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RANDOMNESS, UNCERTAINTY, INCOMPLETENESS, RISK AND ITS MEASUREMENTS

Vladimir Rykov [1,2], Boyan Dimitrov [3], Alexander Bochkov [4],
Elvira Zaripova [5], Olga Kochueva [1]

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[1]Gubkin Oil & Gas Russian State University, Russia,
[2]Peoples Friendship University of Russia (RUDN University), Russia,
[3]Kettering University, Flint, USA,
[4]JSC NIIAS, Moscow, Russia
[5]Moscow University of the Ministry of Internal Affairs of the RF named after V.J.Kikot, Russia
vladimir_rykov@mail.ru
bdimitro@kettering.edu
a.bochkov@vniias.ru
ezarip@gmail.com
olgakoch@mail.ru

Abstract

In this work we analyze different aspects regarding the terminology, understanding concepts and approaches in modeling and measuring components and variables related to safety and risk. This discussion conversation is open for further interpretations and suggestions. We study it with the help of models (images) artificial, descriptions, scientific approaches, discussions, etc., and using the help of poly-semantic languages. Various kinds of risks arise precisely because of the uncertainty of the situation. Mathematical models use uncertainties in several ways: randomness, which is explained and measured by objective probability and estimated using statistical methods. Uncertainty, measured by subjective probabilities, is estimated by expert methods, or by fuzzy uncertainty methods. Each of these approaches has its advantages and disadvantages. Despite the difference in approaches to measuring uncertainties, the development of a risk situation are described in an appropriate way using an event tree. We show the construction of an event tree, that allows us to see details and specify in the development of a risk situation. We also discuss how this approach can be used to evaluate the sensitivity of the output characteristics of the process to the parameters of the initial information.

Keywords: risk, relative risk, uncertainty, certainty, risk tree, incompleteness, modeling, measurements, probability measures, randomness

I. Introduction

We live in a complex world full of indefinite uncertain situations, and incompleteness. In the XVII - XVIII centuries, when there was a discussion about the possibility of studying random phenomena, the main difference between DETERMINISTIC and RANDOM phenomena was formulated:

Deterministic phenomena UNIVERSALLY respond to the repetition of the experiment in the same HOMOGENEOUS conditions, and random ones - MULTIPLE. At the same time, random phenomena suggest the possibility of their MULTIPLE (ideally, infinite) repetition in the same (homogeneous) conditions.

However, we note that not all uncertain phenomena allow a probabilistic interpretation, which requires the possibility of their repeated observation under homogeneous conditions. This led to the need to build other models for the study of uncertain (except random) phenomena.

Since risks are associated with the uncertainty of the situation in which they occur, a significant part of this article is devoted to clarifying concepts and terminology in the field of uncertainty, randomness and risks. Numerous studies are devoted to risks in modern literature. However, a sufficiently definite understanding of this concept among specialists has not yet been achieved. The works [1] — [3] offer a fairly consistent analysis of the interpretation of the concept of risk at the time of their publication.

The concept of risk in the mathematical literature appeared for the first time in connection with the "ruin" problem in insurance and financial models in mathematics, and was the subject of so-called "collective" risk models. These models have brought to life a large stream of deep GENERAL mathematical research [4], [5].

Features of the models of "individual risks", which include risks in oil and gas industries their specificity, and the need for an individual approach to the study of various types of risks. The article [6] gives a detailed study of the origin and development of the concept of risk, which, however, does not help much in formalizing this concept for a clear study of the phenomena associated with it.

In many studies related to the concept of risks, probabilistic terminology is used unreasonably from our point of view. In this sense, it is categorically impossible to agree with the concept of chance reasoning about the chance or "biological determinism" of the emergence of life on Earth [7]. By themselves, studies of the conditions, causes, and development of living matter on Earth are, of course, interesting, and useful and deserve every encouragement. Still, the concept of "INCIDENCE" should not be involved in their description, because in science it is already ENGAGED and strictly defined. The fact that the word chance in "everyday language" occurs in different contexts should not replace it in scientific research. In this regard, when studying such phenomena as reliability, risk, security, etc., it is necessary, first of all, to understand how it is possible to use such concepts as randomness, probability, etc. for their study.

In this regard, a significant part of this article is devoted to clarifying the concepts and terminology in the field of risks. At the same time, we note that not all uncertain phenomena allow a probabilistic interpretation, which requires the possibility of their repeated observation under homogeneous conditions. This led to the need to build other models for the study of uncertain (except random) phenomena.

First of all, we note that the language itself, due to its ambiguity, introduces its share of uncertainty into the interpretation of various concepts and we will try to give an unambiguous interpretation of various terms in their strictly scientific sense.

II. Polysemous of language. Various concept of uncertainty and their mathematical models

According to the complexity of the World and the languages and technology, all languages are poly-semantic. There are at least three different areas of language use: every day, literary and artistic (including poetic), scientific, etc. What is acceptable in ordinary language (different interpretation of words), is good in literature, is absolutely unacceptable in science, where an unambiguous understanding of words and terms is assumed and required. In this regard, we will try to give exact meanings (unambiguous language designations) to various concepts of uncertainties.

Various risks arise precisely because of the uncertainty of the situation. In everyday life, we are constantly confronted with various types of phenomena, in addition to *deterministic* and *unique* phenomena to deal with different types of uncertain phenomena. Various risks arise precisely because of the uncertainty of the situation. In the study of such phenomena as "reliability", "risk", "security", etc., it is necessary, first of all, to understand how it is possible to use such concepts as randomness, probability, etc. for their study.

