

# STARTING MODE OF SYNCHRONOUS MACHINES WITH MASSIVE ROTORS

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## Abstract

*It is known that in synchronous machines with massive rotors, it is required to take into account the change in the equivalent rotor of active resistance depending on the frequency of the current in the rotor at starting of these machines. A three-phase mathematical model of these machines has been compiled, the equations of which are written in axes rotating at the speed of the machine rotor. Study of the start-up modes of these machines and operation in synchronous mode with a load surge (dynamic mode) has been carried out on this model. The studies have allowed for making the following conclusions. When starting synchronous machines with massive rotors, it is most preferable to take into account the change in the equivalent resistance in the form of a linear sliding function. It was found out that in the synchronous mode of operation of these machines, including in stable dynamic modes, there is no need to take into account changes in the rotor resistance-sliding, since the sliding in these modes oscillatory damps around zero, thereby not affecting changes in the value of the equivalent resistance. Therefore, in these modes it is recommended to consider the value of this resistance constant and definite at slip equal to zero.*

**Keywords:** synchronous machines with massive rotor, start mode, dynamic mode, three-coordinate system

## I. Introduction

Synchronous machines with massive rotors are widely used both as generators (turbo generators) and as motors [1,2,3]. For the studying of static and, especially, dynamic modes of their operation, the well-known Park equations are widely used [4,5].

However, the presence of a massive rotor in them requires to take into account the change in the active resistance of the rotor depending on the frequency of the current in the rotor. This question was most completely solved analytically in [6]. Although there we are talking about synchronous motors with massive rotor, but this can, of course, be extended to low-power synchronous generators, since their asynchronous start is identical.

## II. Methods

In [6], it is proposed to change the active resistance of a massive rotor according to the following expression:

$$r_r = r_{r_{s=0}} + (r_{r_{s=1}} - r_{r_{s=0}}) \sqrt{s} \quad (1)$$

where  $r_{r=s=0}$  – active resistance of the rotor at sliding  $s=0$  and short-circuited excitation winding;  $r_{r=s=1}$  – active resistance of the rotor at  $s=1$ , also at short-circuited excitation winding. Further, in [6], as in other works [7], the static characteristic of the asynchronous moment is determined, i.e. on the basis of this characteristic, the start of the electric machine is defined.

For a more accurate idea of the starting modes of synchronous machines with massive rotor, it is necessary to take into account the effect of transient processes during start-up.

On this basis, the purpose of this paper is the issue of mathematical modeling of synchronous machines with massive rotor and the study of the dynamic modes of their operation during direct asynchronous start.

As the basis of the mathematical model, it is proposed to take the equations of synchronous machines written in a three-phase coordinate system, in the axes  $\alpha_r, \beta_r, \gamma_r$ , rotating with the speed of the rotor  $\omega_r$  of the synchronous machine. It should be noted that it is relatively easy to obtain these equations from the equation of a three-phase model of a double-fed machine, also written in the axes  $\alpha_r, \beta_r, \gamma_r$ , rotating at the speed of the machine rotor [8,9] and this can be done since a synchronous machine with a massive rotor is magnetically symmetrical electric machine, the rotor of which is non-salient pole, i.e. the air gap remains practically unchanged along the entire perimeter of the machine rotor.

On this basis, equations of synchronous machines with massive rotor in a three-coordinate (three-phase) system in extensive form will appear as follows:

$$\left. \begin{aligned} p\Psi_{sa} &= U_{sa} \cdot \sin\theta + \frac{1}{\sqrt{3}}\omega_r(\Psi_{s\beta} - \Psi_{s\gamma}) - r_s \cdot i_{sa} \\ p\Psi_{s\beta} &= U_{s\beta} \cdot \sin\left(\theta - \frac{2\pi}{3}\right) + \frac{1}{\sqrt{3}}\omega_r(\Psi_{s\gamma} - \Psi_{sa}) - r_s \cdot i_{s\beta} \\ p\Psi_{s\gamma} &= U_{s\gamma} \cdot \sin\left(\theta + \frac{2\pi}{3}\right) + \frac{1}{\sqrt{3}}\omega_r(\Psi_{sa} - \Psi_{s\beta}) - r_s \cdot i_{s\gamma} \\ p\Psi_f &= U_f - r_f \cdot i_f \\ p\Psi_{ra} &= 0 - r_r \cdot i_{ra} \\ p\Psi_{r\beta} &= 0 - r_r \cdot i_{r\beta} \\ p\Psi_{r\gamma} &= 0 - r_r \cdot i_{r\gamma} \end{aligned} \right\} \quad (2)$$

The system of equations (2) describes the balance of voltages in the stator and rotor circuits of the machine. At that, it is meant that the stator has three symmetrical windings shifted in space by 120 electric degrees, and four windings are placed on the rotor – one excitation winding and three damper (starting) windings equivalent to the massive rotor of the machine, moreover, we agree to assume that the axis of excitation winding coincides with the axis of the damper winding located along the  $\alpha$  axis.

