Reliability and Economic Analysis of Captive Power Plant With Reduced Capacity

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Abstract

This paper reported the performance evaluation of Captive Power plant working in the fertilizer industry with possible production capacities. The idea of reduced capacity and load sharing to use the available system optimally is analyzed. The system works on two STG's (steam turbine generators) and one gridline. Gridline can bear the load of one or both STG's on failures. At the breakdown in gridline and STG, the system work at reduced capacity. Gridline repaired on a priority basis. The semi-Markov processses and regenerative point technique are used to evaluate the reliability and economic measures such as availability, busy period of repairman, and expected no. of repairs. The graphical study shows the relationships between these measures with the failure rates of STG and gridline.

Keywords: Steam Turbine Generators, Regenerative point technique, semi-Markov process, Reduced capacity, Reliability modeling.

I. Introduction

Nowadays, Captive power plants are a reliable and beneficial energy source for power-consuming production industries. Optimizing the operations of the power-producing units in these captive power plants can boost the industry's profit. A good number of researchers have worked on various reliability models with conditions of repair and maintenance. Gupta and Goel [3] studied a two-unit cold standby system working under abnormal weather conditions. Chandrashekhar et al. [1], Goyal et al. [2], Parashar et al. [4] have analyzed two and three-unit systems. Rizwan et al. [5] worked with the reliability of the hot standby industrial system.

Singh and Taneja [7], [8] analyzed power generating systems with various types of inspections. Rajesh et. al [10], [9] studied gas turbine power plants consisting of two and three units. These attempts to the literature create a motivation for the present study, to work with the economic benefits of the captive power plant. The captive power plants are auto producers of electricity, which operates off-grid or in parallel with gridline to make consistent and quality electricity supply for industries at reasonable costs. Availability of these power generating units in any possible way (full or reduced) can make a reliable electricity supply at less cost. Keeping an eye on the above fact economic analysis of Captive Power plant working in National Fertilizer limited, Bathinda, India has done.

The present system comprises two STG's (steam turbine generators) connected in parallel with the Gridline of PSPCL (Punjab state power corporation limited). These two STGs can fulfill the electrical load for the system. On failure of any one or both of STGs, the system operates with the help of gridline. The system will work at reduced capacity when only one STG is working (one STG and gridline failed). The failure of these three units leads to complete system failure. Repair of gridline is done on a priority basis among all units, whereas the FCFS repair pattern is applied on both STGs. The reliability measure MTSF (mean time to system failure) and economic measures such as availability, a busy period of the repairman, and expected no. of repairs have been derived using the semi-Markov processes and regenerative point techniques numerically. Also, graphical plotting was performed for these measures.

I. Assumptions for the model

- All failure time variables follow exponential distribution but repair times distributed generally..
- Every repaired unit works as new one.
- In the given model system initially started working from state S_0.

II. Nomenclature & Model Description

I. Notations & abbreviations

Notations		Discription
λ_1	:	Constant failure rate of STG 1.
λ_2	:	Constant failure rate of STG 2.
λ_3	:	Constant failure rate of Gridline.
α_1	:	Repair rate of STG 1.
α_2	:	Repair rate of STG 2.
α_3	:	Repair rate of Gridline.
$G_1(t)$, $g_1(t)$	:	c.d.f. & p.d.f of repair time of STG 1.
$G_2(t)$, $g_2(t)$	:	c.d.f. & p.d.f of repair time of STG 2.
$G_3(t)$, $g_3(t)$	:	c.d.f. & p.d.f of repair time of Gridline.
a	:	probability of transit from S_7 & S_8 to S_3 respectively after repair .
b	:	probability of transit from S_7 & S_8 to S_4 respectively after repair .
©	:	Laplace Convolution.
Ⓢ	:	Stieltjes Convolution.
$*/**$	:	Laplace Transformation/ Laplace Stieltjes Transformation.
$M_i(t)$	:	Probability that system is working at state S_i during the time interval $(0-t]$.
$W_i(t)$	:	Probability of repairman repairing at state S_i during the time interval $(0-t]$.

II. Symbols for States

Symbols for the states of the system:-
S_i : States of the system with number i, $i = 1, 2, 3, ...8$.
O_I, O_{II}, O_{III} : STG 1, STG 2, Gridline from PSPCL are in operating state.
CS_{III} : Gridline (PSPCL) in cold standby state.
Fr_I, Fr_{II}, Fr_{III} : STG 1, STG 2, Gridline under repair.
FR_I, FR_{II}, FR_{III} : STG 1, STG 2, Gridline under repair from previous state.
Fwr_I, Fwr_{II} :Failed Units STG 1, STG 2 waiting for repair.

III. State Transition Diagram

Figure 1, shows the state transitions diagram of the Captive power plant consisting of two STGs and one gridline from PSPCL. The states S_0, S_1, S_2, S_3, S_4 are operating states. The states $S_0, S_1, S_2, S_3, S_4, S_5, S_6, S_8$ are regenerative states. The states S_5, S_6 are reduced capacity states. The states S_7, S_8 are failed states. Table 1 shows the description of every state of the system.

