RELIABILITY MODELLING OF A PARALLEL-COLD STANDBY SYSTEM WITH REPAIR PRIORITY

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Abstract

This paper deals with the reliability modelling of a parallel cold standby system of four units. The units operate in two phases; phase-I and phase-II. In phase-I, two identical units (called main units) work in parallel and the other two identical units (called duplicate units) have been taken as spare in cold standby. The units of phase-I and of the phase-II are not identical. The priority to repair the units of phase-I has been given over the repair of the units of the phase-II. However, no priority is given for operation of the units of both phases. There is a single repair facility which tackles all types of faults whenever occurred in the system. After repair each unit works as new and the switches devices are considered as perfect. The repair time of the units follows arbitrary probability distribution while the failure time of the units is assumed as constant. The behaviour of mean sojourn time (MST), transition probabilities, mean time to system failures (MTSF), availability, expected number of repairs for both phase-I and phase-II units, expected number of visits of the server, busy period of the server and finally the profit function are obtained in steady state by making use of well-known semi-Markov process (SMP) and Regenerative Point Technique (RPT) for arbitrary values of the parameters in steady state. Novelty and Application: A four-unit system is configured in two phases namely; phase-I and phase-II under some novel assumptions with a practical visualization in metallic bush manufacturing industries.

Keywords: Parallel-Cold Standby System, Phase wise Non-identical Units, Reliability Measures, Priority and Profit Analysis

1. Introduction

The advancement in technology played a great role in economic development and thus in the overall growth of a country. This plays a fundamental role in wealth creation, improvement of the quality of life, and real economic growth and transformation in any society. Technology has a great impact on everyone's life, including industries, which are dependent on machines for their chores. The great challenge for researchers and engineers is to produce highly reliable products at minimum cost. Thus, a basic need in the fast-growing industries is to select highly reliable systems subject to the cost. Many attempts from the researchers, engineers and industrialists have been made to improve the performance and designing of existing machines. Moreover, it is challenging for researchers and engineers to produce high quality products at minimum cost. Thus, reliability and profit analysis play a key role in defining quality of systems. Various techniques for improving performance and reliability of maintainable systems operating under different environmental conditions have been suggested by the researchers from time to time.

Barak and Malik [1] performed the cost benefit analysis of computer system with priority to preventive maintenance using the concepts of maximum operation and repair times. Pundir et al. [2], [3] performed a Bayesian analysis by using some prior information for two non-identical cold standby system and stochastic analysis of two non-identical unit parallel system with priority in repair. Kadyan et al. [4,5] discussed a non-identical repairable system of three units with priority for operation and priority to main unit for operation and repair with the simultaneous working of cold-standby units. Using the concept of periodic switching approach, reliability modelling of two-unit cold standby system is performed by Behboudi et al. [6]. Fryilmaz and Finkelsteil [7] discussed the reliability of two-unit system with the revisit of standby system. Stochastic analysis of a computer system with redundant and priority to hardware and repair subject to failure of service facility have been discussed by Yadav and Malik [8]. Anuradha and Malik [10] have obgtained the reliability measures of a 2-out-3 systems under the conditioner service facility. A cold standby system subject to refreshment was studied by kumar at el. [11]. The models discussed by different researchers focus either on the identical units in parallel or one non- identical unit in spare. But there can be situations where one non-identical unit is not capable enough to work at place of failed unit and two or more units are required to work simultaneously in order to meet the system expectations. Therefore, the reliability analysis of a system model of four unit operating in two phases with simultaneous working of parallel and cold standby units has been analysed to add significant insight into reliability literature.

2. System description and Notations

I. Notations

Table 1: *Symbol Description*

MST	Mean sojourn time
MTSF	Mean time to system failure
O	System is operative
Dc	Cold-standby unit
•	Regenerative point
M/D	Phase-I unit (main units)/ Phase-II unit (duplicate units)
MFur/MFwr	Phase-I unit is failed and under repair/waiting for repair
MFUR/ MFWR	Phase-I unit is failed and under repair/waiting for repair continuously from the previous state
DFur/DFwr	Phase-II unit is failed and under repair//waiting for repair
DFUR/ DFWR	Phase-II unit is failed and under repair/waiting for repair continuously from the previous state
λ/λ_1	Failure rate of phase-I unit/ phase-II unit
g(t)/G(t)	pdf/cdf of the repair rate of the phase-I unit
f(t)/F(t)	pdf/cdf of the repair rate of the phase-II unit
$\overline{G(t)}/\overline{F(t)}$	cdf of repair rate of Phase I /phase II units that repair will not be completed in (0,t]
*/**	Symbol for Laplace transformation/Laplace-Stieltjes
$q_{ij}(t)/Q_{ij}(t)$	pdf/cdf of passage time from regenerative state 'i' to a regenerative state 'j' or to a failed state 'j' without visiting any other regenerative state in (0,t]
$q_{ij.k,r}(t)/Q_{ij.k,r}(t)$	pdf/cdf of direct transition time from regenerative state 'i' to a regenerative state 'j' or to a failed state 'j' visiting state k, r once in (0,t]
q_{ij.{k(r,s)}^n}/ $Q_{ij.k(r,s)^n}$	pdf/cdf of direct transition time from regenerative state 'i' to a regenerative state 'j' or to a failed state 'j' visiting state k once and n-times states r and s
Ⓢ/©	Symbol for Stieltjes convolution / Laplace Convolution.
LIT/ LT/LST	Laplace Inverse Transform/ Laplace Transform/Laplace Stieltjes Transform
$A_i(t)$	Probability that the system is in up-state at instant time 't'
$V_i^S(t)$	Expected number of visits of server
$R_i^M(t)/R_i^D(t)$	Expected number of repairs of phase-I /phase-II Units
$B_i^R(t)$	Busy period of server due to repair
$W_i(t)$	Probability that the server is busy in the state Si up to time 't' without making any transition to any other regenerative state or returning to the same state via one or more non-regenerative states