Then the equations of connection between flux linkages and currents will appear in the form:

$$\left. \begin{aligned} \Psi_{sa} &= x_s \cdot i_{sa} - 0.5 \cdot x_m \cdot i_{s\beta} - 0.5 \cdot x_m \cdot i_{s\gamma} + x_m \cdot i_f + x_m \cdot i_{ra} - 0.5 \cdot x_m \cdot i_{r\beta} - 0.5 \cdot x_m \cdot i_{r\gamma} \\ \Psi_{s\beta} &= -0.5 \cdot x_m \cdot i_{sa} + x_s \cdot i_{s\beta} - 0.5 \cdot x_m \cdot i_{s\gamma} - 0.5 \cdot x_m \cdot i_f - 0.5 \cdot x_m \cdot i_{ra} + x_m \cdot i_{r\beta} - 0.5 \cdot x_m \cdot i_{r\gamma} \\ \Psi_{s\gamma} &= -0.5 \cdot x_m \cdot i_{sa} - 0.5 \cdot x_m \cdot i_{s\beta} + x_s \cdot i_{s\gamma} - 0.5 \cdot x_m \cdot i_f - 0.5 \cdot x_m \cdot i_{ra} - 0.5 \cdot x_m \cdot i_{r\beta} + x_m \cdot i_{r\gamma} \\ \Psi_f &= x_m \cdot i_{sa} - 0.5 \cdot x_m \cdot i_{s\beta} - 0.5 \cdot x_m \cdot i_{s\gamma} + x_f \cdot i_f + x_m \cdot i_{ra} - 0.5 \cdot x_m \cdot i_{r\beta} - 0.5 \cdot x_m \cdot i_{r\gamma} \\ \Psi_{ra} &= x_m \cdot i_{sa} - 0.5 \cdot x_m \cdot i_{s\beta} - 0.5 \cdot x_m \cdot i_{s\gamma} + x_m \cdot i_f + x_r \cdot i_{ra} - 0.5 \cdot x_m \cdot i_{r\beta} - 0.5 \cdot x_m \cdot i_{r\gamma} \\ \Psi_{r\beta} &= -0.5 \cdot x_m \cdot i_{sa} + x_m \cdot i_{s\beta} - 0.5 \cdot x_m \cdot i_{s\gamma} - 0.5 \cdot x_m \cdot i_f - 0.5 \cdot x_m \cdot i_{ra} + x_r \cdot i_{r\beta} - 0.5 \cdot x_m \cdot i_{r\gamma} \\ \Psi_{r\gamma} &= -0.5 \cdot x_m \cdot i_{sa} - 0.5 \cdot x_m \cdot i_{s\beta} + x_m \cdot i_{s\gamma} - 0.5 \cdot x_m \cdot i_f - 0.5 \cdot x_m \cdot i_{ra} - 0.5 \cdot x_m \cdot i_{r\beta} + x_r \cdot i_{r\gamma} \end{aligned} \right\} \quad (3)$$

In equations (2), (3):  $\Psi_{sa}, \Psi_{s\beta}, \Psi_{s\gamma}, i_{sa}, i_{s\beta}, i_{s\gamma}$  are flux linkages and currents of stator windings,  $\Psi_f, \Psi_{ra}, \Psi_{r\beta}, \Psi_{r\gamma}, i_f, i_{ra}, i_{r\beta}, i_{r\gamma}$  are flux linkages and currents of rotor circuits, and  $\Psi_f$  and  $\Psi_{ra}$  are located along the rotor axis  $\alpha_r$ ;  $r_r$  is active resistance equivalent to the rotor circuits;  $x_s, x_r, x_f$  are inductive resistances (in relative units they are equal to inductances) of the stator windings, damper and rotor

excitation windings,  $x_m$  is mutual induction resistance between the stator and rotor circuits (i.e. in relative units mutual inductance).

The relationship between currents and flux linkages is determined using the inverse matrix according to the relations:

$$\begin{bmatrix} i_{sa} \\ i_{sb} \\ i_{sy} \\ i_f \\ i_{ra} \\ i_{rb} \\ i_{ry} \end{bmatrix} = \begin{bmatrix} x_s - 0.5 \cdot x_m - 0.5 \cdot x_m + x_m + x_m - 0.5 \cdot x_m - 0.5 \cdot x_m \\ -0.5 \cdot x_m + x_s - 0.5 \cdot x_m - 0.5 \cdot x_m - 0.5 \cdot x_m + x_m - 0.5 \cdot x_m \\ -0.5 \cdot x_m - 0.5 \cdot x_m + x_s - 0.5 \cdot x_m - 0.5 \cdot x_m + 0.5 \cdot x_m + x_m \\ x_m - 0.5 \cdot x_m - 0.5 \cdot x_m + x_f + x_m - 0.5 \cdot x_m - 0.5 \cdot x_m \\ x_m - 0.5 \cdot x_m - 0.5 \cdot x_m + x_m + x_r - 0.5 \cdot x_m - 0.5 \cdot x_m \\ -0.5 \cdot x_m + x_m - 0.5 \cdot x_m - 0.5 \cdot x_m - 0.5 \cdot x_m + x_r - 0.5 \cdot x_m \\ -0.5 \cdot x_m - 0.5 \cdot x_m + x_m - 0.5 \cdot x_m - 0.5 \cdot x_m - 0.5 \cdot x_m + x_r \end{bmatrix}^{-1} \begin{bmatrix} \Psi_{sa} \\ \Psi_{sb} \\ \Psi_{sy} \\ \Psi_f \\ \Psi_{ra} \\ \Psi_{rb} \\ \Psi_{ry} \end{bmatrix} \quad (4)$$

