wish we could claim that the LST is a powerful test to detect dependence in general. For this, one has to examine its performance against all kinds of dependencies – an impossible mission. We have experimented with several models with serial dependence and the conclusion is similar – except for very small sample sizes, LST is more powerful than KST. Here are two examples.

Binomial model. Let $U_0, U_1, U_2, \ldots$ be iid uniform as before and let $B_1, B_2, \ldots$ be a Bernoulli sequence with parameter p, independent of the U-sequence. The *binomial sequence* Y is defined by $Y_i = B_i Y_{i-1} + (1 - B_i)U_i$ ($i \geq 1$, $Y_0 = U_0$). The marginal distribution of each Y_i is U [0, 1], the first serial correlation is p and so is $P\{Y_i = Y_{i+1}\} = p$. Clusters of equal neighbors are of random (geometric) length (see Figure 5 for a scatter points (i, Y_i)).

Moving-max model. Let $\xi_1, \xi_2, \ldots$ be a sequence of iid $\beta(k^{-1}, 1)$ random variables, where k is a fixed positive integer. Let $Z_i = \max\{\xi_i, \xi_{i+1}, \ldots, \xi_{i+k-1}\}$ ($i \geq 1$). The Z-sequence is called *a moving-max sequence of order k*. For each i, Z_i is U [0, 1]-distributed but neighboring values are dependent. Upper extreme values appear in clusters of size k, which imply that the *extremal index* is equal to k^{-1}. For $k = 2$, the first serial correlation is 3/7 and $P\{Z_i = Z_{i+1}\} = 1/3$.

The two plots in Figure 5 look very similar. In both cases, the experienced practitioner will reject the independence hypothesis just on the basis of the fact that for continuous random variables, the probability of a tie is 0. We brought these cases to see how well the LST and the KST detect the dependence.

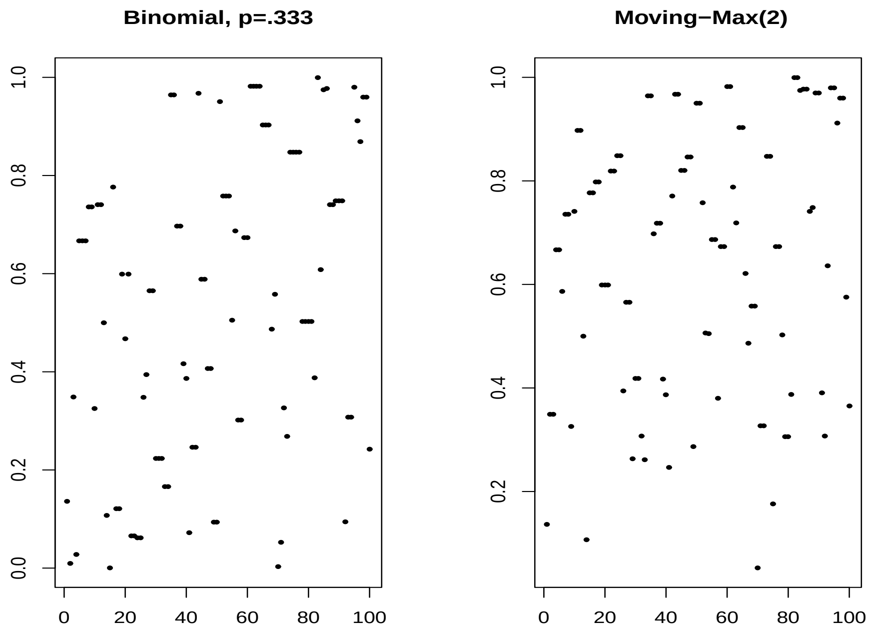

Figure 5. Scatter plot for the two models

Figure 6 shows the (empirical) power functions of the two tests applied to the moving-max models of order $k = 2$ and $k = 3$. The superiority of LST over KST is evident, moreover, LST is even not consistent in this case. A similar Monte Carlo study was carried out on the binomial model. The results are shown in the Appendix. The conclusion is very similar to the conclusion regarding the autoregressive model, i.e., for $n \geq 100$, LST is superior to the KST, and as n increases, it becomes more and more so.

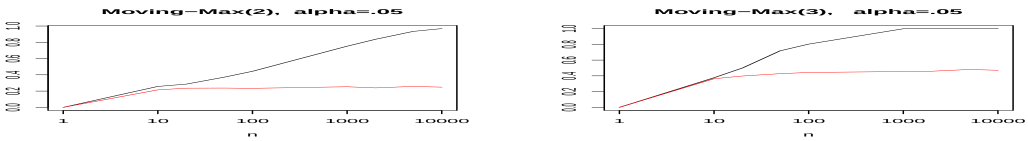

Figure 6. Power Functions, Moving-Max Model (logarithmic scale), LST (black), KST (red)

Remark 4. The binomial model has some resemblance to the autoregressive model (where ρ is replaced by a random variable with mean p). Suppose we apply the LRT of the autoregressive model to a sample from the binomial model with some $p > 0$. Then, for all i such that $B_i = 0$, the T_{1i} are (as in Section 4) iid uniform on [0, 1]. For all i such that $B_i = 1$, one has $T_{1i} = 1$. Hence, given $S = \sum_1^n (1 - B_i)$,

$$P\{T_{1min} > 1 - \alpha^{1/n}\} = \alpha^{S/n}.$$

Since $S/n \to q = 1 - p$ a.s. as $n \to \infty$, the power tends to α^q(= .0675 for $\alpha = .05$ and $p = .1$). Applying the same test to data from the moving-max model yields even lower power.

7 CONCLUSIONS

The main theme of the paper is to show that the largest spacing of a sample is sensitive to serial correlation and is quite powerful in detecting it, more powerful than the Kolmogorov-Smirnov distance. The opposite is true under independence but the true distribution is diαerent from the null distribution. Of course, when the data are generated by a specific (known) parametric model, and there exists a most powerful test, one should use that test. We went into detail in Sections 4 and 5 because we found it interesting to learn (for the first time in the statistical literature, to the best of our knowledge) that the MLE of a serial correlation is a (lower) sample extreme, and a test based on it is most powerful. The overall message is clear. if you worry about serial dependence in your data and you cannot assume a particular model, the LST is a reasonable test to use.

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APPENDIX

The empirical powers of the Monte Carlo simulations for the autoregressive model are given below. Each entry is a result of 10^5 replications. The graphs in Figure 3 are based on Table 5.

Table 5. Empirical Powers, Autoregressive Model

$n = 10$				$n = 20$			
ρ	*SSC*	V_{max}	D_n	ρ	*SSC*	V_{max}	D_n
0.00	0.05133	0.04975	0.05037	0.00	0.05051	0.04971	0.05038
0.05	0.07103	0.05769	0.06590	0.05	0.07656	0.04728	0.05602
0.10	0.08995	0.04648	0.06801	0.10	0.11030	0.03834	0.05934
0.15	0.11316	0.03983	0.07153	0.15	0.15607	0.03559	0.06609
0.20	0.14359	0.03763	0.07762	0.20	0.21530	0.04134	0.07790
0.25	0.17958	0.04137	0.08681	0.25	0.28692	0.05967	0.09602
0.30	0.22447	0.0499	0.09939	0.30	0.37274	0.09604	0.12600
0.35	0.27866	0.06514	0.11846	0.35	0.47051	0.15446	0.17001
0.40	0.34355	0.08843	0.14538	0.40	0.57579	0.23172	0.23157

$n = 50$				$n = 100$			
ρ	*SSC*	V_{max}	D_n	ρ	*SSC*	V_{max}	D_n
0.00	0.05043	0.04969	0.05023	0.00	0.04979	0.05106	0.04822
0.05	0.09709	0.04166	0.05470	0.05	0.12491	0.04251	0.05587
0.10	0.17079	0.04192	0.06556	0.10	0.25663	0.10982	0.07755
0.15	0.27259	0.07875	0.08652	0.15	0.43726	0.29966	0.13949
0.20	0.40277	0.17347	0.12812	0.20	0.63583	0.53433	0.29601
0.25	0.54738	0.31108	0.21007	0.25	0.80235	0.74673	0.57287
0.30	0.68625	0.47260	0.34568	0.30	0.91155	0.89186	0.82717
0.35	0.80513	0.64069	0.52163	0.35	0.96809	0.96080	0.95418
0.40	0.89275	0.78542	0.69733	0.40	0.99113	0.98766	0.99162

n = 200				n = 500			
ρ	SSC	V_{max}	D_n	ρ	SSC	V_{max}	D_n
0.00	0.04640	0.04720	0.04740	0.00	0.04912	0.05077	0.05007
0.05	0.16710	0.08130	0.06220	0.01	0.07674	0.04870	0.05142
0.10	0.40150	0.39340	0.12790	0.02	0.11337	0.06056	0.05484
0.15	0.66820	0.71950	0.38950	0.03	0.16028	0.13689	0.06191
0.20	0.87580	0.91640	0.82570	0.04	0.22103	0.26020	0.07383
0.25	0.96790	0.98460	0.98410	0.05	0.29361	0.39809	0.09249
0.30	0.99340	0.99730	0.99890	0.06	0.37480	0.53170	0.11984
0.35	0.99930	0.99980	1.00000	0.07	0.46123	0.65072	0.16376
0.40	0.99990	0.99990	1.00000	0.08	0.54949	0.74919	0.23214
				0.09	0.63764	0.82675	0.33809
				0.10	0.71659	0.88551	0.48909
				0.15	0.95492	0.99416	0.99432
				0.20	0.99754	0.99950	1.00000

n = 1000				n = 2000			
ρ	SSC	V_{max}	D_n	ρ	SSC	V_{max}	D_n
0.00	0.05050	0.04840	0.04750	0.000	0.05111	0.04949	0.04976
0.01	0.09250	0.05280	0.05080	0.005	0.07938	0.05189	0.05127
0.02	0.15840	0.19390	0.05910	0.010	0.11750	0.15014	0.05435
0.03	0.24100	0.43230	0.07820	0.015	0.16819	0.34179	0.06163
0.04	0.34580	0.63770	0.10970	0.020	0.22958	0.53729	0.07315
0.05	0.46370	0.79240	0.17040	0.030	0.38317	0.80883	0.11937
0.06	0.58670	0.88840	0.28550	0.040	0.55600	0.93420	0.23348
0.07	0.70690	0.94400	0.48920	0.050	0.72345	0.98030	0.51278
0.08	0.80440	0.97450	0.76100	.060	0.85195	0.99537	0.88792
0.09	0.88040	0.98900	0.93690	0.070	0.93184	0.99915	0.99573
0.10	0.93130	0.99680	0.99170	0.080	0.97397	0.99976	0.99998
0.15	0.99780	0.99990	1.00000	0.090	0.99178	0.99998	1.00000
				0.100	0.99759	1.00000	1.00000

n = 5000				n = 10000			
ρ	SSC	V_{max}	D_n	ρ	SSC	V_{max}	D_n
0.0000	0.0509	0.0489	0.0548	0.0000	0.0471	0.0468	0.0481
0.0025	0.0741	0.0589	0.0515	0.0025	0.0800	0.1566	0.0519
0.0050	0.0994	0.1694	0.0554	0.0050	0.1235	0.4980	0.0567
0.0075	0.1356	0.3035	0.0578	0.0075	0.1810	0.6642	0.0654
0.0100	0.1782	0.5778	0.0661	0.0100	0.2580	0.9067	0.0827
0.0125	0.2204	0.6239	0.0724	0.0125	0.3382	0.9331	0.1044
0.0150	0.2794	0.8338	0.0907	0.0150	0.4434	0.9888	0.1505
0.0175	0.3406	0.8456	0.1070	0.0175	0.5470	0.9908	0.2182
0.0200	0.4131	0.9411	0.1350	0.0200	0.6345	0.9988	0.3466
0.0300	0.6922	0.9949	0.4299	0.0300	0.9093	1.0000	0.9982
0.0400	0.8838	0.9999	0.9754	0.0400	0.9908	1.0000	1.0000
0.0500	0.9744	1.0000	1.0000	0.0500	0.9997	1.0000	1.0000
0.0600	0.9961	1.0000	1.0000				
0.0700	0.9997	1.0000	1.0000				

The following plots show the empirical powers of the Binomial Model, based on 10^4 replications for each pair (n, p).

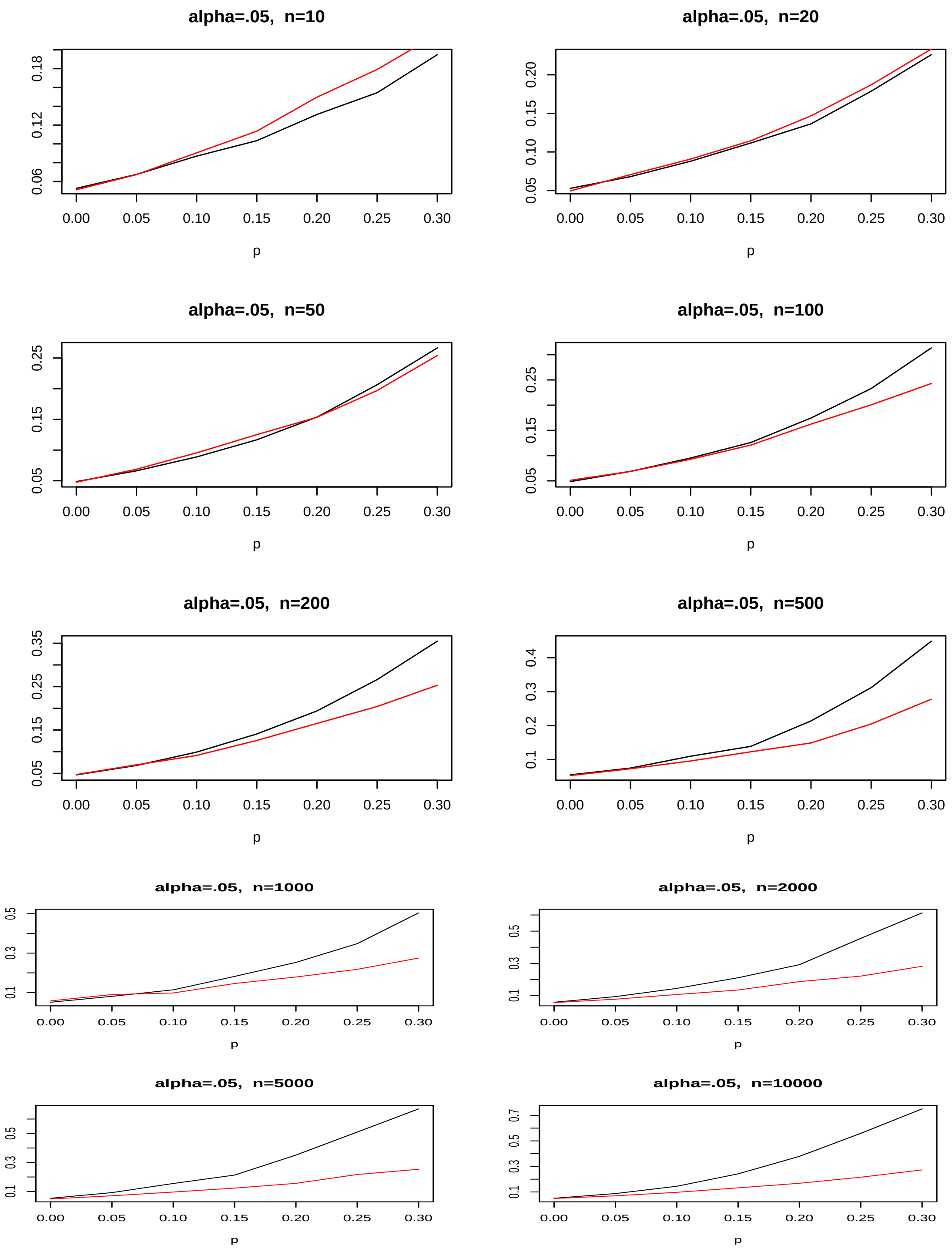

Figure 7. Power Functions, Binomial Model, LST (black), KST (red)

METHODS AND MEANS OF THE ESTIMATION OF INDICATORS OF RELIABILITY OF MECHANICAL AND ELECTROMECHANICAL ELEMENTS OF DEVICES AND SYSTEMS

V. Zhadnov

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Moscow State Institute of Electronics and Mathematics (Technical University), Moscow, Russia

e-mail: jadnov@mitme.ru

ABSTRACT

Features of the application of standards for evaluation (predict) the reliability of mechanical and electromechanical devices and systems are considered. The main characteristics of the models of operational failure rate of mechanical components and provides recommendations for their use are given. The use of software tools that implement the methods for calculating reliability, can cause significant difficulties with the reliability of initial data.

1. INTRODUCTION

Handbook “Reliability ERP“ is the main source for the calculation of reliability indices of elements of sensors, devices and systems at the present time in Russia. The handbook is an official document of the Ministry of Defence of Russian Federation in accordance with standard (RDV 319.01.20-98 1998). Mathematical models of the operational failure rate electro-radio-products (ERP) and the numerical values of the coefficients are given in it. The survey provides data not only for performance reliability of electronic devices, such as integrated circuits, semiconductor devices, etc., and of electromechanical components (such as electrical machines of small capacity) and the purely mechanical components (connections, circuit boards, etc.).

Since the range of such elements is limited, so in practical calculations the reliability of the elements that are missing in the Handbook “Reliability ERP“, not included and considers them an “absolutely reliable“. However, this approach can lead (and leads) to a significant overestimation of the reliability of systems (Markin et al. 2010), primarily of test containing primary transducers (sensors). Evaluation of reliability of such systems requires taking into account not only the reliability of the electronic devices, but also a wide class of mechanical components (connections, details, etc.).

2. MODELS RELIABILITY OF MECHANICAL COMPONENTS

Mathematical models of operational failure rates are given in article (Shavykin & Petruhin 2006) for the following classes of mechanical connections, components, assemblies and equipment manufactured in Russia:

- Non-demountable fasteners
- Threaded fasteners
- Demountable fasteners
- Nodes of transmission of motion
- Bearings
- Seals and Gaskets
- Bellows and tubular springs

- Impeller flow sensors
- Membrane
- Pressure Gauges
- Vacuum gauges with tubular springs
- Springs, couplings, watch mechanisms

All these models are as follows:

$$\lambda_{э} = \lambda_0 \cdot \prod_{i=1}^{I} \alpha_i$$

where: $\lambda_{Э}$ = operational failure rates of seals; λ_0 = base failure rate of seals; α_i = multiplying factors which considers the effect on the base failure rate; I = number of multiplying factors.

In addition to mathematical models (formulas) in article (Shavykin & Petruhin 2006) also shows a table containing the numerical values of their coefficients. The possibility of using these models due to the fact that they are full copies of the “Procedure…” (Shavykin et al. 1998) in terms of mechanical parts, which at present applies to ROSATOM.

As an example we will consider class “Seals and Gaskets”. For this class in article (Shaykin & Petrukhin 2006) the following mathematical model of operational failure rate is resulted:

$$\lambda_{э} = \lambda_0 \cdot \alpha_1 \cdot \alpha_m \quad (1)$$

where: $\lambda_{Э}$ = operational failure rate of seals; λ_0 = base failure rate of seals; α_1 = multiplying factor which considers the effect of service conditions and etc. on the base failure rate; α_m = multiplying factor which considers the effect of type of a material on the base failure rate.

In turn, the multiplying factor α_1 pays off on model:

$$\alpha_1 = k_{11} \cdot k_{12} \cdot k_{13} \cdot k_{14} \cdot k_{15}, \quad (2)$$

where: k_{11} = multiplying factor which considers the effect of service conditions, vibrating loadings and amortization on the base failure rate; k_{12} = multiplying factor which considers the effect of service conditions, shock loadings and amortization on the base failure rate; k_{13} = multiplying factor which considers the effect of type of a climate, presence of the conditioner, heating and premise hermetic sealing on the base failure rate; k_{14} = multiplying factor which considers the effect of a kind of equipment and quality of service on the base failure rate; k_{15} = multiplying factor which considers the effect of a kind of equipment and quality of manufacturing on the base failure rate.

Numerical values multiplying factors k_{11}-k_{15} are given in tables. Due to the limited size of the article yourself a table full list here are not going to give only their fragments (see Table 1-4).

Table 1. Values of multiplying factors k_{11} and k_{12}

Operating conditions	k_{11} (vibration)		k_{12} (strikes)	
	not amortized	amortised	not amortized	amortised
Lab, instrument-making enterprises, assembly plants	1,0	1,0	1,0	1,0
Marine conditions	3,0	1,0	1,2	1,0
…	…	…	…	…
The equipment of rolling mills, forging hammers, presses	10,0	3,0	15,0	5,0

Table 2. Values of multiplying factor k_{13}

Climate	air conditioning	Heated room		Not heated room	
		sealed	not sealed	sealed	not sealed
Cold	1,0	1,0	1,2	1,2	2,0
Moderate, middle band	1,0	1,0	1,2	1,0	1,3
…	…	…	…	…	…
marine subtropical	1,1	-	-	1,5	2,0

Table 3. Values of multiplying factors k_{14} and k_{15}

Equipment	k_{14} (service)	k_{15} (production)
Household	5,0	2,0
Management of individual units	1,0	1,0
…	…	…
Process control	0,8	0,5

Table 4. Values of multiplying factor α_m

Material	Temperature				
	20 ^{0}C	40 ^{0}C	60 ^{0}C	80 ^{0}C	100 ^{0}C
Organic fibrous	1,0	1,5	2,0	4,0	8,0
Rubber	1,0	1,2	1,5	2,5	4,0
…	…	…	…	…	…
Fluoroplastic	1,0	1,0	1,0	1,0	1,2

Even a simple analysis of Tables 1-4 shows that the model (1) is of little use to solve practical problems of reliability. Indeed, the failure rate of the seals does not depend on their size, nor on the quantitative characteristics of the physical and mechanical properties of materials, etc.

For example, in the standard (TU 38 005 1166-98 1998) is defined more than 50 brands of rubber compounds with different physic-chemical properties and the calculation of the model (1) for all of them give the same failure rate (see Table 5)!

Fragment of the table the standard (TU 38 005 1166-98 1998) are as follows (see Table 5).

Table 5. Characteristics of rubber compounds

Mark rubber compound	Operating conditions		The main purpose of	Relative tensile strength, n/m		Elongation at break, % n/m	Hardness, Shore unit
	environment	T, ^{0}C		MPa	kgs/sm^2		
В-14 *НТА*	Air	-45…+100	Molded rubber and rubber parts and immobile joints operating under a static strain	10,8	110	160	72-92
	Oil *АМГ*-10	-60…+100					
	Ethanol	-60…+70					
В-14-1 *НТА*	Air	-45…+100	Molded Rubber and rubber parts and immobile joints operating under a static strain	11,8	120	150	78-85
	Ethanol	-60…+70					
…	…	…	…	…		…	…
ИРП-1354 *НТА*	Air	-70…+250	Shaped syringe electrical parts operating in the unstressed state, Amortisseur, Molded and non-sealing parts (rings, gaskets, etc.) for fixed joints	5,4	55	280	54-66
	Air with high ozone	-76…+125					
	The electric field	-60…+250					
	Lubrication *ВНИИ НП*-279	-60…+150					

In other words, if the resulting failure rate calculations seals (or any other elements) will be high, the number of possible ways to enhance their security in the use of model (1) is limited, and, consequently, the possibility of implementation of relevant activities is unlikely or will require a large material costs.