II. The state transition diagram of the system model

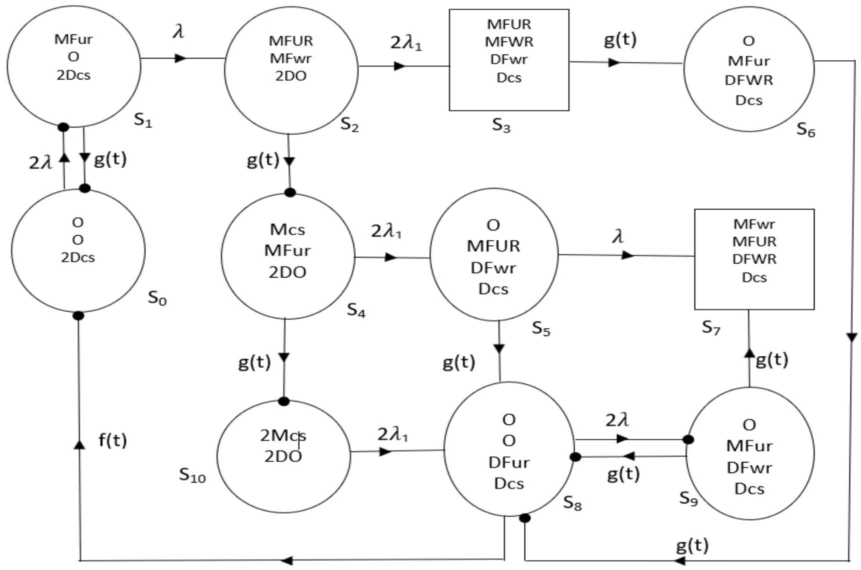

Figure 1: *State Transition Diagram*

3. State Transition Probabilities and Mean Sojourn Time

Table 2: *Transition State Description*

S_0	O O 2Dcs	The initial state in which the phase-I units are in operation and phase-II units are in cold standby
S_1	MFur O 2Dcs	The second state in which one of the phase-I units is in operation and other unit is failed under repair and phase-II units are in cold-standby
S_2	MFUR MFwr 2DO	The third state in which one of the phase-I units is continuously failed under repair and other is failed waiting for repair and the phase-II units are in operation mode
S_3	MFUR MFWR DFwr Dcs	The fourth state in which system is completely failed. One phase-I units are failed under repair and waiting for repair continuously from the previously state and phase-II unit is failed waiting for repair, other is in cold standby mode.
S_4	Mcs MFur 2DO	The fifth state in which one of the phase-I unit is in spare and other unit failed under repair and the phase-II units are in operation
S_5	O MFUR DFwr Dcs	The sixth state in which one phase-I unit is in operation, other is failed under repair continuously and in phase-II one unit is failed waiting for repair, other unit is cold standby mode
S_6	O MFur DFWR Dcs	The seventh state in which system is in operation. One of phase-I unit is operative, other is failed under repair and one of the phase-II unit is failed waiting for repair continuously, other unit is in cold standby mode
S_7	MFwr MFUR DFWR Dcs	The eighth state in which system is completely failed. One unit of phase-I is failed under repair continuously, other unit is failed waiting for repair and one of the phase-II unit is continuously failed under repair, other is in cold standby mode
S_8	O O DFur Dcs	The ninth state in which phase-I units are in operation and in phase-II, one unit is failed under repair and other is in cold-standby mode

S_9	O MFur DFwr Dcs	The tenth state in which one phase-I unit is in operation, other is failed under repair and one of phase-II unit is failed waiting for repair, other is in cold standby.
S_{10}	2Mcs 2DO	The eleventh state in which phase-I units are in spare and phase-II units are in operation

4. Reliability Measures

I. Transition Probabilities

The expressions for transition probabilities from state i to j as follow:

$p_{ij}=Q_{ij}(\infty)=\int_0^\infty q_{ij}(t)dt$ $\quad p_{ij}=\lim_{n\to\infty} Q_{ij}(t)dt$

$dQ_{01}(t)=2\lambda e^{-2\lambda t}dt$ $\quad dQ_{10}(t)=e^{-\lambda t}g(t)\,dt$ $\quad dQ_{12}(t)=\lambda e^{-\lambda t}\overline{G(t)}dt$

$dQ_{23}(t)=2\lambda_1 e^{-2\lambda_1 t}\overline{G(t)}dt$ $\quad dQ_{24}(t)=e^{-2\lambda_1 t}g(t)dt$ $\quad dQ_{36}(t)=g(t)dt$

$dQ_{45}(t)=2\lambda_1 e^{-2\lambda_1 t}\overline{G(t)}dt$ $\quad dQ_{4'10}(t)=e^{-2\lambda_1 t}g(t)dt$ $\quad dQ_{58}(t)=e^{-\lambda t}g(t)\,dt$

$dQ_{57}(t)=\lambda e^{-\lambda t}\overline{G(t)}dt$ $\quad dQ_{68}(t)=\lambda e^{-\lambda t}\overline{G(t)}dt$ $\quad dQ_{67}(t)=e^{-\lambda t}g(t)\,dt$

$dQ_{76}(t)=g(t)dt$ $\quad dQ_{80}(t)=e^{-2\lambda t}f(t)\,dt$ $\quad dQ_{89}(t)=2\lambda e^{-2\lambda t}\overline{F(t)}dt$

