NEW MEDIAN BASED ALMOST UNBIASED EXPONENTIAL TYPE RATIO ESTIMATORS IN THE ABSENCE OF AUXILIARY VARIABLE

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Abstract

The problem of biasness and availability of auxiliary variable for the estimating population mean is a big concern, both can be handled by proposing unbiased estimators in the absence of auxiliary variable. So in this paper unbiased exponential type estimators of population mean have been proposed. The estimators are proposed in the absence of the instrumental variable called the auxiliary variable by taking the advantage of the population and the sample median of the study variable. To about the first order approximation, the theoretical formulations of the bias and mean square error (MSE) are obtained. The circumstances in which the suggested estimators have the lowest mean squared error values when compared to the existing estimators were also deduced. In comparison to the currently used estimators, it was discovered that the suggested estimators of population mean had the lowest MSE, hence highest efficiency. Also least influence from the data's influential observations when it came to accurately calculating the population mean for skewed data. The theoretical findings of the paper are validated by the numerical study.

Keywords: Dual estimator, Median, Study variable, Unbiased Estimator, Mean square error

1. Introduction

In sampling, a representative part of the population called the sample is studied to determine the parameters or the characteristics of the population like the mean, variance, median, correlation coefficient etc. The sample mean estimator is a good device to find the approximate value of the population mean. The population mean can be estimated more precisely by using the auxiliary information as the pioneer work of Cochran [2] proposed a ratio estimator which is more precise than the sample mean estimator when the study and auxiliary variable are positively correlated, though this estimator is biased. Using auxiliary information which is often obtained at an extra survey cost, the authors such as Sisodia and Dwivedi [9], Yadav and Kadilar [10], Ekpenyong and Enang [15] etc. proposed modified ratio estimators which are more efficient than the sample mean and the classical ratio estimator. The pioneering work of Subramani [6] offered a precise ratio estimator to estimate the mean of skewed population without the usage of auxiliary variable by leveraging the median of main variable as the aforementioned estimators are inapplicable in the absence of the auxiliary variable. The median of the study variable may easily be available without having exact information on every data point (See Subramani [6] and these median based estimators are robust in nature since outliers have the least impact on them. The exponential ratio estimator of population mean introduced by Bahl and Tuteja [1] can be applied even when correlations are weak. Later, to be more accurate than the traditional exponential ratio estimator, Singh *et.al* [12], Yasmeen *et al.* [8], Zaman and Kadilar [4], Hussain *et al.* [3] and several others suggested various modified exponential ratio type estimators.

The estimators discussed above are all biassed, which could cause the population mean to

be overestimated or underestimated. As a result, the authors Singh *et al.* [13], Yadav *et al.* [11], Singh *et al.* [14] etc. proposed almost unbiased estimators of population mean in presence of auxiliary variable. The auxiliary variable may not be always avaliable, therefore in the absence of auxiliary variable present study is carried to propose high precision exponential type estimators of population mean which may be unbiased in nature and also able to handle the voluminous data influenced by outliers.

2. Material and Methods

Assume that a simple random sampling without replacement (SRSWOR) sampling strategy is used to select a random sample of size n from a finite population of N number of units. The goal of the study is to estimate the population mean $\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i$ without the use of auxiliary variable information. Assume that data on the study variable Y's correlation with the auxiliary variable X is accessible for every member of the population. The notations and formula used in the paper are as follows

Study Variable

"$C_y = \frac{S_y}{\bar{Y}}$ is the coefficient of variation .

$S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(Y_i - \bar{Y})^2$ is the population mean square.

$s_y^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2$ is the sample mean square.

Auxiliary Variable

: $C_x = \frac{S_x}{\bar{X}}$ is the coefficient of variation.

: $S_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - \bar{X})^2$ is the population mean square.

: $s_x^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$ is the sample mean square.

Further,

M (m) are the population (sample) median of the main variable.

$\bar{M} = \frac{1}{{}^N C_n}\sum_{i=1}^{{}^N C_n} m_i$ is the average of sample medians of the main variable.

m_i is the sample median of ith sample $(i = 1, 2, ..., {}^N C_n)$.

${}^N C_n$ is the number of samples of size n from N.

The study variable dataset is skewed, $M \neq \bar{Y}$.

$C_m = \frac{S_m}{M}$, $C_{ym} = \frac{S_{ym}}{\bar{Y}M}$, $S_{ym} = \frac{1}{{}^N C_n}\sum_{i=1}^{{}^N C_n}(\bar{y}_i - \bar{Y})(m_i - M)$, $S_m^2 = \frac{1}{{}^N C_n}\sum_{i=1}^{{}^N C_n}(m_i - M)^2$.