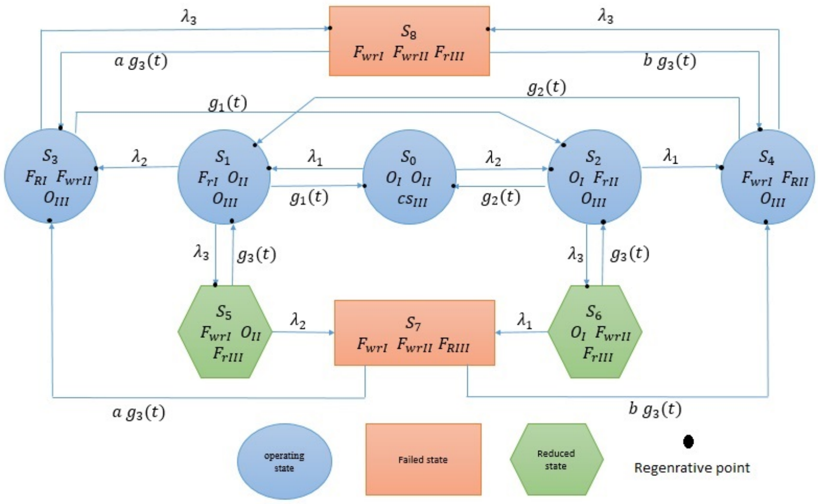

Figure 1: *State Transition Diagram*

Table 1: *State Description*

State notation	States Discription
S_0	This is the initial full capacity working state where both STGs are working. Gridline is in a standby state.
S_1	System working at full capacity where STG 1 and gridline are working. STG 2 is in a failed state under repair.
S_2	System working full capacity where STG 2 and gridline are working. STG 1 is in failed state under repair.
S_3, S_4	System is operating at full capacity with gridline. Both STGs are in a failed state.
S_5	System operating at reduced capacity where only STG 2 is working. STG 1 and gridline are in the failed state.
S_6	System operating at reduced capacity where only STG 1 is working. STG 2 and gridline are in failed states.
S_7, S_8	These are failed states where all units are in a failed state.

IV. Transition Probabilities & Mean Sojourn Times

p_{ij} represents non-zero elements which are given below The non zero elements p_{ij}'s are given as:

$$p_{01} = \frac{\lambda_1}{\lambda_1+\lambda_2}, \quad p_{02} = \frac{\lambda_2}{\lambda_1+\lambda_2}, \quad p_{10} = g_1^*(\lambda_3+\lambda_2),$$

$$p_{13} = \frac{\lambda_2}{\lambda_3+\lambda_2}[1-g_1^*(\lambda_3+\lambda_2)], \quad p_{15} = \frac{\lambda_3}{\lambda_3+\lambda_2}[1-g_1^*(\lambda_3+\lambda_2)], \quad p_{20} = g_2^*(\lambda_1+\lambda_3),$$

$$p_{24} = \frac{\lambda_1}{\lambda_1+\lambda_3}[1-g_2^*(\lambda_1+\lambda_3)], \quad p_{26} = \frac{\lambda_3}{\lambda_1+\lambda_3}[1-g_2^*(\lambda_1+\lambda_3)], \quad p_{32} = g_1^*(\lambda_3),$$

$$p_{38} = [1-g_1^*(\lambda_3)], \quad p_{41} = g_2^*(\lambda_3), \quad p_{48} = [1-g_2^*(\lambda_3)]$$

$$p_{51} = g_3^*(\lambda_2), \quad p_{57} = [1-g_3^*(\lambda_2)], \quad p_{53}^{(7)} = a[1-g_3^*(\lambda_2)],$$

$$p_{54}^{(7)} = b[1-g_3^*(\lambda_2)], \quad p_{62} = g_3^*(\lambda_1), \quad p_{67} = [1-g_3^*(\lambda_1)],$$

$$p_{63}^{(7)} = a[1-g_3^*(\lambda_1)], \quad p_{64}^{(7)} = b[1-g_3^*(\lambda_1)], \quad p_{83} = a,$$

$$p_{84} = b$$

The mean sojourn time μ_i corresponding to regenerative state $'i'$ is given as:

$\mu_0 = \frac{1}{\lambda_1+\lambda_2}$, $\mu_1 = \frac{1}{\lambda_3+\lambda_2}[1-g_1^*(\lambda_3+\lambda_2)]$, $\mu_2 = \frac{1}{\lambda_1+\lambda_3}[1-g_2^*(\lambda_1+\lambda_3)]$,
$\mu_3 = \frac{1}{\lambda_3}[1-g_1^*(\lambda_3)]$, $\mu_4 = \frac{1}{\lambda_3}[1-g_2^*(\lambda_3)]$, $\mu_5 = \frac{1}{\lambda_2}[1-g_3^*(\lambda_2)]$,
$\mu_6 = \frac{1}{\lambda_1}[1-g_3^*(\lambda_1)]$, $\mu_8 = -g_3^{*'}(0)$

The unconditional mean time m_{ij} required by the system to transit from state $'i'$ to any regenerative state $'j'$ when time is counted from the epoch of entrance into the state $'i'$ is mathematically stated as:

$$m_{ij} = \int_a^b t dQij(t) = -q_{ij}^{*'}(0) \tag{1}$$

So we have

$m_{01}+m_{02} = \mu_0$, $m_{10}+m_{13}+m_{15} = \mu_1$, $m_{20}+m_{24}+m_{28} = \mu_2$, $m_{32}+m_{38} = \mu_3$,
$m_{41}+m_{48} = \mu_4$, $m_{51}+m_{53}^{(7)}+m_{54}^{(7)} = k_1$, $m_{62}+m_{63}^{(7)}+m_{64}^{(7)} = k_1$, $m_{83}+m_{84} = \mu_8$

III. Reliability and Economic Measures for System Effectiveness

I. Mean Time to System Failure (MTSF)

Assume $\phi_i(t)$ as a distribution function of variable time (t) lapses during the system transition from a regenerative state S_i to any working or failed state where failed state act as an absorbing state. By probabilistic arguments, the following recursive relations are obtained:

$$\phi_0(t) = Q_{01}(t)Ⓢ\phi_1(t) + Q_{02}(t)Ⓢ\phi_2(t) \tag{2}$$
$$\phi_1(t) = Q_{10}(t)Ⓢ\phi_0(t) + Q_{13}(t)Ⓢ\phi_3(t) + Q_{15}(t)Ⓢ\phi_5(t) \tag{3}$$
$$\phi_2(t) = Q_{20}(t)Ⓢ\phi_0(t) + Q_{24}(t)Ⓢ\phi_4(t) + Q_{26}(t)Ⓢ\phi_6(t) \tag{4}$$
$$\phi_3(t) = Q_{32}(t)Ⓢ\phi_2(t) + Q_{38}(t) \tag{5}$$
$$\phi_4(t) = Q_{41}(t)Ⓢ\phi_1(t) + Q_{48}(t) \tag{6}$$
$$\phi_5(t) = Q_{51}(t)Ⓢ\phi_1(t) + Q_{57}(t) \tag{7}$$
$$\phi_6(t) = Q_{62}(t)Ⓢ\phi_2(t) + Q_{67}(t) \tag{8}$$