Mathematicians model uncertainties in several ways:

- randomness, which is measured by objective probability and estimated using statistical methods due to Kolmogorov [8];
- virtual uncertainty, which is measured by subjective probabilities according to DeFinetti and Savage [9];
- uncertainty that can be estimated by expert methods [10];
- uncertainty that is considered in the framework of fuzzy theory according to Zadeh [11]:
- and may be some other types of uncertainties.

In this regard, bearing in mind the ambiguity of the language, it is proposed to clarify the above concepts. Below a classification of various types of phenomena is presented in the diagram in Fig.1.

It is proposed not to call all uncertain phenomena and events random, but, for example, unique, uncertain, possible, and fuzzy, and measure not by probabilities, but by chances, possibilities, belongings, etc.

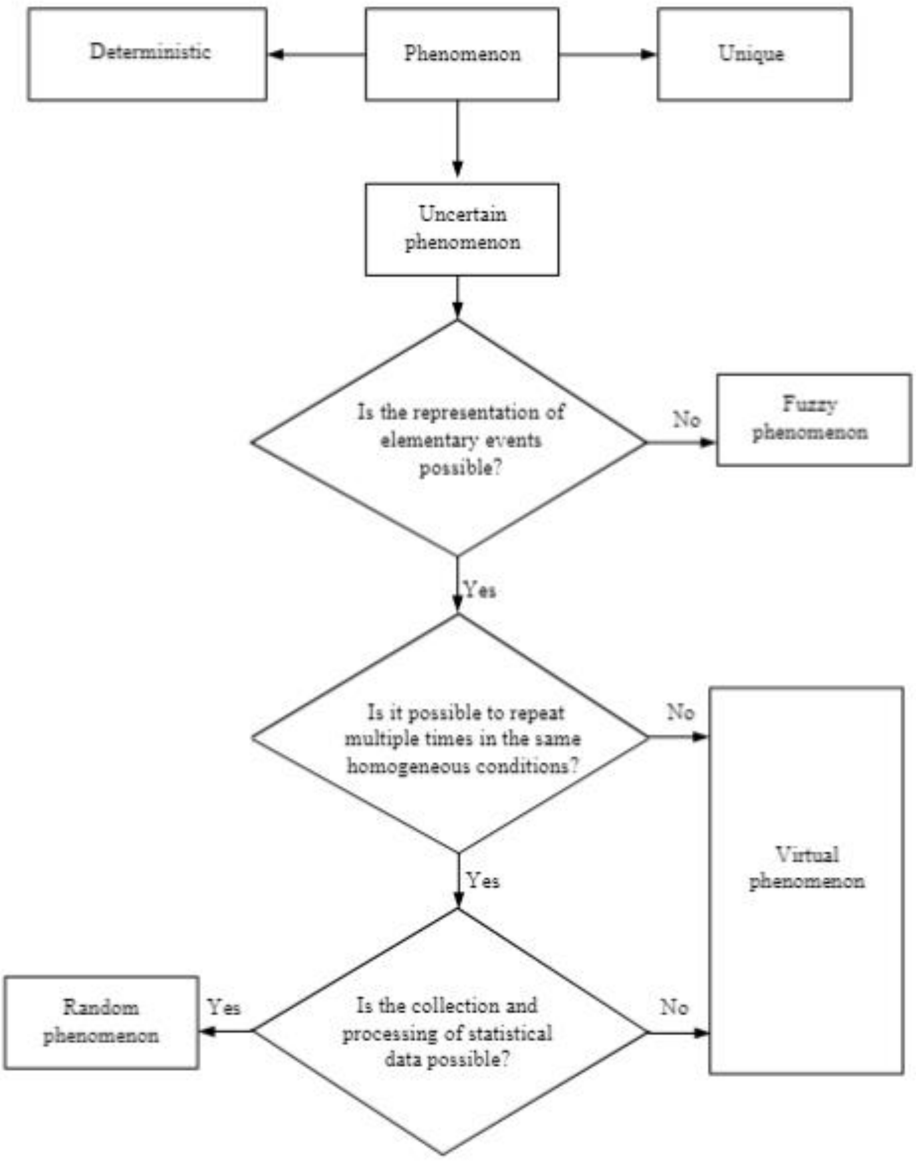


Fig. 1: *Classification of phenomena*

Their features described below:

- *Deterministic* phenomenon is a phenomenon that uniquely responds to the creation of the same (homogeneous) conditions, including when it is repeatedly observed.

- *Unique* An event is an *event* that has only been observed once in the past and for the foreseeable future.
- By a *random* phenomenon we mean a phenomenon that can be observed many times under the same conditions and *reacts* to them with a multi-valued set of possible events, among which the minimal ones can be unambiguously identified. In this case, the measure of randomness is the objective probability, measured by frequency.
- A *virtual* phenomenon is understood as a phenomenon that reacts to the creation of identical conditions with a *multi*-valued set of possible events, among which it is possible to single out the minimal ones, but the possibilities of repeated observation of such phenomena under the same conditions are impossible or severely limited. We will call the measure of a virtual event a chance or a subjective probability, which is set by an expert in compliance with certain rules.
- *Fuzzy* is a phenomenon that reacts to the creation of the same conditions with ambiguous (vague, fuzzy) events, among which elementary (minimal) events are not singled out, and their belonging to one or another (possible) event is characterized by a membership function specified by an expert.

