# Fuzzy Control Charts based on Ranking of Pentagonal Fuzzy Numbers

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**Abstract**

*A Control Chart is a fundamental approach in Statistical Process Control. When uncommon causes of variability are present, sample averages will plot beyond the control boundaries, making the control chart a particularly effective process monitoring approach. Uncertainties are caused by the measuring system, including the gauges operators and ambient circumstances. In this paper, the concept of fuzzy set theory is used for dealing with uncertainty. The control limits are to converted into fuzzy control limits using the membership function. The fuzzy $\bar{X}$-R and $\bar{X}$-S control chart is developed by using the ranking of the pentagonal fuzzy number system. An illustrative example is done with the discussed technique to make fuzzy $\bar{X}$-R and $\bar{X}$-R control charts and increase the flexibility of the control limit.*

**Keywords:** Statistical Process Control, Rank Membership Function, Fuzzy Pentagonal Number

## I. Introduction

Quality has long been recognized as a significant influencing element in the performance and competitiveness of manufacturing and service organizations in both domestic and global markets. The return on capital is the consequence of well-executed strategies. Appropriate quality techniques provide productive outcomes. Fuzzy number ranking is a component of the quality control planning system. The fuzzy mathematical model for transportation of vegetable diet plan based on the ranking function of fuzzy pentagonal number (FPN) and solved by Vogel's approximation method to minimize the cost is discussed by Venkatesh and Manoj [1]. In constructing an FPN and related arithmetic operations, A. Panda and M. Pal [2] established the logical definition. The construction and fundamental features of pentagonal fuzzy matrices (PFMs) are investigated using FPN. The algebraic natures of several particular types of PFMs (trace of PFM, adjoint of PFM, determinant of PFM, etc.) are addressed. Pathinathan and Ponnivalavan [3] discussed FPN in continuation with the other defined fuzzy numbers and addressed some basic arithmetic operations. A. Chakraborty et al. [4] discuss different measures of interval-valued pentagonal fuzzy numbers (IVFPN) associated with assorted membership functions, and the ranking function is the main feature. The ranking functions of FPN develop real application and comprehend the uncertainty of the parameters more precisely in the evaluation process. A.

Chakraborty et al. [5] dealt with the idea of pentagonal neutrosophic number (PNN) from a different frame of reference and discussed some properties of PNN with real-life operational research applications, which is more reliable than the other method. A. Shafqat et al. [6] used the lower record values for developing the $\bar{X}$ control chart for the Inverse Rayleigh Distribution (IRD) is designed under repetitive group sampling. The mean and standard deviation of the Inverse Rayleigh Distribution based on lower record values are used to determine the width and power of the $\bar{X}$ control limits. Lim S. A. H. [7] evolved $\bar{X}$-R and $\bar{X}$-S chart for the food industry in the UK. The control charts developed using triangular and trapezoidal fuzzy control for balanced and unbalanced [8, 9, 10]. Ozdemir [11] developed the fuzzy control chart with a triangular fuzzy system into three phases for $\bar{X}$-S chart and process capability indices using unbalanced data, converted the data for each sample into the fuzzy form and then decided the fuzzy limits and illustrated it for uncertain data. Yeh [12] Shows an example of weighted triangular approximation of fuzzy numbers, which Zheng and Li propose. Senturk et al. [13] researched the most popular control chart for univariate data, the exponential weighted moving average control chart under fuzzy environment and applied this work into real case applications in Turkey. Senturk et al. [14] consider the control chart for fuzzy nonconformities per unit by using alpha cut and applied this technique in real case applications for truck engine manufacture. Alipour and Noorossana [15] created a control chart using a fuzzy multivariate exponentially weighted moving average (F-MEWMA). The proposed technique is developed using a combination of multivariate statistical quality control and fuzzy set theory in this study. Erginel and Şentürk [16] derived the fuzzy exponential weighted moving average and cumulative sum control chart (CUSUM) with a suitable example. Erginel [17] developed a fuzzy P control chart by using the rules that introduces the fuzzy np chart based on the constant sample size and variable sample size. In addition, the decision is taken whether it is under control or out of control. In the uncertainty theory for modelling, fuzzy sets theory plays numerous important roles. An essential consideration is that if somebody wants to take an FPN, what should its graphical representations (uncertainty quantification area) look like? How should the membership functions be defined? From this perspective, we developed the phases of an FPN control chart that may be a good choice for a decision-maker in a real-world scenario. In this study we proposed a Fuzzy control chart by using rank membership function of Pentagonal fuzzy number and the example is done with this proposed technique.