Transforming the equations(2-8) using Laplace Stieltjes Transformations to get $\phi_i^{**}(t)$. Mean Time to System Failure T_0 at steady state S_o is given by

$$T_0 = \lim_{s\to 0} \frac{1-\phi_0{}^{**}(s)}{s} \tag{9}$$

Using L' Hospital's rule here, we get

$$T_0 = N/D \tag{10}$$

where

$$N = \mu_0(1 - p_{15}p_{51} - p_{26}p_{62} - p_{15}p_{26}p_{62}p_{57} + p_{13}p_{24}p_{32}p_{48} + p_{15}p_{26}p_{62} - p_{24}p_{13}p_{32}) + \mu_1(-p_{62}p_{26}p_{01} + p_{02}p_{24}p_{41} + p_{01}) + \mu_2(-p_{51}p_{02}p_{15} + p_{13}p_{32}p_{01} + p_{02}) + \mu_3(p_{13}p_{01} - p_{13}p_{01}p_{26}p_{62} + p_{24}p_{02}p_{13}p_{41}) + \mu_4(p_{24}p_{02} - p_{24}p_{02}p_{15}p_{51} + p_{13}p_{01}p_{24}p_{32}) + \mu_5(p_{15}p_{01} - p_{15}p_{01}p_{26}p_{62} + p_{24}p_{02}p_{15}p_{41}) + \mu_6(p_{26}p_{02} - p_{15}p_{51}p_{26}p_{02} + p_{13}p_{01}p_{26}p_{32}) \tag{11}$$

and

$$D = 1 + p_{51}p_{26}p_{15}p_{62} + p_{51}p_{02}p_{20}p_{15} - p_{15}p_{51} - p_{26}p_{62} + p_{62}p_{26}p_{01}p_{10} - p_{13}p_{32}p_{24}p_{41} - p_{13}p_{32}p_{01}p_{20} - p_{02}p_{10}p_{24}p_{41} - p_{02}p_{20} - p_{01}p_{10} \tag{12}$$

II. Availability Analysis at Full & Reduced capacity

Let $A_i^F(t)$ notates the probability that system is available with full capacity to perform its intended task at a regenerative state S_i at time $t = 0$. The availability of system at successive regenerative state S_j $(j = 1, 2, ..6, 8)$ is independant from its previous transitions made. This phenomenon follows the theory of regenerative process techniques [6]. Thus following recursive relations are obtained:

$$A_0^F(t) = M_0(t) + q_{01}(t)©A_1^F(t) + q_{02}(t)©A_2^F(t) \tag{13}$$
$$A_1^F(t) = M_1(t) + q_{10}(t)©A_0^F(t) + q_{13}(t)©A_3^F(t) + q_{15}(t)©A_5(t) \tag{14}$$
$$A_2^F(t) = M_2(t) + q_{20}(t)©A_0^F(t) + q_{24}(t)©A_4^F(t) + q_{26}(t)©A_6(t) \tag{15}$$
$$A_3^F(t) = M_3(t) + q_{32}(t)©A_2^F(t) + q_{38}(t)©A_8^F(t) \tag{16}$$
$$A_4^F(t) = M_4(t) + q_{41}(t)©A_1^F(t) + q_{48}(t)©A_8^F(t) \tag{17}$$
$$A_5^F(t) = q_{51}(t)©A_1^F(t) + q_{53}^{(7)}(t)©A_3^F(t) + q_{54}^{(7)}(t)©A_4^F(t) \tag{18}$$
$$A_6^F(t) = q_{62}(t)©A_2^F(t) + q_{63}^{(7)}(t)©A_3^F(t) + q_{64}^{(7)}(t)©A_4^F(t) \tag{19}$$
$$A_8^F(t) = q_{83}(t)©A_3^F(t) + q_{84}(t)©A_4^F(t) \tag{20}$$

Where

$$M_0(t) = e^{-(\lambda_1+\lambda_2)t}, M_1(t) = e^{-(\lambda_1+\lambda_2)t}\bar{G_1}(t), M_2(t) = e^{-(\lambda_1+\lambda_2)t}\bar{G_2}(t),$$
$$M_3(t) = e^{-(\lambda_3)t}\bar{G_1}(t), M_4(t) = e^{-(\lambda_3)t}\bar{G_2}(t)$$

Transforming the equations(13-20) using Laplace transformations to get $A_0^{F*}(s)$. we have

$$A_0^F(t) = \lim_{s\to 0}(sA_0^{F*}(s)) \tag{21}$$

The steady state availability A_0^F of the system having full capacity is given by:

$$A_0^F = \lim_{t\to 0} A_0^F(t) = N_1/D_1 \tag{22}$$

where

$$\begin{aligned}
N_1 = \mu_0[(p_{15}p_{51} - 1)(p_{26}p_{32}p_{63} + (1 - p_{62}p_{26})(p_{38}p_{83} + p_{84}p_{48}) + p_{26}p_{62}(1 + p_{15}p_{54}^{(7)}p_{41}p_{38})\\
- p_{83}p_{24}p_{32}p_{48} - 1) + (1 - p_{62}p_{26})((p_{83}p_{54}^{(7)}p_{15} - p_{84}p_{53}^{(7)}p_{15} - p_{84}p_{13})p_{41}p_{38})\\
- p_{84}p_{32}p_{15}p_{26}p_{51}p_{48}(1 - p_{62}(1 - p_{63}^{(7)})) - (1 - p_{26}p_{32})p_{15}p_{54}^{(7)}p_{41} - p_{64}^{(7)}p_{32}p_{26}p_{41}(p_{13} + p_{15}p_{54}^{(7)})\\
- p_{24}p_{32}P_{41}(p_{13} + p_{15}p_{53}^{(7)})] + \mu_1[p_{01}((1 - p_{84}p_{48} - p_{83}p_{38})(1 - p_{26}p_{62}) - p_{63}^{(7)}p_{32}p_{26}(1 - p_{84}p_{48})\\
- p_{83}p_{48}p_{32}(p_{24} + p_{64}^{(7)}p_{26})) + p_{02}((p_{64}^{(7)}p_{41}p_{26} + p_{24}p_{41})(1 - p_{83}p_{38}) + p_{84}p_{63}^{(7)}p_{41}p_{26}p_{38})]\\
+ \mu_2[p_{02}((1 - p_{83}p_{38})(1 - p_{15}p_{51} - p_{15}p_{54}^{(7)}p_{41}) - p_{84}p_{48}(1 - p_{51}p_{15}) - p_{84}p_{38}p_{13}p_{41})\\
+ p_{01}p_{32}((p_{15}p_{53}^{(7)} + p_{13})(1 - p_{84}p_{48}) + p_{15}p_{54}^{(7)}p_{83}p_{48})] + \mu_3[(1 - p_{62}p_{26})(p_{15}p_{53}^{(7)}p_{01} + p_{01}p_{13}\\
- p_{01}p_{84}p_{13}p_{48}) + (p_{84}p_{63}^{(7)} - p_{83}p_{64}^{(7)})(p_{15}p_{51}p_{02}p_{26}p_{48} - p_{02}p_{26}p_{48}) + p_{83}p_{24}p_{02}p_{48}(1 - p_{15}p_{51})\\
- p_{15}p_{26}p_{02}p_{41}(p_{54}^{(7)}p_{53}^{(7)} - p_{64}^{(7)}p_{53}^{(7)}) + p_{15}p_{01}p_{48}(p_{83}p_{54}^{(7)} - p_{84}p_{53}^{(7)}) + p_{32}p_{02}p_{26} - p_{15}p_{26}p_{63}^{(7)}p_{51}p_{02}\\
+ p_{64}^{(7)}p_{02}p_{41}p_{13}p_{26} + p_{24}p_{15}p_{02}p_{41}p_{53}^{(7)} + p_{24}p_{02}p_{41}p_{13}] + \mu_4[(1 - p_{83}p_{38})(p_{24}p_{02} - p_{24}p_{15}p_{51}p_{02}\\
- p_{15}p_{64}^{(7)}p_{51}p_{02}p_{26} + p_{64}^{(7)}p_{02}p_{26} + p_{15}p_{01}p_{54}^{(7)}) + (p_{13}p_{15}p_{53}^{(7)})(p_{01}p_{24}p_{32} + p_{01}p_{64}^{(7)}p_{32}p_{26})\\
+ p_{15}p_{84}p_{53}^{(7)}p_{38}(p_{01} - p_{02}p_{41}) + p_{84}p_{63}p_{02}p_{26}p_{38}(1 - p_{15}p_{51}) + p_{01}p_{54}^{(7)}p_{15}(p_{26}(p_{62}(p_{83}p_{38}\\
- 1) - p_{63}^{(7)}p_{32}))]
\end{aligned} \tag{23}$$

and

$$
\begin{aligned}
D_1 = \mu_0((1-p_{83}p_{38})(p_{24}p_{41}p_{10}+p_{41}p_{64}^{(7)}p_{26}p_{10}+p_{20}-p_{51}p_{20}p_{15})+p_{41}p_{15}p_{38}p_{20}(p_{83}p_{54}^{(7)}- \\
p_{84}p_{53}^{(7)})-p_{84}p_{20}p_{48}(1-p_{15}p_{51})+p_{41}p_{84}p_{26}p_{63}^{(7)}p_{38}p_{10})+\mu_1(p_{41}p_{84}p_{38}(1-p_{26}p_{62})+p_{32}p_{01}p_{20} \\
(1-p_{84}p_{48})+p_{32}p_{24}p_{41}+p_{32}p_{41}p_{64}^{(7)}p_{26}-p_{41}p_{84}p_{38}p_{02}p_{20}+)+\mu_2(p_{32}p_{83}p_{48}(1-p_{01}p_{10}) \\
+p_{41}p_{02}p_{10}(1-p_{83}p_{38})+p_{32}p_{53}^{(7)}p_{41}p_{15}+p_{32}p_{13}p_{41}-p_{32}p_{15}p_{48}p_{51}p_{83})+\mu_3((1-p_{26}p_{62}) \\
(p_{41}p_{84}p_{15}p_{53}^{(7)}+p_{41}p_{84}p_{13}-p_{01}p_{10}p_{83}+p_{83}-p_{83}p_{51}p_{15})-p_{41}p_{83}p_{54}^{(7)}p_{15}(1-p_{02}p_{02}) \\
-p_{41}p_{84}p_{02}p_{20}(p_{13}+p_{53}^{(7)}p_{15})-p_{24}p_{41}p_{83}p_{02}p_{10}-p_{41}p_{64}^{(7)}p_{83}p_{26}p_{02}p_{10}+p_{41}p_{83}p_{54}^{(7)}p_{15}p_{26}p_{62} \\
+p_{83}p_{51}p_{15}p_{02}p_{20}+p_{41}p_{84}p_{26}p_{63}^{(7)}p_{02}p_{10})+\mu_4(p_{32}p_{24}p_{83}(1-p_{15}p_{51}-p_{01}p_{10})+p_{84}(1-p_{01}p_{10} \\
-p_{26}p_{62}-p_{15}p_{51})+p_{32}p_{84}p_{15}p_{26}p_{51}p_{63}^{(7)}+p_{32}p_{83}p_{54}^{(7)}p_{15}p_{01}p_{20}-p_{32}p_{64}^{(7)}p_{26}p_{15}p_{51}p_{83} \\
+p_{84}p_{26}p_{62}(p_{15}p_{51}+p_{01}p_{10})-p_{32}p_{84}p_{01}p_{20}(p_{53}^{(7)}p_{15}+p_{13})-p_{84}p_{02}p_{20}(1-p_{15}p_{51}) \\
+(1-p_{01}p_{10})(p_{32}p_{64}^{(7)}p_{83}p_{26}-p_{32}p_{84}p_{26}p_{63}^{(7)})+k_1(p_{41}p_{84}p_{15}p_{38}(1-p_{26}p_{62}-p_{02}p_{20}) \\
+p_{32}p_{24}p_{41}p_{15}+p_{32}p_{01}p_{20}p_{15}+p_{32}p_{41}p_{64}^{(7)}p_{26}p_{15}-p_{32}p_{15}p_{84}p_{48}p_{01}p_{20})+k_1(p_{32}p_{83}p_{26}p_{48}(1 \\
-p_{01}p_{10})+p_{41}p_{26}p_{02}p_{10}(1-p_{83}p_{38})+p_{32}p_{41}p_{53}^{(7)}p_{26}p_{15}+p_{32}p_{41}p_{26}p_{13})+\mu_8((1-p_{15}p_{51}) \\
(p_{32}p_{24}p_{48}-p_{38}p_{02}p_{20})+p_{32}p_{64}p_{26}p_{48}(1-p_{01}p_{10})-p_{41}p_{54}^{(7)}p_{15}p_{38}(1-p_{02}p_{20}-p_{26}p_{63}^{(7)}) \\
+p_{38}(-p_{26}p_{62}-p_{01}p_{10}-p_{15}p_{51})+p_{26}p_{38}p_{62}(p_{15}p_{51}+p_{01}p_{10})-p_{41}p_{38}p_{02}p_{10}(p_{24}+p_{64}^{(7)}p_{26}) \\
-p_{32}p_{24}p_{48}p_{01}p_{10}-p_{32}p_{64}^{(7)}p_{26}p_{15}p_{48}p_{51}) \quad (24)
\end{aligned}
$$

