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Power Length biased weighted lomax distribution

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Abstract

In this research paper, we have proposed the Power Length Biased Weighted Lomax Distribution(PLBWLD) as a new probability model . Moments, moment generating function, characteristic function, cumulant generating function, and reliability analysis such as survival function, hazard rate, reverse hazard rate, cumulative hazard function, and mills ratio are among the statistical features of PLBWLD that have been obtained here. Order statistics and PLBWLD's generalized entropy are also calculated. Maximum likelihood estimation is used to estimate the parameters of the model. Finally for demonstration purposes an application to the real data sets is provided to understand the new probability model's performance and flexibility.

Keywords: Length biased weighted Lomax distribution, power length biased weighted Lomax distribution, hazard rate function, moments, maximum likelihood estimation, order statistics, generalized entropy.

1. Introduction

Pareto distribution of second type is another name for the Lomax distribution. Lomax distribution was first used to model the failure rate of businesses by Lomax [8]. In the literature, the Lomax distribution has been employed in a variety of ways. According to Balkema and de Haan[3], it has been extensively utilized for life testing and reliability modeling including insurance, actuarial, demographics, economics, medical sciences, finance and engineering. The number of novel models with a high degree of flexibility is growing year after year. As a result, the researchers have shifted their focus to create new families of distributions and propose a variety of new families of distributions in order to better examine and investigate real-world data in various applications. Statistical distributions have gained a lot of attention recently as researchers try to figure out how to create flexible models for modelling a variety of data sets. It is because the classical distributions aren't very good at modelling data sets with a lot of variation. As a result, generalised probability models continue to grow and expand. In recent years, designing a new probability model from previously established models using various methodologies has gained a lot of attention. The power transformation technique, in which an extra parameter is added to the parent distribution, is one such strategy employed by several researchers. The addition of an extra parameter to the parent model usually increases the goodness of fit and gives more flexibility. Krishnarani [6] , Zaka and Akhter [9] are few of the researchers who have worked on power generalization of probability models. The concept of weighted distributions was first developed by Fisher [5].

If X is a non-negative random variable with the probability density function $f(x)$, then the probability density function of the weighted random variable X_w is given by

$$f_w(x) = \frac{w(x)f(x)}{E(w(x))} \; ; \; x \geq 0$$

When $w(x) = x$, the resultant distribution is clearly length biased, with a probability density function given as;

$$f_L(x) = \frac{xf(x)}{E(x)} \; ; \; x \geq 0$$

If the random variable X has the length biased weighted Lomax distribution with shape parameter η and scale parameter λ respectively, then it's probability density function(pdf) and cumulative distribution function (cdf) proposed by Ahmad *et al.* [1] , are respectively given as

$$f(x;\eta,\lambda) = \frac{\eta(\eta-1)}{\lambda^2} x \left(1+\frac{x}{\lambda}\right)^{-(\eta+1)} \; ; \; x > 0, \eta > 1, \lambda > 0$$

$$F(x;\eta,\lambda) = 1 - \left(1+\frac{x}{\lambda}\right)^{-\eta} \left(1+\frac{x\eta}{\lambda}\right) \; ; \; x > 0, \eta > 1, \lambda > 0$$

2. POWER LENGTH BIASED WEIGHTED LOMAX DISTRIBUTION(PLBWLD)

The primary goal of this research paper is to improve the flexibility of the length biased weighted Lomax distribution by developing an expanded version of the model using power transformation technique. Suppose the random variable X assumes the length biased weighted Lomax distribution with parameters η and λ, then the transformed variable $V = X^{\frac{1}{\beta}}$ will follow power length biased weighted Lomax distribution with parameters η, β and λ.
The probability density function of the power length biased weighted Lomax distribution is obtained as;

$$f(v;\eta,\beta,\lambda) = \frac{\eta(\eta-1)\beta}{\lambda^2} v^{2\beta-1} \left(1+\frac{v^\beta}{\lambda}\right)^{-(\eta+1)} \; ; \; x > 0, \eta > 1, \lambda, \beta > 0 \tag{1}$$

The cumulative distribution function of the power length biased weighted Lomax distribution is obtained as

$$F(v;\eta,\beta,\lambda) = 1 - \left(1+\frac{v^\beta}{\lambda}\right)^{-\eta} \left(1+\frac{v^\beta\eta}{\lambda}\right) \tag{2}$$

For the visual illustration of the possible shapes of pdf and cdf of PLBWLD, Figure 1 and Figure 2 have been plotted. Plots of the survival function and hazard rate function of the PLBWLD distribution for different parameter values are also displayed in Figure 3
Remark: For $\beta = 1$ in 1, we obtain the length biased weighted Lomax distrbution.

3. RELIABILITY ANALYSIS OF THE POWER LENGTH BIASED WEIGHTED LOMAX DISTRIBUTION(PLBWLD)

This section focuses on obtaining the reliability (survival function), hazard rate (failure rate), reverse hazard function, cumulative hazard function and mills ratio expressions respectively for PLBWLD.

3.1. Survival function

The survival function or reliability function is the complement of the cumulative distribution function and it is defined as the probability that a system will survive beyond a specified time

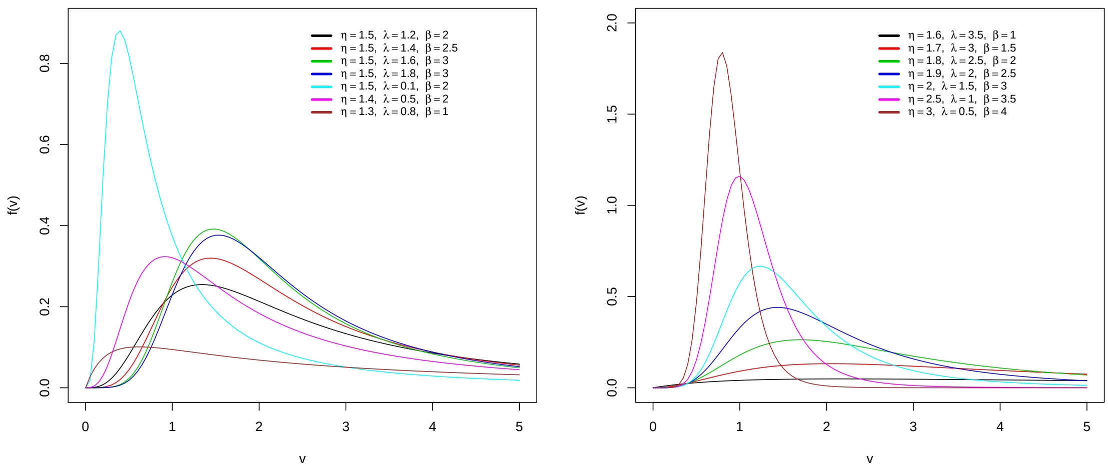

Figure 1: *Pdf Plots of the PLBWLD density for various values of η , β and λ.*

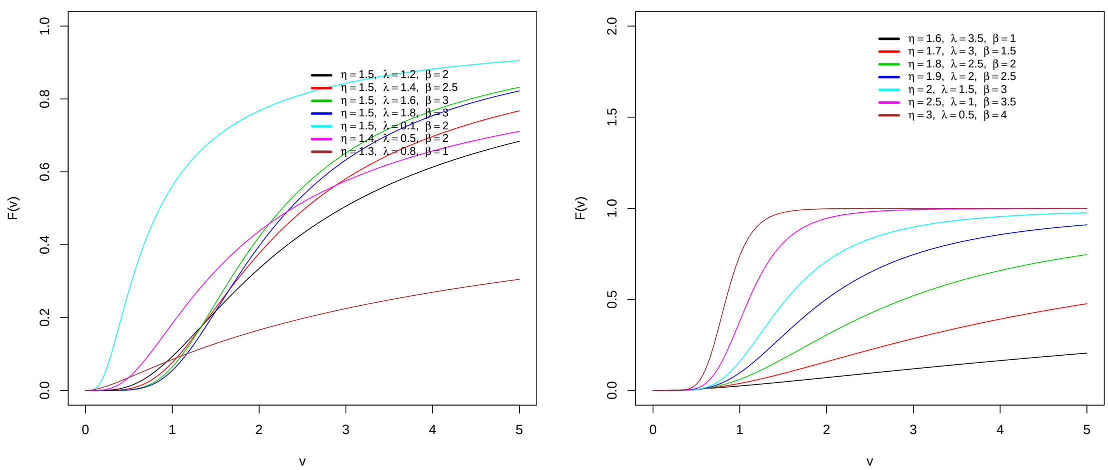

Figure 2: *Distribution function Plots of the PLBWLD for various values of η , β and λ.*