And the "Procedure…" (Shavykin et al. 1998), and the standard TU 38 005 1166-98 were published in the same year. But the data presented in "Procedure…" (Shavykin et al. 1998) almost entirely with the data of the Recommendation (RM 25 446-87 1987). From this it follows that the conduct of the calculation is based on data from 20-years old ago.

From this point of view of greater interest are the models listed in the American standard (NSWC-98/LE1 1998), developed by experts Naval Surface Warfare Center, or rather, its latest edition (NSWC-06/LE10 2006).

The standard includes mathematical models of operational failure rate for the following classes of mechanical equipment:

- Seals and Gaskets
- Springs
- Solenoids
- Contactors
- Valve assemblies,
- Bearings
- Gears and Splines
- Actuators
- Pumps
- Fluid Filters
- Brakes and Clutches
- Compressors
- Electric Motors
- Accumulators, Reservoirs
- Threaded Fasteners
- mecanical Couplings
- Slider cranks mechanisms
- Sensors and Transducers.
- Shafts
- Belt and chain drivers
- Fluid conductors

To give an idea of the mathematical models of operational failure rates present a model for the class “Seals and Gaskets“:

$$\lambda_{SE} = \lambda_{SE,B} \cdot C_P \cdot C_Q \cdot C_{DL} \cdot C_H \cdot C_F \cdot C_v \cdot C_T \cdot C_N, \quad (3)$$

where: λ_{SE} = failure rate of seals in failures/million hours; $\lambda_{SE,B}$ = base failure rate of seals, 2,4 failures/million hours; C_P = multiplying factor which considers the effect of pressure on the base failure rate; C_Q = multiplying factor which considers the effect of allowable leakage on the base failure rate; C_{DL} = multiplying factor which considers the effect of seal size on the base failure rate; C_H = multiplying factor which considers the effect of contact stress and seal hardness on the base failure rate; C_F = multiplying factor which considers the effect of seat smoothness on the base failure rate; C_v = multiplying factor which considers the effect of fluid viscosity on the base failure rate; C_T = multiplying factor which considers the effect of temperature on the base failure rate; C_T = multiplying factor which considers the effect of contaminants on the base failure rate.

Even simple comparison of models (2) and (3) shows that models of standard (NSWC-06/LE10 2006) more exact, than resulted in article (Shavykin & Petruhin 2006). Besides, factors of these models also represent functions of the geometrical sizes, physic mechanical properties of materials, etc. For example, the formula for calculating the factor C_H is shown below:

$$C_H = \left(\frac{M/C}{0,55} \right)^{4,3} \quad (4)$$

where: M = Meyer hardness, lbs/in^2; C = contact pressure, lbs/in^2.

It is obvious that application of models of standard (NSWC-06/LE10 2006) essentially increases labor input of ”manual” calculations in comparison with article (Shavykin & Petruhin

2006). Therefore along with formulas for calculation of factors in it corresponding monograms (**Figure 1** see) are resulted.

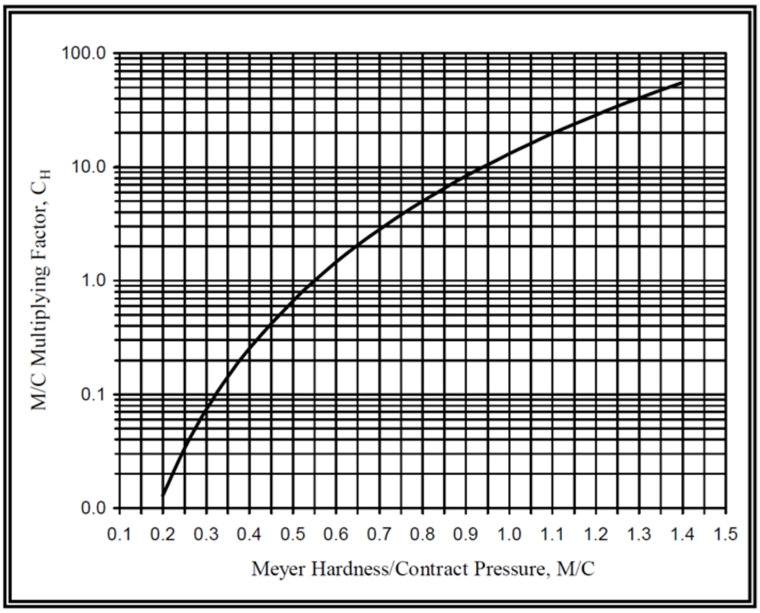

Figure 1. NSWC-06/LE10: Dependence of multiplying factor C_H on relation M/C.

These monograms may be helpful in justifying the choice of measures aimed at improving the reliability, because give a clear idea of the effect of each option on the magnitude of the coefficients and, consequently, the operational failure rate in general. However, these monograms rather to the tradition of information in handbooks on reliability than a means to facilitate the calculations, since with a large number of elements the complexity of calculations, even for simple models of article (Shavykin & Petruhin 2006) increases significantly.

Another feature of the standard (NSWC-06/LE10 2006) is that some elements are considered as a system. For example, the calculation of the failure rate of the electric motor is carried out according to the model:

$$\lambda_M = \lambda_{BE} + \lambda_{WI} + \lambda_{BS} + \lambda_{AS} + \lambda_{ST} + \lambda_{GR}, \qquad (5)$$

where: λ_M = total failure rate of motor system, failures/million hours; λ_{BE} = failure rate of *bearings*, failures/million hours; λ_{WI} = failure rate of electric motor windings, failures/million hours; λ_{BS} = failure rate of brushes, failures/million hours; λ_{AS} = failure rate of the armature shaft, failures/million hours; λ_{ST} = failure rate of the stator housing, failures/million hours; λ_{GR} = failure rate of *gears*, failures/million hours.

As follows from (5), the calculation of the electric motor, you must first calculate the failure rate of the *bearings* and *gears*.

2. SOFTWARE FOR THE CALCULATION OF RELIABILITY

To automate the calculation of reliability of mechanical connections, parts, components and devices on models of article (Shavykin & Petruhin 2006) can be used system ASONIKA-K-ME, which is part of the software complex ASONIKA-K (Zhadnov et al. 2011). The main form of the interface of the user of system is resulted on **Figure 2**.

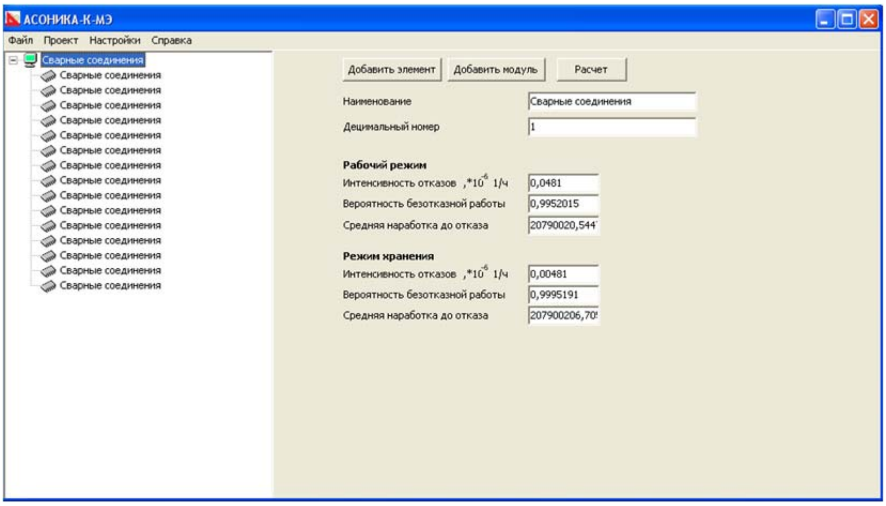

Figure 2. System ASONIKA-K-ME: the Interface of the user.

To automate the calculation of reliability of mechanical equipment on the models standard (NSWC-06/LE10 2006) developers provide a computer program MechReal (see **Figure 3**).

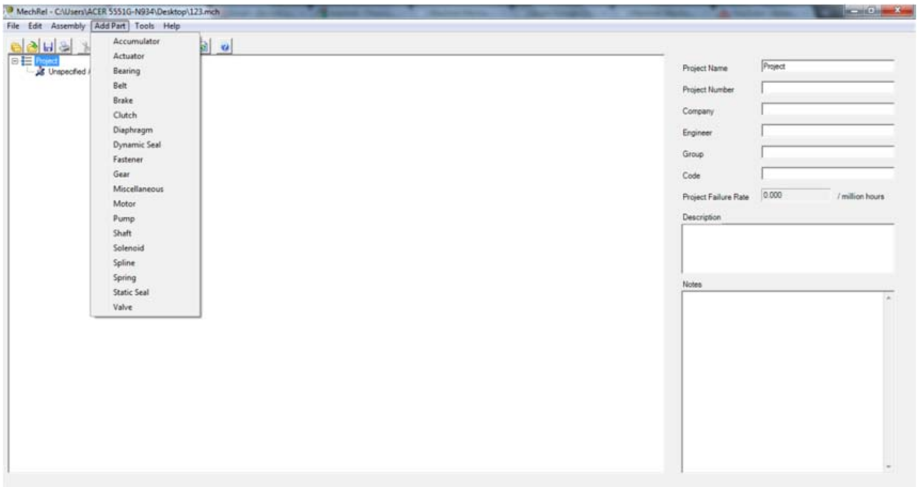

Figure 3. Program MechReal: the Interface of the user.

It is natural that models of this standard are included and into structure of software (modules of Reliability Prediction) well-known manufacturers, such as PTC Corporate Headquarters (Relex visual studio), ReliaSoft Corporation (Lambda predict), ALD Ltd. (RAM Commander), Etc. (Stroganov et al. 2007). And, many are constructed of them in such a manner that allow to consider in one calculation as electronic devices, and mechanical elements, unlike system ASONIKA-K-ME and programs MechReal.

On **Figure 4** is shown the interface of the user of program Lambda Predict of company ReliaSoft Corporation.

But practical application of these programs causes considerable difficulties that is connected not only (and not so much) with the English-speaking interface (**Figure 3, 4** see), how many with search of the initial information. Therefore in standard (NSWC-06/LE10 2006) tables with “typical” values of parameters are resulted.

For example, a fragment of the table “typical” values of is maximum-admissible working temperatures (T_R) of materials of Seal is shown below (see Table 6).

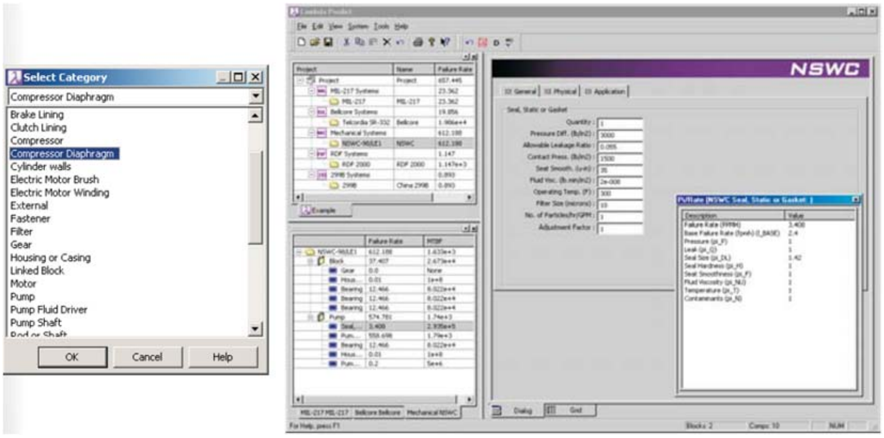

Figure 4. Program Lambda Predict: the Interface of the user.

Table 6. Typical values of is maximum-admissible working temperatures of materials of seal

Seal material	T_R, (^{0}F)
Natural rubber	**160**
Ethylene propylene	**250**
…	**…**
Impregnated poromeric material	**250**

Comparing the first columns of Table 4 and 6 it is easy to notice that search of values is carried out not to concrete marks of materials, and to the more general classification signs. Since these tables are standard (NSWC-06/LE10 2006) and placed in modules "Prediction of Reliability", as stated above, it is clear that, despite the adequacy of the model, calculations with the this data source will be little different from the calculations on model article (Shavykin and Petrukhin, 2006) for accuracy

Therefore for maintenance of accuracy of calculations creation of the database containing all necessary help given (characteristics of materials, maximum-permissible operating modes etc.) is necessary. For the decision of this problem in structure of a help part of a database of the system ASONIKA-K-SCh such database has been added. On **Figure 5** as an example the model of the data for a class "Bearings", created in Sybase PowerDesigner is resulted.

Feature of the developed base is that are stored in it not only numerical values, but also codes of mathematical models (formulas) on which calculation is carried out. In addition, the database stores brand materials and their characteristics are necessary for the calculation of the coefficients of models standard (NSWC-06/LE10 2006). For work with this database the system has been modified ASONIKA-K-SCh.

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In addition, in the system ASONIKA-K-SCh were added to the model, listed in article (Shavykin and Petrukhin, 2006). Thus, the new version of system ASONIKA-K-SCh allows the do calculation of mechanical elements on models of standard (NSWC-06/LE10 2006) and article (Shavykin and Petrukhin, 2006).

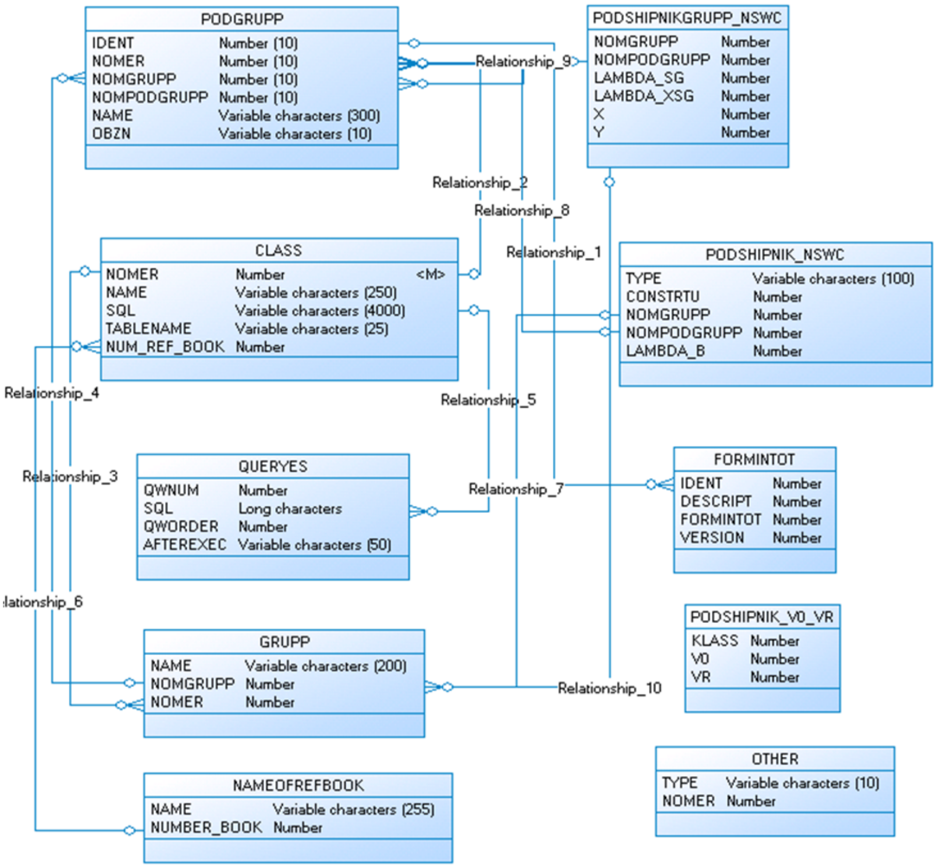

Figure 5. Class “Bearings“: model of the data.

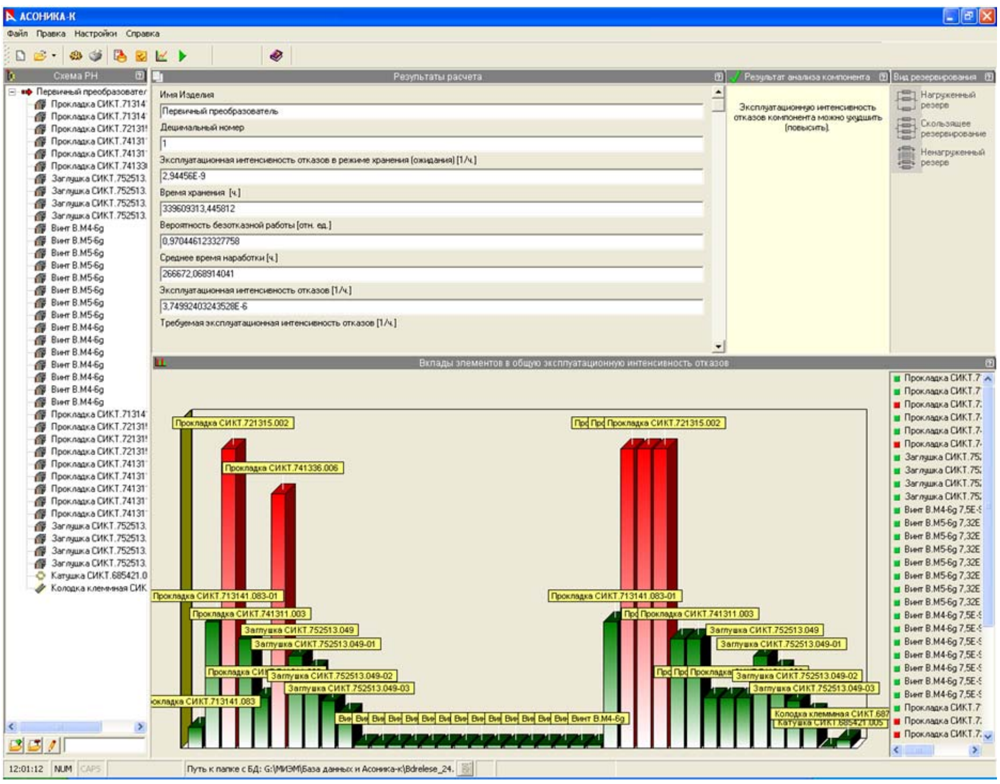

Figure 6. System ASONIKA-K-SCh: the Interface of the user.

3. CONCLUSION

In the conclusion it is necessary to note the following. Even creation of a database doesn't remove all problems with the initial information for calculations. In the model standard (NSWC-06/LE10 2006) enter the variable parameters, depending on the application (allowable leakage, size of gasket, the pressure on the gasket, etc.). Already in itself calculation of these parameters is not a trivial problem and, despite an abundance of software of modeling physical process (mechanical, hydraulic, etc.), that under force only to professionals in area of mechanics, resistance of materials, etc.

In other words, calculation of reliability of such products can be hardly spent one expert. However program realization of system ASONIKA-K-SCh in technology “client-server” allows to carry out calculation of one product from different workstations (i.e. to involve in calculations so much experts, how many it is necessary) that will help to remove the specified problem partly.

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ASYMPTOTIC APPROACH TO RELIABILITY OF LARGE COMPLEX SYSTEMS

Krzysztof Kolowrocki, Joanna Soszynska-Budny.
•
Gdynia Maritime University, Gdynia, Poland

e-mail: katmatkk@am.gdynia.pl, joannas@am.gdynia.pl

"All the results presented in the paper would not be possible to develop without Gnedenko's origin result on limit distributions of minimum and maximum statistics"

1 INTRODUCTION

The paper is concerned with the application of limit reliability functions to the reliability evaluation of large complex systems. Two-state and multi-state ageing large complex systems composed of independent components are considered.

Many technical systems belong to the class of complex systems as a result of the large number of components they are built of and their complicated operating processes. This complexity very often causes evaluation of system reliability and safety to become difficult. As a rule these are series systems composed of large number of components. Sometimes the series systems have either components or subsystems reserved and then they become parallel-series or series-parallel reliability structures. We meet large series systems, for instance, in piping transportation of water, gas, oil and various chemical substances. Large systems of these kinds are also used in electrical energy distribution. A city bus transportation system composed of a number of communication lines each serviced by one bus may be a model series system, if we treat it as not failed, when all its lines are able to transport passengers. If the communication lines have at their disposal several buses we may consider it as either a parallel-series system or an "m out of n" system. The simplest example of a parallel system or an "m out of n" system may be an electrical cable composed of a number of wires, which are its basic components, whereas the transmitting electrical network may be either a parallel-series system or an "m out of n"-series system. Large systems of these types are also used in telecommunication, in rope transportation and in transport using belt conveyers and elevators. Rope transportation systems like port elevators and ship-rope elevators used in shipyards during ship docking and undocking are model examples of series-parallel and parallel-series systems.

Taking into account the importance of the safety and operating process effectiveness of such systems it seems reasonable to expand the two-state approach to multi-state approach in their reliability analysis. The assumption that the systems are composed of multi-state components with reliability states degrading in time gives the possibility for more precise analysis of their reliability, safety and operational processes' effectiveness. This assumption allows us to distinguish a system reliability critical state to exceed which is either dangerous for the environment or does not assure the necessary level of its operational process effectiveness. Then, an important system reliability characteristic is the time to the moment of exceeding the system reliability critical state and its distribution, which is called the system risk function. This distribution is strictly related to the system multi-state reliability function that is a basic characteristic of the multi-state system.

In the case of large systems, the determination of the exact reliability functions of the systems and the system risk functions leads us to very complicated formulae that are often useless for reliability practitioners. One of the important techniques in this situation is the asymptotic approach to system reliability evaluation. In this approach, instead of the preliminary complex formula for the system reliability function, after assuming that the number of system components tends to infinity and finding the limit reliability of the system, we obtain its simplified form.

The mathematical methods used in the asymptotic approach to the system reliability analysis of large systems are based on limit theorems on order statistics distributions considered in very wide literature (Barndorff-Nielsen 1963, Berman 1962, Berman 1964, Fisher, Tippett 1928, Frechet 1927, Galambos 1975, Gniedenko 1943, Gumbel 1935, Gumbel 1962, Leadbetter 1974, Von Mises 1936). These theorems have generated the investigation concerned with limit reliability functions of the systems composed of two-state components. The main and fundamental results on this subject that determine the three-element classes of limit reliability functions for homogeneous series systems and for homogeneous parallel systems have been established by Gniedenko in (Gniedenko 1943). These results are also presented, sometimes with different proofs, for instance in subsequent works (Barlow, Proschan 1975, Castillo 1988, Chernoff, Teicher 1965, De Haan 1970, Kołowrocki 1993c). The generalizations of these results for homogeneous "*m* out of *n*" systems have been formulated and proved by Smirnow in (Smirnow 1949), where the seven-element class of possible limit reliability functions for these systems has been fixed. Some partial results obtained by Smirnow may be found in (Kołowrocki 2001b). As it has been done for homogeneous series and parallel systems classes of limit reliability functions have been fixed by Chernoff and Teicher in (Chernoff, Teicher 1965) for homogeneous series-parallel and parallel-series systems. Their results were concerned with so-called "quadratic" systems only. They have fixed limit reliability functions for the homogeneous series-parallel systems with the number of series subsystems equal to the number of components in these subsystems, and for the homogeneous parallel-series systems with the number of parallel subsystems equal to the number of components in these subsystems. These results may also be found for instance in later works (Barlow, Proschan 1975) and (Kołowrocki 1993d).