## II. Development of Proposed Methodology

There are many articles published related to FPN. Venkatesh and Manoj [1] developed the pentagonal fuzzy model for transportation problems using the ranking membership function. Some definitions of FPN are as follows:

**Definition 1:** (Zadeh [19]) Let $X$ be a fixed set. A fuzzy set $\tilde{A}$ of X is an object having the form $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$ where $\mu_{\tilde{A}}(x) \in [0,1]$ represents the degree of membership of the element $x \in X$ being in $\tilde{A}$, and $\mu_{\tilde{A}} : X \to [0,1]$ is called the membership function.

**Definition 2:** The $\alpha$ -cut of the fuzzy set $\tilde{A}$ is defined as:

$$\bar{A}_{\alpha} = \{x \in x/\mu_{\tilde{A}}(x) \geq \alpha, \text{ where } \alpha \in (0, 1).$$

**Definition 3:** A set $\tilde{A}$ is defined on the real numbers $\Re$ is said to be a fuzzy number if its membership function $\mu_{\tilde{A}} : X \to [0,1]$ follows:

**(i)** $\tilde{A}$ is continuous.

**(ii)** $\tilde{A}$ is normal such that $\mu_{\tilde{A}}(x) = 1$ there exists an $x \in$ R

**Definition 4:** A fuzzy number $\tilde{A}_P$ is an FPN denoted by $\tilde{A}_P = (a_1, a_2, a_3, a_4, a_5)$, where $a_1, a_2, a_3, a_4, a_5$ are real numbers, and its membership function will be:

$$\mu_{\tilde{A}_P}(x) = \begin{cases} 0 & , \ x < a_1 \\ \frac{(x-a_1)}{(a_2-a_1)} & , \ a_1 \leq x \leq a_2 \\ \frac{1}{2}\frac{(x-a_2)}{(a_3-a_2)} & , \ a_2 \leq x \leq a_3 \\ 1 , & x = a_3 \\ \frac{(a_4-x)}{2(a_4-a_3)} & , \ a_3 \leq x \leq a_4 \\ \frac{(a_5-x)}{(a_5-a_4)} & , \ a_4 \leq x \leq a_5 \\ 0 & , \ x > a_5 \end{cases} \tag{1}$$

**Ranking of FPN:** Ranking a fuzzy number entails comparing up to two fuzzy numbers, and defuzzification is a technique for converting a fuzzy number to an estimated crisp number. Just as the decision-maker compares two ideas that are the same, we must convert the fuzzy number to a comparable crisp number and compare the numbers based on crisp values in this problem.

Fuzzy numbers become the real line directly by using the ranking method [1]. Let $\tilde{A}_P$ be a generalized FPN. The ranking of $\tilde{A}_P$ is symbolised by $R(\tilde{A}_P)$ and it is calculated as follows:

$$R(\tilde{A}_P) = \left[\frac{a_1+2a_2+3a_3+2a_4+a_5}{9}\right] \tag{2}$$

**Statistical Process Control:** Many quality attributes may be stated numerically. A bearing's diameter, for example, might be measured using a micrometre and represented in mm. A variable is a single quantifiable qualitative attribute, such as a dimension, weight, or volume. Control charts for variables are widely used. When dealing with a variable quality characteristic, it is frequently required to monitor both the mean value and the variability of the quality characteristic. The control chart for means, or the control chart, is typically used to regulate the process average or mean quality level. Process variability may be tracked using either a control chart for the standard deviation known as an S control chart or a control chart for the range, known as an R control chart. The $\bar{X}$, R-chart and $\bar{X}$, S-chart is the most widely used control chart for the production process [16, 18].

### $\bar{X}$ and R control charts

**Table 1:** *Formula for $\bar{X}$ and R Control Charts*

| Chart | Lower Control Limit (LCL) | Central Line | Upper Control Limit (UCL) |
|---|---|---|---|
| $\bar{X}$ | $\bar{\bar{X}} - A_2\bar{R}$ | $\bar{\bar{X}}$ | $\bar{\bar{X}} + A_2\bar{R}$ |
| $R$ | $\bar{R}D_3$ | $\bar{R}$ | $\bar{R}D_4$ |

where, $\bar{\bar{X}} = \frac{\sum \bar{x}}{m}$, $\bar{x} = \frac{\sum_{i=1}^{5} R(\tilde{A}_P(x_i))}{n}$, $A_2$ is constant tabulated value for n.
where, $\bar{R} = \frac{\sum Range(R(\tilde{A}_P(x_i)))}{m}$, and $D_3, D_4$ are constant tabulated values for n.