and is obtained for the PLBWLD as

$$R(v;\eta,\beta,\lambda) = 1 - F(v;\eta,\beta,\lambda) = \left(1+\frac{v^\beta}{\lambda}\right)^{-\eta}\left(1+\frac{v^\beta\eta}{\lambda}\right) \tag{3}$$

3.2. Hazard Rate

Hazard rate also known as hazard function , force of mortality or failure rate. The Hazard rate assess the ability of a lifetime component to fail or to expire depending on the life completed and thus has wide variety of applications in lifetime distributions. Using (1) and (3), the expression for the hazard rate of PLBWLD is obtained as

$$h(v;\eta,\beta,\lambda) = \frac{f(v;\eta,\beta,\lambda)}{R(v;\eta,\beta,\lambda)} = \frac{\eta(\eta-1)\beta}{\lambda^2}v^{2\beta-1}\left(1+\frac{v^\beta}{\lambda}\right)^{-1}\left(1+\frac{v^\beta\eta}{\lambda}\right)^{-1} \tag{4}$$

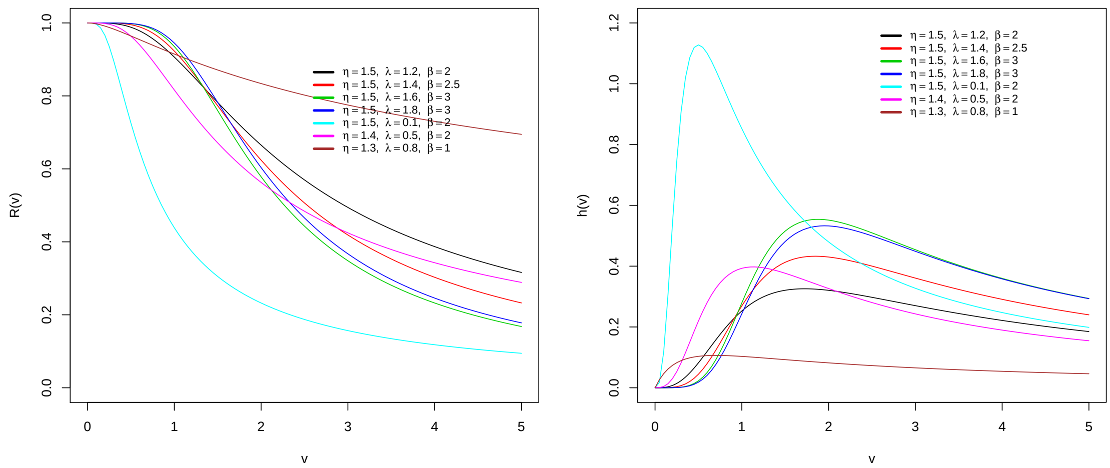

Figure 3: *Survival function and Hazard Rate Plots of the PLBWLD for various values of η , β and λ.*

3.3. Reverse Hazard function

The concept of reversed hazard rate of a random life is defined as the ratio between the life probability density to its distribution function . It is expressed as

$$h_r(v;\eta,\beta,\lambda) = \frac{f(v;\eta,\beta,\lambda)}{F(v;\eta,\beta,\lambda)}$$

Using equation (1) and (2) , the reverse hazard function for the Power length biased weighted Lomax distribution is obtained as

$$h_r(v;\eta,\beta,\lambda) = \frac{\frac{\eta(\eta-1)\beta}{\lambda^2} v^{2\beta-1}\left(1+\frac{v^\beta}{\lambda}\right)^{-(\eta+1)}}{1-\left(1+\frac{v^\beta}{\lambda}\right)^{-\eta}\left(1+\frac{v^\beta\eta}{\lambda}\right)} \tag{5}$$

3.4. Cumulative Hazard function

The cumulative hazard function can be thought of as providing the total accumulated risk of experiencing the event of interest that has been gained by progressing to time t. The cumulative hazard function for the PLBWLD is defined as

$$\Lambda_{PLBWLD}(v;\eta,\beta,\lambda) = -\log R(v;\eta,\beta,\lambda)$$

$$\Lambda_{PLBWLD}(v;\eta,\beta,\lambda) = \log\left\{\frac{\left(1+\frac{v^\beta}{\lambda}\right)^\eta}{\left(1+\frac{v^\beta\eta}{\lambda}\right)}\right\} \tag{6}$$

3.5. Mills Ratio

The mills ratio for the power length biased weighted Lomax distribution is defined as

$$M.R = \frac{F(v;\eta,\beta,\lambda)}{R(v;\eta,\beta,\lambda)} = \frac{1}{\left(1+\frac{v^\beta}{\lambda}\right)^{-\eta}\left(1+\frac{v^\beta\eta}{\lambda}\right)} - 1 \tag{7}$$

4. RESIDUAL AND REVERSED RESIDUAL LIFE FUNCTIONS OF THE POWER LENGTH BIASED WEIGHTED LOMAX DISTRIBUTION(PLBWLD)

4.1. Residual life function

In life testing situations, the additional lifetime given that a component has survived until time t is called residual life function. More specifically, if v is the life of a component, then the random variable $r(t) = (v - t|v > t); t \geq 0$ is used to explain the residual life of a lifetime component. For the PLBWLD, the survival function of the residual life time $r_{(t)}, t \geq 0$ is defined as

$$R_{r_{(t)}}(v;\eta,\beta,\lambda) = \frac{R(v+t)}{R(v)}$$

$$R_{r_{(t)}}(v;\eta,\beta,\lambda) = \left(1+\frac{(v+t)^{\beta}}{\lambda}\right)^{-\eta}\left(1+\frac{(v+t)^{\beta}\eta}{\lambda}\right)\left(1+\frac{t^{\beta}}{\lambda}\right)^{\eta}\left(1+\frac{t^{\beta}\eta}{\lambda}\right)^{-1} \tag{8}$$

For the residual life time random variable $r_{(t)}, t \geq 0$, the cdf and pdf are respectively obtained as

$$F_{r_{(t)}}(v;\eta,\beta,\lambda) = 1 - R_{r_{(t)}}(v;\eta,\beta,\lambda)$$

$$F_{r_{(t)}}(v;\eta,\beta,\lambda) = 1-\left(1+\frac{(v+t)^{\beta}}{\lambda}\right)^{-\eta}\left(1+\frac{(v+t)^{\beta}\eta}{\lambda}\right)\left(1+\frac{t^{\beta}}{\lambda}\right)^{\eta}\left(1+\frac{t^{\beta}\eta}{\lambda}\right)^{-1} \tag{9}$$