$\rho = \frac{cov(x,y)}{S_x S_y}$, is the population correlation coefficient between X and Y.

$\gamma = \frac{1-f}{n}$, where the sampling fraction $f = \frac{n}{N}$.

$\theta = \frac{r\bar{X}}{2(r\bar{X}+s)}$".

3. Examining Current Ratio Type Estimators

The sample mean estimator is the fundamental estimator of the population mean without the usage of an auxiliary variable as

$$t_1 = \frac{1}{n}\sum_{i=1}^{n} y_i.$$

Bias and MSE of the estimator t_1 up to $O(n)^{-1}$ are as

$$Bias(t_1) = 0. \tag{1}$$

$$MSE(t_1) = \gamma \bar{Y}^2 C_y^2. \tag{2}$$

Making the use of auxiliary variable Cochran [2] proposed a ratio estimator which is more efficient than the estimator t_1, if $\frac{C_x}{2C_y} < \rho \leq +1$ as

$$t_2 = \bar{y}\frac{\bar{X}}{\bar{x}}.$$

The Bias and MSE expressions of the estimator t_2 up to $O(n)^{-1}$ are as

$$Bias(t_2) = \gamma \bar{Y}(C_x^2 - C_{yx}). \tag{3}$$

$$MSE(t_2) = \gamma \bar{Y}^2(C_y^2 + C_x^2 - 2C_{yx}). \tag{4}$$

According to Bahl and Tuteja [1] proposal, an effective exponential ratio type estimator may be applied even when X and Y have weak correlations.

$$t_3 = \bar{y} \exp \left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}} \right).$$

The estimator t_3 is more efficient than the sample mean estimator t_1, if $\frac{1}{4} < \rho_{xy} \frac{C_y}{C_x} < \frac{3}{4}$. The Bias and MSE up to $O(n)^{-1}$ are as

$$Bias(t_3) = \gamma \bar{Y} \left(\frac{3}{8}C_x^2 - \frac{1}{2}C_{yx} \right). \tag{5}$$

$$MSE(t_3) = \gamma \bar{Y}^2 \left(C_y^2 + \frac{C_x^2}{4} - C_{yx} \right). \tag{6}$$

A family of modified exponential ratio estimators was presented by Singh et al. [12] employing various well-known auxiliary variable characteristics, such as correlation coefficient, coefficient of variation, skewness, etc. as

$$t_4 = \bar{y} \exp \left[\frac{(r\bar{X} + s) - (r\bar{x} - s)}{(r\bar{X} + s) + (r\bar{x} - s)} \right].$$

The Bias and MSE expressions of up to $O(n)^{-1}$ for the estimator t_4 are as

$$Bias(t_4) = \gamma \bar{Y}(\theta^2 C_x^2 - \theta C_{yx}). \tag{7}$$

$$MSE(t_4) = \gamma \bar{Y}^2(C_y^2 + \theta^2 C_x^2 - 2\theta C_{yx}). \tag{8}$$

A median-based ratio type estimator without the usage of an auxiliary variable was proposed by Subramani [6] as

$$t_5 = \bar{y} \left[\frac{M}{m} \right].$$

The estimator t_5 is biased with the expressions of Bias and MSE up to $O(n)^{-1}$ as

$$Bias(t_5) = \gamma \bar{Y} \left(C_m^2 - C_{ym} - \frac{Bias(m)}{\gamma M} \right). \tag{9}$$

$$MSE(t_5) = \gamma \bar{Y}^2 \left(C_y^2 + \frac{\bar{Y}^2}{M^2}C_m^2 - 2\frac{\bar{Y}}{M}C_{ym} \right). \tag{10}$$

The sample mean estimators is biased while as all other estimators viz. Cochran [2], Bahl and Tuteja [1], Singh *et al.* [12] and Subramani [6] discussed above are biased.