**$\bar{X}$ and S Control Charts**

**Table 2:** *Formula for $\bar{X}$ and S Control Charts*

| Charts | LCL | Central Line | UCL |
|---|---|---|---|
| $\bar{X}$ | $\bar{\bar{X}} - A_3\bar{S}$ | $\bar{\bar{X}}$ | $\bar{\bar{X}} + A_3\bar{S}$ |
| $S$ | $B_3\bar{S}$ | $\bar{S}$ | $B_4\bar{S}$ |

where, $\bar{\bar{X}} = \frac{\sum \bar{x}}{m}$, $\bar{x} = \frac{\sum_{i=1}^{5} R(\tilde{A}_P(x_i))}{n}$, $S = \sqrt{\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{(n-1)}}$, $\bar{S} = \frac{\sum_{i=1}^{m} S}{m}$ and $A_3, B_3, B_4$ are constant tabulated values for n.

The next step will be complete with three steps for the proposed fuzzy control chart by using the following procedure.

**Step 1:** ***Normality Assumption***- Check the normality assumption using Anderson Darling Test (Anderson and Darling, 1954).

**Step 2:** ***Use of FPN Control Chart for $\bar{X}$-R* and *$\bar{X}$-S***

A summary of the work on the FPN Control Chart is as follows:

- (i) The development of FPN
- (ii) The representation of the FPNs in parametric form.
- (iii) Apply ranking and defuzzification of FPN for the data.
- (iv) Put the calculated crisp value into the control chart formula and set up the FPN control limit.

**Step 3:** ***Interpretation of FPN Control Chart for $\bar{X}$-R and $\bar{X}$-S***. The fuzzy CLs of the recommended $\bar{X}$-R and $\bar{X}$-S control charts are used to assess the fuzzy sample mean and standard deviation. If the fuzzy sample mean and standard deviation are inside the control bounds, the process is under control for the sample. Otherwise, the process will spiral out of control.

## III. Illustrative Example

In this article, we choose the simulation data shown in Table 3 are the deviations from milling a slot in an aircraft terminal block. A high rate of rejections for many of the components manufactured in an aviation company's machine shop highlighted the necessity for an investigation into the causes of the problems. Because the majority of the rejections were for failing to satisfy dimensional limits, it was decided to utilize $\bar{X}$-R and $\bar{X}$-S charts to try to pinpoint the source of the problem. These charts, which needed real dimension measurement, were to be utilized just for the dimensions that were creating a high number of rejections. Among many such dimensions, the ones chosen for control charts were those with significant spoilage costs and reworked for those where rejections caused delays in assembly processes. Although the primary objective of all of the $\bar{X}$-R and $\bar{X}$-S charts was to diagnose problems, it was expected that some of the charts would be kept for routine process control and potentially for acceptance inspection.

**Table 3:** *Data for width of slot in an aircraft terminal block.*

| Sample | X1 | X2 | X3 | X4 | X5 |
|---|---|---|---|---|---|
| **1** | 773 | 803 | 780 | 720 | 776 |
| **2** | 757 | 786 | 734 | 740 | 735 |
| **3** | 755 | 774 | 720 | 761 | 746 |
| **4** | 745 | 779 | 755 | 775 | 772 |
| **5** | 800 | 728 | 747 | 759 | 745 |

| | | | | | |
|---|---|---|---|---|---|
| **6** | 784 | 806 | 787 | 763 | 758 |
| **7** | 745 | 765 | 754 | 759 | 768 |
| **8** | 789 | 751 | 782 | 769 | 763 |
| **9** | 757 | 747 | 741 | 746 | 747 |
| **10** | 746 | 731 | 762 | 781 | 744 |
| **11** | 742 | 731 | 754 | 736 | 750 |
| **12** | 748 | 726 | 764 | 735 | 733 |
| **13** | 748 | 763 | 779 | 786 | 770 |
| **14** | 770 | 768 | 784 | 771 | 767 |
| **15** | 772 | 759 | 770 | 772 | 772 |
| **16** | 765 | 768 | 773 | 791 | 787 |
| **17** | 780 | 777 | 742 | 761 | 761 |
| **18** | 775 | 765 | 746 | 782 | 745 |
| **19** | 759 | 748 | 781 | 764 | 756 |
| **20** | 765 | 782 | 739 | 754 | 768 |
| **21** | 770 | 784 | 730 | 759 | 781 |
| **22** | 773 | 765 | 777 | 760 | 750 |
| **23** | 758 | 780 | 761 | 746 | 757 |
| **24** | 761 | 771 | 756 | 772 | 758 |
| **25** | 752 | 785 | 764 | 758 | 773 |
| **26** | 745 | 774 | 786 | 739 | 796 |
| **27** | 759 | 766 | 790 | 730 | 781 |
| **28** | 739 | 795 | 750 | 780 | 776 |
| **29** | 747 | 781 | 763 | 768 | 756 |
| **30** | 767 | 752 | 774 | 746 | 769 |

The first step is to test the normality assumption. It is shown that Figure 1 holds the normality assumption for the above data.