On differentiating the above equation w.r.t v, we obtain

$$f_{r_{(t)}}(v;\eta,\beta,\lambda) = \frac{l-m}{n^2} \tag{10}$$

where

$$l = \left(1+\frac{(v+t)^{\beta}}{\lambda}\right)^{-\eta}\left(1+\frac{(v+t)^{\beta}\eta}{\lambda}\right)\left(1+\frac{t^{\beta}}{\lambda}\right)^{-\eta}\frac{\theta}{\lambda}\beta t^{\beta-1}\left[1-\left(1+\frac{t^{\beta}\eta}{\lambda}\right)\left(1+\frac{t^{\beta}}{\lambda}\right)^{-1}\right]$$

$$m = \left(1+\frac{t^{\beta}}{\lambda}\right)^{-\eta}\left(1+\frac{t^{\beta}\eta}{\lambda}\right)\left(1+\frac{(v+t)^{\beta}}{\lambda}\right)^{-\eta}\frac{\theta}{\lambda}\beta(v+t)^{\beta-1}\left[1-\left(1+\frac{(v+t)^{\beta}\eta}{\lambda}\right)\left(1+\frac{(v+t)^{\beta}}{\lambda}\right)^{-1}\right]$$

$$n = \left[\left(1+\frac{t^{\beta}\eta}{\lambda}\right)\left(1+\frac{t^{\beta}}{\lambda}\right)^{-\eta}\right]^2$$

Also, the associated failure rate of $r_{(t)}, t \geq 0$ for the power length biased weighted Lomax distribution is given by

$$h_{r_{(t)}}(v;\eta,\beta,\lambda) = \frac{f_{r_{(t)}}(v;\eta,\beta,\lambda)}{R_{r_{(t)}}(v;\eta,\beta,\lambda)}$$

$$h_{r_{(t)}}(v;\eta,\beta,\lambda) = \left(\frac{l-m}{n^2}\right)\left(1+\frac{(v+t)^{\beta}}{\lambda}\right)^{\eta}\left(1+\frac{(v+t)^{\beta}\eta}{\lambda}\right)^{-1}\left(1+\frac{t^{\beta}}{\lambda}\right)^{-\eta}\left(1+\frac{t^{\beta}\eta}{\lambda}\right)^{1} \tag{11}$$

where

$$l = \left(1+\frac{(v+t)^{\beta}}{\lambda}\right)^{-\eta}\left(1+\frac{(v+t)^{\beta}\eta}{\lambda}\right)\left(1+\frac{t^{\beta}}{\lambda}\right)^{-\eta}\frac{\theta}{\lambda}\beta t^{\beta-1}\left[1-\left(1+\frac{t^{\beta}\eta}{\lambda}\right)\left(1+\frac{t^{\beta}}{\lambda}\right)^{-1}\right]$$

$$m=\left(1+\frac{t^{\beta}}{\lambda}\right)^{-\eta}\left(1+\frac{t^{\beta}\eta}{\lambda}\right)\left(1+\frac{(v+t)^{\beta}}{\lambda}\right)^{-\eta}\frac{\theta}{\lambda}\beta(v+t)^{\beta-1}\left[1-\left(1+\frac{(v+t)^{\beta}\eta}{\lambda}\right)\left(1+\frac{(v+t)^{\beta}}{\lambda}\right)^{-1}\right]$$

$$n=\left[\left(1+\frac{t^{\beta}\eta}{\lambda}\right)\left(1+\frac{t^{\beta}}{\lambda}\right)^{-\eta}\right]^{2}$$

4.2. Reversed Residual life function

The random variable $\bar{r_{(t)}}=(t-v|v\leq t);t\geq 0$ is used to explain the residual life of a lifetime component . For the power length biased weighted Lomax distribution , the survival function of the reversed residual life time $\bar{r_{(t)}},t\geq 0$ is defined as

$$R_{\bar{r_{(t)}}}(v;\eta,\beta,\lambda)=\frac{F(t-v)}{F(t)}$$

$$R_{\bar{r_{(t)}}}(v;\eta,\beta,\lambda)=\frac{1-\left(1+\frac{(t-v)^{\beta}}{\lambda}\right)^{-\eta}\left(1+\frac{(t-v)^{\beta}\eta}{\lambda}\right)}{1-\left(1+\frac{t^{\beta}}{\lambda}\right)^{-\eta}\left(1+\frac{t^{\beta}\eta}{\lambda}\right)} \tag{12}$$

For the reversed residual life time random variable $\bar{r_{(t)}},t\geq 0$, the cdf of power length biased weighted Lomax distribution is obtained as

$$F_{\bar{r_{(t)}}}(v;\eta,\beta,\lambda)=1-R_{\bar{r_{(t)}}}(v;\eta,\beta,\lambda)$$

$$F_{\bar{r_{(t)}}}(v;\eta,\beta,\lambda)=\left[\frac{\left(1+\frac{(t-v)^{\beta}}{\lambda}\right)^{-\eta}\left(1+\frac{(t-v)^{\beta}\eta}{\lambda}\right)}{1-\left(1+\frac{t^{\beta}}{\lambda}\right)^{-\eta}\left(1+\frac{t^{\beta}\eta}{\lambda}\right)}\right]-\left[\frac{\left(1+\frac{t^{\beta}}{\lambda}\right)^{-\eta}\left(1+\frac{t^{\beta}\eta}{\lambda}\right)}{1-\left(1+\frac{t^{\beta}}{\lambda}\right)^{-\eta}\left(1+\frac{t^{\beta}\eta}{\lambda}\right)}\right] \tag{13}$$

5. STATISTICAL PROPERTIES OF PLBWLD

This section is devoted to discuss the related measures of the new formulated model like raw moments, central moments, measures of skewness, kurtosis, coefficient of variation, index of dispersion, mode and harmonic mean.

5.1. Raw Moments

The r^{th}moment of the PLBWLD about origin $\mu_r^{'}$ is given by

$$\mu_r^{'}=E(V^r)=\int_0^{\infty}v^r f(v;\eta,\beta,\lambda)dv$$

Using (1) and further simplification, r^{th}moment of the PLBWLD about origin $\mu_r^{'}$ is obtained as

$$\mu_r^{'}=\frac{\lambda^{\frac{r}{\beta}}\left(\frac{r}{\beta}+1\right)!}{(\eta-\frac{1}{\beta}-1)(\eta-\frac{2}{\beta}-1)...(\eta-\frac{r}{\beta}-1)} \tag{14}$$

Using equation(14) and substituting $r=1,2,3,4$, the first four moments about origin of the PLBWLD are obtained as

$$\mu_1^{'}=\frac{\lambda^{\frac{1}{\beta}}\left(\frac{1}{\beta}+1\right)!}{(\eta-\frac{1}{\beta}-1)} \tag{15}$$

The equation (15) represents the mean of the PLBWLD.

$$\mu_2^{'} = \frac{\lambda^{\frac{2}{\beta}}\left(\frac{2}{\beta}+1\right)!}{(\eta-\frac{1}{\beta}-1)(\eta-\frac{2}{\beta}-1)} \tag{16}$$

$$\mu_3^{'} = \frac{\lambda^{\frac{3}{\beta}}\left(\frac{3}{\beta}+1\right)!}{(\eta-\frac{1}{\beta}-1)(\eta-\frac{2}{\beta}-1)(\eta-\frac{3}{\beta}-1)} \tag{17}$$

$$\mu_4^{'} = \frac{\lambda^{\frac{4}{\beta}}\left(\frac{4}{\beta}+1\right)!}{(\eta-\frac{1}{\beta}-1)(\eta-\frac{2}{\beta}-1)(\eta-\frac{3}{\beta}-1)(\eta-\frac{4}{\beta}-1)} \tag{18}$$