$dQ_{97}(t)=\lambda e^{-\lambda t}\overline{G(t)}dt$ $\quad dQ_{98}(t)=e^{-\lambda t}g(t)\,dt$ $\quad dQ_{10,8}(t)=2\lambda_1 e^{-2\lambda_1 t}dt$

$dQ_{14.2}(t)=dQ_{12}(t)ⓢdQ_{24}$ $\quad dQ_{18.2,3,6}(t)=dQ_{12}(t)ⓢdQ_{23}ⓢdQ_{36}ⓢdQ_{68}$

$dQ_{18.2,3(6,7)^n}(t)=\frac{dQ_{12}(t)ⓢdQ_{23}ⓢdQ_{36}ⓢdQ_{67}ⓢdQ_{76}ⓢdQ_{68}}{1-dQ_{67}(t)dQ_{76}(t)}$

$dQ_{48.5}(t)=dQ_{45}(t)ⓢdQ_{58}$ $\quad dQ_{48.5,7,6}(t)=dQ_{45}(t)ⓢdQ_{57}ⓢdQ_{76}ⓢdQ_{68}$

$dQ_{48.5(6,7)^n}(t)=\frac{dQ_{45}(t)ⓢdQ_{57}ⓢdQ_{76}ⓢdQ_{67}ⓢdQ_{76}ⓢdQ_{68}}{1-dQ_{67}(t)dQ_{76}(t)}$

$dQ_{98.7,6}(t)=dQ_{97}(t)ⓢdQ_{76}ⓢdQ_{68}$

$dQ_{98.(6,7)^n}(t)=\frac{dQ_{97}(t)ⓢdQ_{76}ⓢdQ_{67}ⓢdQ_{76}ⓢdQ_{68}}{1-dQ_{67}(t)dQ_{76}(t)}$

$p_{ij}=\lim_{t\to\infty} Q_{ij}(t)dt=\lim_{s\to 0} Q_{ij}^{**}(s)$

$p_{01}=p_{36}=p_{76}=p_{76}=1$

$p_{10}=p_{58}=p_{68}=p_{98}=g^*(\lambda)$ $\quad p_{12}=p_{57}=p_{67}=p_{97}=1-g^*(\lambda)$

$p_{24}=p_{4,10}=g^*(2\lambda_1)$ $\quad p_{23}=p_{45}=1-g^*(2\lambda_1)$

$p_{80}=f^*(2\lambda)$ $\quad p_{89}=1-f^*(2\lambda)$

$p_{36}=p_{76}=g^*(0)=1$ $\quad p_{14.2}=p_{12}p_{24}$

$p_{18.2,3,6}=p_{12}p_{23}p_{36}p_{68}$

$P_{18.2,3(6,7)^n}=\frac{p_{12}p_{23}p_{36}p_{67}p_{76}p_{68}}{1-p_{67}p_{76}}=p_{12}p_{23}p_{67}$ $\quad p_{48.5}=p_{45}p_{58}$

$p_{48.5,7,6}=p_{45}p_{57}p_{68}$

$p_{48.5(7,6)^n}=\frac{p_{45}p_{57}p_{76}p_{67}p_{76}p_{68}}{1-p_{67}p_{76}}=p_{45}p_{57}p_{67}$

$p_{98.7,6}=p_{97}p_{68}$

$p_{98.(7,6)^n}=\frac{p_{97}p_{76}p_{67}p_{76}p_{68}}{1-p_{67}p_{76}}=p_{97}p_{67}$

Also, it is verified that

$p_{01}=p_{10}+p_{12}=p_{23}+p_{24}=p_{36}=p_{45}+p_{4,10}=p_{57}+p_{58}=p_{67}+p_{68}=p_{76}=p_{80}+p_{89}$
$=p_{97}+p_{98}=p_{10,8}=p_{10}+p_{14.2}+p_{18.2,3,6}+p_{18.2,3(6,7)^n}=p_{4,10}+p_{48.5}+p_{48.5,7,6}+p_{48.5,7,6}p_{71.10}=p_{98}+p_{98.76}+p_{98.(7,6)^n}=1$

Mean Sojourn Times

$\mu_0=m_{01}=\frac{1}{2\lambda}$ $\quad \mu_1=m_{10}+m_{12}=\frac{1}{\lambda}\left[1-g^*(\lambda)\right]$

$\mu_2=m_{23}+m_{24}=\frac{1}{2\lambda_1}[1-g^*(2\lambda_1)]$ $\quad \mu_3=m_{36}=-g^{*'}(0)$

$\mu_4=m_{45}+m_{4,10}=\frac{1}{2\lambda_1}[1-g^*(2\lambda_1)]$ $\quad \mu_5=m_{57}+m_{58}=\frac{1}{\lambda}\left[1-g^*(\lambda)\right]$

$\mu_6=m_{67}+m_{68}=\frac{1}{\lambda}\left[1-g^*(\lambda)\right]$ $\quad \mu_7=m_{76}=-g^{*'}(0)$

$\mu_8=m_{80}+m_{89}=\frac{1}{2\lambda}\left[1-f^*(\lambda)\right]$ $\mu_9=m_{97}+m_{98}=\frac{1}{\lambda}\left[1-g^*(\lambda)\right]$

$\mu_{10}=\frac{1}{2\lambda_1}$

$\mu_1'=\mu_1+\mu_2 p_{12}+p_{12}p_{23}\left(\mu_3+\mu_6\right)+\frac{p_{12}p_{23}p_{67}(\mu_6+\mu_7)}{p_{68}}$