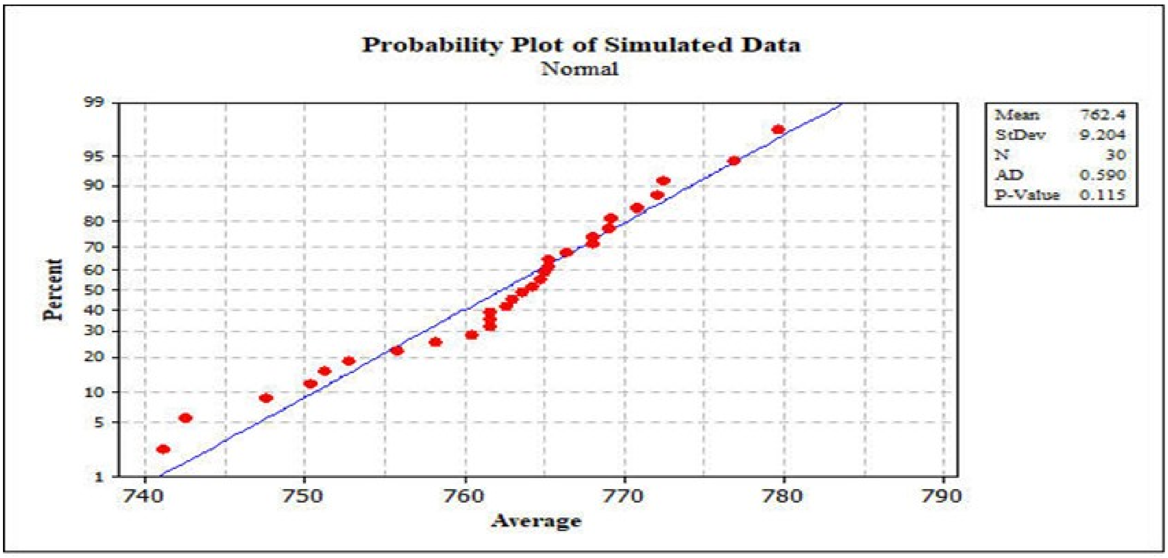


**Figure 1:** *Normal Probability Plot*

Now the data shows normal, and we convert the data into FPN form. Table [4-8] shows the FPN form for the variables X1, X2, X3, X4 and X5.

**Table 4:** *FPN for X1*

| Xa1 | Xb1 | Xc1 | Xd1 | Xe1 | $R(\widetilde{A}_P(x_1))$ |
|---|---|---|---|---|---|
| 768 | 770 | 773 | 777 | 785 | 774 |
| 752 | 754 | 757 | 761 | 771 | 758.2222 |
| 750 | 752 | 755 | 759 | 768 | 756.1111 |
| 740 | 742 | 745 | 749 | 757 | 746 |
| 795 | 797 | 800 | 804 | 812 | 801 |
| 777 | 781 | 784 | 788 | 796 | 784.7778 |
| 738 | 741 | 745 | 749 | 757 | 745.5556 |
| 781 | 786 | 789 | 793 | 801 | 789.6667 |
| 748 | 754 | 757 | 760 | 769 | 757.3333 |
| 740 | 743 | 746 | 749 | 752 | 746 |
| 721 | 727 | 730 | 733 | 741 | 730.2222 |
| 739 | 745 | 748 | 751 | 758 | 748.1111 |
| 740 | 744 | 748 | 751 | 759 | 748.1111 |
| 764 | 767 | 770 | 773 | 780 | 770.4444 |
| 766 | 769 | 772 | 775 | 778 | 772 |
| 757 | 761 | 765 | 769 | 786 | 766.4444 |
| 775 | 777 | 780 | 784 | 792 | 781 |
| 770 | 772 | 775 | 779 | 788 | 776.1111 |
| 749 | 756 | 759 | 763 | 771 | 759.4444 |
| 758 | 763 | 765 | 769 | 778 | 766.1111 |
| 763 | 767 | 770 | 774 | 783 | 770.8889 |
| 768 | 770 | 773 | 777 | 785 | 774 |
| 750 | 755 | 758 | 762 | 770 | 758.6667 |
| 756 | 758 | 761 | 765 | 774 | 762.1111 |
| 745 | 749 | 752 | 756 | 764 | 752.7778 |
| 738 | 743 | 745 | 750 | 759 | 746.4444 |
| 754 | 756 | 759 | 763 | 772 | 760.1111 |
| 734 | 736 | 739 | 743 | 751 | 740 |
| 740 | 745 | 747 | 751 | 760 | 748.1111 |
| 760 | 764 | 767 | 771 | 779 | 767.7778 |

