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APPLICATION OF KANIADAKIS κ -STATISTICS TO EXTREME WIND SPEED LOAD DISTRIBUTIONS

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Abstract

The article considers application of the Kaniadakis' κ-statistics [1-3] (non-extensive statistical mechanics) which introduced in 2001 in the framework of Einstein's special theory of relativity, to the analysis and adequate description of extreme wind loads. The κ-deformed Kaniadakis exponential function is used to introduce new classes of κ-deformed statistical versions of known distributions. These distributions coincide with the original ones with the exception that their κ-deformed tail follows the Pareto power law. This allows converting the original distributions into heavy-tailed distributions that more closely match the experimental data of mixed systems and systems operating under conditions of increased uncertainty. This allows, within the framework of known distributions of loads and impacts, to model above-standard stressors and analyze the near impossible to predict "Black Swan" and "Dragon-King" ultra-rare type of events with humongous consequences.

Keywords: Kaniadakis, κ-statistics, heavy-tailed distributions, black swan, dragon-king, wind speed, uncertainty.

I. Introduction

During the operation of a complex technical system, due to its complexity, no matter how carefully the calculations are carried out during its design, there always will be unforeseen impacts due to beyond- design (extreme) loads, which will eventually lead, at best, to its local damage. In this case, it is very important to know whether these damages will cause catastrophic destruction of the system as a whole or to its unsuitability for further operation. Such out-of-design or extreme loads and impacts fall on the tails of the so-called fat-tailed distributions. An example of such a distribution is shown in Fig.1.

The distribution shown in Fig. 1 is divided into two parts: central (solid thick line) and caudal (dashed thick line). Separation boundary K represents a threshold value determined by the specifics of the task. The central part represents the values at design loading (normal operation), the tail represents the beyond design values. The central part is usually described by standard distributions (exponential, Weibull, Rayleigh, lognormal, etc.), the caudal (tail) part is described by power-law or heavy-tailed distributions.

Below, we consider the application of Kaniadakis κ-statistics [1-3] to known distributions of loads and impacts, which makes it possible to simulate above-norm stressors in the tails of these distributions and analyze the consequences of beyond-design situations (including " Black Swan" and "Dragon-King" type disasters) . The use of κ -statistics makes it possible to obtain simple analytical closed-form expressions for all major statistical functions such as the probability density function, distribution function, survival function, quantile function, risk function, and cumulative risk function.

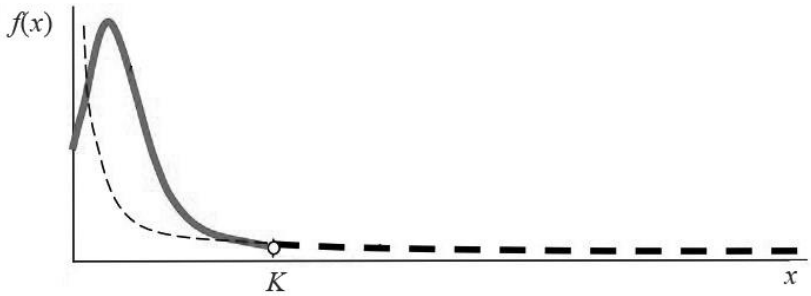

Figure 1: *Example of distribution of an impact, load or damage*

The analysis was made on the example of extreme wind speeds in the Arctic zone of the Russian Federation, obtained at various weather stations. Information about extremely high wind speeds is very important, since knowledge of the true distribution of wind loads is necessary when designing new and operating existing antennas and masts, bridges, wind turbines, high-rise buildings and other objects. This is especially true nowadays, when the extreme events frequency is observed to be constantly increasing [5].

The statistical theory of extremes is based on the idea that extreme identically distributed independent random variables obey one of the three probability distributions (Gumbel, Fréchet and Weibull) [6–8], and the Weibull law is successfully used to approximate the distribution of extreme wind speed. Another approach to studying extreme values is by approximating only the tail of the distribution. For this, the so-called Pareto distribution is used, to which the probabilities of events whose intensity (amplitude) exceeds a certain threshold value [6-8] are subjected. The question of how to assign / determine the point K is not disclosed in this article. This is a separate task, determined by the specifics of the problem under consideration.

The initial data for the analysis are taken from the works [6-8], devoted to the study of anomalous features of the wind regime. The study region includes the coastal zone of the Arctic (from the Kola Peninsula to the Chukchi Peninsula), as well as some inland areas (in total, data from more than 30 meteorological stations was used in the analysis). Standard station wind speed measurements at a height of 10 m, averaged over 10 minutes were used. The analyzed data is for the period of 1966–2013.