$\mu_4'=\mu_4+\mu_5 p_{45}+\frac{p_{45}p_{57}(\mu_6+\mu_7)}{p_{68}}$

$\mu_9'=\mu_9+p_{97}(\mu_6+\mu_7)+\frac{p_{97}p_{67}(\mu_6+\mu_7)}{p_{68}}$

II. Mean Time to System Failure and Reliability

Let $\phi_i(t)$ be the cdf of the first passage time from regenerative state 'i' to a failed state, regarding failed state as absorbing state, we have

$\phi_0(t)=Q_{01}(t)Ⓢ\phi_1(t)$ (1)

$\phi_1(t)=Q_{10}(t)Ⓢ\phi_0(t)+Q_{14.2}(t)Ⓢ\phi_4(t)+Q_{13.2}(t)$ (2)

$\phi_4(t)=Q_{4,10}(t)Ⓢ\phi_{10}(t)+Q_{48.5}(t)Ⓢ\phi_8(t)+Q_{47.5}(t)$ (3)

$\phi_8(t)=Q_{80}(t)Ⓢ\phi_0(t)+Q_{97}(t)$ (4)

$\phi_9(t)=Q_{98}(t)Ⓢ\phi_8(t)+Q_{71.10}(t)Ⓢ\phi_1(t)+Q_{79.10}(t)$ (5)

$\phi_{10}(t)=Q_{10,8}(t)Ⓢ\phi_8(t)$ (6)

By taking LST of the above expressions the reliability of the system model can be obtained as:

$$\phi_0^{**}(s)=\frac{\Delta_1}{\Delta}=\frac{\left(1-Q_{89}^{**}(s)Q_{98}^{**}(s)\right)[\left(Q_{13.2}^{**}(s)Q_{01}^{**}(s)-Q_{01}^{**}(s)Q_{14.2}^{**}(s)Q_{47.5}^{**}\right]+(Q_{01}^{**}(s)Q_{14.2}^{**}(s)Q_{89}^{**}Q_{97}^{**})[Q_{48.5}^{**}(s)+Q_{4,10}^{**}(s)Q_{10,8}^{**}(s)]}{\left(1-Q_{89}^{**}(s)Q_{98}^{**}(s)\right)\left(1-Q_{01}^{**}(s)Q_{10}^{**}(s)\right)-(Q_{14.2}^{**}(s)Q_{01}^{**}(s))[(Q_{48.5}^{**}(s)Q_{80}^{**}(s)+Q_{4,10}^{**}(s)Q_{4,8}^{**}(s)Q_{80}^{**}(s)]}$$

$$\text{MTSF}=\lim_{s\to 0}\frac{1-\phi_0^{**}(s)}{s}=\frac{N_1}{D_1}$$

$N_1=\left(\mu_0+\mu_1'+\mu_{10}p_{14.2}p_{4,10}\right)(1-p_{89}p_{98})+(\mu_4' p_{14.2})(p_{80}+p_{97}p_{89})+\left(\mu_8+\mu_9 p_{89}-\mu_0 p_{80}\right)(p_{4,10}+p_{48.5})p_{14.2}$

$D_1=\left(1-p_{10}\right)\left(1-p_{89}p_{98}\right)-p_{14.2}p_{80}\left(p_{48.5}+p_{4,10}\right)$; *where,*

$\mu_1'=\mu_1+p_{12}\mu_2$

$\mu_4'=\mu_4+p_{45}\mu_5$

The reliability of the system is determined as:

$R^*(s)=\frac{1-\phi_0^{**}(s)}{s}$ and $R(t)=L^{-1}\left(R^*(s)\right)$

III. Long Run Availability of the System

Let $A_i(t)$be the probability that the system is in up-state at instant 't' given that the system entered regenerative state 'i' at time instant t=0. We have

$A_0(t)=M_0(t)+q_{01}(t)©A_1(t)$ (7)

$A_1(t)=M_1(t)+q_{10}(t)©A_0(t)+q_{14.2}(t)©A_4(t)+[q_{18.2,3,6}(t)+q_{18.2,3(6,7)^n}(t)]©A_8(t)$ (8)

$A_4(t)=M_4(t)+q_{4,10}(t)©A_{10}(t)+[q_{48.5}(t)+q_{48.5,7,6}(t)+q_{48.5(7,6)^n}]©A_8(t)$ (9)

$A_8(t)=M_8(t)+q_{80}(t)©A_0(t)+q_{89}(t)©A_9(t)$ (10)

$A_9(t)=M_9(t)+q_{98}©A_8(t)+[q_{98.7,6}+q_{98.(7,6)^n}]©A_8(t)$ (11)

$A_{10}(t)=M_{10}(t)+q_{10,8}©A_8(t)$ (12)

Where $M_i(t)$ is the probability that the system is up initially in state i is up at time t without visiting to any other regenerative state.