All the results so far described have been obtained under the linear normalization of the system lifetimes. Of course, there is a possibility to look for limit reliability functions of large systems under other than linear standardization of their lifetimes. In this context, the results obtained by (Pantcheva1984) and (Cichocki 2001) are exemplary. Pantcheva in (Pantcheva 1984) has fixed the seven-element classes of limit reliability functions of homogeneous series and parallel systems under power standardization for their lifetimes. Cichocki in (Cichocki 2001) has generalized Pantcheva's results to hierarchical series-parallel and parallel-series systems of any order.

The paper contains the results described above and their newest generalizations for large two-state systems and their exemplary developments for multi-state systems' asymptotic reliability analysis under the linear standardization of the system lifetimes and the system sojourn times in the state subsets, respectively.

Generalizations presented here of the results on limit reliability functions of two-state homogeneous series, and parallel systems for these systems in case they are non-homogeneous, are mostly taken from (Kołowrocki 1994c) and (Kołowrocki 2001b). A more general problem is concerned with fixing the classes of possible limit reliability functions for so-called "rectangular" series-parallel and parallel-series systems. This problem for homogeneous series-parallel and parallel-series systems of any shapes, with different number of subsystems and numbers of components in these subsystems, has been progressively solved in (Kołowrocki 1993a,b,c,d), (Kołowrocki 1994c) and (Kołowrocki 1994e). The main and new result of these works was the determination of seven new limit reliability functions for homogeneous series-parallel systems as well as for parallel-series systems. This way, new ten-element classes of all possible limit reliability functions for these systems have been fixed. Moreover, in these works it has been pointed out that the type of the system limit reliability function strongly depends on the system shape. These results allow us to evaluate reliability characteristics of homogeneous series-parallel and parallel-series systems with regular reliability structures, i.e. systems composed of subsystems having the same numbers of components. The extensions of these results for non-homogeneous series-parallel and parallel-series systems have been formulated and proved successively in (Kołowrocki 1993d), (Kołowrocki 1994d,e, Kołowrocki 1995a,b) and (Kołowrocki 2001b). These generalizations additionally allow us to evaluate reliability characteristics of the series-parallel and parallel-series systems with non-regular structures, i.e. systems with subsystems having different numbers of components. In some of the cited works, as well as the theoretical considerations and solutions, numerous practical applications of the asymptotic approach to real technical system reliability evaluation may also be found (Daniels 1945), (Harlow, Phoenix 1991, Harlow 1997, Harris 1970, Kaufman, Dugan , Johnson 1999, Kołowrocki 1994a, Kołowrocki 1995a, Kołowrocki 1998, Smith 1982, Smith 1983, Soszyńska 2006a, Watherhold 1987).

More general and practically important complex systems composed of multi-state and degrading in time components are considered among others in (Xue 1985, 1995a,b). An especially important role they play in the evaluation of technical systems reliability and safety and their operating process effectiveness is defined in the paper for large multi-state systems with degrading components. The most important results regarding generalizations of the results on limit reliability functions of two-state systems dependent on transferring them to multi-state systems with degrading components are given in (Kołowrocki 1999a,b, Kołowrocki 2000a,b,c, Kołowrocki 2001a,b,c, Kołowrocki 2003a,b). Some of these publications also contain practical applications of the asymptotic approach to the reliability evaluation of various technical systems (Kołowrocki 1999a,b, Kołowrocki 2000a,b,c, Kołowrocki 2001a,b, Kołowrocki 2003a,b).

The results concerned with the asymptotic approach to system reliability analysis have become the basis for the investigation concerned with domains of attraction for the limit reliability functions of the considered systems (Kołowrocki 2004). In a natural way they have led to investigation of the speed of convergence of the system reliability function sequences to their limit reliability functions (Kołowrocki 2004). These results have also initiated the investigation of limit reliability functions of "*m* out of *n*"-series, series-"*m* out of *n*" systems and systems with hierarchical reliability structures as well as investigations on the problems of the system reliability improvement and optimization (Cichocki 2001, Kołowrocki 2004, Soszyńska 2007).

The aim of the paper is to present the state of art on the method of asymptotic approach to reliability evaluation for as wide as possible a range of large systems. The paper describes current theoretical results of the asymptotic approach to reliability evaluation of large two-state and multi-state systems. Additionally, some recent partial results on the asymptotic approach to reliability evaluation of large systems reliability analysis in their operation processes called complex technical systems are presented in the paper (Kołowrocki, Soszyńska 2009, Kołowrocki, Soszyńska 2010a,b, Kołowrocki, Soszyńska-Budny 2011, Soszyńska 2004a,b, Soszyńska 2006a,b,c, Soszyńska 2007, Soszyńska 2008, Soszyńska 2010).

2 BASIC NOTIONS

Considering the reliability of two-state systems we assume that the distributions of the component and the system lifetimes T do not necessarily have to be concentrated in the interval $<0,\infty)$. It means that a reliability function

$$R(t) = P(T > t),\ t \in (-\infty, \infty),$$

does not have to satisfy the usually demanded condition

$$R(t) = 1 \quad \text{for } t \in (-\infty, 0).$$

This is a generalisation of the normally used concept of a reliability function. This generalisation is convenient in the theoretical considerations. At the same time, from the achieved results on the

generalised reliability functions, for particular cases, the same properties of the normally used reliability functions appear.
From that assumption it follows that between a reliability function $R(t)$ and a distribution function

$$F(t) = P(T \le t), t \in (-\infty, \infty),$$

there exists a relationship given by

$$R(t) = 1 - F(t) \text{ for } t \in (-\infty, \infty).$$

Thus, the following corollary is obvious.

Corollary 2.1
A reliability function $R(t)$ is non-increasing, right-continuous and moreover

$$R(-\infty) = 1, R(+\infty) = 0.$$

Definition 2.1
A reliability function $R(t)$ is called degenerate if there exists $t_0 \in (-\infty, \infty)$, such that

$$R(t) = \begin{cases} 1, & t < t_0 \\ 0, & t \ge t_0. \end{cases}$$

The asymptotic approach to the reliability of two-state systems depends on the investigation of limit distributions of a standardised random variable

$$(T - b_n)/a_n,$$

where T is the lifetime of a system and $a_n > 0$, $b_n \in (-\infty, \infty)$, are suitably chosen numbers called normalising constants.
Since

$$P((T - b_n)/a_n > t) = P(T > a_n t + b_n) = \boldsymbol{R}_n(a_n t + b_n),$$

where $\boldsymbol{R}_n(t)$ is a reliability function of a system composed of n components, then the following definition becomes natural.

Definition 2.2
A reliability function $\mathfrak{R}(t)$ is called a limit reliability function or an asymptotic reliability function of a system having a reliability function $\boldsymbol{R}_n(t)$ if there exist normalising constants $a_n > 0$, $b_n \in (-\infty, \infty)$, such that

$$\lim_{n\to\infty} \boldsymbol{R}_n(a_n t + b_n) = \mathfrak{R}(t) \text{ for } t \in C_{\mathfrak{R}}. \quad (2.1)$$

Thus, if the asymptotic reliability function $\mathfrak{R}(t)$ of a system is known, then for sufficiently large n, the approximate formula

$$\boldsymbol{R}_n(t) \cong \mathfrak{R}((t - b_n)/a_n), \ t \in (-\infty, \infty). \quad (2.2)$$

may be used instead of the system exact reliability function $\boldsymbol{R}_n$(t).
From the condition

$$\lim_{n\to\infty} \boldsymbol{R}_n(a_n t + b_n) = \mathfrak{R}(t) \text{ for } t \in C_{\mathfrak{R}},$$

it follows that setting

$$\alpha_n = a a_n, \ \beta_n = b a_n + b_n,$$

where $a > 0$ and $b \in (-\infty, \infty)$, we get

$$\lim_{n\to\infty} \boldsymbol{R}_n(\alpha_n t + \beta_n) = \lim_{n\to\infty} \boldsymbol{R}_n(a_n(at+b)+b_n) = \boldsymbol{\mathfrak{R}}(at+b) \text{ for } t \in C_{\boldsymbol{\mathfrak{R}}}.$$

Hence, if $\boldsymbol{\mathfrak{R}}(t)$ is the limit reliability function of a system, then $\boldsymbol{\mathfrak{R}}(at + b)$ with arbitrary $a > 0$ and $b \in (-\infty, \infty)$ is also its limit reliability function. That fact, in a natural way, yields the concept of a type of limit reliability function.

Definition 2.3
The limit reliability functions $\boldsymbol{\mathfrak{R}}_0(t)$ and $\boldsymbol{\mathfrak{R}}(t)$ are said to be of the same type if there exist numbers $a > 0$ and $b \in (-\infty, \infty)$ such that

$\boldsymbol{\mathfrak{R}}_0(t) = \boldsymbol{\mathfrak{R}}(at + b)$ for $t \in (-\infty, \infty)$.

3 RELIABILITY OF LARGE TWO-STATE SYSTEMS

3.1 RELIABILITY EVALUATION OF TWO-STATE SERIES SYSTEMS

The investigations of limit reliability functions of homogeneous two-state series systems are based on the following auxiliary theorem.

Lemma 3.1
If

(i) $\overline{\boldsymbol{\mathfrak{R}}}(t) = \exp[-\overline{V}(t)]$ is a non-degenerate reliability function,

(ii) $\overline{\boldsymbol{R}}_n(t)$ is the reliability function of a homogeneous two-state series system defined by (2.1) (Kołowrocki 2004)

(iii) $a_n > 0,\ b_n \in (-\infty, \infty)$,

then

$$\lim_{n\to\infty} \overline{\boldsymbol{R}}_n(a_n t + b_n) = \overline{\boldsymbol{\mathfrak{R}}}(t) \text{ for } t \in C_{\overline{\boldsymbol{\mathfrak{R}}}}$$

if and only if

$$\lim_{n\to\infty} nF(a_n t + b_n) = \overline{V}(t) \text{ for } t \in C_{\overline{V}}$$

Lemma 3.1 is an essential tool in finding limit reliability functions of two-state series systems. Its various proofs may be found in (Barlow, Proschan 1975, Gniedenko 1943) and (Kołowrocki 1993d). It also is the basis for fixing the class of all possible limit reliability functions of these systems. This class is determined by the following theorem proved in (Barlow, Proschan 1975, Gniedenko 1943) and (Kołowrocki 1993d).

Theorem 3.1
The only non-degenerate limit reliability functions of the homogeneous two-state series system are:

$$\overline{\boldsymbol{\mathfrak{R}}}_1(t) = \exp[-(-t)^{-\alpha}] \text{ for } t < 0,\ \overline{\boldsymbol{\mathfrak{R}}}_1(t) = 0 \text{ for } t \ge 0,\ \alpha > 0,$$

$$\overline{\boldsymbol{\mathfrak{R}}}_2(t) = 1 \text{ for } t < 0,\ \overline{\boldsymbol{\mathfrak{R}}}_2(t) = \exp[-t^{\alpha}] \text{ for } t \ge 0,\ \alpha > 0,$$

$$\overline{\boldsymbol{\mathfrak{R}}}_3(t) = \exp[-\exp[t]] \text{ for } t \in (-\infty, \infty).$$

The next auxiliary theorem is an extension of Lemma 3.1 to non-homogeneous two-state series systems.

Lemma 3.2
If

(i) $\overline{\mathfrak{R}}'(t) = \exp[-\overline{V}'(t)]$ is a non-degenerate reliability function,

(ii) $\overline{\mathbf{R}}'_n(t)$ is the reliability function of a non-homogeneous two-state series system defined by (2.8) (Kołowrocki 2004a),

(iii) $a_n > 0,\ b_n \in (-\infty, \infty)$,

then

$$\lim_{n\to\infty} \overline{\mathbf{R}}'_n(a_n t + b_n) = \overline{\mathfrak{R}}'(t) \text{ for } t \in C_{\overline{\mathfrak{R}}'}$$

if and only if

$$\lim_{n\to\infty} n\sum_{i=1}^{a} q_i F^{(i)}(a_n t + b_n) = \overline{V}'(t) \text{ for } t \in C_{\overline{V}'}.$$

The proof of Lemma 3.2 is given in (Kołowrocki 1993d). From the latest lemma, as a particular case, it is possible to derive the next auxiliary theorem that is a more convenient tool than Lemma 3.2 for finding limit reliability functions of non-homogeneous series systems and the starting point for fixing limit reliability functions for these systems.

Lemma 3.3
If

(i) $\overline{\mathfrak{R}}'(t) = \exp[-\overline{V}'(t)]$ is a non-degenerate reliability function,

(ii) $\overline{\mathbf{R}}'_n(t)$ is the reliability function of a non-homogeneous two-state series system defined by (2.8) (Kołowrocki 2004a),

(iii) $a_n > 0,\ b_n \in (-\infty, \infty)$,

(iv) $F(t)$ is one of the distribution functions $F^{(1)}(t), F^{(2)}(t),...,F^{(a)}(t)$ defined by (2.7) (Kołowrocki 2004a), such that

(v) $\exists N\ \forall n > N\ F(a_n t + b_n) = 0$ for $t < t_0$ and $F(a_n t + b_n) \neq 0$ for $t \geq t_0$, where $t_0 \in <-\infty, \infty)$,

(vi) $\lim_{n\to\infty} \frac{F^{(i)}(a_n t + b_n)}{F(a_n t + b_n)} \leq 1$ for $t \geq t_0$, $i = 1,2,...,a$,

and moreover there exists a non-decreasing function

(vii) $$\bar{d}(t) = \begin{cases} 0 & for\ t < t_o \\ \lim_{n\to\infty} \sum_{i=1}^{a} q_i \bar{d}_i(a_n t + b_n) & for\ t \ge t_o, \end{cases} \tag{3.1}$$

where

(viii) $$\bar{d}_i(a_n t + b_n) = \frac{F^{(i)}(a_n t + b_n)}{F(a_n t + b_n)}, \tag{3.2}$$

then

$$\lim_{n\to\infty} \overline{\boldsymbol{R}}'_n(a_n t + b_n) = \overline{\boldsymbol{\mathfrak{R}}}'(t) \text{ for } t \in C_{\overline{\mathfrak{R}}'}$$

if and only if

$$\lim_{n\to\infty} nF(a_n t + b_n)\bar{d}(t) = \overline{V}'(t) \text{ for } t \in C_{\overline{V}'}.$$

On the basis of Theorem 3.1 and Lemma 3.3 in (Kołowrocki 1993d), the class of limit reliability functions for non-homogeneous two-state series systems has been fixed. The members of this class are specified in the following theorem (Kołowrocki 1993d).

Theorem 3.2

The only non-degenerate limit reliability functions of the non-homogeneous two-state series system, under the assumptions of Lemma 3.3, are:

$$\overline{\boldsymbol{\mathfrak{R}}}'_1(t) = \exp[-\bar{d}(t)(-t)^{-\alpha}] \text{ for } t < 0,\ \overline{\boldsymbol{\mathfrak{R}}}'_1(t) = 0 \text{ for } t \ge 0,\ \alpha > 0,$$

$$\overline{\boldsymbol{\mathfrak{R}}}'_2(t) = 1 \text{ for } t < 0,\ \overline{\boldsymbol{\mathfrak{R}}}'_2(t) = \exp[-\bar{d}(t)t^{\alpha}] \text{ for } t \ge 0,\ \alpha > 0,$$

$$\overline{\boldsymbol{\mathfrak{R}}}'_3(t) = \exp[-\bar{d}(t)\exp[t]] \text{ for } t \in (-\infty, \infty),$$

where $\bar{\bar{d}}(t)$ is a non-decreasing function dependent on the reliability functions of particular system components and their fractions in the system defined by (3.1)-(3.2).

3.2 RELIABILITY EVALUATION OF TWO-STATE PARALLEL SYSTEMS

The class of limit reliability functions for homogeneous two-state parallel systems may be determined on the basis of the following auxiliary theorem proved for instance in (Barlow, Proschan 1975, Gniedenko 1943) and (Kołowrocki 1993d).

Lemma 3.4

If $\overline{\boldsymbol{\mathfrak{R}}}(t)$ is the limit reliability function of a homogeneous two-state series system with reliability functions of particular components $\overline{R}(t)$, then

$$\boldsymbol{\mathfrak{R}}(t) = 1 - \overline{\boldsymbol{\mathfrak{R}}}(-t) \text{ for } t \in C_{\overline{\mathfrak{R}}}$$

is the limit reliability function of a homogeneous two-state parallel system with reliability functions of particular components

$$R(t) = 1 - \bar{R}(-t) \text{ for } t \in C_{\bar{R}} .$$

At the same time, if (a_n, b_n) is a pair of normalising constants in the first case, then $(a_n, -b_n)$ is such a pair in the second case.

Applying the above lemma it is possible to prove an equivalent of Lemma 3.1 that allows us to justify facts on limit reliability functions for homogeneous parallel systems. Its form is as follows (Barlow, Proschan 1975, Gniedenko 1943, Kołowrocki 1993d).

Lemma 3.5
If

(i) $\mathfrak{R}(t) = 1 - \exp[-V(t)]$ is a non-degenerate reliability function,

(ii) $\boldsymbol{R}_n(t)$ is the reliability function of a homogeneous two-state parallel system defined by (2.2) [42],

(iii) $a_n > 0,\ b_n \in (-\infty, \infty)$,

then

$$\lim_{n\to\infty} \boldsymbol{R}_n(a_n t + b_n) = \mathfrak{R}(t) \text{ for } t \in C_{\mathfrak{R}} ,$$

if and only if

$$\lim_{n\to\infty} nR(a_n t + b_n) = V(t) \text{ for } t \in C_V .$$

By applying Lemma 3.5 and proceeding in an analogous way to the case of homogeneous series systems it is possible to fix the class of limit reliability functions for homogeneous two-state parallel systems. However, it is easier to obtain this result using Lemma 3.4 and Theorem 3.1. Their application immediately results in the following issue.

Theorem 3.3
The only non-degenerate limit reliability functions of the homogeneous parallel system are:

$$\mathfrak{R}_1(t) = 1 \text{ for } t \le 0,\ \mathfrak{R}_1(t) = 1 - \exp[-t^{-\alpha}] \text{ for } t > 0,\ \alpha > 0,$$

$$\mathfrak{R}_2(t) = 1 - \exp[-(-t)^{\alpha}] \text{ for } t < 0,\ \mathfrak{R}_2(t) = 0 \text{ for } t \ge 0,\ \alpha > 0,$$

$$\mathfrak{R}_3(t) = 1 - \exp[-\exp[-t]] \text{ for } t \in (-\infty,\infty).$$

The next lemma is a slight modification of Lemma 3.5 proved in (Kołowrocki 1993d). It is also a particular case of Lemma 2, which is proved in (Kołowrocki 1995a).

Lemma 3.6
If $\bar{\mathfrak{R}}'(t)$ is the limit reliability function of a non-homogeneous two-state series system with reliability functions of particular components

$$\bar{R}^{(i)}(t),\ i = 1,2,...,a,$$

then

$$\mathfrak{R}'(t) = 1 - \bar{\mathfrak{R}}'(-t) \ \text{ for } t \in C_{\bar{\mathfrak{R}}'}$$

is the limit reliability function of a non-homogeneous two-state parallel system with reliability functions of particular components

$$R^{(i)}(t) = 1 - \bar{R}^{(i)}(-t) \text{ for } t \in C_{\bar{R}^{(i)}},\ i = 1,2,...,a.$$

At the same time, if (a_n, b_n) is a pair of normalising constants in the first case, then $(a_n, -b_n)$ is such a pair in the second case.

Applying the above lemma and Theorem 3.2 it is possible to arrive at the next result (Kołowrocki 1993d, Kołowrocki 1995b).

Lemma 3.7
If

(i) $\mathfrak{R}'(t) = 1 - \exp[-V'(t)]$ is a non-degenerate reliability function,

(ii) $\boldsymbol{R}'_n(t)$ is the reliability function of a non-homogeneous two-state parallel system defined by (2.10) (Kołowrocki 2004a),

(iii) $a_n > 0$, $b_n \in (-\infty,\infty)$,

then

$$\lim_{n\to\infty} \boldsymbol{R}'_n(a_n t + b_n) = \mathfrak{R}'(t) \ \text{ for } t \in C_{\mathfrak{R}'}$$

if and only if

$$\lim_{n\to\infty} n\sum_{i=1}^{a} q_i R^{(i)}(a_n t + b_n) = V'(t) \ \text{ for } t \in C_{V'}.$$

The next lemma motivated in (Kołowrocki 1993d) that is useful in practical applications is a particular case of Lemma 3 proved in (Kołowrocki 1995b).

Lemma 3.8

If

(i) $\mathfrak{R}'(t) = 1 - \exp[-V'(t)]$ is a non-degenerate reliability function,

(ii) $\mathbf{R}'_n(t)$ is the reliability function of a non-homogeneous two-state parallel system defined by (2.10) (Kołowrocki 2004a),

(iii) $a_n > 0$, $b_n \in (-\infty,\infty)$,

(iv) $R(t)$ is one of the reliability functions $R^{(1)}(t)$, $R^{(2)}(t)$,...,$R^{(a)}(t)$ defined by (2.9) (Kołowrocki 2004a), such that

(v) $\exists\, N\ \forall\, n > N\ \ R(a_n t + b_n) \neq 0$ for $t < t_0$ and $R(a_n t + b_n) = 0$ for $t \geq t_0$,
where $t_0 \in (-\infty,\infty>$,

(vi) $\lim\limits_{n\to\infty} \dfrac{R^{(i)}(a_n t + b_n)}{R(a_n t + b_n)} \leq 1$ for $t < t_0$, $i = 1,2,...,a$,

and moreover there exists a non-increasing function

(vii) $$d(t) = \begin{cases} \lim\limits_{n\to\infty} \sum\limits_{i=1}^{a} q_i d_i(a_n t + b_n) & for\ t < t_0 \\ 0 & for\ t \geq t_0, \end{cases} \qquad (3.3)$$

where

(viii) $$d_i(a_n t + b_n) = \frac{R^{(i)}(a_n t + b_n)}{R(a_n t + b_n)}, \qquad (3.4)$$

then

$$\lim_{n\to\infty} \mathbf{R}'_n(a_n t + b_n) = \mathfrak{R}'(t) \ \text{ for } t \in C_{\mathfrak{R}'}$$

if and only if

$$\lim_{n\to\infty} nR(a_n t + b_n) d(t) = V'(t) \ \text{ for } t \in C_{V'}.$$

Starting from this lemma it is possible to fix the class of possible limit reliability for non-homogeneous two-state parallel systems (Kołowrocki 1993d, Kołowrocki 1995b).