According to [6-8] one of the basic principles underlying the theory of extreme random processes is violated - the requirement that all sample data belong to the same set. It is shown that the analyzed sample data includes representatives of two different distributions, each of which is approximated by its own Weibull function. This is a situation where representatives of fundamentally different distributions occur among identical (by nomenclature) quantities. The main array of “intermediate” extremes is called “White Swans”, and the appearance of the largest and rarest phenomena in this sample is called “Black Swans” [9]. It should be noted that objects belonging to the same distribution have a similar genesis, i.e. large anomalies differ from their "smaller relatives" only in amplitude or degree of impact. Events that belong to a different distribution have a different genesis and characterize fundamentally different objects, called "Dragons" or "Dragons-Kings" [10].

II. Mathematical description of κ -statistics

Consider a random value (RV) X with probability density function (PDF) $f(x)$. In statistical mechanics, the general equation for the rate of change of $f(t)$ is a first-order linear ordinary differential equation (ODE):

$$\frac{df(x)}{dx} = -r(x)f(x), \tag{1}$$

where the function $r(x)$ is the decay rate. The solution to this ODE is exponential

$$f(t) = c\exp\left(-\int_{x_0}^{x} r(t)\,dt\right), \tag{2}$$

with the standard normalization condition defining the constant c:

$$\int_{x_0}^{x} f(t)\,dt = 1 \tag{3}$$

As an exponential decision, consider three simple cases.

1. *Exponential model* for constant decay rate, ie

$$r(x) = \lambda, \tag{4}$$

which from Equ. (1) leads to the exponential PDF

$$f(x) = \lambda e^{-\lambda t}. \tag{5}$$

2. *Pareto distribution* type I. The PDF of the Pareto distribution is obtained from Eq. (1) with a decay rate function equal to

$$r(x) = \frac{p}{x}, \ \ p > 1, \tag{6}$$

with PDF in this case :

$$f(x) = \frac{p-1}{x_0}\left(\frac{x_0}{x}\right)^p, \ p > 1, \ x \in (x_0, +\infty), \ x_0 > 0. \tag{7}$$

3. *The κ-exponential model* [10-13]. has proven useful in many applications. According to [1-4], experimental data indicate that the PDFs should resemble an exponential function for $x \to 0$. However, as $x \to 0$, the Pareto PDF diverges. On the other hand, for high values of x, many experimental results show a Pareto – like PDF with power-law tails instead of tails with exponential decay. Therefore, as $x \to 0$ it follows that $r(x) \sim \lambda$, and as $x \to +\infty$ it follows that $r(x) \sim p/x$. Thus, the actual decay rate function $r(x)$ should smoothly interpolate between these two modes. A good suggestion for $r(x)$ was introduced in the context of special relativity where the function $r(x)$ is given via the Lorentz factor

$$\gamma_\kappa(q) = \sqrt{1+\kappa^2 q^2}. \tag{8}$$

This expression includes the dimensionless momentum q, where the parameter κ is the reciprocal of the dimensionless speed of light c, i.e. $\kappa \propto 1/c$. Then, taking

$$r(x) = \frac{\lambda}{\sqrt{1+\kappa^2\lambda^2 x^2}}, \tag{9}$$

we get for $x \to 0$, the decay rate $r(x)$ corresponds to an exponential distribution, i.e $r(x) \sim \lambda$, and as $x \to +\infty$ corresponds to the Pareto distribution, i.e $r(x) \sim 1/\kappa x$.

The solution of ODE (1) leads to the following PDF:

$$f(x)=\lambda\left(1-\kappa^2\right)\exp_\kappa(-\lambda x), \tag{10}$$

where κ is the deformed exponential function given by

$$\exp_\kappa(x)=\left(\sqrt{1+\kappa^2x^2}+\kappa x\right)^{1/\kappa}. \tag{11}$$

for $0<\kappa<1$.

It is important to note that as $k\to 0$ and as $x\to 0$, the function $\exp_\kappa(x)$ tends to the usual exponent, i.e.

$$\begin{aligned}&\exp_\kappa(x)\underset{\kappa\to 0}{\sim}\exp(x),\\&\exp_\kappa(x)\underset{x\to 0}{\sim}\exp(x).\end{aligned} \tag{12}$$

On the other hand, the function $\exp_\kappa(-x)$ as $x\to+\infty$ is a power tail, i.e.,

$$\exp_\kappa(-x)\underset{t\to+\infty}{\sim}(2\kappa x)^{-1/\kappa}. \tag{13}$$

In addition, the κ-exponential satisfies the following identity

$$\exp_\kappa(x)\exp_\kappa(-x)=1. \tag{14}$$

by analogy with the standard, undeformed exponential.

The κ-exponent is a very powerful tool that can be used to formulate a generalized statistical theory capable of considering systems described by distribution functions that have power tails.

Consider the application of the Kaniadakis κ-statistic to the Weibull distribution. According to [4], the κ-deformed Weibull cumulative distribution function:

$$F_\kappa(x)=1-\exp_\kappa\left(-(x/\tau)^\alpha\right), x\ge 0,\ \alpha,\tau,\kappa>0, \tag{15}$$

where α is the shape parameter, τ is the scale parameter .