Equations (2), (3) and (4) are supplemented with the equations of electromagnetic torque [10], motion and sliding:

$$\left. \begin{aligned} m_{em} &= p_m \frac{\sqrt{3}}{2} x_m \left[ (i_{sa} \cdot i_{ry} + i_{sb} \cdot i_{ra} + i_{sy} \cdot i_f + i_{sy} \cdot i_{rb}) - (i_{sa} \cdot i_{rb} + i_{sb} \cdot i_{ry} + i_{sy} \cdot i_{ra} + i_{sy} \cdot i_f) \right] \\ p\theta &= 1 - \omega_r = s \\ p\omega_r &= \frac{p_m}{J} (m_{em} - m_T) \end{aligned} \right\} \quad (5)$$

here –  $p_m$  – number of pole pairs;  $s=p\theta$  – sliding;  $J$  – moment of inertia of moving parts.

Equations (4) in extensive form will appear in the form:

$$\left. \begin{aligned} i_{sa} &= k_{sa1} \cdot \Psi_{sa} + k_{sa2} \cdot \Psi_{sb} + k_{sa3} \cdot \Psi_{sy} + k_{sa4} \cdot \Psi_f + k_{sa5} \cdot \Psi_{ra} + k_{sa6} \cdot \Psi_{rb} + k_{sa7} \cdot \Psi_{ry} \\ i_{sb} &= k_{sb1} \cdot \Psi_{sa} + k_{sb2} \cdot \Psi_{sb} + k_{sb3} \cdot \Psi_{sy} + k_{sb4} \cdot \Psi_f + k_{sb5} \cdot \Psi_{ra} + k_{sb6} \cdot \Psi_{rb} + k_{sb7} \cdot \Psi_{ry} \\ i_{sy} &= k_{sy1} \cdot \Psi_{sa} + k_{sy2} \cdot \Psi_{sb} + k_{sy3} \cdot \Psi_{sy} + k_{sy4} \cdot \Psi_f + k_{sy5} \cdot \Psi_{ra} + k_{sy6} \cdot \Psi_{rb} + k_{sy7} \cdot \Psi_{ry} \\ i_f &= k_{f1} \cdot \Psi_{sa} + k_{f2} \cdot \Psi_{sb} + k_{f3} \cdot \Psi_{sy} + k_{f4} \cdot \Psi_f + k_{f5} \cdot \Psi_{ra} + k_{f6} \cdot \Psi_{rb} + k_{f7} \cdot \Psi_{ry} \\ i_{ra} &= k_{ra1} \cdot \Psi_{sa} + k_{ra2} \cdot \Psi_{sb} + k_{ra3} \cdot \Psi_{sy} + k_{ra4} \cdot \Psi_f + k_{ra5} \cdot \Psi_{ra} + k_{ra6} \cdot \Psi_{rb} + k_{ra7} \cdot \Psi_{ry} \\ i_{rb} &= k_{rb1} \cdot \Psi_{sa} + k_{rb2} \cdot \Psi_{sb} + k_{rb3} \cdot \Psi_{sy} + k_{rb4} \cdot \Psi_f + k_{rb5} \cdot \Psi_{ra} + k_{rb6} \cdot \Psi_{rb} + k_{rb7} \cdot \Psi_{ry} \\ i_{ry} &= k_{ry1} \cdot \Psi_{sa} + k_{ry2} \cdot \Psi_{sb} + k_{ry3} \cdot \Psi_{sy} + k_{ry4} \cdot \Psi_f + k_{ry5} \cdot \Psi_{ra} + k_{ry6} \cdot \Psi_{rb} + k_{ry7} \cdot \Psi_{ry} \end{aligned} \right\} \quad (6)$$

The coefficient  $k_{sa1} \div k_{ry2}$  is determined from the inverse matrix of machine parameters (4).

Thus, comparing these equations with the equations of double-fed machine given in [8,9,10], it can be noted that their structure remains practically unchanged, but the action of the excitation winding, which is proposed to be placed on the same axis as the damper (starting) winding located on the axis, is additionally taken into account. Naturally, it is also taken into account in the equations of flux linkages of rotor and electromagnetic torque.

Once again it is necessary to emphasize, as it was noted in [6], it is necessary to use the three-phase model only in extreme cases, when it is necessary to bring additional clarity to issues that are impossible (or difficult to implement) to study in two-coordinate models. For example, such as the study of asymmetric modes in rotary circuits, etc.

### III. Results

In this case, the proposed model is used for studying the starting modes and modes of involvement into synchronism of synchronous machines with massive rotor – this is the formation of asynchronous starting torques with the equivalent massive rotors of these machines, as well as load surge in synchronous mode.