$M_0(t)=e^{-2\lambda t}dt$ $M_1(t)=e^{-\lambda t}\overline{G(t)}dt$ $M_4(t)=e^{-2\lambda_1 t}\overline{G(t)}dt$

$M_8(t)=e^{-2\lambda t}dt$ $M_9(t)=e^{-\lambda t}\overline{G(t)}dt$ $M_{10}(t)=e^{-2\lambda_1 t}dt$

Taking LT of above equations and solving for $A_0^*(s)$, the steady state availability is given by:

$A_0(\infty)=\lim_{s\to 0} s\,A_0^*(s)=\frac{N_2}{D_2}$; where

$$N_2=\left(\mu_0+\mu_1\right)p_{80}+p_{80}p_{14.2}\mu_4+\mu_8p_{12}+\mu_9p_{12}p_{89}+\mu_{10}p_{14.2}p_{4,10}p_{80}$$

$$D_2=\left(\mu_0+\mu'_1\right)p_{80}+\mu'_4p_{14.2}p_{80}+\mu_8p_{12}+\mu'_9p_{12}p_{89}+\mu_{10}p_{14.2}p_{4,10}p_{80}$$

$$\mu'_1=\mu_1+\mu_2p_{12}+p_{12}p_{23}\left(\mu_3+\mu_6\right)+\frac{p_{12}p_{23}p_{67}(\mu_6+\mu_7)}{p_{68}}$$

$$\mu'_4=\mu_4+\mu_5p_{45}+\frac{p_{45}p_{57}(\mu_6+\mu_7)}{p_{68}}$$

$$\mu'_9=\mu_9+p_{97}(\mu_6+\mu_7)+\frac{p_{97}p_{67}(\mu_6+\mu_7)}{p_{68}}$$

IV. Expected Number of Repairs of Phase-I Units

Let $R_i^M(t)$be the expected number of repairs of phase-I units given to the server in (0,t] such that the system entered regenerative state 'i' at t=0. We have

$$R_0^M(t)=Q_{01}(t)Ⓢ R_1^M(t) \quad (13)$$

$$R_1^M(t)=Q_{10}(t)Ⓢ\left[1+R_0^M(t)\right]+Q_{14.2}(t)Ⓢ\left[1+R_4^M(t)\right]+[Q_{18.2,3,6}(t)+Q_{18.2,3(6,7)^n}(t)]Ⓢ[1+R_8^M(t)] \quad (14)$$

$$R_4^M(t)=Q_{4,10}(t)Ⓢ[1+R_{10}^M(t)]+[Q_{48.5}(t)+Q_{48.5,7,6}(t)+Q_{48.5(7,6)^n}(t)]Ⓢ[1+R_8^M(t)] \quad (15)$$

$$R_8^M(t)=Q_{80}(t)Ⓢ R_0^M(t)+Q_{89}(t)Ⓢ R_9^M(t) \quad (16)$$

$$R_9^M(t)=[Q_{98}(t)+Q_{98.7,6}(t)+Q_{98.(7,6)^n}(t)]Ⓢ[1+R_8^M(t)] \quad (17)$$

$$R_{10}^M(t)=Q_{10,8}Ⓢ R_8^M(t) \quad (18)$$

Taking LST of above equations and solving for$R_0^{M^{**}}(s)$, by using Cramer's Rule we get the expected number of repairs of phase-I unit as:

$R_0^M(\infty)=\lim_{s\to 0} s\,R_0^{M^{**}}(t)=\frac{\Delta_1}{\Delta'_1}=\frac{N_3}{D_2}$; where

$N_3=P_{80}+p_{12}p_{24}+P_{12}P_{23}P_{89}$ *and*

$$D_2=\left(\mu_0+\mu'_1\right)p_{80}+\mu'_4p_{14.2}p_{80}+\mu_8p_{12}+\mu'_9p_{12}p_{89}+\mu_{10}p_{14.2}p_{4,10}p_{80}$$

V. Expected Numbers of Repairs of Phase-II Units

Let $R_i^D(t)$be the expected number of repairs of phase-II unit given to the server in (0,t] such that the system entered regenerative state 'i' at t=0. We have

$$R_0^D(t)=Q_{01}(t)Ⓢ R_1^D(t) \quad (19)$$

$$R_1^D(t)=Q_{10}(t)Ⓢ R_0^D(t)+Q_{14.2}(t)Ⓢ R_4^D(t)+[Q_{18.2,3,6}(t)+Q_{18.2,3(6,7)^n}(t)]Ⓢ R_8^D(t) \quad (20)$$

$$R_4^D(t)=Q_{4,10}(t)Ⓢ R_{10}^D(t)+[Q_{48.5}(t)+Q_{48.5,7,6}(t)+Q_{48.5(7,6)^n}(t)]Ⓢ R_8^D(t) \quad (21)$$

$$R_8^D(t)=Q_{80}(t)Ⓢ[1+R_0^D(t)]+Q_{89}(t)Ⓢ R_9^D(t) \quad (22)$$

$$R_9^D(t)=[Q_{98}(t)+Q_{98.7,6}(t)+Q_{98.(7,6)^n}(t)]Ⓢ R_8^D(t) \quad (23)$$

$$R_{10}^D(t)=Q_{10,8}Ⓢ R_8^D(t) \quad (24)$$

Taking LST of above equations and solving for $R_0^{D^{**}}(s)$, by using this we get expected number of repairs of phase-II units as:

$$R_0^D(\infty)=\lim_{s\to 0} s\,R_0^{D^{**}}(s)=\frac{N_4}{D_2}$$

Where $N_4=p_{12}p_{80}$ and

$$D_2=\left(\mu_0+\mu'_1\right)p_{80}+\mu'_4p_{14.2}p_{80}+\mu_8p_{12}+\mu'_9p_{12}p_{89}+\mu_{10}p_{14.2}p_{4,10}p_{80}$$

VI. Expected Number of Visits by the Server

LetV_0^S be the expected number of repairs of duplicate unit by the repairman in (0,t] such that the system entered regenerative state 'i' at t=0.We have

$V_0^S(t)$=$Q_{01}(t)$ Ⓢ$[1+V_1^S(t)]$ (25)