Another parameterization of the Weibull distribution is often used. The shape parameter α remains the same, but the scale parameter is replaced with $\beta=\tau^{-\alpha}$. Then

$$F_\kappa(x)=1-\exp_\kappa\left(-\beta x^\alpha\right), x\ge 0,\ \alpha,\beta>0,\ 0\le\kappa<1. \tag{16}$$

It is easy to obtain the κ-deformed Weibull PDF :

$$f_\kappa(x)=\frac{dF_\kappa(x)}{dx}=\frac{e_\kappa\left(-\beta t^\alpha\right)\alpha\beta\kappa x^{\alpha-1}\left(1-\dfrac{\beta\kappa x^\alpha}{\sqrt{1+\kappa^2\beta^2x^{2\alpha}}}\right)}{\kappa\beta\kappa x^\alpha\left(\dfrac{\sqrt{1+\kappa^2\beta^2x^{2\alpha}}}{\beta\kappa x^\alpha}-1\right)}. \tag{17}$$

Introducing a substitution $y=\dfrac{\alpha\beta^2\kappa^2x^{2\alpha-1}}{\sqrt{1+\kappa^2\beta^2x^{2\alpha}}}$, yields

$$\begin{aligned}f_\kappa(x)&=\frac{e_\kappa\left(-\beta x^\alpha\right)\alpha\beta\kappa x^{\alpha-1}(1-y)}{\kappa\beta\kappa x^\alpha\left(\dfrac{1-y}{y}\right)}=\frac{e_\kappa\left(-\beta x^\alpha\right)\alpha\beta\kappa x^{\alpha-1}y}{\kappa\beta\kappa x^\alpha}=\\&=\frac{e_\kappa\left(-\beta x^\alpha\right)\alpha\beta x^{\alpha-1}}{\sqrt{1+\kappa^2\beta^2x^{2\alpha}}}=\frac{\alpha}{\tau}\frac{(x/\tau)^{\alpha-1}e_\kappa\left(-(x/\tau)^\alpha\right)}{\sqrt{1+\kappa^2(x/\tau)^{2\alpha}}}.\end{aligned} \tag{18}$$

The shape parameter (index of power) α quantitatively characterizes the shape of the distribution, which is less (more) pronounced at smaller (larger) values of the parameter. The parameter τ is a scale parameter: if τ is small, then the distribution will be *more concentrated* around the mode; if τ is large, the distribution will be less concentrated and more scattered. Finally, the parameter κ characterizes and measures the heaviness of the right tail: the larger (smaller) its value, the thicker (thinner) the tail. As $\kappa\to 0$, the distribution tends to the standard

Weibull distribution. It can be easily verified [4]

$$\lim_{\kappa\to 0} f_\kappa(x,\alpha,\tau,\kappa)=\frac{\alpha}{\tau}\left(\frac{x}{\tau}\right)^{\alpha-1}\exp\left(-(x/\tau)^\alpha\right),$$
$$\lim_{\kappa\to 0} F_\kappa(x,\alpha,\tau,\kappa)=1-\exp\left(-(x/\tau)^\alpha\right). \tag{19}$$

Since the exponential distribution is a special case of Weibull PDF with a shape parameter equal to 1, then as κ → 0 and $\alpha = 1$, the κ-deformed functions tend to the exponential law. For $x \to 0+$, the distribution behaves similarly to the standard Weibull model, while for large values of x it approaches a Pareto distribution with a scale parameter $\tau(2\kappa)^{-\frac{1}{\alpha}}$ and a shape parameter $\frac{\alpha}{\kappa}$, i.e.,

$$\lim_{x\to+\infty} f_\kappa(x,\alpha,\tau,\kappa)=\frac{\frac{\alpha}{\kappa}\left(\tau(2\kappa)^{-\frac{1}{\alpha}}\right)^{\frac{\alpha}{\kappa}}}{x^{\frac{\alpha}{\kappa}+1}},$$
$$\lim_{x\to+\infty} F_\kappa(x,\alpha,\tau,\kappa)=1-\left(\frac{\tau(2\kappa)^{-\frac{1}{\alpha}}}{x}\right)^{\frac{\alpha}{\kappa}}. \tag{20}$$