On Fig. 1 (a, b, c) are shown, respectively, the fluctograms of the change in the rotational speed  $\omega_r$  of the rotor, the electromagnetic torque  $m_{em}$  and the excitation current  $i_f$  of the model synchronous

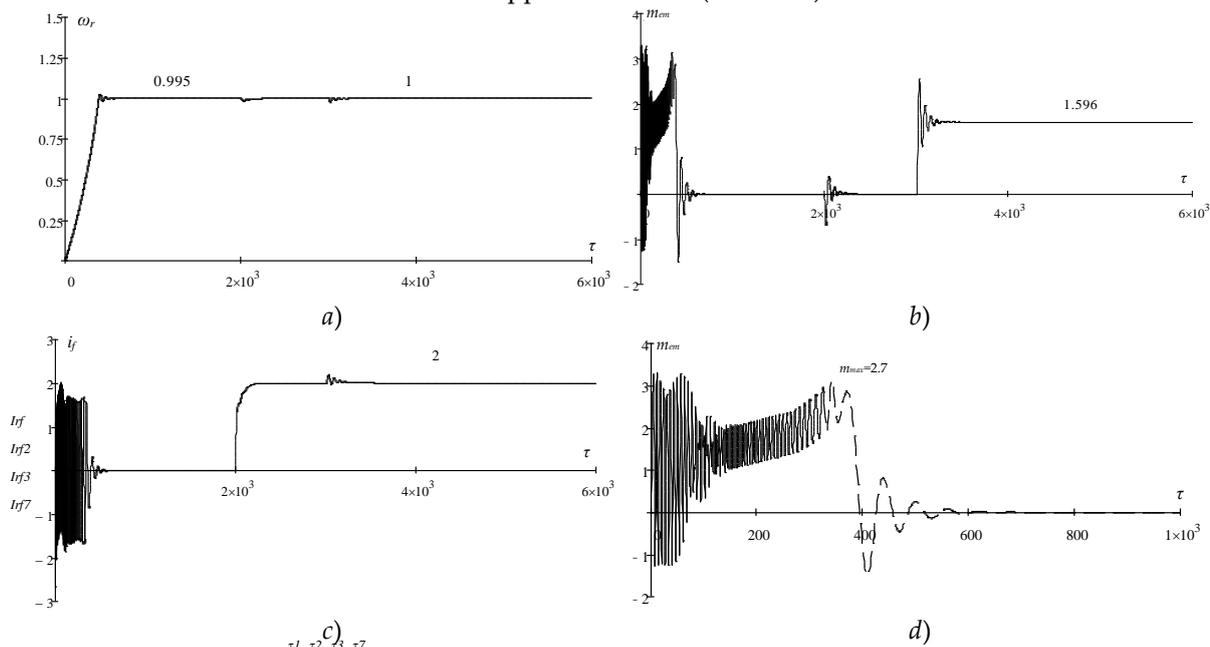
generator, the parameters of which are given in Appendix 1. In Fig. 1 (d) the extensive fluctogram of the starting torque  $m_{em}$  is shown. These fluctograms were obtained at rotor resistance equivalent to massive rotor according to the expression [6]:

$$r_r = r_{r_{s=0}} + (r_{r_{s=1}} - r_{r_{s=0}}) \cdot \sqrt{s} \quad (7)$$

For the generator under study  $r_{r_{s=0}}=0.01$  – the active resistance of the rotor at  $s=0$ ;  $r_{r_{s=1}}=0.05$  – active resistance of the rotor at  $s=1$ ;  $s=(1-\omega_r)$  – sliding, i.e.

$$r_r = 0.01 + 0.04 \cdot \sqrt{1 - \omega_r} .$$

On fluctograms after asynchronous start at  $2 \cdot 10^3$  rad. the voltage is applied to the excitation winding, the machine is involved into synchronism, and at 3000 rad. the moment of resistance equal to the rated moment of the machine is applied abruptly ( $m_n=1.596$ ).