Let $A_i^R(t)$ notates the probability that system is available with reduced capacity to work at a regenerative state S_i at time $t=0$. The following recursive relations are obtained using the above described argument of regenrative process techniques:

$$A_0^R(t) = q_{01}(t)©A_1^R(t) + q_{02}(t)©A_2^R(t) \quad (25)$$

$$A_1^R(t) = q_{10}(t)©A_0^R(t) + q_{13}(t)©A_3^R(t) + q_{15}(t)©A_5^R(t) \quad (26)$$

$$A_2^R(t) = q_{20}(t)©A_0^R(t) + q_{24}(t)©A_4^R(t) + q_{26}(t)©A_6^R(t) \quad (27)$$

$$A_3^R(t) = q_{32}(t)©A_2^R(t) + q_{38}(t)©A_8^R(t) \quad (28)$$

$$A_4^R(t) = q_{41}(t)©A_1^R(t) + q_{48}(t)©A_8^R(t) \quad (29)$$

$$A_5^R(t) = M_5(t) + q_{51}(t)©A_1^R(t) + q_{53}^{(7)}(t)©A_3^R(t) + q_{54}^{(7)}(t)©A_4^R(t) \quad (30)$$

$$A_6^R(t) = M_6(t) + q_{62}(t)©A_2^R(t) + q_{63}^{(7)}(t)©A_3^R(t) + q_{64}^{(7)}(t)©A_4^R(t) \quad (31)$$

$$A_8^R(t) = q_{83}(t)©A_3^R(t) + q_{84}(t)©A_4^R(t) \quad (32)$$

where

$$M_5(t) = e^{-(\lambda_2)t}\bar{G_3}(t), M_6(t) = e^{-(\lambda_1)t}\bar{G_3}(t)$$

Transforming the equations(25-32) using Laplace transformations to get $A_0^{R*}(s)$. we have

$$A_0^R(t) = \lim_{s\to 0}(sA_0^{R*}(s)) \quad (33)$$

The steady state availability A_0^R of the system having reduced capacity is given by:

$$A_0^R = \lim_{s\to 0}(sA_0^{R*}(s)) = N_2/D_1 \quad (34)$$

where

$$N_2 = \mu_5(p_{24}p_{02}p_{15}p_{41} - p_{15}p_{01}p_{26}p_{62} - p_{15}p_{01}p_{38}p_{83} - p_{84}p_{15}p_{01}p_{48} - p_{15}p_{01}p_{26}p_{32}p_{63} + p_{64}^{(7)}p_{15}p_{41}p_{26}p_{02} + p_{15}p_{01} - p_{64}^{(7)}p_{15}p_{41}p_{26}p_{02}p_{38}p_{83} - p_{64}^{(7)}p_{15}p_{01}p_{26}p_{32}p_{83}p_{48} + p_{84}p_{15}p_{41}p_{26}p_{02}p_{38}p_{63}^{(7)} + p_{84}p_{15}p_{02}p_{26}p_{32}p_{63}^{(7)}p_{48} + p_{15}p_{01}p_{26}p_{62}p_{38}p_{83} - p_{24}p_{02}p_{15}p_{41}p_{38}p_{83} + p_{84}p_{15}p_{01}p_{26}p_{62}p_{48} - p_{24}p_{32}p_{83}p_{15}p_{01}p_{48}) + \mu_6(p_{26}p_{02} - p_{26}p_{02}p_{38}p_{83} + p_{26}p_{32}p_{13}p_{01} - p_{15}p_{51}p_{26}p_{02} - p_{84}p_{26}p_{02}p_{48} + p_{15}p_{51}p_{26}p_{02}p_{38}p_{83} + p_{15}p_{01}p_{26}p_{32}p_{53}^{(7)} + p_{54}^{(7)}p_{15}p_{41}p_{26}p_{02}p_{38}p_{83} + p_{54}^{(7)}p_{15}p_{01}p_{26}p_{32}p_{83}p_{48} + p_{84}p_{51}p_{15}p_{26}p_{02}p_{48} - p_{84}p_{15}p_{41}p_{26}p_{02}p_{38}p_{53}^{(7)} - p_{84}p_{15}p_{01}p_{26}p_{32}p_{53}^{(7)}p_{48} - p_{84}p_{26}p_{02}p_{38}p_{13}p_{41} - p_{84}p_{26}p_{32}p_{13}p_{01}p_{48} - p_{54}^{(7)}p_{15}p_{41}p_{26}p_{02}) \quad (35)$$