From Eq. (15) one can find the quantile function (inverse distribution function) and the survival (reliability) function [4]:

$$F_\kappa^{-1}(p,\alpha,\tau,\kappa)=\tau\left(\ln_\kappa\frac{1}{1-p}\right)^{1/\alpha}=\beta^{-\frac{1}{\alpha}}\left(\ln_\kappa\frac{1}{1-p}\right)^{1/\alpha},\ 0<p<1,$$
$$S_\kappa(x,\alpha,\tau,\kappa)=\exp_\kappa\left(-(x/\tau)^\alpha\right)=\exp_\kappa\left(-\beta x^\alpha\right), \tag{21}$$

where the κ-logarithm of $\ln_\kappa(u)$ is the inverse of $\exp_\kappa(u)$, i.e. $\ln_\kappa(\exp_\kappa(x)) = \exp_\kappa(\ln_\kappa(x)) = x$, and is determined by the formula

$$\ln_\kappa(x)=\frac{x^\kappa-x^{-\kappa}}{2\kappa}.$$

The median of κ-deformed Weibull distribution

$$x_{med,\kappa}=\tau\left[\ln_\kappa(2)\right]^{\frac{1}{\alpha}}=\beta^{-\frac{1}{\alpha}}\left[\ln_\kappa(2)\right]^{\frac{1}{\alpha}}. \tag{22}$$

The mode $x_{\text{mode},\kappa}$ of PDF can be obtained analytically as a function of the parameters α, β and κ. If $x_{\text{mode},\kappa}$ is the point of PDF maximum, then by equating the derivative of the PDF to zero, we can find $x_{\text{mode},\kappa}$ [4]:

$$x_{\text{mode},\kappa}=\tau\left[\frac{\alpha^2+2\kappa^2(\alpha-1)}{2\kappa^2(\alpha^2-\kappa^2)}\left(\sqrt{1+\frac{4\kappa^2(\alpha^2-\kappa^2)(\alpha-1)^2}{\left[\alpha^2+2\kappa^2(\alpha-1)\right]^2}}+1\right)\right]^{\frac{1}{2\alpha}}. \tag{23}$$

According to [4], the moments of κ-deformed Weibull distribution can be calculated using the formula:

$$a_{r,\kappa}=\frac{\tau^r}{(2\kappa)^{\frac{r}{\alpha}}}\frac{\Gamma\left(1+\frac{r}{\alpha}\right)}{1+r\frac{\kappa}{\alpha}}\frac{\Gamma\left(\frac{1}{2\kappa}-\frac{r}{2\alpha}\right)}{\Gamma\left(\frac{1}{2\kappa}+\frac{r}{2\alpha}\right)}. \tag{24}$$

Where, in particular, the mean, variance and coefficient of variation:

$$M_\kappa[X] = \frac{\tau}{(2\kappa)^{\frac{1}{\alpha}}} \frac{\Gamma\left(1+\frac{1}{\alpha}\right)}{1+\frac{\kappa}{\alpha}} \frac{\Gamma\left(\frac{1}{2\kappa}-\frac{1}{2\alpha}\right)}{\Gamma\left(\frac{1}{2\kappa}+\frac{1}{2\alpha}\right)},$$

$$D_\kappa[X] = \frac{\tau^2}{(2\kappa)^{\frac{2}{\alpha}}} \frac{\Gamma\left(1+\frac{2}{\alpha}\right)}{1+2\frac{\kappa}{\alpha}} \frac{\Gamma\left(\frac{1}{2\kappa}-\frac{1}{\alpha}\right)}{\Gamma\left(\frac{1}{2\kappa}+\frac{1}{\alpha}\right)} - M_\kappa^2[X],$$

$$V_\kappa[X] = \sqrt{\frac{a_{2,\kappa}}{a_{1,\kappa}^2}-1} = \sqrt{\frac{\frac{\Gamma\left(1+\frac{2}{\alpha}\right)}{1+2\frac{\kappa}{\alpha}} \frac{\Gamma\left(\frac{1}{2\kappa}-\frac{1}{\alpha}\right)}{\Gamma\left(\frac{1}{2\kappa}+\frac{1}{\alpha}\right)}}{\left(\frac{\Gamma\left(1+\frac{1}{\alpha}\right)}{1+\frac{\kappa}{\alpha}} \frac{\Gamma\left(\frac{1}{2\kappa}-\frac{1}{2\alpha}\right)}{\Gamma\left(\frac{1}{2\kappa}+\frac{1}{2\alpha}\right)}\right)^2} - 1}. \tag{25}$$

Denote

$$C_r = \frac{(2\kappa)^{-\frac{r}{\alpha}}}{1+r\frac{\kappa}{\alpha}} \frac{\Gamma\left(\frac{1}{2\kappa}-\frac{r}{2\alpha}\right)}{\Gamma\left(\frac{1}{2\kappa}+\frac{r}{2\alpha}\right)}. \tag{26}$$

Then the connection between the moments of the κ-deformed $a_{r,\kappa}$ and standard $a_r = \tau^r \Gamma\left(1+\frac{r}{\alpha}\right)$ Weibull distribution is given by:

$$a_{r,\kappa} = a_r C_r \,. \tag{27}$$

Now it is not difficult to obtain expressions relating the main numerical characteristics of the standard and κ-deformed Weibull distributions

$$\begin{aligned} &M_\kappa[X] = M[X]C_1, \\ &D_\kappa[X] = a_2 C_2 - M^2[X]C_1^2 = D[X]C_2 + M^2[X]\left(C_2 + C_1^2\right), \\ &V_\kappa^2[X]+1 = \frac{a_{2,\kappa}}{a_{1,\kappa}^2} = \frac{a_2 C_2}{a_1^2 C_1^2} = \left(V^2[X]+1\right)\frac{C_2}{C_1^2}. \end{aligned} \tag{28}$$