$V_1^S(t)$=$Q_{10}(t)$ Ⓢ$V_0^S(t)$+$Q_{14.2}(t)$ Ⓢ$V_4^S(t)$+$[Q_{18.2,3,6}(t)+Q_{18.2,3(6,7)^n}(t)]$Ⓢ$V_8^S(t)$ (26)

$V_4^S(t)$=$Q_{4,10}(t)$ Ⓢ$V_{10}^S(t)$+$[Q_{48.5}(t)+Q_{48.5,7,6}(t)+Q_{48.5(7,6)^n}(t)]$Ⓢ$V_8^S(t)$ (27)

$V_8^S(t)$=$Q_{80}(t)$ Ⓢ$[1+V_0^S(t)]$+$Q_{89}(t)$ Ⓢ$V_9^S(t)$ (28)

$V_9^S(t)$=$[Q_{98}(t)+Q_{98.7,6}(t)+Q_{98.(7,6)^n}(t)]$Ⓢ$V_8^S(t)$ (29)

$V_{10}^S(t)$=$Q_{10,8}$Ⓢ$[1+V_8^S(t)]$ (30)

Taking LST of above equations and solving for $V_0^{s**}(s)$, by using this we get expected number of visits of the server:

$$V_0^s(\infty)=\lim_{s\to 0} s\, V_0^{s^{**}}(s)=\frac{N_5}{D_2}$$

Where $N_5=p_{80}[1+P_{12}P_{24}P_{4,10}]$

And $D_2=\left(\mu_0+\mu'_1\right)p_{80}+\mu'_4 p_{14.2}p_{80}+\mu_8 p_{12}+\mu'_9 p_{12}p_{89}+\mu_{10}p_{14.2}p_{4,10}p_{80}$

VII. Busy Period Analysis for the Server due to Repair

Let $B_i^R(t)$ be the probability that a server is busy at the time point given that the system entered in the regenerative state 'i' at t=0. We have

$B_0^R(t)=q_{01}(t)©B_1^R(t)$ (31)

$B_1^R(t)=W_1^R(t)+q_{10}(t)©B_0^R(t)+q_{14.2}(t)©B_4^R(t)+[q_{18.2,3,6}(t)+q_{18.2,3(6,7)^n}(t)]©B_8^R(t)$ (32)

$B_4^R(t)=W_4^R(t)+q_{4,10}(t)©B_{10}^R(t)+[q_{48.5}(t)+q_{48.5,7,6}(t)+q_{48.5(7,6)^n}]©B_8^R(t)$ (33)

$B_8^R(t)=W_8^R(t)+q_{80}(t)©B_0^R(t)(t)+q_{89}(t)©B_9^R(t)$ (34)

$B_9^R(t)=W_9^R(t)+q_{98}©A_8(t)+[q_{98.7,6}+q_{98.(7,6)^n}]©B_8^R(t)$ (35)

$B_{10}^R(t)(t)=q_{10,8}©B_8^R(t)$; *where* (36)

$W_1^R(t)=e^{-\lambda t}\overline{G(t)}dt$ $\quad W_4^R(t)=e^{-2\lambda_1 t}\overline{G(t)}dt$

$W_8^R(t)=e^{-2\lambda t}dt$ $\quad W_9^R(t)=e^{-\lambda t}\overline{G(t)}dt$

Taking LT of the above expressions and solving for $B_0^{R^*}(s)$ we have

$$B_0^R(\infty)=\lim_{s\to 0} s\, B_0^{R^*}(s)=\frac{N_6}{D_2}$$

$N_6=W_2^{R^*}(0)p_{80}+W_4^{R^*}(0)p_{12}p_{24}p_{80}+W_8^{R^*}(0)P_{12}+p_{12}p_{24}p_{4,10}\left(1+p_{80}\right)+W_9^{R^*}(0)p_{12}p_{89}(p_{23}+p_{24}p_{45})$

$D_2=\left(\mu_0+\mu'_1\right)p_{80}+\mu'_4 p_{14.2}p_{80}+\mu_8 p_{12}+\mu'_9 p_{12}p_{89}+\mu_{10}p_{14.2}p_{4,10}p_{80}$

5. Profit Analysis

The profit (P) incurred to the system model in steady state can be obtained as:

$P=K_0A_0-K_1B_0^R-K_2R_0^D-K_3R_0^M-K_4V_0^S$ (37)

Where,

K_0 = Revenue per unit up time of the system

K_1=Cost per unit time for which server is busy to repair

K_3 = Cost per unit time repairs of phase-I unit

K_2 = Cost per unit time repairs of phase-II unit

K_4= Cost per unit time visit of the server

For graphical presentation of profit, these constants need some values and for this purpose K_0, K_1, K_2, K_3 and K_4 have been taken to be 15000, 3000, 800, 2000 and 1000 respectively.

6. Results and Graphical Representation of Reliability Measures

The reliability characteristics of the system model have been obtained by assuming arbitrary distributions for repairs rates of the units. The results for these measures have also been obtained for the particular situations where the repair rates of the units follow negative exponential distribution. The behavior of MTSF, Availability and Profit function have been shown numerically and graphically respectively in the tables 1, 2, 3 and in the figures 2, 3 and 4. Here, we take the repair time distribution as negative exponential: $g(t)=\xi e^{-\xi t}$ and $f(t)=\Psi e^{-\Psi t}$.