Theorem 3.4

The only non-degenerate limit reliability functions of the non-homogeneous two-state parallel system, under the assumptions of Lemma 3.8, are:

$$\mathfrak{R}'_1(t) = 1 \text{ for } t \leq 0,\ \mathfrak{R}'_1(t) = 1 - \exp[-d(t)t^{-\alpha}] \text{ for } t > 0,\ \alpha > 0,$$

$$\mathfrak{R}'_2(t) = 1 - \exp[-d(t)(-t)^{\alpha}] \text{ for } t < 0,\ \mathfrak{R}'_2(t) = 0 \text{ for } t \geq 0,\ \alpha > 0,$$

$$\mathfrak{R}'_3(t) = 1 - \exp[-d(t)\exp[-t]] \text{ for } t \in (-\infty,\infty),$$

where $d(t)$ is a non-increasing function dependent on the reliability functions of particular system components and their fractions in the system defined by (3.3)-(3.4).

3.3 RELIABILITY EVALUATION OF TWO-STATE "*M* OUT OF *N*" SYSTEMS

The class of limit reliability function for homogeneous two-state "*m* out of *n*" systems may be established by applying the auxiliary theorems proved in (Smirnow 1949) and (Kołowrocki 1993d). The applications of these lemmas allow us to establish the class of possible limit reliability functions for homogeneous two-state "*m* out of *n*" systems pointed out in the following theorem (Kołowrocki 1993d, Smirnow 1949).

Theorem 3.5
The only non-degenerate limit reliability functions of the homogeneous two-state "*m* out of *n*" system are:

Case 1. $m = \text{constant}\,(m/n \to 0 \text{ as } n \to \infty)$.

$$\mathfrak{R}_1^{(0)}(t) = 1 \text{ for } t \le 0,\ \mathfrak{R}_1^{(0)}(t) = 1 - \sum_{i=0}^{m-1} \frac{t^{-i\alpha}}{i!} \exp[-t^{-\alpha}] \text{ for } t > 0,\ \alpha > 0,$$

$$\mathfrak{R}_2^{(0)}(t) = 1 - \sum_{i=0}^{m-1} \frac{(-t)^{i\alpha}}{i!} \exp[-(-t)^{\alpha}] \text{ for } t < 0,\ \mathfrak{R}_2^{(0)}(t) = 0 \text{ for } t \ge 0,\ \alpha > 0,$$

$$\mathfrak{R}_3^{(0)}(t) = 1 - \sum_{i=0}^{m-1} \frac{\exp[-it]}{i!} \exp[-\exp[-t]] \text{ for } t \in (-\infty, \infty).$$

Case 2. $m/n = \mu + o(1/\sqrt{n}),\ 0 < \mu < 1,\ (m/n \to \mu \text{ as } n \to \infty)$.

$$\mathfrak{R}_4^{(\mu)}(t) = 1 \text{ for } t < 0,\ \mathfrak{R}_4^{(\mu)}(t) = 1 - \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{ct^{\alpha}} e^{-\frac{x^2}{2}} dx \text{ for } t \ge 0,\ c > 0, \alpha > 0,$$

$$\mathfrak{R}_5^{(\mu)}(t) = 1 - \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-c|t|^{\alpha}} e^{-\frac{x^2}{2}} dx \text{ for } t < 0,\ \mathfrak{R}_5^{(\mu)}(t) = 0 \text{ for } t \ge 0,\ c > 0, \alpha > 0,$$

$$\mathfrak{R}_6^{(\mu)}(t) = 1 - \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-c_1|t|^{\alpha}} e^{-\frac{x^2}{2}} dx \text{ for } t < 0,\ c_1 > 0, \alpha > 0,$$

$$\mathfrak{R}_6^{(\mu)}(t) = \frac{1}{2} - \frac{1}{\sqrt{2\pi}} \int_{0}^{c_2 t^{\alpha}} e^{-\frac{x^2}{2}} dx \text{ for } t \ge 0,\ c_2 > 0, \alpha > 0,$$

$$\mathfrak{R}_7^{(\mu)}(t) = 1 \text{ for } t < -1,\ \mathfrak{R}_7^{(\mu)}(t) = \frac{1}{2} \text{ for } -1 \le t < 1,\ \mathfrak{R}_7^{(\mu)}(t) = 0 \text{ for } t \ge 0.$$

Case 3. $n - m = \overline{m} = \text{constant}\ (m/n \to 1 \text{ as } n \to \infty)$.

$$\overline{\mathfrak{R}}_8^{(1)}(t) = \sum_{i=0}^{\overline{m}} \frac{(-t)^{-i\alpha}}{i!} \exp[-(-t)^{-\alpha}] \text{ for } t < 0,\ \overline{\mathfrak{R}}_8^{(1)}(t) = 0 \text{ for } t \ge 0,\ \alpha > 0,$$

$$\overline{\mathfrak{R}}_9^{(1)}(t) = 1 \text{ for } t < 0,\ \overline{\mathfrak{R}}_9^{(1)}(t) = \sum_{i=0}^{\overline{m}} \frac{t^{i\alpha}}{i!} \exp[-t^{\alpha}] \text{ for } t \ge 0,\ \alpha > 0,$$

$$\overline{\mathfrak{R}}_9^{(1)}(t) = \sum_{i=0}^{\overline{m}} \frac{\exp[it]}{i!} \exp[-\exp[t]] \text{ for } t \in (-\infty,\infty).$$

3.4 RELIABILITY EVALUATION OF TWO-STATE SERIES-PARALLEL SYSTEMS

Prior to the formulation of the overall results for the classes of limit reliability functions for two-state regular series-parallel systems we should introduce some assumptions for all cases of the considered systems shapes. These assumptions distinguish all possible relationships between the number of their series subsystems k_n and the number of components l_n in these subsystems (Assumption 4.1, (Kołowrocki 2004)).

The proofs of the theorems on limit reliability functions for homogeneous regular series-parallel systems and methods of finding such functions for individual systems are based on the lemma given in (Kołowrocki 1993a) and (Kołowrocki 1993d). The results achieved in (Kołowrocki 1993a,b,c,d, Kołowrocki 1994c) and based on those lemmas may be formulated in the form of the following theorem (Kołowrocki 1993b, Kołowrocki 1994a, Kołowrocki 1995b).

Theorem 3.6

The only non-degenerate limit reliability functions of the homogeneous regular two-state series-parallel system are:

Case 1. $k_n = n$, $|l_n - c \log n| >> s$, $s > 0$, $c > 0$ (under Assumption 4.1 (Kołowrocki 2004)).

$$\mathfrak{R}_1(t) = 1 \text{ for } t \le 0,\ \mathfrak{R}_1(t) = 1 - \exp[-t^{-\alpha}] \text{ for } t > 0,\ \alpha > 0,$$

$$\mathfrak{R}_2(t) = 1 - \exp[-(-t)^{\alpha}] \text{ for } t < 0,\ \mathfrak{R}_2(t) = 0 \text{ for } t \ge 0,\ \alpha > 0,$$

$$\mathfrak{R}_3(t) = 1 - \exp[-\exp[-t]] \text{ for } t \in (-\infty,\infty),$$

Case 2. $k_n = n$, $l_n - c \log n \approx s$, $s \in (-\infty,\infty)$, $c > 0$.

$$\mathfrak{R}_4(t) = 1 \text{ for } t < 0,\ \mathfrak{R}_4(t) = 1 - \exp[-\exp[-t^{\alpha} - s/c]] \text{ for } t \ge 0,\ \alpha > 0,$$

$$\mathfrak{R}_5(t) = 1 - \exp[-\exp[(-t)^{\alpha} - s/c]] \text{ for } t < 0,\ \mathfrak{R}_5(t) = 0 \text{ for } t \ge 0,\ \alpha > 0,$$

$$\mathfrak{R}_6(t) = 1 - \exp[-\exp[\beta(-t)^{\alpha} - s/c]] \text{ for } t < 0,$$

$$\mathfrak{R}_6(t) = 1 - \exp[-\exp[-t^{\alpha} - s/c]] \text{ for } t \ge 0,\ \alpha > 0,\ \beta > 0,$$

$$\mathfrak{R}_7(t) = 1 \text{ for } t < t_1,\ \mathfrak{R}_7(t) = 1 - \exp[-\exp[-s/c]] \text{ for } t_1 \le t < t_2,$$

$$\mathfrak{R}_7(t) = 0 \text{ for } t \ge t_2,\ t_1 < t_2,$$

Case 3. $k_n \to k$, $k > 0$, $l_n \to \infty$.

$$\mathfrak{R}_8(t) = 1 - [1 - \exp[-(-t)^{-\alpha}]]^k \text{ for } t < 0,\ \mathfrak{R}_8(t) = 0 \text{ for } t \ge 0,\ \alpha > 0,$$

$$\mathfrak{R}_9(t) = 1 \text{ for } t < 0,\ \mathfrak{R}_9(t) = 1 - [1 - \exp[-t^{\alpha}]]^k \text{ for } t \ge 0,\ \alpha > 0,$$

$$\mathfrak{R}_{10}(t) = 1 - [1 - \exp[-\exp t]]^k \text{ for } t \in (-\infty,\infty).$$

The proofs of the facts concerned with limit reliability functions of non-homogeneous two-state series-parallel systems are based on the auxiliary theorems formulated and proved in (Kołowrocki 1993d, Kołowrocki 1994c) and (Kołowrocki 1995b). Theorem 3.6 and those lemmas determine the class of limit reliability functions for non-homogeneous regular series-parallel systems whose members are pointed out in the following theorem (Kołowrocki 1993d, Kołowrocki 1994a, Kołowrocki 1994d).

Theorem 3.7
The only non-degenerate limit reliability functions of the non-homogeneous regular two-state series-parallel system are:

Case 1. $k_n = n$, $|l_n - c \log n| >> s$, $s > 0$, $c > 0$ (under Assumption 4.1 (Kołowrocki 2004)).

$$\mathfrak{R}'_1(t) = 1 \text{ for } t \le 0,\ \mathfrak{R}'_1(t) = 1 - \exp[-d(t)t^{-\alpha}] \text{ for } t > 0,\ \alpha > 0,$$

$$\mathfrak{R}'_2(t) = 1 - \exp[-d(t)(-t)^{\alpha}] \text{ for } t < 0,\ \mathfrak{R}'_2(t) = 0 \text{ for } t \ge 0,\ \alpha > 0,$$

$$\mathfrak{R}'_3(t) = 1 - \exp[-d(t)\exp[-t]] \text{ for } t \in (-\infty,\infty),$$

Case 2. $k_n = n$, $l_n - c \log n \approx s$, $s \in (-\infty,\infty)$, $c > 0$ (under Assumption 3.1 (Kołowrocki 2004)).

$$\mathfrak{R}'_4(t) = 1 \text{ for } t < 0,\ \mathfrak{R}'_4(t) = 1 - \exp[-d(t)\exp[-t^{\alpha} - s/c]] \text{ for } t \ge 0,\ \alpha > 0,$$

$$\mathfrak{R}'_5(t) = 1 - \exp[-d(t)\exp[(-t)^{\alpha} - s/c]] \text{ for } t < 0,\ \mathfrak{R}'_5(t) = 0 \text{ for } t \ge 0,\ \alpha > 0,$$

$$\mathfrak{R}'_6(t) = 1 - \exp[-d(t)\exp[\beta(-t)^{\alpha} - s/c]] \text{ for } t < 0,$$

$$\mathfrak{R}'_6(t) = 1 - \exp[-d(t)\exp[-t^{\alpha} - s/c]] \text{ for } t \ge 0,\ \alpha > 0,\ \beta > 0,$$

$$\mathfrak{R}'_7(t) = 1 \text{ for } t < t_1,\ \mathfrak{R}'_7(t) = 1 - \exp[-d(t)\exp[-s/c]] \text{ for } t_1 \le t < t_2,\ \mathfrak{R}'_7(t) = 0 \text{ for } t \ge t_2,\ t_1 < t_2,$$

Case 3. $k_n \to k$, $k > 0$, $l_n \to \infty$ (under Assumption 4.1 (Kołowrocki 2004).

$$\mathfrak{R}'_8(t) = 1 - \prod_{i=1}^{a} [1 - d_i(t)\exp[-(-t)^{-\alpha}]]^{q_i k} \text{ for } t < 0,\ \mathfrak{R}'_8(t) = 0 \text{ for } t \ge 0,\ \alpha > 0,$$

$$\mathfrak{R}'_9(t) = 1 \text{ for } t < 0,\ \mathfrak{R}'_9(t) = 1 - \prod_{i=1}^{a} [1 - d_i(t)\exp[-t^{\alpha}]]^{q_i k} \text{ for } t \ge 0,\ \alpha > 0, \quad (3.80)$$

$$\mathfrak{R}'_{10}(t) = 1 - \prod_{i=1}^{a} [1 - d_i(t)\exp[-\exp t]]^{q_i k} \text{ for } t \in (-\infty,\infty),$$

where $d(t)$ and $d_i(t)$ are non-increasing functions dependent on the reliability functions of the system's particular components and their fractions in the system defined in (Kołowrocki 2004).

3.5 RELIABILITY EVALUATION OF TWO-STATE PARALLEL-SERIES SYSTEMS

Prior to the formulation of the overall results for the classes of limit reliability functions for two-state regular parallel-series systems we should introduce some assumptions for all cases of the considered systems shapes. These assumptions distinguish all possible relationships between the number of their parallel subsystems k_n and the number of components l_n in these subsystems (Assumption 4.1 (Kołowrocki 2004)).
The class of limit reliability functions for homogeneous regular two-state parallel-series systems is successively fixed in (Kołowrocki 1993a,b,c,d, Kołowrocki 1994c) and (Kołowrocki 1994e,f, Kołowrocki 1995). The class of limit reliability functions for homogeneous regular two-state parallel-series system is pointed out in the following theorem (Kołowrocki 1993d, Kołowrocki 1994d).

Theorem 3.8
The only non-degenerate limit reliability functions of the homogeneous regular two-state parallel-series system are:

Case 1. $k_n = n$, $|l_n - c\log n| >> s$, $s > 0$, $c > 0$ (under Assumption 4.1 (Kołowrocki 2004).

$$\overline{\mathfrak{R}}_1(t) = \exp[-(-t)^{-\alpha}] \text{ for } t < 0,\ \overline{\mathfrak{R}}_1(t) = 0, \text{ for } t \ge 0,\ \alpha > 0,$$

$$\overline{\mathfrak{R}}_2(t) = 1 \text{ for } t < 0,\ \overline{\mathfrak{R}}_2(t) = \exp[-t^{\alpha}], \text{ for } t \ge 0,\ \alpha > 0,$$

$$\overline{\mathfrak{R}}_3(t) = \exp[-\exp[t]] \text{ for } t \in (-\infty,\infty),$$

Case 2. $k_n = n$, $l_n - c\log n \approx s$, $s \in (-\infty,\infty)$, $c > 0$;

$$\overline{\mathfrak{R}}_4(t) = \exp[-\exp[-(-t)^{\alpha} - s/c]] \text{ for } t < 0,\ \overline{\mathfrak{R}}_4(t) = 0 \text{ for } t \ge 0,\ \alpha > 0,$$

$$\overline{\mathfrak{R}}_5(t) = 1 \text{ for } t < 0,\ \overline{\mathfrak{R}}_5(t) = \exp[-\exp[t^{\alpha} - s/c]] \text{ for } t \ge 0,\ \alpha > 0,$$

$$\overline{\mathfrak{R}}_6(t) = \exp[-\exp[-(-t)^{\alpha} - s/c]] \text{ for } t < 0, \overline{\mathfrak{R}}_6(t) = \exp[-\exp[\beta t^{\alpha} - s/c]] \text{ for } t \ge 0,\ \alpha > 0,\ \beta > 0,$$

$$\overline{\mathfrak{R}}_7(t) = 1 \text{ for } t < t_1,\ \overline{\mathfrak{R}}_7(t) = \exp[-\exp[-s/c]] \text{ for } t_1 \le t < t_2, \overline{\mathfrak{R}}_7(t) = 0 \text{ for } t \ge t_2,\ t_1 < t_2,$$

Case 3. $k_n \to k$, $k > 0$, $l_n \to \infty$.

$$\overline{\mathfrak{R}}_8(t) = 1 \text{ for } t \le 0,\ \overline{\mathfrak{R}}_8(t) = [1 - \exp[-t^{-\alpha}]]^k \text{ for } t > 0,\ \alpha > 0,$$

$$\overline{\mathfrak{R}}_9(t) = [1 - \exp[-(-t)^{\alpha}]]^k \text{ for } t < 0,\ \overline{\mathfrak{R}}_9(t) = 0 \text{ for } t \ge 0,\ \alpha > 0,$$

$$\overline{\mathfrak{R}}_{10}(t) = [1 - \exp[-\exp[-t]]]^k \text{ for } t \in (-\infty,\infty).$$

The class of limit reliability functions for non-homogeneous regular two-state parallel-series system is pointed out in the following theorem (Kołowrocki 1993d, Kołowrocki 1994d).

Theorem 3.9
The only non-degenerate limit reliability functions of the non-homogeneous regular two-state parallel-series system are:

Case 1. $k_n = n$, $|\, l_n - c \log n \,| >> s$, $s > 0$, $c > 0$ (under Assumption 4.1 (Kołowrocki 2004)).

$$\overline{\mathfrak{R}}'_1(t) = \exp[-\bar{d}(t)(-t)^{-\alpha}] \text{ for } t < 0,\ \overline{\mathfrak{R}}'_1(t) = 0 \text{ for } t \geq 0,\ \alpha > 0,$$

$$\overline{\mathfrak{R}}'_2(t) = 1 \text{ for } t < 0,\ \overline{\mathfrak{R}}'_2(t) = \exp[-\bar{d}(t)t^{\alpha}] \text{ for } t \geq 0,\ \alpha > 0,$$

$$\overline{\mathfrak{R}}'_3(t) = \exp[-\bar{d}(t)\exp[t]] \text{ for } t \in (-\infty,\infty),$$

Case 2. $k_n = n$, $l_n - c \log n \approx s$, $s \in (-\infty,\infty)$, $c > 0$ (under Assumption 4.1 (Kołowrocki 2004)).

$$\overline{\mathfrak{R}}'_4(t) = \exp[-\bar{d}(t)\exp[-(-t)^{\alpha} - s/c]] \text{ for } t < 0,\ \overline{\mathfrak{R}}'_4(t) = 0 \text{ for } t \geq 0,\ \alpha > 0,$$

$$\overline{\mathfrak{R}}'_5(t) = 1 \text{ for } t < 0,\ \overline{\mathfrak{R}}'_5(t) = \exp[-\bar{d}(t)\exp[t^{\alpha} - s/c]] \text{ for } t \geq 0,\ \alpha > 0,$$

$$\overline{\mathfrak{R}}'_6(t) = \exp[-\bar{d}(t)\exp[-(-t)^{\alpha} - s/c]] \text{ for } t < 0,$$

$$\overline{\mathfrak{R}}'_6(t) = \exp[-\bar{d}(t)\exp[\beta t^{\alpha} - s/c]] \text{ for } t \geq 0,\ \alpha > 0,\ \beta > 0,$$

$$\overline{\mathfrak{R}}'_7(t) = 1 \text{ for } t < t_1,\ \overline{\mathfrak{R}}'_7(t) = \exp[-\bar{d}(t)\exp[-s/c]] \text{ for } t_1 \leq t < t_2,$$

$$\overline{\mathfrak{R}}'_7(t) = 0 \text{ for } t \geq t_2,\ t_1 < t_2,$$

Case 3. $k_n \to k$, $k > 0$, $l_n \to \infty$ (under Assumption 4.1 (Kołowrocki 2004)).

$$\overline{\mathfrak{R}}'_8(t) = 1 \text{ for } t \leq 0,\ \overline{\mathfrak{R}}'_8(t) = \prod_{i=1}^{a}[1 - \bar{d}_i(t)\exp[-t^{-\alpha}]]^{q_i k} \text{ for } t > 0,\ \alpha > 0,$$

$$\overline{\mathfrak{R}}'_9(t) = \prod_{i=1}^{a}[1 - \bar{d}_i(t)\exp[-(-t)^{\alpha}]]^{q_i k} \text{ for } t < 0,\ \overline{\mathfrak{R}}'_9(t) = 0 \text{ for } t \geq 0,\ \alpha > 0,$$

$$\overline{\mathfrak{R}}'_{10}(t) = \prod_{i=1}^{a}[1 - \bar{d}_i(t)\exp[-\exp(-t)]]^{q_i k} \text{ for } t \in (-\infty,\infty),$$

where $\bar{d}(t)$ and $\bar{d}_i(t)$ are non-decreasing functions dependent on the reliability functions of particular system components and their fractions in the system defined in (Kołowrocki 2004).

4 RELIABILITY OF LARGE MULTI-STATE SYSTEMS

In the multi-state reliability analysis to define systems with degrading (ageing) components we assume that:

– E_i, $i = 1,2,...,n$, are components of a system,
– all components and a system under consideration have the reliability state set $\{0,1,...,z\}$, $z \geq 1$,
– the reliability states are ordered, the state 0 is the worst and the state z is the best,
– $T_i(u)$, $i = 1,2,...,n$, are independent random variables representing the lifetimes of components E_i in the reliability state subset $\{u,u+1,...,z\}$, while they were in the reliability state z at the moment $t = 0$,
– $T(u)$ is a random variable representing the lifetime of a system in the reliability state subset $\{u,u+1,...,z\}$ while it was in the reliability state z at the moment $t = 0$,
– the system state degrades with time t,
– $e_i(t)$ is a component E_i reliability state at the moment t, $t \in < 0,\infty)$, given that it was in the reliability state z at the moment $t = 0$,
– $s(t)$ is a system reliability state at the moment t, $t \in < 0,\infty)$, given that it was in the reliability state z at the moment $t = 0$.

The above assumptions mean that the reliability states of the system with degrading components may be changed in time only from better to worse. The way in which the components and the system reliability states change is illustrated in Figure 4.1.

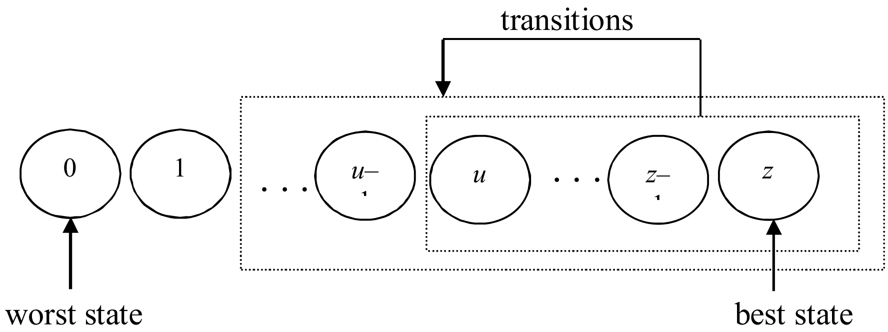

Figure. 4.1. Illustration of reliability states changing in system with ageing components

Definition 4.1
A vector

$$R_i(t,\cdot) = [R_i(t,0),R_i(t,1),...,R_i(t,z)],\ t \in < 0,\infty),\ i = 1,2,...,n,$$

where

$$R_i(t,u) = P(e_i(t) \geq u \mid e_i(0) = z) = P(T_i(u) > t),\ t \in < 0,\infty),\ u = 0,1,...,z,$$

is the probability that the component E_i is in the reliability state subset $\{u, u+1,..., z\}$ at the moment t, $t \in < 0,\infty)$, while it was in the reliability state z at the moment $t = 0$, is called the multi-state reliability function of a component E_i.