**Table 5:** *FPN for X2*

| Xa2 | Xb2 | Xc2 | Xd2 | Xe2 | $R(\widetilde{A}_P(x_2))$ |
|---|---|---|---|---|---|
| 796 | 799 | 803 | 808 | 816 | 803.8889 |
| 778 | 781 | 786 | 790 | 798 | 786.2222 |
| 768 | 771 | 774 | 779 | 785 | 775 |
| 771 | 776 | 779 | 784 | 791 | 779.8889 |
| 719 | 724 | 728 | 735 | 742 | 729.2222 |
| 799 | 802 | 806 | 811 | 819 | 806.8889 |
| 758 | 762 | 765 | 770 | 776 | 765.8889 |
| 746 | 749 | 751 | 758 | 765 | 753.1111 |

| | | | | | |
|---|---|---|---|---|---|
| 741 | 745 | 747 | 752 | 759 | 748.3333 |
| 725 | 729 | 731 | 736 | 744 | 732.4444 |
| 724 | 728 | 731 | 735 | 745 | 732 |
| 719 | 723 | 726 | 730 | 737 | 726.6667 |
| 757 | 760 | 763 | 769 | 775 | 764.3333 |
| 760 | 765 | 768 | 773 | 778 | 768.6667 |
| 752 | 755 | 759 | 764 | 772 | 759.8889 |
| 760 | 765 | 768 | 774 | 780 | 769.1111 |
| 770 | 774 | 777 | 781 | 789 | 777.7778 |
| 760 | 762 | 765 | 769 | 778 | 766.1111 |
| 743 | 745 | 748 | 752 | 760 | 749 |
| 774 | 779 | 782 | 786 | 794 | 782.6667 |
| 777 | 782 | 784 | 788 | 797 | 785.1111 |
| 758 | 762 | 765 | 769 | 777 | 765.7778 |
| 772 | 777 | 780 | 784 | 792 | 780.6667 |
| 764 | 769 | 771 | 775 | 780 | 771.6667 |
| 780 | 782 | 785 | 789 | 796 | 785.8889 |
| 768 | 771 | 774 | 779 | 789 | 775.4444 |
| 759 | 761 | 766 | 771 | 782 | 767 |
| 790 | 793 | 795 | 799 | 808 | 796.3333 |
| 776 | 779 | 781 | 786 | 794 | 782.5556 |
| 747 | 750 | 752 | 757 | 765 | 753.5556 |

**Table 6:** *FPN for X3*

| Xa3 | Xb3 | Xc3 | Xd3 | Xe3 | $R(\widetilde{A}_P(x_3))$ |
|---|---|---|---|---|---|
| 773 | 777 | 780 | 783 | 788 | 780.1111 |
| 727 | 731 | 734 | 740 | 744 | 735 |
| 715 | 717 | 720 | 725 | 728 | 720.7778 |
| 750 | 753 | 755 | 760 | 764 | 756.1111 |
| 741 | 745 | 747 | 753 | 755 | 748.1111 |
| 782 | 785 | 787 | 793 | 796 | 788.3333 |
| 749 | 752 | 754 | 760 | 763 | 755.3333 |
| 777 | 779 | 782 | 788 | 791 | 783.1111 |
| 737 | 740 | 741 | 747 | 750 | 742.6667 |
| 756 | 759 | 762 | 765 | 769 | 762.1111 |
| 748 | 751 | 754 | 760 | 763 | 755 |
| 758 | 761 | 764 | 770 | 773 | 765 |
| 772 | 775 | 779 | 785 | 787 | 779.5556 |
| 778 | 781 | 784 | 791 | 793 | 785.2222 |
| 764 | 767 | 770 | 776 | 779 | 771 |
| 768 | 771 | 773 | 780 | 782 | 774.5556 |
| 737 | 740 | 742 | 748 | 756 | 743.8889 |
| 740 | 744 | 746 | 750 | 758 | 747.1111 |
| 775 | 779 | 781 | 786 | 794 | 782.4444 |
| 734 | 737 | 739 | 746 | 754 | 741.2222 |
| 725 | 728 | 730 | 737 | 746 | 732.3333 |
| 771 | 774 | 777 | 783 | 791 | 778.5556 |

| | | | | | |
|---|---|---|---|---|---|
| 757 | 759 | 761 | 768 | 776 | 763.3333 |
| 751 | 754 | 756 | 760 | 768 | 757.2222 |
| 759 | 762 | 764 | 769 | 777 | 765.5556 |
| 781 | 784 | 786 | 790 | 798 | 787.2222 |
| 785 | 788 | 790 | 795 | 805 | 791.7778 |
| 745 | 747 | 750 | 758 | 768 | 752.5556 |
| 758 | 760 | 763 | 768 | 775 | 764.2222 |
| 768 | 771 | 774 | 779 | 787 | 775.2222 |