The above concepts are fundamental and not subject to strict definition. To understand what we mean, here are a few examples of various phenomena:

- deterministic: sunrise and sunset, pendulum swing;
- unique: "Big Bang" - the birth of the Universe, the birth of living matter on Earth;
- random: life expectancy of residents of various countries, up-time of incandescent lamps of a fixed manufacturer;
- virtual: the presence of an oil (coal, gold) deposit in some fixed area;
- fuzzy: train speed is rather fast, weather is rather overcast, teenager is rather tall.

All of the above is illustrated by the following Fig.2.

Each of these approaches has its advantages and disadvantages:

1. The advantage of the randomness model is the objectivity of estimates, the disadvantage is the difficulty (and sometimes impossibility) of obtaining the necessary initial information;
2. The disadvantage of the second and third approaches is the subjectivity of the estimates of the initial information, and as a result, the subjectivity of the conclusions;
3. In fuzzy logic models, the construction of the membership function is also at the mercy of the researcher, which leads to the subjectivity of the conclusions.

Regardless of which concept to follow when describing and studying risk, it develops over time and is associated with some costs. Therefore, the time of occurrence of a risk event and the costs it brings are its main characteristics. However, depending on the accepted concept of risk analysis, these characteristics can be measured and evaluated objective, subjective probabilities (and distribution functions built on their basis) or studied within fuzzy sets.

III. Understandings and measures of the risks

3.1. Understanding the risk

Risk is inherent in human activity. Activity is a conscious active interaction of the subject with the object, during which the subject purposefully influences the object, satisfying any of his needs, achieving the goal. Only a person who is able to realize possible losses or gains as a result of some action is at risk. Only a person is able to determine the purpose of his activity. Technique does not have subjectivity - it is only a means to achieve goals. Which should be efficient, reliable and safe.

Any activity is a random process aimed at achieving a certain goal, which is a change of states. States can be stable and unstable. Completion of the process - achievement of the final target state. Depending on external and internal factors, the trajectory of achieving the final target state may be different. To control a random process, a functional should be defined, for which an optimization problem should be set (search for the optimal control strategy), taking into account the existing external and internal constraints (Kashtanov).

The change of states is preceded by a decision made by a person (refusal of a decision is also a decision), taking into account the state of the external environment and the control object. Since the decision maker does not, as a rule, have all the information about the external and internal environment, the decision is made under conditions of partial or complete uncertainty.