Definition 4.2
A vector

$$\boldsymbol{R}_n(t,\cdot) = [\boldsymbol{R}_n(t,0),\boldsymbol{R}_n(t,1),...,\boldsymbol{R}_n(t,z)],\ t \in (-\infty,\infty), \tag{4.1}$$

where

$$\boldsymbol{R}_n(t,u) = P(s(t) \geq u \mid s(0) = z) = P(T(u) > t),\ t \in <0,\infty),\ u = 0,1,...,z, \tag{4.2}$$

is the probability that the system is in the reliability state subset $\{u, u+1,..., z\}$ at the moment t, $t \in <0,\infty)$, while it was in the reliability state z at the moment t = 0, is called the multi-state reliability function of a system.

Definition 4.3
A probability

$$\boldsymbol{r}(t) = P(s(t) < r \mid s(0) = z) = P(T(r) \leq t),\ t \in <0,\infty),$$

that the system is in the subset of reliability states worse than the critical reliability state r, $r \in \{1,...,z\}$ while it was in the reliability state z at the moment t = 0 is called a risk function of the multi-state system.

Under this definition, from (4.1)-(4.2), we have

$$\boldsymbol{r}(t) = 1 - P(s(t) \geq r \mid s(0) = z) = 1 - \boldsymbol{R}_n(t,r),\ t \in <0,\infty).$$

and if τ is the moment when the risk exceeds a permitted level δ, then

$$\tau = \boldsymbol{r}^{-1}(\delta),$$

where $\boldsymbol{r}^{-1}(t)$, if it exists, is the inverse function of the risk function $\boldsymbol{r}(t)$.

In the asymptotic approach to multi-state system reliability analysis we are interested in the limit distributions of a standardised random variable

$$(T(u) - b_n(u)) / a_n(u),\ u = 1,2,...,z,$$

where $T(u)$ is the lifetime of the system in the state subset $\{u, u+1,..., z\}$ and

$$a_n(u) > 0,\ b_n(u) \in (-\infty,\infty), u = 1,2,...,z,$$

are some suitably chosen numbers, called normalising constants. And, since

$$P((T(u) - b_n(u)) / a_n(u) > t) = P(T(u) > a_n(u)t + b_n(u)) = \boldsymbol{R}_n(a_n(u)t + b_n(u),u),\ u = 1,2,...,z,$$

where

$$\boldsymbol{R}_n(t,\cdot) = [\boldsymbol{R}_n(t,0),\boldsymbol{R}_n(t,1),...,\boldsymbol{R}_n(t,z)],\ t \in (-\infty,\infty),$$

is the multi-state reliability function of the system composed of n components, then we assume the following definition.

Definition 4.4
A vector

$$\mathfrak{R}(t,\cdot) = [1,\mathfrak{R}(t,1),...,\mathfrak{R}(t,z)],\ t \in (-\infty,\infty),$$

is called the limit multi-state reliability function of the system with reliability function $\boldsymbol{R}_n(t,\cdot)$ if there exist normalising constants $a_n(u) > 0$, $b_n(u) \in (-\infty,\infty)$, such that

$$\lim_{n\to\infty} \boldsymbol{R}_n(a_n(u)t + b_n(u),u) = \mathfrak{R}(t,u) \text{ for } t \in C_{\mathfrak{R}(u)},\ u = 1,2,...,z,$$

where $C_{\mathfrak{R}(u)}$ is the set of continuity points of $\mathfrak{R}(t,u)$.

Knowing the system limit reliability function allows us, for sufficiently large n, to apply the following approximate formula

$$[1,\boldsymbol{R}_n(t,1),...,\boldsymbol{R}_n(t,z)] \cong [1,\mathfrak{R}(\frac{t-b_n(1)}{a_n(1)},1),...,\mathfrak{R}(\frac{t-b_n(z)}{a_n(z)},z)],\ t \in (-\infty,\infty).$$

Similar as in Section 3, auxiliary theorems on limit reliability functions of multi-state systems, which are necessary for their approximate reliability evaluation, can be formulated and proved (Kołowrocki 2004). The classes of limit reliability functions for homogeneous and non-homogeneous series, parallel, series-parallel and parallel-series multi-state systems and for a homogeneous multi-state “m out of n” system can be fixed as well (Kołowrocki 2004).

5 RELIABILITY OF COMPLEX TECHNICAL SYSTEMS

Most real technical systems are structurally very complex and they often have complicated operation processes. The time dependent interactions between the systems’ operation processes operation states changing and the systems’ structures and their components reliability states changing processes are evident features of most real technical systems. The common reliability and operation analysis of these complex technical systems is of great value in the industrial practice. The convenient tools for analysing this problem are presented in (Kołowrocki, Soszyńska-Budny 2011) where the multistate system’s reliability modelling commonly used with the semi-Markov modelling of the systems operation processes, leads to the construction the joint general reliability models of the complex technical systems related to their operation process (Kołowrocki 2006, Kołowrocki 2007a,b, Kołowrocki, Soszyńska 2006, Kołowrocki, Soszyńska 2010a, Kołowrocki, Soszyńska 2011, Soszyńska 2004a,b, Soszyńska 2006, Soszyńska 2007, Soszyńska 2008). In the case of large complex technical systems, one of the important techniques is the asymptotic approach (Kołowrocki 2004, Kołowrocki 2008b, Soszńska 2004a,b, Soszyńska 2006a, Soszyńska 2007, Soszyńska 2008) to their reliability evaluation. .

5.1 RELIABILITY OF MULTISTATE SYSTEMS AT VARIABLE OPERATIONS CONDITIONS

We assume that the changes of the operation states of the system operation process have an influence on the system multistate components E_i, $i = 1,2,...,n,$ reliability and the system reliability structure as well. Consequently, we denote the system multistate component E_i, $i = 1,2,...,n,$ conditional lifetime in the reliability state subset $\{u, u+1,...,z\}$ while the system is at the operation state z_b, $b = 1,2,...,v,$ by $T_i^{(b)}(u)$ and its conditional reliability function by the vector

$$[R_i(t,\cdot)]^{(b)} = [1, [R_i(t,1)]^{(b)}, ..., [R_i(t,z)]^{(b)}],$$

with the coordinates defined by

$$[R_i(t,u)]^{(b)} = P(T_i^{(b)}(u) > t | Z(t) = z_b)$$

for $t \in < 0, \infty)$, $u = 1,2,...,z$, $b = 1,2,...,v$.

The reliability function $[R_i(t,u)]^{(b)}$ is the conditional probability that the component E_i lifetime $T_i^{(b)}(u)$ in the reliability state subset $\{u, u+1,...,z\}$ is greater than *t*, while the system operation process is at the operation state z_b.

Similarly, we denote the system conditional lifetime in the reliability state subset $\{u, u+1,...,z\}$ while the system is at the operation state z_b, $b = 1,2,...,v,$ by $T^{(b)}(u)$ and the conditional reliability function of the system by the vector

$$[\boldsymbol{R}(t,\cdot)]^{(b)} = [1, [\boldsymbol{R}(t,1)]^{(b)}, ..., [\boldsymbol{R}(t,z)]^{(b)}], \quad (5.1)$$

with the coordinates defined by

$$[\boldsymbol{R}(t,u)]^{(b)} = P(T^{(b)}(u) > t | Z(t) = z_b)$$

(5.2)

for $t \in < 0, \infty)$, $u = 1,2,...,z$, $b = 1,2,...,v$.

The reliability function $[\boldsymbol{R}(t,u)]^{(b)}$ is the conditional probability that the system lifetime $T^{(b)}(u)$ in the reliability state subset $\{u, u+1,...,z\}$ is greater than *t*, while the system operation process is at the operation state z_b.

Further, we denote the system unconditional lifetime in the reliability state subset $\{u, u+1,...,z\}$ by $T(u)$ and the unconditional reliability function of the system by the vector

$$\boldsymbol{R}(t,\cdot) = [1, \boldsymbol{R}(t,1), ..., \boldsymbol{R}(t,z)], \quad (5.3)$$

with the coordinates defined by

$$\boldsymbol{R}(t,u) = P(T(u) > t)$$

for $t \in < 0, \infty)$, $u = 1,2,...,z$.

In the case when the system operation time is large enough, the coordinates of the unconditional reliability function of the system defined by (5.3) are given by

$$\boldsymbol{R}(t,u) \cong \sum_{b=1}^{v} p_b [\boldsymbol{R}(t,u)]^{(b)} \text{ for } t \geq 0, \; u = 1,2,...,z,$$

where $[\boldsymbol{R}(t,u)]^{(b)}$, $u = 1,2,...,z$, $b = 1,2,...,\nu$, are the coordinates of the system conditional reliability functions defined by (5.1)-(5.2) and p_b, $b = 1,2,...,\nu$, are the system operation process limit transient probabilities given by (2.22) (Kołowrocki, Soszyńska-Budny 2011).

5.2 ASYMPTOTIC APPROACH TO RELIABILITY OF LARGE MULTISTATE SYSTEMS AT VARIABLE OERATION CONDITIONS

In the case of large complex systems, the possibility of combining the results of the reliability joint models of complex technical systems and the results concerning the limit reliability functions of the considered systems is possible (Kołowrocki 2004, Kołowrocki 2008b, Soszyńska 2004a,b, Soszyńska 2006a, Soszyńska 2008). This way, the results concerned with asymptotic approach to estimation of non-repairable multi-state systems at variable operation conditions may be obtained. Main results concerning asymptotic approach to multi-state large system reliability with ageing components in the constant operation conditions are comprehensively presented in the work (Kołowrocki 2004) and some of these results' extentions to the systems operating at the variable conditions can be found in (Soszyńska 2004a,b, Soszyńska 2006a, Soszyńska 2007, Soszyńska 2008).

In order to combine the results on the reliability of multi-state systems related to their operation processes and the results concerning the limit reliability functions of the multistate systems, and to obtain the results on the asymptotic approach to the evaluation of the large multi-state systems reliability at the variable operation conditions, we assume the following definition (Soszyńska 2007).

Definition 5.1

A reliability function

$$\boldsymbol{\mathfrak{R}}(t,\cdot) = [1, \boldsymbol{\mathfrak{R}}(t,1),..., \boldsymbol{\mathfrak{R}}(t,z)], \; t \in (-\infty,\infty),$$

where

$$\boldsymbol{\mathfrak{R}}(t,u) = \sum_{b=1}^{v} p_b [\boldsymbol{\mathfrak{R}}(t,u)]^{(b)}, \; u = 1,2,...,z,$$

is called a limit reliability function of a complex multistate system with the reliability function sequence

$$\boldsymbol{R}_n(t, \cdot) = [1, \boldsymbol{R}_n(t,1),..., \boldsymbol{R}_n(t,z)], \; t \in (-\infty,\infty), \quad n \in N,$$

where

$$\boldsymbol{R}_n(t,u) \cong \sum_{b=1}^{v} p_b [\boldsymbol{R}_n(t,u)]^{(b)}, \; u = 1,2,...,z,$$

if there exist normalizing constants

$$a_n^{(b)}(u) > 0,\ b_n^{(b)}(u) \in (-\infty, \infty),\ u = 1,2,...,z, b = 1,2,...,v,$$

such that

$$\lim_{n\to\infty}[\boldsymbol{R}_n(a_n^{(b)}(u)t + b_n^{(b)}(u), u)]^{(b)} = [\boldsymbol{\mathfrak{R}}(t,u)]^{(b)}$$

for all t from the sets of continuity points $C_{[\boldsymbol{\mathfrak{R}}(u)]^{(b)}}$ of the functions $[\boldsymbol{\mathfrak{R}}(t,u)]^{(b)}$, $u = 1,2,...,z$, $b = 1,2,...,v$.

Hence, for sufficiently large n, the following approximate formulae are valid

$$\boldsymbol{R}_n(t,\ \cdot) = [1, \boldsymbol{R}_n(t,1), ..., \boldsymbol{R}_n(t,z)],\ t \in (-\infty, \infty),$$

where

$$\boldsymbol{R}_n(t,u) \cong \sum_{b=1}^{v} p_b [\boldsymbol{\mathfrak{R}}(\frac{t - b_n^{(b)}(u)}{a_n^{(b)}(u)}, u)]^{(b)},\ t \in (-\infty, \infty), u = 1,2,...,z.$$

The following theorems concerned with the large complex series-parallel and parallel-series exponential systems operating at the variable operation states are exemplary results that can be worked out on the basis of the results included in (Kołowrocki 2004, Kołowrocki 2008b, Soszyńska 2004b, Soszyńska 2007) for the large systems.

Theorem 5.1

If components of the multistate series-parallel regular system at the operation states z_b, $b = 1,2,...,v$, i.e., the system with the structure shape parameters such that

$$k = k_n^{(b)},\ l_1 = l_2 = ... = l_k = l_n^{(b)},\ b = 1,2,...,v,\ n \in N,$$

have the exponential reliability functions given by (3.15)-(3.16) in (Kolowrocki, Soszynska-Budny, 2011) are homogeneous, i.e.,

$$[\lambda_{ij}(u)]^{(b)} = [\lambda(u)]^{(b)},\ i = 1,2,...,k_n^{(b)},\ j = 1,2,...,l_n^{(b)},\ b = 1,2,...,v,$$

then the system unconditional multistate reliability function is given by the approximate formulae, respectively in the following cases of the system structure shape at the particular operation states:

i) $k_n^{(b)} = n,\ l_n^{(b)} > 0,$

$$\boldsymbol{R}(t,\cdot) = [1, \boldsymbol{R}(t,1), ..., \boldsymbol{R}(t,z)]$$

where

$$\boldsymbol{R}(t,u) \cong 1 - \sum_{b=1}^{v} p_b \exp[-n\exp[-[\lambda(u)]^{(b)} l_n^{(b)} t]] \text{ for } t \in (-\infty,\infty),\ u = 1,2,...,z;$$

ii) $k_n^{(b)} \to k^{(b)},\ l_n^{(b)} \to \infty,$

$$\boldsymbol{R}(t,\cdot) = [1, \boldsymbol{R}(t,1), ..., \boldsymbol{R}(t,z)]$$

where

$$\boldsymbol{R}(t,u) \cong \begin{cases} 1 & \text{for } t < 0, \\ 1 - \sum\limits_{b=1}^{v} p_b [1 - \exp[-[\lambda(u)]^{(b)} l_n^{(b)} t]]^{k^{(b)}} & \text{for } t \ge 0, \end{cases} \quad u = 1,2,\ldots,z.$$

Theorem 5.2
If components of the multistate parallel-series regular system at the operation states z_b, $b = 1,2,\ldots,v$, i.e., the system with the structure shape parameters such that

$$k = k_n^{(b)}, \ l_1 = l_2 = \ldots = l_k = l_n^{(b)}, \ b = 1,2,\ldots,v, \ n \in N,$$

have the exponential reliability functions given by (3.15)-(3.16) in (Kolowrocki, Soszynska-Budny, 2011) are homogeneous, i.e.,

$$[\lambda_{ij}(u)]^{(b)} = [\lambda(u)]^{(b)}, \ i = 1,2,\ldots,k_n^{(b)}, \ j = 1,2,\ldots,l_n^{(b)}, \ b = 1,2,\ldots,v,$$

then the system unconditional multi-state reliability function is given by the approximate formulae, respectively in the following cases of the system structure shapes at the particular operation states:

i) $k_n^{(b)} = n, \ l_n^{(b)} \to l^{(b)}, \ l^{(b)} > 0,$

$$\boldsymbol{R}(t,\cdot) = [1, \boldsymbol{R}(t,1), \ldots, \boldsymbol{R}(t,z)]$$

where

$$\boldsymbol{R}(t,u) \cong \begin{cases} 1 & \text{for } t < 0, \\ \sum\limits_{b=1}^{v} p_b \exp[-n([\lambda(u)]^{(b)} t)^{l^{(b)}}] & \text{for } t \ge 0, \end{cases} \quad u = 1,2,\ldots,z.$$

ii) $k_n^{(b)} \to k^{(b)}, \ l_n^{(b)} \to \infty,$

$$\boldsymbol{R}(t,\cdot) = [1, \boldsymbol{R}(t,1), \ldots, \boldsymbol{R}(t,z)]$$

where

$$\boldsymbol{R}(t,u) \cong \sum_{b=1}^{v} p_b [1 - \exp[-l_n^{(b)} \exp[-[\lambda(u)]^{(b)} t]]]^{k^{(b)}} \ \text{for } t \in (-\infty,\infty), u = 1,2,\ldots,z.$$

It is possible to obtain similar and more general results for other complex multistate systems after some modification of the results included in (Kołowrocki 2004, Kołowrocki 2008b).

6 SUMMARY

In the paper, the asymptotic approach to the reliability evaluation of homogeneous and non-homogeneous series and parallel systems, homogeneous "*m* out of *n*" systems and homogeneous and non-homogeneous regular series-parallel and parallel-series systems has been presented. For these systems, in the case where their components are two-state as well in the case where they are multi-state, the classes of limit reliability functions can be fixed. Moreover, the auxiliary theorems useful for finding limit reliability functions of real technical systems composed of components

having any reliability functions can be formulated and motivated. The series-parallel and parallel-series systems have been considered in the case where their reliability structures are regular. However, this fact does not restrict the completeness of the performed analysis, since by conventional joining of a suitable number of failed components in parallel subsystems of the non-regular parallel-series systems we get the regular non-homogeneous parallel-series systems considered in the book. Similarly, conventional joining of a suitable number of components which do not fail, in series sub-systems of the non-regular series-parallel systems, leads us to the regular non-homogeneous series-parallel systems considered in the book. Thus the problem has been analysed exhaustively.
The results presented in the paper have become the basis of investigations on domains of attraction of system limit reliability functions and initiated the problem of the speed at which system reliability function sequences reach their limit reliability functions (Kołowrocki 2004).
Additionally, the results presented in the paper have initiated and become the basis for the investigations on limit reliability functions of practically important large series-"*m* out of *n*" and "*m* out of *n*"-series systems and hierarchical systems have been recently significantly developed (Kołowrocki 2004, Kołowrocki, Soszyńska 2007, Sun et al 2011). Some further consequences of these results are also given in (Kołowroci, Soszyńska–Budny 2011), where the comprehensive approach to the analysis, identification, evaluation, prediction and optimization of the complex technical systems operation, reliability, availability and safety is presented. Those all tools are useful in reliability, availability and safety optimization and operation cost analysis of a very wide class of real technical systems operating at the varying conditions that have an influence on changing their reliability and safety structures and their components reliability and safety characteristics.

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JOINT IMPORTANCE MEASURES IN NETWORK SYSTEM

V. M. Chacko [1] **and M. Manoharan** [2]

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[1]Department of Statistics, St.Thomas College, Thrissur-680001,Kerala, India
[1]Department of Statistics, University of Calicut, Kerala, India-673635
e-mail: chackovm@gmail.com, [1]

Abstract: Many real world systems (electric power, transportation, telecommunication, etc) are multistate systems composed of multistate components in which system reliability can be computed in terms reliabilities of its components. Such systems may be regarded as flow networks whose arcs (components) have independent, discrete, and multi-valued random capacities. An arc can, at different conditions, be characterized by different performance levels, causing network system to work with different levels of output performance. The criticality of such arcs must be measured with reference to their performance level and reliability, and its contribution to the overall system output performance measure(OPM). In this paper, we introduce a generalized concept of importance measures and joint importance measures for the flow network made up of multistate arcs with respect to output performance measures (expected performance, reliability and availability). An approach based on the universal generating function (UGF) for the evaluation of the proposed joint importance measures is introduced. An illustrative example is given.

Keywords: Network reliability, availability, discrete state arc, joint importance measure, UGF.
MSC 2000. 68M10, 90B25, 90B15.

1. Introduction

Since the very early times of reliability engineering, the network reliability is one of the main subjects of research. The network reliability theory has been applied extensively in many real-world systems such as computer and communication systems, power transmission and distribution systems, transportation systems, oil/gas production systems etc [8]. Network reliability evaluation approaches exploit a variety of tools for system modeling and reliability index calculation. Network reliability problems are generally classified based on the method used to transfer the flow (or signal) and how the flow conservation law is satisfied. Typically there are two categories; the multistate arc network (MAN) and the multistate node network (MNN). In MAN, each arc has a non-negative integer valued discrete random variable capacity (multistate arc) and all flows in the network obey the conservation law. Apparently in MNN, each node is a multistate node with discrete states determined by a set of nodes receiving the signal directly from it without satisfying conservation law. Both have their own applications; for example electrical power distribution system can be modeled by MAN, and computer networks or cellular phone networks can be modeled as MNN.

The standard mathematical and statistical theory of system reliability assumes both system and component behavior are of binary nature, functioning (state 1) and failed (state 0), [1]. However, in some systems, when components may be operating in a degraded state, the system may be operating in degraded state, and the system may still provide an acceptable level of service, [2]. The network reliability evaluation for complex designs relies on enumerative techniques, [12]. The flow reliability problem for the directed capacited-flow network in which the capacity of each arc has $M+1$ value from source to sink is generalized as a multistate system model, [10]. A graph theoretic method is used for the reliability evaluation of multistage interconnection networks with multistate elements, [14].

Importance measure (IM) quantifies the criticality of a particular component within a system design. They have been widely used as tools for identifying system weakness, and to prioritize reliability improvement activities, [6]. They can also provide valuable information for the safety and efficient operation of the system. In multistate system (MSS), IMs characterizes, for a given component, the most important component state with regard to its impact on system reliability. The knowledge about the IM can be used as a guide to provide redundancy so that system reliability is increased. Thus, measures that can differentiate such an impact are highly desirable. In general, there are two ways to improve the reliability of a binary system, 1) increase the reliability of individual components, and/or 2) add redundant components to the system. Composite importance measures are developed with the aim of identifying and ranking particular arcs (components) in a network system depending on their impact on the multistate network reliability behavior, [13]. Joint reliability importance (*JRI*) of two or more components is a quantitative measure of the interactions of two or more components or states of two or more components, [5]. It is investigated to provide information on the type and degree of interactions between two or more components by identifying the sign and size of it, [5,15]. The value of *JRI* represents the degree of interactions between two or more components with respect to system reliability. Joint structural importance (*JSI*) is used when the component reliabilities are not available, [15]. Joint structural and joint reliability importance measures for any number of multistate components in the MSS are useful for the design engineers, [5], [17]. For the MSS with multistate components, the problem related to MSS reliability improvement is still evolving. The problem of finding the joint importance of more than two arcs in a network system with various output performance measures (e.g. reliability, availability, etc) still remains unsolved. To solve this problem, methods dependent on the information obtained from multistate IMs and joint importance measure(JIM)s can be developed for efficient resource allocation. Many of the engineering systems are modeled by networks (electric power generation system, transportation system, telecommunication network system, etc) (see [7], [10], and [14]), hence the development of joint importance measures of two or more arcs with different output performances (e.g. productivity, capacity, etc) in a directed network with multistate performance levels is quite desirable. In order to answer this problem, we introduce the JIMs of two or more arcs in multistate directed network system with various output performance measures (expected performance, reliability and availability). We provide an algorithm based on universal generating function (UGF) for the evaluation of joint importance measure when network system has different output performances.