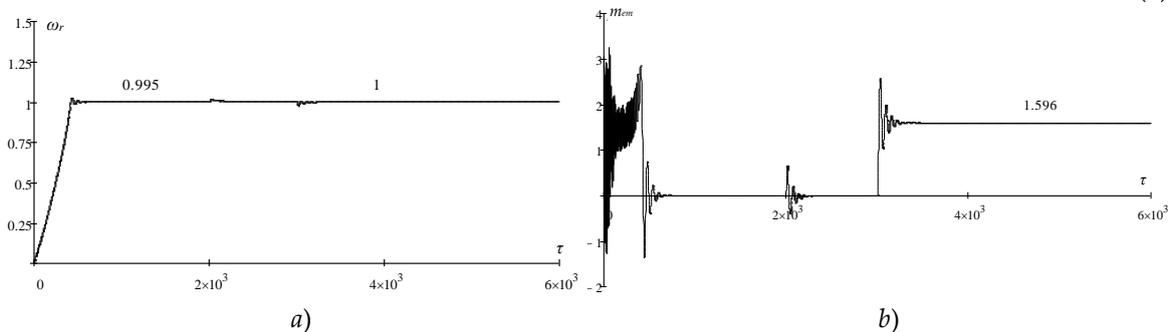


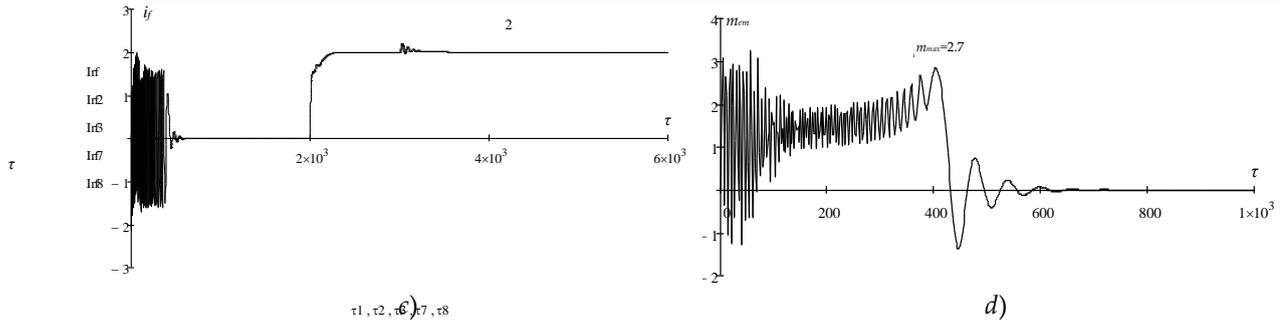
**Figure 1:** Fluctograms of change in the mode parameters of the equivalent massive rotor by the expression

$$r_r = 0.01 + 0.04 \cdot \sqrt{1 - \omega_r}$$

On Fig. 2 (a, b, c, d) the fluctograms of these mode parameters are presented in the same sequence when the massive rotor is equivalent to active resistance, which varies depending on the sliding according to the linear law:

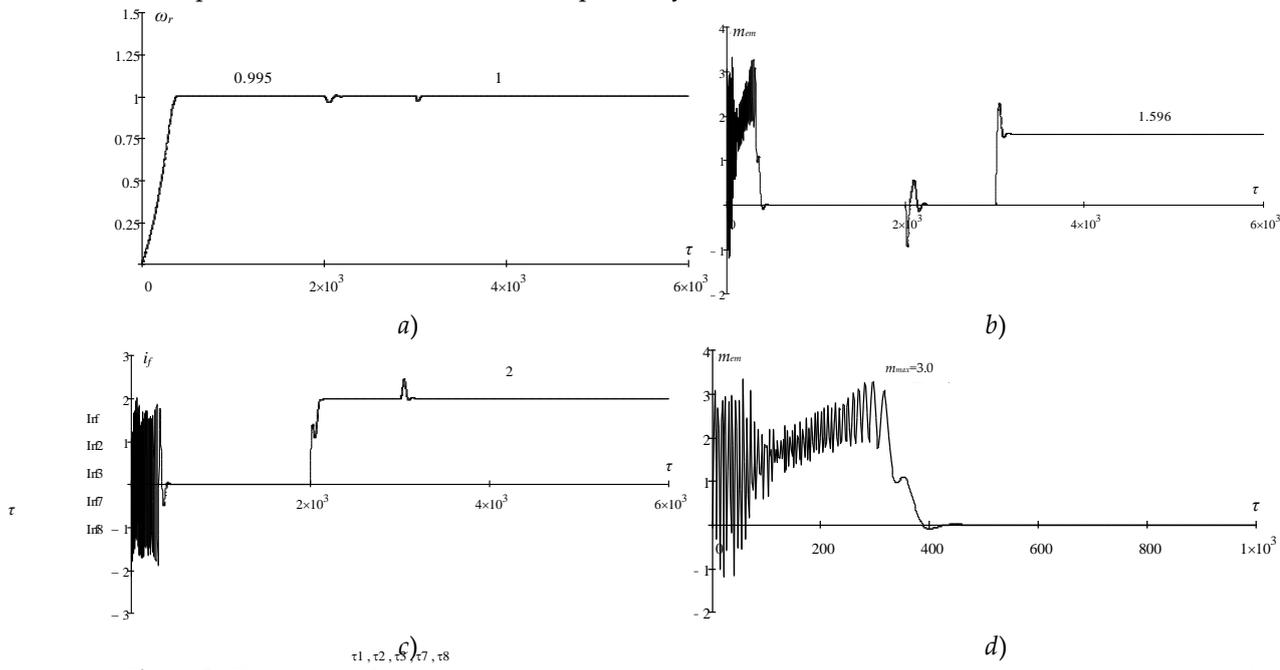
$$r_r = 0.01 + 0.04 \cdot (1 - \omega_r) \quad (8)$$