5.2. Moments about Mean (Central Moments)

The moments about the mean, also known as central moments is defined as

$$\mu_2 = \mu_2^{'} - (\mu_1^{'})^2$$

using equations (15) and (16), we have

$$\mu_2 = \frac{\lambda^{\frac{2}{\beta}}\left(\frac{2}{\beta}+1\right)!}{(\eta-\frac{1}{\beta}-1)(\eta-\frac{2}{\beta}-1)} - \left(\frac{\lambda^{\frac{1}{\beta}}\left(\frac{1}{\beta}+1\right)!}{(\theta-\frac{1}{\beta}-1)}\right)^2 \tag{19}$$

The equation(19) represents the variance of our new formulated model.

$$\mu_3 = \frac{\lambda^{\frac{3}{\beta}}\left(\frac{3}{\beta}+1\right)!}{(\eta-\frac{1}{\beta}-1)(\eta-\frac{2}{\beta}-1)(\eta-\frac{3}{\beta}-1)} - 3\frac{\lambda^{\frac{2}{\beta}}\left(\frac{2}{\beta}+1\right)!}{(\eta-\frac{1}{\beta}-1)(\eta-\frac{2}{\beta}-1)}\frac{\lambda^{\frac{1}{\beta}}\left(\frac{1}{\beta}+1\right)!}{(\eta-\frac{1}{\beta}-1)} + 2\left[\frac{\lambda^{\frac{1}{\beta}}\left(\frac{1}{\beta}+1\right)!}{(\eta-\frac{1}{\beta}-1)}\right]^3 \tag{20}$$

$$\begin{aligned}\mu_4 = {} & \frac{\lambda^{\frac{4}{\beta}}\left(\frac{4}{\beta}+1\right)!}{(\eta-\frac{1}{\beta}-1)(\eta-\frac{2}{\beta}-1)(\eta-\frac{3}{\beta}-1)(\eta-\frac{4}{\beta}-1)} - 4\left(\frac{\lambda^{\frac{3}{\beta}}\left(\frac{3}{\beta}+1\right)!}{(\eta-\frac{1}{\beta}-1)(\eta-\frac{2}{\beta}-1)(\eta-\frac{3}{\beta}-1)}\right)\left(\frac{\lambda^{\frac{1}{\beta}}\left(\frac{1}{\beta}+1\right)!}{(\eta-\frac{1}{\beta}-1)}\right) \\ & +6\left(\frac{\lambda^{\frac{2}{\beta}}\left(\frac{2}{\beta}+1\right)!}{(\eta-\frac{1}{\beta}-1)(\eta-\frac{2}{\beta}-1)}\right)\left(\frac{\lambda^{\frac{1}{\beta}}\left(\frac{1}{\beta}+1\right)!}{(\eta-\frac{1}{\beta}-1)}\right) - 3\left(\frac{\lambda^{\frac{1}{\beta}}\left(\frac{1}{\beta}+1\right)!}{(\eta-\frac{1}{\beta}-1)}\right)^4\end{aligned}$$

The following four coefficients are obtained for the PLBWLD based upon the first four moments about the mean and using the above expressions defined as:

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3}$$

$$\gamma_1 = \sqrt{\beta_1}$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

$$\gamma_2 = \beta_2 - 3$$

$$\beta_1 = \frac{\left\{ \frac{\lambda^{\frac{3}{\beta}}\left(\frac{3}{\beta}+1\right)!}{(\eta-\frac{1}{\beta}-1)(\eta-\frac{2}{\beta}-1)(\eta-\frac{3}{\beta}-1)} - 3\left(\frac{\lambda^{\frac{2}{\beta}}\left(\frac{2}{\beta}+1\right)!}{(\eta-\frac{1}{\beta}-1)(\eta-\frac{2}{\beta}-1)}\right)\left(\frac{\lambda^{\frac{1}{\beta}}\left(\frac{1}{\beta}+1\right)!}{(\eta-\frac{1}{\beta}-1)}\right) + 2\left[\frac{\lambda^{\frac{1}{\beta}}\left(\frac{1}{\beta}+1\right)!}{(\eta-\frac{1}{\beta}-1)}\right]^3 \right\}^2}{\left\{ \frac{\lambda^{\frac{2}{\beta}}\left(\frac{2}{\beta}+1\right)!}{(\eta-\frac{1}{\beta}-1)(\eta-\frac{2}{\beta}-1)} - \left(\frac{\lambda^{\frac{1}{\beta}}\left(\frac{1}{\beta}+1\right)!}{(\eta-\frac{1}{\beta}-1)}\right)^2 \right\}^3} \tag{21}$$

We need another measure that is dependent on the sign of the third central moment since the nature of skewness cannot be estimated using this relation.

$$\gamma_1 = \sqrt{\beta_1}$$

$$\gamma_1 = \frac{\left\{ \frac{\lambda^{\frac{3}{\beta}}\left(\frac{3}{\beta}+1\right)!}{(\eta-\frac{1}{\beta}-1)(\eta-\frac{2}{\beta}-1)(\eta-\frac{3}{\beta}-1)} - 3\left(\frac{\lambda^{\frac{2}{\beta}}\left(\frac{2}{\beta}+1\right)!}{(\eta-\frac{1}{\beta}-1)(\eta-\frac{2}{\beta}-1)}\right)\left(\frac{\lambda^{\frac{1}{\beta}}\left(\frac{1}{\beta}+1\right)!}{(\eta-\frac{1}{\beta}-1)}\right) + 2\left[\frac{\lambda^{\frac{1}{\beta}}\left(\frac{1}{\beta}+1\right)!}{(\eta-\frac{1}{\beta}-1)}\right]^3 \right\}}{\left\{ \frac{\lambda^{\frac{2}{\beta}}\left(\frac{2}{\beta}+1\right)!}{(\eta-\frac{1}{\beta}-1)(\eta-\frac{2}{\beta}-1)} - \left(\frac{\lambda^{\frac{1}{\beta}}\left(\frac{1}{\beta}+1\right)!}{(\eta-\frac{1}{\beta}-1)}\right)^2 \right\}^{\frac{3}{2}}} \tag{22}$$

Also,

$$\beta_2 = \frac{\mu_4}{(\mu_2)^2} \tag{23}$$

where

$$\mu_4 = \frac{\lambda^{\frac{4}{\beta}}\left(\frac{4}{\beta}+1\right)!}{(\eta-\frac{1}{\beta}-1)(\eta-\frac{2}{\beta}-1)(\eta-\frac{3}{\beta}-1)(\eta-\frac{4}{\beta}-1)} - 4\left(\frac{\lambda^{\frac{3}{\beta}}\left(\frac{3}{\beta}+1\right)!}{(\eta-\frac{1}{\beta}-1)(\eta-\frac{2}{\beta}-1)(\eta-\frac{3}{\beta}-1)}\right)\left(\frac{\lambda^{\frac{1}{\beta}}\left(\frac{1}{\beta}+1\right)!}{(\eta-\frac{1}{\beta}-1)}\right)$$
$$+6\left(\frac{\lambda^{\frac{2}{\beta}}\left(\frac{2}{\beta}+1\right)!}{(\eta-\frac{1}{\beta}-1)(\eta-\frac{2}{\beta}-1)}\right)\left(\frac{\lambda^{\frac{1}{\beta}}\left(\frac{1}{\beta}+1\right)!}{(\eta-\frac{1}{\beta}-1)}\right) - 3\left(\frac{\lambda^{\frac{1}{\beta}}\left(\frac{1}{\beta}+1\right)!}{(\eta-\frac{1}{\beta}-1)}\right)^4$$