and D_1 is already specified in equation 24

III. Busy Period for Repairman

Let $B_i(t)$ notates the probability that the repairman is busy on the job when the system is at a regenerative state S_i at time $t = 0$. Using the probabilistic arguments as described above, The following recursive relations are obtained:

$$B_0(t) = q_{01}(t)©B_1(t) + q_{02}(t)©B_2(t) \quad (36)$$
$$B_1(t) = W_1(t) + q_{10}(t)©B_0(t) + q_{13}(t)©B_3(t) + q_{15}(t)©B_5(t) \quad (37)$$
$$B_2(t) = W_2(t) + q_{20}(t)©B_0(t) + q_{24}(t)©B_4(t) + q_{26}(t)©B_6(t) \quad (38)$$
$$B_3(t) = q_{32}(t)©B_2(t) + q_{38}(t)©B_8(t) \quad (39)$$
$$B_4(t) = q_{41}(t)©B_1(t) + q_{48}(t)©B_8(t) \quad (40)$$
$$B_5(t) = W_5(t) + q_{51}(t)©B_1(t) + q_{53}^{(7)}(t)©B_3(t) + q_{54}^{(7)}(t)©B_4(t) \quad (41)$$
$$B_6(t) = W_5(t) + q_{62}(t)©B_2(t) + q_{63}^{(7)}(t)©B_3(t) + q_{64}^{(7)}(t)©B_4(t) \quad (42)$$
$$B_8(t) = W_8(t) + q_{83}(t)©B_3(t) + q_{84}(t)©B_4(t) \quad (43)$$

Where

$$W_0(t) = e^{-(\lambda_1+\lambda_2)t}, W_1(t) = e^{-(\lambda_1+\lambda_2)t}\bar{G_1}(t), W_2(t) = e^{-(\lambda_1+\lambda_2)t}\bar{G_2}(t), W_5(t) = e^{-(\lambda_2)t}\bar{G_3}(t), W_6(t) = e^{-(\lambda_1)t}\bar{G_3}(t), W_8(t) = (a+b)\bar{G_3}(t)$$

Taking Laplace transform of equations (36-43) we get $B_0^*(s)$.
we have

$$B_0(t) = \lim_{s\to 0}(sB_0^*(s))$$

Expected busy period of a repairman is given by

$$B_0 = \lim_{t\to 0}(B_0(t)) = N_3/D_1 \quad (44)$$

where

$$N_3 = \mu_1[(p_{26}p_{64}^{(7)} + p_{24})(p_{02}p_{41} - p_{41}p_{83}p_{38}p_{02} - p_{83}p_{32}p_{48}p_{01}) + (p_{84}p_{48} + p_{83}p_{38})(p_{62}p_{26}p_{01} - p_{01}) - p_{63}^{(7)}p_{32}p_{26}p_{01}(1 - p_{84}p_{48}) - p_{01}(p_{62}p_{26} - 1)] + \mu_2[(1 - p_{84}p_{48})(p_{53}^{(7)}p_{32}p_{15}p_{01}$$

$$+ p_{13}p_{32}p_{01} + p_{02} + p_{15}p_{51}p_{02}) - p_{83}p_{38}p_{02}(1 - p_{51}p_{15}) - p_{41}p_{15}p_{38}p_{02}(p_{53}^{(7)}p_{84} + p_{83}p_{54}^{(7)}) + p_{15}p_{54}^{(7)}(p_{83}p_{32}p_{48}p_{01} - p_{41}p_{02}) - p_{41}p_{13}p_{84}p_{38}p_{02}] + \mu_5[(1 - p_{62}p_{26})(p_{15}p_{01} - p_{84}p_{15}p_{48}p_{01} - p_{83}p_{15}p_{01}p_{38}) + (1 - p_{84}p_{48})(-p_{63}^{(7)}p_{32}p_{15}p_{26}p_{01}) + (p_{02}p_{41}p_{15}p_{26}p_{64}^{(7)} + p_{02}p_{41}p_{15}p_{24}) (1 - p_{83}p_{38}) + p_{15}p_{26}(p_{41}p_{63}^{(7)}p_{84}p_{38}p_{02} - p_{83}p_{32}p_{64}^{(7)}p_{48}p_{01})] + \mu_6[(1 - p_{15}p_{51})(-p_{02}p_{84}p_{26}p_{48} - p_{83}p_{26}p_{38}p_{02} + p_{02}p_{26})(1 - p_{84}p_{48})(p_{53}^{(7)}p_{32}p_{15}p_{26}p_{01}p_{13}p_{32}p_{26}p_{01}) + p_{83}p_{32}p_{15}p_{54}^{(7)}p_{26}p_{48}p_{01}] + \mu_8[(1 - p_{62}p_{26})(p_{13}p_{01}p_{38} + p_{53}^{(7)}p_{15}p_{01}p_{38}) + (1 - p_{15}p_{51})(p_{02}p_{24}p_{48} + p_{02}p_{64}^{(7)}p_{26}p_{48} + p_{63}^{(7)}p_{26}p_{38}p_{02}) + (p_{24} + p_{26}p_{64}^{(7)})(p_{13}p_{32}p_{48}p_{01} + p_{53}^{(7)}p_{32}p_{15}p_{48}p_{01}) - p_{15}p_{26}p_{54}^{(7)}p_{48}p_{01}(p_{62} + p_{63}^{(7)}p_{32}) + p_{41}p_{15}p_{26}p_{38}p_{02}(p_{53}^{(7)}p_{64}^{(7)} - p_{63}^{(7)}p_{54}^{(7)}) + p_{15}p_{54}^{(7)}p_{48}p_{01} + p_{41}p_{13}p_{26}p_{38}p_{64}^{(7)}p_{02}] \quad (45)$$

and D_1 is already specified in equation 24.