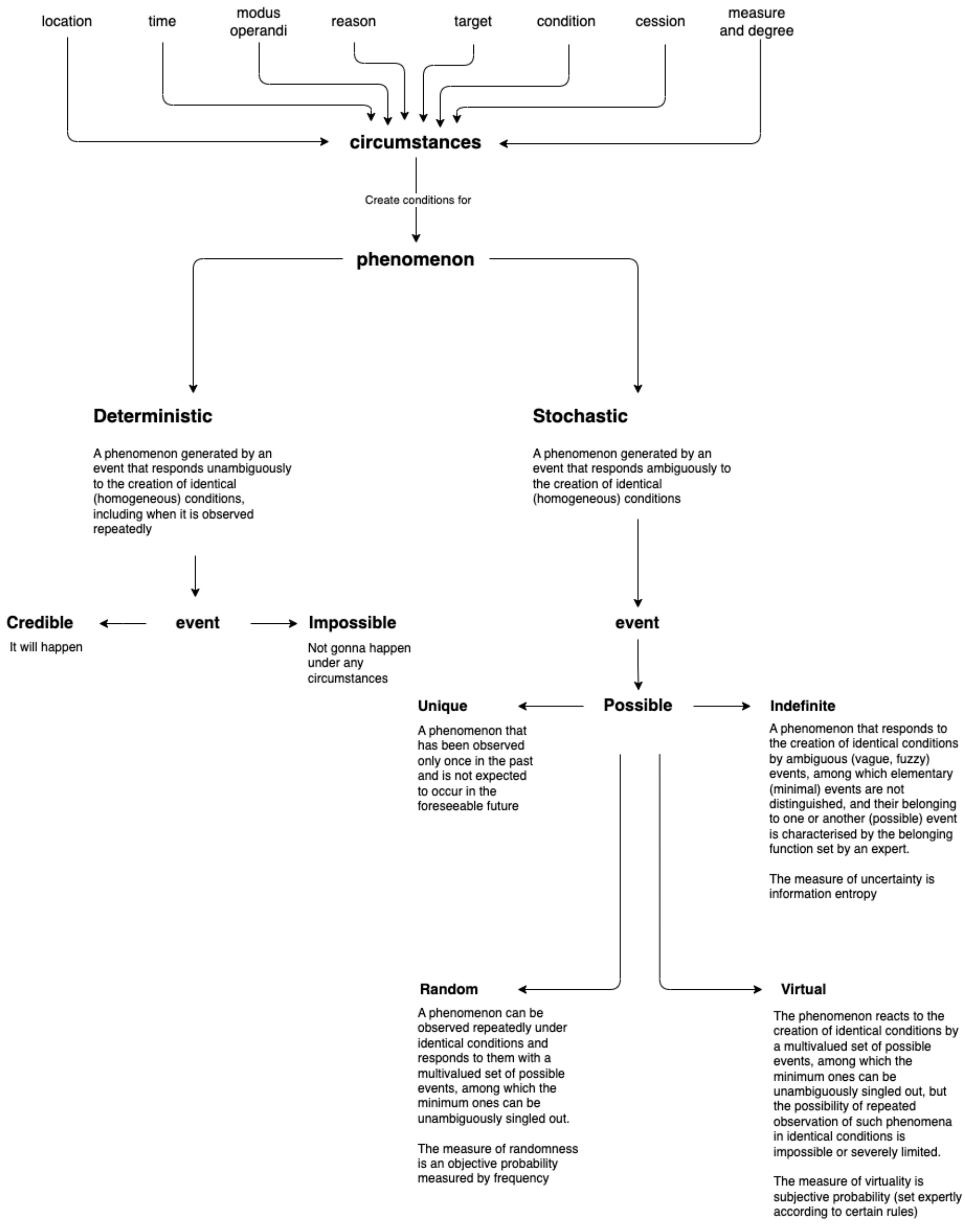


Fig. 2: *Event-occurrence diagram*

States have properties. For example, reliability is the property of an object to function continuously without fail with a 100 efficiency level. Only cancellations are considered. The failure criterion divides everything by yes/no. Functioning efficiency is the property of a system to function continuously, although, possibly, with a reduced level of вЂњoutput parametersвЂќ. Stability - the property of the system to return (within a reasonable time) to the previous 100 percents level of functioning after the failure of individual components. Survivability is the property of an object to continue functioning within acceptable limits after the failure of individual components. Risk is a generalized property of the states of a real or model controlled random process, on the trajectories of which a functional is specified that determines the goal of control and an optimization problem is set (search for an optimal control strategy, effective control) under constraints. Particular properties of states - circumstances (place, time, mode of action, causes, goals, conditions, concessions, measures and degrees, and opportunities), expected loss, probability of an undesirable event, objective and subjective uncertainty, possibility of loss, combination of probability and severity of consequences, consequences, the severity of consequences and their uncertainty, the impact of uncertainty on goals, etc.

3.2. On measuring the risks

Every measure uses numbers. but numbers are met everywhere. Important is to know the meaning of these numbers and what do they measure. In risks measures there is no exception.

Each particular property has its own measure (according to Riemann or Lebesgue). Numerous risk measures have been proposed in the literature. The choice of an appropriate risk measure is critical for the next decision step:

- Risk as a relative value (comparison with a control group, defined as the ratio of the probability of an outcome in the exposed group to the probability of an outcome in the unexposed group). Relative risk is often used in statistical analysis of paired outcomes when the outcome of interest has a relatively low probability [12]. Interesting measure of the risks is the so called relative risk used in boo-statistics and in medical statistics let us explain it in simple way. Assume, B is a risky event, like having a serious disease. and let A be a result of some test. Relative risk of B with respect of the event A is called the ratio called the ratio o the conditional probabilities $P(B|A)P(B|notA)$. This measure may take arbitrary positive values, while most of the risk measures are probabilities - numbers between 0 and 1.
- p-value as measure of risk. any statistical program after finishing the work on some dates gives at the end many numeric results, and there one will see a lots of numbers indicated p-values. This is actually a measure of the risk to admit the null hypothesise is not true, when actually it is.Therefore, it is important to know what is the hypothesis tested behind these p-values.
- Risk as a posterior error estimate (in mathematical statistics and decision theory, Bayesian estimate is a statistical estimate that minimizes the a posterior expectation of a loss function (a posterior loss expectation); ordinary (non-Bayesian) risk is the mathematical expectation of the variance of the posterior distribution) [13].
- Risk as a probability of loss (a consequence of the occurrence of some random event from a possible family of all events or the totality of possible damage in some stochastic situation and its probability (this concept covers the so-called frequency, statistical approach, most often applied to queuing systems, in insurance, theory reliability, etc.)) [14], [15].
- Risk in вЂњgames with natureвЂќ (payment for a decision in case of uncertainty of the response to the chosen decision (this includes the so-called Wald's maximum utility (guaranteed result, minimum gain) or Savage's mini-max regret (maximum loss), Hurwitz criterion (optimism coefficient))) [16].
- Risk as the difficulty of achieving the goal (geometric anti-risk, defined through a functional that describes the evolution of the system on a set of given trajectories, being a measure of assessing the quality of the system in relation to the quality required to achieve the goal) [17].
- Risk as a measure of the difference between states (semi-Hamming measure - a measure of the assessment of the degree of discrepancy between the real and the reference process, a measure of quality) [18].
- Risk as a measure of the stability of the center of the quasi-attract or in the phase space of states (for example, the product of variation ranges (in the sense of Stewart)) [19].
- Risk as an anti-potential for development (risks act as lowers of the rate of reproduction of the entire system) [20].
- Risk as a violation of the sequence of significant factors (violation of the lexicographic order, is estimated as a minimum of the total inconsistency of expert assessments (based on the equality of all participants in the examination) of the options for the development of the system,

measured in inversions of the transitions necessary to restore the lexicographic order of the compared options) [21] and etc.

If an activity leads to uncertain outcomes, then different decisions lead to different stochastic outcome variables. Probabilistic functional are used to compare the results obtained with different solutions. Such functional relate a quality value to a stochastic outcome variable or measure the degree of risk. In the latter case, these functionals are also called risk functional or risk measures. When speaking about risk measurement in this article, we prefer to use the name "risk functional", and we reserve the term "measure" for probability measures.

In our paper, we consider the basic non-atomic probability space (Ω, F, P) and the linear space of real random variables defined on it. This space can be either the space Y of all measurable real functions on (Ω, F), or some subspace of integrable functions.

The probability functional R is an extended real function defined on Y or on some of its subsets, i.e. we assume that R can take the values $+\infty$ or $-\infty$, but not both.

Risk, as a generalized property, is evaluated in two ways:

a) according to Kolmogorov - Borell - Riemann - when the integration is carried out over the damage (we add up the damages from all risks);

b) according to Lebesgue - the damage should be, as it were, fixed, but everything happens with different probability depending on the behavior of the person (we determine what we can lose).