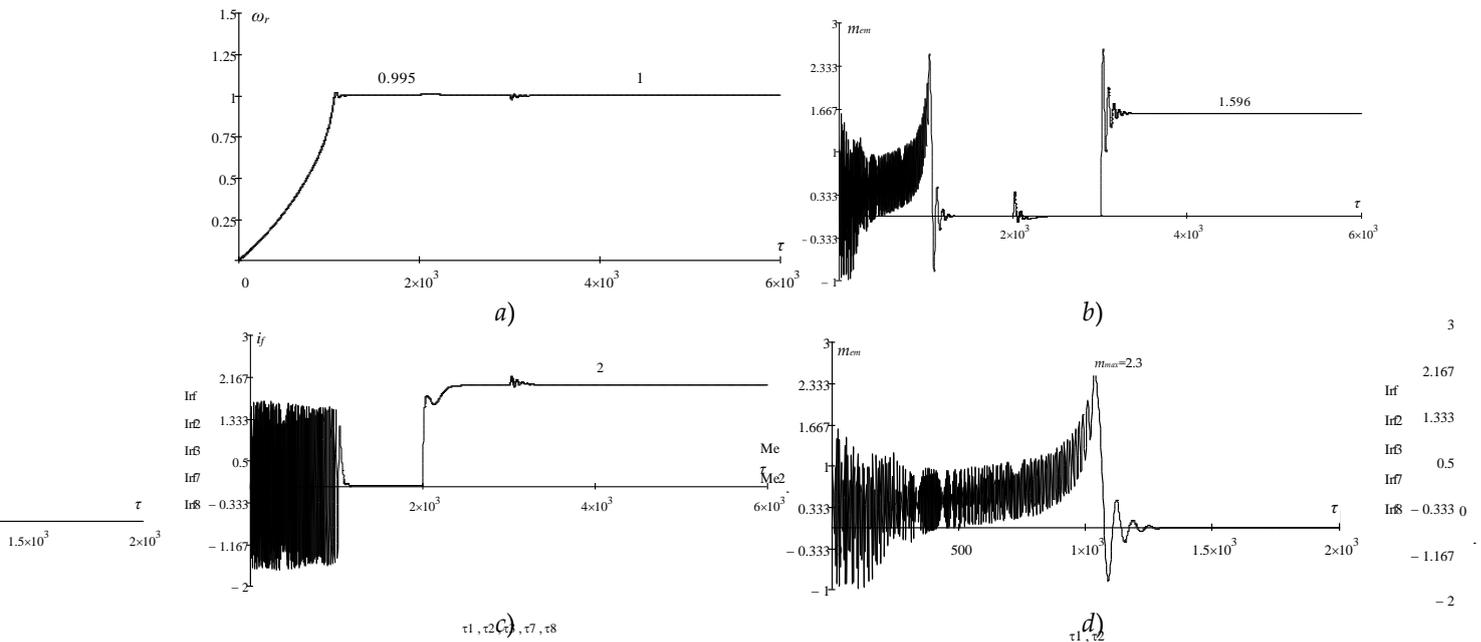


**Figure 2:** Fluctograms of change in mode parameters when massive rotor is equivalent to active resistance, which varies depending on sliding according to linear law

And finally, in Fig. 3 (a, b, c, d) and Fig. 4 (a, b, c, d) the fluctograms are shown at constant values of  $r_r$ , equal to  $r_{rs=1}=0.05$  and  $r_{rs=0}=0.01$  respectively.



**Figure 3:** Fluctograms of change in mode parameters, when the massive rotor is equivalent to active resistance equal to  $r_{rs=1}=0.05$



**Figure 4:** Fluctograms of change in mode parameters, when the massive rotor is equivalent to active resistance equal to  $r_{rs=0}=0.01$

Before proceeding to the analysis of fluctograms, it is necessary to note the following. The implementation of expression (7) on the computer will introduce a significant error, since due to the presence of even a small sliding after the completion of the asynchronous start (in the example given,  $s_{st-st}=0.005$ , the radicand from it will be equal to  $\sqrt{s} = 0.071$ , i.e. expression (7) after the start will be equal not to  $r_{rs=0}=0.01$ , but to  $r_{rs=0}=0.0128$  (i.e. the error will be 28%). When using expression (8), there will also be an error, but it will not exceed 2%, which is quite acceptable. On this basis, it is proposed to simulate expression (7) by a piecewise linear approximation, i.e. in the range of change of  $\omega_r$  from 0 to 0.8 – to represent  $r_r$  with linear dependence of the form  $r_{r1}=0.05 - 0.0275 \cdot \omega_r$ , and in the range of change of  $\omega_r$  from 0.8 to 1 –  $r_{r1} = 0.1 - 0.09 \cdot \omega_r$ . At that, the maximum error will not exceed 12%. Thus, the fluctograms in Fig. 1 (a, b, c, d) are obtained just on the basis of the piecewise linear approximation of expression (7).

Comparative analysis of fluctograms shows the following. The start-up process is a dynamic process, i.e. the acceleration time cannot be determined only by the static characteristic of the asynchronous moment of the synchronous machine, since during the start-up the kinetic energy of the rotating masses of the unit together with the rotor of the synchronous machine takes an active part. However, this characteristic is necessary, since with its help two important parameters of the starting characteristic are specified – the equivalent active resistances of the rotor during  $s=0$  and  $s=1$  slidings.