**Table 7:** *FPN for X4*

| Xa4 | Xb4 | Xc4 | Xd4 | Xe4 | $R(\widetilde{A}_P(x_4))$ |
|---|---|---|---|---|---|
| 715 | 717 | 720 | 727 | 729 | 721.3333 |
| 736 | 738 | 740 | 749 | 751 | 742.3333 |
| 756 | 759 | 761 | 768 | 771 | 762.6667 |
| 770 | 773 | 775 | 781 | 784 | 776.3333 |
| 753 | 756 | 759 | 766 | 768 | 760.2222 |
| 758 | 761 | 763 | 769 | 771 | 764.2222 |
| 753 | 756 | 759 | 766 | 768 | 760.2222 |
| 764 | 767 | 769 | 775 | 778 | 770.3333 |
| 740 | 743 | 746 | 753 | 756 | 747.3333 |
| 776 | 779 | 781 | 787 | 789 | 782.2222 |
| 728 | 733 | 736 | 742 | 743 | 736.5556 |
| 729 | 732 | 735 | 741 | 743 | 735.8889 |
| 781 | 783 | 786 | 793 | 795 | 787.3333 |
| 766 | 769 | 771 | 778 | 781 | 772.6667 |
| 768 | 770 | 772 | 778 | 782 | 773.5556 |
| 787 | 789 | 791 | 797 | 799 | 792.3333 |
| 755 | 758 | 761 | 768 | 775 | 762.7778 |
| 777 | 780 | 782 | 787 | 794 | 783.4444 |
| 757 | 760 | 764 | 769 | 778 | 765 |
| 748 | 752 | 754 | 759 | 767 | 755.4444 |
| 754 | 757 | 759 | 765 | 773 | 760.8889 |
| 755 | 758 | 760 | 765 | 774 | 761.6667 |
| 741 | 744 | 746 | 750 | 758 | 747.2222 |
| 767 | 770 | 772 | 777 | 784 | 773.4444 |
| 753 | 755 | 758 | 764 | 773 | 759.7778 |
| 734 | 736 | 739 | 745 | 755 | 740.8889 |
| 725 | 728 | 730 | 735 | 743 | 731.5556 |
| 775 | 778 | 780 | 785 | 794 | 781.6667 |
| 763 | 765 | 768 | 772 | 780 | 769 |
| 741 | 744 | 746 | 751 | 760 | 747.6667 |

**Table 8:** *FPN for X5*

| Xa5 | Xb5 | Xc5 | Xd5 | Xe5 | $R(\widetilde{A}_P(x_5))$ |
|---|---|---|---|---|---|
| 770 | 773 | 776 | 782 | 784 | 776.8889 |
| 729 | 733 | 735 | 742 | 745 | 736.5556 |
| 741 | 744 | 746 | 753 | 755 | 747.5556 |
| 767 | 769 | 772 | 778 | 780 | 773 |
| 739 | 743 | 745 | 751 | 753 | 746.1111 |
| 752 | 754 | 758 | 764 | 767 | 758.7778 |
| 762 | 766 | 768 | 774 | 776 | 769.1111 |
| 758 | 760 | 763 | 769 | 772 | 764.1111 |
| 741 | 745 | 747 | 754 | 758 | 748.6667 |
| 738 | 741 | 744 | 750 | 752 | 744.8889 |
| 744 | 747 | 750 | 756 | 758 | 750.8889 |
| 727 | 730 | 733 | 739 | 743 | 734.1111 |
| 764 | 768 | 770 | 776 | 779 | 771.2222 |
| 761 | 765 | 767 | 774 | 778 | 768.6667 |
| 767 | 770 | 772 | 778 | 781 | 773.3333 |
| 782 | 785 | 787 | 793 | 795 | 788.2222 |
| 756 | 758 | 761 | 766 | 772 | 762.1111 |
| 740 | 743 | 745 | 750 | 759 | 746.6667 |
| 751 | 754 | 756 | 760 | 768 | 757.2222 |
| 763 | 766 | 768 | 771 | 779 | 768.8889 |
| 776 | 779 | 781 | 785 | 793 | 782.2222 |
| 745 | 748 | 750 | 755 | 762 | 751.4444 |
| 752 | 754 | 757 | 761 | 769 | 758 |
| 753 | 755 | 758 | 763 | 770 | 759.2222 |
| 768 | 770 | 773 | 778 | 784 | 774.1111 |
| 790 | 793 | 796 | 800 | 808 | 796.8889 |
| 776 | 779 | 781 | 786 | 794 | 782.5556 |
| 771 | 774 | 776 | 780 | 788 | 777.2222 |
| 751 | 753 | 756 | 760 | 767 | 756.8889 |
| 765 | 767 | 769 | 773 | 780 | 770.2222 |

After the FPN form, we use the rank membership function (equation (1)) for the crisp value.

