Performance Analysis of System where Service Type for Boiler Depends Upon Major or Minor Failures

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Abstract

In industries, the type of failure sensitively affects the system. So, it is essential to Categorize these failures into different categories to enhance the system performance. In this research, concentration made in differentiating the failure type into major/minor categories with repair/replacement facility for the service by single repairman. Currently, we studied the boiler system of the steam generation plant to perform the task of repair/replacement with a single repairman. A reliability model constructs to compute MTSF(mean time to system failure), availability, Busy period for repair/replacement, and profit evaluation. The above measures were estimated numerically and plotted graphically using semi-Markov processes and regenerative point technique. Various effectiveness measures show how system performance gets affected by major/minor failures & the type of service provided.

Keywords: Regenerative point technique, major/minor failure, repair/replacement, Reliability modeling, semi-Markov processes.

I. Introduction

Proper functioning of any system is a prerequisite of any industrial process, as an interruption in the operation of a system causes not only deterioration in the quality of manufactured products but damages to the plant itself. Thus the reliability of the system becomes much more essential. Many contributors pay their efforts in the literature of reliability. [5], [8], [2] have worked on cost-effectiveness and reliability analysis on different situations. Various situations on repair, replacement, & inspection have been investigated by the authors [6], [4], [9], [3], [1]. However, the distinction between major and minor faults has not been the topmost research topic in reliability. [7] have discussed the concept of major/minor failures subjected to the ordinary and expert repairman. [10] revealed the possibility of immediate repair on minor failure and waiting time for repair on major failure at night hours. But the concept of repair/replacements depending upon the minor/major faults has not been seen in the literature of reliability modeling. In addition to the above idea, the boiler of the Steam Generation Plant studied, in which the type of service provided for a boiler depends upon major and minor faults. Minor faults are repaired easily while major faults are replaceable.

Initially, a three-unit system with a boiler and two FD fans has considered for the study. When a boiler fails system stops working immediately, but if any one of the FD fans fails system goes on reduced capacity. For continuous functioning, a boiler and 2 FD fans should function. On boiler failure, two possibilities arise major and minor faults. Repair facility provided for a small crack in the outer chamber of the boiler that occurs due to overheating & erosion, whereas the repairman performed replacement for the major equipment failures for a boiler. Priority of repair is given to the boiler over fans so that the system can operate for a long time. For the FD fans, repair priority is on an FCFS basis.

The following reliability measures were computed numerically using semi-Markov processes and regenerative point techniques and also plotted graphically based on the information gathered from the industry :

- Mean time to system failure (MTSF).
- Availability analysis at full capacity.
- Availability analysis at reduced capacity.
- Busy period for repair time only.
- Busy period for replacement time only.
- Expected no. of repairs.
- Expected no. of replacements.
- Cost-benefit analysis.

II. Model Description

I. State Transition Diagram

Figure 1, shows the state transitions diagram of the steam generation plant consisting of one boiler and two FD fans.