And,

$$\mu_2 = \frac{\lambda^{\frac{2}{\beta}}\left(\frac{2}{\beta}+1\right)!}{(\eta-\frac{1}{\beta}-1)(\eta-\frac{2}{\beta}-1)} - \left(\frac{\lambda^{\frac{1}{\beta}}\left(\frac{1}{\beta}+1\right)!}{(\eta-\frac{1}{\beta}-1)}\right)^2$$

Again,

$$\gamma_2 = \beta_2 - 3$$

5.2.1 Coefficient of variation

$$CV = \frac{\sqrt{\mu_2}}{\mu_1'}$$

On using the equations (15) and(19), the coefficient of variation can be obtained for PLWLD.

$$C.V = \frac{\sqrt{\frac{\lambda^{\frac{2}{\beta}}\left(\frac{2}{\beta}+1\right)!}{(\eta-\frac{1}{\beta}-1)(\eta-\frac{2}{\beta}-1)} - \left(\frac{\lambda^{\frac{1}{\beta}}\left(\frac{1}{\beta}+1\right)!}{(\eta-\frac{1}{\beta}-1)}\right)^2}}{\frac{\lambda^{\frac{1}{\beta}}\left(\frac{1}{\beta}+1\right)!}{(\eta-\frac{1}{\beta}-1)}} \tag{24}$$

5.2.2 Index of Dispersion

The index of dispersion is defined as

$$D = \frac{\sigma^2}{\mu_1'}$$

Using the formula we obtain the index of dispersion for the PWLBWLD as

$$D = \frac{\frac{\lambda^{\frac{2}{\beta}}\left(\frac{2}{\beta}+1\right)!}{(\eta-\frac{1}{\beta}-1)(\eta-\frac{2}{\beta}-1)} - \left(\frac{\lambda^{\frac{1}{\beta}}\left(\frac{1}{\beta}+1\right)!}{(\eta-\frac{1}{\beta}-1)}\right)^2}{\frac{\lambda^{\frac{1}{\beta}}\left(\frac{1}{\beta}+1\right)!}{(\eta-\frac{1}{\beta}-1)}} \tag{25}$$

5.2.3 Mode

To discuss PLBWLD's monotonicity, we use the logarithm of its probability density function as;

$$\log f(v;\eta,\beta,\lambda) = \log\left\{\frac{\eta(\eta-1)\beta}{\lambda^2} v^{2\beta-1}\left(1+\frac{v^\beta}{\lambda}\right)^{-(\eta+1)}\right\}$$

In order to find the value of mode, we differentiate the above equation w.r.t v and equate to zero, it yields

$$\hat{v} = \left\{\frac{(2\beta-1)\lambda}{(\eta-2)\beta+2}\right\}^{\frac{1}{\beta}} \tag{26}$$

Equation (26) represents the modal value for the PLBWLD.

5.2.4 Harmonic Mean

The harmonic mean for the PLBWLD is defined as

$$E(V^{-1}) = E(\frac{1}{V}) = \int_0^\infty \frac{1}{v} f(v;\eta,\beta,\lambda)dv$$
$$= \int_0^\infty \frac{1}{v}\frac{\eta(\eta-1)\beta}{\lambda^2} v^{2\beta-1}\left(1+\frac{v^\beta}{\lambda}\right)^{-(\eta+1)} dv$$

on solving the integral and further simplification, we obtain the harmonic mean for PLBWLD as

$$H = \frac{1}{\eta(\eta-1)\lambda^{-\frac{1}{\beta}}\sum_{k=0}^{-\frac{1}{\beta}+1}(-1)^{k+1}\binom{-\frac{1}{\beta}+1}{k}\left(\frac{1}{-\frac{1}{\beta}-\eta-k+1}\right)} \tag{27}$$

6. Moment Generating function, Characteristic function and cumulant Generating function OF PLBWLD

6.1. Moment Generating Function

The moment generating function of PLBWLD distribution is defined as

$$M_v(t) = \int_0^\infty e^{tv} f(v)dv$$

using the following series expansion

$$e^x = 1 + x + \frac{x^2}{2!} + ...$$

$$M_v(t) = \sum_{r=0}^{\infty} \frac{t^r}{r!} \int_0^{\infty} v^r f(v; \eta, \beta, \lambda) dv$$

Using equation (14) we obtain the moment generating function for PLBWLD as

$$M_v(t) = \sum_{r=0}^{\infty} \frac{t^r}{r!} \frac{\lambda^{\frac{r}{\beta}} \left(\frac{r}{\beta} + 1\right)!}{(\eta - \frac{1}{\beta} - 1)(\eta - \frac{2}{\beta} - 1)...(\eta - \frac{r}{\beta} - 1)} \tag{28}$$

6.2. Characteristic Function

The characteristic function for the PLBWLD can be obtained using the relation $\phi_v(t) = M_v(it)$

$$\phi(t) = \sum_{r=0}^{\infty} \frac{(it)^r}{r!} \frac{\lambda^{\frac{r}{\beta}} \left(\frac{r}{\beta} + 1\right)!}{(\eta - \frac{1}{\beta} - 1)(\eta - \frac{2}{\beta} - 1)...(\eta - \frac{r}{\beta} - 1)} \tag{29}$$

6.3. Cumulant Function

The cumulant function for the PLBWLD is obtained by using the relation $k_v(t) = \log M_v(it)$

$$k_v(t) = \log \sum_{r=0}^{\infty} \frac{t^r}{r!} \frac{\lambda^{\frac{r}{\beta}} \left(\frac{r}{\beta} + 1\right)!}{(\eta - \frac{1}{\beta} - 1)(\eta - \frac{2}{\beta} - 1)...(\eta - \frac{r}{\beta} - 1)} \tag{30}$$

7. Order Statistics of PLBWLD

The order statistics connected to the power length biased weighted Lomax distribution is devoted in this section. Let $V_{(t;n)}$ be the t^{th} order statistics with the random sample $v_{(1)}, v_{(2)}, v_{(3)}, ...v_{(m)}$ derived from the PLBWLD having the probability density function (pdf) $f(v; \eta, \beta, \lambda)$ and cumulative distribution function (cdf) $F(v; \eta, \beta, \lambda)$. Therefore, the probability density function (pdf) and cumulative distribution function (cdf) of $v_{(t;n)}$ say $f_{(t;n)}(v)$ and $F_{(t;n)}(v)$ are respectively defined as

$$f_{(t;n)}(v) = \frac{n!}{(t-1)!(n-t)!} \left[F(v; \eta, \beta, \lambda)\right]^{t-1} \left[1 - F(v; \eta, \beta, \lambda)\right]^{n-t} f(v; \eta, \beta, \lambda) \tag{31}$$

$$F_{(t;n)}(v) = \sum_{j=t}^{n} \binom{n}{j} \left[F(v; \eta, \beta, \lambda)\right]^j \left[1 - F(v; \eta, \beta, \lambda)\right]^{n-j} \tag{32}$$