The Risk trees discussed further also contain an interesting approach in measuring the risks.

IV. About the risk tree

Despite the different approaches to the measurement of uncertainty and the specificity of the evolution of risk situations, its development can be described in the same way using an event tree. The idea of the event tree dates back to the 1960s when in Bell-Lab it was used to study the reliability of complex objects and systems. Review of earliest studies in this direction one cab find in [22]. Later it was used in other applications, including risk research. The information about the novel investigations in this direction one can find in Internet.

The construction of the event tree makes it possible to detail and specify the evolution of a risk situation. Therefore, in the paper, the technique of creating a complex program of construction and analysis of an event tree is offered, which is applicable to various types of risks and hazards. Regardless of the quality of the source information, event tree analysis allows us to assess the sensitivity of the output characteristics of the considered situation to the type and parameters of the initial information.

In the works [23, 24], a technique for constructing, equipping and analyzing a risk tree was developed. In our previous reports at the conferences Risk-21, Risk-22 [25, 26], as well in [27, 28], where the methodology was applied to analyze the risks of monitoring the subsea pipeline. Formally, the methodology is divided into three parts: building a risk tree, equipping it with information, and analyzing the risk tree.

Building a risk tree is the same for all types of uncertainties. The structure of the tree, the arrangement of elements, gateways for all types of uncertainties are the same. Equipping the risk tree with data for each type of uncertainty is different. For random events, the estimate will be represented by an objective probability, for a virtual phenomenon, expert estimates can be applied, and fuzzy events can be measured by a membership function. Using probability, chances and membership functions, one can obtain an estimate of the considered risk event.

This presentation methodology is detailed in [23, 24].

The risk tree is independent of the chosen concept of risk analysis. However, its equipping with initial information essentially relies on the chosen concept. finally, the last stage, with the first two correctly performed, is implemented automatically.

Complex risk phenomena have a hierarchical (tree-like) structure descending from the main (root) risk event through intermediate events to the minimum (leaf) events that initiate the development of the risk phenomenon. To describe such a structure, it is convenient to use the vector notation. Each minimal (elementary) event is associated with the vector $\vec{\imath} = (i_1, i_2, \dots, i_r)$, where i_0 means the main risk event, i_1 is the number of the first possible risk event under consideration. events of the first level, i_2 is the number of the event of the second level, leading to the event i_1, etc. up to the leaf event i_r, leading along this path to the main risk event $(i_1, i_2, \dots, i_{r-1})$, where r is the hierarchy level of the considered minimum event, and different minimum events can have different hierarchy levels.

To work with the risk tree, we also need truncated vectors $\vec{\imath}_k = (i_1, i_2, \dots, i_k)$ to denote an intermediate event of the k-th level, and denote the j-th subevent of this event via $j(\vec{\imath}_k)$. So, each minimal event of the considered risk phenomenon is completely identified by the vector $\vec{\imath} = (i_1, i_2, \dots, i_r)$, and various intermediate events are truncated vectors $\vec{\imath}_k = (i_1, i_2, \dots, i_k)$.

To work with the risk tree, each event will be characterized by the structural variable $x_{\vec{\imath}_k}$. The structure variable is set to 1, $x_{\vec{\imath}_k} = 1$, when the event $\vec{\imath}_k$ occurs, otherwise $x_{\vec{\imath}_k} = 0$. For each event, using structural variables, it is possible to calculate structural functions in accordance with the rules of reliability theory [23, 24].

When constructing a risk tree, it is convenient to use the notation taken from the monograph by Heinley and Kumamoto [22].Rectangles in the risk tree denote events that can be divided into sub-events, circles denote final <<leaf>> events, event labels in the risk tree are shown in the table 0. The corresponding connections between events in the form of gates, together with the structural functions corresponding to them, are given in the table 1.

Table 1: *Symbol table in the risk tree*

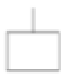
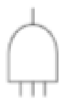
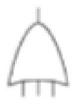
Event Symbols	Description
	Basic event
	Event represented by a gate

Table 2: *Table of gateways in the risk tree*

Gate Symbols	Name	Description
	$\prod_{i=1}^{n} x_i$ AND gate	Output event occurs if all input events occur simultaneously
	$1 - \prod_{i=1}^{n} (1 - x_i)$ OR gate	Output event occurs if any one of input event occur

It is possible to calculate the most dangerous risky path of development, risky by the criterion of maximum risk probability, by highlighting for all components of the event $\vec{\imath}_k$ the number of its component $j^*_{\vec{\imath}_k}$, on which this maximum is reached.

Let the event $\vec{\imath}_k$ contain $n(\vec{\imath}_k)$ components, then the maximum probability is $q^*_{\vec{\imath}_k}$ its implementation $\vec{\imath}_k$ due to the component $j^*_{\vec{\imath}_k}$ is found by the formula 1:

$$q^*_{\vec{\imath}_k} = \max_{1 \le j \le n(\vec{\imath}_k)} q_{(\vec{\imath}_k, j)}; \qquad j^*_{\vec{\imath}_k} = arg \max_{1 \le j \le n(\vec{\imath}_k)} q_{(\vec{\imath}_k, j)}. \tag{1}$$

Starting from the lower level, you can find the most dangerous path to the $\vec{\imath}_k$ subsystem, starting from leaf events, according to the criterion of the maximum risk probability, including for the entire system using the formula 2:

$$\vec{\imath}^*_k(q) = (j^*_{\vec{\imath}^*_k}, j^*_{\vec{\imath}^*_{k+1}}, \dots, j^*_{\vec{\imath}^*_{r-1}}); \quad \vec{\imath}^*_{k+l+1} = (\vec{\imath}^*_{k+l}, j^*_{\vec{\imath}^*_{k+l}}), \quad l \ge 0. \tag{2}$$

Building a dangerous path according to the criteria of the maximum risk probability, the largest penalties, can serve as an additional focus for decision makers.