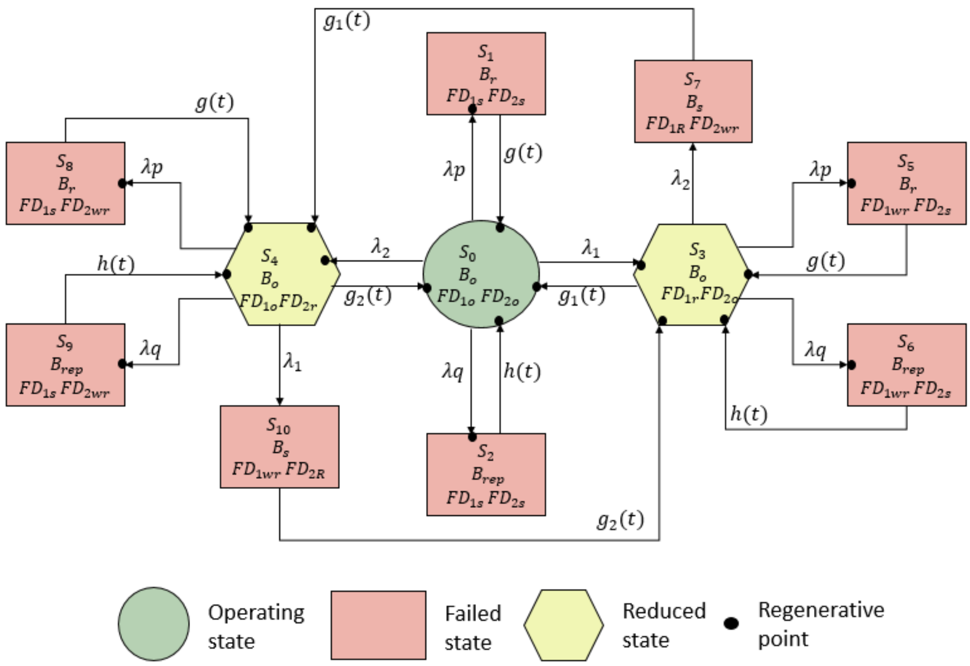

Figure 1: *State Transition Diagram*

Table 1: *State Discription*

States	Discription
S_0	This is the operating state of the system
$S_0, S_1, S_2, S_3, S_4, S_5, S_6, S_8, S_9$	The epoch of entry into these states are regenerative points thus, these states are called regenerative states
S_3, S_4	These are reduced capacity states
$S_1, S_2, S_5, S_6, S_7, S_8, S_9, S_{10}$	These are failed states of the system.

II. Assumptions

The stated model follows these assumptions.

- All the random variables are independent.
- Failure times distribution is exponential, whereas repair times have distributed arbitrarily.
- System works as well as new after every repair.
- when the failure occurs repairman will come immediately.

III. Nomenclature

Table 2: *Notations & symbols for the states of system*

	Notations
Notations	Discription
λ	Constant failure rate of Boiler.
λ_1	Constant failure rate of FD fan 1.
λ_2	Constant failure rate of FD fan 2.
α	Repair rate of Boiler.
α_1	Repair rate of FD fan 1.
α_2	Repair rate of FD fan 2.
γ	Replacement rate of Boiler.
p	probability of minor failure in Boiler.
q	probability of major failure in Boiler.
$G(t)$, $g(t)$	c.d.f. & p.d.f of repair time of Boiler.
$H(t)$, $h(t)$	c.d.f. & p.d.f of replacement time of Boiler.
$G_1(t)$, $g_1(t)$	c.d.f. & p.d.f of repair time of FD fan 1.
$G_2(t)$, $g_2(t)$	c.d.f. & p.d.f of repair time of FD fan 2.
	Symbols for the states of the system
Symbols	Discription
S_i	states of the system, $i = 1, 2, 3, ..., 10$
B_o, FD_{1o}, FD_{2o}	Boiler, FD fans 1 and 2 are in operating state respectively.
B_r	Boiler under repair.
B_{rep}	Boiler under replacement.
B_s, FD_{1s}, FD_{2s}	Boiler, FD fans 1 and 2 in standby state.
FD_{1r}, FD_{2r}	FD fans 1 and 2 under repair.
FD_{1wr}, FD_{2wr}	FD fans 1 and 2 are waiting for repair.
FD_{1R}, FD_{2R}	FD fans 1 and 2 are under repair from previous state.

IV. Transition Probabilities & Mean Sojourn Times

The p_{ij} represents non-zero elements which are given below

$$p_{01} = \frac{\lambda p}{\lambda + \lambda_1 + \lambda_2}, \qquad p_{02} = \frac{\lambda q}{\lambda + \lambda_1 + \lambda_2},$$

$$p_{03} = \frac{\lambda_1}{\lambda + \lambda_1 + \lambda_2}, \qquad p_{04} = \frac{\lambda_2}{\lambda + \lambda_1 + \lambda_2},$$

$$p_{10} = p_{20} = 1, \qquad p_{30} = g_1{}^*(\lambda + \lambda_2),$$

$$p_{35} = \frac{\lambda p}{\lambda + \lambda_2}[1 - g_1{}^*(\lambda + \lambda_2)], \qquad p_{36} = \frac{\lambda q}{\lambda + \lambda_2}[1 - g_1{}^*(\lambda + \lambda_2)],$$