4. Proposed Exponential Ratio Estimators

In the absence of auxiliary variable, the proposed exponential ratio type estimators of the study are as

$$t_{ue1} = \bar{y} \left[\alpha \exp \left(\frac{M - m}{aM} \right) + (1 - \alpha) \exp \left(\frac{m - M}{aM} \right) \right].$$

$$t_{ue2} = \bar{y} \left[\beta \exp \left(\frac{M - m}{bm} \right) + (1 - \beta) \exp \left(\frac{m - M}{bm} \right) \right].$$

The value of non zero constants a and b is chosen such that the estimators t_{ue1} and t_{ue2} are unbiased and the value of constants α and β are chosen such that the MSE of t_{ue1} and t_{ue2} should be minimum.
Consider,

$$\bar{y} = \bar{Y}(1 + e_0) \quad \text{and} \quad m = M(1 + e_1)$$

Therefore,

$$E(e_0) = 0, \quad E(e_1) = \frac{Bias(m)}{M}$$

$$E(e_0^2) = \gamma C_y^2; \quad E(e_1^2) = \gamma C_m^2; \quad E(e_0 e_1) = \gamma C_{ym}$$

Transforming the estimator t_{ue1} and t_{ue2} in terms of $e_i (i = 0, 1)$, the equations obtained are as

$$t_{ue1} = \bar{Y}(1 + e_0) \left[\alpha \exp\left(\frac{-e_1}{a}\right) + (1 - \alpha) \exp\left(\frac{e_1}{a}\right) \right]. \tag{11}$$

$$t_{ue2} = \bar{Y}(1 + e_0) \left[\beta \exp\left(\frac{-e_1}{b(1 + e_1)}\right) + (1 - \beta) \exp\left(\frac{e_1}{b(1 + e_1)}\right) \right]. \tag{12}$$

Solving the equations (11) & (12) and retaining the terms only up to up 2nd degree, the reduced equations are as

$$t_{ue1} = \bar{Y} \left[1 + e_0 + (1 - 2\alpha)\frac{e_1}{a} + \frac{e_1^2}{2a^2} + (1 - 2\alpha)\frac{e_0 e_1}{a} \right].$$

$$\Rightarrow t_{ue1} - \bar{Y} = \bar{Y} \left[e_0 + (1 - 2\alpha)\frac{e_1}{a} + \frac{e_1^2}{2a^2} + (1 - 2\alpha)\frac{e_0 e_1}{a} \right] \tag{13}$$

$$t_{ue2} = \bar{Y} \left[1 + e_0 + (1 - 2\beta)\frac{e_1}{b} + \left(\frac{1}{2b} + 2\beta - 1\right)\frac{e_1^2}{b} + (1 - 2\beta)\frac{e_0 e_1}{b} \right].$$

$$\Rightarrow t_{ue2} - \bar{Y} = \bar{Y} \left[e_0 + (1 - 2\beta)\frac{e_1}{b} + \left(\frac{1}{2b} + 2\beta - 1\right)\frac{e_1^2}{b} + (1 - 2\beta)\frac{e_0 e_1}{b} \right] \tag{14}$$

The bias of the estimators t_{ue1} and t_{ue2} is obtained by taking expectation on both sides of (13) and (14) as

$$Bias(t_{ue1}) = \gamma \bar{Y} \left[\frac{1}{2a^2} C_m^2 + \frac{1}{a}(1 - 2\alpha)\left(C_{ym} + \frac{Bias(m)}{\gamma M}\right) \right]. \tag{15}$$

$$Bias(t_{ue2}) = \gamma \bar{Y} \frac{1}{b} \left[\left(\frac{1}{2b} + 2\beta - 1\right) C_m^2 + (1 - 2\beta)\left(C_{ym} + \frac{Bias(m)}{\gamma M}\right) \right]. \tag{16}$$

Taking the expectation of the square of the equations (13) & (14), the mean square error of t_{de1} and t_{de2} is obtained as

$$MSE(t_{ue1}) = \gamma \bar{Y}^2 \left[C_y^2 + (1 - 2\alpha)^2 \frac{C_m^2}{a^2} + \frac{2}{a}(1 - 2\alpha)C_{ym} \right]. \tag{17}$$

$$MSE(t_{ue2}) = \gamma \bar{Y}^2 \left[C_y^2 + (1 - 2\beta)^2 \frac{C_m^2}{b^2} + \frac{2}{b}(1 - 2\beta)C_{ym} \right]. \tag{18}$$

The estimator t_{ue1} is unbiased, if

$$a = \frac{C_m^2}{2(2\alpha - 1)\left(C_{ym} + \frac{Bias(m)}{\gamma M}\right)}. \tag{19}$$

Whereas the estimator t_{ue2} is unbiased, if

$$b = \frac{C_m^2}{2(2\beta - 1)\left(\frac{Bias(m)}{\gamma M} + C_{ym} - C_m^2\right)}. \tag{20}$$