IV. Expected No. of Repairs

Let $V_i(t)$ notate no. of repairs performed by repairman in the time interval (0 to t] when the system is at regenerative state S_i at time $t = 0$. The general formula for $V_i(t)$ is given by

$$V_i(t) = \sum_j Q_{ij}^{(n)}(t)Ⓢ[\alpha_j + V_i(t)] \quad (46)$$

Where $Q_{ij}(t)$ is the probability of system transition from the regenerative state i to regenerative j and $\alpha_j = 1$ if the repairman starts new job at regenerative state j, otherwise $\alpha_j = 0$. Using Equation 46 the following recursive relations are obtained:

$$V_0(t) = Q_{01}(t)Ⓢ[1 + V_1(t)] + Q_{02}(t)Ⓢ[1 + V_2(t)] \quad (47)$$
$$V_1(t) = Q_{10}(t)ⓈV_0(t) + Q_{13}(t)ⓈV_3(t) + Q_{15}(t)ⓈV_5(t) \quad (48)$$
$$V_2(t) = Q_{20}(t)ⓈV_0(t) + Q_{24}(t)ⓈV_4(t) + Q_{26}(t)ⓈV_6(t) \quad (49)$$
$$V_3(t) = Q_{32}(t)ⓈV_2(t) + Q_{38}(t)ⓈV_8(t) \quad (50)$$
$$V_4(t) = Q_{41}(t)ⓈV_1(t) + Q_{48}(t)ⓈV_8(t) \quad (51)$$
$$V_5(t) = Q_{51}(t)ⓈV_1(t) + Q_{53}^{(7)}(t)ⓈV_3(t) + Q_{54}^{(7)}(t)ⓈV_4(t) \quad (52)$$
$$V_6(t) = Q_{62}(t)ⓈV_2(t) + Q_{63}^{(7)}(t)ⓈV_3(t) + Q_{64}^{(7)}(t)ⓈV_4(t) \quad (53)$$
$$V_8(t) = Q_{83}(t)ⓈV_3(t) + Q_{84}(t)ⓈV_4(t) \quad (54)$$

Taking Laplace Stieltjes Transformations of the equations (47-54) to get $V_0{}^{**}(s)$.
we have

$$V_0(t) = \lim_{s\to 0}(sV_0^{**}(s)) \quad (55)$$

The expected no. of repairs by repairman are given by

$$V_0 = \lim_{t\to 0}(V_0(t)) = N_4/D_1$$

where

$$N_4 = (1 - p_{15}p_{51})(p_{38}p_{83}p_{26}p_{62} - p_{24}p_{32}p_{83}p_{48} + p_{84}p_{48}p_{26}p_{62} - p_{64}^{(7)}p_{26}p_{32}p_{83}p_{48} - p_{26}p_{62} - p_{83}p_{38} - p_{84}p_{48} + 1) + (1 - p_{26}p_{62})(-p_{84}p_{38}p_{13}p_{41} + p_{54}^{(7)}p_{15}p_{41}p_{38}p_{83} - p_{84}p_{15}p_{41}p_{38}p_{53}^{(7)} - p_{54}^{(7)}p_{15}p_{41}) + p_{26}p_{32}p_{63}^{(7)}(p_{54}^{(7)}p_{15}p_{41} + p_{15}p_{51} + p_{84}p_{48} - p_{84}p_{48}p_{15}p_{51} - 1) - p_{64}^{(7)}p_{15}p_{41}p_{26}p_{32}p_{53}^{(7)} - p_{64}^{(7)}p_{26}p_{32}p_{13}p_{41} - p_{24}p_{32}p_{53}^{(7)}p_{15}p_{41} - p_{24}p_{32}p_{13}p_{41} \quad (56)$$

and D_1 is already specified in equation 24.

IV. Profit Analysis

The expected total profit per unit time incurred to the system in steady state is given by

$$P_0 = C_0 A_0^F + C_1 A_0^R - C_2 B_0 - C_3 V_0 \tag{57}$$

Where
C_0 = revenue per unit up time at full capacity.
C_1 = revenue per unit up time at reduced capacity.
C_2 = cost per unit time when repairman is busy.
C_3 = cost per repair.

V. Particular Cases

For evaluation of above described various system performance measures and their graphical representation, the following particular cases are considered, where distribution of repair times has been taken as exponential. Let us assume that $g_1(t) = \alpha_1 e^{-\alpha_1 t}, g_2(t) = \alpha_2 e^{-\alpha_2 t}, g_3(t) = \alpha_3 e^{-\alpha_3 t}$ and remaining distributions same as in general case. Therefore we have

$$p_{01} = \frac{\lambda_1}{\lambda_1+\lambda_2}, \quad p_{02} = \frac{\lambda_2}{\lambda_1+\lambda_2}, \quad p_{10} = \frac{\alpha_1}{\lambda_2+\lambda_3+\alpha_1}, \quad p_{13} = \frac{\lambda_2}{\lambda_2+\lambda_3+\alpha_1},$$

$$p_{15} = \frac{\lambda_3}{\lambda_2+\lambda_3+\alpha_1}, \quad p_{20} = \frac{\alpha_2}{\lambda_1+\lambda_3+\alpha_2}, \quad p_{24} = \frac{\lambda_1}{\lambda_1+\lambda_3+\alpha_2}, \quad p_{26} = \frac{\lambda_3}{\lambda_1+\lambda_3+\alpha_2},$$

$$p_{32} = \frac{\alpha_1}{\alpha_1+\lambda_3}, \quad p_{38} = \frac{\lambda_3}{\lambda_3+\alpha_1}, \quad p_{41} = \frac{\alpha_2}{\alpha_2+\lambda_3}, \quad p_{48} = \frac{\lambda_3}{\lambda_3+\alpha_2},$$

$$p_{51} = \frac{\alpha_3}{\alpha_3+\lambda_2}, \quad p_{57} = \frac{\lambda_2}{\lambda_2+\alpha_3}, \quad p_{53}^{(7)} = a\left[\frac{\lambda_2}{\alpha_3+\lambda_2}\right], \quad p_{54}^{(7)} = b\left[\frac{\lambda_2}{\alpha_3+\lambda_2}\right],$$

$$p_{63}^{(7)} = a\left[\frac{\lambda_1}{\alpha_3+\lambda_1}\right], \quad p_{64}^{(7)} = b\left[\frac{\lambda_1}{\alpha_3+\lambda_1}\right], \quad p_{62} = \frac{\alpha_3}{\alpha_3+\lambda_1}, \quad p_{67} = \frac{\lambda_1}{\lambda_1+\alpha_3},$$