$$p_{37} = \frac{\lambda_2}{\lambda+\lambda_2}[1-g_1{}^*(\lambda+\lambda_2)], \qquad p_{34}^{(7)} = \frac{\lambda_2}{\lambda+\lambda_2}[1-g_1{}^*(\lambda+\lambda_2)],$$
$$p_{40} = g_2{}^*(\lambda+\lambda_1), \qquad p_{48} = \frac{\lambda p}{\lambda+\lambda_1}[1-g_2{}^*(\lambda+\lambda_1)],$$
$$p_{49} = \frac{\lambda q}{\lambda+\lambda_1}[1-g_2{}^*(\lambda+\lambda_1)], \qquad p_{4,10} = \frac{\lambda_1}{\lambda+\lambda_1}[1-g_2{}^*(\lambda+\lambda_1)],$$
$$p_{4,3}^{(10)} = \frac{\lambda_1}{\lambda+\lambda_1}[1-g_2{}^*(\lambda+\lambda_1)], \qquad p_{53} = 1,$$
$$p_{63} = 1, \qquad p_{74} = 1,$$
$$p_{84} = 1, \qquad p_{94} = 1,$$
$$p_{10,3} = 1$$

It can be verified by these probabilities that

$$p_{01}+p_{02}+p_{03}+p_{04} = 1, \qquad p_{10} = 1, \qquad p_{20} = 1,$$
$$p_{30}+p_{35}+p_{36}+p_{37} = 1, \qquad p_{30}+p_{35}+p_{36}+p_{34}^{(7)} = 1, \qquad p_{40}+p_{48}+p_{49}+p_{4,10} = 1,$$
$$p_{40}+p_{48}+p_{49}+p_{43}^{(10)} = 1, \qquad p_{53} = p_{63} = p_{74} = 1, \qquad p_{84} = p_{94} = p_{10,3} = 1$$

Also μ_i, the mean sojourn times in state S_j are

$$\mu_0 = \frac{1}{\lambda+\lambda_1+\lambda_2}, \qquad \mu_1 = -g^{*\prime}(0), \quad \mu_2 = -h^{*\prime}(0), \quad \mu_3 = \frac{1}{\lambda+\lambda_2}[1-g_1{}^*(\lambda+\lambda_2)],$$
$$\mu_4 = \frac{1}{\lambda+\lambda_1}[1-g_2{}^*(\lambda+\lambda_1)], \quad \mu_5 = -g^{*\prime}(0), \quad \mu_6 = -h^{*\prime}(0), \quad \mu_7 = -g_1{}^{*\prime}(0),$$
$$\mu_8 = -g^{*\prime}(0), \qquad \mu_9 = -h^{*\prime}(0), \quad \mu_{10} = -g_2{}^{*\prime}(0),$$

The unconditional mean time taken by the system to transit for any regenerative state $'j'$ when it (time) is counted from the epoch of entrance into state $'i'$ is mathematically represented as

$$m_{ij} = \int_0^\infty t dQ_{ij}(t) = -q_{ij}{}^{*\prime}(0)$$

$$m_{01}+m_{02}+m_{03}+m_{04} = \mu_0, \qquad m_{10} = \mu_1,$$
$$m_{20} = \mu_2, \qquad m_{30}+m_{35}+m_{36}+m_{37} = \mu_3,$$
$$m_{30}+m_{35}+m_{36}+m_{34}^{(7)} = \mu_3+K_1, \qquad m_{40}+m_{48}+m_{49}+m_{4,10} = \mu_4,$$
$$m_{40}+m_{48}+m_{49}+m_{43}^{(10)} = \mu_4+K_2, \qquad m_{53} = \mu_5,$$
$$m_{63} = \mu_6, \qquad m_{74} = \mu_7,$$
$$m_{84} = \mu_8, \qquad m_{94} = \mu_9,$$
$$m_{10,3} = \mu_{10}$$

where

$$K_1 = \frac{\lambda_2}{\lambda}\int_0^\infty t g_1(t) dt, \qquad K_2 = \frac{\lambda_1}{\lambda}\int_0^\infty t g_2(t) dt, \tag{1}$$