**Table 9:** *The crisp value for X1, X2, X3, X4, X5*

| $R(\widetilde{A}_P(x_1))$ | $R(\widetilde{A}_P(x_2))$ | $R(\widetilde{A}_P(x_3))$ | $R(\widetilde{A}_P(x_4))$ | $R(\widetilde{A}_P(x_5))$ |
|---|---|---|---|---|
| 774 | 803.8889 | 780.1111 | 721.3333 | 776.8889 |
| 758.2222 | 786.2222 | 735 | 742.3333 | 736.5556 |
| 756.1111 | 775 | 720.7778 | 762.6667 | 747.5556 |
| 746 | 779.8889 | 756.1111 | 776.3333 | 773 |
| 801 | 729.2222 | 748.1111 | 760.2222 | 746.1111 |
| 784.7778 | 806.8889 | 788.3333 | 764.2222 | 758.7778 |
| 745.5556 | 765.8889 | 755.3333 | 760.2222 | 769.1111 |
| 789.6667 | 753.1111 | 783.1111 | 770.3333 | 764.1111 |
| 757.3333 | 748.3333 | 742.6667 | 747.3333 | 748.6667 |
| 746 | 732.4444 | 762.1111 | 782.2222 | 744.8889 |

| | | | | |
|---|---|---|---|---|
| 730.2222 | 732 | 755 | 736.5556 | 750.8889 |
| 748.1111 | 726.6667 | 765 | 735.8889 | 734.1111 |
| 748.1111 | 764.3333 | 779.5556 | 787.3333 | 771.2222 |
| 770.4444 | 768.6667 | 785.2222 | 772.6667 | 768.6667 |
| 772 | 759.8889 | 771 | 773.5556 | 773.3333 |
| 766.4444 | 769.1111 | 774.5556 | 792.3333 | 788.2222 |
| 781 | 777.7778 | 743.8889 | 762.7778 | 762.1111 |
| 776.1111 | 766.1111 | 747.1111 | 783.4444 | 746.6667 |
| 759.4444 | 749 | 782.4444 | 765 | 757.2222 |
| 766.1111 | 782.6667 | 741.2222 | 755.4444 | 768.8889 |
| 770.8889 | 785.1111 | 732.3333 | 760.8889 | 782.2222 |
| 774 | 765.7778 | 778.5556 | 761.6667 | 751.4444 |
| 758.6667 | 780.6667 | 763.3333 | 747.2222 | 758 |
| 762.1111 | 771.6667 | 757.2222 | 773.4444 | 759.2222 |
| 752.7778 | 785.8889 | 765.5556 | 759.7778 | 774.1111 |
| 746.4444 | 775.4444 | 787.2222 | 740.8889 | 796.8889 |
| 760.1111 | 767 | 791.7778 | 731.5556 | 782.5556 |
| 740 | 796.3333 | 752.5556 | 781.6667 | 777.2222 |
| 748.1111 | 782.5556 | 764.2222 | 769 | 756.8889 |
| 767.7778 | 753.5556 | 775.2222 | 747.6667 | 770.2222 |

All the crisp calculated value of the simulated data is in Table 9. Now we calculate the mean of crisps value and then put it into the formula of $\bar{X}$- R and $\bar{X}$-S, which is given in Table [1&2]. Now we found that the Control limits for the data are as follows:
UCL=785.65, LCL=741.39 and CL=763.52 for $\bar{X}$ chart,
UCL=81.11, LCL=0 and CL=38.36 for R chart and
UCL=32.38, LCL=0 and CL=15.50 for S chart.