In the next section, we will use this technique to build, equip, and analyze a geological risk tree based on expert research from [29, 30, 31, 32, 33, 34]. The methodology for assessing risk events is proposed in the articles [25, 26, 27, 28].

V. Risk tree for assessing the success of geological exploration

As an example, let's consider the construction of a tree of success (failure) of geological exploration for one promising object. The papers [29, 30, 31, 32, 33, 34] propose a system of events affecting exploration, statistical data and expert assessments of negative events.

The key event, let's denote it (0), for which the risk tree is built, is the failure to receive economic profit in the development of a promising facility for oil and gas production. The tree will be built from top to bottom (left to right), implying two scales at once: by time (unsuccessful events that occur first) and by development stages (from the top layer of the earth inward to the discovery of hydrocarbons):

- the presence or absence of a successful natural reservoir;
- the presence or absence of an effective trap;
- presence or absence of hydrocarbons in the trap;
- safety / absence of hydrocarbons in the trap, as well as full financial support / failure of the project.

(0) — Exploration at site is not successful.

This key event in the first stage of geological exploration associated with an effective natural reservoir can be broken down into 4 possible sub-events for which the geological project may not be successful. Firstly, the failure of the project will be revealed immediately if an effective natural reservoir is not found. Two indicators are responsible for this: the presence of facies (rocks) for which the formation of reservoirs is favorable, and sufficient porosity of these rocks. Porosity reflects the percentage of pore space in relation to the total rock volume. Through the communicating pores, a sufficient amount of hydrocarbons enters the well, an example of a porous rock with non-communicating pores can be pumice. The following non-overlapping combinations of these options arise.

(1) — lack of favorable faces in the presence of acceptable porosity in the rock;

(2) — the presence of favorable faces with unacceptable porosity, eg shale;

(3) — lack of favorable rock and porosity not acceptable;

(4) — The failure of the project arose as a result of a deeper geological analysis, which we will reveal later, now the event can be described as <<another reason for the failure of the project>>.

Thus, three events (1), (2) and (3) end immediately and are leaf events, and event (4) can be represented through sub-events, due to which the failure of the project is possible during the subsequent (deeper) geological analysis.

Event (4) is divided into four sub-events. This stage of the geological analysis establishes the presence of suitable rock and acceptable porosity, and then an effective hydrocarbon trap is required for the success of the project. A trap is a part of the reservoir bounded from above and from the sides by impermeable rocks (the so-called impermeable seal, tire), in which oil and gas can theoretically collect. Seals have low permeability and are capable of retaining hydrocarbons at very high pressures. There are false seals without oil deposits, in which there are only minor oil shows. Here and below, commas in the vector representation of elements will be omitted. Consider the events:

(41) — no trap, although seal quality is acceptable;

(42) — there is an oil show, but the seal is of poor quality and does not retain hydrocarbon, it does not accumulate;

(43) — no trap and no seal;

(44) — the trap is effective, but the failure of the project is due to other reasons, you can call this event <<failure of the project for other reasons>>.

Events (41), (42), (43) are final and are leaf events in the risk tree. Event (44) occurs at the stage when both the reservoir and the impermeable trap are effective, but the causes of failure are in the underlying geological layers. Let's consider the third stage of the exploration study, which is called "the presence of hydrocarbons in the trap". This requires two conditions: the presence of mature oil and gas source strata, which, under the influence of time, temperature and pressure, generate hydrocarbons, and the presence of favorable conditions for the migration of hydrocarbons into traps. Thus, we distinguish four sub-events of event (44):

(441) — there are no mature oil and gas source strata, but there are pores and cracks along which hydrocarbons can theoretically move into traps;

(442) — there are mature oil and gas source strata, but unfavorable conditions for the movement of hydrocarbons into traps;

(443) — no mature oil and gas source strata and unfavorable conditions for the movement of hydrocarbons into traps;

(444) — there are mature suitable oil and gas source sequences and favorable conditions for moving hydrocarbons into traps, but the event is not successful for reasons that will be revealed in the next level.

Let's consider the fourth stage of development of the geological project. At this stage of development, project development is looking at "holding/preservation of hydrocarbons in the trap", and we will add here <<other reasons, for example, the economic nature of the legal entity>> that the development of the facility is not economically viable. Event (444) is divided into two sub-events:

(4441) — Hydrocarbon is not stored in the trap, migrates;

(4442) — financial problems of a legal entity.

In the current example, exploration is not successful if at least one risk event occurs.

Let's equip the events of the risk tree in the figure 2 with an expert assessment of practicing geologists [29, 30, 33, 34], fix the probabilities of phased failure in the Table 2 .