III. Reliability Measures for System Effectiveness

I. Mean Time to System Failure (MTSF)

When the system starts from the initial state S_0, Mean time to system failure (MTSF) of the system is determined by considering failed state as absorbing state as given below

$$MTSF = T_0 = \lim_{s\to 0}\frac{1-\phi_0{}^{**}(s)}{s} \tag{2}$$

Using L' Hospital Rule & putting the value of $\phi_0^{**}(s)$, we have

$$T_0 = \frac{N}{D} \tag{3}$$

where
$N = \mu_0 + \mu_3 p_{03} + \mu_4 p_{04}$
$D = 1 - p_{03}p_{30} - p_{04}p_{40}$

II. Availability Analysis at Full Capacity

Using the theory of regenerative processes, the availability AF_0 of the system at full capacity is given by

$$A_{F0} = \lim_{s \to 0}(s\, A_{F0}{}^*(s)) = \frac{N_1}{D_1}$$

where

$$N_1 = \mu_0[(1 - p_{48} - p_{49})(1 - p_{35} - p_{36}) - p_{34}^{(7)} p_{43}^{(10)}] \tag{4}$$

$$\begin{aligned} \& \quad D_1 = \mu_0[p_{40} - p_{30}p_{43} - p_{40}p_{35}p_{53} - p_{40}p_{63}p_{36}] + (\mu_1 p_{01} + \mu_2 p_{02})[(1 - p_{48}p_{84} - p_{49}p_{94}) \\ (1 - p_{36}p_{63} - p_{35}p_{53}) - p_{43}p_{34}] + [\mu_3 + K_1][p_{03} + p_{04}p_{43} - p_{03}p_{48}p_{84} - p_{03}p_{49}p_{94}] \\ + [\mu_4 + K_2][p_{04} + p_{03}p_{34} - p_{04}p_{35}p_{53} - p_{04}p_{36}p_{63}] \\ + (\mu_5 p_{35} + \mu_6 p_{36})[(1 - p_{01}p_{10} - p_{02}p_{20})(1 - p_{48}p_{84} - p_{49}p_{94}) - p_{04}p_{40}] \\ + (\mu_8 p_{48} + \mu_9 p_{49})[(1 - p_{01}p_{10} - p_{02}p_{20})(1 - p_{35}p_{53} - p_{63}p_{36}) - p_{03}p_{30}] \end{aligned} \tag{5}$$

III. Availability Analysis at Reduced Capacity

Using the theory of regenrative processes, the availability AR_0 of the system at reduced capacity is given by

$$A_{R0} = \lim_{s \to 0}(s\, A_{R0}{}^*(s)) = \frac{N_2}{D_1}$$

where

$$N_2 = \mu_3[p_{03}(1 - p_{48} - p_{49}) + p_{04}p_{43}^{(10)}] + \mu_4[p_{04}(1 - p_{35} - p_{36}) + p_{03}p_{34}^{(7)}] \tag{6}$$

and D_1 is already specified in equation (5).

IV. Busy Period for Repair Time only

In steady state, busy period for repair time is defined as the time for which system is under repair by repairman and is given by

$$B_{R0} = \lim_{s \to 0}(sB_{R0}{}^*(s)) = \frac{N_3}{D_1} \tag{7}$$

Where

$$\begin{aligned} N_3 = \mu_1 p_{01}[(1 - p_{48} - p_{49})(1 - p_{35}p_{36}) - p_{43}{}^{(10)}p_{34}{}^{(7)}] + \mu_3[p_{03}(1 - p_{48} - p_{49}) + p_{04}p_{43}{}^{(10)}] \\ + \mu_4[p_{04}(1 - p_{35} - p_{36}) + p_{03}p_{34}{}^{(7)}] + \mu_5 p_{35}[p_{03}p_{40} + (p_{03} + p_{04})p_{43}{}^{(10)}] \\ + \mu_8 p_{48}[p_{04}(1 - p_{35} - p_{36}) + p_{03}p_{34}{}^{(7)}] \end{aligned} \tag{8}$$

& D_1 is already specified in equation (5).

V. Busy Period for Replacement Time only

In steady state, busy period for replacement time is defined as the time for which system is busy under replacements and is given by

$$B_{RP0} = \lim_{s \to 0} (sB_{RP0}{}^{*}(s)) = \frac{N_4}{D_1} \tag{9}$$

where

$$N_4 = \mu_2 p_{02}[(1 - p_{48} - p_{49})(1 - p_{35}p_{36}) - p_{43}{}^{(10)} p_{34}{}^{(7)}] + \mu_6 p_{36}[p_{03}(1 - p_{48} - p_{49}) + p_{43}{}^{(10)}] + \mu_9 p_{49}[p_{04}(1 - p_{35} - p_{36}) + p_{34}{}^{(7)}] \tag{10}$$

& D_1 is already specified in equation (5).