Table 3: *MTSF Vs Failure Rate of Phase-I Unit*

λ	λ1=0.001,Ψ=2,ξ=1.5	λ1=0.002,Ψ=2.0,ξ=1.5	λ1=0.001,Ψ=2,ξ=2.5	λ1=0.001,Ψ=2.5,ξ=1.5
0.1	76879.65	37210.85	136435.40	91728.39
0.11	64441.61	31295.15	112598.00	77338.17
0.12	54852.02	26717.79	94594.12	66138.10
0.13	47300.84	23100.57	80660.17	57248.89
0.14	41247.20	20190.9	69651.85	50075.23
0.15	36318.62	17814.55	60800.75	44202.02
0.16	32251.73	15848.03	53575.41	39332.56
0.17	28856.03	14201.76	47598.75	35250.15
0.18	25990.91	12809.45	42597.23	31793.53
0.19	23550.81	11621.14	38368.47	28840.73

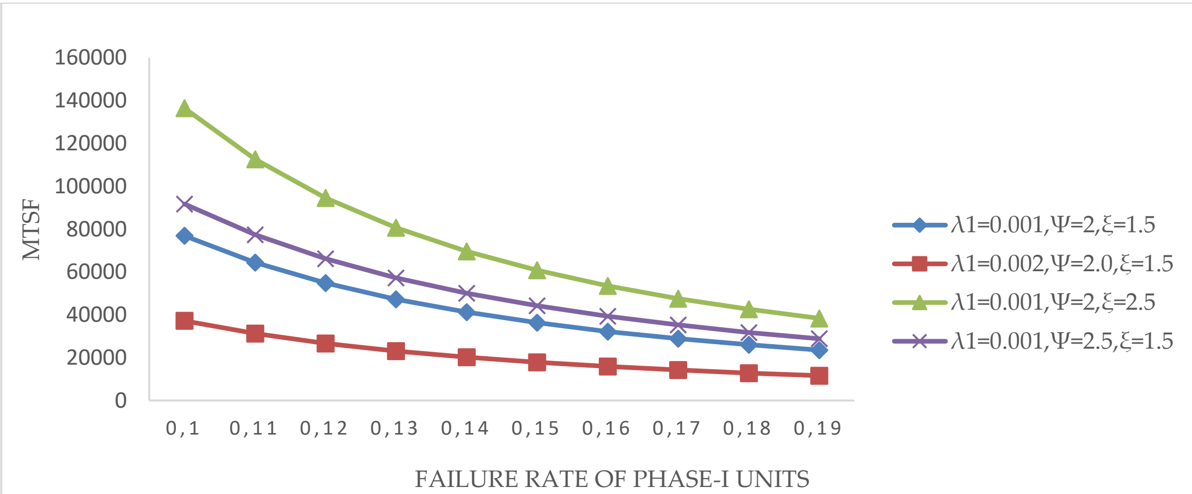

Figure 2: *MTSF Vs Failure Rate of Phase-I Unit*

Table 4: *Availability Vs Failure Rate of Phase-I Unit*

λ	λ1=0.001,Ψ=2,ξ=1.5	λ1=0.002,Ψ=2.0,ξ=1.5	λ1=0.001,Ψ=2,ξ=2.5		λ1=0.001,Ψ=2.5,ξ=1.5
0.1	0.99885337	0.99800616	0.999369592		0.998856064
0.11	0.99882166	0.997912792	0.999344996		0.998825033
0.12	0.99879522	0.997833532	0.999324552		0.998799332
0.13	0.99877269	0.997765415	0.999307327		0.998777608
0.14	0.99875309	0.997706120	0.999292617		0.998758883
0.15	0.99873570	0.997653834	0.999279881		0.998742441
0.16	0.99872001	0.997607137	0.999268708		0.998727751
0.17	0.99870561	0.997564913	0.999258778		0.998714414
0.18	0.99869219	0.997526279	0.999249842		0.998702124
0.19	0.99867952	0.997490532	0.999241707		0.998690646