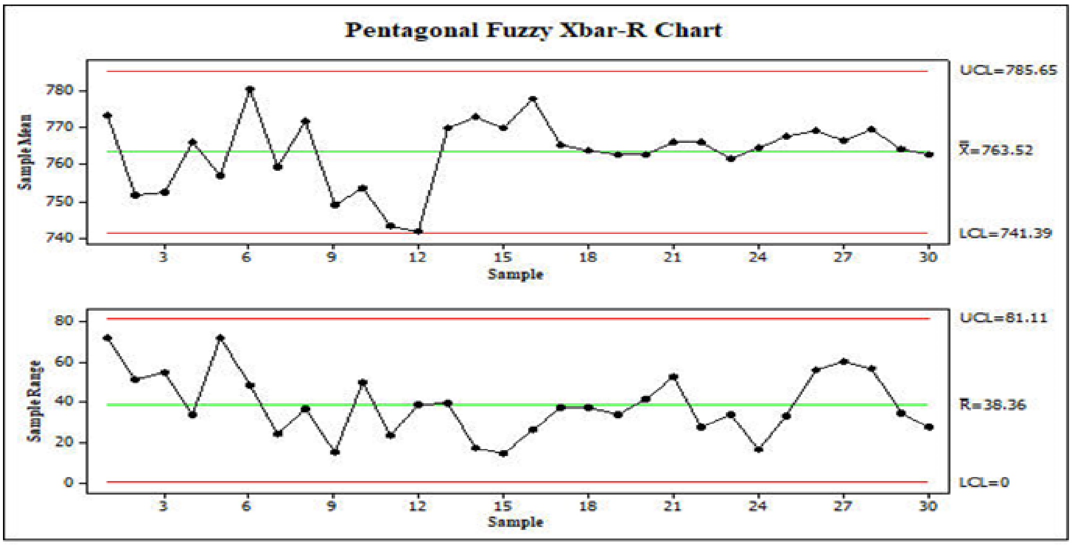


**Figure 2:** *Pentagonal Fuzzy* $\bar{X} - R$ *Chart*

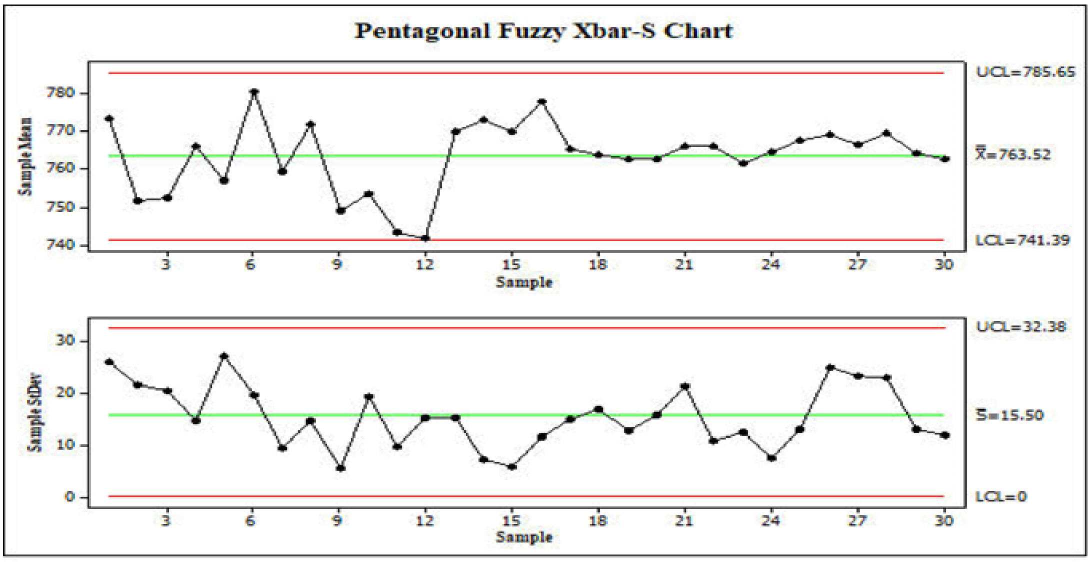


**Figure 3:** *Pentagonal Fuzzy $\bar{X} - S$ Chart*

It is shown that there is no point out of control after plotting the $\bar{X}$-R and $\bar{X}$-S control charts.

## IV. Conclusion

In this paper, it is shown that FPN is suitable for traditional variable control charts. If uncertainty is presented in the data, the FPN control chart theory should control the process. In this study, the population parameter ($\mu$ *and* $\sigma$) is unknown, and we develop the theory for fuzzy $\bar{X}$-R and $\bar{X}$-S chart by using the rank membership function of FPN. More fuzzy control charts for variable and attribute data were done by $\alpha - cut$, triangular and trapezoidal fuzzy numbers in several published articles. The result of the illustrative example is done with this proposed technique, and it is shown that the fuzzy process is under control and capable. The proposed FPN control chart effectively increases the process flexibility. For further studies, the process capability indices, p-np chart, fuzzy CUSUM and fuzzy EWMA chart can be used to detect the slight shifting in the FPN process control with fuzzy data.

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