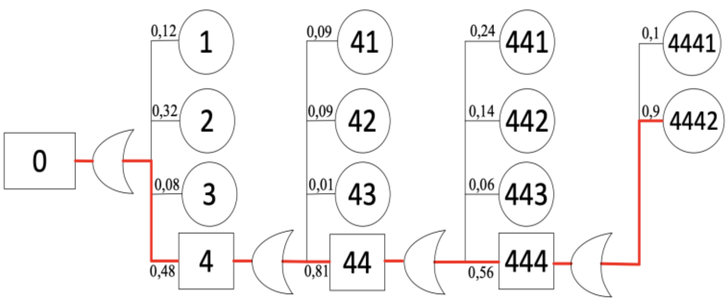


Fig. 3: *A dangerous path according to the criterion of the maximum probability of failure*

Table 3: *Probabilities of failure by events*

Events	q_i	**Events**	q_i
(1)	0.12	(44)	0.81
(2)	0.32	(441)	0.24
(3)	0.08	(442)	0.14

(4)	0.48	(443)	0.06
(41)	0.09	(444)	0.56
(42)	0.09	(4441)	0.1
(43)	0.01	(4442)	0.9

In the example under study, the probabilities $q_{(4441)} = 0.1$ and $q_{(4442)} = 0.9$ are given, thus, using the formulas (1) and (2) we get: $q^*_{(444)} = \max\limits_{1\le j\le 2} q_{(444j)} = 0.9$, and $j^*_{(444)} = \operatorname*{argmax}\limits_{1\le j\le 2} q_{(444j)} = 2$. At the level above $q_{(441)} = 0.24, q_{(442)} = 0.14, q_{(443)} = 0.06$ and $q_{(444)} = 0.56$ i.e. probability $q^*_{(44)} = \max\limits_{1\le j\le 4} q_{(44j)} = 0.56$ and $j^*_{(44)} = \operatorname*{argmax}\limits_{1\le j\le 4} q_{(44j)} = 4$. At the next level $q_{(41)} = 0.09, q_{(42)} = 0.09, q_{(43)} = 0.01$ and $q_{(44)} = 0.81$ i.e. probability $q^*_{(4)} = \max\limits_{1\le j\le 4} q_{(4j)} = 0.81$ and $j^*_{(4)} = \operatorname*{argmax}\limits_{1\le j\le 4} q_{(4j)} = 4$. At the upper level $q_{(1)} = 0.12, q_{(2)} = 0.32, q_{(3)} = 0.08$ and $q_{(4)} = 0.48$ i.e. probability $q^*_{(0)} = \max\limits_{1\ j\le 4} q_{(j)} = 0.48$ and $j^*_{(0)} = \operatorname*{argmax}\limits_{1\le j\le 4} q_{(j)} = 4$. The dangerous path according to the criterion of the maximum probability of failure leads through the elements (4),(44),(444),(4442), in the figure 2 the found dangerous path in the risk tree is highlighted in red.

The dangerous path according to the criterion of the events The next Diagram of events explains the ways of analyzing a process to achieve the risk target.

VI. Conclusion

The current work shows the features of random phenomena. The concepts of random, virtual and fuzzy phenomena are classified. Depending on the classification of phenomena, one can end up with an assessment that can be both objective for random phenomena and subjective due to the subjectivity of the initial information. To assess the risk events of the described phenomena, a risk tree is built and, using the mathematical apparatus, an assessment for the situation can be obtained.

In the current work, an example with expert assessments from practicing geologists is given. A technique for choosing a dangerous path is described. For other risk events of complex engineering structures with average service life of elements, by changing, for example, the coefficients of variation, it is possible to change the direction of the dangerous path. This toolkit can serve as additional advice for decision makers.

The authors invite to further joint work experts with real data and requests related to the assessment of risk events of complex engineering systems and structures to calculate the main and alternative dangerous routes, fines.

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References

[1] Rykov, V. and Yastrebenetsky, M. (1999) Risk: notion and measurement. Proceedings of the International Conference <<Probabilistic analysis of rare events: Theory and problems of safety, insurance and ruin>>. Riga, Latvia, Riga Avian University, 63–70.

[2] Rykov, V. and Yastrebenetsky, M. (2007) Back to the definition of Risk notion. Abstract to MMR- Conference, Glasgow.

[3] Rykov, V. (2011) Reliability, Safety, Risk: Control and Management. Proceedings of the 7-th International Conference <<Mathematical Methods in Reliability >> Beijing, China. Ed. by Lirong C., Xian Z. Beijing: Institute of Technology Press.

[4] Cramer, H. (1930) On the mathematical theory of risk. Forsakringsaktiebolaget Skandias Festskrift; Centraltryckeriet: Stockholm, Sweden, pp. 7–84.

[5] CramΓ©r, H. (1955) Collective Risk Theory: A Survey of the Theory from the Point of View of the Theory of Stochastic Processes; Ab Nordiska Bokhandeln: Stockholm, Sweden.

https://doi.org/10.1007/978-3-642-40607-2_17.

[6] Aven, T. (2012) The risk conceptвЂ"historical and recent development trends. Reliability Engineering & System Safety. 99. 33вЂ"44. 10.1016/j.ress.2011.11.006.

[7] Brooks M. (edd) (2015) Chance. The science and secrets of luck, randomness and probability. New Scientist, 2915

[8] Kolmogorov, A.N. (1974) Basic concepts of the theory of probabilities. Grundbegriffe der Wahrscheinlichkeitsrechnung, 2nd ed.; Translated from A.N. Kolmogoroff; Fizmatgiz: Moscow, Russia, (In Russian) .

[9] Savage, L.J. (1954) The Foundations of Statistics. John Wiley: New York, NY, USA, .

[10] Cooke, R.M. (1991) Experts in Uncertainty: Opinion and Subjective Probability in Science. Oxford University Press: Oxford, UK. 336p.

[11] Zadeh, L.A. (1965) Fuzzy sets. Information and Control; ; Volume 8, pp. 338-353. https://doi.org/10.1016/S0019-9958(65)90241-X.