This paper is organized as follows. In section 2, we define the JIMs in network system with various output performance measures (expected performance reliability, and availability). Section 3 considers the application of UGF for the JIM evaluation. Illustration is given in section 4 followed by conclusions in the last section.

2. Joint importance measures of arcs in multistate network system

Consider a directed multistate network made up of *n* arcs. Each arc *i* may be in one of M_i+1 states, $\{0,1,...,M_i\}$, $i \in \{1,2,...,n\}$. Let $W(t)$ output performance of the multistate network at time *t* which takes the values w_i, i=0, 1, 2, … M, where $M = \max_i\{M_i\}$, depending on the system state *i* at time *t*. The two vectors of the system performance realizations, w={w_i, $0 \le i \le M$}, and of the system state probabilities, p={p_i, $0 \le i \le M$}, define the system output performance distribution. Let $\phi(t)$ is the state of the MSS at time t.

We use some measures of the performance of a MSS for obtaining joint importance measures.

The steady-state the probability distribution of the system states is:

$$p_i = \lim_{t\to\infty} \Pr\{\phi(t) = i\} = \lim_{t\to\infty} \Pr\{W(t) = w_i\}, \quad 0 \le i \le M .$$

An associated simple measure of system output performance is its expected value of system state defined, in the steady-state, as:

$$E_s[\phi(t)] = \sum_i ip_i \ .$$

A similar measure of system output performance is its expected value of system output performance, in the steady-state, as:

$$E = \sum_i w_i p_i \ .$$

When applied to MSS, the concept of availability is related to the ability of the system to meet a required demand w_k, corresponding to state k. The general definition of instantaneous multistate system availability is, then:

$$A(t) = \Pr\{\phi(t) \geq k\} = \Pr\{W(t) \geq w_k\} \ .$$

If the system is under operation without break up to time t, then $A(t)$ is the system reliability:

$$R(t) = \Pr\{\phi(t) \geq k\} = \Pr\{W(t) \geq w_k\} \ .$$

The MSS stationary availability is defined as

$$A = \sum_{i=0}^{M} p_i \, 1(\mathrm{w_i} - w_k) \ .$$

Let $G = (N, A)$ represent a stochastic capacited network with known demand d from a specified source node s to a specified sink node t. N represents the set of all nodes and $A = \{a_i \mid 1 \leq i \leq n\}$ represents the set of all arcs. The current state (capacity) of arc a_i, represented by $x_i \in \{0,1,...,M_i\}$, the range of states of arc a_i. The vector $p_i = (p_{i0}, p_{i1}, p_{i2},..., p_{iM_i})$ represents the probability associated to each of the values taken by x_i. The system state vector $x = (x_1, x_2,..., x_n)$ denotes the state of all the arcs of the network system. Function $\phi(x): Z^n \to Z$, where $Z = \{0,1,2,...,M\}$, $M = \max_i\{M_i\}$, maps the system state vector into system state. That is, $\phi(x)$, is the network capacity from source to sink under system state vector x, which represents a multistate structure function, [2]. Network reliability may be defined as the probability that a demand of d units can be supplied from source to sink through the multistate arcs. We shall make the following assumptions for the network reliability system.

1. Arc states are stochastically independent.
2. The structure function $\phi(x)$ is statistically coherent. That is, improving an arc performance cannot cause to degrade the performance of the network system and all arcs are relevant.

Joint reliability importance (*JRI*) of the two edges in an undirected network in binary nature is an extension of the marginal reliability importance (*MRI*) of edges, [6]. In an undirected network, reliability is the probability that source and terminal are connected by working edges, [6].

For an undirected stochastic network $G(N,E)$, where $E = \{e_i \mid 1 \leq i \leq n\}$ is set of all edges, N, the set of nodes, let $R(G)$ represents the probability that the source and terminal are connected by working edges and $q = (q_1, q_2,..., q_n)$ where $q_i = Pr\{e_i \in E \text{ is working}\}$. *MRI* of edge e_i in an undirected network is defined as $I_G(i) = \dfrac{\partial R(G)}{\partial q_i}$, [6]. Again *JRI* of two edges is defined as follows.

Definition 1. The *JRI* of two edges e_i and e_j is the second order partial derivative of reliability of an undirected network with respect to reliabilities of both edges:

$$I_G(i,j) = \frac{\partial^2 R(G)}{\partial q_i \partial q_j}.$$

An explicit expression for this *JRI* of two edges is

$$I_G(i,j) = R(G^* i^* j) - R(G^* i - j) - R(G^* j - i) + R(G - i - j)$$

where $G^* i - j$ represents G with edge e_i contracted and edge e_j is deleted. The *JRI* is expressed in terms of *MRI* of edges in same sub-graphs as,

$$I_G(i,j) = I_{G^*j}(i) - I_{G-j}(i) \text{ and } I_G(i,j) = I_{G^*i}(j) - I_{G-i}(j).$$

Alternatively the following relationships are obtained.

$$I_G(i,j) = \frac{I_G(i) - I_{G-j}(i)}{p_j} \text{ and } I_G(i,j) = \frac{I_G(j) - I_{G-i}(j)}{p_i}.$$

We now proceed with the problem of measuring joint importance in the directed network system with respect to expected performance. First we find the *JRI* of any number of arcs in the network where the arc capacity is represented as finite discrete state in nature. That is each arc can take the value in a discrete state space $\{0,1,...,M_i\}$ where M_i represents the maximum flow (best state) through the arc *i*. For finding the *JRI* of more than two arcs, we follow the method for finding *JRI* in MSS, [5]. Suppose for instance the probability distribution of each arc is unknown, then we use the joint structural importance of multistate system (*JSIM*) of more than two components, [5]. The *JSIM* (*i*, *j*), for two components *i* and *j* is given by,

$$JSIM(i,l) = \sum_{m=1}^{M_i} \sum_{k=1}^{M_l} \{ SIM(i,l;m,k) - SIM(i,l;m,\hat{k}) \}$$

where $SIM(i,l;m,k) = \dfrac{\sum_{\vec{X}_{il}} \sum_{q=1}^{j} \chi(\phi(m_i, k_l, \vec{X}_{il}) = j, \phi(\hat{m}_i, k_l, \vec{X}_{il}) = j - q)}{(M+1)^{n-2}}$, $\hat{m} = m - 1$, and

$\overline{X}_{il} = (x_1, x_2, ..., m_i, ..., k_l, ..., x_n)$, the state space vector of system components.

Here χ(true)=1 and χ(false)=0, $\phi(m_i, k_l, \overline{X}_{il}) = j, \phi(\hat{m}_i, k_l, \overline{X}_{il}) = j - q$ determines the critical path vector to the level j with state *m* of component *i*. The *JSIM* (*i,j,k*) for three components is

$$JSIM(i,l,r) = \sum_{k=1}^{M_i} \sum_{n=1}^{M_l} \sum_{m=1}^{M_r} \{ JSIM(i,l,r;m,k,n) - JSIM(i,l,r;m,k,\hat{n}) \},$$

where $JSIM(i,l,r;m,k,n) = SIM(i,l,r;m,k,n) - SIM(i,l,r;m,\hat{k},n)$. So in order to find the *JSIM* of three arcs we have to find *JSIM* of two arcs for each state of third arc and, take successive difference and total sum. Again the change in *JSIM* of three components with fourth component provides *JSIM* of four components. Thus proceeding like this we can find *JSIM* of any number of arcs.

Suppose that the arc probabilities are known. Then to find the joint reliability importance of more than two multistate arcs for the network, one may proceed as follows. The joint reliability importance (*JRIM*) of MSS, for *k* components is defined as follows, [5]. The joint reliability importance of state b_1 of component a_1, state b_2 of component a_2,..., state b_k of the component a_k $(k \leq n)$ of the MSS is

$$JRIM(a_1,...,a_k;b_1,...,b_k)=\frac{\partial^k E_s}{\partial R_{a_1}b_1\partial R_{a_2}b_2...\partial R_{a_k}b_k},\ k=2,...,n,$$

where $E_s=\sum_{j=0}^{M}P(\phi(x)\geq j)$ is the expected performance of system and $\forall i\in\{1,2,...,k\}$, $R_{a_i}b_i=P(x_{a_i}\geq b_i)$ are the component reliabilities. Thus in finding the *JRIM* of *k* multistate arcs, we have to find the k^{th} partial derivative of the overall expected performance of network with respect to the reliabilities of each arc under consideration. For instance we consider the *JRIM* of states of three components. Let $P_{mkn}=P(\phi(m_i,k_l,n_r,\overline{X}_{ilr})\geq j)$ and $p_{im}=P(X_i\geq m)$. For *k*=3, i.e. differentiating E_s partially with respect to p_{im}, p_{lk} and p_{rn}, we get,

$$\frac{\partial^3 E_s}{\partial p_{im}\partial p_{lk}\partial p_{rn}}=[\,P_{mkn}-P_{m\bar{k}n}-P_{\bar{m}kn}+P_{\bar{m}\bar{k}n}\,]-[\,P_{mk\bar{n}}-P_{m\bar{k}\bar{n}}-P_{\bar{m}k\bar{n}}+P_{\bar{m}\bar{k}\bar{n}}\,].$$

But observe that $P_{mkn}-P_{m\bar{k}n}-P_{\bar{m}kn}+P_{\bar{m}\bar{k}n}$ is *JRIM* of states of two components when third component is in state *n*. Therefore *JRIM* of states three components are expressed in terms of *JRIM* of states of two components as follows:

$$\frac{\partial E_s^{\,3}}{\partial R_i m\,\partial R_l k\partial R_r n}=[\,\frac{\partial^2 E_s}{\partial R_i m\,\partial R_l k}\,]_{n_r}-[\,\frac{\partial^2 E_s}{\partial R_i m\,\partial R_l k}\,]_{\bar{n}_r}$$

JRI working states for edges can be written as $\frac{\partial^2 R(G)}{\partial R_i 1\partial R_l 1}$ where $R_i 1=p_i$. It shows that the above result holds with binary nature of edges*, i. e., M*=1. Hence, the results of *JRI* of two edges in a binary network, [5], can be considered as a generalization of the results *JRIM*, [6], to any number of binary and multistate edges when considering undirected network system. Thus we have the following theorem for three arcs of a directed network system.

Theorem 1. The joint reliability importance of three arcs in a multistate network with multistate arcs is

$$\sum_{m=1}^{M_i}\sum_{k=1}^{M_l}\sum_{n=1}^{M_r}\frac{\partial E_s^{\,3}}{\partial R_i m\,\partial R_l k\partial R_r n}=\sum_{m=1}^{M_i}\sum_{k=1}^{M_l}\sum_{n=1}^{M_r}\left([\,\frac{\partial^2 E_s}{\partial R_i m\,\partial R_l k}\,]_{n_r}-[\,\frac{\partial^2 E_s}{\partial R_i m\,\partial R_l k}\,]_{\bar{n}_r}\right)$$

where E_s is the expected output performance of network, $R_i m$, $R_l k$, *and* $R_r n$ are the reliabilities of arcs *i, l,* and *r* with respect to performance level *m, k,* and *n* respectively.□

In the above discussed joint reliability importance measures and joint structural importance measures, we used the expected performance of the network as output performance measure. But in order to find the JIMs with respect to other output performance measures, reliability and availability, of the multistate network systems, we proceed as follows.

When the generic j-th multistate arc is considered, one can introduce a performance threshold α and divide the complete ordered set of its states into two ordered subsets corresponding to the arc performance above and below the level α, respectively. By so doing, we re-introduce a collectively binary logic for the arc's states. Let arc *j* be constrained to performance below α, while the rest of arcs of the network system are not constrained: we denote by $OS_j^{\leq\alpha}$ the network system OPM (reliability or availability) obtained in this situation. Similarly, we denote by $OS_j^{>\alpha}$ the network system OPM resulting from the dual situation in which arc j is constrained to performances above α. The network system performance measures so introduced rely on a restriction of the achievable performance of the network arcs. Different modeling assumptions in the enforcement of this restriction will lead to different performance values. Using the measures $OS_j^{\le\alpha}$ and $OS_j^{>\alpha}$, we can define Birnubaum importance measures for multistate elements

$$OS^{\alpha}{}_j = OS^{>\alpha}{}_j - OS^{\le\alpha}{}_j .$$

Suppose that $OS^{>\beta,\alpha}{}_{i,j}$ represents the Birnubaum importance of the component *i* when component *j* is restricted to the performance above level β. Similarly define $OS^{\le\beta,\alpha}{}_{i,j}$ the Birnubaum importance of the component *i* when component *j* is restricted to below level β. Thus we can define the joint importance of two components *i* and *j* to the network system performance as

$$OS^{\alpha,\beta}{}_{i,j} = OS^{\ge\beta,\alpha}{}_{i,j} - OS^{\le\beta,\alpha}{}_{i,j} .$$

Similarly we can obtain the higher order joint importance measures for more than two arcs, i.e., for example, to measure the improvement of joint importance of two arcs with respect to the interactive effect of more than two arcs, at first we shall calculate change in the joint importance of two arcs with respect to the change of third arc. If there is any change in the joint importance of two arcs due to change in performance of third arc from upper states to lower states we can say that there is an interactive effect for three arcs for the network OPM improvement. We shall find the joint importance measures at steady state system performance in the following section.

3. Application of UGF

In MSS modeled by networks with respect to various output performances, we modify the above joint importance measures. The UGF is found to be a useful tool in assessment of output performance measures of the network systems, [11]. The method of UGF generalizes the technique that is based on using a well known ordinary generating function. The basic ideas are introduced by Ushakov in 1987,. [11]. The approach proved to be very convenient for numerical realization. It requires relatively small computational resources for evaluating MSS reliability indices and, therefore, can be used in complex reliability operations. Importance measure evaluation in MSSs using UGF can be see in Ref. [9].

The MSS model includes the performance distribution of all arcs and the system structure function: $x_j,\ p_j, 1 \le j \le n,\quad \phi(x_1, x_2, ..., x_n)$, where any system element *j* can have finite number, M_j of discrete states, and its performance distribution is represented by ordered sets $x_j : (x_{j1}, x_{j2}, ..., x_{jM_j})$ and $p_j : (p_{j1}, p_{j2}, ..., p_{jM_j})$ that relate the probability of each state with performance corresponding to this state.

The UGF of a discrete variable *X* corresponding to the state of an arc is defined as the polynomial

$$U(Z) = \sum_{k=0}^{M} p_k Z^{x_k}$$

where the discrete random variable *X* has *M* possible values and p_k is the probability that *X* is equal to x_k. In order to represent all the possible combinations of states of the two arcs a_1 and a_2, one has to relate the corresponding probabilities of states of two multistate arcs subsystem with values of the vector $\psi(x_{a_1}, x_{a_2})$ in these states. For these purpose, we consider a composition operator Ω over UGFs of individual multistate arcs which takes the following form for a pair (i, j) of multistate arcs.

$$U_{i,i+1}(Z)=\Omega(U_i(Z),U_{i+1}(Z))=\Omega(\sum_{k=1}^{M_i}p_{ik}Z^{x_{ik}},\sum_{n=1}^{M_{i+1}}p_{i+1,n}Z^{x_{i+1n}})=\sum_{k=1}^{M_i}\sum_{n=1}^{M_{i+1}}p_{ik}p_{i+1n}Z^{\psi(x_{ik},x_{i+1n})}$$

The resulting polynomial $U_{i,i+1}(Z)$ represents the probability distribution of the subsystem containing arcs i and i+1. Applying the operator Ω to all other arcs one by one we get the resulting polynomial that takes the form,

$$U_{1,2,\ldots,n}(Z)=\sum_{k=0}^{M}q_kZ^{y_k}$$

The polynomial represents distribution of state of connections between source and sink of the entire network. This polynomial relates the probabilities of all the possible states of whole network, q_k, with the output performance corresponding to these states, y_k. Thus we can obtain the reliability of network as

$$R=\sum_{k=0}^{M}q_k.I(demand\ of\ d\ units\ sup\ plied\ from\ s\ to\ t)$$

where $I(.)$ is the indicator function. To evaluate the joint importance measures we need the steady state distribution of the observed performance of the network system under some constraints. In order to use the UGF in joint importance measure evaluation, we use the following approach.

Let O_{jk} be the output performance of multistate network system when arc j is in fixed state k while the rest of the arcs evolve stochastically among their corresponding states with steady-state performance distributions $\{x_{il,}p_{il}\}$, $1\le i\le n, 0\le l\le M_i$. Assume that the arc j is in one of its states, k, with performance not greater than α. We denote by $k_{j\alpha}$ the state in the ordered set of states of arc j whose performance $x_{jk_{j\alpha}}$ is equal or immediately below α, i.e., $x_{jk_{j\alpha}}\le\alpha<x_{jk_{j\alpha}+1}$. The conditional probability of the arc j being in a generic state k characterized by a performance $X_j=x_{jk}$ not greater than a pre-specified level threshold α $(k\le k_{j\alpha})$ is:

$$\Pr[X_j=x_{jk}\mid k\le k_{j\alpha}]=p_1{}^*{}_{jk}=\frac{p_{jk}}{\sum_{r=0}^{k_{j\alpha}}p_{jr}}=\frac{p_{jk}}{p^{\le\alpha}{}_j}.$$

Similarly, the conditional probability of arc j being in a state k when it is known that $k>k_{j\alpha}$ is

$$Pr[X_j=x_{jk}\mid k>k_{j\alpha}]=p_2{}^*{}_{jk}=\frac{p_{jk}}{Pr[k>k_{j\alpha}]}=\frac{p_{jk}}{\sum_{r=k_{j\alpha}+1}^{M_j}p_{jr}}=\frac{p_{jk}}{p^{>\alpha}{}_j}.$$

Now consider the joint probability distribution of two arcs i and j, for $X_i=x_{ik},X_j=x_{jh}$ given four additional restrictions, (1) $k>k_{i\alpha},h>h_{j\beta}$, (2) $k\le k_{i\alpha},h>h_{j\beta}$, (3) $k>k_{i\alpha},h\le h_{j\beta}$ and (4) $k\le k_{i\alpha},h\le h_{j\beta}$. Thus now under the consideration that the arcs are independent we could arrive at probability distributions given below. That can be computed as earlier result for independent arcs as follows. Let

$$Pr[X_{ik}=x_{ik},X_j=x_{jh}\mid k\le k_{i\alpha},h\le h_{j\beta}]=p_1{}^{**}{}_{hk}=\frac{p_{ik}p_{jh}}{\sum_{r=0}^{k_{i\alpha}}p_{ir}\sum_{m=0}^{h_{j\beta}}p_{jm}},$$

$$Pr[\,X_i = x_{ik}, X_j = x_{jh} \mid k \le k_{i\alpha}, h > h_{j\beta}\,] = p_2{}^{**}{}_{hk} = \frac{p_{ik}\, p_{jh}}{\sum_{r=0}^{k_{i\alpha}} p_{ir} \sum_{m=h_{j\beta}+1}^{M_j} p_{jm}},$$

$$Pr[\,X_i = x_{ik}, X_j = x_{jh} \mid k > k_{i\alpha}, h \le h_{j\beta}\,] = p_3{}^{**}{}_{hk} = \frac{p_{ik}\, p_{jh}}{\sum_{r=k_{i\alpha}+1}^{M_i} p_{ir} \sum_{m=0}^{h_{j\beta j}} p_{jm}},$$

and

$$Pr[\,X_i = x_{ik}, X_j = x_{jh} \mid k > k_{i\alpha}, h > h_{j\beta}\,] = p_4{}^{**}{}_{hk} = \frac{p_{ik}\, p_{jh}}{\sum_{r=k_{i\alpha}+1}^{M_i} p_{ir} \sum_{m=h_{j\beta}+1}^{M_j} p_{jm}}.$$

Similarly we can find joint distributions of any number of arcs with the specified restrictions. We define $OS^{\le\alpha}{}_j$ as the network output performance measure (e.g. reliability or availability) obtained when arc j is forced to visit only states with performance not greater than α:

$$OS^{\le\alpha}{}_j = \sum_{k=0}^{k_{j\alpha}} \frac{p_{jk}}{p^{\le\alpha}{}_j} O_{jk}\ .$$

Similarly, we define as $OS^{>\alpha}{}_j$ the network output performance measure obtained under the condition that the arc j stays only in states with performance greater than α:

$$OS^{>\alpha}{}_j = \sum_{k=k_{j\alpha}+1}^{M_j} \frac{p_{jk}}{p^{>\alpha}{}_j} O_{jk}\ .$$

Thus the Birnubaum importance takes the form,

$$OS^{\alpha}{}_j = OS^{>\alpha}{}_j - OS^{\le\alpha}{}_j\ .$$

In order to compute the joint importance of two arcs i and j, i.e., let $OS^{\alpha,\beta}{}_{ij}$ denote the joint importance of two arcs with respect to the network output performance measure OS, then

$$OS^{\alpha,\beta}{}_{ij} = OS^{>\beta,\alpha}{}_{ij} - OS^{\le\beta,\alpha}{}_{ij}$$

i.e.,

$$OS^{\alpha,\beta}{}_{ij} = \sum_{k=0}^{k_{i\alpha}} \sum_{h=0}^{h_{j\beta}} p_1{}^{**}{}_{hk} O_{ik,jh} - \sum_{k=0}^{k_{i\alpha}} \sum_{h=h_{j\beta}+1}^{M_j} p_2{}^{**}{}_{hk} O_{ik,jh} - \sum_{k=k_{i\alpha}+1}^{M_i} \sum_{h=0}^{h_{j\beta}} p_3{}^{**}{}_{hk} O_{ik,jh} + \sum_{k=k_{i\alpha}+1}^{M_i} \sum_{h=h_{j\beta}+1}^{M_j} p_4{}^{**}{}_{hk} O_{ik,jh}$$

Similarly by finding change in joint importance of two arcs with respect to third arc, we get the joint importance of three arcs. Continuing like this we get the joint importance of any number of arcs with respect to network output performance measure and state space restricted probabilities of all arcs.