The extensive fluctograms of the electromagnetic moment  $m_{em}$ , presented in Fig. 1 and Fig. 2, according to the average values of starting moment  $m_{s1}$  and maximum moment  $m_{max}$  practically coincide  $m_{s1}=m_{s2} \approx 1.1$  ( $\sim 0.7 \cdot m_n$ )  $\cdot m_{max1}=m_{max2} \approx 2.7$  ( $\sim 1.7 \cdot m_n$ ), the start time at change of  $r_r$  according to expressions (7) and (8) is also the same and equal to  $\tau_p \sim 600$  rad. On the fluctograms in Fig. 3 and 4 at constant values  $r_r=r_{rs=1}=0.05$  and  $r_r=r_{rs=0}=0.01$ , the average values of the starting torque are respectively equal to  $m_{s3} \approx 1.25$  and  $m_{s4} \approx 0.4$ , and the maximum moments  $m_{max3}=3$  and  $m_{max4}=2.3$ , the start time, respectively, is equal to  $\tau_{s3}=400$  rad. and  $\tau_{s4}=1250$  rad. That is, the options of Fig. 3 and 4 cannot reflect the real dynamics of the starting mode of a synchronous machine.

Thus, a comparative analysis of the fluctograms shows that since the options in Fig. 1 and 2 in terms of the dynamic starting torque practically coincide with each other, then, based on the simplicity of implementation, it is necessary to stop at the 2<sup>nd</sup> option, when the resistance  $r_r$  changes linearly as a function of sliding (8).

Analysis of the synchronous mode of fluctograms (we remind that at the 2000<sup>th</sup> rad. the excitation winding opens and connects to the power source – the machine enters synchronism  $\omega_r=1$ , and at the 3000<sup>th</sup> rad. the moment of resistance equal to the rated moment of the machine  $m_r=1.596$  is supplied abruptly) shows, that during the rated load surge on the synchronous motor in Fig. 1, 2, and 4, the process of steadiness of the moment after the transient mode is almost the same on all the above fluctograms (i.e. the maximum radius and the number of oscillations are the same). This, of course, also confirms the adequacy of the model, since in this mode the synchronous machine “operates” with a steady value of the equivalent rotor active resistance equal to  $r_r=r_{rs=0}$  (for the simulative motor under study  $r_{rs=0}=0.01$ ).

In Fig. 3, the load surge process is almost aperiodic. This mode does not exist in reality, since the equivalent rotor resistance  $r_r$  is assumed to be equal to  $r_r=r_{rs=1}$  (which leads to incorrect results in the calculations both in the asynchronous mode and in the synchronous mode of operation of the synchronous machine).

## IV. Discussion

### I. Subsection One

A three-coordinate (three-phase) digital model of synchronous machines with massive rotors is

proposed, which makes it possible to study in one structure asynchronous and synchronous modes of operation of these machines.

## II. Subsection Two

In the starting mode and synchronous mode of operation, it is possible to represent practically without error the formula for the equivalent active resistance of the rotor, which varies as a function of sliding by a linear expression (8).

## III. Subsection Three

When studying only synchronous modes of operation of these machines, it is sufficient to equivalent the active resistance of the massive rotor with a constant value equal to  $r_r=r_{rs=0}$ , i.e. oscillations in sliding in this mode does not affect the average value of this resistance

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Appendices

**Appendix 1.** Model generator parameters

$$\begin{array}{lll}
 x_s = x_d = x_q = 2.78(\text{r.u.}) & U_{\text{baz}} = 310 \text{ B} & r_f = 0.03(\text{r.u.}) \\
 x_m = 2.69(\text{r.u.}) & I_{\text{baz}} = 30.44 \text{ A} & r_s = 0.045(\text{r.u.}) \\
 x_r = x_{dr} = x_{qr} = 2.82(\text{r.u.}) & R_{\text{baz}} = 10.2 \text{ Ohm} & r_{r_{s=0}} = 0.01(\text{r.u.}) \\
 x_f = 3(\text{o.e}) & \Psi_{\text{baz}} = 0.987 \text{ Bc} & r_{r_{s=1}} = 0.05(\text{r.u.}) \\
 J_{o6} = 0.3 \text{ kgm}^2 & P_{\text{baz}} = 14.15 \text{ kW} & r_r = 0.01 + 0.04 \cdot \sqrt{1 - \omega_r}(\text{r.u.}) \\
 J_{\text{baz}} = \frac{M_{\text{baz}}}{\omega_{\text{baz}}^2} = 0.00046 & \omega_{\text{baz}} = 314 & r_{r2} = 0.01 + (0.05 - 0.01) \cdot (1 - \omega)(\text{r.u.}) \\
 J^* = \frac{J}{J_{\text{baz}}} = \frac{0.3}{0.0006} \approx 656.5 & M_{\text{baz}} = 45.1 \text{ Hm} & r_{r3} = r_{r_{s=1}} = 0.05(\text{r.u.}) \\
 U_{fn} = 0.06(\text{r.u.}) & p_m = 1 & r_{r4} = r_{r_{s=0}} = 0.01(\text{r.u.})
 \end{array}$$