VI. Expected No. of Repairs

Expected number of repairs per unit time for the system is given by

$$V_{R0} = \lim_{s \to 0} [sV_{R0}{}^{**}(s)] = \frac{N_5}{D_1} \tag{11}$$

where

$$N_5 = (1 - p_{02})(1 - p_{35} - p_{36})(1 - p_{48} - p_{49}) - (1 - p_{02}) p_{34}{}^{(7)} p_{43}^{(10)} + p_{03} p_{48} p_{34}^{(7)} + p_{04} p_{35} p_{43}^{(10)} + p_{04} p_{48}(1 - p_{35} - p_{36}) + p_{03} p_{35}(1 - p_{48} - p_{49}) \tag{12}$$

& D_1 is already specified in equation (5).

VII. Expected No. of Replacements

Expected number of replacements per unit time for the system is given by

$$V_{RP0} = \lim_{s \to 0} [sV_{RP0}{}^{**}(s)] = \frac{N_6}{D_1} \tag{13}$$

where

$$N_6 = p_{02}((1 - p_{35} - p_{36})(1 - p_{48} - p_{49}) - p_{34}{}^{(7)} p_{43}^{(10)}) + p_{03}(p_{36}(1 - p_{48} - p_{49}) + p_{49} p_{34}^{(7)}) + p_{04}(p_{49}(1 - p_{35} - p_{36}) + p_{36} p_{43}^{(10)}) \tag{14}$$

& D_1 is already specified in equation (5).

IV. Cost-Benefit Analysis

The expected total profit incurred to the system is

$$P_0 = C_0 A_{F0} + C_1 A_{R0} - C_2 B_{R0} - C_3 B_{RP0} - C_4 V_{R0} - C_5 V_{RP0}$$

where
C_0 = revenue per unit up time at full capacity
C_1 = revenue per unit time at reduced capacity
C_2 = cost per unit time when repairman is busy in doing repair
C_3 = cost per unit time when repairman is busy in doing replacement
C_4 = cost per repair.
C_5 = cost per replacement.

V. Particular Cases

For the numerical evaluation and graphical plotting for various reliability measures, the following particular cases are considered.
Let us assume that $g(t) = \alpha e^{-\alpha t}, h(t) = \gamma e^{-\gamma t}, g_1(t) = \alpha_1 e^{-\alpha_1 t}, g_2(t) = \alpha_2 e^{-\alpha_2 t}$, and the remaining distributions are the same as in the general case. Therefore, we have

$$p_{01} = \frac{\lambda p}{\lambda + \lambda_1 + \lambda_2}, \quad p_{02} = \frac{\lambda q}{\lambda + \lambda_1 + \lambda_2}, \quad p_{03} = \frac{\lambda_1}{\lambda + \lambda_1 + \lambda_2},$$

$$p_{04} = \frac{\lambda_2}{\lambda + \lambda_1 + \lambda_2}, \quad p_{10} = 1, \quad p_{20} = 1,$$

$$p_{30} = \frac{\alpha_1}{\lambda + \alpha_1 + \lambda_2}, \quad p_{35} = \frac{\lambda p}{\lambda + \alpha_1 + \lambda_2}, \quad p_{36} = \frac{\lambda q}{\lambda + \alpha_1 + \lambda_2},$$

$$p_{37} = \frac{\lambda_2}{\lambda + \alpha_1 + \lambda_2} = p_{34}{}^{(7)}, \quad p_{40} = \frac{\alpha_2}{\lambda + \lambda_1 + \alpha_2}, \quad p_{48} = \frac{\lambda p}{\lambda + \alpha_2 + \lambda_1},$$

$$p_{49} = \frac{\lambda q}{\lambda + \alpha_2 + \lambda_1}, \quad p_{4,10} = \frac{\lambda_1}{\lambda + \alpha_2 + \lambda_1} = p_{43}{}^{(10)}, \quad p_{53} = p_{63} = p_{74} = 1$$