III. Applications of κ -statistics to the analysis of extreme wind speeds

The analysis of extreme values of wind speeds was carried out separately for the cold and warm seasons, which in the Arctic are characterized not only by sharply contrasting temperatures, but also by features of the atmospheric circulation. The summer months here are, in fact, July and August, while the winter season covers not only December, January and February, but usually includes November, March and April as well.

To approximate the distribution function of the frequency of wind speeds, we used the Weibull law, which has the distribution function [6, 7]

$$\frac{n}{N} \approx F_\kappa(u) = 1-\exp\left(-\beta u^\alpha\right), u \ge 0,\ \alpha, \beta > 0,$$

where *u* is the wind speed module, *n* / *N* characterizes the accumulated frequency.

This expression can be transformed

$$ln\left[-ln(N-n)/N\right] = \ln u \,.$$

It follows that in special coordinates $\{ln\left[-ln\left(N-n\right)/N\right],\ \ln u\}$ the Weibull probability distribution is represented by a straight line. The degree of deviation of empirical points from it characterizes, together with known statistical criteria, the applicability of the theoretical distribution law.

Consider examples of the empirical distribution of the frequency of wind speeds (Fig. 2) [6]. It can be seen from Fig. 2 that the set of empirical points has a rectilinear section, however, when moving to especially large values, the regression line bends down. *This feature turned out to be typical for the data of all considered meteorological stations* [6]. In this case, the selection of a general linear dependence by the least squares' method is possible since most of the points fill the "linear" section. However, such an approximation will not satisfactorily describe the largest values of wind speed (located in the tail of the distribution) and as a result, *their probability will be significantly underestimated*. It was assumed that the sample included representatives of two different distributions, and each of them (since this is the case of extreme values) can be approximated by the Weibull law.

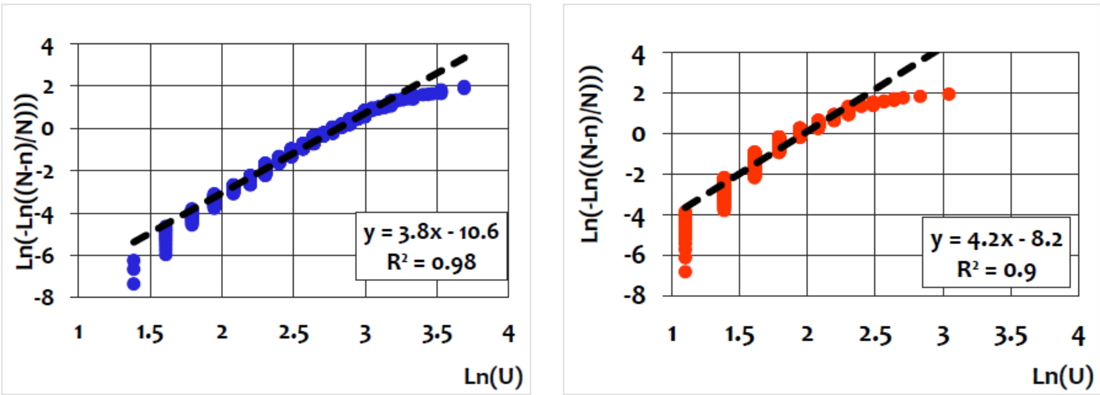

Figure 2: *Empirical distributions of extreme wind speeds (1966 - 2013) according to measurements at Teriberka station (left) in cold periods and at Okunev Nose station (right) in warm periods of the year, plotted in the coordinates of the Weibull probability distribution*

Thus, the selection of two independent straight lines (Weibull distributions) has been carried out. This situation is shown as an example in Fig. 3 [6] for two stations and different seasons.