**Appendix 2.** Equations of three-phase model of simulative synchronous generator

$$\begin{aligned}
 p\psi_{s\alpha} &= 1 \cdot \sin \theta + 0.577 \cdot \omega_r (\Psi_{s\beta} - \Psi_{s\gamma}) - 0.045 \cdot i_{s\alpha} \\
 p\psi_{s\beta} &= 1 \cdot \sin(\theta - 2.09) + 0.577 \cdot \omega_r (\Psi_{s\gamma} - \Psi_{s\alpha}) - 0.045 \cdot i_{s\beta} \\
 p\psi_{s\gamma} &= 1 \cdot \sin(\theta + 2.09) + 0.577 \cdot \omega_r (\Psi_{s\alpha} - \Psi_{s\beta}) - 0.045 \cdot i_{s\gamma} \\
 p\psi_f &= U_f - 0.03 \cdot i_f \\
 p\psi_{r\alpha} &= 0 - r_m \cdot i_{r\alpha} \\
 p\psi_{r\beta} &= 0 - r_m \cdot i_{r\beta} \\
 p\psi_{r\gamma} &= 0 - r_m \cdot i_{r\gamma} \\
 p\omega_r &= \frac{1}{656.46} (m_{em} - m_T) = 0.0015(m_{em} - m_T) \\
 p\theta &= s = (1 - \omega_r) \\
 i_{s\alpha} &= 7.689 \cdot \Psi_{s\alpha} + 2.108 \cdot \Psi_{s\beta} + 2.108 \cdot \Psi_{s\gamma} - 1.165 \cdot \Psi_f - 2.856 \cdot \Psi_{r\alpha} + 1.428 \cdot \Psi_{r\beta} + 1.428 \cdot \Psi_{r\gamma} \\
 i_{s\beta} &= 2.108 \cdot \Psi_{s\alpha} + 7.346 \cdot \Psi_{s\beta} + 2.451 \cdot \Psi_{s\gamma} + 0.582 \cdot \Psi_f + 1.428 \cdot \Psi_{r\alpha} - 3.088 \cdot \Psi_{r\beta} + 1.66 \cdot \Psi_{r\gamma} \\
 i_{s\gamma} &= 2.108 \cdot \Psi_{s\alpha} + 2.451 \cdot \Psi_{s\beta} + 7.346 \cdot \Psi_{s\gamma} + 0.582 \cdot \Psi_f + 1.428 \cdot \Psi_{r\alpha} + 1.66 \cdot \Psi_{r\beta} - 3.088 \cdot \Psi_{r\gamma} \\
 i_f &= -1.165 \cdot \Psi_{s\alpha} + 0.582 \cdot \Psi_{s\beta} + 0.582 \cdot \Psi_{s\gamma} + 2.968 \cdot \Psi_f - 0.789 \cdot \Psi_{r\alpha} + 0.395 \cdot \Psi_{r\beta} + 0.395 \cdot \Psi_{r\gamma} \\
 i_{r\alpha} &= -2.856 \cdot \Psi_{s\alpha} + 1.428 \cdot \Psi_{s\beta} + 1.428 \cdot \Psi_{s\gamma} - 0.789 \cdot \Psi_f + 6.13 \cdot \Psi_{r\alpha} + 0.967 \cdot \Psi_{r\beta} + 0.967 \cdot \Psi_{r\gamma} \\
 i_{r\beta} &= 1.428 \cdot \Psi_{s\alpha} - 3.088 \cdot \Psi_{s\beta} + 1.66 \cdot \Psi_{s\gamma} + 0.395 \cdot \Psi_f + 0.967 \cdot \Psi_{r\alpha} + 5.972 \cdot \Psi_{r\beta} + 1.125 \cdot \Psi_{r\gamma} \\
 i_{r\gamma} &= 1.428 \cdot \Psi_{s\alpha} + 1.66 \cdot \Psi_{s\beta} - 3.088 \cdot \Psi_{s\gamma} + 0.395 \cdot \Psi_f + 0.967 \cdot \Psi_{r\alpha} + 1.125 \cdot \Psi_{r\beta} + 5.972 \cdot \Psi_{r\gamma} \\
 m_{em} &= 2.3 \left[ (i_{s\alpha} \cdot i_{r\gamma} + i_{s\beta} \cdot i_{r\alpha} + i_{s\gamma} \cdot i_{r\beta}) - (i_{s\alpha} \cdot i_{r\beta} + i_{s\beta} \cdot i_{r\gamma} + i_{s\gamma} \cdot i_{r\alpha} + i_{s\gamma} \cdot i_f) \right]
 \end{aligned}$$

where  $r_m$  – equivalent resistance of rotor circuits taking the value  $r_{r1}$ , (Figure 1),  $r_{r2}$  (Figure 2),  $r_{r3}$  (Figure 3) и  $r_{r4}$  (Figure 4).