$$p_{84} = p_{94} = p_{10,3} = 1, \quad \mu_0 = \frac{1}{\lambda + \lambda_1 + \lambda_2}, \quad \mu_1 = \frac{1}{\alpha},$$

$$\mu_2 = \frac{1}{\gamma}, \quad \mu_3 = \frac{1}{\lambda + \alpha_1 + \lambda_2}, \quad \mu_4 = \frac{1}{\lambda + \alpha_2 + \lambda_1},$$

$$\mu_5 = \frac{1}{\alpha}, \quad \mu_6 = \frac{1}{\gamma}, \quad \mu_7 = \frac{1}{\alpha_1},$$

$$\mu_8 = \frac{1}{\alpha}, \quad \mu_9 = \frac{1}{\gamma}, \quad \mu_{10} = \frac{1}{\alpha_2},$$

Table 3: *Computation of various rates/costs on the basis of actual data collected from industry*

Various rates/ cost associated	corresponding values
Failure rate of Boiler (λ)	0.0001186/hr
Failure rate of FD fan 1 (λ_1)	0.0001171/hr
Failure rate of FD fan 2 (λ_2)	0.000101295/hr
Repair rate of Boiler (α)	0.00738/hr
Replacement rate of Boiler (γ)	0.0008733 /hr
Repair rate of FD fan 1 (α_1)	0.024272/hr
Repair rate of FD fan 2 (α_2)	0.048544/hr
Expected cost per repair (C_4)	Rs. 14282
Expected cost per replacement (C_5)	Rs. 1579627

Hypothetical values have been taken for remaining rates/costs. arious reliability measures for system performance have been computed in table 4 by putting the values given in the table 3 based on particular cases.

Table 4: *Computation of various measures of system effectiveness*

Mean time to system failure	08376.82/hr
Availability of the system at full capacity	0.344986/hr
Availability of the system at reduced capacity	0.639788/hr
Busy period of repairman for repair time only	0.644142/hr
Busy period of repairman for replacement time only	0.010379/hr
Expected no. of repairs	0.000107/hr
Expected no. of replacements	0.000009/hr