Using equation(1) and equation(2) in equation(31) and equation(32), the pdf and cdf of t^{th} ordered statistics for the PLBWLD are derived and are expressed as

$$f_{(t;n)}(v) = \frac{n!}{(t-1)!(n-t)!} \left[1 - \left(1 + \frac{v^\beta}{\lambda}\right)^{-\eta} \left(1 + \frac{v^\beta \eta}{\lambda}\right)\right]^{t-1} \left[1 + \frac{v^\beta}{\lambda}\right)^{-\eta} \left(1 + \frac{v^\beta \eta}{\lambda}\right]^{n-t} \frac{\eta(\eta-1)\beta}{\lambda^2} v^{2\beta-1} \left(1 + \frac{v^\beta}{\lambda}\right)^{-(\eta+1)}$$

$$F_{(t;n)}(v) = \sum_{j=t}^{n} \binom{n}{j} \left[1 - \left(1 + \frac{v^\beta}{\lambda}\right)^{-\eta} \left(1 + \frac{v^\beta \eta}{\lambda}\right)\right]^j \left[\left(1 + \frac{v^\beta}{\lambda}\right)^{-\eta} \left(1 + \frac{v^\beta \eta}{\lambda}\right)\right]^{n-j}$$

In order to obtain the expression for pdf of smallest(minimum) order statistics $v_{(1)}$ and the largest (maximum) order statistics $v_{(m)}$ of PLBWLD , we assume $t = 1$ and n respectively and are expressed in the form as

$$f_{(1;n)}(v) = n\left[1+\frac{v^{\beta}}{\lambda}\right)^{-\eta}\left(1+\frac{v^{\beta}\eta}{\lambda}\right]^{n-1}\frac{\eta(\eta-1)\beta}{\lambda^2}v^{2\beta-1}\left(1+\frac{v^{\beta}}{\lambda}\right)^{-(\eta+1)} \tag{33}$$

$$f_{(n;n)}(v) = n\left[1-\left(1+\frac{v^{\beta}}{\lambda}\right)^{-\eta}\left(1+\frac{v^{\beta}\eta}{\lambda}\right)\right]^{n-1}\frac{\eta(\eta-1)\beta}{\lambda^2}v^{2\beta-1}\left(1+\frac{v^{\beta}}{\lambda}\right)^{-(\eta+1)} \tag{34}$$

7.1. Median order statistics

The pdf of median order statistics, $v_{(n+1)}$ is defined as

$$f_{(n+1;n)}(v) = \frac{(2n+1)!}{n!n!}\left[F(v;\eta,\beta,\lambda)\right]^n\left[1-F(v;\eta,\beta,\lambda)\right]^n f(v;\eta,\beta,\lambda)$$

$$f_{(n+1;n)}(v) = \frac{(2n+1)!}{(n)!(n)!}\left[1-\left(1+\frac{v^{\beta}}{\lambda}\right)^{-\eta}\left(1+\frac{v^{\beta}\eta}{\lambda}\right)\right]^n\left[1+\frac{v^{\beta}}{\lambda}\right)^{-\eta}\left(1+\frac{v^{\beta}\eta}{\lambda}\right]^n \frac{\eta(\eta-1)\beta}{\lambda^2}v^{2\beta-1}\left(1+\frac{v^{\beta}}{\lambda}\right)^{-(\eta+1)}$$

8. Characterization of PLBWLD

Theorem 1. Let $v_{(1)}, v_{(2)}, ...v_{(n)}$ be n independently and identically distributed random samples selected from PLBWLD having a sample mean of $\bar{v_n}$ and sample variance of s_n^2 then,

$$\lim_{n\to\infty} E\left(\frac{s_n^2}{\bar{v_n}}\right) = \left(\frac{\sigma}{\mu}\right)^2$$

Proof: $E(\bar{v_n}) = \mu$ and $var(\bar{v_n}) = \frac{\sigma^2}{n}$

We know that

$$E(\bar{v_n})^2 = var(\bar{v_n}) + [E(\bar{v_n})]^2$$

$$E(\bar{v_n})^2 = \frac{1}{n}\left[\frac{\lambda^{\frac{2}{\beta}}\left(\frac{2}{\beta}+1\right)!}{(\eta-\frac{1}{\beta}-1)(\eta-\frac{2}{\beta}-1)} - \left(\frac{\lambda^{\frac{1}{\beta}}\left(\frac{1}{\beta}+1\right)!}{(\theta-\frac{1}{\beta}-1)}\right)^2\right] + \left[\frac{\lambda^{\frac{1}{\beta}}\left(\frac{1}{\beta}+1\right)!}{(\eta-\frac{1}{\beta}-1)}\right]^2$$

since

$$E(s_n^2) = \sigma^2 = \left[\frac{\lambda^{\frac{2}{\beta}}\left(\frac{2}{\beta}+1\right)!}{(\eta-\frac{1}{\beta}-1)(\eta-\frac{2}{\beta}-1)} - \left(\frac{\lambda^{\frac{1}{\beta}}\left(\frac{1}{\beta}+1\right)!}{(\theta-\frac{1}{\beta}-1)}\right)^2\right]$$

Therefore

$$E\left(\frac{s_n^2}{\bar{v_n}^2}\right) = \frac{\left[\frac{\lambda^{\frac{2}{\beta}}\left(\frac{2}{\beta}+1\right)!}{(\eta-\frac{1}{\beta}-1)(\eta-\frac{2}{\beta}-1)} - \left(\frac{\lambda^{\frac{1}{\beta}}\left(\frac{1}{\beta}+1\right)!}{(\theta-\frac{1}{\beta}-1)}\right)^2\right]}{\frac{1}{n}\left[\frac{\lambda^{\frac{2}{\beta}}\left(\frac{2}{\beta}+1\right)!}{(\eta-\frac{1}{\beta}-1)(\eta-\frac{2}{\beta}-1)} - \left(\frac{\lambda^{\frac{1}{\beta}}\left(\frac{1}{\beta}+1\right)!}{(\theta-\frac{1}{\beta}-1)}\right)^2\right] + \left[\frac{\lambda^{\frac{1}{\beta}}\left(\frac{1}{\beta}+1\right)!}{(\eta-\frac{1}{\beta}-1)}\right]^2}$$

On taking the limits to both sides of the above equation , we have

$$\lim_{n\to\infty} E(\frac{s_n^2}{\bar{v_n}}) = \frac{\left[\frac{\lambda^{\frac{2}{\beta}}\left(\frac{2}{\beta}+1\right)!}{(\eta-\frac{1}{\beta}-1)(\eta-\frac{2}{\beta}-1)} - \left(\frac{\lambda^{\frac{1}{\beta}}\left(\frac{1}{\beta}+1\right)!}{(\theta-\frac{1}{\beta}-1)}\right)^2\right]}{\left[\frac{\lambda^{\frac{1}{\beta}}\left(\frac{1}{\beta}+1\right)!}{(\eta-\frac{1}{\beta}-1)}\right]^2}$$

$$\lim_{n\to\infty} E(\frac{s_n^2}{\bar{v_n}}) = \left(\frac{\sigma}{\mu}\right)^2$$

Hence , the above theorem is proved

9. Information measure of PLBWLD

Entropy is a quantitative measures of the amount of uncertainty in a random variable. This section is dedicated to obtaining the PLBWLD generalized entropy expression.