Substituting the values of (19) and (20) in equations (7) and (8) respectively, the following equations are obtained as

$$MSE(t_{ue1}) = \gamma \bar{Y}^2 \left[C_y^2 + 4(1-2\alpha)^4 \frac{\left(C_{ym} + \frac{Bias(m)}{\gamma M}\right)^2}{C_m^2} - 4(1-2\alpha)^2 \frac{C_{ym}\left(C_{ym} + \frac{Bias(m)}{\gamma M}\right)}{C_m^2} \right]. \quad (21)$$

$$MSE(t_{ue2}) = \gamma \bar{Y}^2 \left[C_y^2 + 4(1-2\beta)^4 \frac{\left(C_{ym} + \frac{Bias(m)}{\gamma M} - C_m^2\right)^2}{C_m^2} - 4(1-2\beta)^2 \frac{\left(C_{ym} + \frac{Bias(m)}{\gamma M} - C_m^2\right)}{C_m^2} C_{ym} \right]. \quad (22)$$

Differentiating equations (21) and (22) with respect to α and β respectively and equating to zero, the optimum value of α and β is obtained as

$$\alpha = \frac{1}{2} \pm \frac{1}{2} \sqrt{\frac{C_{ym}}{2\left(C_{ym} + \frac{Bias(m)}{\gamma M}\right)}} \quad \text{and} \quad \beta = \frac{1}{2} \pm \frac{1}{2} \sqrt{\frac{C_{ym}}{2\left(C_{ym} + \frac{Bias(m)}{\gamma M} - C_m^2\right)}}.$$

The constants a, b, α and β contain the unknowns C_m and C_{ym} whose value is considered to be known well in advance and if unknown, they can be determined from past surveys, experience carried by the researcher in the due course of time or from the pilot survey (See Srivenkataramana & Tracy [17], Singh & Kumar [16] and the references cited therein). Now on using the value of α and β in equation (21) and (22) respectively, the minimum value of MSE of the estimator t_{ue1} and t_{ue2} up to $O(n)^{-1}$ is obtained as

$$MSE_{min}(t_{uei}) = \gamma \bar{Y}^2 \left[C_y^2 - \frac{C_{ym}^2}{C_m^2} \right]. \quad (23)$$

5. Theoretical Efficiency Comparisons

From equations (2), (4), (6), (8), (10) and (23), the circumstances and conditions in which the suggested estimators outperform the sample mean estimator, the existing estimators of Cochran [2], Bahl and Tuteja [1], Singh *et al.* [12] and Subramani [6] are obtained as

$$MSE_{min}(t_{uei}) < MSE(t_1)$$

$$\Rightarrow \gamma \bar{Y}^2 \left[C_y^2 - \frac{C_{ym}^2}{C_m^2} \right] < \gamma \bar{Y}^2 C_y^2, \text{ if } C_{ym}^2 > 0. \quad (24)$$

$$MSE_{min}(t_{uei}) < MSE(t_2)$$

$$\Rightarrow \gamma \bar{Y}^2 \left[C_y^2 - \frac{C_{ym}^2}{C_m^2} \right] < \gamma \bar{Y}^2 (C_y^2 + C_x^2 - 2C_{yx}), \text{ if } C_{ym}^2 > C_m^2(2C_{yx} - C_x^2). \quad (25)$$

$$MSE_{min}(t_{uei}) < MSE(t_3)$$

$$\Rightarrow \gamma \bar{Y}^2 \left[C_y^2 - \frac{C_{ym}^2}{C_m^2} \right] < \gamma \bar{Y}^2 \left(C_y^2 + \frac{C_x^2}{4} - C_{yx} \right), \text{ if } C_{ym}^2 > \frac{C_m^2}{4}(4C_{yx} - C_x^2). \quad (26)$$

$$MSE_{min}(t_{uei}) < MSE(t_4)$$

$$\Rightarrow \gamma \bar{Y}^2 \left[C_y^2 - \frac{C_{ym}^2}{C_m^2} \right] < \gamma \bar{Y}^2 (C_y^2 + \theta^2 C_x^2 - 2\theta C_{yx}), \text{ if } C_{ym}^2 > C_m^2(2\theta C_{yx} - \theta^2 C_x^2). \quad (27)$$

$$MSE_{min}(t_{uei}) < MSE(t_5)$$

$$\Rightarrow \gamma \bar{Y}^2 \left[C_y^2 - \frac{C_{ym}^2}{C_m^2} \right] < \gamma \bar{Y}^2 \left(C_y^2 + \frac{\bar{Y}^2}{M^2} C_m^2 - 2\frac{\bar{Y}}{M} C_{ym} \right), \text{ if } C_{ym}^2 > C_m^2 \left(2\frac{\bar{Y}}{M} C_{ym} - \frac{\bar{Y}^2}{M^2} C_m^2 \right). \quad (28)$$

The proposed median based unbiased exponential ratio type estimators t_{uei} are more precise than the estimators t_1 to t_6 under the conditions (24) to (28).