VI. Results and Discussion

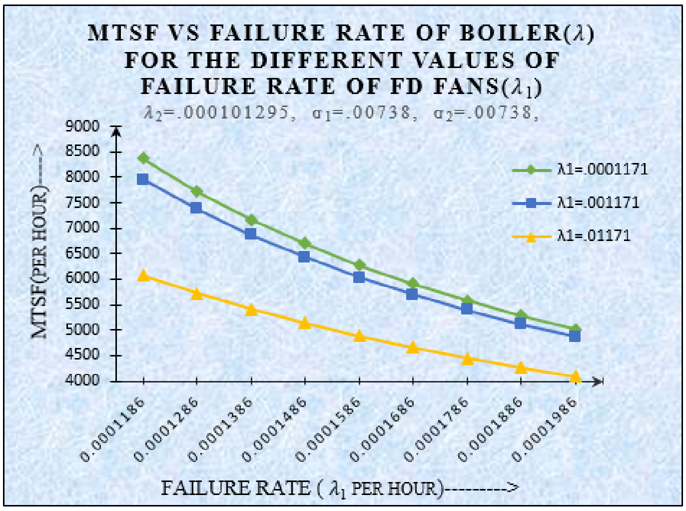

Figure 2: *MTSF vs Failure rate of Boiler*

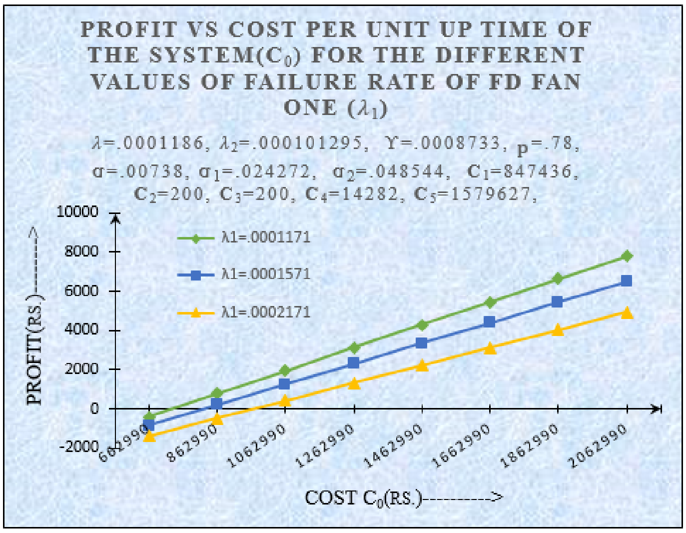

Figure 3: *Profit vs Revenue up-time for the system*

Table 5: *Cut Point for profit w.r.t. Revenue Up-time of the system.*

Failure rate of FD fan one (/hr)	Revenue per unit up time (Rs.)	Profit (Rs.)
$\lambda_1 = .0001171$	$C_0 < or = or > 731316.24$	negative or zero or positive
$\lambda_1 = .0001571$	$C_0 < or = or > 826502.352$	negative or zero or positive
$\lambda_1 = .0002171$	$C_0 < or = or > 969279.171.$	negative or zero or positive

In figure 2, the effect of the failure rate of Boiler(λ) on MTSF has shown for the different values of the failure rate of FD fan one (λ_1). As the failure rate (λ) increases, the MTSF of the system decreases. Also, as the failure rate (λ_1) increases MTSF of the system decreases. In figure 3, the effect of cost per unit up time of the system (C_0) w.r.t. profit has shown for the different values of failure rate of FD fan one (λ_1). As the cost C_0 increases, profit of the system increases. Also, as the failure rate of FD fan one (λ_1) increases profit decreases. Various cut point formed from the graph of profit w.r.t. revenue Up-time of the system as shown in table 5.

VII. Conclusion

Reliability modeling established for steam generation plant that shows the effect of service type & type of failures on system performance. The study reveals that the busy period of repairman for repair time is more as compared to replacement time. Also, cut-off points formed from profit helps the industrialist to maintain the economy of their system. In addition, any industry can consider the stated model to enhance the performance of their system using the different rates for repair/replacement.

References

[1] Agrawal, A. et. al (2021), Performance Analysis of the Water Treatment Reverse Osmosis Plant, *Reliability: Theory & Applications*, 16(3(63)): 16-25.

[2] Al Balushi, N. (2021), A Review of the Reliability Analysis of the Complex Industrial Systems, *Advances in Dynamical Systems and Applications*, 16(1):257–297.

[3] Al Rahbi, Y., & Rizwan, S. M. (2020), A Comparative Analysis between the Models of a Single Component with Single Repairman & Multiple Repairmen of an Aluminium Industry, *In 2020 International Conference on Computational Performance Evaluation (ComPE), IEEE*, 132–135.

[4] Bhardwj, R. K., & Malik, S. C. (2010). MTSF and Cost effectiveness of 2-out-of-3 cold standby system with probability of repair and inspection, *Int. J. of Eng. Sci. and Tech*, 2(1):5882–5889.

[5] Carrasco, J. A. (2003), Solving dependability/performability irreducible Markov models using regenerative randomization, *IEEE Transactions on Reliability*, 52(3):319–329.

[6] Li, W. et. al (1998), Stochastic analysis of a repairable system with three units and two repair facilities, *Microelectronics Reliability*, 38(4):585–595.

[7] Parashar, B., & Taneja, G., (2007), Reliability and profit evaluation of a PLC hot standby system based on a master-slave concept and two types of repair facilities, *IEEE Transactions on reliability*, 56(3):534–539.

[8] Rani, P., & Duhan, M. (2020), Reliability Modeling, Cost-Benefit Analysis with Inspection, Repair and Replacement Facility for Optical Fiber Network in DCRUST, *IUP Journal of Telecommunications*, 12(2):51-61.

[9] Sattiraju, R. et. al (2014), Reliability analysis of a wireless transmission as a repairable system, *In 2014 IEEE Globecom Workshops (GC Wkshps), IEEE*, 1397–1401.

[10] Sheetal, D. S., & Taneja, G. (2018), Reliability Analysis of a System Working in High Temperature Zones with Fault-Dependent Repair during Night hours. *International Journal of Applied Engineering Research*, 13(20):14650-14656.