In order to obtain the state space restricted measures, one has to modify the UGF of arcs as follows,

$$U^{\le}{}_i(Z)=\sum_{k=0}^{k_{i\alpha}}\frac{p_{ik}}{p^{\le\alpha}{}_i}Z^{x_{ik}},\qquad U^{>}{}_i(Z)=\sum_{k=k_{i\alpha}+1}^{M_i}\frac{p_{ik}}{p^{>\alpha}{}_i}Z^{x_{ik}},\qquad U^{\le}{}_j(Z)=\sum_{h=0}^{h_{j\beta}}\frac{p_{jk}}{p^{\le\beta}{}_j}Z^{x_{jk}},$$

$$U^{>}{}_j(Z)=\sum_{h=h_{j\beta}+1}^{M_j}\frac{p_{jk}}{p^{>\beta}{}_j}Z^{x_{jk}},\; U^{\le,\le}{}_{ij}(Z)=\sum_{k=0}^{k_{i\alpha}}\frac{p_{ik}}{p^{\le\alpha}{}_i}Z^{x_{ik}}\sum_{h=0}^{h_{j\beta}}\frac{p_{jh}}{p^{\le\beta}{}_j}Z^{x_{jh}}=\sum_{k=0}^{k_{i\alpha}}\sum_{h=0}^{h_{j\beta}}\frac{p_{ik}p_{jh}}{p^{\le\alpha}{}_ip^{\le\beta}{}_j}Z^{\psi(x_{ik},x_{jh})},$$

$$U^{>,\le}{}_{ij}(Z)=\sum_{k=k_{i\alpha}+1}^{M_i}\frac{p_{ik}}{p^{>\alpha}{}_i}Z^{x_{ik}}\sum_{h=0}^{h_{j\alpha}}\frac{p_{jh}}{p^{\le\beta}{}_j}Z^{x_{jh}}=\sum_{k=k_{i\alpha}+1}^{M_i}\sum_{h=0}^{h_{j\beta}}\frac{p_{ik}p_{jh}}{p^{>\alpha}{}_ip^{\le\beta}{}_j}Z^{\psi(x_{ik},x_{jh})},$$

$$U^{>,>}{}_{ij}(Z)=\sum_{k=k_{i\alpha}+1}^{M_i}\frac{p_{ik}}{p^{>\alpha}{}_i}Z^{x_{ik}}\sum_{h=h_{j\beta}+1}^{M_j}\frac{p_{jh}}{p^{>\beta}{}_j}Z^{x_{jh}}=\sum_{k=k_{i\alpha}+1}^{M_i}\sum_{h=h_{j\beta}+1}^{M_j}\frac{p_{ik}p_{jh}}{p^{>\alpha}{}_ip^{>\beta}{}_j}Z^{\psi(x_{ik},x_{jh})}$$

and $$U^{\le,>}{}_{ij}(Z)=\sum_{k=0}^{k_{i\alpha}}\frac{p_{ik}}{p^{\le\alpha}{}_i}Z^{x_{ik}}\sum_{h=h_{j\beta}+1}^{M_j}\frac{p_{jh}}{p^{>\beta}{}_j}Z^{x_{jh}}=\sum_{k=0}^{k_{i\alpha}}\sum_{h=h_{j\beta}+1}^{M_j}\frac{p_{ik}p_{jh}}{p^{\le\alpha}{}_ip^{>\beta}{}_j}Z^{\psi(x_{ik},x_{jh})},$$

when evaluating UGF of $OS^{\le\alpha}{}_i,\ OS^{>\alpha}{}_i,\ OS^{\le\beta}{}_j,\ OS^{>\beta}{}_j,\ OS^{\le\beta,\alpha}{}_{ij},\ and\ OS^{>\beta,\alpha}{}_{ij}$. We use the following algorithm for evaluation of $OS^{\le\alpha}{}_i, OS^{>\alpha}{}_i, OS^{\le\beta}{}_j, OS^{>\beta}{}_j, OS^{\le\beta,\alpha}{}_{ij}, and\ OS^{>\beta,\alpha}{}_{ij}$.

Obtain the u-functions of all of the system elements. If the system contains a pair of elements connected in parallel or in series, replace this pair with an equivalent macro-element with u-function obtained by 'sum' or 'min' operator for $\psi(\cdot)$. If the system contains more than one element, do it again and again. Then, determine the u-function of the entire series-parallel system as the u-function of the remaining single equivalent macro-element. The system probability and performance distributions are represented by the coefficients and exponents of this u-function, corresponding to the state probabilities and performance levels, respectively. Compute the system OPM for the given level with the given vectors of the state probabilities and performance levels.

4. Illustrative example

For the network in figure 1, it is desired to obtain the probability that a demand of 10 units can be supplied from source to sink, [13]. Here the system can be considered as the MSS in Ref. [5]. Table 1 presents arc probabilities. In table 2, we computed *JRIM* of pair of arcs with α=2, β=2 which influences system most with respect to system output met the demand or not.

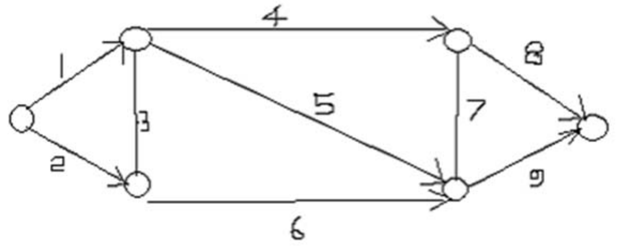

Fig1

It shows the pair (8,9) has largest *JRIM* with respect to system reliability.

4. Conclusion

In this paper joint importance measures of two or more arcs in multistate arc network with various output performance measures are developed. The procedure of evaluating joint importance measures using UGF is proposed. The proposed measures can be used in any systems modeled as multistate networks having various output performance measures with multistate arcs.

Table 1

Arc	State	State probabilities
1	0 3 4 8	0.005 0.005 0.01 0.98
2	0 3 4 6	0.02 0.01 0.015 0.955
3	0 3	0.02 0.98
4	0 3 4	0.01 0.015 0.975
5	0 3	0.02 0.98
6	0 3 6	0.005 0.02 0.975
7	0 3	0.01 0.99
8	0 3 4 6	0.01 0.015 0.005 0.97
9	0 3 4 8	0.02 0.01 0.01 0.96

Table 2

Pair	(1,2)	(8,9)	(1,4)	(2,6)
JRI	0.000394	0.089946	-0.894983	-0.01469

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A BAYESIAN ANALYSIS ON WEIBULL MODEL ALLOWING NEARLY INSTANTANEOUS FAILURES

K. Muralidharan[1], Rajiv Parikh[2] and C. D. Lai[3]

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[1] Department of Statistics, Faculty of Science,
The M. S. University of Baroda, Vadodara- 390 002, India.
Email: muralikustat@gmail.com

[2] Department of Statistics,
Bhavnagar University, Bhavnagar- 364 002, India.
Email: parikh.rajeev@gmail.com

[3]Institute of Fundamental Sciences,
Massey University, Palmerston North, New Zealand
c.lai@massey.ac.nz

Abstract: In this article, we study, the maximum likelihood as well as Bayes estimation on parameters of Mixture of Weibull with 'nearly instantaneous failure' as introduced in Lai et.al. (2007). For Maximum likelihood estimation, the EM algorithm is used. For Bayes estimation of parameters, we used three different algorithms namely, Population Monte Carlo method (PMC), Mixture version of Metropolis-Hasting and Gibbs sampler. The methods are compared using a simulation study. A numerical example is also discussed at the end of the paper.

Key Words: Maximum Likelihood, Expectation-Maximization, Metropolis-Hasting, Gibbs, Population Monte Carlo method, Bayes Estimation, Mixture of Non-Degenerate and Weibull Model.

1. Introduction

The mixed failure time distribution arises frequently in many different contexts in statistical literature. For instance, when we put units in a life testing experiment, then some of the units fail instantaneously and thereafter the life time of units follow a distribution such as exponential, Weibull, gamma etc. Such situations may be represented as a mixture of singular distribution at zero and a positive continuous random variable distribution. Lai et. al. (2007) has proposed a model as a mixture of generalized Dirac delta function and the 2-parameter Weibull having a closed form expression for survival function and hazard rate. The density of such a mixture can be shown as

$$f(x)=p\delta_d(x-x_0)+q\,\alpha\,\lambda^{\alpha}\,x^{\alpha-1}\,e^{-(\lambda x)^{\alpha}},\quad p+q=1, q=1-p, 0<p<1 \qquad (1)$$

Where

$$\delta_d(x-x_0)=\begin{cases}\dfrac{1}{d} & , x_0\le x\le x_0+d\\ 0, & otherwise\end{cases}$$

For sufficiently small *d*. Here *p* > 0 is the mixing proportion. We note that

$$\delta(x-x_0)=\lim_{d\to 0}\delta_d(x-x_0),$$

where δ(.) is the Dirac delta function that is well known in mathematical analysis. One can view the Dirac delta function as a normal distribution having a zero mean and standard deviation that tends to 0. For a fixed value of *d*, the distribution becomes a mixture of a Weibull with Uniform distribution.

As a special case, the model presented in (1) for $x_0 = 0$ becomes a mixture of Weibull with a uniform distribution (See Muralidharan and Lathika, 2007). The pdf of mixture of Weibull with 'nearly instantaneous failure' occurring uniformly over [0, *d*] can be given as

$$f(x)=\frac{p}{d}+(1-p)\alpha\lambda^{\alpha}x^{\alpha-1}e^{-(\lambda x)^{\alpha}},0<p<1 \tag{2}$$

Taking $f_1 \sim U(x_0, x_0+d)$ and $f_2 \sim$ Weibull (α,λ), then (2) can be written as

$$f(x) = p f_1(x_i \mid d) + (1-p) f_2(x_i \mid \alpha,\lambda) \tag{3}$$

If $X_1, X_2, \ldots, X_n$ are the random sample of size n from (3), then the likelihood function is given by

$$L(p,\alpha,\lambda,d \mid \underline{x}) = \prod_{i=1}^{n} f(x_i)$$

Introducing Latent variable Z_i in the likelihood function, we have

$$L(p,\alpha,\lambda,d \mid \underline{x},\underline{z})=\prod_{i=1}^{n}\{p f_1(x_i \mid d)\}^{z_i}\{(1-p)f_2(x_i \mid \alpha,\lambda\}^{1-z_i} \tag{4}$$

where

$$P(z_i=1)=p f_1(x_i \mid d)\ , \quad P(z_i=0)=(1-p) f_2(x_i \mid d)$$

After some simplification, the Log-likelihood can be written as

$$\ln(L(p,\alpha,\lambda,d \mid \underline{x},\underline{z}))=\sum_{i=1}^{n} z_i \ln\left(\left[\frac{p}{1-p}\right]\left[\frac{f_1(x_i \mid d)}{f_2(x_i \mid \alpha,\lambda)}\right]\right)+\ln((1-p)f_2(x_i \mid \alpha,\lambda)) \tag{5}$$

For further development in the study, we make use of the likelihood given in (5). One may refer to Lai et.al. (2007) paper for various characteristics of the model and the parametric estimation of the model.

Here, we compute the Maximum likelihood estimator using EM algorithm (see section-2), while in section-3, the Bayes estimators of the parameters are calculated. The posterior samples have been generated using three different approaches namely, Population Monte Carlo, Mixture version of Metropolis-Hasting and Gibbs sampler. The performance of the algorithms is evaluated using a simulation study. The last section discusses a practical example.

2. Maximum Likelihood Estimators

To find MLE of the parameters of an underlying distribution we use EM algorithm given by Dempster, Laird and Rubin (1977). The EM algorithm has two advantages here: The first occurs when the data indeed has missing values, due to problems with or limitations of the observation process. The second occurs when optimizing the likelihood function is analytically intractable but when the likelihood function can be simplified by assuming the existence of and values for additional but missing (or hidden) parameters. There is much literature devoted to extensions and applications of the EM algorithm, and this is summarized in McLachlan & Krishnan (1997).

The expectation step or E-step computes the expected likelihood for the complete data. Let θ be the complete collection of parameters occurring in the mixture, i.e. $\theta = (d,\alpha,\lambda)$.

E-Step: We take expectation and get Q function

$$Q(\theta|\theta^{(t)}) = E[\ln(L(p,\theta|Z,X))|x,\theta^{(t)}]$$

$$= \sum_{i=1}^{n} E(Z_i|x_i,\theta^{(t)}) \ln\left(\left[\frac{p}{1-p}\right]\left[\frac{f_1(x_i|d)}{f_2(x_i|\alpha,\lambda)}\right]\right) + \ln\left((1-p) f_2(x_i|\alpha,\lambda)\right) \tag{6}$$

where

$$E(Z_i|x_i,\theta^{(t)}) = P(z_i = 1|x_i,\theta^{(t)})$$

$$= \frac{p f_1(x_i|d^{(t)})}{p f_1(x_i|d^{(t)}) + (1-p) f_2(x_i|\alpha^{(t)},\lambda^{(t)})}$$

$$= \frac{\dfrac{p^{(t)}}{d^{(t)}}}{\dfrac{p^{(t)}}{d^{(t)}} + (1-p^{(t)})\,\alpha^{(t)}\,\lambda^{(t)^{\alpha^{(t)}}}\,x_i^{\alpha^{(t)}-1}\,e^{-(\lambda^{(t)}x_i)^{\alpha^{(t)}}}} \tag{7}$$

M-Step: Here we maximize the expectation, i.e., the Q-function that we computed in the E-step. The two steps may be repeated as per requirement

Maximizing $Q(\theta|\theta^{(t)})$ w.r.t. θ yields the update equations

$$\frac{\partial Q}{\partial p} = 0 \;\Rightarrow\; p^{(t+1)} = \frac{1}{n}\sum_{i=1}^{n} E(z_i) \tag{8}$$

$$\frac{\partial Q}{\partial d} = 0 \Rightarrow -\frac{1}{d}\sum_{i=1}^{n} E(z_i) = 0 \tag{9}$$

$$\frac{\partial Q}{\partial \lambda} = 0 \Rightarrow \lambda^{(t+1)} = \left[\frac{\sum_{i=1}^{n}(1-E(z_i)}{\sum_{i=1}^{n}(1-E(z_i)\ x_i^{\alpha^{(t)}}}\right]^{1/\alpha^{(t)}} \tag{10}$$

$$\frac{\partial Q}{\partial \alpha} = 0 \Rightarrow \sum_{i=1}^{n}(1-E(Z_i))[\frac{1}{\alpha^{(t)}} + \ln(\lambda^{(t)}) + \ln(x_i) - (\lambda^{(t)}\ x_i)^{\alpha^{(t)}} \ln(\lambda^{(t)}\ x_i)] = 0 \tag{11}$$

Since equation (9) cannot solved analytically, we use $d^{(t+1)} = Max(x_i)$ since $x_0 \le x_i \le x_0 + d$. To obtain $\alpha^{(t+1)}$, we solve equation (11) numerically using Newton-Raphson method.

The EM algorithm starts by assigning initial values to all parameters to be estimated. It then iteratively alternates between two steps, the E-step and M-step. The E-step computes the expected likelihood and the M-step re-estimates all the parameters by maximizing the Q-function. If the convergence criteria is not met, then the parameters p,α,λ and d are updated. We can repeat E-step followed by the M-step until the likelihood converges. Every iteration is guaranteed to increase the log-likelihood and the algorithm is guaranteed to converge to a local maximum of the likelihood function.

3. Bayesian Estimate when all parameters are unknown

Let the prior distributions be

$$\pi(p) = \frac{1}{B(\eta,\beta)}\ p^{\eta-1}\ (1-p)^{\beta-1}\ ,\ \eta > 0, \beta > 0$$

$$\pi(d) = \frac{1}{\xi},\ 0 < d < \xi$$

$$\pi(\alpha) = \theta e^{-\alpha\theta},\ \theta > 0$$

and

$$\pi(\lambda) = \frac{b^a}{\Gamma(a)}\ \lambda^{-(a+1)}\ e^{-b/\lambda}$$

Then the joint pdf becomes

$$f(\underline{x}; p, d, \alpha, \lambda) = \prod_{i=1}^{n} f(x_i)\ \pi(p)\pi(d)\pi(\alpha)\pi(\lambda)$$

$$= \prod_{i=1}^{n} \left(\frac{p}{d} + (1-p)\,\alpha\,\lambda^{\alpha}\, x_i^{\,\alpha-1}\, e^{-(\lambda x_i)^{\alpha}} \right) \frac{1}{\xi}\,\theta\, e^{-\alpha\theta}\, \frac{1}{B(\eta,\beta)}\, p^{\eta-1}(1-p)^{\beta-1}\, \frac{b^{a}}{\Gamma(a)}\, \lambda^{-(a+1)}\, e^{-b/\lambda} \qquad (12)$$

Thus, the posterior density is proportional to

$$\Pi(p,d,\alpha,\lambda\,|\,\underline{x}) \propto \prod_{i=1}^{n} \left(\frac{p}{d} + (1-p)\,\alpha\,\lambda^{\alpha}\, x_i^{\,\alpha-1}\, e^{-(\lambda x_i)^{\alpha}} \right) e^{-\alpha\theta}\, p^{\eta-1}(1-p)^{\beta-1}\, \lambda^{-(a+1)}\, e^{-b/\lambda} \qquad (13)$$

To generate sample from (13), we use Population Monte Carlo (PMC) algorithm and Mixture version of Metropolis-Hasting algorithm. The PMC algorithm is an iterated importance sampling scheme, it is an adaptive algorithm that calibrates the proposal distribution to the target distribution at each iteration by learning from the performance of the previous proposal distributions. A complete detail about PMC can be found in Capp'e et al. (2004). Letting $\theta = (p,d,\alpha,\lambda)$, the PMC algorithm can be given as

Algorithm-1

Initialize $\theta_0^{(1)},\ldots,\theta_0^{(M)}$ *; set t=1.*

Step – 1 For i=1,….,M

1.1.1 Generate $\theta_{(t)}^{(i)} \sim q_{it}(\theta_{(t-1)}^{(i)})$

1.1.2 Compute

$$\rho^{(i)} = \frac{\Pi(\theta_{(t)}^{(i)})}{q_{it}(\theta_{(t-1)}^{(i)})}$$

1.2 Compute $w^{(i)} = \dfrac{\rho^{(i)}}{\sum_{l=1}^{M} \rho^{(l)}}$

1.3 Resample M values with replacement from the $\theta_{(t)}^{(i)}$ *'s using the weights* $w^{(i)}$*.*

Step – 2 Repeat step – 1 for t=2 to N

Below we present Metropolis-Hasting (MH) algorithm corresponding to the mixture distribution shown in (13).

Algorithm-2

Initialize $\theta^{(0)}$ *; set t=0.*

Step 1: Generate $\tilde{\theta} \sim q(\theta\,|\,\theta^{(t)})$ *and u ~ Uniform (0, 1)*

Step 2: Compute

$$r = \frac{f(\underline{x}\,|\,\tilde{\theta})\ \pi(\tilde{\theta})\ q(\theta^{(t)}\,|\,\tilde{\theta})}{f(\underline{x}\,|\,\theta^{(t)})\ \pi(\theta^{(t)})\ q(\tilde{\theta}\,|\,\theta^{(t)})}$$

Step 3: If u < r then

$$\theta^{(t+1)} = \tilde{\theta}$$

else

$$\theta^{(t+1)} = \theta^{(t)}$$

Step 4: Repeat steps 1 to 2 for t = 1,2,...,N and return the values $\theta^{(1)},...,\theta^{(T)}$.

It is to be noted that in both the algorithms, the instrumental distributions q, for (p, d,α,λ) is taken as their prior distributions. As seen above, Population Monte Carlo method is a combination of the Markov Chain Monte Carlo algorithms (for the construction of the proposal), Importance Sampling (for the construction of appropriate estimators), Sampling Importance Resampling (for sample equalization) and Iterated particle systems (for sample improvement). Thus the Population Monte Carlo (PMC) algorithm is in essence an iterated importance sampling scheme that simultaneously produces, at each iteration, a sample approximately simulated from a target distribution and provide unbiased estimates for that distribution. The sample is constructed using sample dependent proposals for generation and importance sampling weights for pruning the proposed sample.

Now, we define another approach to carry out Bayes estimation in mixture context. Introducing latent in the structure, Posterior density can be written as

$$\prod_{i=1}^{n}\left\{\frac{p}{d}\right\}^{z_i}\left\{(1-p)\ \alpha\lambda^{\alpha}x_i^{\alpha-1}\ e^{-(\lambda x_i)^{\alpha}}\right\}^{1-z_i}\ \theta e^{-\alpha\theta}p^{\eta-1}(1-p)^{\beta-1}\ \lambda^{-(a+1)}\ e^{-b/\lambda} \tag{14}$$

Then the full conditional distribution of (p, d,α,λ) is given by

$$\Pi(d\,|\,z) \propto d^{-n_1},$$

$$\Pi(p\,|\,z) \propto p^{n_1+\eta-1}\ (1-p)^{n_2+\beta-1},$$

$$\Pi(\alpha\,|\,z,\lambda) = \alpha^{n_2}\ e^{-\alpha\theta}\prod_{\{i:\ z_i=0\}}^{n} x_i^{\alpha-1}\ e^{-(\lambda x_i)^{\alpha}}$$

and

$$\Pi(\lambda\,|\,z,\alpha) = \lambda^{n_2\alpha-a-1}\ e^{-b/\lambda}\prod_{\{i:\ z_i=0\}}^{n} e^{-(\lambda x_i)^{\alpha}} \tag{15}$$

Where $n_1 = \sum_{i=1}^{n} \mathrm{I}_{z_i=1}$, $n_2 = \sum_{i=1}^{n} \mathrm{I}_{z_i=0}$ and $n = n_1 + n_2$

For generating a sample from this full conditional distribution, we use following algorithm.

Algorithm-3

Initialize $p^{(0)}$, $d^{(0)}$, $\alpha^{(0)}$ and $\lambda^{(0)}$; Set t=1.
Step – 1 Generate $z_i^{(t)}$ (i=1,..., n) from

$$P(z_i^{(t)}=1) \propto p^{(t-1)} / d^{(t-1)}$$

$$P(z_i^{(t)}=0) \propto (1-p^{(t-1)})\, \alpha^{(t-1)} \lambda^{(t-1)^{\alpha^{(t-1)}}} x_i^{\alpha^{(t-1)}-1} e^{-(\lambda^{(t-1)} x_i)^{\alpha^{(t-1)}}}$$

Step – 2 Compute $n_1^{(t)} = \sum_{i=1}^{n} \mathrm{I}_{z_i^{(t)}=1}$ and $n_2^{(t)} = \sum_{i=1}^{n} \mathrm{I}_{z_i^{(t)}=0}$

Step – 3 Generate $p^{(t)} \sim Beta(n_1^{(t)} + \eta - 1, n_2^{(t)} + \beta - 1)$

Step – 4 Generate $d^{(t)} \sim Uniform(d^{(t-1)})$

Step – 5 Generate $\alpha^{(t)} \sim \Pi(\alpha^{(t)} | n_2^{(t-1)}, \lambda^{(t-1)})$ using M-H algorithm.

Step – 6 Generate $\lambda^{(t)} \sim \Pi(\lambda^{(t)} | n_2^{(t-1)}, \alpha^{(t-1)})$ using M-H algorithm.