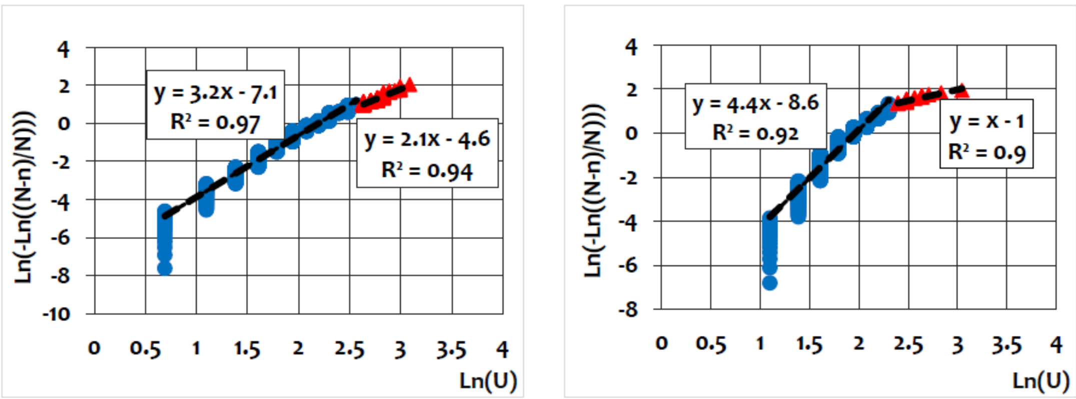

Figure 3: *Empirical distributions of extreme wind speeds of the year (1966 - 2013) according to measurements at Lovozero (left) stations during cold periods and Okunev Nose (right) during warm periods, constructed in the coordinates of the Weibull probability distribution, where the line segments correspond to two different laws of distribution.*

Each group of points with values greater than ($u > U_{th}$) and less than ($u < U_{th}$) is well approximated by its straight line in a special coordinate system, showing that the set of extremes is formed from values belonging to *different* general populations.

The obtained parameters of the Weibull distribution for the data of the three stations considered above are presented in Table 1 [7]. For the data of each station for the summer and winter periods, the parameters of two Weibull distributions were determined, describing: the main wind speeds distribution ("white and black swans" - "WS and BS") and the distribution of large (tail) values ("dragons" - "DR").

Table 1: *Weibull distribution parameters calculated for the period 1966–2013, separately for two groups of extremes corresponding to two families of distributions "WS and BS" and "DR" (for different periods and speeds, in m/s)*

Station	Family membership	winter period		Summer period	
		Parameter α	Parameter β	Parameter α	Parameter β
Teriberka	WS and BS	3.97	$1.6 \cdot 10^{-5}$	4.39	$3.1 \cdot 10^{-5}$
	DR	1.77	0.012	2.12	0.0081
Okunev Nose	WS and BS	3.40	0.0017	4.40	0.0002
	DR	0.52	1.1722	0.98	0.3816
Lovozero	WS and BS	3.19	0.0013	4.45	0.0003
	DR	1.69	0.0429	2.30	0.0202

The PDFs of the obtained distributions are shown in Figs. 4-6. Thus, in order to adequately describe the extremes of wind speed with one distribution, it is necessary to combine the PDFs as follows: before the intersection point of the PDF, the main distribution family is used ("WS and BS" - solid line), after the intersection point, the distribution of large (tail) values ("DR " - dotted line).

We modify the main distribution ("WS and BS") so that its tail part adequately describes large values of extreme wind speed. To do this, we construct a κ-deformed Weibull PDF, for which we choose the parameter κ so that the right tail corresponds to the tail of the "dragons" distribution. The results obtained are presented in Figs. 7-9.

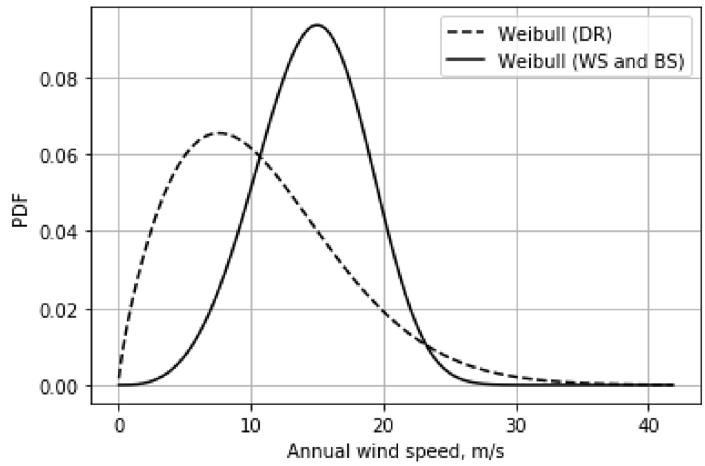

Figure 4: *PDF of extreme wind speeds (1966 - 2013) according to measurements at Teriberka station in winter, corresponding to two families of distributions "WS and BS" and "DR"*

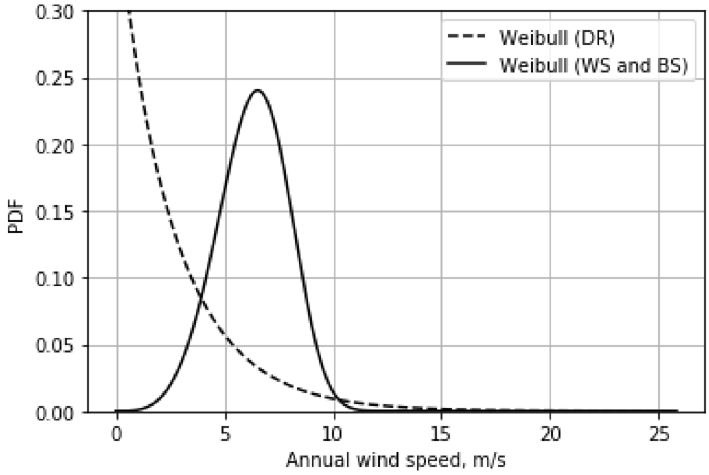

Figure 5: *PDF of extreme wind speeds (1966 - 2013) according to measurements at Okunev Nose station in summer, corresponding to two families of distributions "WS and BS" and "DR"*