6. Numerical Study Comparisons

For the numerical study, data of populations P1 and P2 containing influential observations have been considered as given in Table-1. The data set of population P1 is sourced from Singh and Chaudhary [7] where the study variable is to estimate area of wheat under cultivation in the year 1974 and the auxiliary variable is the cultivated area under wheat in the year 1971. The data set of population P2 is taken from Mukhopadhyay [5] where the study variable is to estimate the ammount of raw materials for 20 jute mills and the auxiliary variable is the number of workers.

Table 1: *Summary statistics of the population data sets.*

Data constants	Populations	
	P1	P2
N	34	20
n	5	5
$^N C_n$	278256	15504
$\bar{Y}$	856.4118	41.50000
$\bar{M}$	736.9811	40.05520
M	767.5000	40.50000
$\bar{X}$	208.8824	441.9500
$\frac{\bar{Y}}{\bar{M}}$	1.115800	1.024700
C_y^2	0.125014	0.008338
C_x^2	0.088563	0.007845
C_m^2	0.100833	0.006606
C_{ym}	0.073140	0.005394
C_{yx}	0.047257	0.005275
ρ_{yx}	0.449100	0.652200

It can be observed from Table-1 that a sample of 5 units has been drawn from two populations P1 and P2 having size 34 and 20 respectively. The values of different parameters like population mean, population median, coefficient of variation etc. of the study and auxiliary variable are obtained and can be seen from the table.

Table 2: *MSE, Bias and PRE of the estimators t_1, t_2, t_3, t_4, t_5 and t_{uei}.*

Estimator	Population					
	P1			P2		
	MSE	| Bias |	PRE	MSE	| Bias |	PRE
t_1	15641.306	0.000	100.000	2.154	0.000	100.000
t_2	14896.738	6.035	104.998	1.455	0.016	148.041
t_3	12498.850	1.399	125.142	1.297	0.002	166.076
t_4	12499.093	0.227	125.139	1.298	0.004	165.948
t_5	10926.773	38.100	143.147	1.090	0.463	197.615
t_{uei}	9003.545	0.000	173.724	1.016	0.000	212.007

The suggested median based exponential ratio estimators t_{uei} $(i = 1, 2)$ have the lowest MSE values for both population data sets P1 and P2, as can be seen from Table 2. When comparing with the sample mean estimators, estimators of Cochran [2], Bahl and Tuteja [1], Singh *et al.* [12], and Subramani [6], the PRE of the suggested estimators is found highest. Furthermore, as the estimators suggested in the paper are unbiased, they may be used to address the issue of under or overestimating the population mean.

7. Discussion

The paper presents two estimators of population mean as t_{ue1} and t_{ue2} based on population median. Since median is a type of parameter which is least influenced by the effect of outliers, so the proposed estimators may work efficiently for skewed data as evident from numerical study. It can be observed from empirical study that the MSE value for t_{uei} is 9003.545 and 1.016 for populations P1 and P2 respectively which can be observed as minimum value among all other estimators considered for comparision. The minimu value of MSE highlights that the estimators t_{uei} are most efficient. Further looking at the bias values, t_{uei} can be found as unbiased so will take care of under or over estimation problem . The population and sample median used in the construction of estimators t_{uei} are of study variable only, so do have a good advantage as the auxiliary variable may not be always available

8. Conclusion

- In the absence of an auxiliary variable, the suggested median-based almost-unbiased exponential ratio estimators of population mean are as follows

$$t_{ue1} = \bar{y} \left[\alpha \exp \left(\frac{M - m}{aM} \right) + (1 - \alpha) \exp \left(\frac{m - M}{aM} \right) \right].$$

$$t_{ue2} = \bar{y} \left[\beta \exp \left(\frac{M - m}{bm} \right) + (1 - \beta) \exp \left(\frac{m - M}{bm} \right) \right].$$

- The proposed estimators t_{ue1} and t_{ue2} are median based and therefore have least influence of outliers present in the data set.

- The estimators proposed in the study are more precise for a skewed data set than the estimators considered.

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