Step – 7 Repeat step-1 to 6 for t = 2, 3, …, N

To implement the M-H algorithm, it is necessary that a suitable candidate-generating density be specified. If the domain explored is too small, compared with the range of *f*, the Markov chain will have difficulties in exploring this range and thus will converge very slowly so *q* may be (for both α and λ, candidate-generating density is taken as exponential distribution) chosen in such a way that it converge to target distribution. Wide range of choices of *q* has been given in the literature; see Tierney (1994) and Chib and Greenberg (1995) and references contained therein.

4. Simulation Study

Here we have used the entire three algorithms to generate a posterior sample. A simulation study is carried out to compare the performance of different algorithms. A sample of size 50 was drawn from the population by taking different values of the parameters. On the basis of this sample, Bayes estimates are calculated using these algorithms. It was seen that, the number of iterations necessary to reach convergence for PMC is below 5000, while for M-H, it is below 15000 and for Gibbs, and it is above 20,000. A deeper difficulty in implementing Algorithm 3 is the existence of computational trapping states. Table-1 shows the ML estimators and Bayes estimators using three different algorithms.

Table 1 MLE and Bayes estimates

Parameter	MLE	PMC	M-H	Gibbs
p=0.2	0.2567	0.2084	0.2156	0.1692
d=0.05	0.0486	0.0467	0.0532	0.0594
α=2.0	1.8345	1.8857	1.8678	1.8523
λ=1.5	1.4976	1.3885	1.3563	1.6974
p=0.6	0.5678	0.6132	0.5817	0.6679
d=0.05	0.0497	0.0522	0.0423	0.0526
α=2.0	1.8754	1.8804	1.7923	1.9069
λ=1.5	1.6925	1.4760	1.5398	1.6856

5. Application to a real life data

Here, we consider the wood dryness data of 40 boards analyzed in Lai et.al. (2007) with $t_i=0$, i=1,2,…,28 and the other positive observations are 0.0463741, 0.0894855, 0.4, 0.42517, 0.623441, 0.6491, 0.73346, 1.35851, 1.77112, 1.86047, 2.12125, 2.12. Here we spread the zeros uniformly over an interval taking d=0.042 so that $t_1 = 0$, $t_2 = 0.0015$, $t_3 = 0.003$,…, $t_{28} = 0.042$. We obtain MLE estimates using EM algorithm as $\hat{p}_{MLE} = 0.7625$, $\hat{d} = 0.0418$, $\hat{\alpha}_{MLE} = 1.5689$ and $\hat{\lambda}_{MLE} = 1.2458$. PMC method is used to find Bayesian estimates. Below we present two Bayes estimates related to same data set but with different prior values.

(i) $\hat{p}_{BAYES} = 0.6899$, $\hat{d}_{BAYES} = 0.0404$, $\hat{\alpha}_{BAYES} = 0.9363$ and $\hat{\lambda}_{BAYES} = 1.9704$ for $\theta = 1.6$, η=1.1, ξ=0.9, β=2.3, a=1.5, b=1.9

(ii) $\hat{p}_{BAYES} = 0.7187$, $\hat{d}_{BAYES} = 0.0649$, $\hat{\alpha}_{BAYES} = 0.9015$ and $\hat{\lambda}_{BAYES} = 1.9704$ for $\theta = 2.5$, η=4.0, ξ=0.05, β=2.3, a=2.0, b=2.0

6. Sensitivity analysis.

As can be seen from tables 2 and 3 that the estimate of the estimates depends on the choice of *d*. In practice, the value of *d* can be manually estimated quite accurately from the dataset. Even if the value of *d* is inside the peak of the target distribution (here it is Weibull), we are still able to estimate all four parameters of the model. The estimates are consistent for both parametric and Bayesian set up.

Table 2. Uniform spread of "nearly instantaneous failures" with *d* = 0.135

	$\hat{p}$	$\hat{d}$	$\hat{\alpha}$	$\hat{\lambda}$
Maximum likelihood estimates	0.78960	0.135	1.6852	1.2398
Bayes Estimates using PMC	0.8729	0.1265	1.2351	1.4388

Table 3. Uniform spread of "nearly instantaneous failures" with *d* = 0.2

	$\hat{p}$	$\hat{d}$	$\hat{\alpha}$	$\hat{\lambda}$
Maximum likelihood estimates	0.6125	0.2000	1.2341	1.3563
Bayes Estimates using PMC	0.7142	0.2764	1.5434	1.7649

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I.S. BLIOH (1836 – 1901) RAILWAY MAGNATE AND PEACEMAKER, PROMINENT SCIENTIST-RAILROADER: ECONOMIST, STATISTICIAN AND FINANCIER

Lapin V., Ph.D., Kryvopishyn O., Ph.D., Katsman M., Ph.D.

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South-Western Railway, Kyiv, Ukraine
e-mail: vnbsp@sw.uz.gov.ua

Wretched presence of *Ivan Stanislavovich Blioh (1836 – 1901)* in modern historical and scientific memory of Poland, Russia and Ukraine, that in those countries in which he was born, was working, and lived extraordinarily creative life specifies on possible, in course of time, danger of complete disappearance of his name not only from the scientific inheritance of these states but also from European collective memory. Unfortunately Blioh, in understanding of nationalistic idea, was very Russian for the Polish nationalists, very bourgeois for soviet communists, very much asimilyant (adopted Christianity) for anti-semites, that in course of time resulted in a partial loss, from these reasons, his scientific acquisition and his merits as military theorist and figure of international peaceful motion. At the same time Blioh was a successful businessman and exceptionally gifted person who knew French, German, Russian and Polish languages. His ideas became the basics for creation OUN. Blioh the first in theory formulated and practically carried out important direction in the sphere of social partnership between businessmen and workers by creation of railway pension cash desks of insurance type. On the whole his life is a whole epoch in technical, financial, intellectual and nationally-cultural development of the Russian empire of the second half of XIX century. In fact Ivan Blioh is not only a railway magnate and banker but also scientist, known economist, statistician and financier, military theorist, peacemaker, figure of international peaceful motion. He was nominated on the Nobel Prime of the world. He was born on June, 24, 1836 in the Polish town of Radom, was a seventh child in a poor, having many children family. His education he received in Warsaw and Berlin [1,2,3].

In 1850 I. Blioh arrived to Warsaw and began to work in the bank of family of Teplica and studied in the Real gymnasium. In 1856 I. Blioh leaves Warsaw and moves to search the best fate to Petersburg. He began to work as a subcontractor on constructing of railway between Warsaw and St. Petersburg. First time the name of I. Blioh appears in the Russian archives in 1860 year. In these documents Blioh is remembered as an owner of company «enterprise of Blioh» and honored businessman [1].

In 1866 I. Blioh got a concession on constructing of railway in Lodz. Lodz railway was built by Blioh independently and only for his money. Railway by a length in 26 versts connected the center of textile industry, which developed quickly, with the station of Kolyushki of Warsaw-Viennese railway. After the sizes of gross and clean profit, in a count on a verst, Lodz railway excelled the Mykolaiv railway even. After high-quality building of this railway in short terms I. Blioh became a famous person among the builders of railways. The following in his career of railway builder was construction of the part of Libavska railway by length in 294 versts. The construction lasted more than two years (1869-1871) and did not bring the considerable financial achievements for Blioh, but he got considerably anymore due to it, – Ivan Stsnislavovich received a reputation of professional specialist, reliable and timely performer of contracts.

Blioh after becoming an actual proprietor of Kiev-Brest railway put beginning on it pension cashdesk with principles of self-finance and insurance. In this he saw the way of providing of deserving financial position of workings on a railway transport at the end of labour life. He confirmed practical actions by scientific work, published in 1875 in a coauthor with O.I. Vishnegradskiy. The purpose for decision of which it was written by two known domestic scientists, - exposure of effective economic mechanisms of creation of the unique, but not separate, system of pension cashdesks which would operate on a railway transport on principles of self-finance and life-insurance. It is necessary to notice that future ministers of finance of Russian Empire I. Vishnegradskiy and S. Vitte at one time, before the ministerial brief-cases, were related to work on the railway transport (South-Western railways) and both submitted to I. Blioh. Ideas that were conducted by them for reformation of the financial system of the Russian Empire were got from collaboration with Blioh [1].

Among colleagues-railroaders, economists, financiers I. Blioh, differed thirst to scientific-practical activity. In 1864, in the age of 28 years Blioh published his first scientific work on questions of constructing and exploitation of railways. This work had a form of message, but paid attention on itself by leading spheres in the state and helped to it's author to take the prominent place among other known figures that time in the area of construction and exploitation of railways. At the beginning of 70-th I. Blioh had a mass popularity as a connoisseur of railway business, he was known as a serious economist and gifted publicist. Since the publication of the first scientific article Ivan Stanislavovich did not limit his life only by business, scientific work became the norm of his everyday life [2].

In 1890 appears his extraordinarily valuable scientific research „About an agricultural reclamative credit in Russia and abroad”. The basis of research is made by the collected materials about organization of this important, but weaker than other kinds of credit, developed in Russia. He is an author of more than ten multivolume, fundamental scientific works in macroeconomics, railway statistics, finances, problems of war and peace, land-tenure, demography. Most of his scientific works were translated into foreign languages. Come into the special notice and have a scientific value the works of Ivan Stanislavovich on questions of railway statistics, determination of cost of railway tariffs and prime price of railway transportations, influence of new type of transport, – railways on the economic and financial position of Russia. Blioh was one of the first, who laid beginning to the scientific analysis of influence of railways and railway construction on economic position of country and began constantly work above this theme [1, 2].

Minister for finance the Russian Empire M.H. Reytern, acknowledging useful to appoint commerce of adviser Blioh, by the member of scientist of Committee of ministry of finance in his presentation to the emperor in relation to setting, gave such description to Ivan Stanislavovich: «In the environment of commercial estate pays attention on itself literary and by scientific works commerce adviser Blioh. The work about the Russian railways and systematic tables about trading in cattle in Russia, published by him deserved the most complimentary reviews of persons, specially acquainted with this business. Presently Bliokh is busy at collecting of materials for the research undertaken by him about development of productive forces of Russia for the last twenty years, part of which appeared already in the print... Commerce adviser Blioh, religions of evangelistic-reforming, is deliberative member of the Veterinary committee of ministry of internal affairs from April, 24, 1874 and has orders of Saint Ann with a crown second stage, Saint Stanislav second stage, Saint Vladimir fourth stage and Austrian order’s Franca Joseph Commander Cross with a star. Position of member of scientific Committee of ministry of finance behaves to the first class».

By his work «Influence of railways on economical state of Russia» (5 volumes with a graphic atlas, 1878) I.S. Blioh extended the limits of financial science, the first analyzing influence of building of railways on expense part of the state budget, marking that the state must not only find out the methods of mobilization of necessary it money but also study, as expense part influences on the different aspects of public life in that number during transition from one form of state ménage to other. Blioh conducted the analysis of financial reforms, directed on strengthening of monetary item, touched state profits and painting of profits and charges.

In his works Blioh proved that true analysis of charges of exploitation has large importance not only for the proprietors of railways but also for national interests, which considerably suffer from complete absence of the analytically exhaust basic beginnings of railway exploitation. He selected 4 groups of running expenses, dividing them into two categories: proportional (dependent) and permanent (independent) from the sizes of motion and also offered comparative measuring devices of volumes of works for establishment of rational norms of exploitation. Determination of own charges of railways on unit of carrying passengers and loads was needed for I.S. Blioh for subsequent determination of utility or unprofitability of passenger and freight tariffs and also for perfection of account of own charges of railways with the purpose of estimation of rightness and necessity of the sustained losses.

Research and conclusions of I. Blioh had a determining influence on the decision of problems of forming and management charges on a railway transport and became the beginning of development of prime's price calculations of transportations on domestic railways. Except for determination of prime price of transportations, I. Blioh, as a research worker and proprietor of railways, was interested in the question of formation of tariffs. In this area he tested «certain difficulties» at reasoning about that, in what dependence charges of exploitation are from motion on this railway on how many advantageously and it is possible to attract on it loads by the greater or less decline of tariff.

The sense of the last years of his life I.S. Blioh saw in distribution of idea of creation the world's peace, which took him approximately in 1890, an in the organization of the first museum of war and peace. He undertook to serious research of the noted problem and as a result I.S. Blioh became an author of «the bible of pacifism», - multivolume work with the title «Future war in technical, economical and political relations» and other popular scientific works on this question. In 70th of XX century the works of 360 prominent figures of peacemaking motion were reprinted in the USA, including 6 volume "Future war in its technical, political and economic relations.

Ivan Stanislavovich Blioh died because of aneurysm of heart in December, 1901. At such intensive way of life there is nothing strange. He was known as titan of work, who dedicated night-time to scientific researches, and in a day conducted the usual economic and financial businesses. For such lifestyle Ivan Stanislavovich got the considerable squall of criticism from contemporaries, they could not understand and accept such way of life – combination of successful business and prominent scientific activity.

Getting considerable financial resources on railway concessions, Blioh actively joined to public life of the Russian state. He productively worked in a commission on the revision of legislation about jewries, headed by count Palen, except for scientific works from an economy and finances came forward as a theorist and practical figure in a fight for world disarmament, accepted active voice in organization of world conference from general disarmament, going out from financial-economical expedience and production necessity founded society of the South-Western railways and long time was the chairman of Rule.

I.S. Blioh is one of founders of railway statistics, which is component part of statistical industry that studies a railway transport. Basic directions of his scientific work there were also problems of railway tariffs, political economy and statistics of national economy.

Appointed the member of Scientist committee of ministry of finance, Blioh printed a work: «About penalty by Russian railways the transport pays in metallic currency» (1877) and published wide work «Finances of Russia of the XIX century» (4 volumes, 1882). This work is translated into French, German and Polish languages and filled up by a volume which contains history of finances of Reign Polish till confluence of them with general state budget. In 1890 there was published Blioh's book: «About an agricultural land-reclamation credit in Russia and foreign states», in 1898 – his research «Future war and it's economic consequences» (6 volumes and 1 volume of cartograms; reprinted also in Polish, German, French and English languages).

In 1901 was printed his scientific work: «Comparison of financial and ethics welfare of provinces western, great russian and at Wisl's» (5 volumes, with the atlas of charts and cartograms, except for 25 copies destroyed the fire of printing-house, now rare book).

In his works «Russian railways in relation to profits and charges of exploitation and motion of cargoes» (1875) and «About penalty by Russian railways the transport pays in metallic currency» (1877) he in detail considered the question of determination of transport pay on the example of the state of businesses on Kiev-Brest railway which belonged to him.

By him there were analyzed volumes of transportations of different types of products of industries of industrial and agricultural production, duration of their delivery and tariffs, necessities of population in railway transportations, degree of satisfaction of necessities of shippers and consignees.

Tasks that were put by him in relation to the analysis of activity of railway transport and it's influence on the economy of country until now are actual and are deciding on railways with the purpose of effective organization of transportations operating work and is a mean operatively prescriptive to activity of guidance of railways.

Considering a necessity to accept certain economic measures against the removal of losses of railways which arose up as a result of depreciation of Russian national currency, Blioh in his scientific work "About penalty by Russian railways the transport pays in metallic currency" proposed the project of increasing the profitability of railways by the increase of passenger tariff on 30%, and freight on 20%. But this suggestion of Ivan Blioh was not supported by shippers and government [1, 2].

Comparing present population on provinces, their financial incomes on one habitant of province I. Blioh came to the conclusion, that from point of justice, expense, that the state carries because of construction of railways it is necessary to translate not on all population of country, which in the majority does not use railways, and only on that part of population, which with appearance of railways extracts additional benefits.

Examining railways «as industrial enterprises for a movement or transportation of cargoes and passengers», I. Blioh analyzed possibility of application of laws of free competition to the railways and came to the conclusion, that railways must carry cargoes and passengers on a maximum tariff. A free competition for railways is possible only in exceptional cases, as railways make a monopoly of transportations in this locality.

Determining the degree of influence of tariffs on passenger and freight motion, Blioh comes to the conclusion that «decline of tariffs with the purpose of bringing in the passengers and loads from other ways of report, it is impossible not to notice that in that behalf passenger and freight motion is presented quite by different characters». For passenger motion the declining of transport pays don't increase the amount of passengers, so as passage of passengers on railways is caused by their vital necessities and only on occasion by travel satisfactions.

Blioh considered that establishment of freight tariffs is impossible without the studying of direction of motion of cargoes. «Without the previous studying straight of motion of cargoes managements of railways indispose the proper information for establishment of rational tariffs». Low tariffs are advantageous to national economy only at their protracted actions. Not variable, but permanent, low tariffs are of the use as to the state so to the railways.

Working out the totals of his work, Blioh considered appropriately, that this work would «serve grain from which, at a good supervision ... in the future not only rational statistics of commercial motion on railways but also rational looks, will be worked out for the correct estimation of activity of railways».

On defence of domestic railway policy which was represented in the increased railway building (for 11 years 20 thousand versts) I.S. Blioh published in «Announcer of Europe» for 1877 row of the articles under the name «Economic position of Russia in the past and today», which later were reworked and in 1878 were published under the name «Influence of railways into economic position of Russia» in 5 volumes with the detailed graphic atlas [4].

In this work first to the analysis of the systems statistical material was subject during great while accumulated in the ministry of ways of report about motion of loads. This significant research was soon translated into the French and Polish languages. On the World exhibition in Paris in 1878 this work of Blioh was recipient by large gold medal. By this work the beginning of development of railway statistics was fixed, – I. Blioh processed and used present materials, applying a new method – grouping and cartograms.

«In the published by me book «Russian railways» I wanted to explain fact sheets and external of our railways environments by comparison to foreign such as: attitude of profits toward charges, freightage, direction of different industries of trade on the different arteries of the railway system, etc. A favorable reception is given this research encouraged me to engage in subsequent development of questions which behave to economic and financial influence of railways in Russia, questions untouched in the first work» [4].

That fact is not subject a doubt, wrote I. Blioh, that diminishing by railways the value of transportation, increase the net income of producers and the same promote to the increasing of the productivity and raising of prices on earth and labour, helping distribution of welfare thus. Construction of 20 thousand versts of railways and their exploitation brought to country great capitals and delivered to different classes of population very considerable earnings [4].

Analyzing reasons of economic backwardness of the Russian Empire before Crimean war, consequences of the Crimean company and reforms before the crisis of 1866, transformation and general financial position in 60-th years of XIX age an author comes to the conclusion, that among main reasons of defeat of Russia it is possible to name three groups of factors: political, technical and social-economic.

For research of the put questions Blioh used current and statistical materials of railways, ministry of ways of reports, in particular published by the government of country in publication «Types of foreign trade».

After systematizing, grouping and dividing into groups the enormous amount of statistical and current material for period from 1802 to 1876 Blioh conducted an external economic point-of-sale statement of the Russian Empire with the subsequent purpose of determination of role of railways in the improvement of economy of country and external point-of-sale relations of the empire.

The last book of 5 volume work has the name «Financial results». Widespread opinion, that to Crimean war in Russian Empire there was brilliant economic position Blioh named a «mirage». Russian public servants that time, insulated political and economic life of country from the other whole world and put little attention on economic position of population, did not give itself a report in constantly growing power of other countries. Force of the states was measured only the quantity of army and forgot that with the improvement of ways of report and military technique, the fight of people between themselves decide accumulated financial and mental resources and [8, p.5-6]. Nevertheless that Russia from 1831 to 1853 years did not conduct wars, the state during 23 years was not able to cover the charges ordinary profits, no expenses were though done on needed works neither on raising of level of education nor on driving of administration to more correct, proper the requirements of time of position [4].

Thus, Ivan Stanislavovich draws conclusion, that state administration was in a self calming in relation to the economic resources of country to the that time, while Crimean war did not lead to all insolvency of the domestic political system [4].

All of it induced to the radical changes in the financial and administrative system of the state. To the measures the necessity of which was realized society and government, belonged aspiration to find the money for possibly rapid constructing of railways. But the decline of state credit was the phenomenon unfavorable for railway construction [4].

Diminishing of federal deficit by the cutback of the soldiery spending was impossible as a result of active armament of the European countries in the period of french-italian-avstrian war in 1859. Russia is not only for the sake of the influence on the European states but also for the sake of the safety had to increase the quantity of army and perfect a military armament [4]. The unique methods of coverage of federal deficit were the external and internal borrowings.

Such was financial and economic position of the Russian empire before the increased construction of railways gave an enormous shove to the productivity and to the considerable receipt of capitals from abroad, changing a fund and financial market condition and improved the position of state finances. Analyzing in the 4 sections of fifth volume the financial position of state credit, in the fifth section «Financial history of railway construction» Blioh notices that building of railways needs money, and as a result of declining of state credit construction of new railway lines was halted. «It became clearly for all that railways must improve economic and financial position of the state». «The question about new railways arose up only then, when Crimean war by unprofitable a perceptible and fatal method showed unprofitable, as a result of large distances, terms in which the put state at defense of the borders» [4].

Working out the totals of the VIII section, Blioh notices: Comparing the numbers of commodity motion to the numbers shown out by us in the work «Russian railways» after 1871, 1872 and 1873 years easily to mark, as position of our railways gets better considerably. From 1871 to 1874 years the median income from a pood and verst diminished only from 1/25 on 1/26 kop., while charges fallen down from 1/32 to 1/43 kop., and it is possible to hope that on this way of progress of railway will not stop [4].

Active construction of railways became the reason of considerable increase of money circulation in the state. A money in an economy acted as paying for labour of great number of office workers, busy on construction and exploitation of railways, and also as paying for different build materials which before the beginning of construction of railways quite not had a value and outlaid unproductively or had an useless value.

Blioh I.S. considered that influence of railways on economic position of Russia had been expressed in dual sense: from one side enormous part of influence belonged to them on development of productive forces, strengthening of consumption, export and import. If not inevitable offerings on them from the side of the state, that increased a national debt abroad, such concept of economic way of life of country would be reflected also on the improvement of rates of exchange in a greater degree, what it could take place at the increase of sum of Russian foreign payments. From the other side, construction of railways made a very large turn, in 1,5 milliards of roubles, which was distributed mainly on earnings of different layers of population and enabled them not only more in good condition to bring in taxes but also strengthen a consumption, in general to improve the way of life and create an economy if to not all the people on the average to mass, in the environment of which again private capitals appeared in great number. In that sense also operated charges for exploitations of railways which made in the period of 11 years (1865-1875) the sum in 535 million roubles.

Ivan Stanislavovich finished his 5 volume work by words which are actual until today: «So, the shown picture certifies by the way of indisputable arguments, which are the numbers, that the welfare of the state and development of it's force is not in the stagnation, not in separate from other nations, not in difficulty of terms of internal way of life, and opposite – in a gradual, but courageous continuous forward movement, in sent forward other people on their way of development and national freedom» [4].

CONCLUSIONS

In the article there is described the production and scientific activity of I.S. Blioh in the industry of building and exploitation of railway transport, creation of railway pension cash desks of insurance type, determination of role of railway statistics is on the cost of construction of tariffs, an analysis of influence of railways is on economic development and finances of Russian Empire in the second part of XIX – beginning of XX centuries, and also public activity from the problems of war and peace, world's disarmament, prevention of soldiery conflicts, land-tenure and demography

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