$$p_{83} = a, \quad p_{84} = b, \quad \mu_0 = \frac{1}{\lambda_1+\lambda_2}, \quad \mu_1 = \frac{1}{\lambda_2+\lambda_3+\alpha_1},$$

$$\mu_2 = \frac{1}{\lambda_1+\lambda_3+\alpha_1}, \quad \mu_3 = \frac{1}{\lambda_3+\alpha_1}, \quad \mu_4 = \frac{1}{\lambda_3+\alpha_2}, \quad \mu_5 = \frac{1}{\lambda_2+\alpha_3},$$

$$\mu_6 = \frac{1}{\lambda_1+\alpha_3}, \quad \mu_8 = \frac{1}{\alpha_3}$$

Estimation of Parameters

The various parameters regarding failure and repair rates involved in our studies are estimated as follows in table 2

Table 2: *Failure & repair rates*

Various rates	corresponding values
Failure rate of STG 1 (λ_1)	0.00043/hr
Failure rate of STG 2 (λ_2)	0.00043/hr
Failure rate of gridline (λ_3)	0.0067/hr
Repair rate of STG 1 (α_1)	0.0065/hr
Repair rate of STG 2 (α_2)	0.0063/hr
Repair rate of gridline (α_3)	0.34/hr

The various costs/revenue amounts involved in our studies are assumed hypothetically. The computed values of Various reliability measures for system performance are given in table 3.

Table 3: *Evaluation of various system effectiveness measures*

Mean time to system failure	39140 hrs.
Availability of the system at full capacity	0.50093/hr
Availability of the system at reduced capacity	0.001044/hr
Busy period of repairman for repair time only	0.12964/hr
Expected no. of repairs	0.000350/hr

VI. Results and Discussion

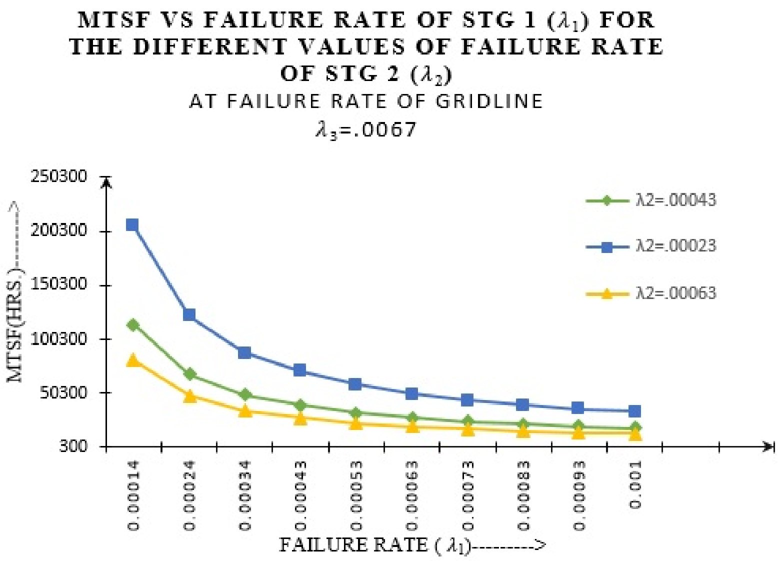

Figure 2: *MTSF vs* (λ_1)*Failure rate of STG*

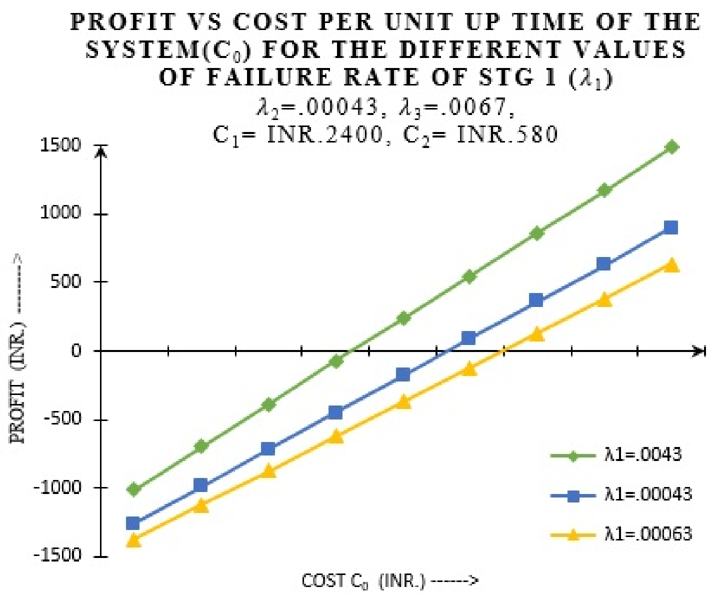

Figure 3: *profit vs cost per unit up time of system*

The figure 2, indicates that MTSF decreases as the falure rate of STG 1 increases and also gives lowering values for the greater values of failure rates of gridline. The graph in figure 3 interpreted that the profit increases with increasing the cost per unit up time of the system and decreases when failure rate of the STG 1 increases.

Table 4: *Cut Point for profit w.r.t. Revenue per unit up-time of the system.*

Failure rate of STG(/hr)	Revenue per unit up time(C_0) (Rs.)	Profit (Rs.)
$\lambda_1 = .00034$	$C_0 < or = or > 161$	negative or zero or positive
$\lambda_1 = .0034$	$C_0 < or = or > 380$	negative or zero or positive
$\lambda_1 = .034$	$C_0 < or = or > 606.$	negative or zero or positive

VII. Conclusion

In this paper self electricity generating System is Studied. The graphical study reveals the negative relationship between failure rates of units of captive power plant and profit gained by the plant. Adding the working of system at reduced capacity results in increasing its availability and profit. The derived results enable us to find acceptable values of revenue per unit up time of the system (Table 4) corresponding to failure rates of units of system. By using this analysis and graphical representations one can procure various system effectiveness measures for similar electricity generation plant.

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