Theorem 2. The generalized entropy for the PLBWLD is expressed as

$$I(\alpha) = \frac{1}{\alpha(\alpha-1)}\left\{\frac{\left(\frac{\alpha}{\beta}+1\right)!}{(\eta-\frac{1}{\beta}-1)(\eta-\frac{2}{\beta}-1)...(\eta-\frac{\alpha}{\beta}-1)}\left\{\frac{\left(\eta-\frac{1}{\beta}-1\right)}{(\frac{1}{\beta}+1)!}\right\}^{\alpha} - 1\right\}$$

Proof: The generalized entropy is defined as

$$I(\alpha) = \frac{v_\alpha \mu^{-\alpha} - 1}{\alpha(\alpha-1)}$$

where

$$v_\alpha = \int_{-\infty}^{\infty} v^\alpha f(v) dv$$

and μ represents mean. For PLBWLD, we have

$$v_\alpha = \frac{\lambda^{\frac{\alpha}{\beta}}\left(\frac{\alpha}{\beta}+1\right)!}{(\eta-\frac{1}{\beta}-1)(\eta-\frac{2}{\beta}-1)...(\eta-\frac{\alpha}{\beta}-1)}$$

$$\mu = \frac{\lambda^{\frac{1}{\beta}}\left(\frac{1}{\beta}+1\right)!}{(\eta-\frac{1}{\beta}-1)}$$

Therefore, the expression for the generalized entropy of PLBWLD is obtained as

$$I(\alpha) = \frac{1}{\alpha(\alpha-1)}\left\{\frac{\left(\frac{\alpha}{\beta}+1\right)!}{(\eta-\frac{1}{\beta}-1)(\eta-\frac{2}{\beta}-1)...(\eta-\frac{\alpha}{\beta}-1)}\left\{\frac{\left(\eta-\frac{1}{\beta}-1\right)}{(\frac{1}{\beta}+1)!}\right\}^{\alpha} - 1\right\} \quad (35)$$

10. Estimation of Parameters

This section is devoted to maximum likelihood estimation technique for estimating the unknown parameters η, β, λ of PLBWLD.

10.1. Maximum Likelihood Estimation(MLE)

Suppose $v_1, v_2, v_3, ...v_m$ be the random sample derived from the PLBWLD having the probability density function (pdf) $f(v;\eta,\beta,\lambda)$. Therefore, for m observations , the likelihood function of PLBLWD is obtained as

$$L(v;\eta,\beta,\lambda) = \left[\frac{\eta(\eta-1)\beta}{\lambda^2}\right]^m \prod_{i=1}^{m} v^{2\beta-1}\left(1+\frac{v^\beta}{\lambda}\right)^{-(\eta+1)}$$

Maximizing the log likelihood function yields estimates $\hat{\eta}, \hat{\beta}, \hat{\lambda}$ estimations of the unknown parameters η, β, λ. The log likelihood function is given by

$$\log L(v;\eta,\beta,\lambda) = m\log\eta + m\log(\eta-1) + m\log\beta - 2m\log\lambda + \sum_{i=1}^{m}\log(v_i)^{2\beta-1} - (\eta+1)\sum_{i=1}^{m}\log\left(1+\frac{v_i^\beta}{\lambda}\right) \tag{36}$$

The MLE's of η, β and λ are derived after partially differentiating (36) with respect to the corresponding parameters and equating to zero. We obtain the three normal equations as

$$\frac{m(2\eta-1)}{\eta(\eta-1)} = \sum_{i=1}^{m}\log\left(1+\frac{v_i^\beta}{\lambda}\right) \tag{37}$$

$$\frac{2m}{\lambda} = \frac{(\eta+1)}{\lambda^2}\sum_{i=1}^{m}\frac{v_i^\beta}{\left(1+\frac{v^\beta}{\lambda}\right)} \tag{38}$$

$$\frac{m}{\beta} + 2\sum_{i=1}^{m}\log v_i = (\eta+1)\sum_{i=1}^{m}\frac{\beta v_i^{\beta-1}}{(1+v_i^\beta)} \tag{39}$$

The above three non-linear equations (37),(38) and (39) are not in closed form. Therefore, we shall solve these equations numerically using Newton-Raphson technique of solving equations iteratively and numerically .

11. SIMULATION ILLUSTRATION

The performance of maximum likelihood estimates are examined in this section. To demonstrate the behavior of maximum likelihood estimates (MLEs) in terms of random generating sample sizes n= 100, 150 and a simulation research was conducted using R software. The procedure was repeated 100 times with various parameter combinations selected. The average MLE values and accompanying empirical mean squared errors (MSEs) were calculated in each scenario. Table 1 and table 2 shows the simulation findings. The estimates are stable and near to the genuine parameter values, as shown in table 1 and 2 . In all circumstances, the MSE drops as the sample size increases.

12. APPLICATION

For illustrating the flexibility, adaptability, and suitability of the PLBWLD, we use two actual data sets to show that the power length biased weighted lomax distribution (PLBWLD) can be better model than lomax distribution (LD) and length biased weighted lomax distribution (LBWLD).
To demonstrate how the proposed distribution can be effective in a real-world situation, two real life data sets have been examined . The following models have been investigated for comparison.

- Length biased weighted Lomax distribution (LBWLD) With pdf given in

$$f(v;\eta,\lambda) = \frac{\eta(\eta-1)}{\lambda^2}v\left(1+\frac{v}{\lambda}\right)^{-(\eta+1)} \;;\; v>0, \eta>1, \lambda>0$$

- Lomax distribution (LD) with pdf given as

$$f(v;\eta,\lambda) = \frac{\eta}{\lambda}\left(1+\frac{v}{\lambda}\right)^{-(\eta+1)} \;;\; v>0, \eta>1, \lambda>0$$

Here, several goodness-of-fit criterion such as Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), Akaike Information Criterion Corrected (AICC) , Hannan Quinn Information Criterion (HQIC) and Kolmogorov -Smirnov (KS) statistics are used. The statistic with the lowest value is considered the best fit. The numerical results are produced using R programme for analysis purposes.

Table 1: *Average values of MLEs and the corresponding MSEs(n=100).*

Parameter			MLE			MSE		
η	λ	β	$\hat{\eta}$	$\hat{\lambda}$	$\hat{\beta}$	$\hat{\eta}$	$\hat{\lambda}$	$\hat{\beta}$
1.5	1.2	2	1.60342	1.46263	1.88991	0.10386	0.29586	0.07998
		2.5	1.61165	1.52594	2.37138	0.04400	0.32747	0.11551
		3	1.60230	1.44918	2.80023	0.04089	0.23487	0.18081
	1.8	2	1.60342	2.13739	1.88990	0.03866	0.54490	0.07997
		2.5	1.56354	2.06030	2.41250	0.01490	0.28395	0.08610
		3	1.60906	2.06175	2.79079	0.039382	0.29147	0.19585
2	1.2	2	2.11303	1.37128	2.10674	0.33824	0.53606	0.17352
		2.5	2.29354	1.64115	2.61302	1.94607	3.27331	0.27052
		3	2.24369	1.54492	3.05438	0.88963	1.57343	0.42315
	1.8	2	2.15325	2.11382	2.08488	0.40547	1.02721	0.19684
		2.5	2.10288	1.95731	2.54206	0.20766	0.57590	0.21280
		3	2.17188	2.13039	3.11005	0.45798	1.41945	0.32907

Table 2: *Average values of MLEs and the corresponding MSEs(n=150).*

Parameter			MLE			MSE		
η	λ	β	$\hat{\eta}$	$\hat{\lambda}$	$\hat{\beta}$	$\hat{\eta}$	$\hat{\lambda}$	$\hat{\beta}$
1.5	1.2	2	1.57382	1.40020	1.92791	0.02535	0.20046	0.06029
		2.5	1.57460	1.35389	2.36473	0.02172	0.10933	0.08347
		3	1.55781	1.36290	2.89490	0.01292	0.11236	0.08685
	1.8	2	1.56615	2.00830	1.90932	0.01953	0.24642	0.04964
		2.5	1.56273	1.92833	2.38188	0.01404	0.16196	0.08055
		3	1.58063	2.05766	2.87854	0.02763	0.23273	0.08986
2	1.2	2	2.12305	1.36716	2.05878	0.25457	0.38323	0.12918
		2.5	2.08487	1.37785	2.59013	0.15097	0.28118	0.16941
		3	2.05282	1.30232	3.10483	0.14772	0.29009	0.22523
	1.8	2	2.06271	1.90663	2.04340	0.14480	0.42436	0.085475
		2.5	2.01103	1.84029	2.62634	0.10312	0.31950	0.20608
		3	2.13476	2.06073	3.04121	0.23962	0.70488	0.26052