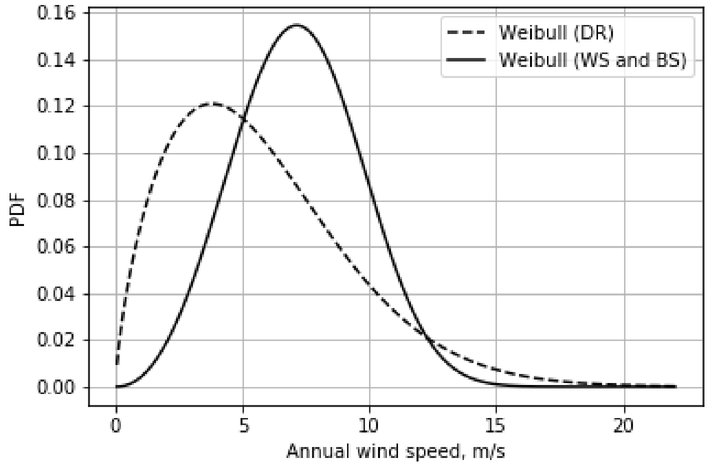

Figure 6: *PDF of extreme wind speeds (1966 - 2013) according to measurements at Lovozero station in winter, corresponding to two families of distributions "WS and BS" and "DR"*

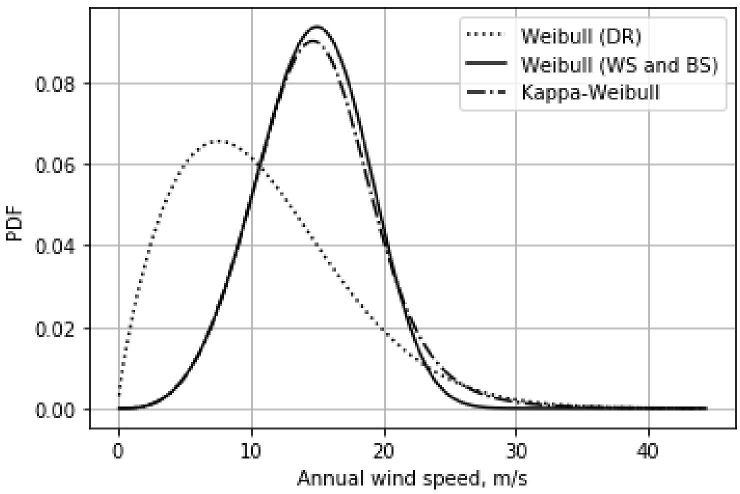

Figure 7: *PDF of extreme wind speeds according to measurements at Teriberka station (winter period) corresponding to three families of Weibull distributions: "WS and BS", "DR" and κ –deformed.*

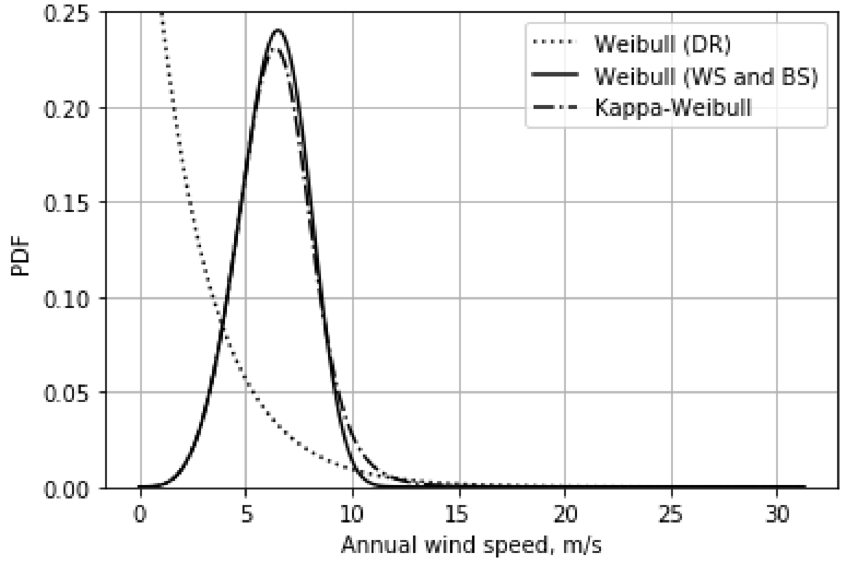

Figure 8: *PDF of extreme wind speeds according to measurements at Okunev Nose station (summer period) corresponding to three families of Weibull distributions: "WS and BS", "DR" and κ-deformed*