[12] Sistrom CL, Garvan CW (2004). "Proportions, odds, and risk". Radiology. 230 (1): 12вЂ"9. doi:10.1148/radiol.2301031028. PMIDВ 14695382

[13] Nozer D. Singpurwalla (2006) Reliability and Risk: A Bayesian Perspective // Wiley Series Probability and Statistics, ISBN: 978-0-470-06033-9, 2006.- 400В p.

[14] Rykov V. V., Itkin V. (2017) Reliability of technical systems and technogenic risk: textbook. - Moscow: INFRA-M, 2017. - 192 p. - (Higher education).

[15] Korolev V.Yu., Bening V.E., Shorgin S.Ya. Mathematical foundations of risk theory. - Moscow: Fizmatlit, 2011. - 620 СЃ. ISBN: 978-5-9221-1267-3

[16] Zhukovsky V.I., Zhukovskaya L.V. (2010) Risk in multicriteria and conflict systems under uncertainty / ed. Ed. 2nd ed. Moscow: Publishing House LKI, 2010. - 272 p.

[17] Russman, I., A. Kaplinsky, and V. Umyvakin (1991). Modeling and algorithmization of poorly formalized problems of choosing the best system options / Voronezh: Voronezh State University, 1991.- 168 p. (in Russian).

[18] Bochkov, A.V. (2019) Methodology of ensuring safe functioning and sustainability of the Unified system of gas supply in emergency situations: dissertation ... Doctor of Technical Sciences: 05.26.02 / Bochkov Alexander Vladimirovich; [Place of protection: LLC "Research Institute of natural gases and gas technologies - Gazprom VNIIGAZ"], 2019. - 385 p.

[19] Bochkov, A.V. (2023), Applied issues of chaotic dynamics in the management of unique evolutionary systems, Editor(s): Mangey Ram, Liudong Xing, In Advances in Reliability Science, Reliability Modeling in Industry 4.0, Elsevier, 2023, Pages 45-81, ISBN 9780323992046, https://doi.org/10.1016/B978-0-323-99204-6.00014-5.

[20] Zhigirev, N.; Bochkov, A.; Kuzmina, N.; Ridley, A. (2022) Introducing a Novel Method for Smart Expansive Systems' Operation Risk Synthesis. Mathematics 2022, 10, 427. https://doi.org/10.3390/math10030427

[21] Bochkov A., Zhigirev N., Kuzminova A. (2022) Inversion Method of Consistency Measurement Estimation Expert Opinions // Reliability: Theory & Applications, vol. 17, no. 3 (69), 2022, pp. 242-252. doi:10.24412/1932-2321-2022-369-242-252.

[22] Henley, E.,G. and Kumamoto, H. (1981) Reliability Engineering and Risk Assessment. Prentice-Hall: New Jersey, NJ, USA. p. 07632. ISBN-0-13-772251-6.

[23] Rykov, V. V. and Itkin, V.Yu (2016) Reliability of technical systems and technogenic risk. Textbook. INFRA-M, 214 p.

[24] Rykov V.V. Reliability of Engineering Systems and Technological Risks. ISTE. WILEY, 2016, 212p.

[25] Rykov, V., Ivanova, N. and Farhadov, M. (2022) On principles of risk analysis with a practical example. RT&A, Special Issue 3 (66), Volume 17, pp. 38–41.

[26] Rykov, V., Farkhadov, M., Kochueva, O. and Zaripova, E. (2022) Sensitivity analysis of risk characteristics to the type of initial information based on a pipeline monitoring system. Reliability: Theory and Applications. Special Issue в„– 4 (70) Volume 17.

[27] Rykov, V., Ivanova, N. and Kozyrev, D. (2021) Risk tree as an assistant tool for the

decision-maker. 13th International Congress on Ultra Modern Telecommunications and Control Systems and Workshops (ICUMT) pp. 109-114. DOI: 10.1109/ICUMT54235.2021.9631604

[28] Rykov, V., Kochueva, O., Farkhadov, M., Zaripova, E. and Zhaglova, A. (2023) Sensitivity Analysis of Risk Characteristics of Complex Engineering Systems: An Application to a Subsea Pipeline Monitoring System. J. Mar. Sci. Eng. 11, 352. https://doi.org/10.3390/jmse11020352

[29] Рўhe CCOP guidelines for risk assessment of petroleum prospects. (2000).

[30] Polyakov, A. A. and Murzin Sh. M. (2012) International experience in geological risk analysis. NeftegasovaГў geologiГў. TeoriГў i practika, 7 (4).

[31] Hope, S., Bjordal, E.N., Diack, H.M., Eddershaw, B.W., Joanny, L., Ortone, G., Payne, F.G., Searson, A.H., Sedlacek, K.W. and van Strien, W. (1982) Methodologies for hazard analysis and risk assessment in the petroleum refining and storage industry.

[32] Hope, S., Eddershaw, B. W., Joanny, L., Bjordal, E. N., Diack, H. M., van Strien, W. (1984). Methodologies for hazard analysis and risk assessment in the petroleum refining and storage industry" Part I. Fire Technology, 20(3), 23-38.

[33] Emelyanova, N.M., Poroskun, V.I. (2018) Geological-economical evaluation and analysis of plays risks. NeftegasovaГў geologiГў. TeoriГў i practika. 13(2).

[34] Kerimov, V. Yu., Bondarev, A. V., Mustaev, R. N., Khoshtaria, V. N. (2017) Estimation of geological risks in searching and exploration of hydrocarbon deposits. Neftyanoje hozyajstvo. 8, pp.36-43.