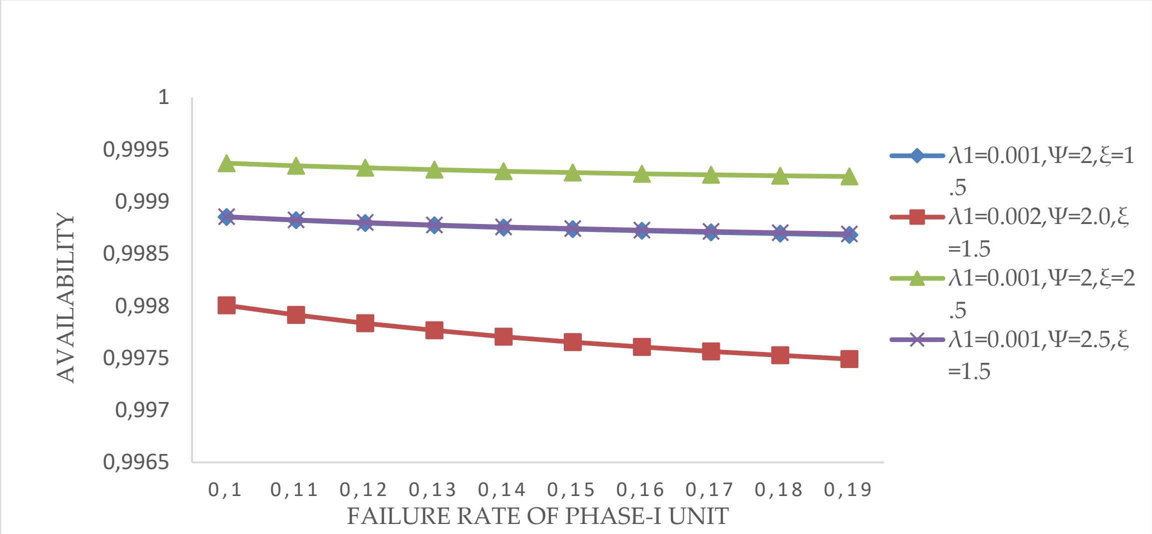

Figure 3: *Availability Vs Failure Rate of Phase-I Unit*

Table 5: *Profit Vs Failure Rate of Phase-I Unit*

λ	λ1=0.001,Ψ=2,ξ=1.5	λ1=0.002,Ψ=2.0,ξ=1.5	λ1=0.001,Ψ=2,ξ=2.5	λ1=0.001,Ψ=2.5,ξ=1.5
0.1	14955.29	14922.42	14961.05	14958.79
0.11	14953.85	14918.44	14959.35	14957.51
0.12	14952.63	14915.00	14957.91	14956.42
0.13	14951.55	14911.99	14956.66	14955.48
0.14	14950.60	14909.32	14955.56	14954.64
0.15	14949.73	14906.92	14954.59	14953.88
0.16	14948.94	14904.76	14953.72	14953.20
0.17	14948.2	14902.77	14952.93	14952.56
0.18	14947.50	14900.93	14952.2	14951.97
0.19	14946.85	14899.22	14951.52	14951.41

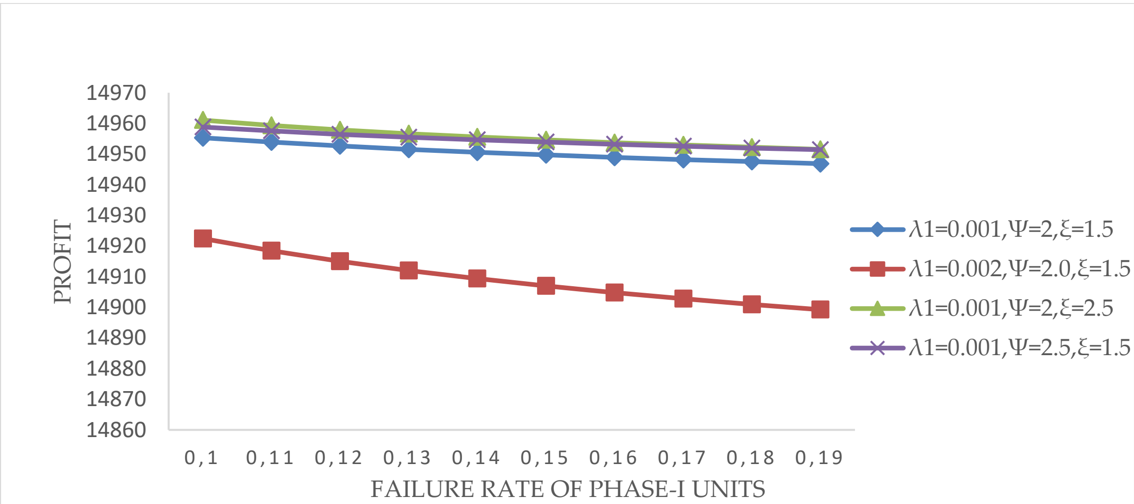

Figure 4: *Profit Vs Failure Rate of Phase-I Unit*

7. Application

The present study has the application in the system of turning and boring operation for making gun metal bushes to be required in production line. To make India as a developed country many industries have been established in last two decades. In industries there is huge requirement of machinery, one of them is CNC (for automatic programmed operation) and Lathe machine (for manual operation) which are used to produce a metallic bush. To meet up the heavy requirement of metallic bush two CNC machine (Phase-I units) and Lathe machine (Phase-II units) are installed. The single CNC machine can fulfil the requirement of production line in case of failure another CNC machine. In case of electric/mechanical failure of CNC machine (Phase-I units), additional arrangement of two lathe machines (Phase-II units) are installed to achieve the same. The system can be shown in the following figure 5:

Fig. 5: Manufacturing of Metallic Components

8. Conclusion

A Parallel-Cold Standby system of four units has been analyzed stochastically with the idea of priority for repair to the phase-I units. There are four units in system comprising two units as phase-I units which work initially in parallel mode and the other two units (called phase-II units) which remain as spare in cold standby mode. The phase-II units can be installed to work simultaneously at the failure of the phase-I units. The important reliability characteristics have been obtained and analyzed for arbitrary values of the parameters. Graphical and tabulated presentations have been studied by taking exponential distributions for the repair time i.e.,$g(t)=\xi e^{-\xi t}$ and $f(t)=\Psi e^{-\Psi t}$. The results are shown graphically in the figures 2,3, and 4 respectively. The following conclusions can be made from the graphical study:

1. From Fig. 2 it is quite evident that MTSF has downward trend with the increase of failure rate of the phase-I unit and phase-II unit while it increases with the increase in repair rate of phase-I. There is little change in the (almost negligible) in the values of availability with the increase of repair rate of Phase-II units. We conclude that a repairable parallel-cold standby system of four units can be made to use in a batter way in terms of reliability by increasing repair rate (from 1.5 to 2.5) of phase-I units.
2. Fig.3 depicts that availability of the system keeps on decreasing with increase of failure rates (0.1 to 0.19) of phase -I units. However, the system availability is more when repair of phase -I units is kept as priority as compare to phase-II units. Availability is increased slightly from 0.998 to 0.999 in case of increased repair rate of phase-I unit. Hence, keeping the repair priority policy for phase-I units is beneficial in case of availability.

3. Fig.4 of profit analysis represents the same trend as that of availability. Repair of phase-I units gives more profit as compare to phase-II units. Therefore, spending money on increasing repair rate of phase-II units will not be fruitful and thus, one should avoid the use of low-quality units in standby.

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