From Fig. 7-9 it can be seen that the κ-deformed distribution ("dot-dash" line) adequately describes not only the main distribution ("WS and BS" - solid line), but also almost completely coincides with the distribution of "dragons" - large (tail) extreme values (dotted line). Thus, κ-statistics allows describing data in which representatives of two different distributions are mixed with a *single κ-deformed distribution.*

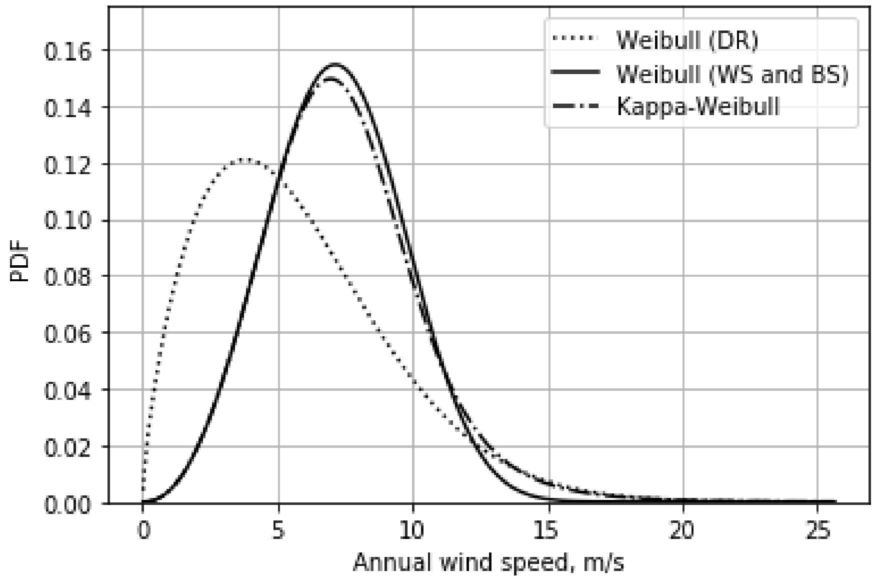

Figure 9: *PDF of extreme wind speeds according to measurements at Lovozero station (winter period) corresponding to three families of Weibull distributions: "WS and BS", "DR" and κ-deformed*

The processing of data from all weather stations showed similar results, which convincingly demonstrate the success of applying *κ-statistics* to the analysis of wind speed extremes, which consist of a mixture of values belonging to different general populations.

In [7], to characterize the geography of extremes and compare summer and winter conditions, quantile speed values were constructed using the formula:

$$U(p) = ln\left[\frac{1}{\beta} ln \frac{1}{1-p}\right]^{\frac{1}{\alpha}},$$

where p is the probability value and $U(p)$ is the corresponding quantile value of the wind speed.

The *p value* is expressed in terms of the "return time", which characterizes the time interval after which the same (or larger) speed anomalies reappear: $T = 1/(1- p)$. For the analysis of extremes, the value $p = 0.99$ was used. For the summer period (62 days - July and August), the total sample size for 48 years (1966–2013), subject to sieving every 3 days, is ~974 days. The share of events (1 – 0.99 = 0.01) is approximately 10 days. For 48 years, this corresponds to the situation of the appearance of an extreme $U(0.99)$ once in 5 warm seasons. For the cold season, $p = 0.99$ corresponds to the average time of extreme value $U(0.99)$ appearance once in two cold periods of the year. The values calculated from the station data (separately for "WS and BS" and "DR" and for the winter period) are presented in Table 2. The values calculated from the κ-deformed distribution are also presented (see formula (21)).

Table 2: *Quantile values of extreme wind speeds (winter period), m/s, U (0.99), calculated separately for three distribution families: "WS and BS", "DR" and κ-deformed*

Station	Distribution family		
	WS and BS	DR	κ-deformed
Teriberka	24	29	28.98
Lovozero	13	16	1 5.81

As can be seen from Table 2, the same quantile values are significantly larger (by 10–30%) in the distribution of "Dragons" than those of "Swans". Moreover, the values of the κ-deformed distribution *almost coincide* with the values of the "Dragons".

IV. Conclusion

The study of wind speed extremes based on standard observations in the coastal regions of the Russian Arctic shows that they represent two sets of data with different statistical properties, each of them obeying its own Weibull law. The κ-statistical approach presented in this paper made it possible to construct a κ-deformed Weibull distribution that adequately describes both sets of extreme wind speed data with different statistical characteristics.

It stands to reason to claim that the developed above approach is applicable to any other type of meteorological loads and impacts.

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