12.1. Data Set 1

Data set 1: The first data is on the breaking stress of carbon fibres of 50 mm length (GPa). The data has been previously used by [2] . The data is as follows:

0.39, 0.85, 1.08, 1.25, 1.47, 1.57, 1.61, 1.61, 1.69, 1.80, 1.84, 1.87, 1.89, 2.03, 2.03, 2.05, 2.12, 2.35, 2.41, 2.43, 2.48, 2.50, 2.53, 2.55, 2.55, 2.56, 2.59, 2.67, 2.73, 2.74, 2.79, 2.81, 2.82, 2.85, 2.87, 2.88, 2.93, 2.95, 2.96, 2.97, 3.09, 3.11, 3.11, 3.15, 3.15, 3.19, 3.22, 3.22, 3.27, 3.28, 3.31, 3.31, 3.33, 3.39, 3.39, 3.56, 3.60, 3.65, 3.68, 3.70, 3.75, 4.20, 4.38, 4.42, 4.70, 4.90

12.2. Data set 2

Data set 2: The following data represent the survival times (in days) of 72 guinea pigs infected with virulent tubercle bacilli, observed and reported by [4]. The data are as follows:

0.1, 0.33, 0.44, 0.56, 0.59, 0.72, 0.74, 0.77, 0.92, 0.93, 0.96, 1, 1, 1.02, 1.05, 1.07, 1.07, 1.08, 1.08, 1.08,1.09, 1.12, 1.13, 1.15, 1.16, 1.2, 1.21, 1.22, 1.22, 1.24, 1.3, 1.34, 1.36, 1.39, 1.44, 1.46, 1.53, 1.59, 1.6, 1.63, 1.63,1.68, 1.71, 1.72,

Table 3: *$-2ln(l)$, AIC, AICC, BIC for the first data set.*

Model	$-2ln(l)$	AIC	AICC	BIC	HQIC	K-S
PLBWLD	175.004	181.004	181.391	187.573	183.6	0.078
LBWLD	224.008	228.008	228.199	232.388	229.731	0.250
LD	265.990	269.989	270.180	274.369	271.720	0.358

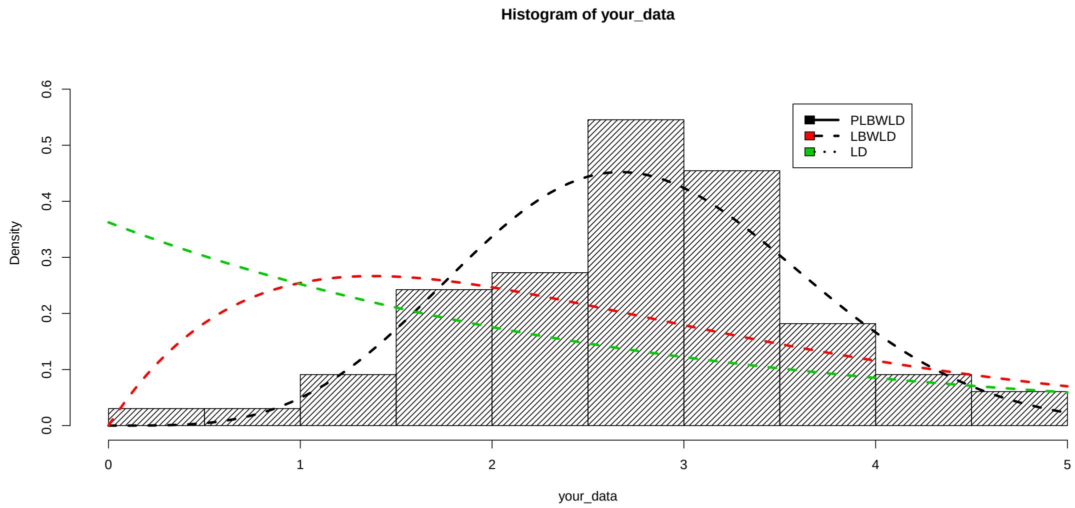

Figure 4: *Fitted density plots for dataset1*

1.76, 1.83, 1.95, 1.96, 1.97, 2.02, 2.13, 2.15, 2.16, 2.22, 2.3, 2.31, 2.4, 2.45, 2.51, 2.53, 2.54,2.54, 2.78, 2.93, 3.27, 3.42, 3.47, 3.61, 4.02, 4.32, 4.58, 5.55

Table 4: *$-2ln(l)$, AIC, AICC, BIC for the second data set.*

Model	$-2ln(l)$	AIC	AICC	BIC	HQIC	K-S
PLBWLD	187.753	193.753	194.106	200.583	196.472	0.084
LBWLD	195.049	199.049	199.223	203.602	200.861	0.168
LD	226.075	230.075	230.249	234.628	231.888	0.294

13. CONCLUSION

This research paper uses power transformation to develop a novel life time probability model called power length biased weighted Lomax distribution. Ordinary moments, moment generating function, hazard rate, order statistics, and generalized entropy are among the significant aspects of PLBWLD that are obtained here. In addition, two real data sets are used to highlight the practical value.
The three-parameter PLBWLD distribution has been introduced here to have more flexibility in terms of the hazard rate function and density function. Using goodness of fit criteria, the suggested model's effectiveness is compared with other competing distributions. The new distribution can exhibit a much more flexible model for life time data . The new model was fitted to two different real-life data sets and showed that it

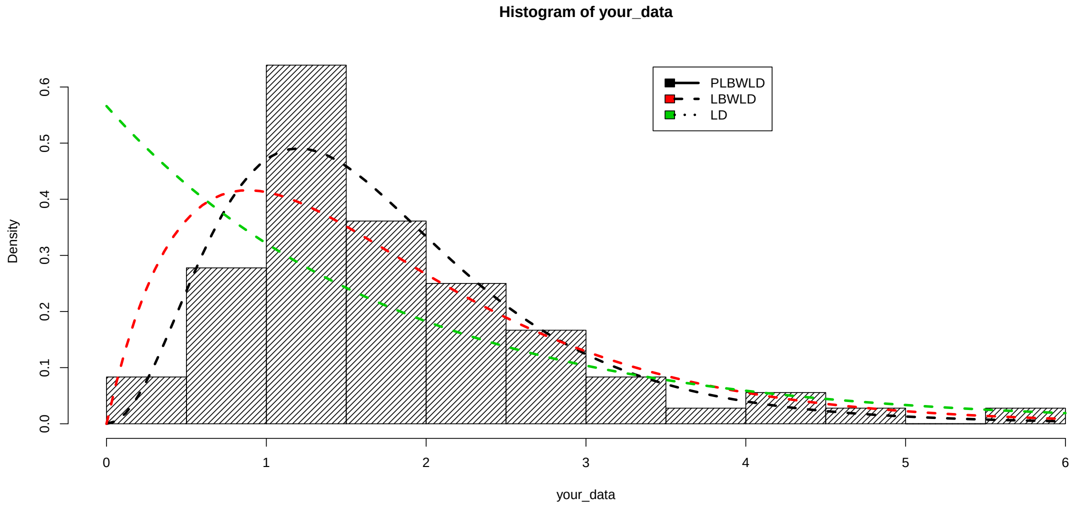

Figure 5: *Fitted density plots for dataset2*

could offer a better fit than a set of extensions of Lomax distribution. We believe that the suggested model will have broader statistical applications.

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